DISTORTION OF FACTOR LOADINGS AS A FUNCTION OF THE NUMBER OF FACTORS ROTATED UNDER VARYING LEVELS OF COMMON VARIANCE AND ERROR

By

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This study examined factor loading stability as a function of the number of factors rotated for four problem matrices under three levels of common variance: 30%, 45%, and 60%; and three levels of sample size: 100, 200, and 500. The sample sizes correspond to standard error values of .10, .07, and .04 respectively.

Four representative problem factor matrices were selected from the literature. Each was treated in the following manner. The matrix was adjusted to account for each of the three specified levels of common variance. The intercorrelation matrix was obtained for each adjusted matrix; the latter was factor analyzed by the principal axes method and a criterion common-factor matrix obtained. For each problem, the three criterion matrices, adjusted to the three levels of common variance, had the same number of factors as that of the original problem matrix.

Computer-generated pseudorandom error at each of the three levels specified was added to the intercorrelation matrices mentioned above, and the error-laden matrices were factor analyzed, principal axes extracted, and several factor rotations performed. The factor rotations involved a

series of successive under and overrotations below and above the correct number of factors for a given problem matrix. Root-mean-square (RMS) deviation values were calculated between the factor loadings of each criterion matrix and the corresponding factor loadings in each of the successively rotated factor matrices. The RMS values were computed for only the initial two or three rotated factors for each problem. The procedures of the addition of random error to the intercorrelation matrix, the factor extraction, the successive rotations, and the calculation of the RMS discrepancies were replicated ten times under each of the nine conditions of common variance by error.

The obtained RMS mean values were plotted and tested for significance using a multifactor repeated measures ANOVA design. Linear, quadratic, and cubic trend analyses were performed. Goodness of fit of the plotted curves of the RMS means for the four problems was examined by computing predicted RMS means and comparing them with the observed RMS means.

For all four problems at the .05 level, the ANOVA results were significant for the number of factors rotated; this was also true for the rotation x common variance and rotation x error interactions. The three trend analyses were also found significant at the .05 level.

The polynomial cubic equation

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_1^2 + \beta_3 X_1^3 + \varepsilon$$

gave the best approximation for the trend of the data for all four selected problem matrices.

This study provided support for the literature's position on underrotation; namely, it was not recommended. The view on overrotation, which advocated overrotation by one or two extra factors, could be neither supported nor rejected by the findings of the study.

There did not seem to be a clear relationship between the number of variables and/or factors for a given matrix and its factor loading stability. Factors with large amounts of common variance and low levels of error were found to be the most stable.

Chairman

CHAPTER I

Introduction

The number of factors to rotate has been long recognized as a problem in factor analysis, since to a certain extent the decision involves the skill and subjective judgment of the analyst (Fruchter, 1954). With the advent of computers most decisions are made automatically, but the decision on the number of factors to rotate and interpret ultimately rests upon the investigator (Guertin & Bailey, 1970).

Even for the classic Holzinger and Harman 24 psychological tests problem, "Harman answers. . .unequivocally that the best number of factors for the problem is either four or five but refuses to commit himself as to which of these two answers is better" (Kaiser, 1970, p. 412-413). Regarding factor analytic methodology, it was Kaiser's (1970) position that "the most important future work, as I see it, should continue to concentrate on the number-of-factors question" (p. 414).

A survey of the literature revealed Kaiser's concern was not unique. Many studies have focused upon the number-of-factors problem. Suggestions, rationales,and solutions abound in the literature (e.g., Cattell, 1966b; Horn, 1965; Humphreys & Ilgen, 1969; Linn, 1965, 1968; Mosier, 1939).

The issue of the number of factors to rotate is directly related to that of the number of factors to extract from the intercorrelation matrix. However, widely available computer programs for the principal axes factoring procedure extract simultaneous factors that account for all the variance of the intercorrelation matrix (Guertin & Bailey, 1970). The question then centers upon the number of extracted principal axes factors which must be carried into rotation to yield the final interpretable factors.

This task would be relatively simple if the earlier extracted large factors contained only common-factor variance and the later small ones contained nothing but error. Unfortunately this is not the case, for "in the extraction process one does not begin to extract only substantive factors until one suddenly gets to 'error' factors, but that some degree of error variance is present from the beginning" (Cattell, 1966a, p. 201).

It is the process of rotation that enables the researcher to separate out substantive 'real' factors from those of error. The aim is to rotate so that the maximum number of 'real' factors is retained "while cutting off as much as possible of the error variance as will not simultaneously carry away too much real variance" (Cattell, 1966a, p. 204). The goal, then, is to rotate substantive factors and ignore the error-laden ones.

The Purpose

While several methods and criteria have been suggested to determine the correct number of factors, none of these seems to have gained unanimous acceptance. As Guilford (1974) has pointed out, "The need for rotation of axes in factor analysis is. . .the most serious weakness of this very useful method of reduction of numerical data. . . ." (p. 498). In light of this weakness, it would be desirable to employ an empirical approach in examining this aspect of factor analysis. The goal was to gain further insight into this troublesome area.

This study examined factor loading stability for four different factor matrices of known solutions under the following conditions: (a) variation of the number of factor rotations, below and above the known number for the criterion matrices; (b) three levels of common variance accounted for by the factor matrices; (c) three levels of random error added to the matrices, i.e., for three different sample sizes.

This stability would be reflected, in part at least, by the characteristics of the shape of plotted curves of the means of root-mean-square discrepancies between each criterion matrix and its corresponding manipulated matrix under the experimental conditions identified above.

The Procedure: An Overview

Four matrices of known factor solutions were selected. These matrices were based upon different numbers of variables. Each factor matrix was adjusted to account for three proportions of common variance; the intercorrelation matrix for each factor matrix was obtained, factored, and rotated orthogonally by the Varimax method (Kaiser, 1958). Thus for the four selected problem matrices, a total of twelve adjusted criterion matrices was obtained -- three per problem.

Error representing three levels of sample size was added to each of the four intercorrelation matrices. These error-laden matrices were subsequently factor analyzed by the principal axes method and several rotations to the Varimax criterion were tried (Kaiser, 1958). The number of factors rotated ranged from two or three factors less to two or three factors more than that of the original problem matrix.

The root-mean-square (RMS) discrepancies between the first two or three factors of each criterion matrix and their corresponding factors in each trial rotation of the error-added factor matrices were calculated.

The RMS statistic is an appropriate, common statistical measure used to make direct comparisons of corresponding factor loadings (Harman, 1976, p. 297).

Ten replications were performed under each condition of common variance by level of error, so that, for example, for a four-factor problem, with five trial rotations, 450 RMS values were obtained for later analysis.

The means of the total RMS's for the trial rotations for each problem were plotted. Null hypotheses about these means were tested for significance at the .05 level by the analysis of variance multifactor repeated measures design (Winer, 1971).

Further analyses were completed to examine the trends in the plotted curves of the means of the RMS's, and, finally, orthogonal polynomial coefficients were used to solve for the predicted values of these means.

These procedures were conducted with the aid of several "software" options. The errors were produced by the use of the Fortran Subroutine NDIST which generates pseudo-random error as specified. The factor analyses were performed by using a modified version of factor analytic program ED 501, (Guertin & Bailey, 1970), available through the University of Florida Educational Evaluation Library, as adapted for the IBM 360. The multifactor repeated measures analysis of variance was completed by using computer program BMD 80V (Dixon, 1974).

For each representative problem examined the research questions to be answered were:

1. What effect does under and overrotation have on the loadings of a known factor matrix given three levels of sample size and three levels of common variance? Since the RMS's are measures

of deviation, a test of differences about their means should be an indicator of this effect.

2. If the F test, at the .05 level of significance indicates a trend in the data, what is the nature of this trend? What degree equation best fits the trend of the data, i.e., what is the shape obtained when the means of the RMS's are plotted?

Some Limitations

This study had several limitations. Only one problem (matrix) of each size was used. To do otherwise would have been impractical in terms of cost and presentation.

When each intercorrelation matrix was factored, only common-factor analysis was employed where communalities were known, i.e., neither principal components analysis nor image analysis was used. All rotations of the principal axes were performed to the Varimax criterion regardless of the original rotation of the input matrices.

Another limitation to the study was that only the first two or three factors in each criterion matrix were matched by the trial rotation factors and their RMS's calculated. Only a limited number of the factors contributing to the shape of the line or curve of discrepancies were investigated. Specifically, underrotation and overrotation were examined as they affect only the first two or three factors of the criterion matrix. The overall effects on all the factors were not assessed. The reason for this limitation was the logistics of being unable to calculate the RMS discrepancies when only two or three trial factors are rotated. This is perhaps the most important limitation to the study.

Because of the practical limitations for the number of trial rotations to be examined and reported, only two or three factors below and two or three factors above the ideal number in the criterion matrices were examined.

For each problem, the variable of sample size was limited to three levels judged to be fairly representative of those used in factor analytic studies. The proportion of common variance accounted for was confined to only three levels, also for reasons of representativeness. Matrices intermediate to, or outside these ranges might give different results.

Significance of the Study

One focus of common-factor analytic methodology has been the number-of-factors problem, i.e., the optimum number of common factors that should be carried into rotation to yield a meaningful solution. A recurrent, though not unanimous, theme has been that it is best to rotate one or two additional factors than to underrotate. It seemed, therefore, that an empirical examination of this issue was appropriate. New information might be gained that could aid the researcher in deciding on the ideal number of factors to retain and interpret.

Organization of the Study

Chapter I has dealt with the purpose of and the background to the study; an overview of the procedure; the research questions; the limitations and the significance of the study. A review of the related literature is presented in Chapter II. The complete procedure and a detailed description of the problem matrices are discussed in Chapter III. Results are presented in Chapter IV. A discussion of the results, conclusions, and the summary appear in Chapter V.

CHAPTER II

Related Literature

The purpose of this study was to examine factor loading stability as a function of under and overrotation of common factors, under three levels of common variance and three levels of error. In this Chapter is reviewed the literature concerned with the issue of the number of factors to rotate as it affects factor stability in common-factor analysis.

For the purpose of presentation, the Chapter is divided into three major sections: (a) the procedural rules and criteria for determining the number of factors; (b) the research findings and conclusions on the effects of under and overrotation; (c) the factor analytic methods and procedures relevant to this investigation.

Procedural Rules and Criteria for Rotation

The issue of the number of factors to rotate and interpret is fairly straight-forward. The goal in factor analysis is to arrive at a small number of common factors which maximally account for the common variance of an intercorrelation matrix (Linn, 1968). Procedural rules have been developed to determine this number. These rules generally fall into two categories: statistical and psychometric (Cliff & Hamburger, 1967; Linn, 1968). The former attempt to generalize from the data to a population of subjects, while the latter seek to generalize from the data to a domain of interest, i.e., a universe of measures. Both approaches have their vigorous proponents. Obviously, rules and procedures espoused by one camp do not necessarily yield factor solutions identical to those obtained from the other's (Hakstian & Muller, 1973).

The statistical approach. A number of rigorously derived statistical procedures have been developed. Among the workers in this area were Bartlett (1950); Joreskog (1963); Lawley (1951); and Rao (1955). Tests of significance have been developed for the hypothesis that a given number of factors is necessary to account for a set of data. Unfortunately, these methods are of narrow applicability. For example, "Barlett's χ^2 test is limited to the principal components model with unities in the diagonal and thus is not applicable to the usual communality model" (Linn, 1968, p. 38).

The status of the statistical approach was described by Cliff and Hamburger (1967):

> The results available from statistical theory, while useful, leave a larger area where the needs of the investigator are unsatisfied. The statistical tests for the number of factors are all tied, naturally enough, to the respective methods for estimating factors. Moreover, these methods of estimation are either computationally arduous, as in the case of Lawley's (1953) or Rao's (1955) method, or unfamiliar, as in the case of Joreskog's[1] (1963) method. Consequently, they are rarely used and so the statistical tests are rarely applied. More important than this is the fact that the number of factors in a given matrix is only one of many concerns of the investigator. He is interested in a wide variety of statistical questions, and he is interested in them as they arise in the methods of factor analysis currently in use (p. 431).

Hakstian and Muller (1973) expressed some reservations about the use

[1]Joreskog's K statistic has been extensively investigated by Monte Carlo techniques and seems to give good results except when the N's are rather low (100) (Cliff & Hamburger, 1967).

of inferential procedures in factor analysis. One concern was the "lack of rigorous control over Type I error" (p. 465). Their major objection, however, centered around the "dependence of this approach upon the total sample size, with the resulting problem of 'statistical but not practical significance' with particularly large N" (p. 465).

Linn (1968) pointed to another problem regarding the statistical approach, namely, the "almost complete lack of knowledge concerning the distribution and standard error of individual factor loadings and their differences. The mathematical difficulty of developing the necessary distribution theory has proven to be exceedingly great" (p. 37). Nevertheless, investigations of sampling error continue to be made. Since the 1960's several of these studies have employed Monte Carlo techniques. A review of these appears in Cliff and Hamburger (1967).

Just as in the statistical case, the psychometric approach has led to several procedural rules to establish the number of interpretable factors that must be retained. Generally, it is the latter methodological approach, used in this study, that is frequently encountered in factor analytic literature. This seems to be related to the previously mentioned limitations and objections to the available statistical tests.

<u>The psychometric approach</u>. A number of psychometric rules of thumb have been developed: (a) the Kaiser-Guttman (Guttman, 1954; Kaiser, 1960) latent root greater than one criterion; (b) Cattell's (1958) rule for computing the percentage of common variance, which led to (c) Cattell's (1966b) scree test. The last two will be considered together since they are interrelated.

The Kaiser-Guttman criterion (Guttman, 1954; Kaiser, 1960) has been adopted by the psychometric approach, although it clearly applies only to correlations of a population and not to those of a universe of measures (Cliff & Hamburger, 1967; Linn, 1968). The rule states that, with unities in the diagonal of a correlation matrix, the number of common factors is equal to the number of latent roots greater than one (Harman, 1976).

Some factor analysts take exception to the application of this rule to obtain rotated common factors (Cattell, 1966b;Gorsuch, 1974; Guertin & Bailey, 1970). Their position is that when unities are used in the diagonal, the principal components obtained should not be rotated; components retain unique as well as common variance and therefore cannot be expected to yield interpretable common factors. Hakstian and Muller (1973) stated that the application of the Kaiser-Guttman rule, with its "procedural implications. . .for only the component model, is seen as theoretically inappropriate when a common-factor. . .analysis is being performed" (pp. 470-471).

Linn (1968) used a Monte Carlo approach to develop criteria for the number of factors. He augmented observed intercorrelation matrices with generated random normal deviates and factor analyzed the augmented matrices. The factoring method, sample size, number of variables,and the estimates of communalities were varied. The latent root criterion correctly estimated the number of factors in only six cases, underestimated in five cases, and grossly overestimated the correct number of factors in four instances. He concluded that the application of this rule in deciding upon the correct number of common factors "cannot be recommended on the basis of the results of the present study" (p. 67).

Humphreys (1964) analyzed intercorrelations of the 21 variables of the 1944 Air Crew Classification Battery based on 8158 cases. He obtained ten rotated interpretable factors corresponding to those obtained by previous studies on the same variables. Had he used the Kaiser-Guttman rule, he would have had to retain only five factors, since, with unities in the diagonals, only five factors had latent roots larger than one. Humphreys (1964) concluded that "The Kaiser [-Guttman] criterion, when N is very large is clearly too conservative with respect to the number of factors" (p. 466).

It would seem, then, that the Kaiser-Guttman rule, statistically sound as it is, has been applied in an inappropriate manner in common-factor analysis. As Gorsuch (1974) said, "The major criticism of the root ≥ 1 criterion lies more in its use than in its concept" (p. 149).

Computing the percentage of variance extracted as a basis for the number of factors to rotate stipulates that rotation should not be terminated until 95 to 98% of the complete principal axes variance is accounted for (Cattell, 1966b; Guertin & Bailey, 1970). This principle subsequently led to the development of the scree test.

Cattell's (1966b) scree test probably best exemplifies the psychometric approach in factor analysis. He stated that ". . .it should be left to rotation to separate substantive [real] and error of measurement factors" (p. 246). Cattell's (1966b) main reservations about the use of statistical tests was that factor extraction may be terminated too soon, resulting in the rejection of substantive variance that may be needed for subsequent rotation.

In the scree test, the latent roots of the principal axes are plotted. At first, the roots fall off rapidly because common variance is extracted early. Subsequently, the roots level off in a linear fashion when almost nothing but measurement error is extracted. The cut-off point that indicates the number of factors is just before the linear descent (Cattell and Jaspers, 1967). Thus the scree test gives the minimum number of factors for the maximum amount of variance (Gorsuch, 1974).

Several empirical studies have evaluated the scree test. Linn's (1968) Monte Carlo study, mentioned earlier, found that the scree technique identified the correct number of factors in seven instances, underestimated in two cases, and overestimated in one. In the remaining six cases, the results of the scree were not clear cut. Tucker, Koopman, and Linn (1969) reported that the scree technique correctly identified the number of factors in 12 out of 18 instances. Similar findings made by others led Gorsuch (1974) to conclude that "the scree test is in the correct general area" (p. 155).

In a study that compared several statistical and psychometric factor analytic rules for determining the number of interpretable factors, Hakstian and Muller (1973) reanalyzed 17 published correlation matrices. Their results suggested "that the appropriate number of factors . . . depends, in part, upon the view held regarding factors and factor analysis and the consequent linear model employed in the analysis" (p. 470).

The scree test, for example, was found to yield too few factors in many cases, while the latent root ≥ 1 criterion was seen theoretically inappropriate when either common-factor or image analysis is performed. They recommended for the common-factor case, at least, that the number of factors be found for rotation "so that an optimally clear solution results" (p. 473). Regarding the number of common factors to rotate, they suggested rotating more factors than will ultimately be interpreted.

<u>Alternative procedures for determining the number of factors</u>. In addition to the Kaiser-Guttman rule (Kaiser, 1960; Guttman, 1954) and

Catell's (1966b) scree test, several investigators have recommended alternative procedures for arriving at the correct number of interpretable factors.

Horn (1965) developed a procedure as a correction for the latent root ≥ 1 criterion for determining the number of factors. His rationale for the necessity for this correction was that the criterion overestimates the number of factors. The technique he presented was designed to determine the number of non-error latent roots.

Horn (1965) used a Monte Carlo procedure to generate random normal deviates for the same number of subjects and variables as ones in an observed 297 x 65 raw score matrix. The latent roots were calculated for the raw score intercorrelation matrix and the randomly-generated data. Horn (1965) stated that the correct number of factors is equal to the number of latent roots of the real data that are larger than their counterparts in the random data. He proposed that this procedure be routinely incorporated in computer programs.

By counter-example, Cliff and Hamburger (1966, p. 433) showed that Horn's (1965) method can underestimate the number of common factors. Linn (1968) found Horn's results "while interesting, can only be taken as suggestive, due to the fact that they consist of only one example" (p. 39).

In a generalization of Horn's (1965) procedure to the common-factor model, Humphreys and Ilgen (1969) recommended what seems to be a promising technique for determining the number of factors to rotate and interpret. Using procedures by Horn (1965) and Linn (1968) as points of departure, the authors employed parallel analysis on matrices of real and random data. Latent roots for the intercorrelation matrices of the real and the random data were plotted and the point at which the latent root curves crossed were assumed to indicate the number of common factors.

Their results compared favorably with maximum likelihood statistical solutions for the same matrices. In fact, with squared multiples in the diagonals, parallel analysis seemed to give a bit more accurate results than maximum likelihood. (Neither unities nor the highest r - adjusted in the diagonal gave equally satisfactory results.) The writers recommended the routine use of their technique since it can be used concurrently with latent root inspection for breaks, and as incorporated in Cattell's scree test (1966b).

Humphreys and Ilgen's (1969) findings were confirmed by sampling studies conducted by Humphreys and Montanelli (1975). They concluded that when the common-factor model provided a good fit to the data, parallel analysis was more accurate than maximum likelihood in determining the number of common factors.

Howard and Gordon (1963) presented an illustration and extension of a method proposed by Wrigley (1960) for identifying common factors. Wrigley (1960) recommended overfactoring, then rotating successive numbers of factors either by Varimax or Quartimax. Next, the rotated factor matrix is searched for specific factors. These are factors that have high loadings of only one variable. If such a specific factor is found, the last principal axes factor is dropped and Varimax or Quartimax reapplied. This procedure is repeated until a factor solution is obtained where each factor has high loadings of at least two variables. By this method, only common factors are obtained.

Howard and Gordon (1963) offer an illustration and a refinement of Wrigley's (1960) method since ". . .there may still be certain ambiguities associated with some of these factors" (p. 245). They analyzed 37 activities variables taken on 598 street-corner gang boys. The 37 variables were intercorrelated, communality estimates were inserted in the diagonal; factoring was done by the principal axes method and eleven factors extracted. "Varimax rotations were performed using the first two, three, four, five, and so on, up to eleven of the principal axes factors" (p. 248). Howard and Gordon (1963) report that, for this illustration, only five common factors are meaningful. But had they followed Wrigley's (1960) criterion, six factors would have had to be retained and interpreted. They recommend that Wrigley's (1960) procedure be followed up to the point when one specific factor emerges. At that time, "an evaluation is made of the stability of the loadings of the remaining common factors" (p. 250), and only the maximum number of stable common factors is retained.

While several approaches and procedural rules have been suggested, there does not seem to be a unanimous agreement on any single way to determine the number of factors to rotate. As a consequence several researchers have investigated the stability of the rotated factors as more factors are carried into rotation. Their findings are now examined.

Studies on the Effects of Under and Overrotation

An empirical investigation of special interest to this study was Mosier's (1939)[1]. The procedure followed in the present study parallels to some extent that of Mosier's. The purpose of the Mosier (1939) study was to assess the influence of chance error and communality estimates on

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[1]Mosier's (1939) study is considered the "earliest paper on the subject of procrustes rotation" (Harman, 1976, p. 336).

simple structure. Mosier constructed a representative hypothetical factor matrix for 20 variables and four orthogonal factors, "satisfying the criterion of simple structure" (p. 34). The intercorrelation matrix was obtained by postmultiplying the factor matrix by its transpose.

Just as was done in the present study, normally distributed "chance error" (p. 35) assuming N = 100, was added to each off diagonal coefficient in the matrix. "This matrix with unknown diagonal entries, represents the situation met in. . .factor analysis, where the individual r_{jk}'s are subject to error and the communalities must be estimated" (p. 36).

The intercorrelation matrix was factored using two different estimates of the communality. Four factors were rotated and root-mean-square discrepancies calculated between the hypothetical factor loadings and those of the error-added matrix. Mosier (1939) concluded that neither the added error nor the estimated communalities prevent accurate determination of a factor solution "provided that the rank of the centroid matrix is equal to or greater than that of the underlying primary trait matrix" (p. 43).

In addition Mosier (1939) investigated several criteria for "completion of the analysis" (p. 39) under the two conditions of estimated communalities and added error. Using the same hypothetical four-factor matrix, he took out three, four, then six factors. None of the criteria tested was considered wholly satisfactory in determining the correct number of factors, although he recommended that "It is safer to have too many than too few factors" (p. 43).

Kiel and Wrigley (1960) used analytical rotational procedures to compare solutions from successive factor rotations. They found that, initially, the existing factors will subdivide when another factor is carried into rotation, and an interpretable factor emerges. A point of stability is assumed to be reached when no further acceptable factors result with further rotation. An acceptable factor, according to Kiel and Wrigley, is one on which at least two variables have their highest loadings. They recommended that the point of stability be used as a criterion for terminating factor rotation.

Dingman, Miller, and Eyman (1964) studied the effect of rotating too many factors for both the orthogonal and the oblique case. Only the former is pertinent to this study. The data were based on three aptitude factors, each with three levels of difficulty. Tests representing the aptitude levels were administered to 479 male college students. Factor extraction was by the centroid method and communality estimates were iterated until stability was reached. Varimax was used for rotating first the three "ideal" factors; this was followed by four, five, and six factors carried into rotation.

Dingman et al. (1964) reported that in the orthogonal case, "as the number of factors rotated was increased over the optimum number of 3; simple structure progressively got worse and more factors tended to appear in the over-all dimension of common factor space up to and including the 5-factor solution" (p. 78). Nevertheless, the authors maintained that meaningful factors can be obtained even when there is overrotation. They admitted, however, that this conclusion may be peculiar to their highly structured data; the three optimum factors remained fairly recognizable in spite of overrotations.

Levonian and Comrey (1966) stated that rotating too few factors can result in a distortion of the rotated matrix. They pointed out that when factoring is stopped too soon, the extracted variance will be crowded into a lesser number of factors than are necessary to represent the underlying factor structure, nor will rotation of these factors clarify the structure of their matrix. It is possible, they stated, that none of the real factors of the matrix will emerge, and those that appear will be severely distorted by "foreign" variance (p. 101).

Levonian and Comrey (1966) pointed out that the effect of rotating too few factors "would seem to become more serious as the degree of under-extraction increases" (p. 401). They also suggested that ". . .the consequences of rotating too many factors is less clear" (p. 401); a possible consequence may be an instability in the common factor loadings.

The authors investigated factorial stability as related to the number of orthogonally rotated factors for two separate problems. The problems were treated differently in terms of the correlations computed and the method of factor extraction. However, both sets of factors were rotated to the Varimax criterion.

The number of rotations was varied for each problem. For example, for the first study, of the first 25 centroid factors extracted, the first 6 were rotated, then the first 10, 14, 18,and finally all 25.

Levonian and Comrey (1966) concluded that, though generalization was not possible from only two studies, "stability considerations suggest the rotation of many, rather than few, factors" (p. 404). Further, that if the number of variables is not small, reasonable stability may be achieved; the ratio of factors to variables should approach 1/3 or possibly larger.

In a study to determine the number of principal axes factors to carry into rotation, Veldman (1974) used the Varimax criterion to rotate successively greater numbers of factors for nine published problems. The

Varimax criterion value, C, was considered by Veldman to be "an index of the degree to which the rotation process has approximated 'simple structure' -- the goal of analytic rotation" (p. 193).

Veldman found that the Varimax criterion value C appears to be useful in identifying the rank of a factor matrix. The C values were found to be unimodel and peaked at the correct number of factors. Another finding was that overrotation was not necessarily disastrous, when the principal axes were rotated. Moreover, overrotation when image analysis was used did not disturb the major factors. However, the criterion values C fluctuated erratically when the latent structure of a matrix was weak.

A general theme in the literature seems to be that retaining one or two additional factors for rotation does little harm and is advocated by some investigators (e.g., Gorsuch, 1974; Mosier, 1939). Underrotation is discouraged since it forces common-factor variance to be compressed into too few factors, thus distorting common-factor space (Guertin & Bailey, 1970).

On the other hand, "factor fission" as Cattell (1952) refers to it, can result when too many factors beyond the scree point are rotated (p. 334). As the number of rotated factors is increased the common variance is redistributed across too many factors, causing some factors to split. The resulting factor matrix degenerates into an uninterpretable, psychologically meaningless solution (Guertin & Bailey, 1970).

Because of the lack of agreement on any one approach or method in determining the correct number of factors to rotate, several kinds of factor solutions have been developed. The differences among these solutions "correspond to the different mathematical theories in the explanation of a particular scientific problem" (Harman, 1976, p. 10).

Comprehensive presentations of the various rationales and procedures may be found in a number of available books (Comrey, 1973; Gorsuch, 1974; Harman, 1976; Mulaik, 1972). In a 1972 investigation, Dielman, Cattell and Wagner included a summary of comparative studies of rotational procedures since 1954. Hakstian and Muller (1973) presented a tabular summary of the views, models, bases for inference, the rationales, and procedures that have been traditionally employed in factor analytic studies. Of the various methods and procedures recommended, the following were selected as most appropriate for this investigation.

Relevant Factor Analytic Methods and Procedures

Because this study was concerned with only common-factor analysis, the principal axes method was used, with squared multiple correlations inserted as communality estimates. Iterations and refactoring were performed until satisfactory convergence was achieved. The resulting principal axes were rotated to approximate simple structure by the use of the Varimax method. The literature pertaining to each of these phases of the analysis will be examined. The use of the RMS mean deviations in factor analytic studies will be presented.

Common factors, principal axes and communalities. The principal axes method developed by Thurstone (1932) has remained popular because it extracts the maximum amount of common-factor variance from a reduced intercorrelation matrix. Additionally, it has the virtue of producing "a lower-valued final residual matrix" (Guertin & Bailey, 1970, p. 62). In a comparison with Harman's (1967) Minres and Lawley's (1951) Maximum likelihood factor extraction procedures, with different communality estimates, the principal axes method gave very similar results after the factor matrices were rotated to Varimax (Guertin, 1971).

A reduced intercorrelation matrix is one where the values in the diagonal are estimated (communalities) prior to factor extraction. Several values for the initial estimates have been proposed. Wrigley (1956) performed an empirical iteration-by-refactoring study in which he compared fifteen different methods of initial communality estimates. He concluded that, with the use of computers, the squared multiple correlation of each variable with the remaining ones is the best initial estimate of the communality. Wrigley (1957) pointed out that "Various objections raised against communalities can be met. . .by the use of the S.M.C.'s" since "the S.M.C. measure variance common to a test [variable] and the remaining p-1 tests in the selection" (p. 94). Guttman (1956) viewed the squared multiple correlations not only as the best possible estimates of communalities, but also as the lower bound for these estimates.

Other workers in the field share similar views on the merits of the squared multiple correlations (Gorsuch, 1974; Harman, 1976). Humphreys and Ilgen (1969) found the use of the squared multiple correlations an "objective useful way of estimating communalities," with the additional advantage of remaining "stable from sample to sample since they depend upon all the data" (p. 572).

A concluding word on the use of communality estimates might be Harman's (1976), "It has been argued, and substantiated by empirical evidence, that it matters little what values are placed in the principal diagonal of the correlation matrix when the number of variables is large (say, n>20)" (p. 86).

The iteration-by-refactoring procedure used in this study has much to recommend it (Gorsuch, 1974). Harman (1976) viewed it as one method for estimating communalities "which has the semblance of" objectivity (p. 86).

In performing common-factor analysis, then, the chief emphasis is upon obtaining the maximum amount of common-factor variance. The use of the communalities in the diagonal prior to factor extraction makes this possible; a definition of the communality is that it is "the amount of variance a test [variable] shares with all others in common-factor space" (Guertin & Bailey, 1970, p. 165).

Simple structure and the Varimax method. The aim of rotating the extracted principal axes factors is to gain the clearest view of common-factor space. Since the principal axes factors extract the maximum possible common variance from the intercorrelation matrix, the question now becomes that of the number of those factors that must be carried into rotation and to what criterion.

The universally accepted criterion that is followed is Thurstone's (1947, p. 335) principle of simple structure which yields factors that are relatively invariant across studies (Guertin & Bailey, 1970).

This criterion is doubly parsimonious: in rotating factors in common-factor space, simple structure dictates that both variables and factors should be described by a minimum number of sizable loadings. Although other criteria have been proposed, none has become as widely used (Gorsuch, 1974). To approximate the ideal of simple structure for a given factor matrix, the factors may be rotated in either an oblique[1] or an orthogonal fashion.

[1]Since only orthogonal rotation was employed in this study, oblique solutions will not be discussed. See Harman (1976) for a comprehensive treatment.

Several analytical orthogonal rotation methods have been developed, all of which were referred to collectively by Harman (1976) as Quartimax. With Kaiser's (1958) development of the Varimax method for orthogonal rotation (used in this study), the Quartimax approach was abandoned (Comrey, 1973).

The Varimax method is the best known and most popular rotational procedure used today (Butler, 1969; Comrey, 1973; Guertin & Bailey, 1970). It is available at most computer centers, and is included in Dixon's (1974) BMD package of computer programs. It has become so important that special sections are devoted to it in several texts (Comrey, 1973; Gorsuch, 1974; Harman, 1976).

Several studies have compared different rotational procedures (Dielman et al., 1972; Gorsuch, 1970; Guertin & Bailey, 1970). The findings were fairly consistent. The Varimax method was found to satisfy the principle of simple structure and that of factorial invariance. These two criteria are considered fundamental to a successful rotational method (Harman, 1976).

In their review of analytic methodology, Glass and Taylor (1966) concluded that "The search for an acceptable analytical orthogonal rotation procedure for attaining simple structure was effectively ended in 1958 with the publication of Kaiser's Varimax procedure" (p. 570). In Glass and Taylor's (1966) view, future interest in improving on Varimax is not expected since "Those who apply factor analysis appear to be content with Varimax" (p. 570).

<u>The use of the root-mean-squares (RMS)</u>. Of the various methods available for comparing the factor loadings of one matrix with those of another, the RMS deviations method was the most appropriate one to use in this study (Harman, 1976). As stated previously (Chapter I, p. 3)

the RMS deviation is a common statistical measure for comparing pairs of corresponding factor loading in two studies "since the variables are the same" (Harman, 1976, p. 343). In comparing Varimax, Quartimax, and subjective solutions for the same factor matrix, Harman (1976) used the RMS index of deviation.

In a Monte Carlo study, Hamburger (1965) computed the RMS deviations between corresponding loadings of rotated factors from sample and population matrices. A similar use of the RMS is in an empirical study by Joreskog (1963) who compared unrotated common-factor loadings from samples and populations.

In an empirical investigation, emulated somewhat by this study, Mosier (1939) calculated the RMS deviations to compare error-free hypothetical factor solutions with error-added ones.

Bailey (1969) computed the RMS discrepancies to compare variable dependence in several oblique solutions.

Because of its simplicity, ease in calculation, and its common use in factor analytic methodology, the RMS index seemed an appropriate measure of deviation to use in this study. A root-mean-square value of zero would mean perfect agreement between two corresponding values (Harman, 1976). Successive increases in the RMS values away from zero should indicate greater degrees of disagreement. The data thus obtained become suitable for further analysis, e.g., as a dependent variable in an analysis of variance design.

Summary

A variety of rationales, rules, and procedures were found in the literature as to the correct number of factors that must be rotated. A small number of studies was noted that examined the effects of under and overrotation on factor structure. The general results were fairly consistent. It is better to overrotate by one or two factors but not much more, otherwise factor fission occurs (Cattell, 1966a). Underrotation was not recommended.

The survey revealed no specific empirical study that dealt with factor loading stability as a function of under and overrotation under all the conditions proposed for the present study. Different investigators dealt with certain aspects of the problem, with no attempt at a broad empirical examination. There also appeared to be few investigations in this troublesome area of factor analysis. In addition, within the investigations noted, the number of intercorrelation matrices examined was quite small. It seemed, therefore, appropriate to employ an empirical approach, with a large number of replicated matrices, under varying experimental conditions representative of observed data. The results may contribute some insights to this aspect of factor analysis.

CHAPTER III

Methodology

Because of the length and complexity of the procedure in this study, Chapter III has been divided into five main sections: (a) the statistical hypotheses generated by the research questions; (b) the selection of the problem matrices; (c) the selection of and the matrices' adjustments to the specified levels of common variance; (d) the selection and generation of the chosen random error levels; (e) the application of the procedures to the problem matrices.

Two major research questions were to be answered by this study. First, what effect does under and overrotation of factors have on the loadings of a factor matrix under the specified levels of common variance and error? A test of the differences in the obtained RMS means should be an indicator of this effect. Second, what are the trends in the data, i.e., what is the form of the equation(s) that best describe(s) the plotted RMS mean values? These questions led to the formulation of several statistical hypotheses.

The Statistical Hypotheses

The research questions generated the following statistical hypotheses, tested at $\alpha = .05$:

a. For each selected problem, there are no differences among the RMS mean values under the various levels of the number of factors rotated, i.e.:

H_0: $\mu_1 = \mu_2 = \mu_3 \cdot \cdot \cdot \cdot \cdot \cdot \cdot \mu_k$.

H_1: some μ_j's are unequal.

Any significant differences that exist among the selected levels of common variance and among the selected levels of error were not a focus of concern for this study. Those levels were chosen to allow for the generalizability of the findings. However, the effect of these levels on the number of factors to rotate is of importance to the investigation.

b. For each problem, there is no linear, quadratic, or cubic component of the model:

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_1^2 + \beta_3 X_1^3 + \varepsilon.$$

That is:

1. H_0: $\beta_1 = 0$

   H_1: $\beta_1 \neq 0$

2. H_0: $\beta_2 = 0$

   H_1: $\beta_2 \neq 0$

3. H_0: $\beta_3 = 0$

   H_1: $\beta_3 \neq 0$.

Selection of the Matrices

Four factor matrices with different numbers of variables and factors were chosen. The matrices were selected to be fairly representative of ones reported in the literature.

Though hypothetical matrices could have been constructed, it was decided that, in keeping with a common practice in factor analytic studies, only published matrices be included (Gorsuch, 1974). Logistical considerations prevented the selection of matrices of extreme size. The ones chosen were Fruchter's (1954) 11x5 matrix, Harman's (1976) 24x4 matrix, Mulaik's (1972) 36x5 matrix, and Whimbey and Denenberg's (1966) 23x6 matrix. The Fruchter and Harman matrices have been used extensively in factor analytic methodological investigations. Harman's matrix, in particular, has become a classic in factor analytic demonstrations.

The Common Variance Adjustment

Since each of the four selected problem matrices accounted for a different amount of common variance, it was necessary to adjust this variance so as to allow comparisons among the matrices. Three levels of the proportion of common variance were chosen. The rationale for choosing .30, .45, and .60 (30%, 45%, and 60% of the common variance accounted for) was to make the study as generalizable as possible by including proportions that were representative of those found in published works. A proportion of .30 is fairly low but not infrequently encountered; .45 is rather commonly and typically reported; and the .60 proportion is somewhat high but also found.

For each factor matrix chosen, several operations were performed. Regardless of the original amount of common variance for which the input matrix accounted, it was adjusted to conform to each of the three prespecified levels. Each factor matrix was "stretched" or "compressed" as follows: every value (factor loading) in the matrix was squared and then the values were summed across the rows to give the communalities; all the communalities thus calculated were then summed and the total divided by the number of variables for the particular matrix.

The value thus obtained was then divided into each of the three prespecified proportions of common variance, yielding constants by which every squared value in the original factor matrix was multiplied. Then the square root values were obtained. The adjusted matrices would now conform to the amount of common variance specified.

The Choice and Generation of Error

Because external validity was a concern, three levels of pseudo-randomly-generated error were chosen, each of which represented a different sample size (N). The N's were 100, 200, and 500. The corresponding standard error (S.E.) for each N was computed by the use of $\frac{1}{\sqrt{N}}$ when the correlation is zero. This is an acceptable formula for determining the S.E. when samples are fairly large (Guilford and Fruchter, 1973).

For an N = 100, the S.E. is .10 with a mean of zero; N = 200, the S.E. is .07 with a mean of zero; and for N = 500, the S.E. is .04 with a mean of zero. The errors were computer-generated from a specified normal distribution for each level with a mean of zero, and the above specified standard deviations.

To assess the effectiveness of Fortran routine NDIST for generating the specified random error, the means and standard deviations of the requested error values were examined. The number of error values generated per intercorrelation matrix was 10 $\frac{m(m-1)}{2}$ where 10 is the number of replications per condition, and m is the number of variables. For example, for Fruchter's (1954) 11x5 matrix, for each level of error, 550 random values were generated and their means and standard deviations computed. Comparisons of the means and standard deviations of the random error values requested and those actually generated for the three levels of error under the three levels of common variance for the four problem matrices are shown in Tables A1 and A2 in Appendix A.

Procedures

All four problem matrices were analyzed in a similar fashion. Differences in the analyses, when they existed, were related to the number of factors that were carried into rotation. This number depended not only on the original rank of each of the problem matrices, but also on the logistical limitations to the analyses.

Because the analytical procedures were complex and lengthy, the first problem matrix will be presented in detail to illustrate completely the technique used in the analyses. A less detailed description of the procedures is given for the other three selected matrices.

Problem One. Problem One was chosen from Fruchter (1954, p. 147), and is the smallest matrix used in this study. It is a five-factor problem based upon eleven variables. The latter were eleven tests, part of a larger battery[1] used by the U.S. Army Air Force during World War II. The five oblique reference-factors, which accounted for 21.33% of the common variance were rotated by Harris's (1948) direct method. This 11x5 factor matrix is shown in Table 1.

Fruchter's (1954) matrix was adjusted by the procedure described earlier to account for 30%, 45%,and 60% of the common variance; the three resulting matrices are shown in Table 2. Each of these adjusted matrices was postmultiplied by its transpose to yield an intercorrelation matrix R.

R', the intercorrelation matrix based on 30% common variance, is shown above the principal diagonal in Table 3. The matrix, R', was

[1]See Guilford, 1947, for a complete description of these tests, including reliability and validity statistics.

TABLE 1

PROBLEM ONE

The Input Factor Matrix Taken from Fruchter[a]

Variable	Factor I	II	III	IV	V
1	.07	.32	-.13	.16	.21
2	.43	.04	-.06	.00	.05
3	-.03	.12	-.05	.42	.05
4	-.03	.03	.03	.08	.30
5	-.03	.02	.35	.03	.09
6	.45	.00	.00	.00	.00
7	.00	.63	.00	.00	.00
8	.09	.68	.11	.02	-.13
9	.01	-.01	.39	-.01	-.05
10	.00	.00	.00	.41	.00
11	.00	.00	.00	.00	.33

[a]Fruchter (1954, p. 147)

TABLE 2

PROBLEM ONE

Fruchter's Matrix Adjusted to Account for Three Levels of Common Variance

Variance	Variable	Factor I	II	III	IV	V
	1	.08[a]	.38	.15	.19	.25
	2	.51	.05	.07	.00	.06
	3	.04	.14	.06	.50	.06
	4	.04	.04	.04	.09	.36
	5	.04	.02	.42	.04	.11
30%	6	.53	.00	.00	.00	.00
	7	.00	.75	.00	.00	.00
	8	.11	.81	.13	.02	.15
	9	.01	.01	.46	.01	.06
	10	.00	.00	.00	.49	.00
	11	.00	.00	.00	.00	.39
	1	.10	.46	.19	.23	.31
	2	.62	.06	.09	.00	.07
	3	.04	.17	.07	.61	.07
	4	.04	.04	.04	.12	.44
	5	.04	.03	.51	.04	.13
45%	6	.65	.00	.00	.00	.00
	7	.00	.92	.00	.00	.00
	8	.13	.99	.16	.03	.19
	9	.01	.01	.57	.01	.07
	10	.00	.00	.00	.60	.00
	11	.00	.00	.00	.00	.48
	1	.12	.54	.22	.27	.35
	2	.72	.07	.10	.00	.08
	3	.05	.20	.08	.70	.08
	4	.05	.05	.05	.13	.50
	5	.05	.03	.59	.05	.15
60%	6	.75	.00	.00	.00	.00
	7	.00	.99	.00	.00	.00
	8	.15	.99	.18	.03	.22
	9	.02	.02	.65	.02	.08
	10	.00	.00	.00	.69	.00
	11	.00	.00	.00	.00	.55

[a]Values only to second decimal place accuracy were retained to facilitate inspection.

TABLE 3

PROBLEM ONE

Intercorrelation Matrices R' and R'' for Replication One with 30% of Common Variance[a]

Variable	1	2	3	4	5	6	7	8	9	10	11
1	.2727[b]	.0861	.1757	.1286	.1093	.0443	.2835	.3779	.0938	.0923	.0975
2	.1130	.2709	.0326	.0435	.0551	.2722	.0354	.1111	.0430	.0000	.0232
3	.0548	.1043	.2767	.0768	.0533	.0190	.1063	.1473	.0390	.2422	.0232
4	.0971	.0675	.1911	.1394	.0582	.0190	.0266	.0942	.0395	.0461	.1392
5	-.0466	.0750	-.1154	.0113	.1868	.0190	.0177	.0944	.1994	.0173	.0418
6	.1045	.1837	.0426	.0394	-.0186	.2848	.0000	.0570	.0063	.0000	.0000
7	.4123	.0955	-.0231	.0048	-.0036	.1097	.5582	.6025	.0089	.0000	.0000
8	.3952	.1367	.1013	.0437	-.0929	-.0203	.5705	.7031	.0806	.0115	.0603
9	.0829	-.0543	.0824	.2023	.1880	-.0046	-.0756	.1035	.2179	.0058	.0232
10	.0813	.0983	.2658	.1333	.1148	-.1706	.1739	-.0898	.0166	.2364	.0000
11	.0687	.0777	-.0090	.0499	-.0183	-.1657	-.1133	.1040	-.0470	-.0957	.1532

[a]The entries above the principal diagonal are error-free correlations of R' while those below the diagonal are correlations of R'' with .10 error added.

[b]The underlined entries in the principal diagonal are unaltered communalities.

factored by the principal axes method and the extracted factors were rotated to the Varimax criterion. The obtained rotated five-factor solution was now the criterion matrix. The latter is shown in the upper third of Table 4. To create the first condition of error, the intercorrelation matrix, R', was subjected to the addition of the pseudo-randomly-generated error level of .10 which represents a sample size of 100. No error was added to the principal diagonal (the exact communalities) so as not to alter the rank of the matrix. The adjusted, error-added intercorrelation matrix R" for Fruchter's (1954) original factor matrix appears in Table 3 below the principal diagonal. The computer-generated error added to the first replication is shown in Table 5.

The R" matrix was factor analyzed by the principal axes method yielding seven factors. The first three principal axes were rotated to Varimax and the differences between their loadings and those of their counterparts in the criterion factor matrix (Table 4) were calculated by the RMS method. By adding the three RMS's for the differences between paired loadings on the three factors, a single value, the total RMS was obtained.

A fourth principal axis was carried into rotation and, in a similar fashion, the total RMS's obtained. The same was done with five, six, then seven factor rotations,and total RMS's calculated. Therefore, for the first replication, five values were obtained each of which represented the total RMS discrepancy between the first three factors of the criterion matrix and the first three factors of the five trial rotation matrices.

Beginning again with the error-free intercorrelation matrix, R',

TABLE 4

PROBLEM ONE

The Criterion Matrices with Three Levels of Common Variance

Variance	Variable	Factor I	II	III	IV	V
	1	.08	.39	.16	.19	.23
	2	.51	.06	.07	.01	.05
	3	.03	.13	.06	.50	.07
	4	.03	.05	.06	.08	.35
	5	.04	.04	.42	.03	.08
30%	6	.53	.01	.00	.01	.00
	7	-.01	.75	-.03	.02	-.03
	8	.09	.82	.11	.05	.11
	9	.02	.03	.46	.01	.03
	10	.01	-.01	.00	.49	.02
	11	.00	.02	.03	-.01	.39
	1	.09	.48	.19	.24	.28
	2	.62	.07	.08	.01	.06
	3	.03	.16	.07	.61	.08
	4	.04	.06	.07	.10	.43
	5	.05	.05	.51	.04	.09
45%	6	.65	.01	-.01	.01	.00
	7	-.02	.91	-.04	.03	-.04
	8	.12	.99	.13	.06	.13
	9	.02	.04	.57	.01	.03
	10	-.01	-.02	.00	.59	.02
	11	.00	.02	.03	-.02	.48
	1	.11	.55	.22	.28	.32
	2	.72	.09	.10	.01	.07
	3	.04	.19	.08	.71	.10
	4	.05	.07	.08	.12	.50
	5	.05	.06	.59	.05	.11
60%	6	.75	.01	-.01	.01	.00
	7	-.02	.99	-.04	.03	-.05
	8	.13	.99	.15	.07	.15
	9	.02	.04	.66	.02	.04
	10	-.01	-.02	.00	.69	.03
	11	.00	.03	.04	-.02	.55

TABLE 5

PROBLEM ONE

Error Values for Replication One
Pseudorandomly Generated Error Values Added to the Adjusted Intercorrelation Matrix R'
and Reflected in R''. The Standard Error is .09 and the Mean is -.01[a].

Variable	1	2	3	4	5	6	7	8	9	10	11
1	.0000[b]	.0269	-.1206	-.0315	-.1559	-.0602	.1288	.0173	-.0109	-.0110	-.0288
2		.0000	.0717	.0240	.0199	-.0885	.0601	.0256	-.0973	.0983	.0545
3			.0000	.1143	-.1687	.0236	-.1294	-.0460	.0434	.0236	-.0322
4				.0000	-.0469	.0204	-.0218	-.0505	.1628	.0872	-.0893
5					.0000	-.0376	-.0213	-.1873	-.0114	.0975	-.0601
6						.0000	.1097	-.0773	-.0109	-.1706	-.1657
7							.0000	-.0320	-.0845	.1739	-.1133
8								.0000	.0229	-.1013	.0437
9									.0000	.0108	-.0702
10										.0000	-.0957
11											.0000

[a]Actual values called for were Standard Error = .10 and Mean = 0.0.
[b]No error was added to the communalities.

the whole process of adding newly-generated error, factoring, rotating, and calculating the total RMS's was completed nine more times for a total of ten replications. The total RMS's for the ten replications for Problem One, where the common variance is 30%, the error is .10, for the five rotations tried, are shown in the upper portion of Table 6.

Fruchter's (1954) matrix was examined under the second level of error, .07, with the common variance remaining at 30%. The same number of rotations was performed and the total RMS's similarly computed. The procedure was replicated ten times. The same operations were repeated with the third error level of .04. Therefore, for one level of common variance and three levels of error, with ten replications each, 150 total RMS values were obtained for Problem One. These values are shown in Table 6.

Fruchter's (1954) original matrix was similarly examined under the condition of 45% of the common variance accounted for and a second criterion factor matrix was obtained. This matrix is shown in the middle portion of Table 4. Again, error, generated at the three levels, was added to the intercorrelation matrix in each instance and the same procedure followed in obtaining total RMS's for five different trial rotations. Ten replications were performed each time and another 150 total RMS values were thus obtained.

The last 150 total RMS's for Problem One were the result of adjusting Fruchter's (1954) matrix to account for 60% of the common variance (see the lower portion of Table 2) and examining it under the three levels of error. The criterion matrix for this experimental condition appears in the lower third of Table 4.

The overall means of the RMS values for the five different rotations

TABLE 6

PROBLEM ONE

Total Root-Mean-Squares for 30% Common Variance and Three Levels of Error

Error	Replication	Number of Factors Rotated 3	4	5	6	7
	1	.13[a]	.13	.08	.09	.09
	2	.14	.13	.13	.10	.09
	3	.12	.10	.08	.07	.07
	4	.15	.12	.10	.10	.11
.10	5	.13	.09	.06	.08	.08
	6	.15	.13	.12	.12	.09
	7	.15	.12	.10	.09	.08
	8	.16	.13	.10	.08	.09
	9	.18	12	.13	.10	.10
	10	.13	.12	.09	.08	.09
	1	.11	.10	.10	.07	.09
	2	.11	.05	.04	.04	.04
	3	.12	.09	.06	.05	.05
	4	.12	.07	.05	.05	.05
.07	5	.13	.07	.07	.09	.08
	6	.12	.08	.06	.06	.06
	7	.12	.08	.08	.07	.07
	8	.12	.11	.06	.07	.08
	9	.10	.06	.05	.04	.04
	10	.13	.08	.07	.08	.08
	1	.11	.05	.04	.04	.04
	2	.07	.05	.04	.03	.04
	3	.09	.04	.04	.04	.03
	4	.12	.06	.05	.04	.04
.04	5	.11	.07	.04	.04	.04
	6	.10	.03	.03	.03	.02
	7	.11	.04	.04	.03	.04
	8	.09	.05	.03	.03	.03
	9	.10	.04	.04	.04	.04
	10	.10	.05	.04	.04	.04

Note. Only this sample of the raw RMS data is included in this study. All the data are available from the author upon request.

[a]Values only to second decimal place retained.

were calculated and plotted. Plotted also were the RMS mean values under the nine conditions of common variance/error level. To test the differences in the RMS means under the conditions of the five different rotations and the nine common variance/error levels, an analysis of variance was completed for a multifactor repeated measures design (Winer, 1971). In this type of design the experimental unit is observed under more than one treatment. As a consequence these repeated observations (measures) will be correlated, i.e., dependent (Winer, 1971).

The element of dependence which necessitated analysis by the repeated measures design stems from the fact that for each replication, the factors rotated were drawn from the same principal axes factor matrix. For Problem One there were five rotations per replication. At every rotation, each additional factor rotated was dependent for its loadings upon the ones preceding it. Therefore, the obtained RMS's for Problem One were analyzed by means of the repeated measures multifactor design, where there were three levels of common variance (A), within each of which there were three levels of error (C); there were ten "subjects" (replications) (S) per experimental condition and five "measures" (rotations) (B) on each replication. This represented a 3x3x5 factorial design with ten subjects per cell. The number of trial rotations, i.e., measures B, for each problem, was considered a quantitative variable with an underlying continuum, having equal treatment levels and equal N's (Winer, 1971; Kirk, 1968). Hence, where there were significant main effects and interactions, trend analyses were performed.

Linear, quadratic,and cubic orthogonal polynomial coefficients were used to calculate the predicted values for the RMS means. Goodness of fit of the polynomial equations was determined by comparing the predicted and obtained values for these means (Kirk, 1968).

Problem Two. The second problem was Harman's (1976, p. 296), twenty-four psychological tests four-factor matrix, rotated to the Varimax criterion. The input matrix accounted for 47.50% of the common variance. The same procedures described for Problem One were followed for this problem. The original matrix appears in Table 7. For this problem, two factors were rotated and RMS's calculated, then similarly three, four, five,and finally six rotations tried. This was done under each combination of the three levels of common variance adjustment and three levels of error. The three matrices reflecting the variance adjustments and the three criterion matrices used for the RMS calculations are shown in Tables B1 and B2 in Appendix B.

As in Problem One,five trial rotations were performed for Problem Two under the nine combinations of common variance and levels of error. The obtained RMS values were plotted, then analyzed by means of a 3x3x5 multifactor repeated measures design as was the case for Problem One. Trend analyses were performed, and goodness of fit of the equations to the data determined.

Problem Three. Problem Three was a modification of a matrix appearing in Mulaik (1972, p. 395). It is a Varimax factor matrix of 35 tests of the Language Modalities Test for Aphasia. The original authors, Jones and Wepman (1961), included two more variables, age and education, in the analysis and rotated six factors. Since the variable of education had a single high loading of .59 on factor six, and loadings of .16 or less on the remaining five factors, it was deleted from the matrix.

Factor six, in turn, was a very weak factor, except for its education variable loading, and it, also, was excluded from the analysis. The

TABLE 7

PROBLEM TWO

The Input Factor Matrix Taken from Harman[a]

Variable	Factor I	II	III	IV
1	.14	.19	.67	.17
2	.10	.07	.43	.10
3	.15	.02	.54	.08
4	.20	.09	.54	.07
5	.75	.21	.22	.13
6	.75	.10	.23	.21
7	.82	.16	.21	.08
8	.54	.26	.38	.12
9	.80	.01	.22	.25
10	.15	.70	-.06	.24
11	.17	.60	.08	.36
12	.02	.69	.23	.11
13	.18	.59	.41	.06
14	.22	.16	.04	.50
15	.12	.07	.14	.50
16	.08	.10	.41	.43
17	.14	.18	.06	.64
18	.00	.26	.32	.54
19	.13	.15	.24	.39
20	.35	.11	.47	.25
21	.15	.38	.42	.26
22	.36	.04	.41	.36
23	.35	.21	.57	.22
24	.34	.44	.22	.34

[a]Harman (1976, p. 296)

matrix finally used for Problem Three was based upon a 36-variable five-factor matrix that accounted for 79.75% of the common variance, the highest amount for all four problems. The input matrix appears in Table 8.

The procedures outlined for Problems One and Two were followed for Problem Three. The number of factors that were rotated was a minimum of three rotations to a maximum of seven inclusive. Therefore, the number of "measures" on each principal axes matrix was five. The obtained RMS's were analyzed by means of a 3x3x5 repeated measures model as was done in the two previous problems. The adjusted and criterion matrices for Problem Three are shown in Appendix B in Tables B3 and B4.

Problem Four. Problem Four, the last one examined, was based upon a six-factor orthogonal solution taken from Whimbey and Denenberg (1966, p. 284). The variables for the matrix were 23 behavioral tests administered to a group of Purdue-Wistar rats. The input matrix, which accounted for 72.74% of the common variance, is shown in Table 9.

Although this 23x6 matrix was smaller than the 36x5 matrix of Problem Three, the number of the total RMS values obtained was much larger. This was because more rotations were tried for this matrix than for any of the other three problems. A total of seven trial rotations was performed: three to nine rotations inclusive. Because the rank of this matrix was the highest of all the matrices selected, it was logistically possible to examine a wider range of the effects of successive rotations.

Therefore, for Problem Four, seven RMS "measures" were computed per replication under each experimental condition. The RMS means were plotted, then analyzed by a repeated measures 3x3x7 factorial design. The

TABLE 8

PROBLEM THREE

The Input Factor Matrix Taken from Mulaik[a]

Variable	Factor I	II	III	IV	V
1	.16	-.25	-.26	.02	-.49
2	.83	.35	.18	.04	.28
3	.89	.26	.10	.11	.19
4	.88	.27	.12	.13	.25
5	.88	.29	.10	.14	.20
6	.86	.28	.13	.16	.24
7	.88	.25	.10	.18	.20
8	.83	.32	.12	.16	.25
9	.85	.27	.14	.06	.26
10	.82	.26	.17	.19	.25
11	.81	.31	.11	.26	.23
12	.84	.27	.14	.19	.23
13	.24	.12	.81	.10	.33
14	.15	.07	.83	.13	.36
15	.11	.18	.81	.14	.30
16	.10	.10	.77	.08	.38
17	.25	.05	.28	.04	.78
18	.30	.01	.25	.11	.70
19	.28	.13	.22	.20	.58
20	.40	.10	.18	.18	.69
21	.34	.08	.11	.18	.71
22	.46	.77	.18	.15	.21
23	.47	.77	.15	.12	.13
24	.36	.76	.17	.09	.24
25	.43	.76	.14	.06	.11
26	.48	.78	.08	-.01	.16
27	.63	.60	.03	.23	.08
28	.61	.63	-.01	.24	.12
29	.50	.13	.30	.59	.42
30	.47	.20	.23	.60	.42
31	.54	.26	.21	.65	.28
32	.55	.19	.21	.62	.30
33	.26	.18	.10	.27	.60
34	.28	.12	.36	.09	.64
35	.16	.36	.25	.18	.68
36	.17	.12	.31	.02	.57

[a]Mulaik (1972, p. 395)

TABLE 9

PROBLEM FOUR

The Input Factor Matrix Taken from Whimbey and Denenberg[a]

Variable	Factor I	II	III	IV	V	VI
1	.697	.004	.192	-.210	.309	-.057
2	-.028	-.178	-.096	.040	-.840	.242
3	.394	-.280	.051	-.461	.436	.001
4	-.046	.671	.341	-.103	.061	.102
5	.117	.083	.013	-.728	-.021	-.080
6	.134	-.047	.073	-.562	.220	.235
7	-.529	-.060	-.435	.431	-.188	-.046
8	-.418	.276	-.371	.057	-.636	.229
9	-.051	.033	-.115	-.059	.234	-.819
10	-.228	.324	-.201	-.132	-.777	.186
11	-.037	-.233	-.940	-.007	-.142	.056
12	.175	.070	.783	-.165	.005	.312
13	-.391	-.376	-.072	-.641	.084	-.196
14	.743	-.157	-.015	.043	.039	-.406
15	-.007	-.006	-.547	-.062	-.124	.135
16	.096	.253	-.341	.315	-.044	.787
17	-.905	-.081	.012	-.036	-.123	-.120
18	-.493	.098	-.028	.060	.012	.711
19	.067	.573	-.675	.191	-.020	.194
20	-.090	.073	.250	-.769	-.129	-.374
21	.081	.414	-.256	-.617	-.335	-.175
22	.504	.120	-.083	-.005	.424	.275
23	-.330	-.335	-.053	.105	-.651	-.366

[a]Whimbey and Denenberg (1966, p. 284)

remainder of the analyses followed procedures identical to those used in examining the other three problems. The adjusted and criterion matrices for Problem Four are shown in Appendix B in Tables B5 and B6.

Because four F statistics were computed for the hypotheses of the four problems, it was necessary to determine the probability of obtaining four significant F's, all by chance. This was done by following the procedure outlined in Jones and Fiske (1953) and Levitt (1961). Both sources stated the rationale and assumptions for the use of Wilkinson's (1951) table. Wilkinson (1951) provided probability tables based upon the expansion of the binomial distribution $(p + q)^n$, where p is the specified level of significance, $q = (1 - p)$, and n is the total number of tests of significance. "The fundamental assumption for the binomial model is that the several experimental results are independent, that the probability value for any one result in no way influences the value for any other result" (Jones & Fiske, 1953, p. 376). Certainly, the assumption of statistical independence was met by this study by using four totally unrelated input matrices.

In applying Wilkinson's (1951) method to this study, the probability of making four Type I errors in four tests at the .05 level was found to be .00000625.

Summary

The research questions generated several statistical hypotheses which were tested at $\alpha = .05$. Four representative factor matrices, selected from the literature were analyzed in a similar fashion. Each was adjusted to three selected levels of common variance and subsequently three criterion matrices were obtained. Three levels of randomly-generated error were selected. Random error from each level was added to intercorrelation matrices of the adjusted factor matrices.

For each problem, the intercorrelation matrices were factor analyzed, principal axes extracted, and several orthogonal factor rotations tried. These rotations were a series of successive under and overrotations of the factors of each selected matrix.

Ten replications were performed for each trial rotation, under each of the nine conditions of common variance/level of error. Root mean square (RMS) deviation values were obtained at each replication. The RMS's represented differences between factor loadings for the initial factors in each criterion matrix and their counterpart loadings on the corresponding factors in the trial rotations.

The means of the RMS values thus obtained were plotted, then analyzed by a multifactor repeated measures design. Trend analyses were performed where indicated. Orthogonal polynomial coefficients were used to compute predicted RMS mean values so as to compare them to the obtained RMS means. This was done to test the goodness of fit of the trends to the data.

Wilkinson's (1951) method was used to calculate the probability of making four Type I errors in four tests at the .05 level.

CHAPTER IV

Results

The aim of this investigation was to examine the stability of factor loadings as a function of the number of factors rotated under specified levels of common variance and error. To this end, four problem matrices were selected and examined under the specified experimental conditions. Because of the nature and length of the analyses performed, the obtained results for each of the four selected problems are presented separately.

For each problem the following will be given: descriptive data including the plotted curves of the RMS mean values; ANOVA summary table and the hypothesis tested; results of the trend analyses; comparisons between the observed and the computed RMS mean values.

Problem One

The means and the standard deviations for the RMS values for each ten replications under the stated experimental conditions of common variance, error, and rotations are presented in Table 10. The overall RMS means for the five rotations are plotted in Figure 1. The mean values of the RMS's under the three levels of common variance and the three levels of error are plotted in Figures 2 and 3, respectively.

Results of the ANOVA test for the hypothesis of equality of the RMS means are summarized in Table 11. It can be seen that at the .05 level

TABLE 10

PROBLEM ONE

Means ($\overline{X}$) and Standard Deviations (S.D.) for Each Ten Replications Under Three Levels of Common Variance, Three Levels of Error and Five Rotations

		Number of Factors Rotated									
		3		4		5		6		7	
Variance	Error	$\overline{X}$	S.D.	$\overline{X}$	S.D.	$\overline{X}$	S.D.	$\overline{X}$	S.D.	$\overline{X}$	S.D.
	.10	.1421	.0186	.1181	.0129	.1001	.0222	.0920	.0149	.0882	.0117
30%	.07	.1173	.0074	.0798	.0163	.0642	.0160	.0627	.0167	.0640	.0184
	.04	.1002	.0142	.0478	.0096	.0375	.0048	.0360	.0049	.0355	.0048
	.10	.1450	.0230	.1067	.0175	.0711	.0120	.0702	.0115	.0649	.0077
45%	.07	.1236	.0140	.0746	.0205	.0561	.0116	.0552	.0101	.0559	.0136
	.04	.1073	.0160	.0415	.0071	.0303	.0051	.0295	.0045	.0287	.0051
	.10	.1396	.0201	.0796	.0172	.0635	.0061	.0604	.0076	.0587	.0095
60%	.07	.1377	.0239	.0596	.0116	.0467	.0057	.0450	.0048	.0450	.0048
	.04	.1208	.0159	.0450	.0086	.0271	.0044	.0266	.0045	.0262	.0042

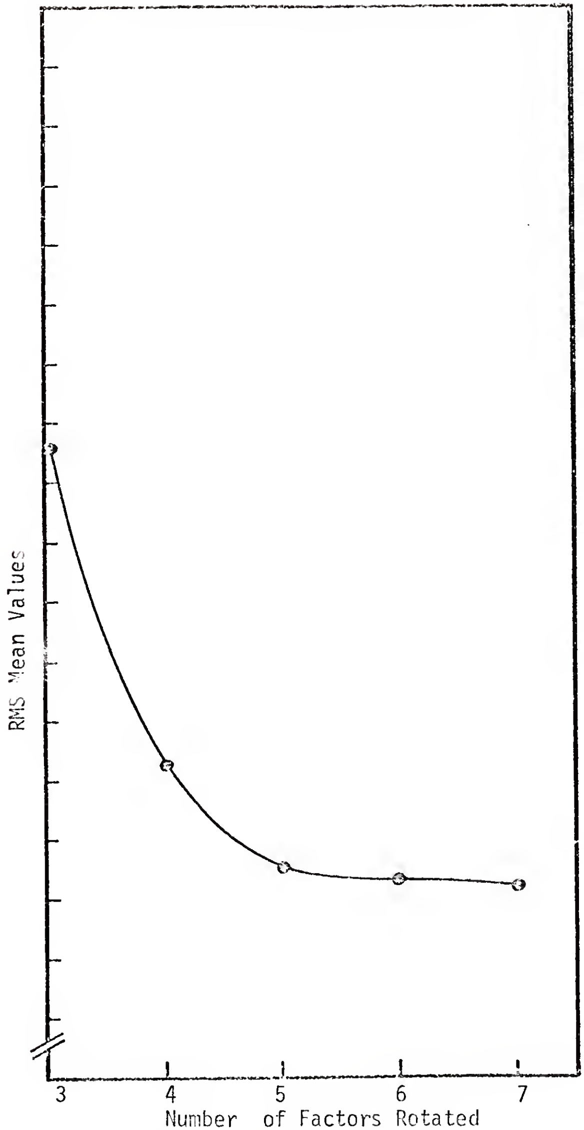

Figure 1. RMS means for the five different rotations for Problem One.

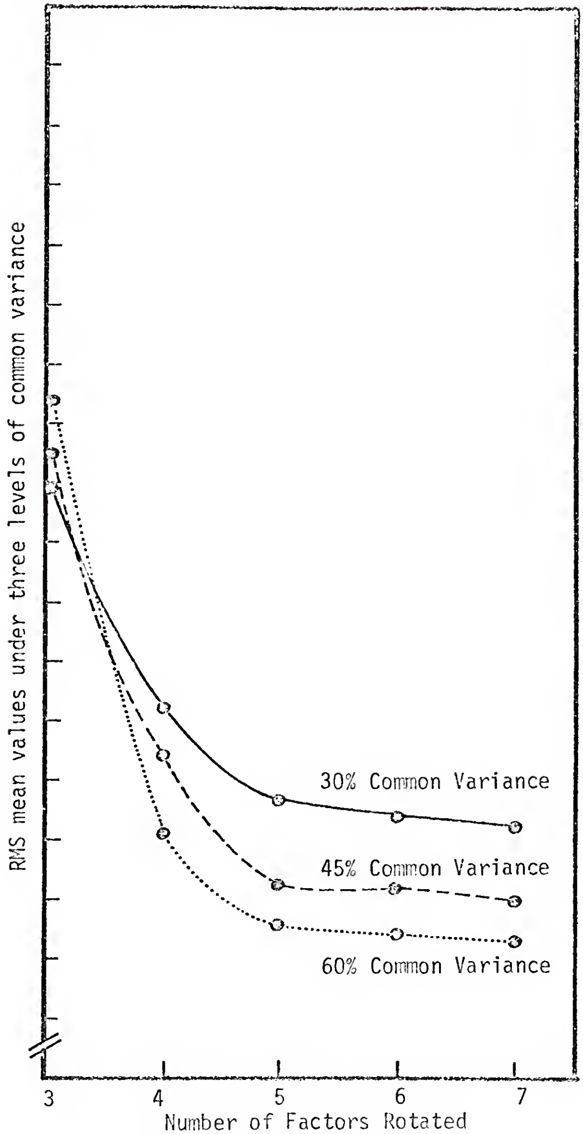

Figure 2. RMS means for the interaction of the five different rotations with the three levels of common variance for Problem One.

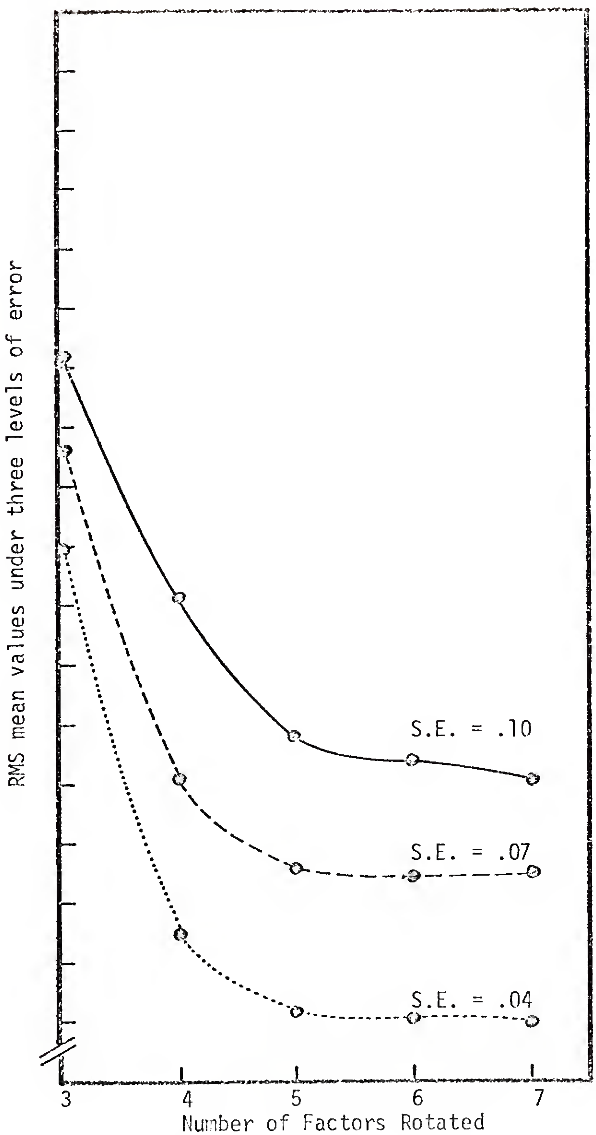

Figure 3. RMS means for the interaction of the five different rotations with the three levels of error for Problem One.

TABLE 11

PROBLEM ONE

ANOVA Summary Table for RMS Mean Values for Five Different Rotations

Source of Variation	SS	*df*	MS	*F*[a]
Between rotations	.2022	89		
Common variance (A)	.0142	2	.0071	18.94*
Error (C)	.1453	2	.0727	193.50*
Common variance x error (AC)	.0087	4	.0022	5.78*
Replications within common variance and error (S/AC)	.0340	81	.0004	
Within rotations	.4140	360		
Rotations (B)	.3566	4	.0892	793.45*
Common variance x rotations (AB)	.0137	8	.0017	15.23*
Error x rotations (CB)	.0050	8	.0006	5.55*
Common variance x rotations x error (ABC)	.0023	16	.0001	1.27
Rotations x replications within common variance and error (BS/AC)	.0364	324	.0001	
Total	.6162	449		

[a]Values rounded to four places for presentation which accounts for apparent inconsistencies in the *F* ratios.

*$p < .05$.

the main effect of the different rotations B is significant. The hypothesis of no difference is therefore rejected in favor of the alternative hypothesis.

The effect of the three levels of common variance on the number of factors rotated, i.e., the variance and rotations interaction AB is found significant. This is also true of the interaction between the different factor rotations and the three levels of error BC. The overall interaction between common variance, rotations and error ABC is not found significant.

The other three significant F's in the ANOVA summary table are those for the common variance A, error levels C, and their interaction AC. It should be noted that the significance of the latter three terms is not of primary concern to this investigation. The three levels of common variance and the three levels of error were selected as independent variables only to allow the generalization of the findings. Therefore, future references to the levels of common variance and error will not be made in the remainder of this Chapter. However, these terms can be found in each ANOVA summary table as part of the overall analysis.

Results of the analysis of variance of the linear, quadratic, and cubic trends of the RMS mean values are shown in Table 12. The results of the trend analyses indicate that the means of the RMS deviation values are a curvilinear function of the number of factors rotated. The three trend components are included in the polynomial cubic equation:

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_1{}^2 + \beta_3 X_1{}^3 + \varepsilon.$$

It seems that an equation of this form represents the simplest description of the curve connecting the five RMS means in Figure 1.

A comparison of the observed RMS mean values for the five rotations

TABLE 12

PROBLEM ONE

ANOVA Summary Tables for Linear, Quadratic and Cubic Trends

Source of variation	SS	df	MS	F
Analysis of Linear Trend				
Within rotations linear	.2749	90		
Rotations (B) linear	.2534	1	.2435	1499.41*
Rotations x common variance (BA)	.0063	2	.0032	18.64*
Rotations x error (BC)	.0003	2	.0002	<1.00
Rotations x common variance x error (BAC)	.0012	4	.0057	33.58*
Rotations x replications within common variance x error (BS/AC)	.0137	81	.0002	
Analysis of Quadratic Trend				
Within rotations quadratic	.1092	90		
Rotations (B) quadratic	.0931	1	.0931	847.31*
Rotations x common variance (BA)	.0049	2	.0025	25.00*
Rotations x error (BC)	.0004	2	.0002	2.00
Rotations x common variance x error (BAC)	.0019	4	.0005	4.75*
Rotations x replications within common variance x error (BS/AC)	.0089	81	.0001	
Analysis of Cubic Trend				
Within rotations cubic	.0240	90		
Rotations (B) cubic	.0111	1	.0111	111.00*
Rotations x common variance (BA)	.0019	2	.0010	10.00*
Rotations x error (BC)	.0018	2	.0009	9.00*
Rotations x common variance x error (BAC)	.0004	4	.0001	1.0
Rotations x replications within common variance x error (BS/AC)	.0088	81	.0001	

*$p < .05$.

and the predicted ones are in Table 13. The three trend components used in computing the predicted RMS mean values are also included. (The computations were done by the use of orthogonal polynomial coefficients and solving for Y.) It can be seen from Table 13 that adequate approximations of the RMS mean values as a function of the number of different factor rotations are those given by the cubic components. It should be noted, however, that the plotted curve of the RMS mean values in Figure 1 shows no visible inflection corresponding to the significant cubic trend.

Problem Two

The means and standard deviations for the RMS mean values for each ten replications under the prespecified experimental conditions are in Table 14. The RMS mean values B for the five different rotations are plotted in Figure 4; the interactions of common variance with rotation AB and error with rotations CB are shown in Figure 5 and 6, respectively.

In Table 15 is shown the summary of the ANOVA completed with the RMS mean values for the five different rotations for Problem Two. The main effect of rotations B is significant at the .05 level, as are the AB and BC interaction terms. The rest of the significant terms are indicated in the table. Therefore, the hypothesis of the equality of the B's is rejected in favor of the alternative hypothesis.

The three trend analyses performed are presented in Table 16. The results of the analyses indicate the presence of a cubic trend in the data. Therefore, a polynomial cubic equation of the form:

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_1^2 + \beta_3 X_1^3 + \varepsilon$$

is accepted as best representing the curvilinear nature of the RMS mean

TABLE 13

PROBLEM ONE

Trend Components, Observed and Predicted RMS Means for Five Different Rotations (B)

Trend Components	
Linear	-1.68×10^{-2}
Quadratic	$.85 \times 10^{-2}$
Cubic	$-.35 \times 10^{-2}$

Number of Factors Rotated	Predicted B's Linear	Predicted B's Quadratic	Predicted B's Cubic	Observed B's
3 (B_1)	.1053	.1223	.1258	.1259
4 (B_2)	.0885	.0800	.0730	.0725
5 (B_3)	.0552	.0717	.0547	.0552
6 (B_4)	.0530	.0549	.0464	.0530
7 (B_5)	.0518	.0381	.0551	.0518

deviation values for the five different rotations.

In Table 17 are found the three trend components and comparisons between the observed RMS mean values for the five different rotations and the RMS values predicted. The best approximation of the data is that of the cubic trend, although, in Figure 4, no indication of this cubic trend is observed.

Problem Three

The means and standard deviations for each ten replications under the experimental conditions of common variance and error are shown in Table 18. In Figures 7, 8,and 9 are the plotted curves of the RMS mean values.

Results of the ANOVA test for the hypothesis of equality of the RMS means are summarized in Table 19. At the .05 level, the main effect of rotations B is significant, as are the interaction terms AB and BC. The null hypothesis is rejected in favor of the alternative hypothesis.

The results of the linear, quadratic, and cubic trend analyses, given in Table 20, indicate the presence of a significant cubic trend in the data. Therefore, a polynomial cubic equation of the form:

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_1{}^2 + \beta_3 X_1{}^3 + \varepsilon$$

is accepted as the best representative of the curvilinear trend of the RMS mean values under the condition of five different rotations.

The three trend components and a comparison between the observed and the predicted RMS mean values are presented in Table 21. It can be seen that the observed B's compare best with the predicted B's when the cubic components are included in the equations. Figure 7, however, shows no inflection in the curve to indicate the cubic trend, significant though it is.

TABLE 14

PROBLEM TWO

Means ($\overline{X}$) and Standard Deviations (S.D.) for Each Ten Replications Under Three Levels of Common Variance, Three Levels of Error and Five Rotations

		Number of Factors Rotated									
		2		3		4		5		6	
Variance	Error	$\overline{X}$	S.D.	$\overline{X}$	S.D.	$\overline{X}$	S.D.	$\overline{X}$	S.D.	$\overline{X}$	S.D.
	.10	.1542	.0102	.1139	.0135	.1013	.0125	.1013	.0168	.0993	.0151
30%	.07	.1431	.0053	.0871	.0156	.0699	.0159	.0711	.0117	.0733	.0152
	.04	.1352	.0041	.0590	.0169	.0372	.0044	.0389	.0058	.0393	.0063
	.10	.1729	.0075	.0920	.0144	.0744	.0055	.0772	.0089	.0791	.0055
45%	.07	.1648	.0030	.0696	.0130	.0518	.0072	.0509	.0073	.0517	.0061
	.04	.1630	.0032	.0573	.0155	.0286	.0034	.0293	.0037	.0292	.0039
	.10	.1930	.0079	.0853	.0138	.0639	.0082	.0653	.0059	.0678	.0085
60%	.07	.2046	.0554	.0688	.0134	.0459	.0062	.0457	.0057	.0464	.0057
	.04	.1863	.0034	.0493	.0146	.0235	.0025	.0242	.0026	.0245	.0026

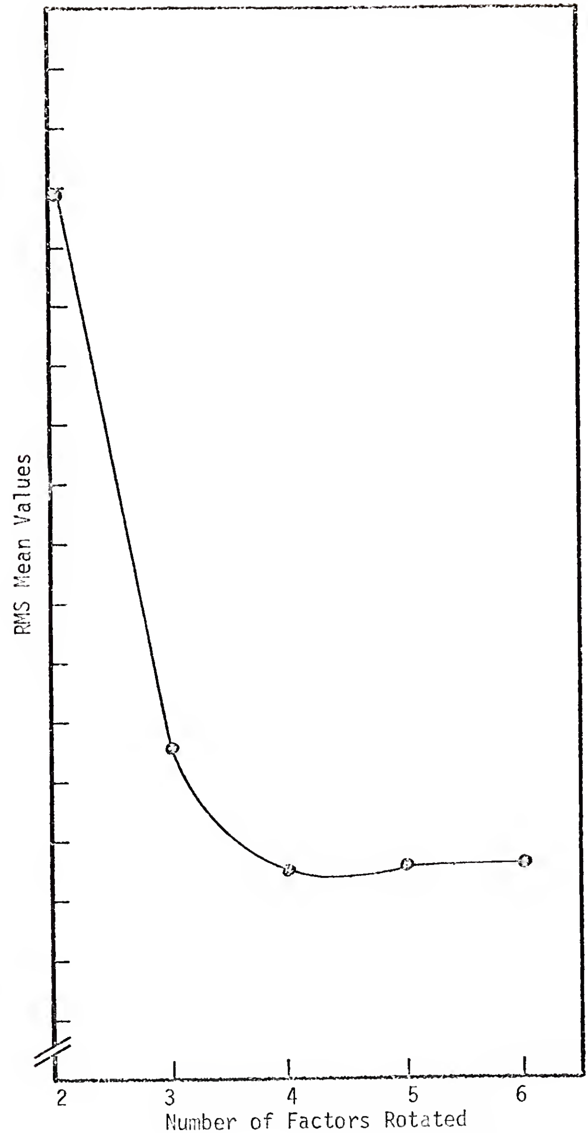

Figure 4. RMS means for the five different rotations for Problem Two.

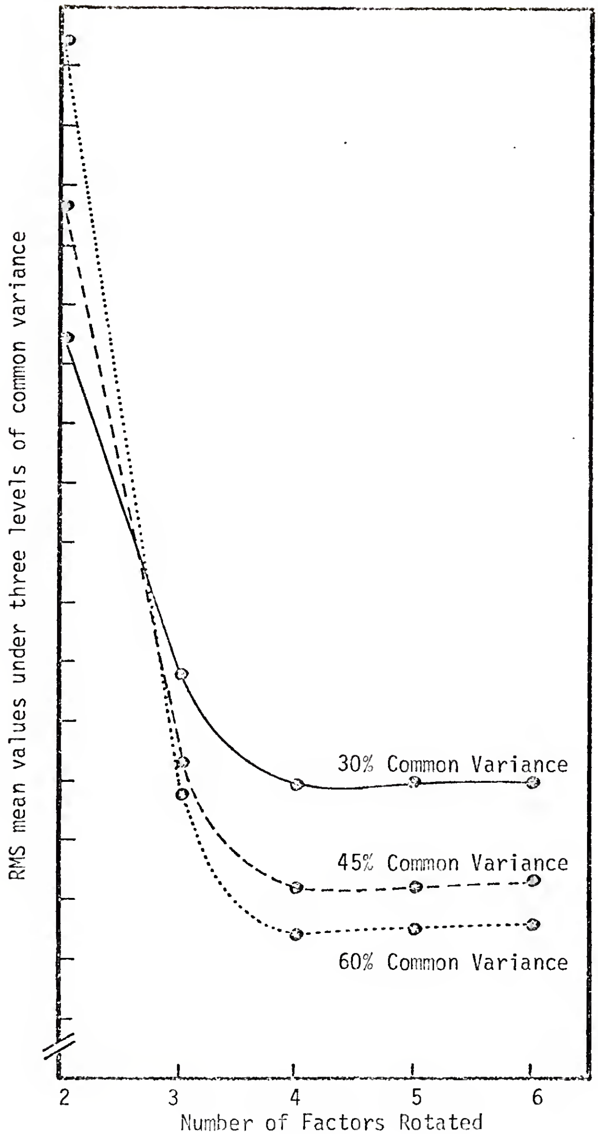

Figure 5. RMS means for the interaction of the five different rotations with the three levels of common variance for Problem Two.

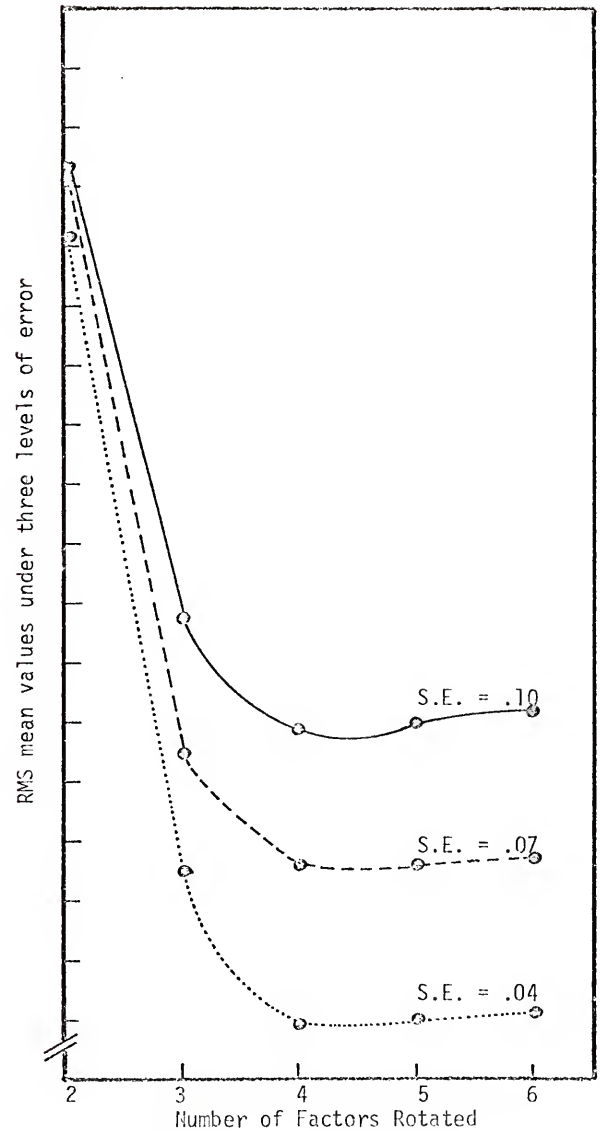

Figure 6. RMS means for the interaction of the five different rotations with the three levels of error for Problem Two.

TABLE 15

PROBLEM TWO

ANOVA Summary for RMS Mean Values for Five Different Rotations

Source of variation	SS	df	MS	F
Between rotations	.1593	89		
Common variance (A)	.0077	2	.0038	15.67*
Error (C)	.1266	2	.0633	259.52*
Common variance x error (AC)	.0053	4	.0013	5.61*
Replications within common variance and error (S/AC)	.0197	81	.0002	
Within rotations	.9917	360		
Rotations (B)	.8614	4	.2154	1538.21*
Common variance x rotations (AB)	.0659	8	.0082	58.52*
Error x rotations (CB)	.0169	8	.0021	15.11*
Common variance x rotations x error (ABC)	.0016	16	.0001	<1.00
Rotations x replications within common variance and error (BS/AC)	.0459	324	.0001	
Total	1.1510	449		

*$p < .05$.

TABLE 16

PROBLEM TWO

ANOVA Summary Table for Linear, Quadratic and Cubic Trends

Source of variation	SS	df	MS	F
Analysis of Linear Trend				
Within rotations linear	.5994	90		
Rotations (B) linear	.5341	1	.5341	2670.50*
Rotations x common variance (BA)	.0367	2	.0184	92.00*
Rotations x error (BC)	.0115	2	.0058	29.00*
Rotations x common variance x error (BAC)	.0007	4	.0002	1.00
Rotations x replications within common variance x error (BS/AC)	.0164	81	.0002	
Analysis of Quadratic Trend				
Within rotations quadratic	.3218	90		
Rotations (B) quadratic	.2795	1	.2795	1615.60*
Rotations x common variance (BA)	.0230	2	.0115	66.47*
Rotations x error (BC)	.0048	2	.0024	13.87*
Rotations x common variance x error (BAC)	.0005	4	.0001	<1.00
Rotations x replications within common variance x error (BS/AC)	.0140	81	$.0017 \times 10^{-1}$	
Analysis of Cubic Trend				
Within rotations cubic	.0639	90		
Rotations (B) cubic	.0478	1	.0478	402.05*
Rotations x common variance (BA)	.0054	2	.0027	22.67*
Rotations x error (BC)	.0010	2	.0005	4.21*
Rotations x common variance x error (BAC)	.0001	4	$.0003 \times 10^{-1}$	<1.00
Rotations x replications within common variance x error (BS/AC)	.0096	81	$.0012 \times 10^{-1}$	

*$p < .05$.

TABLE 17

PROBLEM TWO

Trend Components, Observed and Predicted RMS Means for Five Different Rotations (B)

Trend Components	
Linear	-2.44×10^{-2}
Quadratic	1.49×10^{-2}
Cubic	$-.72 \times 10^{-2}$

Number of Factors Rotated	Predicted B's			Observed B's
	Linear	Quadratic	Cubic	
2 (B_1)	.1312	.1610	.1682	.1685
3 (B_2)	.1068	.0919	.0775	.0758
4 (B_3)	.0824	.0526	.0526	.0552
5 (B_4)	.0580	.0431	.0575	.0559
6 (B_5)	.0336	.0634	.0562	.0567

TABLE 18

PROBLEM THREE

Means ($\overline{X}$) and Standard Deviations (S.D.) for Each Ten Replications Under Three Levels of Common Variance, Three Levels of Error and Five Rotations

		Number of Factors Rotated									
		3		4		5		6		7	
Variance	Error	$\overline{X}$	S.D.	$\overline{X}$	S.D.	$\overline{X}$	S.D.	$\overline{X}$	S.D.	$\overline{X}$	S.D.
	.10	.1345	.0100	.1250	.0135	.1344	.0119	.1290	.0122	.1282	.0101
30%	.07	.1099	.0149	.1007	.0129	.0930	.0120	.0891	.0107	.0899	.0112
	.04	.0846	.0137	.0511	.0087	.0438	.0054	.0414	.0063	.0420	.0064
	.10	.1253	.0227	.1260	.0271	.1103	.0198	.1053	.0178	.1025	.0144
45%	.07	.1013	.0201	.0814	.0142	.0710	.0120	.0667	.0080	.0676	.0087
	.04	.0870	.0091	.0443	.0093	.0346	.0036	.0344	.0033	.0348	.0035
	.10	.1358	.0331	.1165	.0281	.0971	.0216	.0908	.0121	.0904	.0131
60%	.07	.1094	.0198	.0786	.0222	.0594	.0131	.0576	.0070	.0576	.0064
	.04	.1008	.0195	.0417	.0047	.0303	.0026	.0301	.0024	.0298	.0017

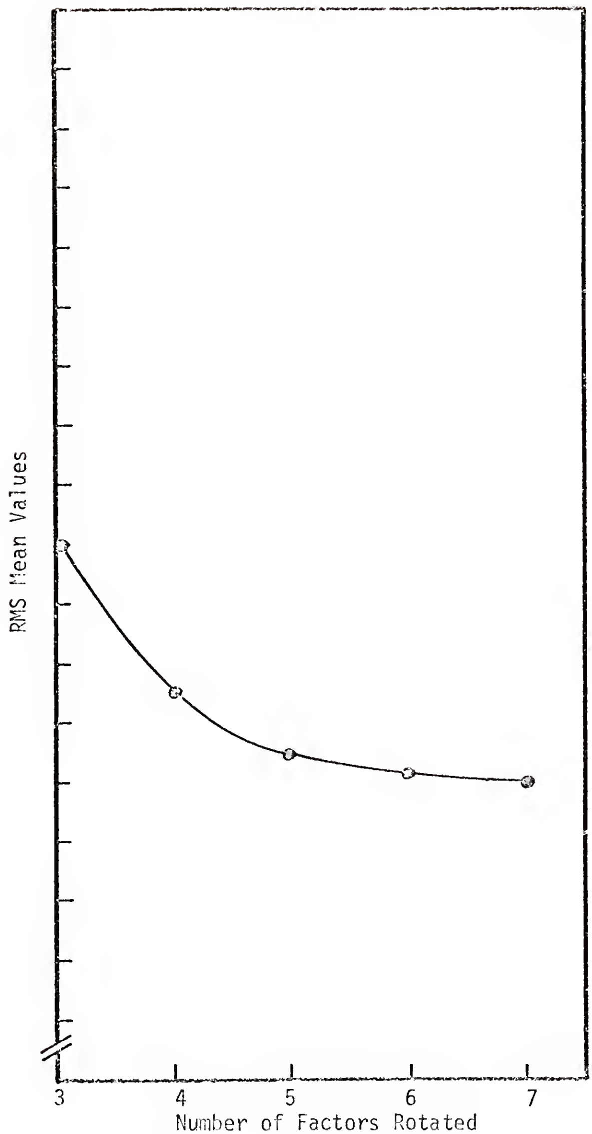

Figure 7. RMS means for the five different rotations for Problem Three.

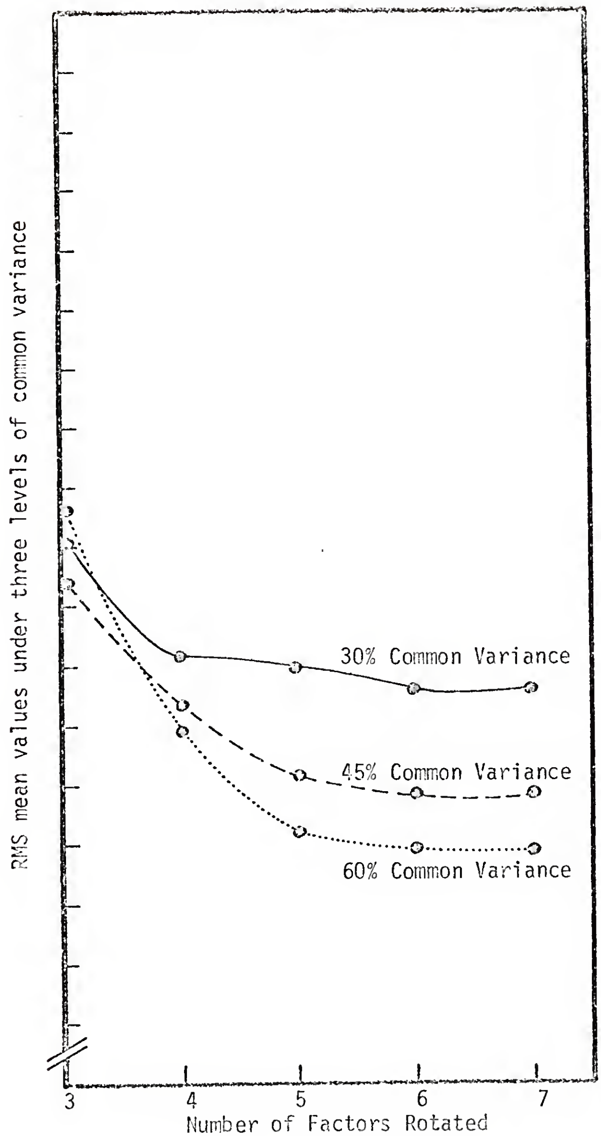

Figure 8. RMS means for the interaction of the five different rotations with the three levels of common variance for Problem Three.

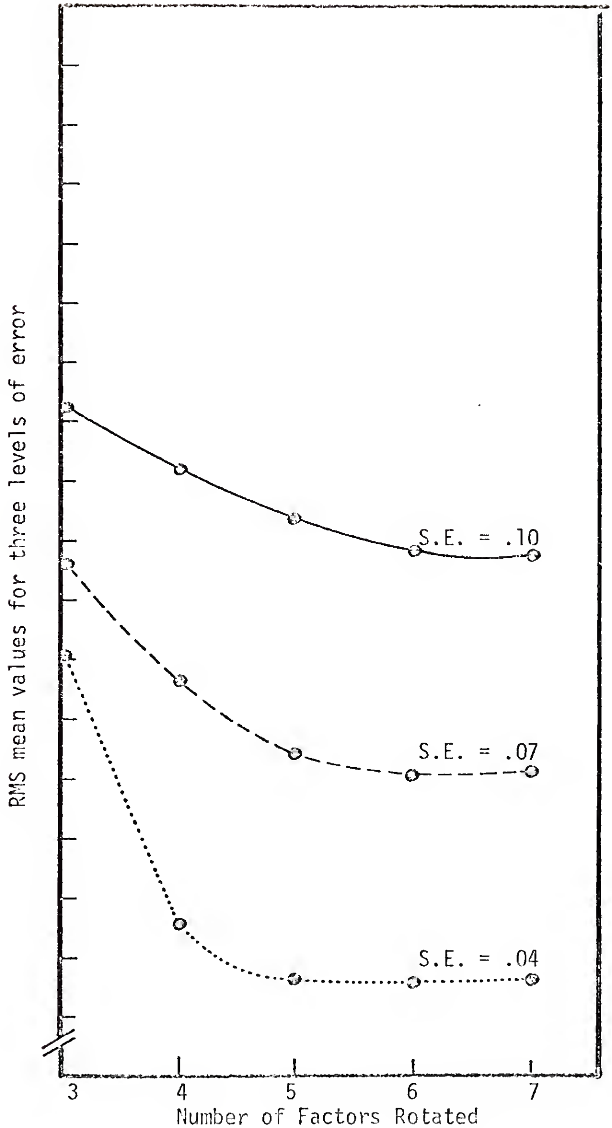

Figure 9. RMS means for the interaction of the five different rotations with the three levels of error for Problem Three.

TABLE 19

PROBLEM THREE

ANOVA Summary for RMS Mean Values for Five Different Rotations

Source of variation	SS	df	MS	F
Between rotations	.4227	89		
Common variance (A)	.0265	2	.0132	24.63*
Error (C)	.3469	2	.1734	322.57*
Common variance x error (AC)	.0057	4	.0014	2.67
Replications within common variance and error (S/AC)	.0436	81	.0005	
Within rotations	.1648	360		
Rotations (B)	.0948	4	.0237	190.90*
Common variance x rotations (AB)	.0132	8	.0017	13.34*
Error x rotations (CB)	.0145	8	.0018	14.65*
Common variance x rotations x error (BAC)	.0020	16	.0001	1.024
Rotations x replications within common variance and error (BS/AC)	.0402	324	.0001	
Total	.5874	449		

*$p < .05$.

TABLE 20

PROBLEM THREE

ANOVA Summary Tables for Linear, Quadratic and Cubic Trends

Source of variance	SS	df	MS	F
Analysis of Linear Trend				
Within rotations linear	.1060	90		
Rotations (B) linear	.0733	1	.0733	366.50*
Rotations x common variance (BA)	.0096	2	.0048	24.00*
Rotations x error (BC)	.0050	2	.0025	12.50*
Rotations x common variance x error (BAC)	.0007	4	$.0018 \times 10^{-1}$	<1.00
Rotations x replications within common variance x error (BS/AC)	.0174	81	.0002	
Analysis of Quadratic Trend				
Within rotations quadratic	.0418	90		
Rotations (B) quadratic	.0203	1	.0203	145.00*
Rotations x common variance (BA)	.0031	2	.0016	11.43*
Rotations x error (BC)	.0069	2	.0035	25.00*
Rotations x common variance x error (BAC)	.0002	4	$.0005 \times 10^{-1}$	<1.00
Rotations x replications within common variance x error (BS/AC)	.0113	81	$.0014 \times 10^{-1}$	
Analysis of Cubic Trends				
Within rotations cubic	.0113	90		
Rotations (B) cubic	.0012	1	.0012	14.99*
Rotations x common variance (BA)	.0002	2	.0001	1.25
Rotations x error (BC)	.0024	2	.0012	15.13*
Rotations x common variance x error (BAC)	.0008	4	.0002	2.38
Rotations x replications within common variance x error (BS/AC)	.0067	81	$.0008 \times 10^{-1}$	

*$p < .05$.

TABLE 21

PROBLEM THREE

Trend Components, Observed and Predicted RMS Means for Five Different Rotations (B)

Trend Components	
Linear	$-.90 \times 10^{-2}$
Quadratic	$.40 \times 10^{-2}$
Cubic	$-.12 \times 10^{-2}$

Number of Factors Rotated	Predicted B's			Observed B's
	Linear	Quadratic	Cubic	
3 (B_1)	.1006	.1086	.1098	.1098
4 (B_2)	.0916	.0876	.0853	.0850
5 (B_3)	.0826	.0745	.0745	.0749
6 (B_4)	.0735	.0695	.0718	.0716
7 (B_5)	.0645	.0725	.0714	.0714

Problem Four

The means and standard deviations of the RMS mean values for each ten replications under the specified three levels of common variance and error and the seven different rotations tried are presented in Table 22. In Figure 9 is the curve of the RMS means for the seven different rotations. The profiles of the interactions of the three levels of common variance with rotations are seen in Figure 10. In Figure 11 are depicted the three interaction terms of error with rotations.

It can be seen from the ANOVA summary in Table 19 that, at the .05 level, rotations B, interaction terms AB, and CB are significant. The null hypothesis is, therefore, rejected and the alternative one is not rejected.

The three trend analyses performed to test the trend in the overall RMS mean values under the seven rotations are shown in Table 24. At the .05 level, the tests for trends are found significant. A polynomial cubic equation of the form $Y = \beta_0 + \beta_1 X_1 + \beta_2 X_1^2 + \beta_3 X_1^3 + \varepsilon$ is accepted as the best representative of the curvilinear trend of the RMS mean values under the seven rotations.

In Table 25 are shown the three trend components and the observed and predicted RMS mean values B. The best approximation of the observed B's is given by the cubic component. However, the significant cubic trend is not reflected in the plotted curve shown in Figure 10.

Summary

This study investigated factor stability as a function of the number of factors carried into rotation under specified levels of common

TABLE 22

PROBLEM FOUR

Means ($\overline{X}$) and Standard Deviations (S.D.) for Each Ten Replications Under Three Levels of Common Variance, Three Levels of Error and Seven Rotations

		Number of Factors Rotated													
		3		4		5		6		7		8		9	
Variance	Error	$\overline{X}$	S.D.	$\overline{X}$	S.D.	$\overline{X}$	S.D.	$\overline{X}$	S.D.	$\overline{X}$	S.D.	$\overline{X}$	S.D.	$\overline{X}$	S.D.
30%	.10	.1487	.0170	.1234	.0164	.1140	.0115	.1105	.0120	.1105	.0077	.1151	.0113	.1124	.0100
	.07	.1272	.0149	.0869	.0119	.0805	.0159	.0771	.0089	.0789	.0108	.0803	.0137	.0854	.0101
	.04	.1121	.0131	.0641	.0128	.0439	.0043	.0424	.0068	.0435	.0070	.0450	.0073	.0467	.0078
45%	.10	.1438	.0132	.1036	.0150	.0882	.0140	.0915	.0113	.0903	.0150	.0931	.0133	.0933	.0148
	.07	.1434	.0080	.0940	.0130	.0603	.0063	.0591	.0042	.0613	.0068	.0656	.0104	.0659	.0121
	.04	.1387	.0108	.0737	.0124	.0364	.0030	.0332	.0022	.0352	.0030	.0353	.0035	.0360	.0042
60%	.10	.1712	.0135	.1178	.0246	.0795	.0087	.0778	.0057	.0788	.0077	.0810	.0060	.0857	.0049
	.07	.1594	.0101	.0889	.0148	.0544	.0062	.0523	.0051	.0521	.0043	.0544	.0046	.0561	.0057
	.04	.1515	.0068	.0723	.0144	.0344	.0047	.0303	.0038	.0307	.0029	.0313	.0039	.0314	.0037

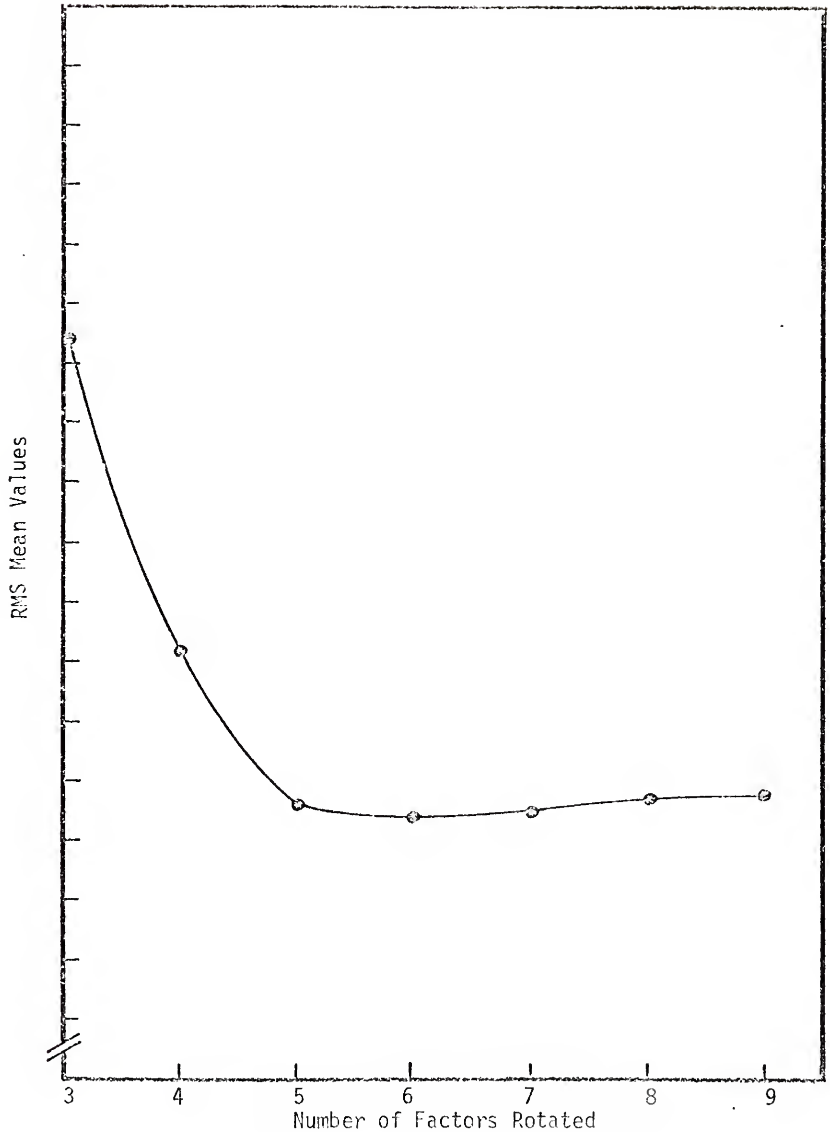

Figure 10. RMS means for the seven different rotations for Problem Four.

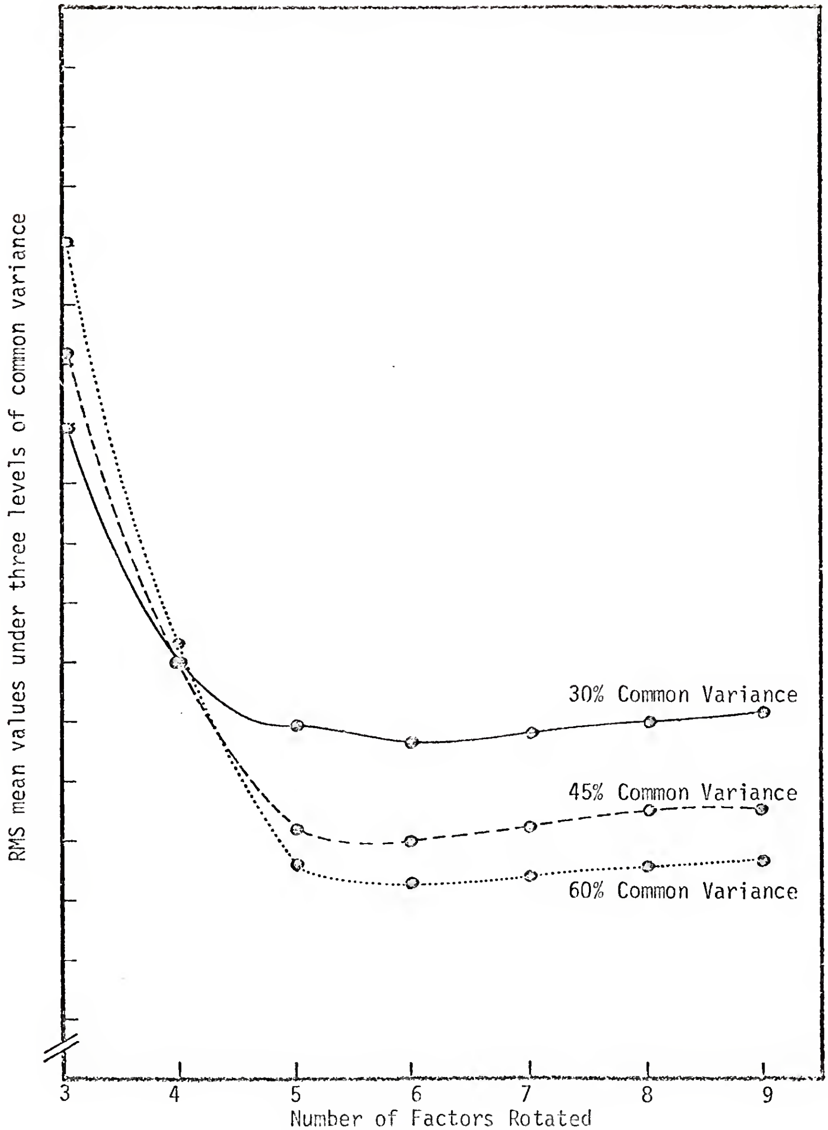

Figure 11. RMS means for the interaction of the seven different rotations with the three levels of common variance for Problem Four.

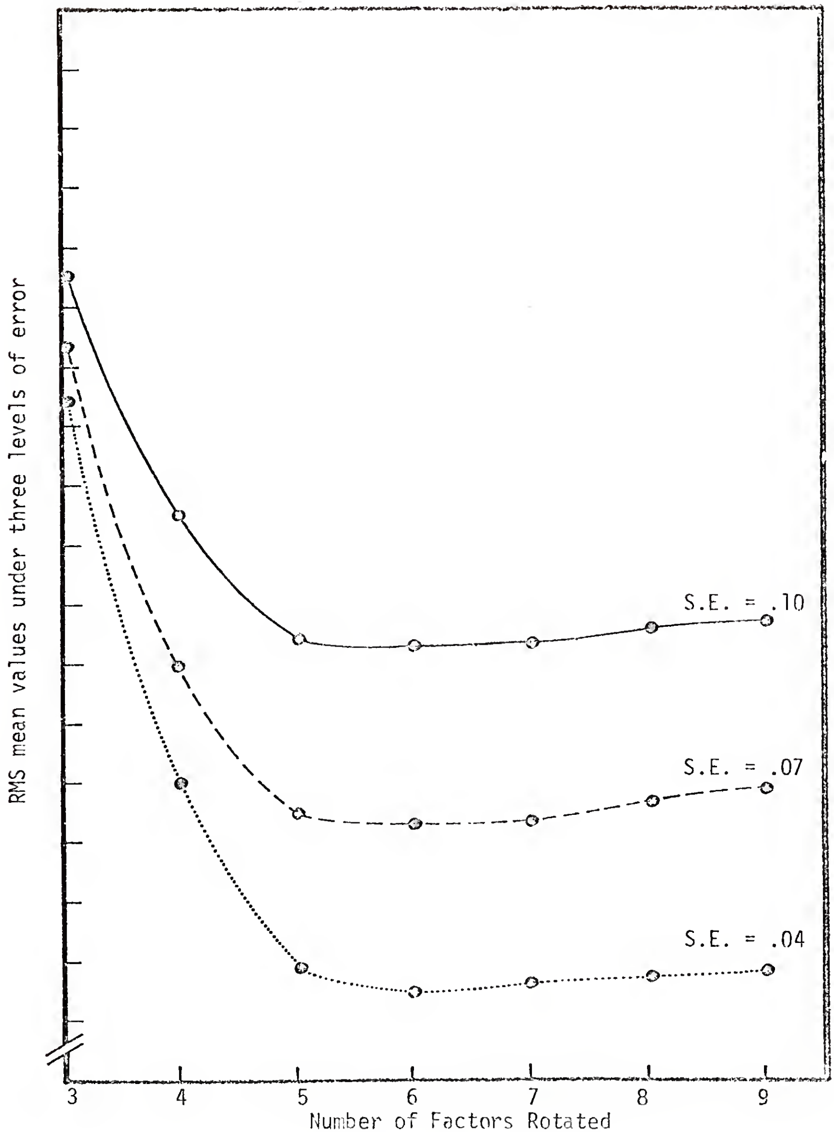

Figure 12. RMS means for the interaction of the seven different rotations with the three levels of error for Problem Four.

TABLE 23

PROBLEM FOUR

ANOVA Summary for RMS Mean Values for
Seven Different Rotations

Source of variation	SS	df	MS	F
Between rotations	.3232	89		
Common variance (A)	.0177	2	.0088	24.54*
Error (C)	.2686	2	.1343	372.53*
Common variance x error (AC)	.0077	4	.0019	5.32*
Replications within common variance and error (S/AC)	.0292	81	.0003	
Within rotations	.5674	540		
Rotations (B)	.4721	6	.0787	1183.16*
Common variance x rotations (AB)	.0415	12	.0035	51.94*
Error x rotations (CB)	.0183	12	.0015	22.87*
Common variance x rotations x error (ABC)	.0032	24	$.0013 \times 10^{-1}$	2.00
Rotations x replications within common variance and error (BS/AC)	.0323	486		
Total	.8906	629		

*p < .05.

TABLE 24

PROBLEM FOUR

ANOVA Summary Table for Linear, Quadratic, and Cubic Trends

Source of variation	SS	df	MS	F
Analysis of Linear Trend				
Within rotations linear	.2997	90		
Rotations (B) linear	.2493	1	.2493	1712.23*
Rotations x common variance (BA)	.0254	2	.0127	87.23*
Rotations x error (BC)	.0115	2	.0058	39.49*
Rotations x common variance x error (BAC)	.0017	4	.0004	2.92
Rotations x replications within common variance x error (BS/AC)	.0118	81	$.0015 \times 10^{-1}$	
Analysis of Quadratic Trends				
Within rotations quadratic	.2094	90		
Rotations (B) quadratic	.1838	1	.1838	2326.58*
Rotations x common variance (BA)	.0132	2	.0066	83.54*
Rotations x error (BC)	.0053	2	.0027	33.54*
Rotations x common variance x error (BAC)	.0007	4	.0002	2.22
Rotations x replications within common variance x error (BS/AC)	.0064	81	$.0008 \times 10^{-1}$	
Analysis of Cubic Trend				
Within rotations cubic	.0447	90		
Rotations (B) cubic	.0377	1	.0377	838.60*
Rotations x common variance (BA)	.0019	2	.0009	22.00*
Rotations x error (BC)	.0013	2	.0006	13.98*
Rotations x common variance x error (BAC)	$.0068 \times 10^{-2}$	4	$.0017 \times 10^{-2}$	<1.00
Rotations x replications within common variance x error (BS/AC)	.0037	81	$.0045 \times 10^{-2}$	

*$p < .05$.

TABLE 25

PROBLEM FOUR

Trend Components, Observed and Predicted RMS
Means for Seven Different Rotations (B)

Trend Components

Linear	$-.99 \times 10^{-2}$
Quadratic	$.49 \times 10^{-2}$
Cubic	$-.83 \times 10^{-2}$

Number of Factors Rotated	Predicted B's Linear	Predicted B's Quadratic	Predicted B's Cubic	Observed B's
3 (B_1)	.1104	.1349	.1432	.1440
4 (B_2)	.1005	.1005	.0921	.0916
5 (B_3)	.0906	.0759	.0675	.0657
6 (B_4)	.0807	.0611	.0611	.0638
7 (B_5)	.0708	.0561	.0644	.0646
8 (B_6)	.0609	.0609	.0692	.0668
9 (B_7)	.0510	.0755	.0671	.0681

variance and error. The RMS deviation measures obtained were used as a dependent variable in a series of statistical tests. The RMS mean values were plotted and the trends of the obtained curves were analyzed. The shape of these curves was considered to be an index of the stability of factor loadings when the number of factor rotations was varied. It was assumed that the lowest RMS values would be those associated with the correct number of rotated factors for a given matrix. The lowest point in a plotted curve should, therefore, graphically illustrate this assumption.

The findings of this study neither support nor reject this assumption. In this respect the results are inconclusive. In only Problems Two and Four are the lowest RMS mean values obtained associated with the correct number of factors.

The results indicate, however, that factor loading stability is a curvilinear function of the number of factors rotated. There is also a significant interaction between the amount of common variance for which the matrices account and the number of factors that are carried into rotation. The interaction between the error added to the matrices and the number of rotations is also found significant. For all four problems, a form of the cubic polynomial equation represents the best approximation of the plotted curves of the RMS means.

CHAPTER V

Discussion and Summary

This study investigated factor loading stability as a function of the number of common factors carried into rotation under specified levels of common variance and error. A survey of the literature revealed fairly consistent recommendations as to the correct number of factors that must be rotated to obtain interpretable common factors. It is best to over-rotate by one or two factors, but no more, otherwise factor fission may result. Underrotation was not recommended because it results in the compression of common variance into too few factors, thus distorting the factor structure.

The results of this investigation provided support for the literature's position on underrotation but were inconclusive in relation to the view on overrotation. Factor loading stability was found to be significantly affected by the number of factors that are rotated. Furthermore a statistically significant interaction was found between the number of factors that are rotated and the amount of common variance for which a matrix accounts. The interaction between the number of rotations and the amount of error added to a matrix was found significant also. The RMS mean deviation values, used as indicators of the effects of the number of rotation on factor loading stability, were found to be a curvilinear function of the number of factors rotated.

Discussion

Each of these findings warrants further discussion. This is divided into four main sections: (a) interpretation of the results in relation to the existing literature; (b) comparisons among the problem matrices; (c) examination of the effects of common variance and error on the number of factors rotated; (d) suggestion for future research.

A summary concludes this chapter.

The findings in relation to the literature. It should be noted that a major limitation to this study was that only the first few factors in the selected matrices were examined for the effects of under and over-rotation under the experimental conditions of common variance and error. Also, only four representative matrices were selected for examination from the large number of matrices available in the literature. Within the confines of this and the limitations stated previously in Chapter I, the results of this investigation seem to confirm some, but not all, of the positions in the literature regarding under and overrotation.

For all four problem matrices, the highest RMS values are those associated with underrotation. An examination of Figure 13 clearly shows this result. This finding is as it should be. When only a few of the total number of factors are rotated, the common variance is compressed and the factors are overdetermined. This results in underdimensioned distorted common-factor space. Because of this distorting effect on the factor loadings, the general position in the literature neither supports nor recommends underrotation. In this respect the study confirms this position.

As the number of factors rotated approaches the correct number of factors for each of the four problem matrices, the RMS values begin to

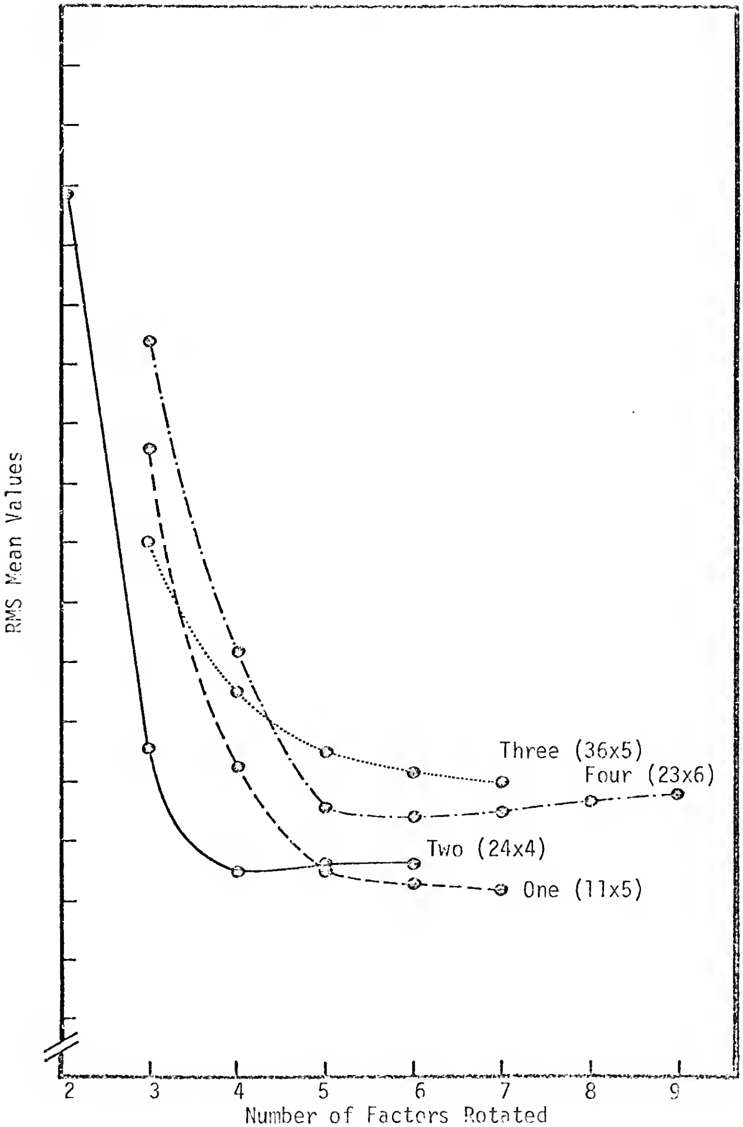

Figure 13. Comparison of the RMS mean values for the different rotations for the Four Problems.

decrease. In this study it has been assumed that the lowest RMS mean deviation values obtained would be those identified with the correct number of factors for each of the selected matrices. This assumption is confirmed in only two of the four problem matrices.

In Problems Two and Four the lowest RMS values obtained are the ones corresponding to the correct number of factors. This is not true, however, of Problems One and Three. In this case, the RMS values continue to decrease as the number of successive factor rotation increases. Because of these conflicting results each two similar matrices are now compared.

Comparison of the Problem Matrices. The results of Problems Two and Four tend to lend support to the literature's position on overrotation. It is obvious from Figure 13 that the lowest points in the plotted RMS mean value curves are those associated with the known number of factors for these two matrices. The upward inclination at the right terminus of the two curves, corresponding to a successive increase in the number of factors rotated, indicates an increase in the RMS deviation values. This increase is interpreted as the beginning of distortion effects on the loadings of the factors examined. It is probable that factor fission may result eventually with an increase in the number of successive overrotations.

Problems Two and Four have a comparable number of variables, 24 and 23 respectively. The original matrix of the former accounts for 47.50% of the common variance and it is a four-factor solution; the latter accounts for 72.74% of the common variance and is a six-factor matrix. However, this amount of common variance each matrix accounts for originally should have no bearing on the results because these matrices were adjusted prior to the analyses. The number of factor rotations tried for each of the two matrices differs drastically, as can be seen from Figure 13. Nevertheless the overall profile of the two matrices is very similar.

Problem Two, the 24x4 matrix, is of special interest since it is the only matrix where two to six successive factor rotations were tried. (The number of factors that were carried into rotation was determined by logistical considerations related to the rank of each matrix.) The plotted curves for Problem Two, found in Figures 4, 5, 6,and 13, show an interesting pattern. When only two factors are rotated, the RMS mean deviation value that is plotted begins at a much higher point on the Y axis than was the case for the other three problems. This is as expected: when all the common variance is compressed in only two factors the RMS deviation value is extremely high. It could be higher still if all the variance was compressed in one factor only.

The factors of Problem Four, the 23x6 matrix, were treated by three under and three overrotations. Yet the shape of the plotted RMS mean values curve for this problem is very similar to that for the 24x4 problem. The profiles of these two problems agree with expectations. One can speculate that as an increasing number of successive overrotations is performed, there would be a corresponding increase in the RMS deviation values obtained.

Just as there are notable resemblances between Problems Two and Four, there are striking similarities between the curves for Problems One and Three. An overall comparison of these matrices shows that they are both five-factor matrices; the number of trial rotations performed is the same for both; the two problems have the most shallow curves of all the four problems. This is seen in the RMS mean deviation curves in Figure 13.

For both these problems, the curves are shown to begin at lower levels than is the case for Problems Two and Four. The curves continue to descend with each successive overrotation, indicating a continued corresponding decrease in the RMS mean deviation values.

The pattern of the curves of the RMS values for these two problems, therefore, does not seem to support the findings in the literature regarding overrotation. Since it is assumed that the ideal factor number is that associated with the lowest RMS mean deviation values obtained, then Problems One and Three violate this assumption. It would be of interest if future studies were to examine matrices such as these when additional successive overrotation can be performed. (Because of the limited number of principal axes obtained in this study, further overrotation was not possible.)

An examination of the differences between the matrices of Problems One and Three, indicates that they differ greatly in terms of the number of variables upon which each is based. The original amount of common variance for which each accounts is also dissimilar. However, the amount of original common variance should have no effect on the findings because of the adjustment of the matrices prior to the analyses.

The inconsistencies in the results of the effects of overrotation on factor stability makes it difficult to reach any general conclusions on this aspect of the study. There does not seem to be any reasonable common denominator relating each two of the problem matrices having similar RMS profiles. The resemblances in profiles do not appear to be a function of either the number of variables or the number of factors of the matrices. Since both the amount of common variance and error are controlled in this study, the effect of these two variables on the results is discounted.

Although this study has strived for generalizability through its empirical approach to the analyses, it is probable that the results obtained are peculiar to the matrices selected and to the limitations imposed.

The effects of common variance and error on the number of factors rotated. There is a remarkable but not unexpected resemblance across the four problems in terms of the profiles of the curves representing the interactions of the number of factors rotated and the amount of common variance to which each matrix was adjusted. A consistent pattern is seen in Figures 2, 5, 8, and 11. The most extreme initial RMS discrepancies are those associated with the 60% common variance level and the fewest number of factors rotated. The curves are steepest and descend the lowest at the 60% level than they do for the other two levels. This is reasonable and consistent with the expected compression that occurs when a comparatively large amount of variance is confined to a smaller number of factors than is ideal. As a successively increasing number of factors is rotated, the variance redistributes itself accordingly across the factors.

The curves for the 45% and the 30% common variance levels begin at correspondingly lower points on the Y axis than was the case for the 60% curve, then level off as expected. The less the initial amount of common variance for which a matrix accounts, the more shallow the curve representing the RMS mean deviation values is expected to be. Only in Problem Three is there a reversal of the points of origin on the Y axis for the curves representing the 30% and 45% common variance. The reason for this reversal is not obvious. (It may be related to properties unique to this particular matrix.)

These results indicate the positive relationship that appears to exist between factor loading stability and the amount of common variance for which a matrix accounts. The larger this amount, the more stable the factor loadings tend to be, and the less subject are they to the vagaries of overrotation.

The implications of these findings for future research are obvious. Special attention needs to be given in factor analytic investigations to the proportion of common variance for which a given matrix accounts. To assure factor loading stability, this proportion must be large.

There is also a resemblance in the RMS mean deviation curves representing the interaction between the levels of the added error and the number of factors rotated. This is seen in Figures 3, 6, 9, and 12. Obviously, the larger the amount of the error added to a matrix, the higher are the RMS discrepancy values. This pattern is observable in all four problems. The highest RMS deviation values are those associated with the .10 added error, and the lowest are those for the .04 level. This finding simply reinforces the importance of the size of the sample of subjects in factor analytic research.

It is of interest to note that the RMS curves for the successive factor rotations is found to be best represented by third degree polynomial equations in all four matrices. Although the significant cubic components give better approximations of the observed RMS mean values throughout, there is no observable indication of this cubic trend in any of the plotted means. The cubic trend can be considered to have statistical but not practical significance.

In conclusion, within the stated limitations, the results of this study confirm the literature's position regarding underrotation of common factors. The evidence indicates that distortion occurs in the factor loadings when the common variance is compressed and common-factor space is underdimensioned.

The literature's view on overrotation is neither clearly supported nor obviously rejected. In two of the problems, the results confirm the position; in the other two instances they do not.

The more common variance for which a factor matrix accounts, the more stable its factor loadings appear to be. The reverse seems to be also true. The less error added to a matrix the more it is stable and vice versa. There does not appear to be a clear relationship between the number of variables and/or factors in a given matrix and its factor loading stability.

The shape of the RMS mean value deviations curves under successive rotations can be best represented by cubic polynomial equations. However, this trend is not demonstrated by the plotted curves. Although the study has attempted generalizability, it is acknowledged that the conclusions drawn must be considered only within the confines of the stated limitations.

A direction for future research. Several questions remain unanswered. The results regarding overrotation are inconclusive and warrant further investigation. This study examined only the first few factors in each matrix. Future work needs to examine the effects of rotation on a different number of factors in selected factor matrices. The RMS deviation or other appropriate measures for factor congruence can be used for this type of investigation (Harman, 1976).

The effects of alternative factor analytic techniques on the number of factors to rotate merit further examination. It would be of interest to investigate the stability of factor loadings, e.g., when image analysis is used as compared to the use of the principal axes method. The results of such a comparison should be of special interest to advocates of both the statistical as well as the psychometric approaches to factor analysis.

Other studies might investigate the effects of rotational procedures on factor loading stability when the number of rotated factors is varied. Since this study employed only orthogonal rotation, it would be desirable to examine matrices when oblique rotation is used.

Matrices considerably larger than the ones selected for this study need to be examined, with perhaps a greater number of replications. The effects of other levels of error and/or common variance on selected matrices need also to be investigated.

Summary

This study examined the effects of under and overrotation on common factor loading stability under three levels of common variance and three levels of error. Four representative factor matrices were selected. In each case, the factor matrix was adjusted to account for 30%, 45%, and 60% of common variance. Each of the adjusted matrices was postmultiplied by its transpose and the intercorrelation matrix obtained was factor analyzed to obtain the criterion matrix.

Randomly-generated error based on 100, 200, and 500 subjects was added to the intercorrelation matrix and this was refactored. Ten replications were completed for each experimental condition. Several rotations were tried below, including, and above the correct number of factors for each problem matrix chosen. Root-mean-square (RMS) mean values were obtained between the first few factors of each criterion matrix and the corresponding factors from the successive rotations.

The RMS mean deviation values were plotted and a multifactor repeated measures ANOVA performed. The number of factors rotated, the interaction of rotations with common variance, and the interaction of rotations with error were found significant at the .05 level in all four problems.

For the four problems linear, quadratic, and cubic trend analyses were completed yielding significant trend components at the .05 level of significance. Polynomial cubic equations represented the best approximation of the plotted curves of the RMS mean values.

The results support the literature's position on underrotation but there was no clear-cut support for the view on overrotation. Matrices with the largest amounts of common variance adjustment and the least-added error tended to be more stable than ones with less variance and more error. There did not seem to be a clear relationship between factor loading stability and the number of variables and/or factors in a given matrix.

Although these findings must be considered within the stated limitations imposed upon this study, it appears, nevertheless, that matrices which account for large amounts of common variance are less susceptible to the vagaries of overrotation. Hence, these matrices tend to have stable factor loadings. Furthermore, common-factor space is clearly distorted in the case of underrotation.

APPENDIX A

TABLE A1

Means of Random Error Under Three Levels of Standard Error and Three Levels of Common Variance (Expected Mean Value = 0.0)

Standard deviation requested	Common Variance			Common Variance		
	30%	45%	60%	30%	45%	60%
	Problem One N = 550			Problem Two N = 2760		
.10	.00276	.00227	.00066	-.00043	-.00068	-.00053
.07	-.00019	.00006	.00146	.00255	.00269	.00263
.04	.00085	.00135	.00101	.00101	.00094	.00105
	Problem Three N = 6300			Problem Four N = 2530		
.10	.00139	.00014	.00151	-.00051	-.00044	-.00029[a]
.07	.00055	.00068	.00046	.00231	.00252	.00259
.04	-.00075	-.00075	-.00069	.00146	.00132	.00139

[a]The means of the generated means at the three error levels are: .10 = .0005; .07 = .0015; .04 = .0007.

TABLE A2

Standard Deviations for Three Levels of Random Error Under Three Levels of Common Variance

Standard deviation requested	Common Variance			Common Variance		
	30%	45%	60%	30%	45%	60%
	Problem One N = 550			Problem Two N = 2760		
.10	.09817	.09867	.10002	.10021	.10039	.10032
.07	.07238	.07218	.07169	.06933	.06929	.06930
.04	.04086	.04131	.04121	.04017	.04016	.04017
	Problem Three N = 6300			Problem Four N = 2530		
.10	.09958	.09954	.09969	.10004	.10002	.09989[a]
.07	.07068	.07076	.07071	.06901	.06909	.06917
.04	.03934	.03934	.03934	.04053	.04049	.04046

[a]The means of the standard deviations requested for the four problems under three levels of error are: .10 = .09971; .07 = .07029; .04 = .04028.

APPENDIX B

TABLE B1

PROBLEM TWO

The Matrix Adjusted for Three Levels of Common Variance

Variance	Variable	Factor I	II	III	IV
	1	.11	.15	.53	.14
	2	.08	.06	.34	.08
	3	.12	.02	.43	.06
	4	.16	.07	.43	.06
	5	.60	.17	.17	.10
	6	.60	.08	.18	.17
	7	.65	.13	.17	.06
	8	.43	.21	.30	.10
	9	.64	.01	.17	.20
	10	.12	.56	.05	.19
	11	.14	.48	.06	.29
30%	12	.02	.55	.18	.09
	13	.14	.47	.33	.05
	14	.17	.13	.03	.40
	15	.10	.06	.11	.40
	16	.06	.08	.33	.34
	17	.11	.14	.05	.51
	18	.00	.21	.25	.43
	19	.10	.12	.19	.31
	20	.28	.09	.37	.20
	21	.12	.30	.33	.21
	22	.29	.03	.33	.29
	23	.28	.17	.45	.17
	24	.27	.35	.17	.27

TABLE B1 - Continued

Variance	Variable	Factor I	II	III	IV
	1	.14	.19	.65	.17
	2	.10	.07	.42	.10
	3	.15	.02	.53	.08
	4	.19	.09	.53	.07
	5	.73	.20	.21	.13
	6	.73	.10	.22	.20
	7	.80	.16	.20	.08
	8	.53	.25	.37	.12
	9	.78	.01	.21	.24
	10	.15	.68	.06	.23
	11	.17	.58	.08	.35
45%	12	.02	.67	.22	.11
	13	.18	.57	.40	.06
	14	.21	.16	.04	.49
	15	.12	.07	.14	.49
	16	.08	.10	.40	.42
	17	.14	.18	.06	.62
	18	.00	.25	.31	.53
	19	.13	.15	.23	.38
	20	.34	.11	.46	.24
	21	.15	.37	.41	.25
	22	.35	.04	.40	.35
	23	.34	.20	.56	.21
	24	.33	.43	.21	.33

TABLE B1 - Continued

Variance	Variable	Factor I	II	III	IV
	1	.16	.21	.75	.19
	2	.11	.08	.48	.11
	3	.17	.02	.61	.09
	4	.22	.10	.61	.08
	5	.84	.24	.25	.15
	6	.84	.11	.26	.24
	7	.92	.18	.24	.09
	8	.61	.29	.43	.13
	9	.90	.01	.25	.28
	10	.17	.79	.07	.27
	11	.19	.67	.09	.40
60%	12	.02	.78	.26	.12
	13	.20	.66	.46	.07
	14	.25	.18	.05	.56
	15	.13	.08	.16	.56
	16	.09	.11	.46	.48
	17	.16	.20	.07	.72
	18	.00	.29	.36	.61
	19	.15	.17	.27	.44
	20	.39	.12	.53	.28
	21	.17	.43	.47	.29
	22	.40	.05	.46	.40
	23	.39	.24	.64	.25
	24	.38	.49	.25	.38

TABLE B2

PROBLEM TWO

The Criterion Matrices with Three Levels of Common Variance

Variance	Variable	Factor I	II	III	IV
	1	.11	.17	.53	.13
	2	.08	.07	.34	.08
	3	.12	.03	.43	.06
	4	.16	.09	.43	.05
	5	.60	.17	.17	.11
	6	.60	.08	.18	.17
	7	.65	.13	.16	.07
	8	.43	.22	.29	.10
	9	.64	.01	.17	.20
	10	.12	.56	.03	.19
	11	.14	.48	.05	.29
30%	12	.02	.55	.16	.09
	13	.15	.48	.31	.05
	14	.17	.13	.03	.40
	15	.09	.06	.11	.40
	16	.06	.09	.32	.34
	17	.11	.14	.05	.51
	18	.00	.21	.25	.43
	19	.10	.12	.19	.31
	20	.28	.10	.37	.20
	21	.12	.31	.32	.21
	22	.29	.04	.33	.29
	23	.28	.18	.45	.17
	24	.27	.35	.16	.27

TABLE B2 - Continued

Variance	Variable	Factor I	II	III	IV
	1	.14	.21	.65	.16
	2	.10	.08	.42	.10
	3	.15	.04	.52	.08
	4	.20	.11	.52	.07
	5	.73	.21	.21	.13
	6	.63	.10	.22	.21
	7	.80	.16	.20	.08
	8	.53	.26	.36	.12
	9	.78	.01	.21	.25
	10	.15	.68	.03	.24
	11	.17	.58	.06	.35
45%	12	.02	.68	.20	.11
	13	.18	.59	.38	.06
	14	.21	.15	.04	.49
	15	.11	.07	.14	.49
	16	.08	.11	.40	.42
	17	.13	.17	.06	.62
	18	.00	.26	.30	.53
	19	.13	.15	.23	.38
	20	.34	.12	.45	.24
	21	.15	.38	.40	.25
	22	.35	.05	.40	.35
	23	.34	.22	.55	.21
	24	.33	.43	.20	.33

TABLE B2 - Continued

Variance	Variable	Factor I	II	III	IV
	1	.16	.24	.75	.19
	2	.11	.10	.48	.11
	3	.17	.04	.61	.09
	4	.23	.12	.60	.08
	5	.84	.24	.24	.15
	6	.84	.12	.25	.24
	7	.92	.18	.23	.09
	8	.61	.30	.42	.14
	9	.90	.01	.25	.28
	10	.17	.79	.04	.27
	11	.19	.67	.07	.41
60%	12	.03	.78	.23	.13
	13	.21	.68	.44	.07
	14	.25	.18	.04	.56
	15	.13	.08	.16	.56
	16	.09	.13	.46	.48
	17	.15	.20	.06	.72
	18	.00	.30	.35	.61
	19	.15	.18	.27	.44
	20	.39	.14	.52	.28
	21	.17	.44	.46	.29
	22	.40	.06	.46	.40
	23	.40	.26	.63	.25
	24	.38	.50	.23	.39

TABLE B3

PROBLEM THREE

The Matrix Adjusted to Account for Three Levels of Common Variance

Variance	Variable	Factor I	II	III	IV	V
	1	.10	.15	.16	.01	.30
	2	.51	.21	.11	.02	.17
	3	.55	.16	.06	.07	.12
	4	.54	.17	.07	.08	.15
	5	.54	.18	.06	.09	.12
	6	.53	.17	.08	.10	.15
	7	.54	.15	.06	.11	.12
	8	.51	.20	.07	.10	.15
	9	.52	.17	.09	.04	.16
	10	.50	.16	.10	.12	.15
	11	.50	.19	.07	.16	.14
	12	.52	.17	.09	.12	.14
	13	.15	.07	.50	.06	.20
	14	.09	.04	.51	.08	.22
	15	.07	.11	.50	.09	.18
	16	.06	.06	.47	.05	.23
	17	.15	.03	.17	.02	.48
30%	18	.18	.01	.15	.07	.43
	19	.17	.08	.13	.12	.36
	20	.25	.06	.11	.11	.42
	21	.21	.05	.07	.11	.44
	22	.28	.47	.11	.09	.13
	23	.29	.47	.09	.07	.08
	24	.22	.47	.10	.06	.15
	25	.26	.47	.09	.04	.07
	26	.29	.48	.05	.01	.10
	27	.39	.37	.02	.14	.05
	28	.37	.39	.01	.15	.07
	29	.31	.08	.18	.36	.26
	30	.29	.12	.14	.37	.26
	31	.33	.16	.13	.40	.17
	32	.34	.12	.13	.38	.18
	33	.16	.11	.06	.17	.37
	34	.17	.07	.22	.06	.39
	35	.10	.22	.15	.11	.42
	36	.10	.07	.19	.01	.35

TABLE B3 - Continued

Variance	Variable	Factor I	II	III	IV	V
	1	.12	.19	.20	.02	.37
	2	.62	.26	.14	.03	.21
	3	.67	.20	.08	.08	.14
	4	.66	.20	.09	.10	.19
	5	.66	.22	.08	.11	.15
	6	.65	.21	.10	.12	.18
	7	.66	.19	.08	.14	.15
	8	.62	.24	.09	.12	.19
	9	.64	.20	.11	.05	.20
	10	.62	.20	.13	.14	.19
	11	.61	.23	.08	.20	.17
	12	.63	.20	.11	.14	.17
	13	.18	.09	.61	.08	.25
	14	.11	.05	.62	.10	.27
	15	.08	.14	.61	.11	.23
	16	.08	.08	.58	.06	.29
	17	.19	.04	.21	.03	.59
45%	18	.23	.01	.19	.08	.53
	19	.21	.10	.17	.15	.44
	20	.30	.08	.14	.14	.52
	21	.26	.06	.08	.14	.53
	22	.35	.58	.14	.11	.16
	23	.35	.58	.11	.09	.10
	24	.27	.57	.13	.07	.18
	25	.32	.57	.11	.05	.08
	26	.36	.59	.06	.01	.12
	27	.47	.45	.02	.17	.06
	28	.46	.47	.01	.18	.09
	29	.38	.10	.23	.44	.32
	30	.35	.15	.17	.45	.32
	31	.41	.20	.16	.49	.21
	32	.41	.14	.16	.47	.23
	33	.20	.14	.08	.20	.45
	34	.21	.09	.27	.07	.48
	35	.12	.27	.19	.14	.51
	36	.13	.09	.23	.02	.43

TABLE B3 - Continued

Variance	Variable	Factor I	II	III	IV	V
	1	.14	.22	.23	.02	.43
	2	.72	.30	.16	.03	.24
	3	.77	.23	.09	.10	.16
	4	.76	.23	.10	.11	.22
	5	.76	.25	.09	.12	.17
	6	.75	.24	.11	.14	.21
	7	.76	.22	.09	.16	.17
	8	.72	.28	.10	.14	.22
	9	.74	.23	.12	.05	.23
	10	.71	.23	.15	.16	.22
	11	.70	.27	.10	.23	.20
	12	.73	.23	.12	.16	.20
	13	.21	.10	.70	.09	.29
	14	.13	.06	.72	.11	.31
	15	.10	.16	.70	.12	.26
	16	.09	.09	.67	.07	.33
	17	.22	.04	.24	.03	.68
60%	18	.26	.01	.22	.10	.61
	19	.24	.11	.19	.17	.50
	20	.35	.09	.16	.16	.60
	21	.29	.07	.10	.16	.62
	22	.40	.67	.16	.13	.18
	23	.41	.67	.13	.10	.11
	24	.31	.66	.15	.08	.21
	25	.37	.66	.12	.05	.10
	26	.42	.68	.07	.01	.14
	27	.55	.52	.03	.20	.07
	28	.53	.55	.01	.21	.10
	29	.43	.11	.26	.51	.36
	30	.41	.17	.20	.52	.36
	31	.47	.23	.18	.56	.24
	32	.48	.16	.18	.54	.26
	33	.23	.16	.09	.23	.52
	34	.24	.10	.31	.08	.56
	35	.14	.31	.22	.16	.59
	36	.15	.10	.27	.02	.49

TABLE B4

PROBLEM THREE

The Criterion Matrices with Three Levels of Common Variance

Variance	Variable	Factor I	II	III	IV	V
	1	.08	.16	.16	.01	.31
	2	.50	.23	.11	.03	.19
	3	.54	.17	.06	.07	.14
	4	.53	.18	.07	.08	.18
	5	.53	.19	.06	.09	.15
	6	.52	.18	.08	.10	.17
	7	.53	.17	.06	.11	.15
	8	.50	.21	.07	.10	.18
	9	.51	.18	.09	.04	.18
	10	.49	.17	.10	.12	.18
	11	.48	.20	.07	.16	.16
	12	.50	.18	.09	.12	.16
	13	.13	.08	.49	.06	.22
	14	.08	.05	.51	.08	.23
	15	.05	.11	.49	.08	.19
	16	.05	.06	.47	.05	.24
	17	.13	.04	.17	.02	.49
30%	18	.16	.01	.15	.06	.44
	19	.15	.09	.13	.12	.37
	20	.22	.07	.11	.11	.44
	21	.19	.06	.06	.10	.45
	22	.26	.48	.11	.09	.14
	23	.27	.48	.09	.08	.09
	24	.20	.47	.10	.06	.16
	25	.25	.47	.09	.04	.08
	26	.28	.49	.05	.01	.11
	27	.37	.38	.02	.14	.06
	28	.36	.40	.01	.15	.09
	29	.29	.09	.18	.36	.28
	30	.27	.13	.14	.37	.28
	31	.32	.17	.13	.40	.19
	32	.32	.12	.13	.38	.21
	33	.14	.12	.06	.16	.38
	34	.15	.08	.22	.05	.40
	35	.07	.23	.15	.10	.42
	36	.09	.08	.19	.01	.36

TABLE B4 - Continued

Variance	Variable	Factor I	II	III	IV	V
	1	.10	.19	.19	.01	.37
	2	.61	.28	.13	.03	.24
	3	.66	.21	.08	.09	.17
	4	.65	.22	.09	.10	.22
	5	.65	.23	.08	.11	.18
	6	.63	.23	.10	.12	.21
	7	.65	.20	.08	.14	.18
	8	.61	.26	.09	.12	.22
	9	.62	.22	.10	.05	.22
	10	.60	.21	.13	.14	.22
	11	.59	.25	.08	.20	.20
	12	.62	.22	.10	.15	.20
	13	.16	.10	.61	.07	.26
	14	.10	.06	.62	.09	.28
	15	.07	.14	.61	.10	.24
	16	.06	.08	.57	.06	.30
	17	.16	.05	.20	.02	.60
45%	18	.20	.02	.18	.08	.54
	19	.19	.11	.16	.14	.45
	20	.28	.09	.13	.13	.53
	21	.23	.07	.08	.13	.55
	22	.32	.59	.13	.11	.17
	23	.33	.59	.11	.09	.11
	24	.25	.58	.13	.07	.19
	25	.31	.58	.11	.05	.09
	26	.34	.60	.06	.01	.13
	27	.46	.46	.02	.18	.08
	28	.44	.48	.01	.18	.11
	29	.36	.11	.22	.44	.34
	30	.33	.16	.17	.45	.34
	31	.39	.21	.16	.49	.24
	32	.40	.15	.16	.47	.25
	33	.17	.14	.07	.20	.46
	34	.19	.10	.26	.06	.49
	35	.09	.28	.18	.13	.52
	36	.11	.10	.23	.01	.44

TABLE B4 - Continued

Variance	Variable	Factor I	II	III	IV	V
	1	.11	.22	.22	.01	.43
	2	.70	.32	.16	.04	.27
	3	.76	.24	.09	.10	.20
	4	.75	.25	.10	.12	.25
	5	.75	.27	.09	.13	.21
	6	.73	.26	.11	.14	.24
	7	.75	.24	.09	.16	.21
	8	.70	.30	.10	.14	.25
	9	.72	.25	.12	.05	.26
	10	.69	.24	.15	.17	.25
	11	.68	.29	.09	.23	.23
	12	.71	.25	.12	.17	.23
	13	.19	.11	.70	.08	.31
	14	.11	.07	.72	.11	.33
	15	.08	.16	.70	.12	.27
	16	.07	.09	.66	.07	.34
	17	.19	.05	.23	.03	.69
60%	18	.23	.02	.21	.09	.62
	19	.22	.12	.18	.17	.52
	20	.32	.10	.15	.15	.62
	21	.27	.08	.09	.15	.63
	22	.37	.68	.15	.13	.20
	23	.39	.68	.13	.11	.13
	24	.29	.67	.15	.08	.22
	25	.35	.67	.12	.05	.11
	26	.39	.69	.07	.01	.15
	27	.53	.53	.03	.20	.09
	28	.51	.56	.01	.21	.13
	29	.41	.12	.26	.51	.39
	30	.38	.18	.19	.52	.39
	31	.45	.24	.18	.56	.27
	32	.46	.18	.18	.54	.29
	33	.20	.16	.08	.23	.53
	34	.22	.11	.31	.07	.57
	35	.10	.32	.21	.15	.60
	36	.12	.11	.26	.01	.50

TABLE B5

PROBLEM FOUR

The Matrix Adjusted to Account for
Three Levels of Common Variance

Variance	Variable	Factor I	II	III	IV	V	VI
	1	.45	.00	.12	.13	.20	.04
	2	.02	.11	.06	.03	.54	.16
	3	.25	.18	.03	.30	.28	.00
	4	.03	.43	.22	.07	.04	.07
	5	.08	.05	.01	.47	.01	.05
	6	.09	.03	.05	.36	.14	.15
	7	.34	.04	.28	.28	.12	.03
	8	.27	.18	.24	.04	.41	.15
	9	.03	.02	.07	.04	.15	.53
	10	.15	.21	.13	.08	.50	.12
	11	.02	.15	.60	.00	.09	.04
30%	12	.11	.05	.50	.11	.00	.20
	13	.25	.24	.05	.41	.05	.13
	14	.48	.10	.01	.03	.03	.26
	15	.00	.00	.35	.04	.08	.09
	16	.06	.16	.22	.20	.03	.51
	17	.58	.05	.01	.02	.08	.08
	18	.32	.06	.02	.04	.01	.46
	19	.04	.37	.43	.12	.01	.12
	20	.06	.05	.16	.49	.08	.24
	21	.05	.27	.16	.40	.22	.11
	22	.32	.08	.05	.00	.27	.18
	23	.21	.22	.03	.07	.42	.24

TABLE B5 - Continued

Variance	Variable	Factor I	II	III	IV	V	VI
	1	.55	.00	.15	.17	.24	.04
	2	.02	.14	.08	.03	.66	.19
	3	.31	.22	.04	.36	.34	.00
	4	.04	.53	.27	.08	.05	.08
	5	.09	.07	.01	.57	.02	.06
	6	.11	.04	.06	.44	.17	.18
	7	.42	.05	.34	.34	.15	.04
	8	.33	.22	.29	.04	.50	.18
	9	.04	.03	.09	.05	.18	.64
	10	.18	.25	.16	.10	.61	.15
	11	.03	.18	.74	.01	.11	.04
45%	12	.14	.06	.62	.13	.00	.25
	13	.31	.30	.06	.50	.07	.15
	14	.58	.12	.01	.03	.03	.32
	15	.01	.00	.43	.05	.10	.11
	16	.08	.20	.27	.25	.03	.62
	17	.71	.06	.01	.03	.10	.09
	18	.39	.08	.02	.05	.01	.56
	19	.05	.45	.53	.15	.02	.15
	20	.07	.06	.20	.60	.10	.29
	21	.06	.33	.20	.49	.26	.14
	22	.40	.09	.07	.00	.33	.22
	23	.26	.26	.04	.08	.51	.29

TABLE B5 - Continued

Variance	Variable	Factor I	II	III	IV	V	VI
	1	.63	.00	.17	.19	.28	.05
	2	.03	.16	.09	.04	.76	.22
	3	.36	.25	.05	.42	.40	.00
	4	.04	.61	.31	.09	.06	.09
	5	.11	.08	.01	.66	.02	.07
	6	.12	.04	.07	.51	.20	.21
	7	.48	.05	.40	.39	.17	.04
	8	.38	.25	.34	.05	.58	.21
	9	.05	.03	.10	.05	.21	.74
	10	.21	.29	.18	.12	.71	.17
	11	.03	.21	.85	.01	.13	.05
60%	12	.16	.06	.71	.15	.00	.28
	13	.36	.34	.07	.58	.08	.18
	14	.67	.14	.01	.04	.04	.37
	15	.01	.01	.50	.06	.11	.12
	16	.09	.23	.31	.29	.04	.71
	17	.82	.07	.01	.03	.11	.11
	18	.45	.09	.03	.05	.01	.65
	19	.06	.52	.61	.17	.02	.18
	20	.08	.07	.23	.70	.12	.34
	21	.07	.38	.23	.56	.30	.16
	22	.46	.11	.08	.00	.39	.25
	23	.30	.30	.05	.10	.59	.33

TABLE B6

PROBLEM FOUR

The Criterion Matrices with Three Levels of Common Variance

Variance	Variable	Factor I	II	III	IV	V	VI
	1	.44	-.04	.12	.17	.19	-.01
	2	.02	.00	.07	.07	.56	.10
	3	.23	.10	.03	.33	.29	-.05
	4	.03	.42	.20	.10	.13	.05
	5	.05	.03	.01	.48	-.01	.02
	6	.07	-.02	.06	.38	.13	.11
	7	.33	.01	.28	.30	.11	-.02
	8	.27	.10	.24	.09	.44	.08
	9	.05	-.02	.10	.08	.20	.50
	10	.14	.10	.13	.14	.53	.05
	11	.03	.17	.60	.02	.12	.00
30%	12	.12	.07	.51	.12	.02	.17
	13	.23	.20	.04	.45	.08	.09
	14	.48	.08	.01	.08	.07	.24
	15	.01	.01	.36	.05	.08	.06
	16	.07	.15	.23	.25	.09	.47
	17	.58	.02	.01	.07	.10	.05
	18	.33	.05	.03	.09	.06	.44
	19	.05	.38	.42	.15	.09	.09
	20	.04	.01	.17	.52	.07	.19
	21	.04	.20	.16	.43	.24	.06
	22	.33	.02	.06	.05	.30	.13
	23	.21	.12	.04	.13	.47	.18

TABLE B6 - Continued

Variance	Variable	Factor I	II	III	IV	V	VI
	1	.54	-.05	.15	.21	.23	-.02
	2	.02	.00	.09	.09	.69	.12
	3	.29	.13	.03	.41	.35	-.07
	4	.04	.52	.24	.12	.15	.06
	5	.06	.03	.01	.58	-.01	.03
	6	.09	-.02	.07	.47	.16	.14
	7	.40	.02	.34	.37	.13	-.03
	8	.33	.12	.29	.11	.54	.10
	9	.06	-.02	.12	.10	.24	.61
	10	.17	.12	.16	.17	.65	.07
	11	.04	.20	.73	.02	.15	.00
45%	12	.14	.08	.62	.15	.03	.20
	13	.29	.25	.05	.55	.10	.10
	14	.59	.10	.02	.09	.08	.29
	15	.01	.01	.44	.06	.10	.07
	16	.09	.18	.29	.30	.11	.58
	17	.71	.03	.01	.08	.12	.06
	18	.40	.06	.04	.11	.07	.54
	19	.06	.46	.51	.19	.11	.11
	20	.05	.01	.21	.63	.09	.24
	21	.04	.25	.19	.53	.30	.07
	22	.40	.02	.07	.06	.36	.16
	23	.26	.14	.04	.16	.57	.22

TABLE B6 - Continued

Variance	Variable	Factor I	II	III	IV	V	VI
	1	.62	-.06	.18	.24	.27	-.02
	2	.02	.00	.10	.10	.79	.14
	3	.33	.15	.04	.47	.41	-.08
	4	.05	.60	.28	.14	.18	.06
	5	.07	.04	.01	.67	-.01	.03
	6	.10	-.02	.08	.54	.18	.16
	7	.46	.02	.39	.43	.15	-.03
	8	.38	.14	.34	.13	.63	.12
	9	.07	-.02	.14	.11	.28	.71
	10	.20	.14	.18	.20	.75	.08
	11	.04	.23	.84	.03	.17	.00
60%	12	.17	.09	.72	.18	.03	.23
	13	.33	.29	.05	.63	.12	.12
	14	.68	.12	.02	.11	.10	.33
	15	.01	.01	.50	.07	.12	.08
	16	.10	.21	.33	.35	.13	.67
	17	.82	.04	.01	.09	.13	.06
	18	.47	.07	.05	.12	.09	.62
	19	.07	.53	.59	.22	.12	.13
	20	.06	.01	.24	.73	.11	.27
	21	.05	.29	.22	.61	.34	.08
	22	.46	.02	.08	.07	.42	.19
	23	.30	.16	.05	.18	.66	.25

BIBLIOGRAPHY

Bailey, J. P., Jr. Test item dependence of several oblique factor solutions. Doctoral dissertation, University of Florida, 1969.

Bartlett, M. S. Tests of significance in factor analysis. British Journal of Psychology, 1950, 3, 77-85.

Butler, J. M. Simple structure reconsidered: Distinguishability and invariance in factor analysis. Multivariate Behavioral Research, 1969, 4, 5-28.

Cattell, R. B. Factor analysis. New York: Harper, 1952.

Cattell, R. B. Extracting the correct number of factors in factor analysis. Educational and Psychological Measurement, 1958, 18, 791-838.

Cattell, R. B. (Ed.) Handbook of multivariate experimental psychology. Chicago: Rand McNally, 1966a.

Cattell, R. B. The scree test for the number of factors. Multivariate Behavioral Research, 1966b, 1, 245-276.

Cattell, R. B. & Jaspers, J. A general plasmode (No. 30-10-5-2) for factor analytic exercises and research. Multivariate Behavioral Research Monographs, 1967, 3.

Cliff, N. & Hamburger, C. D. The study of sampling errors in factor analysis by means of artificial experiments. Psychological Bulletin, 1967, 68, 430-445.

Comrey, A. L. A first course in factor analysis. New York: Academic Press, 1973.

Dielman, T. E., Cattell, R. B. & Wagner, A. Evidence on the simple structure and factor invariance achieved by five rotational methods on four types of data. Multivariate Behavioral Research, 1972, 7, 223-242.

Dingman, H. F., Miller, C. R., & Eyman, R. K. A comparison between two analytic rotational solutions where the number of factors is indeterminate. Behavioral Science, 1964, 9, 76-80.

Dixon, W. J. (Ed.) Biomedical Computer Programs. Berkeley: University of California Press, 1974.

Fruchter, B. Introduction to factor analysis. New York: Van Nostrand, 1954.

Glass, G. V., & Taylor, P. A. Factor analytic methodology. Review of Educational Research, 1966, 36, 566-587.

Gorsuch, R. L. A comparison of Biquartimin, Maxplane, Promax, and Varimax. Educational and Psychological Measurement, 1970, 30, 861-872.

Gorsuch, R. L. Factor analysis. Philadelphia: W. B. Saunders Company, 1974.

Guertin, W. H. Do the different methods of factor analysis still give almost identical results? Perceptual and Motor Skills, 1971, 33, 600-602.

Guertin, W. H., & Bailey, J. P. Introduction to modern factor analysis. Ann Arbor: Edwards Brothers, 1970.

Guilford, J. P. (Ed.) Printed Classification Tests. Army Air Forces Aviation Psychology Research Report No. 5. Washington: U.S. Government Printing Office, 1947.

Guilford, J. P. Rotation problems in factor analysis. Psychological Bulletin, 1974, 81, 498-501.

Guilford, J. P., & Fruchter, B. Fundamental statistics in psychology and education. (5th ed.). New York: McGraw-Hill, 1973.

Guttman, L. Some necessary conditions for common factor analysis. Psychometrika, 1954, 19, 149-161.

Guttman, L. Best possible systematic estimates for communalities. Psychometrika, 1956, 21, 273-285.

Hakstian, A. R., & Muller, V. G. Some notes on the number of factors problem. Multivariate Behavioral Research, 1973, 8, 461-476.

Hamburger, C. D. Factorial stability as a function of analytical rotational method, type of simple structure and size of sample (Doctoral dissertation, University of Southern California, 1965). Dissertation Abstracts International, 1965, 27, 4551B-4552B. (University Microfilms No. 65-3106)

Harman, H. H. Modern factor analysis. (2nd ed.) Chicago: University of Chicago Press, 1967.

Harman, H. H. Modern factor analysis. (3rd ed. rev.) Chicago: University of Chicago Press, 1976.

Harris, C. W. Direct rotation to primary structure. Journal of Educational Psychology, 1948, 39, 449-468.

Horn, J. L. A rationale and test for the number of factors in factor analysis. Psychometrika, 1965, 30, 179-185.

Howard, K. G., & Gordon, R. A. Empirical note on the "number of factors" problem in factor analysis. Psychological Reports, 1963, 12, 247-250.

Humphreys, L. G. Number of cases and number of factors: An example where N is very large. Educational and Psychological Measurement, 1964, 24, 457-466.

Humphreys, L. G., & Ilgen, D. R. Note on a criterion for the number of common factors. Educational and Psychological Measurement, 1969, 29, 571-578.

Humphreys, L. G., & Montanelli, R. G. An investigation of the parallel analysis criterion for determining the number of common factors. Multivariate Behavioral Research, 1975, 10, 193-205.

Jones, L. V., & Fiske, D. W. Models for testing significance of combined results. Psychological Bulletin, 1953, 50, 375-382.

Jones, L. V., & Wepman, J. M. Dimensions of language performance in aphasia. Journal of Speech and Hearing Research, 1961, 4, 220-232.

Joreskog, K. G. Statistical estimation in factor analysis. Stockholm: Almquist & Wiksell, 1963.

Kaiser, H. F. The Varimax criterion for analytic rotation in factor analysis. Psychometrika, 1958, 23, 187-200.

Kaiser, H. F. The application of electronic computers to factor analysis. Educational and Psychological Measurement, 1960, 20, 141-151.

Kaiser, H. F. A second generation Little Jiffy. Psychometrika, 1970, 35, 401-415.

Kiel, D. F., & Wrigley, C. Effects upon the factorial solution of rotating varying numbers of factors. American Psychologist, 1960, 15, 487-488.

Kirk, R. E. Experimental design: Procedures for the behavioral sciences. Belmont: Brooks/Cole Publishing Company, 1968.

Lawley, D. N. The maximum likelihood method of estimating factor loadings. In G. H. Thompson (Ed.) The factorial analysis of human ability (5th ed.). Boston: Houghton-Mifflin, 1951.

Levitt, E. E. Clinical research design and analysis in the behavioral science. Springfield: Charles C. Thomas Publisher, 1961.

Levonian, E., & Comrey, A. L. Factorial stability as a function of the number of orthogonally-rotated factors. Behavioral Science, 1966, 11, 400-404.

Linn, R. L. A Monte Carlo approach to the number of factors problem (Doctoral dissertation, University of Illinois, 1965). Dissertation Abstracts International, 1965, 26, 480-481. (University Microfilms No. 65-7127)

Linn, R. L. A Monte Carlo approach to number of factors problem. Psychometrika, 1968, 33, 37-71.

Mosier, C. I. Influence of chance error on simple structure. Psychometrika, 1939, 4, 33-44.

Mulaik, S. A. The foundations of factor analysis. New York: McGraw-Hill, 1972.

Rao, C. R. Estimation and tests of significance in factor analysis. Psychometrika, 1955, 20, 93-111.

Thurstone, L. L. Theory of multiple factors. Ann Arbor: Edwards Brothers, 1932.

Thurstone, L. L. Multiple-factor analysis. Chicago: University of Chicago Press, 1947.

Tucker, L. R., Koopman, R. F., and Linn, R. L. Evaluation of factor analytic research procedures by means of simulated correlation matrices. Psychometrika, 1969, 34, 421-459.

Veldman, D. J. Simple structure and the number of factors problem. Multivariate Behavioral Research, 1974, 9, 191-200.

Whimbey, A. E., & Denenberg, V. H. Programming life histories: Creating individual differences by the experimental control of early experiences. Multivariate Behavioral Research, 1966, 1, 279-286.

Wilkinson, B. A statistical consideration in psychological research. Psychological Bulletin, 1951, 48, 156-158.

Winer, B. J. Statistical principles in experimental design (2nd ed.). New York: McGraw-Hill Book Company, 1971.

Wrigley, C. An empirical comparison of various methods for estimating communalities. Contract Report, 1956, No. 1, Berkeley: University of California.

Wrigley, C. The distinction between common and specific variance in factor theory. British Journal of Statistical Psychology, 1957, 10, 81-98.

Wrigley, C. A procedure for objective factor analysis. Paper presented at the first annual meeting of the Society of Multivariate Experimental Psychology, 1960. Sited in Howard, K. G., & Gordon, R. A. Empirical note on the "numbers of factors" in factor analysis. Psychological Reports, 1963, 12, 247-250.

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Azza Showket Guertin was born on December 3, 1935, in Baghdad, Iraq. She received her early education there, and later attended and was graduated from South High School in Worcester, Massachusetts, in June, 1953. In June, 1957, she received a Bachelor of Arts degree with a major in biology from the University of Baghdad. She received the Master of Education in science teaching degree in December, 1969 from the University of Florida.

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I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

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