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A Critical Summary of New and Previously Reported Research

BY

WILLIAM A. BROWNELL

WITH THE ASSISTANCE OF

ROY A. DOTY AND WILLIAM C. REIN

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Ohio
- Marysville
- Wayne

Pennsylvania
- Canton Township
- Charleston
- Davidson Township
- Dushore
- Erie (Lawrence Park)
- Estella
- Forest City
- Jenkins Township
- New Brighton
- Northumberland
- Smethport
- Summerhill Township
- Susquehanna (Thompson)
- Williamsport

Virginia
- Carson
- Petersburg (Brown, Jackson, Lee)
- Prince George
- Rives
- Woodlawn

Wisconsin
- Green Bay

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W. A. B.

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ARITHMETIC IN GRADES I AND II

CHAPTER I

THE PRESENT STATUS OF NUMBER IN THE PRIMARY GRADES

Few problems relating to the elementary school curriculum are as troublesome as are those associated with the kind and amount of arithmetic to be taught in the primary grades. These problems are essentially new problems; they did not exist, or at least they were not generally recognized, a quarter century ago when practices with regard to primary number were relatively more uniform. The last two decades have witnessed a decided break from tradition, but as yet no satisfactory solution has been found. Instead, there is such variety of practice as to amount almost to confusion.

The extent of this confusion is readily noted if one but compares different course of study offerings in primary arithmetic. For example, one school expects children to learn the addition combinations with sums to 10 in Grade I; another defers systematic instruction on these facts to Grade II; still another postpones such instruction to Grade III. Needless to say, were other arithmetical topics included in these comparisons, the variations suggested in the case of a single topic would be greatly enhanced.

REASONS FOR PRESENT CONFUSION

Reasons for confusion with respect to primary grade number are not hard to find. In the first place, evidence from school surveys, from local testing programs, from investigations of the learning of individual children, from research on adult usage—evidence from all of these and other sources made it incontestably clear some years ago that the anticipated results of arithmetical competence were not being attained. Children proved to be inaccurate in computation and unintelligent in problem solving; adults were found to employ neither frequently nor effectively the arithmetic they had been taught, even when it was obviously useful.

While most of these surveys and most of the experimentation dealt with arithmetic in the intermediate and higher grades, search for an explanation of the unsatisfactory conditions inevitably led to an examination of the state of affairs in the primary grades. Still, it should be observed that while attention was directed to arithmetic in the first grades (a tendency greatly accelerated by other influences discussed below), the resulting studies did not in themselves provide an answer for the questions they raised. True, instruction, as measured by its results, was ineffective, but what was to be done? It was possible from the data collected to argue in favor of widely different programs.

A second factor making for change was the spread of the educational philosophy epitomized in the phrase "the child-centered school." Clearly, the kind of arithmetic content and teaching found commonly in the primary grades fifteen or twenty years ago was inconsistent with this conception of education. Little effort was expended to utilize children's interests; classroom situations in the arithmetic class were typically "unnatural"; few indeed were the opportunities for individual creative and exploratory activity; children were told what to learn and how to "learn" it and then required to "learn" precisely as told.

In a word, many practices in the primary arithmetic class violated the tenets of the new conception of education; but, unfortunately, this new conception did not in itself contain a single unambiguous plan for the needed improvement. To illustrate, it was perfectly consistent with the new views (or with some of them) to abolish all planned experiences with number, on the one hand, and, on the other, to adopt a new and different kind of planned arithmetic and teaching to start in Grade I. Like the research mentioned in earlier paragraphs, the new conception of education pointed the need for change but found equally congenial actual changes which were quite unlike each other.

A third factor leading to change, if not for agreement in that change, was psychological in character. Reference here is to learning theory. Both educational and psychological experimentation on learning during the last fifteen years or so has raised objections to the oversimplification of the learning process which had theretofore been generally accepted. This research called attention to aspects of the learning situation which had been neglected when the experimenter artificially isolated the particular "S's" and "R's" in which he was interested. Nowadays psychologists tend to stress the importance of the subjective conditions of learning—the previous relevant experiences of the learner, his present goals and motivation, and the like; they point out that when learning involves relationships and rich understandings, learning does not take place all at once, but is rather long continued; they insist that however consistent the learning process in the acquisition of meanings and understandings may be basically and ultimately with the facts of simple conditioning, learning in such cases is not very helpfully viewed in these terms.

Briefly, then, learning theory now emphasizes increasingly: (1) the allowance of sufficient time for the completion of learning, (2) the necessity that the learner have the essential intellectual capacity to learn, and (3) the presence of a felt purpose or a goal for learning. Unfortunately, these new psychological emphases, important and welcome as they are, furnish guidance no less equivocal for the determination of the content and instructional procedures in primary number than do the factors already considered. Advocates of early systematic instruction insist that by providing the requisite foundational experiences from the start and by deferring mastery perhaps to Grade III they are making proper allowance for time for learning. At the same time those who would restrict all number experiences in the primary grades to those which appear incidentally and by chance, hold that they alone are acting in accordance with modern psychology, for they are waiting for the child to develop adequate intellectual power for learning.

Confusion with regard to primary arithmetic is worse confounded by the operation of a fourth factor, namely, different conceptions of the purpose of arithmetic in the elementary curriculum. These different conceptions of course quite largely reflect the influence of factors mentioned above, and their differences arise from the ways in which these factors and the principles deduced therefrom have been combined. Be that as it may, these various conceptions provide teacher and administrator with concrete patterns of thinking which in turn influence the kind and amount of arithmetic assigned to the lower grades.

According to one view, for example, arithmetic is a tool subject; it is in the elementary curriculum to equip children to deal effectively with the quantitative problems they can hardly avoid in later life. Consequently, attention is given chiefly to efficiency (speed and accuracy), and there is little concern that children shall understand what they are taught. The arithmetic of abstract numbers is introduced from the start of Grade I; instruction takes the form of telling children what and how to think; and children "learn" by mastering the prescribed formulas and skills. According to a second conception, arithmetic is first of all a mathematical system; the crucial element in learning is the understanding of the number system and its operation.

Obviously, both in content and emphasis the courses in primary number which arise from these two conceptions must be quite dissimilar.[1]

THE COMMONER PROGRAMS OF PRIMARY NUMBER INSTRUCTION

The situation with respect to primary number has been described as one of confusion, in the sense that no single program of instruction is predominantly favored at present. Nevertheless, it would be misleading to say that the situation is chaotic, for in the confusion one can detect at least four programs of instruction which are fairly common.

(1) The first program is in one sense not a program at all, but rather the negation of programs: it abolishes all systematic instruction in number in Grade I, or in Grades I and II, or in Grades I, II, and III. This program may be designated as the *incidental approach* and may be interpreted as a most vigorous reaction against the formalism of traditional arithmetic teaching. It is not supposed that children will have no number experiences in the primary grades; on the contrary, it is assumed that children, after entering school, will continue to find number a part of their natural activities as they did before entering school; and it is further assumed that these informal and unplanned contacts are sufficient to provide a basis for later systematic instruction—if indeed such instruction is ever offered. Arguments in favor of the incidental approach stress the mental immaturity of children in the primary grades, the waste inherent in efforts to teach arithmetic to children before they are ready for it, the dangers (largely emotional) which must result from unseemly haste and forcing, and the much greater value of other types of experience than those involving number, in the development of the primary pupil.[2]

(2) The second program is planned from the outset, but it is planned chiefly to acquaint children with number as a normal part of their environments. That is to say, the center of interest in the planning is the social setting or application of number, and children are accordingly encouraged to establish and operate grocery stores, school

[1] Nor does the confusion stop merely with adherence to different conceptions of elementary school arithmetic, only two of which have been mentioned. Each conception may be differently interpreted and differently implemented. Thus, one recent writer, while stoutly protesting his conviction that all arithmetic must be "meaningful," objects vigorously to teaching children in Grade III the rationale of the number system. How else it is possible to get children to understand computational operations with numbers, this writer does not say.

[2] For example: Howard A. Lane, "Child Development and the Three R's," *Childhood Education*, XVI (Nov., 1939), 101-104.

banks, post offices, and the like. Because of the large place given such experiences in this program, Program (2) may, for the purposes of this report, be designated as the *social approach*.[3] It should be noted that, as part of this plan, teachers make certain that the socially centered activities will provide plenty of contacts with number, and contacts of a known kind. At this point Program (2) is markedly different from Program (1). It is, however, like Program (1) in that it seeks to make arithmetic functional, and many of the arguments in favor of Program (1) and in opposition to traditional practices may be cited in support of Program (2).

(3) Mention has just been made of "traditional practice," and this may be regarded as the third program of primary number instruction. According to this program, number is systematically presented from the start (or from the beginning of Grade II). Little attention is given to the social applications of number and to the psychological factors so prominent in Programs (1) and (2). Indeed, it is assumed that children when they enter Grade I (or shortly thereafter) are ready for abstract number. After giving some practice in counting, perhaps, teachers begin on the simple addition and subtraction facts. These they present orally and later in written form, but always with the number symbols and with a minimum of concrete experience. The dominating purpose of the *traditional program* is to get children in the primary grades to "know" these number facts in the briefest possible time as the surest foundation for economical use in the computation of Grade III.

(4) Unlike the first three programs—the incidental, the social, and the traditional—the fourth program cannot be neatly described by a simple name. One reason is that the fourth program gives place to various features of the first three and is therefore somewhat eclectic in nature. Perhaps a better reason is that this program has no single striking characteristic, except that it recognizes the element of mathematical meaning which is, or tends to be, neglected in the other programs. Nevertheless, to call this program the "meaning" program is to slight other features that are equally essential parts. Advocates of the fourth program are no less concerned than are the exponents of the first two with making arithmetic socially functional, and no less concerned than are the exponents of the traditional program

[3] For example: G. M. Wilson, "New Standards in Arithmetic: A Controlled Experiment in Supervision," *Journal of Education Research*, XXII (Dec., 1930), 351-360. This is reference 55 in the research bibliography at the end of this monograph. Henceforth citations to this bibliography will be by number only.

with developing eventually a high degree of efficiency in the use of arithmetic; but they hold that these are not enough—that children must understand what they learn, and so must have a grasp upon the mathematics of arithmetic. Briefly, then, according to the fourth program full advantage is taken of all incidental occurrences of number in "natural" situations; moreover, other social settings involving number are deliberately arranged to increase real contacts with number. But these informal and planned social experiences are supplemented with learning activities deliberately designed (a) to make number and number operations sensible and (b) to encourage children as rapidly as they safely may to adopt the procedures which make for arithmetical proficiency.[4]

THE CRUCIAL QUESTIONS

Four programs of primary arithmetic have been briefly described. Actually, of course, still other patterns of content and instruction result from various combinations of the four outlined. This is not the place for a critique of the programs. The purpose in mentioning them here is mainly to point out that the different programs represent different solutions for the troublesome problems which relate to arithmetic in the primary curriculum. These troublesome problems may be reduced to three crucial questions, all of which must be answered before a satisfactory solution can ultimately be found. These questions are:

(1) Is the primary grade pupil intellectually capable of profiting from systematic instruction in arithmetic? That is, has he the mental powers requisite to this learning?

(2) If the primary grade pupil *can* learn much about arithmetic, *should he be asked* to do so at this time; is instruction in number wasteful if given in the primary grades, or does it produce gains which justify its being given?

(3) If the primary grade child *can* and *should* learn arithmetic from the start of his school career, what should be the *content and form of this teaching?*

The four programs of instruction which have been analyzed obviously are based upon different answers to these crucial questions. Thus, Program (1) answers question (1) in the negative, while the

[4] This program is essentially that of the National Council Committee on Arithmetic, sponsored by the National Council of Teachers of Mathematics. The point of view has been set forth by the chairman of the Committee, Professor R. L. Morton, in the *Mathematics Teacher* for Oct., 1938, and in the *Curriculum Journal* for Nov., 1938. The forthcoming (1941) yearbook of this Committee, the sixteenth in the series of the National Council of Teachers of Mathematics, should be especially helpful in implementing this program.

other three programs answer it in the affirmative. Likewise Programs (2), (3), and (4) answer the second question in the affirmative, but answer question (3) in quite different ways, while Program (1) consistently says No to question (2) and dismisses the problem of content and method by relying upon incidental number experiences.

But at the fewest here are four quite different programs of instruction which arise from different answers to the crucial questions and from different ways of implementing those answers. The fact that there can be different answers and programs arises in part from lack of research data and in part from disagreement with respect to educational philosophy. Question (2) is fundamentally a question of values and so of philosophy, but an answer to the question, even if philosophical, is a better answer if it is based upon some kind of research data. Thus, an adequate answer to question (1) should make easier the answering of question (2). Likewise, data on the outcomes of various programs of primary number instruction should be helpful in answering question (3), and indirectly in answering question (2).

PURPOSE OF THIS MONOGRAPH

It is the purpose of this monograph to supply data with which to help answer the crucial questions just posed. The question as to the "readiness" of the primary pupil for number instruction is partly answerable from evidence as to the number knowledge and skill with which he enters school. Several investigations have already been made in this area. These will be summarized, and to their data will be added the results of an original investigation not previously reported. The question as to the best system of instruction (it being assumed that instruction is to be given) is answerable in the light of evidence of the effects of various programs. A few research reports contain information on different approaches to instruction, and this information will be summarized and supplemented by new data to show the results of a particular program of instruction. If it can be demonstrated that children actually do profit from number instruction from the start, then further evidence is provided for an affirmative answer for question (1) as well.

Chapter II will canvass previous investigations and report a new study of the number knowledge and ability of children when they enter Grade I. Chapter III will present data on a new study, designed to show the results of one particular program of instruction in Grades I and II. Chapter IV will summarize other research on the effects both of postponing and of initiating instruction in the primary grades. Chapter V contains a final interpretation of research findings and sets forth the writer's conception of arithmetic for the primary grades.

Not the least useful feature of the monograph should be the bibliography of sixty research studies in the field of primary arithmetic, with which the monograph ends. Throughout the monograph citations to this bibliography will be by number only. Facing the first page of the bibliography is a classification of research studies according to the problems with which they deal. This classification should be of service to the careful student of arithmetic who wishes to check the writer's reviews and interpretations by consulting the original sources.

CHAPTER II

ARITHMETIC IN THE POSSESSION OF SCHOOL BEGINNERS

This chapter is devoted to a critical review of both new and previously reported research on the arithmetical skills and knowledge of children just entering Grade I.[1] The treatment will be topical. That is to say, all research which relates to the skill, item of knowledge, or other category in question will be brought together and discussed together. The list of topics to be considered in this chapter follows:

The chapter is divided into three parts. Topics 1-7 are treated as a group in Part I, because new data relating to these topics are to be reported here for the first time. Other research dealing with these same topics will be summarized in Part I also, but the separate consideration of this group of seven topics assures some unity to the original investigation. The rest of the topics listed above are considered in Part II. Part III consists in an appraisal of the research in this general area, the purpose being to point out various limitations on research techniques which have been used, to stress the consequent necessity for caution in interpreting the results of the re-

[1] The published research available before 1932 has been elsewhere summarized by T. G. Foran in "Early Development of Number Ideas," *Catholic Educational Review*, XXX (Dec., 1932), 598-609.

search, and to direct notice to aspects of number knowledge upon which research is needed.

Not all the available research data are canvassed in Parts I and II. (1) Unpublished material is not included, partly because such reports are not accessible to the average reader, more because an exhaustive inventory of these reports was impracticable. (2) All foreign studies, except British, are omitted. Most of these too are useless to the average reader, since they cannot be obtained, or read if obtained. Moreover—and what is more important—the significance of such studies for American education is ambiguous. Dissimilar environmental circumstances almost certainly introduce factors which make for large differences in number knowledge and skill. For these reasons even such well-known studies as those of Decroly and Degand[2] and of Descœudres[3] are not here considered. (3) Studies made with few subjects, as in the case of Baldwin and Stecher,[4] or made under conditions insufficiently described to justify evaluation, as in the case of Hall,[5] are likewise omitted. (4) It is possible too that a few significant investigations printed in periodicals not covered by the *Education Index* or by bibliographies like those in the *Review of Educational Research* may have escaped notice and so are not here summarized.

PART I. ORIGINAL INVESTIGATION, WITH RELATED RESEARCH
(Topics 1-7; see p. 11)

New data on the "readiness" of first-grade children for arithmetic were collected in the fall of 1938 and the fall of 1939 from twenty-four schools (thirty-two classes) in four states. The instrument used for this testing was the series of pretests which are published in connection with the "Jolly Numbers" primary materials of the *Daily-Life Arithmetics*.[6]

These pretests consist in (a) a group of sub-tests based upon pictures and administered to the class as a whole, (b) a second series of sub-tests intended to be given to smaller, selected groups of chil-

[2] Dr. M. le Decroly and Mlle. Julia Degand, "Observations relatives à l'évolution des notions de quantités, continues et discontinues, chez l'enfant," *Archives de Psychologie*, XII (May, 1912), 81-121.

[3] Alice Descœudres, *Le Développement de l'Enfant* (Paris: Delachauz et Niestle, S. A., n.d.), pp. 271-294.

[4] B. T. Baldwin and Lorle Stecher, *The Psychology of the Preschool Child* (New York: D. Appleton and Co., 1924), pp. 152-169.

[5] G. Stanley Hall, "Contents of Children's Minds on Entering School," *Pedagogical Seminary*, I (1891), 139-173.

[6] Guy T. Buswell, William A. Brownell, and Lenore John, *Daily Life Arithmetics* (Boston: Ginn and Co., 1938).

dren who previously have had sub-test (a), and (c) a third series of sub-tests to be given individually to a still smaller sample of pupils. In this particular instance, however, sub-tests (b) and (c) were administered in their entirety to all pupils of a class or to as many pupils of a class as circumstances permitted. In the latter case directions were to select the first, fourth, seventh pupil, etc., on the teacher's alphabetical roll, or the first, third, fifth pupil, etc.—in other words, to give the tests so as to insure a representative sample of the total enrolment. In all, 365 rural and 327 city pupils (see Table 1) took sub-test (a) and all or a usable part of the other sub-tests.

The testing was regularly done within the first two months of the term, in Grade I only, and with pupils who had just entered school for the first time. (Repeaters were thus eliminated.) In no class had any number instruction been offered.

TABLE 1

GRADE I PUPILS TESTED

State	*Number of Pupils by Schools*			
	Rural		*City*	
Florida	....		Sanford:	
			School 16	12
North Carolina	Durham County:		Durham:	
	School 1	30	School 17	72*
	School 2	40	School 18	62*
	School 3	25	Raleigh:	
	School 4	35	School 19	28
			School 20	47
Pennsylvania	North Central area:		Williamsport:	
	School 5	52*	School 21	25
	School 6	25		
	School 7	22*		
	School 8	9		
	School 9	23*		
	School 10	10		
	School 11	33		
Virginia	Vicinity of Petersburg:		Petersburg:	
	School 12	7	School 22	14
	School 13	13	School 23	36
	School 14	12	School 24	31
	School 15	29		
Totals†		365		327

* Schools marked thus supplied pupils to the testing both in 1938 and in 1939.

† These totals include all pupils whose records, even when incomplete, were used in the tabulations of this chapter. The tables reporting data from the individual tests show totals smaller than the figures above, for the reason that not all pupils in every class were given the individual tests.

The testers were graduate students in the writer's course, Investigations in Arithmetic, in the case of schools 1-4, 12-15, 17 and 18 (for the most part), 19 and 20, and 22-24. In the other schools (5-11, 16, 17 and 18 [in part], and 21) the tester was Miss Hulda Kilmer,[7] a teacher in the Dushore, Pennsylvania, schools, or the classroom teacher whose assistance Miss Kilmer had secured. In the latter cases great care was taken to have the teachers understand and follow the test directions.

The results obtained from this testing are given below according to the order of the topics as outlined on page 11. This order of topics differs from the order of the sub-tests themselves, but is here adopted as making for greater clarity and convenience to the reader. For each topic there will be given a description of the testing instrument, the directions for administering the sub-test, and the quantitative data secured. These data will then be compared with the corresponding findings of others who have investigated the same topic. In this way it will be possible to synthesize the new and the older information with respect to each of the seven arithmetical ideas and skills considered in this part of the chapter.

1. Rote Counting

The test of ability in rote counting was of course given individually. The child was told, "I want to see how well you can count. Will you count for me?" If the child was timid or was uncertain of the nature of the task, he was encouraged, and the task was illustrated in some such fashion as: "I know you can count. Now listen to me: one, two, three. Can you count from there?" All children were stopped when they reached 20. Children who did not try to count that far or who made mistakes before that point in the number series were credited with having counted to the last correctly stated number. For practically all the children one trial was enough. When, however, the examiner had reason to believe that a child had erred accidentally, he was given a second trial.

The first half of Table 2 summarizes the findings. It will be noted that 47.8 per cent of the rural children and 57.4 per cent of the city children counted to 20 and that 52.3 per cent of the 631 children in the total group attained this same point. Only about one tenth stopped short of 10.

[7] Miss Kilmer had previously had the above-mentioned course with the writer, and collected her data for her A.M. thesis, which (unpublished) is on file in the Duke University Library.

TABLE 2

Present Study: Results of the Tests of Rote Counting and Enumeration

Ability in	*Per cents of Pupils* Rural (N = 335)	City (N = 296)	Total (N = 631)
Rote counting by 1's to			
20	47.8	57.4	52.3
15-19	57.1	67.2	61.8
10-14	88.4	94.9	90.5
1- 9	100.0	100.0	100.0
Enumeration of			
10 objects	83.3	88.5	85.7
9 objects	87.5	91.2	89.2
8 objects	90.4	93.2	91.7
7 objects	91.3	93.9	92.5
6 objects	93.3	96.0	94.6
5 objects, or fewer	100.0	100.0	100.0

The figures obtained in the present study are to be compared with those of other investigators, as summarized in Table 3. The first three of these studies have been reported by Buckingham and MacLatchy (reference **12** in the research bibliography at the end of the monograph). For two of these studies Buckingham and MacLatchy are themselves directly responsible. One of their studies, which will hereinafter be referred to as their "major" or "main" study, involved a total of 1,356 school entrants in one Texas and sixteen Ohio communities. Their second study, herein designated the "Cincinnati" study, exactly paralleled their main study but was restricted to Cincinnati schools, where 1,067 children, about two thirds of whom had previously attended kindergarten, were interviewed. (In spite of the difference in earlier educational experience among the Cincinnati children, they will all be referred to here as school entrants.) The third study reported by Buckingham and MacLatchy was conducted independently in Cleveland by O'Connor, whose subjects were 1,242 kindergarten children, only 313 of whom were however tested on rote counting. O'Connor's study will be named the "Cleveland" study for the purposes of this report. According to Table 3, the figures for the Cleveland subjects run high, probably because of the influence of some selective factor, or because of some special instruction not enjoyed by other subjects, or because of both of the factors mentioned.

Data for the Woody study (**56, 58**; see the research bibliography at the end of the monograph) were gathered from 2,895 children in

TABLE 3

SUMMARY OF FINDINGS WITH RESPECT TO ROTE COUNTING BY 1'S

Investigation	*Subjects*	*Per cents Able to Count Through*			
		10	*20*	*50*	*100*
Buckingham and MacLatchy:					
Main Study	1,290 school entrants	90	60	20	10
Cincinnati Study	1,067 school entrants	88	60	..	10
Cleveland Study*	313 kindergarteners	97	76	18	..
Woody†	2,895 pupils in 5 half-grade groups, kindergarten - IIA	..	..	..	26, 38, 66, 76, 94
Present Study	631 school entrants	90	52	..	..

* O'Connor's figures reported by Buckingham and MacLatchy (12).

† The total number of children interviewed in the kindergarten was 94; in Grade IB, 607; in Grade IA, 1,897; in Grade IIB, 80; and in Grade IIA, 237. The five per cents reported above in the 100-column correspond to the records made by these groups (Woody, 58).

five half-grade groups in thirty-nine systems, these half-grade groups being: kindergarten, IB (beginning half of Grade I), IA, IIB, and IIA. In each instance children were tested in the half-grade just prior to that in which systematic instruction in arithmetic was begun. In so far as Woody has published data for the five groups they will be included in the summary tables of this monograph.

Four studies contain data for counting by 1's. The two made by Buckingham and MacLatchy agree in showing that about one pupil in ten has developed this ability at least to 100 as a limit, without specific school instruction, by the time he enters Grade I. Woody's figure is much higher, 38 per cent, or better than one pupil in three. The reason for this large discrepancy is problematic, but it is not improbable that many of Woody's Grade IB subjects had received instruction in this ability in connection with kindergarten schooling.

For counting by 1's to 10, 20, and 50 the data from the studies which are comparable with respect to subjects and testing procedures are in fairly substantial agreement. It seems safe to conclude that nearly all children upon entering school are in command of the number series to 10 and that between half and two thirds know the series to 20.[8] For the purposes of instruction, the significance of this ability

[8] An early study, that by Yocum (59), found that 20 per cent of fifty boys and fifty girls just entering Grade I could count to 20 (data from Buckingham and MacLatchy).

in rote counting may be either exaggerated or too greatly minimized. The child who can count to 20, for example, (a) knows the number names and (b) knows them in their correct order. Having this knowledge, he (c) has the basis for understanding the relative size of numbers. Thus, in knowing that 8 comes after 7 and before 9 he has the basis for knowing that 8 is more than 7 and less than 9. Furthermore, when this ability to repeat the number sequence is coupled with the ability to apply the number names to groups of objects, he (d) has a means of satisfying by the very immature procedure of enumeration most of the practical number needs which he will encounter in and out of school.

At the same time, it is to be noted that rote counting in itself does not make (c) and (d) possible. On the contrary, in both instances rote counting must be supplemented by other kinds of learning. As a matter of fact, rote counting guarantees only (a) and (b) and may actually be quite devoid of quantitative meaning. The truth of this statement is revealed in the child's inability to put the number terms he knows to any useful service. He may "count" a group of six objects and find "eight," or "three," or "sixteen"; whatever number he assigns the group, he is entirely complacent about its accuracy. For such a child "five" is merely the word to say after saying "four" and before saying "six"; for him "five" possesses no more mathematical meaning than does "e" in the alphabetical series "a, b, c, d, e, f."

Thus far ability in rote counting by 1's only has been discussed. Some attempts have been made to ascertain how well school entrants can count by other units, as by 10's. Buckingham and MacLatchy (12) report that (a) about 50 per cent of their pupils could not count by 10's at all, even though they were helped by the examiner's starting them with "Ten, twenty, thirty"; (b) about a fourth (28 per cent) could count by 10's to 40, and (c) another fourth (24 per cent) could count in this way to 100. The figures for the separate Cincinnati study are extraordinarily alike, being 52 per cent for (a), 29 per cent for (b), and 22 per cent for (c).

In Woody's investigation (58) about the same proportion of children could count to 100 by 10's as by 1's. For his five half-grade groups (kindergarten through IIA) the per cents who could reach 100 were 32, 46, 69, 76, and 93. Woody's second group, composed of children in Grade IB, were most like those in the main Buckingham and MacLatchy study and in their Cincinnati study; yet Woody reports a percentage figure of 46 for his subjects, and the latter two investigations, a percentage figure of about 28. As mentioned before, the most reasonable explanation for the difference is the probability of specialized training by many of the children in Woody's groups, though there is no proof that this hypothesis is valid.

As for other forms of rote counting, Woody (**56**) reports 34 per cent of his 1,897 Grade IA pupils as able to count by 2's to 30, 9 per cent as able to count by 3's to 30, and 12 per cent as able to count backward from 20 by 2's.

2. *Enumeration*

The second half of Table 2 (p. 15) summarizes the results of the test in enumeration. The differences as between rural and city children are slight and probably of no educational significance. According to the last column, nearly seven eighths of the children could enumerate ten objects and about 92 per cent could enumerate at least eight. Their ability in enumeration therefore was but slightly less than their ability in rote counting by 1's, a finding which confirms that reported by Buckingham and MacLatchy.

In this study the testing procedure was as follows: the examiner scatters ten small objects (pegs, paper clips, pennies) on the table top in front of each child and says, "Here are some pegs. I want you to tell me how many you have." The child is required to lay his finger or hand on each object as he tells it off, thus to show that he actually has established a one-to-one correspondence between the series of language terms and the objects in the group. This procedure has been rather generally used in the investigations of others, though the objects used and the numbers of objects have differed. Thus, Buckingham and MacLatchy used twenty instead of ten objects; Woody presented a row of twenty-one circles in a printed booklet; and in the Cleveland study made by O'Connor and reported by Buckingham and MacLatchy the children were given two trials in pushing twenty tacks into a board.

It is difficult to assess the influence of variations such as those just described, but internal inconsistencies in bodies of data from the same study and discrepancies as between different studies probably mean that these and other variations in testing procedure introduce factors which, however negligible they appear to adults, cause marked differences in the responses of children.

Comparative data on enumeration from other investigations are summarized in Table 4. The Cleveland and the Woody data are out of line with the data from the other three studies. In the case of

TABLE 4

SUMMARY OF FINDINGS WITH RESPECT TO ENUMERATION

Investigation	*Subjects*	*Per cents Able to Enumerate Through*				
		5	*8*	*10*	*15*	*20*
Buckingham and MacLatchy:						
Main Study	1,222 school entrants	97	93	90	70	58
Cincinnati Study	1,067 school entrants	..	..	90	70	58
Cleveland Study*	1,242 kindergarteners	26 (both trials) 22 (more, on one trial only)			.. ..	.. ..
Woody	2,895 pupils, kindergarten to IIA	..	..	..	..	71, 79, 93, 98, 97†
Present Study	631 school entrants	99	93	89	..	..

* O'Connor's figures reported by Buckingham and MacLatchy (12).
† Figures are for subjects as follows: kindergarten (94), IB (607), IA (1,877), IIB (80), and IIA (207). (Woody, 58)

the Cleveland study the discrepancy is explained partly by the fact that kindergarteners served as subjects, but probably much more by differences in the procedure for testing enumeration. These children were required not alone to select the correct number of tacks (20) but also to stick them into a board. Clearly, lack of success may be as reasonably attributed to distraction or to difficulties in fixing the tacks in the board as to difficulties of a purely quantitative character. The high per cents in the Woody study may be explained by the hypothesis suggested twice before (p. 18), namely, that his subjects actually benefited by planned kindergarten experiences before the supposed start of systematic instruction in Grade I or II.

If conclusions are drawn only from the data for school entrants it appears that nine out of ten children may be expected at the start of their schooling to be able to enumerate ten objects and that seven of the ten will be able to enumerate successfully fifteen objects and six of the ten, twenty objects.[9] This conclusion is of considerable

[9] The 1916 Stanford-Binet placed the enumeration of four pennies at age four. Enumeration to 4 appears in the new (1938) test at age five, where children must successfully enumerate objects in two of three trials, the objects varying from trial to trial. In the validation of this item it was found that 40 per cent were successful at age four, 68 per cent at age five, and 91 per cent at age six. The 1916 item calling for enumeration of thirteen pennies and placed then at age six has been dropped in the revision.

Yocum (59) in 1901 reported that 30 per cent of his fifty boys and 38 per practical significance. Unlike the ability to count by rote to 10, the ability to enumerate ten objects correctly is possible only to children whose number concepts have really begun to take on content. That is to say, such children can use the number names in functional quantitative situations. Ability to enumerate to 10 or some other such point does not warrant the inference that children have well-developed number concepts, but it does warrant the inference that their number concepts are in process of quantitative development.

3. Identification

Closely related to ability in enumeration is the ability to identify or name the number of objects in groups of various sizes. In the present study this ability was tested with classes as wholes by means of pictured groups (see Part A, p. 21). Instructions were to "put a mark on the man with four balloons; on Mary's birthday cake with seven candles; on the pot with ten flowers." A similar procedure was employed by Grant (**15**), who made use of Test 5 of the Metropolitan Readiness Tests for this purpose. His subjects had to select from among four choices the box that contained three dots, six dots, and eight dots. In both of these studies, therefore, something more than mere enumeration was involved; the children tested were presented with a variety of number representations and had to select the appropriate one.

Quite a different procedure was employed by Buckingham and MacLatchy (**12**). In their two investigations (in Cincinnati and elsewhere) the examiner threw five, six, seven, eight, and ten objects in random patterns on the table top before the child and asked, "How many pegs (or what not) are here?" In their main study, three trials were given and data are reported for successes on one trial only, on two trials, and on three trials. Also, they translated successes into a "mean index of difficulty" for the different numbers in such a way as to make the resulting index figures represent the per cents of success as compared with the total number of opportunities. In their Cincinnati study evidently but one trial was given. It might be argued that both of these studies are better classified under enumeration than under identification, but they are included under the latter, in part because they are so classified by the authors and in part because of discrepancies between the percentage figures for these tasks and those for enumeration (Table 4).

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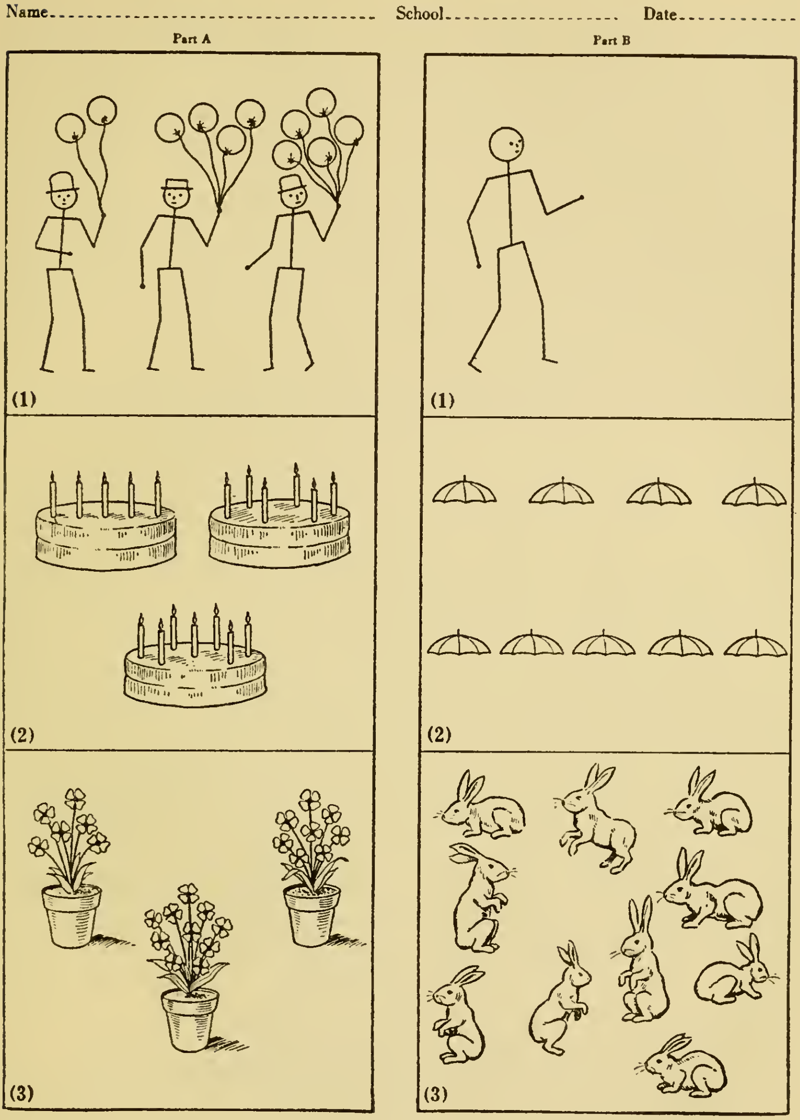
cent of his fifty girls on entering school could count more than twenty objects (data from Buckingham and MacLatchy).



Woody (56) presented his subjects with three sets of grouped objects, the sets being made up of domino figures, of groups of dots, and of groups of lines. Each set consisted of five groups. The subject was first instructed to point to the particular domino among the five dominoes given which contained five dots; then the one which contained four, etc., etc., until he had used four of the five dominoes in that set. The subject then proceeded similarly with the set of dot figures, and then with the set of line figures, in each case identifying the number in four of the five groups. Three other items in the Woody test required the subject to divide a line of stars into groups of three, another line of stars into groups of five, and a third, into groups of four. The per cents of accuracy for 1,897 IA pupils were 90, 79, and 76, respectively. Obviously, this last task differs considerably from that in the other studies in this section, and for this reason the data are not included in the summary table or statements.

Table 5 contains the detailed summary of findings from the present study. Again the results of rural and city children are about the same, the rural children being slightly superior (about 5 per cent) on the numbers 4 and 7 and slightly inferior (4.3 per cent) on the number 10. About 6 per cent more of the city children were successful on all three numbers than is true in the case of the rural children. As to the relative difficulty of the three numbers 4, 7, and 10 for the purposes of identification, the data are anomalous: the per cents of children who identified 4 and 10 were very much alike, but the per cents who identified 7 were, both for city and for rural children, less than the per cents for 10. On the assumption that the figures for 4 and 10 are correct, and that more than three fourths of school beginners can identify 10 as well as 4, then the figures for 7 are difficult to account for. On the assumption that the per cents for 4 and 7 are correct and that numbers increase in difficulty of identification as they increase in size, then the per cent for 10 is too high. A tentative explanation is as follows: In the third picture the children were instructed to mark the pot which contains ten flowers; they knew that 10 is a large number, and the pot showing ten flowers is rather obviously better supplied with flowers than the other two pic-

TABLE 5

Present Study: Results on Group Test of Identification—Four, Seven, and Ten Objects

Type of Pupil	*Per cents Identifying 0, 1, 2, or 3 Numbers Correctly*				*Per cents Identifying Specific Numbers Correctly*		
	0	1	2	3	4	7	10
Rural (365)	6.3	12.1	31.2	50.3	86.7	72.6	76.4
City (327)	5.5	14.9	22.9	56.5	80.7	69.1	80.7
Total (692)	5.9	13.4	27.3	53.3	83.8	70.9	78.5

tured, which contain but six and eight flowers respectively. That is to say, the decision in favor of the 10-pot may well have been made in terms of gross comparison rather than in terms of actual and exact enumeration and comparison. That children do so discriminate between groups of objects has been amply demonstrated by Russell (42).

Comparative data on identification (in the different meanings of this term as already described) have been previously reported by Buckingham and MacLatchy (12) (their main and their Cincinnati studies), by Woody (56), and by Grant (15). The relevant facts are presented in Table 6. If one takes the median figures reported for each number (and uses only the Grant per cents for children of average intelligence), the numbers are found to have the following approximate per cents of success:

3	4	5	6	7	8	9	10
(90)	(90)	(80)	(66)	(71)	(68)	(75)	(70)

The anomaly found in the present study in the case of 7 and 10 is to be noted in the Grant study in the case of 6 and 8. If Grant's

TABLE 6

SUMMARY OF FINDINGS WITH RESPECT TO IDENTIFICATION

Investigation	*Subjects*	*Per cents Able to Identify*							
		3	*4*	*5*	*6*	*7*	*8*	*9*	*10*
Buckingham and MacLatchy:									
Main Study*	1,356 school entrants	..	..	72	63	60	58	..	56
		..	..	82	75	74	72	..	70
		..	..	63	52	46	45	..	42
Cincinnati Study	1,123 school entrants	..	..	80	69	63	63	..	60
Woody†	1,897 IA pupils	..	..	80	..	83	81	..	79
		99	96	92	95	..	..	..	..
		..	..	91	..	86	..	75	80
Grant‡	563 school entrants	54	..	..	19	..	54	..	..
		82	..	..	43	..	73	..	..
		96	..	..	61	..	86	..	..
Present Study	692 school entrants	..	84	..	..	71	..	..	79

* First row of figures is in terms of "mean index of difficulty"; second row shows per cents successful in at least one trial; third row shows per cents successful in all three trials.

† Data are presented for Grade IA only. First row is for responses to domino pattern; second row, to groups of dots; third row, to group of lines.

‡ First row for 145 pupils with IQs below 90; second row for 252 pupils with IQs between 90 and 109; third row for 166 pupils with IQs of 110 and higher.

per cents for 3 and 6 are correct, that for 8 seems to be too high; if the per cents for 3 and 8 are correct, that for 6 seems to be much too low. An examination of the pictured groups supplies no explanation to favor either hypothesis. All groups of dots, from three to nine, are distributed over squares of the same size (half-inch). The number 8 must be identified from pictures containing four, nine, six, and eight dots; the number 6, from pictures containing three, six, four, and five dots.

Whatever be the true explanation for the two curious inconsistencies noted, the data on identification as a whole seem to warrant the conclusion that six out of ten children on entering Grade I can identify or name the numbers to 10 when these numbers are represented concretely and without regularity of pattern. This figure is to be compared with that for enumeration, in which nine out of ten children were able to enumerate ten objects correctly. The difference in relative success in enumeration and identification may be ascribed to the extra task in identification as here tested, namely, to the necessity of selecting the correct number from among several others present at the time.

This conclusion and this explanation are by no means certain. All measures for enumeration were secured from individual children. The data on identification, however, were obtained, in the present study and in the Grant study, by means of group tests. Unfamiliarity with the test situation on the part of school beginners almost certainly would have made for an excessive number of mistakes. On the other hand, another factor in this testing would have had the opposite effect. In both of the studies named the subject made his selection from among three or four pictures, and he could therefore have scored some unearned successes through the operation of chance. How much one of these two factors—test unsophistication and chance—compensated for the other it is impossible to say.

4. Reproduction

Identification is the activity by which one answers the question, "How many apples have I?" Reproduction, on the other hand, is the activity in which one engages to comply with the request, "Give me five apples." In the case of identification a group of objects is given and their number must be found; in the case of reproduction the number is given or announced and the corresponding group of objects must be found. The mental processes required in the two number feats are markedly different (though both obviously employ enumeration), and so both need to be tested.

In the present study reproduction was tested for classes as wholes by means of three pictures (Part B, p. 21). The directions were:

1. This boy wants to play marbles. Draw five marbles for him.
2. Look at the umbrellas. They have no handles. Put handles on six of them.
3. Do you see the rabbits? They have no tails. Put tails on nine of the rabbits.

A single trial was given with each picture, and the numbers for which ability to reproduce was tested were 5, 6, and 9. The results of the testing are summarized in Table 7.

TABLE 7

PRESENT STUDY: RESULTS ON GROUP TEST OF NUMBER REPRODUCTION—5, 6, AND 9

Type of Pupil	*Per cents Reproducing 0, 1, 2, or 3 Numbers Correctly*				*Per cents Reproducing Specific Numbers Correctly*		
	0	1	2	3	5	6	9
Rural (365)....	17.8	20.8	27.1	34.2	73.7	55.1	49.0
City (327).....	5.2	22.9	25.9	45.9	86.9	66.7	59.0
Total (692)..	11.8	21.8	26.6	39.7	79.9	60.5	53.8

Nearly half (45.9 per cent) of the city children, compared with about a third (34.2 per cent) of the rural children, were successful with all three numbers; and only 5.2 per cent of the city children, compared with 17.8 per cent of the rural children, failed on all numbers. So far as the separate numbers 5, 6, and 9 are concerned, difficulty in reproduction is seen to have increased with the size of the groups to be reproduced.

There are at least four other related investigations. Buckingham and MacLatchy (12) report data for the numbers 5, 6, 7, 8, and 10. In their study the examiner began with 5 and worked upward if the child was successful, and downward if he was unsuccessful. Three trials were given, and their data are here reproduced for those subjects who were successful in one trial only, in two trials, and in all three trials. In addition, their figures for "mean index of difficulty" (see p. 20) are quoted.

TABLE 8

SUMMARY OF FINDINGS WITH RESPECT TO NUMBER REPRODUCTION

Investigation	*Subjects*	*Per cents Successful in Reproducing*						
		4	*5*	*6*	*7*	*8*	*10*	*13*
Buckingham and MacLatchy:								
Main Study*........	1,355 school entrants	..	74	67	66	63	63	..
Cincinnati...........	1,123 school entrants	..	83	73	72	68	67	..
Grant†..................	513 school entrants	57	..	..	39	..	..	19
		64	..	..	58	..	..	40
		87	..	..	83	..	..	64
Present Study...........	692 school entrants	..	80	61	..	..	54	..

* Figures are for "mean index of difficulty." Per cents of subjects successful on one trial only were, for the five members respectively, 14, 15, 16, 17, and 16; per cents successful on two trials only were: 7, 10, 11, 12, 10; per cents successful on all three trials were: 64, 56, 54, 49, 50.

† First row of figures is for 145 children with IQs below 90; second row, for 252 children with IQs between 90 and 109; third row, for 116 children with IQs above 110.

Table 8 summarizes the data from the Buckingham and MacLatchy studies. No facts for O'Connor's study with Cleveland kindergarten children are given here or in the Buckingham and MacLatchy report, for the reason that the methods of testing and of treating the results are not comparable. Buckingham and MacLatchy's statement may be quoted:

> Of 1,242 children at Cleveland, 32 per cent were completely successful in both trials of a test in which they were to reproduce 5, 7, 9, and 11 by putting the required number of marks in designated spaces on a sheet of paper. According to the marking plan for this test, the highest obtainable score was 10, and the median of all the scores actually made was 8. This is a very creditable performance.

Grant's procedure (**15**) with Test 5 of the Metropolitan Readiness Tests was much like that in the present study. Pictures of objects were given, with directions to put marks on four objects (houses) in one picture, on seven in the second, and on thirteen in the third. His test results were then tabulated according to the IQs of the subjects, and the per cents so obtained are included in the summary table (Table 8).

The per cents obtained in the present study and in the Grant study are, with one exception (the number 5, in the present study), lower than the corresponding per cents obtained in the other two studies. If it is assumed that the true abilities of the four different groups of subjects were actually equivalent, then it follows that the children tested by group tests were thereby put at a disadvantage and were unable to show their real abilities. This inference is not improbable. In the first place, school entrants would suffer from their unfamiliarity with the techniques of group testing: they lose their place, misunderstand directions, etc., thus making errors which should not arise in the case of individual testing. In the second place, their scores with the larger numbers would be especially influenced for the worse by the rather crowded appearance of the objects pictured. Certainly this is true in the case of the test picture for 10 (rabbits) in the present study (see p. 21). Likewise it seems to have been true in the case of the pictures for 9 and 13 in the Grant testing procedure.

For these reasons the two sets of figures from the Buckingham and MacLatchy studies should be given special weight in assessing the ability of school entrants to reproduce numbers. But in this case which of Buckingham and MacLatchy's four sets of measures should be adopted? These authors point out that no less than three successes in as many trials can be accepted as guaranteeing real ability, but they also stress the danger of disregarding or minimizing the significance of smaller degrees of success. After all, the child who can produce five objects in two of three trials is well on his way in the development of a good reproduction-concept of 5, and even the child who can correctly produce five objects only once in three trials is not without some degree of ability.

In the listing below the numbers are arranged in the order from 4 to 13, and with each is given the per cent of successful reproduction in the same way as had already been done for identification. Each per cent figure represents the median of the measures available for each number, only the Buckingham and MacLatchy mean indices of difficulty being used from their study:

4	5	6	7	8	10	13
(64)	(80)	(67)	(66)	(66)	(63)	(40)

If these figures could be accepted at face value, it would be possible to conclude that about two thirds of school entrants are able to reproduce all the numbers to 10 (the Grant figure for 4 is almost certainly much too low).[10] Obviously, however, various limitations

[10] The 1938 Stanford-Binet places a reproduction test at age six. On demand children must be able to supply three, nine, five, and seven objects (three successes required for credit). Percentages for the various numbers are not given.

with respect to testing procedures and with respect to the determination of accurate single indices from the data obtained render this conclusion debatable; it must be regarded as purely tentative, though it is the best guess, on the basis of the evidence available.

5. Crude Quantitative Comparison

Concrete objects or pictures.—Under this caption are included terms like *largest, shortest, most,* and so on, a total of eleven such terms by means of which one describes crude differences in size or amount without attempting to fix exactly the degree of the difference. In the present study single trials were given in the use of six of these words or phrases. Three of these same terms appear among the eleven terms of the Metropolitan Readiness Tests, Test 5, which was used by Grant (15).

In both of these studies the testing employed pictures. The picture test of the present study is Part C of page 29. The specific directions are:

1. ...... Put a mark on the *largest* one [cat].
2. ...... Put a mark on the *smallest* one [doll].
3. ...... Put a mark on the kite with the *longest* tail.
4. ...... Put a mark on the pan with the *shortest* handle.
5. ...... Put a mark on the nest that has the *most* eggs in it.
6. ...... Put a mark on the tree that has the *most* apples on it.
7. ...... Put a mark on the table that has the *smallest* number of cups, or the *fewest* cups, on it.

As shown in the first half of Table 9, less than half the children tested knew all the terms well enough to score consistent success.

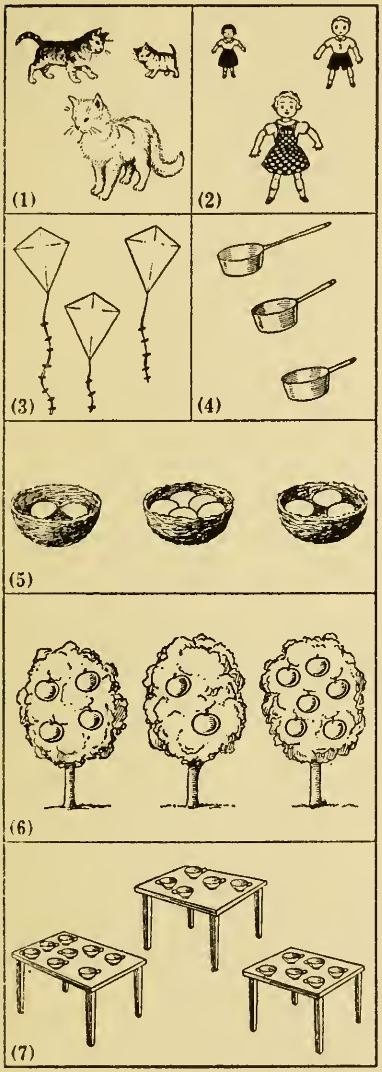
TABLE 9

Present Study: Results on Group Test of Crude Comparison—Concrete Numbers

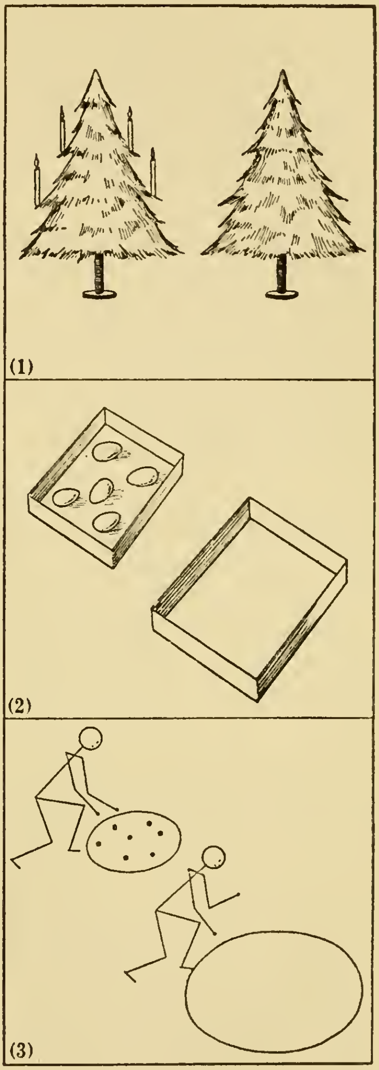
Type of Pupil	*Per cents Answering 0-7 Items Correctly*							
	0	1	2	3	4	5	6	7
Rural (365)	0.8	0.8	0.5	2.7	5.8	16.2	30.7	42.5
City (327)	0.0	0.0	0.6	2.1	4.6	14.1	32.4	46.2
Total (692)	0.4	0.4	0.6	2.5	5.2	15.2	31.5	44.2
	Per cents Answering Specific Items Correctly							
	Largest	*Smallest*	*Longest*	*Shortest*	*Most*	*Most*	*Fewest*	
Rural (365)	80.0	78.6	92.3	86.8	96.2	96.2	65.2	
City (327)	87.8	79.8	95.7	92.4	97.6	98.8	62.1	
Total (692)	83.7	79.2	93.9	89.5	96.8	97.4	63.7	

Name................................ School.................. Date............

Part C



Part D



Three fourths of them, however, identified either six or seven terms, and nine out of ten identified at least five. Differences between rural and city children were conspicuous only in the case of *largest* and *shortest,* where the city children were slightly superior (about 6 percentage points). The best known terms in both groups of children were *most* and *longest;* the least well known was *fewest* (or *smallest* number).

The comparisons in the present study, as will be noted from the picture, regularly involved three objects; the comparisons in the Grant study with equal regularity involved four objects, and his results are therefore less subject to chance success and error. The specific directions for the Grant test were:

Mark the other board that is *just as long as the first one.*

Two tests involve *half,* and it is impossible to tell from the report which of the two yielded the data reported for the term.

Mark the *longest* pencil.
Mark the *middle* hat.
Mark the *shortest* flower.
Mark the *smallest* tree.
Mark the *tallest* boy.
Mark the *widest* board.

The three terms studied in both the present and the Grant investigations are *longest, shortest,* and *smallest.* Both groups of children knew the first two of these terms exceedingly well, the per cents who were successful running at least to 90. The term *smallest* was about as well known to Grant's pupils as were the other two, but in the present study the per cent amounted to 79, perhaps because of unfavorable factors relating to the test picture (see above).

TABLE 10

SUMMARY OF DATA WITH RESPECT TO CRUDE COMPARISON, CONCRETE NUMBERS

Term of Comparison	*Per cents of School Entrants Successful with Various Terms*			
	Grant Study			*Present Study*
	IQ 90− (N = 145)	IQ 90-110 (N = 252)	IQ 110+ (N = 166)	(N = 692)
as long as	24	50	68	..
fewest (smallest number)	..	..	..	64
half	55	76	83	..
largest	..	..	..	84
longest	92	99	100	94
middle	84	96	98	..
most	..	..	..	97, 98*
shortest	74	94	98	90
smallest	66	91	93	79
tallest	71	89	98	..
widest	72	85	93	..

* Two tests—5 compared with 3 and 2; 6 compared with 2 and 4.

The figures of Table 10 seem to support the conclusion that most children on entering school are in control of the process of crude comparison, at least as comparison was involved in the test situations used in these studies. Exceptions should be noted in the case of *half* (though, as stated, Grant's data are ambiguous) and *fewest,* or *smallest* number. It is a fair guess that the latter term was less troublesome because of an as yet undeveloped kind of comparison than because of difficulty associated with the language for expressing the results of the comparison.

The most extensive data collected with respect to crude comparison have been summarized by Woody (**56**) in a way which greatly reduces their value for the present purpose. Woody's inventory test, given to children from the kindergarten through Grade IIA, contained twenty-two items relevant to crude comparison. The first eight were concrete in type—e.g., "Which is more, 3 apples or 5 apples?" "Which is most, 11 cookies, 7 cookies, or 9 cookies?" and "Which is less, 33 marbles or 29 marbles?" Ten items dealt with abstract numbers and involved the questions, "Which is more (e.g., 3 or 5)?" "Which is less?" "Which is 'biggest'?" "Which is 'smallest'?" some of the numbers being as large as 229. In another exercise children had to cross out all numbers "bigger" than 9 in a set of eight numbers, all numbers "bigger" than 11 in another set, all numbers "smaller" than 17 in a third set, and all numbers "bigger" than 25 in a fourth. On these twenty-two tasks combined, the per cent of success was 77, for Grade IA pupils.

The most careful study yet made of children's procedures in comparing numbers represented by groups of objects has been reported by Russell (**42**). Unfortunately for the purposes of this monograph his data are not tabulated to show quantitatively the ability of school entrants to make the kinds of comparisons he studied. In his first experiment he used thirteen kindergarten, twelve first-grade, and four second-grade children, who were asked to identify which of two groups of blocks (such as 4 and 7, 5 and 5, 6 and 8) was "more" or "most." Having found this to be a "leading" or a "misleading" question, in his second experiment with ten kindergarten, ten first-grade, and five second-grade children, he asked for the identification of groups that were "alike," or "the same," or "equal in size."

The value of this study lies in Russell's psychological analysis of children's thought processes when required to compare quantities. Russell reports that children of age five have good concepts of *most, both,* and *biggest;* that not even seven-year-old children know *same* and *equal* at all generally; that in making crude comparisons children of the ages of his subjects rarely use counting to detect differences in amount, but rather estimate directly when they can or break up large groups into identifiable smaller groups which they then use by some such process as matching. The introduction of blocks of different sizes or different colors resulted in marked increase in error, a fact which clearly reveals the instability of children's thought processes in quantitative comparisons of the kind Russell investigated.

Abstract numbers.—The crude comparisons thus far considered all dealt with groups of actual or pictured concrete objects. It is of course possible to compare abstract numbers, such as 4 and 6, 3 and 7, etc. The mental demands of such comparisons are more arduous than when actual or pictured objects are present. In the latter case, for example, the comparer has direct evidence of difference in size or amount in the external groupings. In comparing abstract numbers, on the other hand, the comparer must himself supply whatever content he uses.

So far as could be ascertained by diligent bibliographical search, this ability has been studied only twice—and then not very satisfactorily—in the investigation here reported for the first time, and in Woody's study. The procedure in the present case was to ask which of two given numbers is *more* and which of two other numbers is *less*. The exact questions asked of the subjects, individually in each case, were:

1. Which is *more,* 2 or 4?
2. Which is *more,* 7 or 3?
3. Which is *more,* 5 or 8?
4. Which is *less,* 3 or 5?
5. Which is *less,* 6 or 4?
6. Which is *less,* 10 or 8?

To be given credit for a "pass" the child had to answer correctly twice in the three trials for *more* and, similarly, twice in the three trials for *less*. This method of scoring is open to criticism, first of all, because it by no means eliminates the factor of chance. It can be criticized, in the second place, because really neither success nor failure tells very much. Even when chance is ruled out, probably many children were given credit for knowing *more* or *less* or both merely because they employed correctly their knowledge of the serial order of the numbers. Moreover, it is not improbable that children scored successes with particular pairs of numbers whose relations as to size they knew, whereas testing with other and less familiar pairs of numbers might have revealed ignorance or inability. For these reasons not much weight can be attached to the results which are summarized in Table 11.

TABLE 11

PRESENT STUDY: RESULTS ON INDIVIDUAL TESTS OF CRUDE COMPARISON—ABSTRACT NUMBERS

Type of Pupil	*Per cents Comparing Correctly*	
	More	*Less*
Rural (337)	81.6	48.7
City (296)	84.8	40.9
Total (633)	83.1	45.0

The difference in per cents of success for *more* and *less,* and in favor of *more,* probably is sufficiently large, in spite of the limitations mentioned above, to be regarded as a fact. Whether this means greater ignorance of the word *less* than of the word *more,* or less ability to compare abstract numbers in descending than in ascending order, or both, it is impossible to say from the data at hand.[11]

6. Exact Quantitative Comparison, or Matching

The requirement in exact quantitative comparison, as this ability was measured in the present study, is essentially that of matching a given number of objects. For this purpose Part D of the group test (see p. 29) was used. The directions are as follows:

1. Put as many candles on this tree [pointing to the tree with no candles] as there are on this one [pointing to the first tree]. [The number to be matched is 4.]
2. Here are two boxes. One has eggs in it; the other one is empty. Put as many eggs in this box [pointing to the empty box] as there are in this one [pointing to the full one]. [The number to be matched is 5.]
3. Draw marbles in this ring, so that this boy [pointing to the second one] will have as many as this one [pointing to the first boy]. [The number to be matched is 7.]

To score a correct response the child must first enumerate the objects presented in the picture and then reproduce this number by drawing. In each instance the drawing requirement is simple—almost any kind of mark satisfies the demand—so that errors are probably quantitative in character, the result of incorrect enumera-

[11] Woody (56) asked his subjects, "Which is more, 3 or 5 (5 or 8) (13 or 17)?" and "Which is less, 15 or 22?" but the response data are not reported in a way which permits comparison with the figures from the present study.

tion, of failure to hold in mind the numerical total, of inability to reproduce that total, or of some combination of elements. Correct matching, in other words, is possible only after rather complicated mental feats.

The results obtained in the present study (the only one yet reported in this area)[12] are to be found in Table 12. Three fourths, slightly more or less, were able to match 4 and 5, and about half, to match 7. The matching of 5 was easier than the matching of either of the other numbers, even 4. Whether this fact is to be explained as evidence of a richer concept of 5 than of 4, or as the result of differences in the test situations which unintentionally made the group of five eggs easier than the group of four candles to apprehend or reproduce or both, it is impossible to say.

TABLE 12

PRESENT STUDY: RESULTS ON GROUP TEST OF EXACT COMPARISON—CONCRETE NUMBERS 4, 5, 7

Type of Pupil	*Per cents Answering 0-3 Items Correctly*				*Per cents Matching Specific Numbers Correctly*		
	0	1	2	3	4	5	7
Rural (365)	13.9	13.9	32.3	39.7	69.0	79.5	49.3
City (327)	12.8	11.6	30.3	45.3	72.5	80.1	55.4
Total (692)	13.4	12.9	31.4	42.3	70.7	79.8	52.2

The city children were better able to match the three numbers in the test than were the rural children; 45.3 per cent of the former scored successes on all three numbers, as compared with 39.7 per cent of the rural children. Moreover, this advantage is consistently in the favor of the city children in the case of each of the numbers, though the difference in the case of the number 5 is negligible.

7. *Number Combinations or Facts, and Their Use in Problems*

The ability of school entrants to deal with number combinations or facts (the terms will be used interchangeably) has been studied in various ways, and the investigations unfortunately have for the most part dealt with different combinations. Comparisons of results are therefore of limited value. For the purposes of the present summary the studies are grouped and the findings reported under three

[12] Russell (42) had his subjects in his second experiment tell whether groups of objects were the "same" in size or "equal." Possibly success means the ability to match; but in any case his data as reported cannot be compared with those obtained in the present study.

heads: (a) the number combinations when represented as a whole or in part by means of concrete objects; (b) the number combinations when presented in verbal problems, and (c) the number combinations when presented abstractly.

a. The number combinations when presented concretely.—Both in the main Buckingham and MacLatchy study (12) and in their Cincinnati study ten addition combinations were presented individually to children, first, in an "invisible" test and, then, for children who failed in the "invisible" test, in a "visible" test. The procedure was as follows: In the case of the combination 2 + 2, for example, a child was first shown two small objects which were then covered; next, he was shown two more small objects which were also immediately covered; then he was asked, "How many oranges are two oranges and two more oranges?" If the child scored a success in his answer, he was given the next combination in the same way. If, however, a child could not give the answer, he was given a "visible" test on the same combination. This time groups of small objects representing both terms of the combination were exposed and left exposed until the child arrived at an answer. Thus in the "visible" test both numbers involved were present together for the child's use.

The Cleveland study (12) included only five combinations which are identical with those in the preceding studies, and data are reported for these combinations alone. The first trial in this study was similar to the "invisible" test already described.

Grant (15) made use of pictures instead of real objects. The subjects were instructed to look at a row of ten apples, after which the examiner said, "I had one apple and mother gave me two more; think how many I would have, and mark the number of apples I would have then." The requirements of the subjects here were more like those of the Buckingham and MacLatchy "visible" test than of their "invisible" test in that all the objects were continuously present before them. On the other hand, the Grant test was probably harder in that (a) the groups of objects for the two terms of the combination were not separated and identifiable as such and (b) the total had to be isolated from a larger total which was also continuously present. On the other hand, the Grant subjects did not have to record their answers in any numerical form, either written or oral, and this fact probably compensated in part for the greater difficulty caused by (a) and (b) just mentioned.

Table 13 contains the per cents of correct responses in the four studies described above. For the Buckingham and MacLatchy studies the first column of figures refers to the "invisible" test and the second column to the total of successes on both the "invisible" and the "visible" test. The assumption here is that those who passed the "invisible" test would certainly have passed the "visible" test also, had they taken the latter test as well.

TABLE 13
SUMMARY OF FINDINGS WITH RESPECT TO NUMBER COMBINATIONS WHEN CONCRETELY PRESENTED

Number Combination	*Per cents of Subjects Successful*					
	Buckingham and MacLatchy					
	Main Study (1,356 school entrants)		*Cincinnati Study (1,123 school entrants)*		*Cleveland Study* (313 kindergarteners)*	*Grant† (563 school entrants)*
	Invisible	*Visible*	*Invisible*	*Visible*	*Invisible*	
2 + 2.....	66	89	70	90	66	....
8 + 1.....	45	76	46	76	..	....
6 + 1.....	51	77	55	81	..	....
1 + 7.....	53	80	54	82	53	....
3 + 1.....	64	89	71	90	..	....
2 + 4.....	40	72	39	82	40	....
2 + 8.....	37	73	36	74	37	....
2 + 6.....	50	78	37	76	50	....
3 + 7.....	33	73	27	72	..	....
4 + 6.....	32	72	27	73	..	....
1 + 2.....	..	..	..	..	..	32, 56, 67
3 + 4.....	..	..	..	..	..	12, 36, 51
6 + 6.....	..	..	..	..	..	11, 30, 50
5 — 2.....	..	..	..	..	..	17, 39, 54
3 — 1.....	..	..	..	..	..	19, 44, 69

* O'Connor's figures reported by Buckingham and MacLatchy (12).
† The three figures for each combination stand for the records made by the three intelligence groups: 145 pupils with IQs less than 90, 252 with IQs between 90 and 109, and 166 with IQs of 110 and higher.

The first three studies in the table are roughly comparable as to procedure, and the results obtained in them are in extraordinarily close agreement. A fair summary is that a third or more of the children (except in the Cincinnati study, for 3 + 7 and 4 + 4, which were "hard" combinations) were able to get answers for the combinations in the "invisible" test, and that this proportion of children increased to about three fourths when the combinations were "visibly" presented. The per cents for the addition combinations in the Grant study are lower if they are compared with the figures for the "visible" tests in the other three studies, but are about the same if they are compared with the results of the "invisible" test. The same may be said with respect to the two subtraction combinations in the Grant study.

None of the investigators in commenting upon his results interprets these facts to mean that such-and-such per cents of children "knew" these facts; they stay well within the conditions under which their data were obtained when they say that such-and-such per cents were able to "get the answers" or were "able to handle" the combinations as they were presented. Others in citing these figures have implied that this research "shows that children on entering Grade I already know many of their addition (or addition and subtraction) combinations." Statements of this kind are wholly unwarranted. It is one thing for a child to "get a correct answer" for a combination or for a problem containing a combination when he is allowed to see or to manipulate representative objects and quite another thing to do so when he can automatically and meaningfully supply a correct abstract answer for a combination abstractly stated.

b. The number combinations when presented in verbal problems.—In six studies number combinations, predominantly in addition and for the most part differing considerably from study to study, have been presented to school entrants as parts of verbal problems.

Five investigations on the use of number combinations in verbal problems will be later considered together. The sixth study, by Woody (56), is treated first and separately, chiefly because the findings cannot be broken down for comparison with those in the other five. The Woody inventory, given to children from the kindergarten through Grade II (but always to the class immediately preceding that in which systematic instruction was offered), contains eighteen verbal problems. These range from simple one-step problems, five in addition and five in subtraction (like "How many pennies are 2 pennies and 1 penny?" and "If you have 2 pennies and give 1 penny away, how many pennies do you have left?")—from problems as easy as these to two one-step problems in column addition, to four one-step problems in higher-decade addition, to two two-step problems in addition. The per cent of success for the 1,666 Grade IA pupils is 39, but obviously this measure tells little about success or failure on particular combinations as presented in verbal problems.

In the present study two addition combinations ($2 + 2$, $2 + 3$) and two subtraction combinations ($3 - 2$, $4 - 3$) were presented individually to 633 school entrants. The actual problems used follow:

1. On Mary's birthday her daddy gave her 2 dollars and her aunt gave her 2 dollars more. How many dollars did she have then?
2. Mary had 3 dolls. She put 2 of them to bed. How many stayed up?
3. Two boys came to play with Tom one day. Then 3 more boys came. How many boys in all came to play with Tom?

4. Tom put 4 pencils on the table. Three rolled off. How many were left on the table?

The results obtained from this testing are summarized in Table 14. Only a few more than 10 per cent of the children were able to give correct answers for all four problems, and slightly less than a third were able to answer as many as three. The city children were somewhat superior to the rural children, both in the function as a whole and in each of the separate problems. Two of the problems were correctly solved by about half the children, one of these being in addition (2 + 2) and the other in subtraction (3 — 2). The other two problems (2 + 3 and 4 — 3) were solved by a few more than a third of the children.

TABLE 14

PRESENT STUDY: RESULTS ON NUMBER COMBINATIONS IN VERBAL PROBLEMS

Type of Pupil	*Per cents Solving 0-4 Verbal Problems Correctly*					*Per cents Answering Specific Combinations Correctly*			
	0	1	2	3	4	2 +2	3 —2	2 +3	4 —3
Rural (337)	22.3	24.6	24.0	19.9	9.2	51.6	47.5	35.0	35.0
City (296)	15.9	20.6	29.4	21.9	12.2	56.4	57.4	39.9	40.2
Total (633)	19.3	22.7	26.5	20.9	10.6	53.9	52.1	37.3	37.4

As has already been explained, the test used in the present study was originally planned for the special purpose of classifying children very crudely at the beginning of a particular course of instruction. The sampling of number combinations is therefore much too limited to serve as a means of inventorying the ability of children in general with respect to the number combinations. The Buckingham and MacLatchy and the Cincinnati studies, on the other hand, were intended primarily for this wider purpose.

Buckingham and MacLatchy (12) made use of ten number combinations, all in addition, in as many verbal problems.[13] The problems deal with familiar situations and with easily imagined objects (pencils, apples, marbles, paper dolls, books, oranges, pennies, jacks, and the like). The ten problems are read or stated to children one at a time, and responses are scored, as in the present study, as successes or failures. The data from these two studies are reproduced in Table 15, along with the corresponding data for the four identical number combinations used in the Cleveland study.

[13] MacLatchy (32) has summarized these and other data for a hypothetical group of thirty-five first graders.

TABLE 15

SUMMARY OF FINDINGS WITH RESPECT TO NUMBER COMBINATIONS WHEN PRESENTED IN VERBAL PROBLEMS

Number Combination	*Buckingham and MacLatchy*			*Grant (563 school entrants)*			*Present Study*
	Main Study (1,356) school entrants)	*Cincinnati Study (1,123) school entrants)*	*Cleveland Study* (313) kindergarteners)*	IQ 90— (N = 145)	IQ 90-109 (N = 252)	IQ 110+ (N = 166)	*(633 school entrants)*
5 + 1......	72	72	48	..	..	..	..
7 + 1......	64	65	30	..	..	..	..
1 + 9......	49	53	..	..	..	..	..
4 + 4......	37	35	37	..	..	..	..
1 + 6......	49	51	..	..	..	..	..
5 + 2......	44	43	..	..	..	..	..
8 + 2......	44	43	36	..	..	..	..
4 + 5......	22	22	..	..	..	..	..
5 + 3......	32	34	..	..	..	..	..
3 + 5......	27	28	..	..	..	..	..
10 — 2†......	..	..	..	17	26	35	..
3 x 2......	..	..	..	0.4	18	31	..
2 + 2......	..	..	..	..	..	..	54
2 + 3......	..	..	..	..	..	..	37
3 — 2......	..	..	..	..	..	..	52
4 — 3......	..	..	..	..	..	..	37

* O'Connor's figures reported by Buckingham and MacLatchy (12).
† The test item actually involves 10 — 2, though Grant has reported data for 10 — 1. It is assumed that 10 — 1 is a typographical error.

Data for two of Grant's number combinations (15), 10 — 2 and 3 x 2, are included in Table 15. Yet, these data are not exactly comparable to those from the other studies. Grant's subjects, once they had decided upon an answer, were required to identify that answer from among five numbers printed in the booklet. Thus the answer for the multiplication problem (6, for 3 x 2) had to be selected from among 9, 7, 10, 6, and 15; and the answer 8, for 10 — 2, had to be selected from among 8, 9, 2, 6, and 4. Errors might therefore have resulted, not from inaccurate solutions, but from inability to recognize the written symbols for the answers.

As has been noted before, the per cent figures for the two Buckingham and MacLatchy studies are in remarkable agreement. The figures for the Cleveland subjects are the same for 4 + 4, slightly lower for 8 + 2, and considerably lower for 5 + 1 and 7 + 1. One explanation for the lower successes is suggested by Buckingham and MacLatchy: apparently the Cleveland problems dealt with less familiar situations or with less easily imagined objects. Thus, the problem for 7 + 1 was:

Bobbie is seven years old. Tom is one year older. How old is Tom?

None of the four combinations employed in the present study were also employed in any other investigation, but the figures for the two addition combinations (54 per cent and 37 per cent) are not seriously out of line with those in the first three columns of the table.

The facts in the table as a whole may be summarized as follows: for the addition combinations involving 1 (e.g., 5 + 1, 1 + 9) the median per cent of successes is slightly more than 50; for the addition combinations involving 2, the median per cent is better than 40; for the remaining addition combinations, the median per cent is about 30. These medians for addition combinations in verbal problems are almost identical with the medians for similar groups of addition combinations when they were presented in "invisible" tests (Table 14), but they are of course much lower than the median per cents for the same combinations when presented in "visible" tests. (In the latter case, the median per cents are 80, 75, and 72, respectively, for the three groups of combinations.) To conclude this summary: obviously the number combinations in subtraction and multiplication are too few to furnish data of any importance.

Success in dealing with number combinations in verbal problems would seem to be much more difficult than success when the number combinations are presented concretely by means of groups. In solving verbal problems of the kind covered by Table 15 the child is without external cues as to the content of the numbers involved and must therefore supply their content himself. This he may do (a) by using abstract concepts of the numbers as units (thus, "five," for example, as a group), (b) by developing clear mental images of groups of objects (including two or more subgroups for each number) which he then combines directly, or (c) by substituting in thought aggregates of single objects totaling the two numbers and then by counting together these substitute mental objects.

In the "visible" tests no such abstract concepts, images of groups, or images of separate objects were required: actual objects were present to sense, and, accordingly, the children were in general much more successful in this kind of test. The fact that some children were equally competent in the "invisible" tests and in solving verbal problems (it being assumed that the number combinations were of about equal difficulty) may mean that they actually employed about the same mental processes in the two situations. In the "invisible" tests the children saw the first group, remembered that (had a clear image of it), saw the second group, remembered that (had another clear image), and then combined these images directly or by counting. In the case of the verbal problems they could not of course start with external groups, but they could have developed clear images of the groups and dealt with these images by direct combination or by counting. It will be noted that practically all the problems used in the various studies were carefully prepared to enable children to conjure up precisely this kind of subjective substitute for actual objects.

Unfortunately, in none of the investigations summarized in Table 15 were data collected on children's thought processes in dealing with the number combinations. Woody (**56, 58**), however, reports a few facts in this connection and stresses the importance of evaluating performance, not merely in terms of correctness of response, but also in terms of the maturity of the process used by the child. Data of this kind are badly needed in order to combat misinterpretation of the results of the investigations which have been summarized above. This point was made in the section immediately preceding, but it can bear restatement. It is a serious error to infer that because a child can furnish the correct answer for the combination $3 + 4$ in a problem, he "knows" that combination. If the child has substituted separate mental objects and counted them ("one, two, three, . . . seven," or "three, four, five, six, seven"), he has not, in strict literalness, reacted to the combination as such at all. What he has done is merely to reveal that he has *some* way of dealing with the quantitative situation in an effective, if immature, manner. Only further observation and questioning can reveal the precise nature of that "way." And this observation and questioning are indispensable if teachers are to understand how children develop number concepts and how they come to an understanding and meaningful mastery of the number combinations (**5**).

What has just been said has reference to the child who approaches number in what may be called a "natural" way, through the extension and refinement of procedures which are at first crude, uneconomical, and immature, and who later surrenders these procedures for others which are more refined, economical, and mature. But not all children take this route to number knowledge. They are sometimes misdirected by well-meaning efforts on the part of older children or adults. In such cases rather peculiar consequences follow. In the present study several children were found who were better able to supply answers for abstract combinations than for combinations in verbal problems. When the combinations were presented in abstract form, they gave the answers glibly and correctly; but when combinations of supposedly equal difficulty were presented in verbal problems, they failed.

These children appear to have been coached to memorize the combinations as abstract facts (a practice which is by no means uncommon in classroom instruction, but which was hardly anticipated in the case of school beginners). When the combinations are presented in abstract form, they are able immediately to produce answers; but when the combinations occur, not as simple uncomplicated number statements, but as parts of verbal descriptions as in problems, they cannot identify the combinations as such; and, having no other means of dealing with the numbers, they cannot supply the answers.

c. The number combinations when presented in abstract form.[14]— Eight number combinations, half in addition and half in subtraction, were presented to the subjects in the present investigation. In addition, the question took the form, "How many are . . . and . . .?" In subtraction this question was, "How many are . . . take away . . .?" No objects were used; the combinations did not occur in verbal problems; they were merely stated abstractly as questions.

According to Table 16 nearly a third (29.5 per cent) gave the correct answers for half of the facts. The per cent for the city children was 32.9, and for rural children, 26.8. As a group, the addition combinations were easier than the subtraction combinations; the mean per cents of children succeeding on the two groups of facts are 42.8 and 27.3, respectively.

As would be expected, the abstract combinations were in general too difficult for these school beginners to negotiate. Only two combinations, $1 + 2$ and $2 + 2$, were correctly dealt with by as many as half of the children. Moreover, it would be stretching probability to infer from these results that even those relatively few children who answered three or more, actually "knew" the combinations in any real sense of the word. Attention has already been called to the fact that many of them evidently recalled memorized (if meaningless) answers which they had picked up somewhere. Others of them undoubtedly counted some kind of images to secure the answers. Still others were lucky in guessing.

[14] Woody (56) collected data both on the abstract addition and on the abstract subtraction facts. In both cases, however, the facts were included in a test which involved more advanced forms of the two processes. It is impossible from his figures to secure measures of "knowledge" of the subtraction combinations, but his later report (58) carries facts on the addition combinations. These are summarized in the section above.

TABLE 16

PRESENT STUDY: RESULTS ON THE NUMBER COMBINATIONS IN ABSTRACT FORM

No. of Different Combinations Answered Correctly	*Per cents, According to Type of Pupil*		
	Rural (N = 337)	City (N = 296)	Total (N = 633)
0........	14.5	12.5	13.6
1........	18.4	14.9	16.7
2........	21.7	19.9	20.9
3........	18.7	19.9	19.3
4........	8.0	7.8	7.9
5........	6.5	6.2	6.3
6........	4.2	5.4	4.7
7........	3.6	5.1	4.3
8........	4.5	8.4	6.3

No. Combinations in Test	*Per cents Answering Correctly, According to Type of Pupil*		
	Rural (N = 337)	City (N = 296)	Total (N = 633)
1 + 2.....	49.9	53.1	51.3
2 + 2.....	52.5	55.5	53.9
1 + 3.....	34.1	38.9	36.3
3 + 2.....	26.4	33.4	29.7
2 — 1.....	37.4	42.2	39.7
3 — 2.....	29.1	35.8	32.2
4 — 3.....	19.6	25.7	22.4
5 — 2.....	19.0	27.9	22.7
....	...	...	...

No data were recorded with respect to the incorrect answers given, but certain interesting facts were observed. There were wide differences among the children in the extent to which their answers erred. Some of them guessed blindly: 3 + 2 were "sixteen," or "twelve," or "twenty"; 4 — 3 were "ten," or "fifteen," or "nine." Others were able to locate their answers very close to the true sums or differences: 3 + 2 were "four," or "six"; 4 — 3 were "two," or "three." The latter children seemed to have mapped out the number system well enough to be able to approximate answers. The former children, on the other hand, had little idea of the comparative size of numbers or of their relative places in the number series. At this point valuable research might well be undertaken, to determine the commonness of the practice of "locating answers in their appropriate area" and the relation between different degrees of this ability and subsequent success in learning the number combinations. It is not unlikely that experiences in "approximating" answers might well be started very early, as a preliminary to the closer study of the number combinations as such.

Data on sixteen addition combinations are reported by Woody (58) for the five half-grade groups to whom his inventory test was administered. Since interest is largest in his Grade IB subjects who were just entering school, the figures for this group are briefly summarized below in terms of per cents of correct responses, and the records for these 604 children are compared with those for Woody's 1,897 Grade IA children. Of the sixteen combinations in the inventory, the best "known" was 2 + 1 (52 per cent in Grade IB, 78 per cent in Grade IA); next in order of increasing difficulty was 2 + 2 (42 per cent in Grade IB, 72 per cent in Grade IA); next, 1 + 7

(36 per cent and 65 per cent); then, 0 + 2 and 7 + 1 (35 per cent in Grade IB, 56 per cent and 65 per cent, respectively, in Grade IA); 6 + 2 (27 per cent and 57 per cent); 9 + 1 and 5 + 5 (25 per cent in Grade IB, and 85 per cent and 54 per cent, respectively, in Grade IA); 6 + 3 (24 per cent and 53 per cent). The combinations 5 + 4, 6 + 4, and 3 + 9 were correctly responded to by between 10 and 18 per cent in Grade IB; the combinations 5 + 7, 5 + 8, 7 + 6, and 9 + 8, by between 5 per cent and 9 per cent. The only combination contained both in the present study and in the Woody investigation was 2 + 2, which was correctly answered by 54 per cent of school entrants in the former and by 42 per cent in the latter investigation.

Conclusions with respect to the number combinations.—It is possible to read both too much and too little significance into the research findings on the ability of school entrants to deal with the number combinations. One person emphasizes the proportions of children who secured answers to the various combinations, pays no attention to the methods used by children to get their answers, and recommends immediate instruction on the abstract combinations as such. The other points to the fact that the combinations included in the testing are all among the simpler ones, stresses the proportion of children who can do nothing with the combinations, and argues for postponement of all instruction on the combinations as such.

Neither extreme view is calculated to promote sound instructional procedures. It is true that rather surprisingly large numbers of these school entrants managed somehow to get answers to the combinations presented; it is true that these combinations are among the easiest; and it is true that probably few, if any, of the subjects found answers by mature methods of thinking. These statements lead, so it seems to the writer, to the conclusion that to be profitable, the experiences (if any) which first-grade children should have with the combinations must be well chosen and wisely directed. The adjectives in this sentence are worth noting.

PART II. TOPICS 8-17

8. *Fractions*

Three studies are summarized in this section, namely, those made by Woody, by Grant, and by Polkinghorne.

Woody's study (56, 58) involved far more subjects than did either of the other two, a total of 3,902 children, divided among five groups as shown below, but none of them as yet exposed to systematic arithmetic instruction. The first series of three items called for the identification of pictures showing apples cut into halves, into fourths, and into thirds. His data for these three items (**56**) are given below in per cents of successful responses, the results for boys and girls being combined:

Fraction	*Kindergarten (N = 94)*	*Grade IB (N = 594)*	*Grade IA (N = 1896)*	*Grade IIB (N = 80)*	*(N = 238) Grade IIA*
$\frac{1}{2}$	64	67	77	82	88
$\frac{1}{4}$	45	52	67	69	74
$\frac{1}{3}$	40	45	65	61	76

Stated briefly, two thirds of school entrants (Grade IB) could deal satisfactorily with $\frac{1}{2}$, about one half with $\frac{1}{4}$, and nearly one half with $\frac{1}{3}$.

The second series of three items took the form, "Into how many halves (thirds, fourths) can you cut an apple?" The results for these items, reported as are those for the first three items above, are:

halves	48	44	48	44	45
thirds	26	29	35	43	46
fourths	21	22	34	29	47

Again, the concept of halves was better understood than that of either of the other fractions, and the concept of fourths better than that of thirds. The per cents for thirds and fourths are considerably below those for halves except in Grade IIA.

The third series of Woody's items called for the comparison of the fractions $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{3}$ as to relative size. Eighty per cent or more of his subjects, regardless of half-grade, answered correctly that a whole is larger than a half; between 40 and 50 per cent up to Grade IIB knew that a half is larger than a fourth; the other comparisons ($\frac{1}{2}$ and $\frac{1}{3}$, $\frac{1}{4}$ and $\frac{1}{3}$, and $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{3}$) were successfully dealt with by only about one third prior to Grade IIB. Woody wisely cautions against inferring from these objective data alone that children in the primary grades actually possess rich concepts of these unit fractions even in the limited ways in which they were tested, and he reports remarks made by his subjects which indicate that success in not a few instances was more or less accidental or the result of but very superficial knowledge.

Item 27 of Test 5 of the Metropolitan Readiness Test used by Grant (**15**) reads as follows:

Look at the row of four circles. Mark *half* of all the circles—just half of the circles.

Item 28 reads:

Look at the row of circles with lines in them. Find the circle that is marked in *halves,* that is cut in *half,* and mark it.

Grant has reported data for "half," but whether his data refer to the results on item 27, on item 28, or on the two combined, it is impossible to say. Whatever the basis, his figures show that "half" was successfully negotiated by 55 per cent of his 145 "dull" pupils (IQ below 90), by 76 per cent of his 252 "average" pupils (IQ 90-109), and by 83 per cent of his "bright" pupils (IQ 110 and above). Item 27 applies the fraction ½ to a group of objects; item 28, to single objects. Obviously, these few data, particularly since their reference is ambiguous, tell little about the ability of school entrants to deal with fractions.

The most thoroughgoing investigation in this area is that made by Polkinghorne (40); yet, good as was this study, it is subject to limitations which affect the general validity of the results. In the first place, only part of her 226 subjects were school entrants. Part were kindergarten pupils, and part were enrolled in Grades II and III. Nevertheless, the data for the group as a whole are here treated together, since whatever the children could do with the test, they had learned to do without systematic instruction. In the second place, the test has not been printed. Only a sample of the forty-two items is available, and lack of information at this point makes interpretation difficult. In the third place, the subjects were undoubtedly of superior ability, having been drawn from the University Laboratory School of the University of Chicago. In the fourth place, in the brief article where the findings are reported, the data are treated in a way which makes impossible the kind of analysis desirable for the present purpose.

The forty-two items of the test, which was given individually to each child, are classified under the heads: unit fractions (such as ½ and ¼), other proper fractions (e.g., ⅖, ¾), improper fractions, identification of fractions, and equivalent fractions. The response for each item was either numerical or a performance which could be scored objectively. Examples are: "Here are two pencils. Will you give me one half of them?"; "I am going to draw a line [draws a line about six inches long]. Draw another line one half as long"; "Here is a drawing of four marbles. Color three-fourths of them."

Fifty-three per cent of the unit fraction items were correctly answered by the group as a whole. The next largest per cent was for other proper fractions—18 per cent. The per cents for improper fractions and identification of fractions were both 8, and that for equivalent fractions, 0. The kindergarten children averaged 3.7 correct items, all dealing with unit fractions; the first-grade children averaged 5.9 correct items, all of them again on unit fractions. The average score on unit fractions for second-graders was 9.4, and for third-graders, 11.9. With other proper fractions, the averages were, respectively, 1.7 and 3.8. The Grade II children had negligible success with the other three classes of items. In the case of Grade III children, the average scores on the three last classes were 1.2, 1.6, and 0.0. When the subjects were grouped according to mental age (5-7, 7-9, 9-11, and 11-13), scores comparable to those for grade groups were made.

Polkinghorne concludes that in the first grade unit fractions may be safely presented, first in connection with single objects, next in connection with groups, and finally for comparing, first, two objects, and, then, two groups of objects. In Grade II fraction concepts may be extended to include proper fractions other than unit fractions, and a beginning may be made in the understanding of improper fractions. Whether or not these recommendations, made on the basis of testing rather than of teaching and on the basis of testing superior children, are valid for schools in general is a question for future research. Data will be presented in Chapter III to show that at least some of these concepts are well within the powers of first- and second-grade children.

9. Ordinals

Grant (15) has reported the only sizeable study of school entrants' knowledge of ordinals. For this purpose he used items 20 and 21 of Test 5 of the Metropolitan Readiness Test.

20. See the farmer feeding the chickens. Mark the *third* chicken from the farmer.
21. See the pigs running to the fence. Mark the *sixth* pig from the fence.

The results of the testing may be summarized as follows:

Ordinal	*"Dull" pupils IQ less than 90 (N = 145)*	*"Average" pupils IQ 90-109 (N = 252)*	*"Bright" pupils IQ 110+ (N = 166)*
third	19	39	71
sixth	21	44	67

10. Reading and Writing Numbers

Reading numbers.—The study by Grant (**15**) samples the number series to 100, testing only on the numbers 4, 9, 16, and 75. The study by Wheeler and Wheeler (**54**) gives data on all the numbers to 100. The item for 4 in the Grant investigation is typical: "Look at the row of numbers where the hand is. Mark the *4*." To be successful the child must select the 4 from among the numerals 6, 3, 4, 8, and 9. Grant's findings, in per cents of successful responses, are as follows:

	"Dull" pupils (N = 145)	*"Average" pupils (N = 252)*	*"Bright" pupils (N = 166)*
4	26	60	83
9	34	48	63
16	19	40	67
75	21	43	62

The data from Wheeler and Wheeler are too numerous to present in full. They were obtained individually from 157 first-grade children within a month of the start of the first term. The numbers from 1 to 9 were known by an average of 64 per cent of the children (4, by 68 per cent; 9, by 33 per cent); the numbers 10 to 19, by 15 per cent (16, by 8 per cent); those for 20 to 29, by 13 per cent; and so on. The per cents for the numbers for the decades above the 20's range from 6 (for the 50's) and 8 (for the 30's) to 13 (for the 70's and 80's). The median score obtained was 9.21 (one point being allowed for each correct response); the range was 0-100, with Q_1 at 0.99 and Q_3 at 11.8.

Woody's inventory test (**56**) contained thirty items on the reading of numbers. Six exercises were based upon the numbers in a calendar and involved the numbers 3, 8, 6, 11, 23, and 28. In another exercise the subject was asked to identify six of seven numbers printed in a row; these numbers, in the testing presented at random, were: 37, 19, 72, 41, 131, and 113. The next exercise consisted in twelve numbers, ranging from 13 to 10,103, which had to be read exactly, with all such words as "thousands" and "hundreds" correctly placed. Finally the subject had to locate correct pages in a book, these pages being numbered 13, 21, 70, 57, 89, and 123. The thirty items were presented to 1,897 pupils in Grade IA, but before they had been subjected to systematic instruction. The results are not analyzed for the separate numbers, only the per cents of success by city (there were eleven cities) being given, together with that for the total, which was 63 per cent.

So far, only the ability to recognize or identify the numbers as such has been considered. A child had only to say "thirty-six" upon being shown "36" to score a success. Knowledge of the meaning of the number played no part. The Grant study contains data on the ability of school entrants to "interpret" the numerals 4 and 7. The item for 7 is much like that for 4, which is: "Now look at the next number [it is 4]. Make as many dots after the number as it tells you to make." Under these conditions Grant's pupils were less successful with 4 than they had been in "reading" 4: the per cent of "dull" pupils fell from 26 to 10; of "average" pupils, from 60 to 40; and of "bright" pupils from 83 to 70. Unfortunately, there are no such comparable "reading" data for 7; as "interpreted" by the three groups of children, the per cents were 9, 33, and 61. Nevertheless, the effect of calling for "interpretation" in the case of the numeral 4 may be enough of a caution against the acceptance of ability to "read" numerals as equivalent to a grasp upon their meaning.

Writing numbers.—Grant (15) had his subjects try to make the numerals for 4, 7, 2, 5, 9, and 6. The per cents of success for his "average" pupils were: 30, 17, 27, 22, 9, and 21 (mean, 21). Arranged in order, the numerals were, from easiest to hardest: 4, 2, 5, 6, 7, and 9. The mean per cent of the "dull" pupils for the six numerals was 6.5, and for the "bright" 49.8. For the last named group, the order of difficulty was: 5, 4, 2, 6, 7, and 9. Grant's figures reveal a rather surprising degree of ability to write the numbers prior to school instruction, particularly in the case of the "bright" pupils.

11. Recognition of Geometric Forms

Grant's study (15) reports data on the ability of his 563 subjects to recognize figures in the shape of a *square,* a *circle,* and a *triangle.* In each case the correct form had to be selected from a group of four diagrams, the correct one and three others. The per cents of success follow:

Term	*"Dull" pupils IQ below 90 (N = 145)*	*"Average" pupils IQ 90-109 (N = 252)*	*"Bright" pupils IQ 110+ (N = 166)*
square	54	79	92
circle	48	76	85
triangle	10	19	34

Three fourths or more of Grant's school entrants already knew the terms *square* and *circle* well enough to use them intelligently. The case with respect to *triangle* is quite different. Clearly, these children did not know the term, even to the extent required by the test. How much of a task it would be to teach its meaning to first-grade children is not known. Their ignorance, according to Grant's data, may be the result (a) of lack of experience with the term and the corresponding form, (b) of real difficulty in acquiring the concept, or (c) of both (a) and (b).

12. Time, U. S. Money, and Measures

Included among the 204 items in Woody's inventory (**56**) are eight dealing with time, thirteen with U. S. coins, and eight with linear and liquid measures. Table 17 summarizes the results of testing 1,897 IA pupils who had had no systematic instruction in arithmetic, the items in the table having to do with the questions on time and on U. S. coins. About half of these children could tell time on the hour, and about four out of ten could locate the position of the long hand on the clock at the even hour. Time at the half- or quarter-hour was pretty well beyond their powers. Practically nine out of ten could identify each of the five coins to the half dollar, and

TABLE 17

PER CENTS OF 1,897 GRADE IA PUPILS WITHOUT INSTRUCTION ON NUMBER, WHO HAD VARIOUS CONCEPTS OF TIME AND U. S. MONEY (AFTER WOODY)

Time Items	*Per cent*	*Money Items*	*Per cent*
Tells time on clock 1 (9 o'clock)	51	Which buys more:	
Tells time on clock 2 (1 o'clock)	53	a penny or a nickel?	99
Tells time on clock 3 (12 o'clock)	68	a penny or a dime?	98
Tells time on clock 4 (3:30 o'clock)	15	a dime or a nickel?	89
Places long hand to show 7 o'clock	44	a dime or a quarter?	98
Places long hand to show 5 o'clock	42	a half dollar or a quarter	95
Places long hand to show 10 o'clock	43	How many .... buy the same amount as a ....?	
Places long hand to show 11:45 o'clock	12	pennies ....nickel	85
Money Items		pennies ....dime	75
Points to penny	99	pennies ....quarter	31
Points to dime	97	nickels ....quarter	23
Points to quarter	86	nickels ....dime	59
Points to nickel	95	quarters ......half-dollar	37
Points to half-dollar	88	dimes ....half-dollar	12

about the same ratio knew the gross comparative worth of pairs of these coins; but the precise relationships between the coins, as between nickels and quarters, were not so well known. As would be expected, the numbers of pennies in the nickel and the dime were best known.

Unfortunately, Woody does not report the corresponding data for children in the other half-grades tested, except in the case of telling time (58). In general, pupils in his IIB and IIA groups showed ability beyond that of his IA pupils, for whom the facts are stated above. Only the facts for his 594 IB pupils are given here. About 30 per cent of these children correctly identified the clock showing nine o'clock (data for the sexes are combined), about 49 per cent identified that showing one o'clock, 40 per cent that showing twelve o'clock, 7 per cent that showing half-past three, 24 per cent that showing eight o'clock, 20 per cent that showing five o'clock, 22 per cent that showing ten o'clock, and 8 per cent that showing a quarter to twelve o'clock.

Woody's findings for linear measure and for liquid measure are consolidated into per cents for his Grade IA pupils as a whole and by cities. Forty per cent of the responses to linear measures were correct (the range by cities was 20 to 60 per cent), and 33 per cent of the responses to liquid measures (range, 33 to 67 per cent). The items on linear measures called for the drawing of lines of the following lengths: a foot, six inches, three inches, two feet, and a foot and a half. The items on liquid measure were: "Which holds more, a pint bottle, or a quart bottle?" "How many pints will a quart bottle hold?" "How many quarts in a gallon?" Obviously, success on these items, small as it was, indicates little of the meaning and significance which these measures held for the children tested.

13. Sex Differences

Buckingham and MacLatchy (12) found a small but rather consistent superiority of girls over boys in the various functions which they studied. In rote counting by 1's the median limit attained by boys was 24.5, and by girls, 29, while 27.3 per cent of the girls achieved the limit of 50 as compared with 19.1 per cent of the boys. Similarly, in rote counting by 10's the boys fell behind the girls, but by a very narrow margin. In enumerating twenty objects 58.2 per cent of the boys and 67.0 per cent of the girls were successful; and 85.3 per cent of the boys and 94.1 per cent of the girls enumerated at least ten objects.

More girls than boys were able to reproduce each of the numbers 5, 6, 7, 8, and 10, the margin of difference being about 5 percentage points (thus, 81.0 and 85.8 per cent for 5). This superiority for girls was maintained, and by the same margin, in identifying each of the same five numbers.

The median percentage scores made by boys and girls were the same in solving ten verbal problems with simple addition combinations and in finding answers for ten other addition facts when "visibly" presented. When the "visible" and the "invisible" presentations were combined, the girls excelled the boys by from 1 to nearly 8 percentage points on all ten of the latter set of addition combinations. Buckingham and MacLatchy conclude that, however consistent the advantage held by the girls, it is small and completely without significance from the standpoint of classroom instruction.

Woody's findings (**56, 58**) with respect to sex differences are unlike those just presented, for in his investigation the advantage lay with the boys. When the scores for the whole test were tabulated, the boys surpassed the girls in all half-grade groups except Grade IIA. None of the differences, however, were statistically reliable. About the same relationships were found when the tabulations were restricted to six-year-olds regardless of grade. The superiority of the boys was more marked in the parts of the test devoted to time, money, etc. Higher per cents of boys than of girls passed each of the eight items on time; in the identification of coins (five items) the boys equaled or surpassed the girls in each case; and the same was true on the twelve items relating to the comparative value of coins.

In general, the problem of sex differences is of less interest now than it was some two decades and more ago. The differences found between the sexes are much less in amount than the differences between individuals within a given sex. That is to say, usually nearly 50 per cent of one sex will be found to equal the median for the other sex. Where differences are found, they are slight and can reasonably be attributed to differences in cultural stimulation, rather than to intrinsic differences of a biological sort.

14. City versus Rural Children

In the original research reported in Part I of this chapter the city children surpassed the rural children rather consistently in the arithmetical functions tested. The advantage lay clearly with the city children in: rote counting by 1's, enumeration of ten objects, reproduction of five, six, and nine objects, crude comparisons with concrete or pictured objects, exact comparison of pictured objects, and the number combinations whether presented in verbal problems or as the abstract facts. The advantage for the city children was not so clear in the identification of four, seven, and ten objects and in crude comparisons involving abstract numbers.

MacLatchy (28) has reported an analysis of the data on ten addition combinations obtained from 1,226 school entrants in ten Ohio cities, 1,123 school entrants in Cincinnati, and 130 school entrants in a number of one-room schools. In every combination except one, the city children excelled by margins of from 6 to 17 percentage points. Thus, the per cents for the three groups just named on the combination $7 + 1$ were 64, 65, and 58; on the combination $1 + 6$, the per cents were 49, 41, and 32. The one exception to this general condition was in the case of the combination $5 + 1$, which was "known" by 91 per cent of the rural children as compared with 72 per cent for the two city groups. No reason either for this exception or for the high degree of success in the case of the rural children is offered by the investigator.

The comparative arithmetical abilities of city and rural children have been treated in other references than the two cited, but for the most part such references and all of the research relate to arithmetic at higher levels than that here under consideration. Future studies may or may not confirm the superiority of city children as mentioned in the foregoing paragraphs, but it is fairly certain (a) that the differences found will not be large, so large as to demand markedly different programs of instruction for the two types of children, and (b) that the reasons for the differences will lie in unlike experiences having their origin in unlike environments.

15. Differences in Levels of Intelligence

As would be expected, such research findings as are available reveal a positive relation between intelligence on the one hand and various arithmetical abilities on the other hand. Evidence with regard to this relation is furnished in two studies in particular.

MacLatchy (30) classified 291 six-year-old school entrants on the basis of their ratings on the Pintner-Cunningham Primary Mental Test, dividing them into the lowest fourth, the middle half, and the highest fourth. The brightest group on the average could count to 27 and "knew" the sums of 6.8 of ten fairly easy addition combinations. The corresponding figures for the other two intelligence groups were: counting to 21 and 4.8 sums for the average group, and counting to 12 and 1.5 sums for the slowest group. The median superior child could reproduce groups of five, six, seven, eight, and ten objects correctly three out of three trials, and the average child, only once in three trials. Ability to reproduce numbers was limited to groups of five for the slowest group, and then only once in three trials.

Grant's data (**15**) have been reported in full in various preceding sections. His subjects were regarded as "dull" if their IQ's were 89 or less, as "average" if between 90 and 109, and as "bright" if 110 or higher. In every one of his comparisons the per cents of accuracy increase from the "dull" through the "average" to the "bright" group. The comparisons cover: identification of groups of three, six, and eight objects; reproduction of groups of four, seven, and thirteen; crude comparisons (such concepts as "as long as," "largest," etc.); number combinations presented concretely ($1 + 2$, $3 + 4$, $6 + 6$, $5 - 2$, and $3 - 1$) and in verbal problems ($10 - 2$ and 3×2); the fraction ½ as applied to a group and to a single object; ordinals (*third* and *sixth*); reading, "interpretation," and writing of numbers, and knowledge of three geometric forms.

Wheeler and Wheeler (**54**) report a coefficient of correlation of .51 between M. A. and ability of uninstructed first-graders to read the numbers to 100.

16. Effect of Kindergarten Instruction

It will be recalled that Buckingham and MacLatchy (**12, 29**) include in their report data from about one thousand school entrants in Cincinnati. Almost exactly two thirds of these entered Grade I after some time in the kindergarten. A comparison of the test results of these kindergarten children with the results of nonkindergarteners should provide some evidence on the effect of the preschool training. Admittedly, nothing is known of the nature of this training in general, to say nothing of the extent to which arithmetical skills were involved.

In rote counting by 1's, 13.4 per cent of the K-group (kindergarten children) and 4.6 per cent of the NK-group (nonkindergarteners) reached 100. The medians for the two groups were 29.3 and 19.1. In rote counting by 10's the K-group held comparable advantages. Of the K-group, 70.3 per cent enumerated twenty objects correctly, compared with 47.7 per cent of the NK-group. Ten objects were correctly enumerated by 93.7 per cent of the K-children and by 82.3 per cent of the NK-children. The K-children both reproduced and identified or named each of the numbers 5, 6, 7, 8, and 10 more successfully than did the NK-children.

The K-children gave correct answers to 5.4 combinations and the NK-children to 3.5 when ten addition combinations were presented in verbal problems. Their margin of superiority ranged from 5 percentage points for $3 + 5$ (29.3 and 24.3) to 20.7 for $7 + 1$ (72.2 and 51.5). When ten other addition combinations were presented "visibly" and "invisibly" the K-group excelled on every combination and by about equal margins.

These figures bespeak far greater ability on the part of the K-children. However, no simple conclusion can be drawn from these figures with respect to the effects of kindergarten instruction. Buckingham and MacLatchy report that the average mental age of the K-children was six years six months, compared with a mental age of five years ten months for the NK-children. Stated in terms of IQs, the difference in average brightness was the difference between IQ 104 and IQ 93. Whatever may be the contribution of each factor, the K-children had advantages not merely in their kindergarten training but also in mental maturity (MA) and in brightness (IQ).

17. Miscellaneous Studies

Preschool studies.—A number of research investigations have dealt with the development of arithmetical abilities of children prior to entering school. These can be but briefly referred to here, since their value is greater for the psychologist than for the practical teacher and administrator.

In the developmental biographies of individual children more or less attention is commonly given to the appearance and use of various arithmetical concepts and skills. Two such studies which deal primarily with aspects of quantitative development are those by Court[15] and by Drummond.[16]

Douglass (14) administered three tests to children aged four and a half to six. The first test consisted in recognizing the number of

[15] Sophie Ravitch Altshiller Court, "Numbers, Time, and Space in the First Five Years of a Child's Life," *Pedagogical Seminary*, XXVII (March, 1920), 71-89. Also by the same author: "Self-Taught Arithmetic from the Age of Five to the Age of Eight," *Pedagogical Seminary*, XXX (March, 1923), 51-68.

[16] Margaret Drummond, *Five Years or Thereabouts* (London: Edward Arnold and Co., 1921), pp. 119-135.

dots in patterns of different sizes; the second, of selecting certain patterns as representing the numbers announced by the experimenter; the third, of estimating the number of marbles exposed momentarily by the experimenter. The conclusion drawn was that the experimental children had "completely accurate concepts of 1 and 2, very serviceable and accurate concepts of 3, and a very serviceable concept of 4, and of 5, 6, 7, 8, 9, and 10 rather vague concepts, though serviceable to a slight degree."

McLaughlin (33) gave individual tests to 125 children enrolled in nursery schools and kindergartens. Their ages ranged from thirty-six to seventy-two months. One series of tests measured ability in rote counting; another, ability to recognize small aggregates of objects; a third, ability to combine numbers under varying conditions. The data, reported only in part in the reference cited, are tabulated for three age groups. Children aged thirty-six to forty-eight months could count by rote to 4.5 on the average, could enumerate an average of 4.4 objects, and could count backwards not at all. The corresponding limits for children aged forty-eight to sixty months were 17.6, 14.5, and 1.6, respectively; for the older children, 33.4, 28.2, and 5.5, respectively. Practically all of the five-year-olds could regularly recognize groups of two and three objects, and sometimes groups of four. The combining of numbers was too hard for three-year-olds, was somewhat easier for four-year-olds, who used counting predominantly, and was much easier for five-year-olds, who used counting to supplement grouping. The reference contains a fairly extensive discussion of the relation between success on the number tasks imposed and the mental processes of the children in arriving at answers.

Ability to compare groups of objects numbering ten or fewer was studied intensively by Russell (42). His subjects in one experiment were thirteen kindergarteners, twelve first-graders, and four second-graders. In this experiment the children were instructed to tell which of two exposed groups of objects was "more." Variations were introduced by using blocks of different sizes and of different colors. These changes seemed to confuse the children, a fact which is perhaps best interpreted (though it is not so interpreted by the experimenter) as meaning that to the subjects the situation had really ceased to be a quantitative one in the ordinary sense. (Binet[17]

[17] Alfred Binet, "La Perception des Longueurs et des Nombres," *Revue Philosophique,* XXX (July, 1890), 68-81.

nearly fifty years before had noted the same phenomenon.) Russell's second experiment involved twenty-five subjects (ten kindergarten, ten Grade I, and five Grade II). This time the children were required to tell when the two exposed groups were "the same" or "equal" in size, the word "more" in the first experiment having proved to furnish false leads. Russell concluded (a) that "manyness" is the first quantitative concept, (b) that cardinal and ordinal number concepts develop together, (c) that counting is not a reliable measure of this development, (d) that children under five years of age understand "most," "both," and "biggest," but not "equal," (e) that seven-year-olds know "more" but not "same" and "equal" at all fully, and (f) that counting as a means of differentiating groups develops late. The last conclusion is understandable, since counting is hardly as direct and serviceable a procedure even with adults as the matching of small groups which can be really apprehended at a glance. The author's views with regard to the late development of counting seem therefore to be in error, since his experimental situations were such as to place counting at a serious disadvantage.

Readiness testing.—The Metropolitan Readiness Tests were administered by Hildreth (18) to two groups totaling about one hundred children in the first month of school. A year and a half later, when one of these groups was in the second grade, they were given a special arithmetic test designed for the program of instruction to which they had been subjected. From Hildreth's description this program appears to have been one based upon "activities" in which considerable use was made of number situations which occurred naturally in the classroom. Comparison of the readiness and the achievement test scores for twenty-six children remaining in this group yielded a correlation coefficient of .50. The other of the two groups, then numbering thirty-three, was given its arithmetic test two and a half years after taking the readiness test. The coefficient of correlation in this case was .58. Neither coefficient suggests much usefulness for this particular readiness test as a means of predicting achievement, however valuable it may be for inventorying purposes. Hildreth points out reasons why the predictions of the readiness test were no higher, but does not mention the fact that to be most effective the readiness test must be closely keyed to the program of instruction which is to follow, as closely as must the achievement test by which the effects of that program are later assessed.

PART III. APPRAISAL OF RESEARCH FINDINGS

Brief Summary of Findings

The research on the arithmetical knowledge and skills of children just entering school is impressive both in its extent and in the facts which it has revealed. Parts I and II of this chapter summarize data from twelve separate investigations relating to twelve different arithmetical topics. As will be pointed out in a later section, some of the studies are open to question on one ground or another; nevertheless, the fact remains that children when they come to school know a great deal about number. It is worth while to classify the research findings in three categories, always remembering that certain limitations attach to this research.

1. The following skills and concepts seem to be quite well developed by the time most children start school:

Rote counting by 1's: through 20 at least
Enumeration: through 20 at least
Identification: through 10 at least (the limit studied in research), and probably through 20
Crude comparisons: a) with objects: the concepts "longest," "middle," "most," "shortest," "smallest," "tallest," "widest"
b) with abstract numbers: "more," with the numbers through 10
Exact comparison or matching: at least through 5 or 7 (the limit of research)
Number combinations: a) with objects: to sums of 10
b) in verbal problems with easily imagined objects and situations: adding 1 and 2, and probably most facts with sums to 6 or 7
Fractions: unit fractions through halves and fourths as applied to single objects, and perhaps halves as used with small groups in even division.
Ordinals: through *sixth*
Geometric figures: "circle" and "square"
Telling time: at the hour
U. S. coins: recognition of all coins to the half dollar, and some understanding of relative values of pennies and other smaller coins

2. The following skills and concepts are not so fully known to school entrants, but are fairly well started among a reasonably large per cent of children of this age:

Rote counting: by 1's, to 100
by 10's, to 100
by 2's, to 20 or 30

Crude comparison: a) with objects: "as long as," "fewest" (or "the smallest number")
b) with abstract numbers: "less," with the abstract numbers to 10

Number combinations: a) in verbal problems: probably all the facts with sums to 9 or 10
b) with abstract numbers: few research data available, but apparently less than 50 per cent able to deal successfully even with the easiest facts (e.g., those involving the addition or subtraction of 1)

Reading numbers: only a few know the numerals to 10

3. The following skills and concepts are possessed by less than a third of school entrants, and then in limited degrees of richness or proficiency:

Rote counting: by 3's, to 30

Crude comparison of objects: "same" or "equal"

Fractions: a) proper fractions other than unit fractions, applied to single objects and small groups
b) improper fractions
c) relative size of fractions

Reading and writing numbers: virtually no ability to read beyond 10; virtually none to write, even to 10

Geometric figures: "triangle"

Telling time: at the half- and quarter-hour

U. S. money: relative value of coins other than pennies

Liquid and linear measures: relative size of units

Research has contributed little or nothing with respect to several traditional topics in the primary course of study. For example, crude comparisons are sometimes made by the use of such terms as "youngest," "oldest," "heaviest," "lightest," "darkest," and the like, on none of which are there research data. Likewise, research is virtually silent in the matter of the subtraction combinations. Too few of these have been included in inventory tests to reveal first-graders' familiarity with them. At other points, for example in the case of fractions, research seems to indicate possible success in teaching ideas now generally withheld to the later grades. In such cases more research is needed, to determine whether the inferences drawn with regard to the probable effects of instruction are sound or unsound.

Limitations of Research

In the sentence immediately preceding the lists of concepts and skills above, a caution was stated to the effect that these research findings must not be accepted too quickly. The investigations which have been summarized are sometimes limited in value because of errors in technique or of insufficient coverage.

In the first place, few arithmetical skills and concepts have been at all exhaustively studied. Reference here is to what may be called the horizontal dimension. Most skills and concepts have been merely sampled, which is to say that they have been studied only in part, in but some of the situations in which they function. An example is the number combinations, only a few of which have been included in research inventories. The few concepts and skills that have been studied more completely, for example, counting and enumeration, are among the simplest and least complicated in the field of arithmetic.

In the second place, a rather large part of the data has been collected by means of group tests. The technique of group testing is somewhat uncertain particularly with pupils as immature as children just beginning Grade I. Every first-grade teacher knows the difficulties of controlling attention of all pupils throughout the testing period; he knows also the extraneous errors introduced by inability to understand and to follow directions and to enter answers in the right places. Group testing (save for multiple-choice types) has, however, one distinct advantage: the results obtained almost certainly understate the actual extent of knowledge. In part, but only in part, this advantage compensates for the incomplete sampling of concepts and skills as these are represented in the tests.

In the third place, in very few investigations have data been obtained on children's procedures in dealing with the number tasks tested. That is, experimenters report the answers children give, but not how the answers have been arrived at. Of course in the case of rote counting and enumeration children necessarily reveal their procedures, but not so in such quantitative feats as comparing numbers, solving combinations,[18] estimating lengths, and the like. Yet, the procedures used by children are essential to a true appraisal of their developmental status. Correct answers can be obtained by immature procedures; incorrect answers may merely mean incomplete command of mature procedures. On this account future research may well include the technique of careful observation and of the interview

[18] The complexity of knowledge involved in meaningful mastery of the number combinations has been analyzed by Brownell. William A. Brownell, "Teaching Meanings," *Foundations in Arithmetic* (Bulletin of the Association for Childhood Education, 1937), pp. 11-15. The whole bulletin will be found of value to teachers interested in developing quantitative meanings.

or conference as means of supplementing the information procurable through group testing.[19]

Cautions in Applications

The research findings summarized in this chapter have been obtained from pupils supposedly typical of all children who enter Grade I year after year. This assumption may not be valid in every case; atypical classes may unintentionally have been selected for testing. But if so, the error is not probably as serious as that arising from a second assumption, namely, that the first-grade pupils of a single class or of a community are necessarily comparable with the research subjects. On the contrary, a local entering class or the classes of a whole system may be quite unlike the experimental subjects. It is therefore unwise, apart from supporting evidence, to regard research findings as directly applicable to any and all school situations indiscriminately.

Research findings tell the teacher or administrator little about the class as a whole, but they tell very much less about the number abilities of particular children. If educational psychology has established any fact, it has established the fact of individual differences. Instruction in Grade I (and for that matter in any other grade) should be based upon accurate knowledge of the abilities of each pupil in the class. On this account careful inventories should be made of the arithmetical concepts and skills of every pupil before teaching is undertaken. The inventory need not—indeed, should not—cover all concepts and skills and be completed all at once. Rather, the inventory should be carried on progressively, by stages. Thus, information should first be obtained concerning ability in the first arithmetical skills or concepts to be taught; the children deficient therein can then be formed into a group for instruction. Meanwhile (or afterwards) the inventory can be extended to cover the next topics. In this way children can eventually by small-group instruction be brought to the point where the class can be taught as a whole.

[19] What is probably the most thorough and penetrating study yet made of the readiness of primary grade children for instruction in arithmetic is unfortunately not available for this summary. It is the doctoral research of Miss Doris Carper and may shortly be obtained from the Duke University Library under the title, "A Study of Some Aspects of Children's Number Knowledge Prior to Instruction." Miss Carper herself interviewed 270 school entrants on a considerable range of number tasks, and her report contains not only the factual data she collected but also a searching inquiry into the procedures employed by children in arriving at their answers. Evidence at this point was obtained of course from the observations and questioning which accompanied the testing.

Significance of Research Findings

It is theoretically possible to take three different positions with respect to the capacity of first-grade children to profit from instruction in arithmetic. According to the first position, first-grade children are too immature for arithmetic; they simply are unable to learn arithmetic. Considerable doubt is thrown on this position by the facts which have been assembled in this chapter. Admittedly, these facts do not completely undermine the position: only direct proof that children when taught arithmetic in Grade I actually learn arithmetic could do this. Nevertheless, the facts concerning school entrants' knowledge of arithmetic warrant the rather confident inference that systematic instruction in this field should yield good returns. In the absence of more direct evidence of an experimental character, it is not unreasonable to believe that children who already know a considerable amount in a given area have thereby demonstrated their ability to learn more.[20]

The second position is to recognize the fact that first-graders already possess a considerable fund of arithmetical knowledge and hence are probably able to extend their learning, but to entrust to "maturation" and to incidental experiences the acquisition of further knowledge. The argument is that since children have already done so well "on their own," they should be allowed to continue their learning on the same basis. Systematic instruction is not needed and may be very harmful.

Nothing in the findings of research is in conflict with this view. As a matter of fact, these research findings are hardly relevant to the issue; they tell what children know and can do when they come to school, but they tell nothing about the way children have come by their knowledge and ability. It is possible to infer, as the supporters of this view seem to do, that children, left to themselves, somehow *grow* into number knowledge. If so, there is every reason to believe that the passing of another year or two will give opportunity for more of this same growth.

But number knowledge is hardly the result of any such growth or inner maturing: it is the result of directed experience, frequently taking the form of direct teaching. Woody (56, 58) has reported certain facts which have not received wide enough attention. Questionnaires were sent to the parents of the first-grade children in the

[20] Dickey in a recent article has rightly challenged the tendency to accept this inference uncritically. John W. Dickey, "Readiness in Arithmetic," *Elementary School Journal,* XL (April, 1940), 592-598.

seven elementary schools of Ann Arbor, Michigan. Replies came from a total of 164 homes. Eighty-three per cent of the parents replying stated that in the home instruction was given in rote counting (54 per cent of the parents taught counting to 100, for example); 78 per cent of these parents taught their children to count objects; 87 per cent, to recognize coins; 68 per cent, to know the value of money; about two thirds, to read and write numbers and to solve simple verbal problems; about one half, to understand the size of numbers, to perform simple additions, and to tell time; about one third, to deal with simple situations involving subtraction.

It would be possible to continue to absolve teachers of the responsibility for number teaching and to leave to parents the task of encouraging further development of number ability. But if Woody's facts mean anything, they mean that *some one* must assume this duty. Mere attainment of more and more birthdays cannot be expected to bring the needed increases in number knowledge, apart from directed experience. It would not of course be inconsistent with the second position to agree that this responsibility is the school's, but to deny that it should be met in the primary grades and to insist that it may more easily and profitably be met in Grade III or even later.

Two positions with regard to the ability of first-grade children to learn arithmetic have been considered. The third position is that the duty of extending and enriching children's number experiences is properly the function of *primary* teachers. Long before this point the reader may have detected evidence that this position is the one entertained by the writer. Briefly, the position is this: Research has shown that school entrants already know much about number; the inference is that they can learn more; society requires that children must know arithmetic; nothing is gained, and much may be lost, if the school delays to later grades the discharging of its obligation.

At later points in the monograph, particularly in Chapter V, this position will be developed and supported at some length. At this point it must suffice to say that one does not need to advocate a return to the unintelligible, abstract drill now happily on the way out; there are other kinds of learning than memorization, and other kinds of learning activities than repetitive practice. Provided that experiences are adjusted to their interests and capacities, first-grade children can and will extend their number knowledge happily, intelligently, and usefully. Evidence in support of this belief will be presented in the next chapter.

CHAPTER III

RESULTS OF A PARTICULAR PROGRAM OF SYSTEMATIC ARITHMETIC INSTRUCTION IN GRADES I AND II

The inference drawn from the research findings reviewed in Chapter II is that children in Grades I and II are ready for systematic instruction in arithmetic. The data reviewed do not in themselves prove that this instruction if given will produce measurable evidence of sound and valuable learning, for the data relate only to the arithmetical equipment with which children start school. Proof that this inference is valid must come from a different kind of research. It must be shown that instruction actually does yield the kind of effects which are desired.

It is the purpose of this chapter to report the results obtained from a particular program for primary grade arithmetic, a program which, after two years of experimentation, was finally employed under circumstances which permitted evaluation by means of tests.[1] It is not the purpose of this chapter to try to prove that this program is the only possible program, nor that it is the best program available at present. As a matter of fact, there are probably many different programs now in use, and some of them may be better than that here tested. The intention in this chapter is, rather, to show what *can* be accomplished when reasonable outcomes are set up, half-grade by half-grade, and when appropriate pupil activities are discovered and arranged with a view to the realization of these outcomes.

In the writer's opinion no apology is needed for reporting completely the results of *any* program of primary arithmetic. As the reader well knows, the literature contains many statements to the effect that children in Grade I (or Grade II) can (or cannot) learn this or that, these statements generally being based upon nothing more substantial than opinion or limited experience. The reports of experiments thus far made available supply essential information neither with regard to outcomes assumed nor with regard to instruc-

[1] This program is built around Teachers' Manuals and pupils' workbooks and readers published under the general title of "Jolly Numbers," as part of the Daily-Life Arithmetic Series for Grades I through VIII inclusive. This series, the work of Guy T. Buswell, Lenore John, and the writer, is published by Ginn and Co.

tional methods employed. Lacking this knowledge, the reader is hardly competent to pass critical judgment on the significance of an investigation, and certainly he must be cautious in accepting the evidence either as proving or as refuting the case for systematic arithmetic in Grades I and II. In the long run, programs should be evaluated, not as wholes, but rather for their success or failure in furthering certain ends. Programs which at most points are very bad indeed may be excellent at other points, so good in fact that these features should be adopted and widely used. There is every reason to believe that from experience with different programs will eventually emerge a program far superior to any we now have. This hope for the ultimate emergence of a superior program, and not the desire to establish special merits in the particular program here under investigation, is responsible for this chapter.

THE PLAN OF INSTRUCTION

The plan of instruction followed in the present investigation is fully described in the Teachers' Manuals which can be consulted by interested readers. In this place, therefore, only the most important aspects of the program are outlined.

Outcomes.—The chart which begins on the next page contains a concise summary of the outcomes by half-grades. The reader who is acquainted with the traditional course of study for the first two grades will find in this chart many familiar outcomes—or at least he will tend to identify those given as familiar. The outcomes with respect to counting and enumeration, for example, are not unusual for the primary grades. Moreover, it has been customary to assign some part of the number combinations to the first two grades. When this has been done, however, the course of study usually has required nothing short of "mastery." In this program, however, the term used is "intelligent control over," and the difference in terminology is a matter of considerable moment. "Intelligent control over" implies that children must be able to deal with the number combinations in some effective, sensible manner; it does not imply that children will from the start, or even very soon thereafter, have the ability automatically to recall correct answers as does the adult. Such mastery is viewed as the product of a long period of growth, the result of development through a series of stages in thinking.

The present list of outcomes differs perhaps more noticeably in the presence of such terms as "understanding," "appreciation," "social values," "disposition to use," and the like, all of which imply that the arithmetic learned must possess meaning and apparent usefulness for the learner; the child must see sense in what he learns and he must have experience in using what he learns.

Methods of teaching.—In general, teachers who follow this system of instruction rely little upon telling their pupils and much upon showing them, or, better, upon having their pupils under guidance make discoveries and then verify those discoveries for themselves. The authority made use of is not their authority as teachers, but rather the authority of truth revealed by their pupils' own practical concrete experiences in number situations. Understandings and generalizations evolve from pupil activity; they are not given out by teachers as neat formulations arrived at ahead of time.

Perhaps the clearest and briefest way to summarize the general principles of method which are recommended for this program of instruction is to quote directly from the Teachers' Manual:

1. We must insure orderly development in quantitative thinking.
2. The child must see sense in what he learns.
3. The child's activities and the purposes of arithmetic must harmonize.
4. Meanings must precede symbols; understandings must precede drill.
5. The way children think of numbers is as important as is the result of their thinking.
6. We must teach at the rate at which the child learns.
7. We must present arithmetic as an object of "natural" interest.
8. Instructional materials should be organized "spirally."
9. Children must know both what they are to learn and how well they are learning it.

CHART 1

OUTCOMES BY HALF-GRADES

	Grade IB	*Grade IA*	*Grade IIB*	*Grade IIA*
1. Counting and enumeration:				
by 1's..................	to 20	to 100	X*	X*
by 10's..................	....	to 100	X	X
by 2's..................	....	....	to 20	X
by 5's..................	....	....	to 100	X
by 3's..................	....	....	....	to 30
2. Understanding of place value of numbers in the series to	10	100	X	X
3. Understanding and use of the ordinals to..........	sixth or seventh	eighth	tenth	thirty-first
4. Reading and writing numerals to..................	10	100	X	X
5. Understanding of significance of 10 as the basic unit in the larger numbers to...	....	20	100	X
6. Recognition of regular groups of objects to.....	6 or 7	9 or 10	X	X

7. Intelligent control over the number combinations through	6	9	12	18
8. Intelligent control over the 0-combinations	....	....	X	X
9. Reading and writing combinations vertically and horizontally through	6	9	12	18
10. Understanding and use of the processes of addition and subtraction	X	X	X	X
11. Understanding of relationship between addition and subtraction and between the addition combinations and the related subtraction combinations	....	X	X	X
12. Higher-decade addition and subtraction, no carrying or borrowing, sums and minuends to	....	....	19	99
13. Column and horizontal addition of more than two numbers with and without 0	....	3 digits, sums to 9	3 digits, sums to 12	4 digits, sums to 18
14. Use of fractions as applied to single objects and to groups of objects (even division)	....	½	¼	⅓
15. Use of number in connection with	....	....	U. S. money Telling time	X X Simple linear and weight measures
16. Reading and writing Roman numerals to	....	....	XII	X*
17. Functional reading vocabulary of number words and symbols	X	X	X	X
18. Appreciation of social values of arithmetic, disposition to use arithmetic, and eagerness to learn more arithmetic	X	X	X	X

* X means that the given skill or concept is extended, enriched, and/or practiced.

Materials of instruction.—If natural and planned number occurrences in the classroom may for convenience be classed under the heading "materials of instruction," teachers in this investigation had the following sources upon which to draw: (1) Teachers' Manuals, (2) pupils' workbooks, (3) pupils' number readers, (4) unplanned number situations which appeared spontaneously, and (5) planned or prearranged number situations.

(1) As has already been explained, every teacher had the manual for her grade. These manuals are unusually complete, containing both theoretical discussions to make clear the underlying philosophy and practical suggestions with regard to teaching procedures and materials. In the case of the last three half-grades (Grade IA through Grade IIA) the pages of the pupils' workbooks are reproduced in the manuals, so that teachers are able to relate the manual suggestions for teaching directly to the page to which they correspond.

(2) Pupils in Grade IA had a workbook of seventy-two pages, about twelve of which contained review and testing material and the rest, developmental material. The workbooks for Grades IIB and IIA comprised eighty pages each, of which about twelve are devoted to reviews and tests. In Grade IB the pupils had no workbooks,[2] and, accordingly, their teachers made much greater use of (4) and (5) in the list above.

The typical page in each workbook starts with an interesting picture, especially designed to reveal the number ideas and relationships set for that unit of instruction. Children are expected, under the teacher's guidance, to arrive at generalizations or relationships on a concrete basis and to express their discoveries in the language of abstract number. Spaces are left in the workbook pages for entering the results of the various number experiences. The intention is not to hurry the immediate memorization of the facts discovered, but, rather, to show number as a shorthand method of translating and recording both the quantitative process used and the results of this process. As will be explained below under (4) and (5), workbook lessons regularly follow experiences of a less artificial character.

(3) Some, but by no means all, of the pupils in this investigation had number readers,[3] which in their content parallel the development of number ideas and relationships outlined in the manuals. These storybooks are planned to show children that number is a natural or normal element in reading matter. The stories contain numbers and quantitative situations in unobtrusive ways which do not interfere with the story proper. After reading a story, children come to number questions, they turn back to the appropriate context, select the relevant quantitative material, and work out the required relationships.

(4) and (5) Each new phase of developmental instruction starts with an actual number experience apart from storybook or workbook.

[2] A workbook for this half-grade has since been published under the title *Jolly Numbers Primer*.

[3] *Jolly Number Tales, Book One*, and *Jolly Number Tales, Book Two*, for Grade IA and the whole of Grade II, respectively.

(Indeed, in Grade IB in this study these extra-book number experiences provided the sole basis for instruction.) Instruction is begun in this way in order to impress children with the need for the new idea or skill. Sometimes a fortuitous or chance occurrence serves the purpose; at other times a situation must be prearranged.

Chance happenings and prearranged situations involving number are utilized not alone to initiate developmental instruction, but also to provide opportunities for children to use the arithmetic they have learned. As a consequence, arithmetic is not confined to the arithmetic period, but is part of the whole school day. It is expected that by encountering number at all sorts of times and in all kinds of ways children will become more sensitive to the values of arithmetic and will develop habits of use that will function in many practical ways. (Outcome 18, p. 67).

Experimental subjects.—Usable returns were received for 223 pupils in Grades IB and IA and for 280 pupils in Grades IIB and IIA. By "usable" is meant (1) that test records were available for each child for the two terms in his grade and (2) that the test blanks showed awareness of the purpose of the test and intention to follow directions. In spite of the second requirement some exceedingly poor test papers were retained, as will subsequently appear. "Repeaters" were not included in the study.

The 223 pupils in the first grade came from ten classrooms located in four states—Massachusetts, North Carolina, Ohio, and Pennsylvania. The 280 second-grade pupils came from eleven classrooms in the same four states.

Tests.—The tests were of two kinds: (1) group tests and (2) individual tests. The first kind was administered to all pupils in each half-grade; the latter, to a sample of the pupils, selected so as to be representative of the whole class.

(1) Group tests. The group tests were specially printed blanks which reproduced the tests provided in the manuals—pages 74 and 75 (Grade IB) and pages 142 and 143 (Grade IA) of the Teachers' Manual for the Beginners' Course; pages 100 and 101 (Grade IIB) and pages 153 and 154 (Grade IIA) of the Teachers' Manual for *Jolly Numbers, Book Two.* Samples of these tests appear as needed on the following pages, altered as required to show the problems used and the directions for administering the tests. In each test the verbal problems were read or stated by the teachers and did not appear on the test blanks. In the test for Grade IB all directions

were given orally to avoid reading difficulties, but Part II of the tests for the other half-grades regularly involved considerable reading.

(2) Individual tests. The contents of the individual tests and the procedure in administering them are described at a later point (beginning with p. 90).

RESULTS FROM THE GROUP TESTS

Gross results.—Table 18 contains a tabulation of the scores made by the experimental subjects on the group tests for the four half-grades. In the group test for Grade IB one point was allowed for each correct response, so that the highest possible number of points was 55; 65 pupils made scores of 53, 54, or 55 (actually 26 had perfect scores); 48 had scores of 50, 51, or 52, and so on. The median score was 50, which represents 92.7 per cent of the possible score. One fourth made scores below 42 (a percentage maximum of 76), and one fourth made scores of 53 or better (a percentage minimum of 96).

TABLE 18

SCORES ON TERM GROUP TESTS, GRADE IB THROUGH GRADE IIA

Score	*Grade I*		*Grade II*	
	First term (55)	*Second term (73)*	*First term (83)*	*Second term (81)*
83	..	..	7	..
80	..	..	53	63
77	..	..	45	71
74	..	..	37	35
71	..	61	30	37
68	..	40	32	22
65	..	30	15	9
62	..	25	11	5
59	..	10	6	10
56	..	11	10	7
53	65	13	3	2
50	48	7	4	3
47	24	5	3	4
44	19	4	7	5
41	17	6	4	1
38	17	0	2	3
35	12	1	2	1
32	6	3	2	1
31, and below	15*	7	7	1
N	223	223	280	280
Median†	50	66	74	76
Q_1‡	42	59	66	70
Q_3‡	53	71	79	79

* The 15 scores distribute as follows: 29-31 (4); 26-28 (5); 23-25 (0); 20-22 (1); 20 and below (5).

† Medians obtained from the raw scores without grouping. The percentage equivalents of the medians are: Grade IB, 92.7 (50 out of possible 55 points); Grade IA, 90.4; Grade IIB, 89.2; Grade IIA, 93.8.

‡ Quartiles obtained from raw scores without grouping.

The figures for the other half-grades were about equally good, with median per cents of 90.4, 89.2, and 93.8, respectively, for Grades IA, IIB, and IIA. There were nineteen perfect papers among those for the Grade IA pupils, seven perfect papers for the Grade IIB pupils, and thirty-four perfect papers for the Grade IIA pupils. There were, however, a number of very poor papers. Eleven (5 per cent) of the Grade IB pupils made percentage scores of less than 50, and there were eleven such pupils (5 per cent) in Grade IA, fifteen such pupils (5 per cent) in Grade IIB, and five such pupils (2 per cent) in Grade IIA. Still, if it be granted that the tests sample achievement satisfactorily and that the method of scoring was adequate, there seems to be sound evidence that the experimental sub-

Grade IB Term Test, Part I

A.

$6-1=$	$5-4=$	$4+1=$
$2+3=$	$5-2=$	$1+5=$
$4-3=$	$6-4=$	$4+2=$

B.

$\begin{array}{r}3\\ \underline{+2}\end{array}$	$\begin{array}{r}6\\ \underline{-5}\end{array}$	$\begin{array}{r}3\\ \underline{-1}\end{array}$	$\begin{array}{r}6\\ \underline{-3}\end{array}$	$\begin{array}{r}5\\ \underline{-3}\end{array}$	$\begin{array}{r}4\\ \underline{-1}\end{array}$
$\begin{array}{r}3\\ \underline{-2}\end{array}$	$\begin{array}{r}1\\ \underline{+2}\end{array}$	$\begin{array}{r}3\\ \underline{+3}\end{array}$	$\begin{array}{r}5\\ \underline{+1}\end{array}$	$\begin{array}{r}2\\ \underline{-1}\end{array}$	$\begin{array}{r}6\\ \underline{-2}\end{array}$

C. (I saw 4 birds on the telephone wire. Then 2 flew away. How many birds were left on the wire?)

D. (I had 5 pennies. I spent 1 penny for a piece of candy. How many pennies did I have left?)

E. (Ann had a party. If 4 other girls came, how many girls in all were at the party?)

F. (I put 2 yellow books and 4 green books on the table. How many books were on the table then?)

G. (Card for 3, 2, and 5 shown; children to write one story or fact.)

H. (Same as G, but card for 1, 3, and 4.)

I. (Same as G, but card for 2, 2, and 4.)

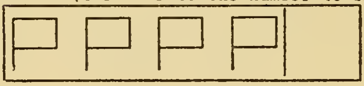
J. (Same as G, but card for 1, 2, and 3.)

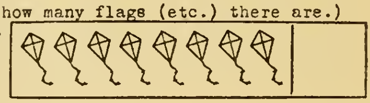
GRADE IB TERM TEST, PART II

A. (1. Draw 7 balls in a row.
2. Put X on the second ball.
3. Put C on the fifth ball.)

B. (. 1. Make 10 crosses in a row.
2. Draw a ring around the fourth cross.)
3. Put a box around the sixth cross.)

(C-F. Write the number to show how many flags (etc.) there are.)

C. 

D. 

E. 

F. 

(G. Write the numerals from 1 - 10 in order.)

G.

(H. Write the figures for the number words.)

H. one ________________ three ________________ two ________________

four ________________ five ________________ six ________________

jects in all the half-grades learned much of what they had been taught. Three fourths of the IB-pupils made scores equivalent to 77 per cent or more of the possible score. In Grade IA three fourths made a percentage score of 81 or better; in Grade IIB the corresponding percentage score was 80, and in Grade IIA, 86.

Results in Grade IB.—The extent to which the Grade IB test covered the outcomes for that half-grade is shown in the upper half of Table 19. The ability to enumerate to 10 (Outcome 1) was tested in items A, B, and C-F of Part II; knowledge of the place values of the numbers to 10 (Outcome 2) was tested in item G of Part II; ability to read and write the numerals to 10 (Outcome 3) was tested in items A-J of Part I and in items C-H of Part II, and so on. All the outcomes are represented by test items, though obviously in each case by a small sample of the many possible items. No outcome was tested, or could be tested by the usual group techniques, in its entirety. Outcome 6, "Intelligent control of the combinations," is, for example, deliberately vague: it means little more than the ability to get the correct answers for combinations by any method whatever. Scores in this part of the test tell nothing about the degree to which the combinations were habituated (they were not supposed to be habituated until the second half-grade or even later). Outcome 10 cannot be tested adequately by any paper-and-pencil procedure. Sensitiveness to the quantitative in life is to be observed only as one avoids or meets satisfactorily the number situations in which one finds oneself. Accordingly, measurement, as here made, through success on the usual kind of verbal problems is very indirect and probably none too valid.

TABLE 19

CLASSIFICATION OF GRADE IB TEST ITEMS BY OUTCOMES, AND SCORES BY TEST PARTS

	Test Items, by Parts	
	Part I	*Part II*
Outcomes:		
1. Enumeration to 10	....	A, B, C-F
2. Place value to 10	....	G
3. Numerals to 10	A-J	C-H
4. Ordinals to *sixth* or *seventh*	....	A, B
5. Recognition of regular groups	G-J	....
6. Intelligent control of combinations through 6	A, B, G-J	....
7. Reading and writing combinations horizontally and vertically	A, B	H
8. Understanding of addition and subtraction	C-F	....
9. Reading vocabulary	A, B	H
10. Appreciation of social values, etc.	C-F	....
Scores:		
Possible	29	26
Median	25	26
Q_1	17	24
Q_3	25	26
Percentage equivalents of:		
Median	86.3	100.0
Q_1	58.6	92.3
Q_3	96.6	100.0

The lower half of Table 19 reveals that the children tested were much more successful with Part II than with Part I. On Part II three fourths or more of the pupils made percentage scores of 92 or better. On Part I, on the other hand, the corresponding percentage score was but 58.6.

The results on the various items of the test are analyzed in Table 21 for a sample of one hundred Grade IB pupils, so selected that they are truly representative of all the pupils tested. The degree to which this representativeness was attained is shown in Table 20 below. It will be seen that 10 per cent both of the total group and of the sample group secured scores of 55 or better, 20 per cent secured scores of at least 54, and so on. The medians and quartile points are identical (or nearly so) for both groups.[4]

TABLE 20

Comparison of Total Grade IB Group and the Selected Sample of 100 Pupils

Percentile Point	*Scores* *Total Group*	*Sample Group*
90	55	55
80	54	54
70	52	52+
60	51	51
50 (Median)	50	50
40	47	47
30	44	44
20	40	40
10	35	36
Q_3	53	53
Q_1	42	42+

To return to Table 21, the per cents of success on Part II of the Grade IB test speak for themselves. Clearly the children had acquired high degrees of skill and knowledge in dealing with arithmetical tasks set for them in this part of the test.

The story for Part I is different. All the items in Part I relate to the number combinations with sums and minuends to 6. In A are nine abstract combinations in horizontal form; in B, twelve abstract combinations in vertical form; in C-F, four combinations presented orally in verbal problems; in G-J, still other combinations to be recognized from pictured groups.

The situation in Grade IB may be fairly well summarized by saying that the children demonstrated substantial growth toward all outcomes, except possibly those connected with the number combinations as such (and possibly those associated with the development of habits of use, an outcome none too well tested).

[4] In the treatment of the results for Grades IA, IIB, and IIA the same practice is followed, of analyzing the data for a selected sample of one hundred cases instead of for the half-grade group as a whole. In each instance the sample was as closely matched with the total group as above in the case of Grade IB.

TABLE 21

NUMBER OF ERRORS AND PER CENTS OF SUCCESS ON ITEMS OF GRADE IB TEST; SAMPLE OF 100 CASES

Test Item	*Outcomes Tested*	*Number of Errors and Omissions*	*Per cents of Success*
Part I			
A	6, 7, 9	214	76.2
B	6, 7, 9	331	72.4
C-F	8, 10	59	85.3
G-J	5, 6	114	71.5
Part II			
A-B	1, 4	33*	91.8
C-F	1, 3	5	98.8
G	2, 3	11	99.9
H	3, 7, 9	43†	92.9

* Out of 100 possible errors on each of the four ordinals tested, 5 errors were made on *second*, 7 on *fourth*, 11 on *fifth*, and 10 on *sixth*.

† Out of 100 possible errors on each of the six number words tested, 6 were made on *one*, 5 on *two*, 4 on *three*, 9 on *four*, 12 on *five*, and 7 on *six*.

The significance attached to the figures for the number combinations varies with what one expects from children in this half-grade. If one calls for automatic mastery of all the combinations taught, then the low per cents signify a wholly unsatisfactory state of affairs: the children had not "learned" these simple number facts.

On the other hand, if one is willing to wait for mastery and if one views the experiences these children had with the number combinations as being exploratory in character, then one is not at all disturbed by the apparent lack of mastery in Table 21. As a matter of fact, on the assumption that the subjects in these experimental classes were typical of those studied in investigations of the number knowledge of children on entering school (Chapter II), these children show unmistakable evidence of growth. A median of about 35 per cent of school entrants "knew" the abstract combinations presented to them (Table 16); the corresponding figure from the Grade IB test is 72 per cent. A median of about 42 per cent "knew" combinations in verbal problems when they entered school (Table 15), though some of these combinations were harder than any here tested; the corresponding per cent for these Grade IB children is 87 (only four combinations, however). The amount of growth revealed by these comparisons is enough to satisfy the student of arithmetic who judges growth in terms other than those of mastery. Such a person is not disturbed by the rather superficial mastery shown by these children since he is confident that the understandings they acquire through meaningful experience will eventually make for a more usable kind of knowledge.

Results in Grade IA.—The Grade IA test is analyzed in terms of outcomes in Table 22 as was done for the Grade IB test in Table 19. It will be noted that all outcomes except two (namely, 1 and 2) are represented in the Grade IA test, though with varying degrees of completeness. Moreover, as in the case of the Grade IB test, the measurement of Outcome 12 is very imperfectly and only indirectly

Grade IA Term Test, Part I

A. Write the answers:

1.	9 − 3	2 + 5	2 + 7	8 − 5	6 − 2	7 − 3	9 − 7
2.	8 − 3	5 + 4	8 − 4	2 + 3	9 − 2	7 − 4	3 + 5
3.	5 + 3	9 − 4	3 + 4	5 − 3	6 + 3	2 + 4	9 − 8
4.	7 − 5	6 + 2	4 + 5	4 + 3	4 + 2	8 − 7	2 + 6
5.	8 − 6	6 − 4	9 − 5	5 + 2	3 + 6	8 − 2	9 − 6

B. Add:

a.	*b.*	*c.*	*d.*	*e.*
6 0 3	2 4 2	4 5 0	0 7 0	3 2 3

***C.** Write just the answers:

a. ____________ *b,* ____________ *c.* ____________

d. ____________ *e.* ____________ *f.* ____________

**Problems for Ex. C*

a. Pat has 3 marbles in his right hand and 6 in his left hand. How many marbles has he in both hands?

b. Ann counted the eggs she found in the barn. There were 5. She put them with the 4 eggs in the icebox. Then there were how many eggs in the icebox?

c. If you take 8 sandwiches to the picnic and only 6 are eaten, how many sandwiches will be left?

d. There were just 7 leaves on one branch of a tree. Along came the wind and blew off all but 2. How many leaves were blown off the branch?

e. Jane has 5 school dresses and 2 party dresses. How many dresses has she in all?

f. Eight of Mabel's drawings were hanging on the wall of her bedroom. She took down the ones that got dirty. Now there are 4 drawings on the wall. How many of the drawings did Mabel take down?

Grade IA Term Test, Part II

A. Draw a ring around the right answers:

a. How do you find how many in all?

subtract add

b. Which picture shows a half?

c. How many ones in 16?

1 4 6

d. Which number comes just before 18?

17 19 7

e. Which means to add?

= + ¢ −

f. Which number is less than 75?

79 80 63 92

g. Which means one half?

2 four $\frac{1}{2}$

h. How do you add?

up down

i. How many tens has 15?

5 1 2

j. Which are parts of 8?

2 and 3 5 and 1 6 and 2

k. How many parts has the whole story about 3, 6, and 9?

2 4 5

l. Which means 7?

nine eight seven

m. How do you find how many are gone?

subtract add

n. Which shows $\frac{1}{2}$?

o. Which is the answer for 7 − 2?

9 5 3

p. Which number is more than 50?

57 29 36 48

q. How many parts has the whole story about 4, 4, and 8?

5 2 4

B. Write the numbers that are left out:

a. 36 37 ______ 39 ______.

b. 58 ______ ______ 61 ______.

c. 10 20 30 ______ ______ 60.

d. 40 ______ 60 ______ 80 ______.

provided for. The omission of items for Outcomes 1 and 2, most of the skills in which can be tested only by individual interviews, is hardly serious. In the first place, practically all children may be assumed to be able to enumerate and to count to 100 by 1's at the end of Grade IA if they receive any instruction at all on these skills. It will be recalled that about one tenth of school entrants already possess this ability (Table 3). In the second place, the ability to count to 100 by 10's is also rather easily acquired; about one fourth have the ability when they enter school (Table 3 again). In the third place, the high degree of success of these same children at the end of Grade IB in dealing with the lower ordinals (about 90 per cent knew *fifth* and *sixth*) warrants the belief that, a half-grade later, they would be equally successful with the new ordinals *seventh* and *eighth*.

The median score on Part I of this test (the lower section of Table 22) amounted to 93.5 per cent of the possibility, and three fourths secured scores of about 85 per cent or better. Part II was harder for these children, the median dropping to 88.9 per cent and Q_1 to 74.1 per cent. The reason for the lower scores on Part II cannot be certainly ascribed to any one cause: (1) the form of this

TABLE 22

CLASSIFICATION OF GRADE IA TEST ITEMS BY OUTCOMES, AND SCORES BY TEST PARTS

Outcomes, by Test Items:	*Part I*	*Part II*
1. Enumeration to 100 by 1's; ordinals to *eighth*	....	....
2. Counting to 100 by 1's; by 10's	....	....
3. Reading and writing numbers to 100	....	A: d, f, p; B: a-d
4. Place of numbers to 100	....	A: d, f, p; B: a-d
5. Significance of 10 in teens numbers	....	A: c, i
6. Intelligent control of combinations with sums and minuends to 9	A (35 facts) C (6 verbal problems)	A: j, k, q
7. Understanding of addition and subtraction as processes	C	A: a, m, o
8. Understanding of the relation between addition and subtraction	....	A: j, k, q
9. Column addition, sums to 9, three addends	B (5 examples)	A: h
10. Understanding of ½ as applied to objects and even groups	....	A: b, n
11. Reading vocabulary (all of Part II, but especially)	....	A: e, g, l
12. Appreciation of values of arithmetic, etc.	C	....
Scores:		
Possible	46	27
Median	43	24
Q_1	39	20
Q_3	45	26
Percentage equivalents of:		
Median	93.5	88.9
Q_1	84.8	74.1
Q_3	97.8	96.3

test may have been relatively unfamiliar to the children; (2) the reading requirements may have baffled some; (3) the ideas contained in the test items may not actually have been possessed by the children. Only the third of these three sources of error is the one which was actually proposed for testing; the purpose, in other words, was to ascertain the degree to which children had acquired the concepts and understandings represented in the items of Part II (uncomplicated by deficiencies in reading skill and in the technique of taking tests). The influence of the first two sources of error is to depress unduly the scores obtained. There is at least one counteracting factor, namely, the chance selection of answers: by pure guessing correct answers could be marked for one out of each three or four items. It is felt, however, that the favorable effect of this last influence was more than offset by the unfavorable effects of the first two factors, so that the scores here reported are almost certainly too low.

According to Table 23 for a sample of one hundred Grade IA pupils 91.5 per cent of the thirty-five abstract combinations were correctly answered, as well as 81.8 per cent of five examples in column addition, and 84.0 per cent of the six verbal problems. (One hundred children each attempted 35 abstract combinations; 297 answers were wrong and 3,203 were correct, to give a figure of 91.5 as the per cent of success.)

The various groups of items in Part II varied, as to per cents of success, from 71.5 to 93.0. The lowest items, separately considered,

TABLE 23

Number of Errors and Per Cents of Success on Items of the Grade IA Test; Sample of 100 Cases*

Test Item	*Outcomes Tested*	*Number of Errors and Omissions*	*Per cents of Success*
Part I			
A	6	297	91.5
B	9	91	81.8
C	6, 7, 12	96	84.0
Part II			
A: d, f, p; B: a-d	3, 4	197	84.8
A: c, i	5	57	71.5
A: j, k, q	6, 8	66	78.0
A: a, m, o	7	49	83.7
A: h	9	19	81.0
A: b, n	10	21	89.5
A: e, g, l	11	21	93.0

* See footnote, p. 74.

are A: c, f, h, i, k, q and B: d. A: c and A: i (81 and 62 per cent respectively) relate to the meaning of *ones* and *tens* in the numbers 11 to 19. Sixteen of the 19 errors made on A: c came from the selection of 4 instead of 6 as the number of ones in 16; 34 out of 38 errors made on A: i represented the selection of 5 instead of 1 as the number of *tens* in 15. Item A: f required the selection of the number (79, 80, 63, or 92) which is "less than 75." The errors were about evenly distributed among the wrong alternatives, a fact which seems to indicate ignorance of the meaning of "less." Item A: h ("How do you add? up . . . down") probably should not have been included in the scoring. The "correct" answer according to the manual directions for teaching is "down," but whole classes tended to mark "up," thus suggesting that in those particular groups the manual instructions had not been adopted. In these classes "up" was of course the correct answer. Items A: k and A: q deal with the number of parts in the "whole story" about 3, 6, and 9 and about 4, 4, and 8, respectively. Seventy-nine per cent answered A: k correctly, but only 57 per cent, A: q, which contains a "catch." Apparently this idea of the "whole story" was none too well understood by these children.

The general conclusions to be drawn with respect to the Grade IA test results must be somewhat guarded, in view of the uncertainty surrounding the results on Part II of the test. So far as the abstract combinations are concerned, these children indicated very satisfactory progress in learning. In column addition, which was introduced in this half-grade, the results were none too good, but the newness of the process may account for the comparatively low degree of success. It is difficult to account for the failure of these children to do better with the verbal problems, in view of their supposed familiarity with this kind of arithmetic. The system of instruction outlined for the experimental schools calls for a great many experiences with described quantitative situations, and if these were actually supplied to the children, they should have been able to solve correctly more of the test problems.

The results on Part II of the test have already been considered in some detail. If it is a fair interpretation to regard the per cents of success reported as really too low, one could infer that these children were acquiring verbal statements of mathematical principles, generalizations, and relationships of large value in their understanding of arithmetic.

Results in Grade IIB.—According to Table 24 all Grade IIB outcomes except one, namely, Outcome 11 (Roman numerals) are represented by at least two items in the group test. The coverage of Outcome 13 is, as in the case of the other tests already considered, very inadequate indeed, and the types of behavior associated with the other outcomes can of course be considered only as sampled rather than as exhaustively tested. This limited sampling is inevitable, so far as the present study is concerned, for the test was devised for the measurement of achievement over a half-grade of instruction and not for the purpose of diagnostic analysis.

Grade IIB Term Test, Part I

***A.** Your teacher will tell you what to do.

a. ________ *b.* ________ *c.* ________ *d.* ________ *e.* ________

B. Look at the sign. Then add or subtract.

1.	11	3	6	12	10	2	5	11
	− 7	+ 9	+ 5	− 9	− 4	+ 8	+ 0	− 4
2.	9	10	9	3	12	7	11	9
	+ 2	− 7	+ 3	+ 7	− 5	+ 4	− 6	− 9
3.	12	0	11	7	11	10	12	5
	− 8	+ 7	− 8	+ 5	− 3	− 8	− 3	+ 6
4.	7	12	8	4	12	10	4	11
	− 0	− 4	+ 3	+ 6	− 7	− 6	+ 8	− 9

C. Look at the sign. Then add or subtract.

1.	13	2	6	17	13	19	10
	− 3	+ 15	+ 10	− 5	+ 4	− 7	+ 8
2.	4	17	18	19	10	19	12
	+ 15	− 6	− 8	− 3	+ 7	− 0	+ 6

D. Add:

1. 4	2. 3	3. 0	4. 5 + 2 + 2 =
0	4	6	5. 3 + 0 + 7 =
5	3	2	6. 0 + 2 + 8 =

* *Problems for Ex. A*

a. Harry counted his toy soldiers. He had 11 toy soldiers with guns and 3 toy soldiers with drums. How many more of his toy soldiers had guns than had drums?

b. The milkman dropped a box of bottles. When he picked them up, only 5 of the 12 bottles were good. How many bottles had been broken?

c. Emily has 7 pieces of peppermint candy and 4 pieces of chocolate candy in her bag. How many pieces of candy has she in all?

d. A farmer was mending his fence. He had to put in 12 new posts. After he had put in 9 posts, how many more did he have to put in?

e. Mrs. Spider spun a fine web. The first day she caught 2 flies and the next day 8 flies. How many flies did she catch in both days?

Grade IIB Term Test, Part II

A. Draw a ring around the answer:

1. Which means to add?

   − ¢ + 卌

2. Which means one fourth?

   1 + 3 4 $\frac{1}{4}$ 4 − 0

3. To find "how many more" you

   subtract. add.

4. Which shows $\frac{1}{4}$?

5. Which is less than 57?

   67 90 58 49

6. Which means cents?

   − ¢ + 1

7. Which number is 卌 |||||·||?

   17 12 26 15

8. How many tens has 80?

   3 0 8 7

9. Which number is largest?

   70 62 89 45

10. Which number is twelve?

    2 36 12 17

11. To find "how many gone" you

    add. subtract.

B. Write the missing number or numbers:

1. After 15, 20, 25, come_____ and _____.

2. The whole story about 7, 5, and 12 has ______ parts.

3. 5 is one part of 11; the other part is ______.

4. At ten o'clock the short hand is on ______. The long hand is on ______.

5. If you take away all of a number, ______ is left.

6. After 8, 10, 12 come ______ and ______.

7. The ring is around one ______ of the dots.

8. 17 has ______ ten and ______ ones.

9. The whole story about 5, 5, and 10 has ______parts.

10. The 卌-picture for 30 is ____________.

11. A dime is the same as ______ cents.

The scores in the lower section of the table indicate the degree to which achievement, broadly considered, was satisfactory among the experimental subjects. One fourth of the children secured scores equivalent to more than 98 per cent of the possibility on Part I, one half, scores equivalent to 93 per cent or better, and three fourths, scores equivalent to 82.5 per cent or better. The corresponding percentage figures for Part II are lower—92.3, 84.6, and 69.2, respec-

TABLE 24

Classification of Grade IIB Test Items by Outcomes, and Scores by Test Parts

Outcomes, by Test Items:	*Part I*	*Part II*
1. Understanding of numbers to 100, ordinals to *tenth*	....	A: 5, 7-9; B-8
2. Counting by 2's to 20 and by 5's to 100	....	B: 1, 6
3. Intelligent control of combinations through 12	B	B: 2, 3, 9
4. Understanding of the relationship between addition and subtraction and the corresponding number combinations	....	B: 2, 3, 9
5. Intelligent control of the 0-combinations	B	....
6. Understanding of the processes of addition and subtraction	A	A: 3, 11; B: 3, 5
7. Column and horizontal addition, three digits, sums to 12	D	....
8. Higher-decade addition, to 19	C	....
9. ¼ as applied to single objects and even groups	....	A: 2, 4; B: 7
10. U. S. money and telling time	....	A: 6; B: 4, 11
11. Reading and writing Roman numerals to XII	....	....
12. Reading vocabulary (all of Part II, but especially)	....	A: 1, 2, 6, 10; B: 10
13. Appreciation of values of arithmetic, etc.	A	....
Scores:		
Possible	57	26
Median	53	22
Q_1	47	18
Q_3	56	24
Percentage equivalents of:		
Median	93.0	84.6
Q_1	82.5	69.2
Q_3	98.3	92.3

tively. Part I is devoted to abstract arithmetic, chiefly computation; Part II, to mathematical relationships, generalizations, and the like.

A sample of one hundred Grade IIB pupils (Table 25) solved 82.2 per cent of the verbal problems (5 problems per child, a total of 500 problems, of which 89 were missed). On the abstract facts with sums and minuends from 10 to 12 (twenty-eight facts in the test), the per cent of success was 87.7, and on the 0-facts (only four in the test) it was 96.3. In higher-decade addition and subtraction with sums and minuends through 19 (fourteen examples), the per cent of accuracy was 82.8. In horizontal and column addition, three digits, sums to 12 (six examples), the per cent of accuracy was 92.5.

Again, as in the case of the Grade IA test, it is difficult to account for the comparatively low per cent in problem solving, except on the ground that these children did not have all the expected experience in dealing with verbally described quantitative situations. The per cent of success on the new addition and subtraction combinations is entirely satisfactory in an instructional program which does not hurry mastery. The figure for the 0-combinations (though based unfortunately on only four combinations) is especially interesting in view of the still common belief that such combinations are especially hard. According to the scheme of instruction here under consideration all 0-facts are presented by means of four generalizations and are taught as groups (0 + n, n + 0, n — 0, n — n). Under these conditions the combinations are readily understood, and the learning is accordingly made easier. As for higher-decade addition and subtraction, the process was new to these children, and 82.8 per cent seems to represent entirely satisfactory learning at this stage. The last item in Part I consists of examples in horizontal and column addition, and the children were very successful with the six examples given them.

TABLE 25

NUMBER OF ERRORS AND PER CENTS OF SUCCESS ON ITEMS OF THE GRADE IIB TEST; SAMPLE OF 100 CASES

Test Items	*Outcomes Tested*	*Number of Errors and Omissions*	*Per cents of Success*
Part I			
A	6, 12	89	82.2
B	28 (facts to 12)	345	87.7
	4 (0-facts)	15	96.3
C	8	241	82.8
D	7	45	92.5
Part II			
A: 5, 7-9; B: 8	1	129	78.5
B: 1, 6	2	76	81.0
B: 2, 3, 9	3, 4	82	72.0
A: 3, 11; B: 3, 5	6	84	79.0
A: 2, 4; B: 7	9	74	72.0
A: 6; B: 4, 11	10	37	90.8
A: 1, 2, 6, 10; B: 10	12	51	89.8

On Part II, success varied from 72.0 per cent for the third group of items (the "whole story" idea) to 90.8 per cent on the sixth group (U. S. money and telling time). As already mentioned, Part II seemed to be harder for these children than was Part I, but the reason for this greater difficulty (if indeed there *was* greater difficulty) is uncertain. Like Part II of the Grade IA test, Part II of this test called (1) for reading and (2) for knowledge of the technique of marking answers. Either or both of these factors, extraneous to the real purpose of the test, may have introduced an undue number of errors.

The following items were missed by 10 per cent or fewer of the children: A: 1, 2, 4, 6, 10; B: 5. The following items were missed by 20 per cent or more: A: 3; B: 2, 3, 6, 7, 8, 9, 10. Forty per cent stated that to find "how many more" one should add (item A: 3), an error that may well mean the early but undesirable adoption of "more" as a specific cue for addition. B: 2, 3, and 9, missed by 26 per cent, 22 per cent, and 34 per cent, respectively, all involve the "whole story" idea, a fact which suggests the need for special attention to the relationship between combinations. B: 6 (25 per cent made errors) involves counting by 2's, but the framing of the test item may not have suggested this fact. B: 7 calls for the identification of a group of three dots as one fourth of twelve dots. Only one third of the children (36 per cent) succeeded on this item as compared with 93 per cent in the case of A: 4, where one fourth of a single object was identified. The difference in per cents confirms the belief that the application of fractions to groups of objects is much more difficult than to single objects, though in this test the low per cent may reflect ambiguity in the test item itself. B: 8, missed by 36 per cent, requires two entries, and one of these was frequently omitted. B: 10 tests ability to use a special symbol for 10 in the construction of a "picture" for 30. Forty-two per cent could not draw the correct "picture"; this probably should not be surprising in view of comparatively slight experience in this kind of activity.

The form of analysis adopted in the foregoing paragraphs and in the corresponding sentences for the other tests has the disadvantage of stressing deficiencies rather than progress in learning. The reader may therefore be predisposed to object to any statement which implies complacency about the situation as revealed by the Grade IIB test results. It is true that the showing at some few places (especially those mentioned in the preceding paragraph) is none too good, but there are many evidences that these children were advancing toward the outcomes for their half-grade. After all, outcomes must be viewed as directions for development to take, and not goals which either are or are not attained, the quicker they are attained, the better. In the instruction to follow in Grade IIA and in later grades there are many opportunities to enrich and deepen the learning which is begun in Grade IIB and to carry that learning to the desired limit.

The crucial point is to make sure that early learning is of the right kind—meaningful, intelligent, based upon understanding—and the experimental subjects in this investigation were approaching arithmetic in this way.

Results in Grade IIA.—The use of ordinals (Outcome 1) is not represented among the items of the Grade IIA test (Table 26) ; and Outcomes 3, 4, 9, 10, and 12 are but slightly represented. However,

Grade IIA Term Test, Part I

***A.** Write just the answers:

1. .... 2. .... 3. ....

4. .... 5. .... 6. ....

B. Do what the sign tells you to do:

1. 15	5	7	16	7	18	14	15
− 6	+ 9	+ 6	− 7	+ 8	− 9	− 6	− 7
2. 17	14	4	17	9	15	14	6
− 9	− 8	+ 9	− 8	+ 7	− 9	− 5	+ 9
3. 9	13	16	8	15	9	14	13
+ 8	− 7	− 9	+ 9	− 8	+ 6	− 9	− 8

C. Do what the sign tells you to do:

1. 94	5	47	37	26	49	23
− 4	+ 34	− 3	− 0	+ 3	− 4	+ 5
2. 25	78	53	4	38	32	99
+ 3	− 3	+ 6	+ 84	− 6	+ 7	− 2

D. Add:

1. 3 + 5 + 0 + 6 = ...

2. 4 + 2 + 3 + 5 = ...

3.	4.	5.
3	2	1
5	2	3
0	3	2
7	9	7

E. Count by 3's:

1. 9 12 .... 21 2. 18 .... 27 ....

F. Write the missing numbers:

1. 37 38 .... 41 2. 69 .... 71 73

* *Problems for Ex. A*

1. If you have 15 peanuts in a bag and eat 9 of them, how many will be left?

2. A book costs 8¢ and a tablet 6¢. How much do the book and tablet cost together?

3. There were 12 berries on a strawberry plant. Along came some birds, and soon only 5 berries were left. How many berries had the birds eaten?

4. Ella wrote 14 words on the board. Tom wrote 6 words. How many fewer words did Tom write?

5. Dorothy has 8 hair ribbons. Her sister Julia has 9. How many hair ribbons have the two girls in all?

6. Mother put a pan of 16 cookies in the oven. The fire was too hot and 7 cookies got burned. How many cookies did not get burned?

Grade IIA Term Test, Part II

A. Draw a ring:

1. Which number comes just after 79?

60 78 80 45

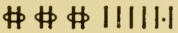
2. Which shows $\frac{1}{3}$?

3. 5 is one part of 14.
Which is the other part?

7 9 19 8

4. Which is the largest part of an apple?

$\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{4}$

5. Which number is  ?

63 52 36 26

6. How many tens has 89?

8 9 4 6

7. Which means cents?

+ = ¢

8. Which has a remainder?

2 + 2 $\frac{1}{2}$ of 12 12 + 5 12 − 5

9. Which means 11?

IX II XI VI

10. Which number is more than 41?

50 39 14 28

B. Draw a line under Yes or No:

1. Has the whole story about 8, 8, and 16 four parts? . . . . **Yes** **No**

2. Do you get a sum when you subtract? . . . . **Yes** **No**

3. Do you weigh more than 26 pounds? . . . . **Yes** **No**

4. To find the other part of a number should you add? . . . . **Yes** **No**

5. Are $\frac{1}{3}$ in the ring? . . . . . . **Yes** **No**

6. At half past 7 o'clock is the long hand on 7? . . . . **Yes** **No**

7. Has every teens number a ten? **Yes** **No**

8. To divide a thing into thirds must you make 4 parts? . . . . . . **Yes** **No**

9. Is 16 the sum of these numbers?
3 2 4 6 . . . . **Yes** **No**

10. To find how much less, should you subtract? . . . . **Yes** **No**

11. Does 73 come just after 72? . . **Yes** **No**

12. Has 38 eight ones? . . . . **Yes** **No**

13. Should you add down? . . . . **Yes** **No**

14. Did you like this workbook? . . **Yes** **No**

even with these limitations the test covers fairly well the abilities and knowledge set for the half-grade, well enough at least to serve as a fair achievement test.

The scores made on this test are higher than on any of the other half-grade tests. On Part I, the median score is equivalent to nearly 97 per cent of the possibility, and three fourths made scores equivalent to 90 per cent. On Part II the corresponding percentage equiva-

TABLE 26

CLASSIFICATION OF GRADE IIA TEST ITEMS BY OUTCOMES, AND SCORES BY TEST PARTS

Outcomes, by Test Items:	*Part I*	*Part II*
1. Increased understanding of the numbers to 100; ordinals to *thirty-first*	F	A: 1, 5, 6, 10; B: 7, 11, 12
2. Counting by 3's to 30	E	....
3. Understanding of the processes of addition and subtraction	....	B: 2, 4, 10
4. Relation between addition and subtraction and corresponding combinations	....	A: 3; B: 1
5. Intelligent control of the combinations through 18	B	....
6. Column and horizontal addition, four digits, sums to 18	D	B: 9, 13
7. Higher-decade addition and subtraction, sums and minuends to 99, no carrying or borrowing	C	....
8. Use of ⅓ with single objects and even groups	....	A: 2, 5; B: 8
9. Use of ½, ⅓, and ¼ to show relative size	....	A: 4
10. Time, U. S. money, simple weight and linear measures	....	B: 3, 6
11. Reading vocabulary (all of Part II, but especially)	....	A: 7, 8, 9; B: 2, 9
12. Appreciation of values of arithmetic, etc.	A	....
Scores:		
Possible	58	23
Median	56	20
Q_1	52	18
Q_3	58	22
Percentage equivalent of:		
Median	96.7	87.0
Q_1	89.7	78.3
Q_3	100.0	95.7

lents are 87.0 and 78.3, respectively. These figures testify to learning of a high order.

The sample of one hundred subjects earned the per cents of success represented in Table 27. In the abstract items of Part I (verbal problems, abstract combinations, higher-decade addition and subtraction, column and horizontal addition of four digits, counting by 3's, and completion of number series) per cents of 90 were regularly made, the one exception being in counting by 3's, a new skill.

The groups of items in Part II were answered successfully by from 78 per cent to 97 per cent. The A-items are of the multiple-

TABLE 27

NUMBER OF ERRORS AND PER CENTS OF SUCCESS ON ITEMS OF THE GRADE IIA TEST; SAMPLE OF 100 CASES

Test Item	*Outcomes Tested*	*Number of Errors and Omissions*	*Per cents of Success*
Part I			
A	12	62	89.7
B	5	202	91.2
C	7	129	90.8
D	6	30	94.0
E	2	73	84.4
F	1	10	97.5
Part II*			
A: 1, 5, 6, 10; B: 7, 11, 12	1	78	88.9
B: 2, 4, 10	3	67	77.7
A: 3; B: 1	4	28	86.0
B: 9, 13	6	43	78.5
A: 2, 4, 5; B: 8	8	18	97.3
A: 4	9	13	87.0
B: 3, 6	10	31	84.5
A: 7, 8, 9; B: 2, 9	11	67	86.6

* Item B: 14 is omitted as not being a test of arithmetical knowledge. (See the test form, p. 87.)

choice variety, but the thirteen B-items are cast in the true-false form, and should therefore really be corrected (R — W) for chance successes. This step has not been taken in reporting the results in Table 27. The 88.9 per cent for the first group of items, which deal with understanding of the number series to 100, would be lowered somewhat (about 10 points) if the correction formula were applied; the second set would be considerably lowered (more than 30 points); the third, 5 points; the fourth, more than 20 points; the fifth, 10 points; the seventh, about 15 points; and the eighth, about the same amount. With these corrections the per cents still remain above 70 except for the second and fourth sets.

The figure for the second set was lowered especially by items B: 2 and B: 7, which have to do with two uses of subtraction, one of them but recently presented. The figure for the fourth set was lowered chiefly by item B: 9: "Is 16 the sum of these numbers? 3 . . . 2 . . . 4 . . . 6." The unusual arrangement of the numbers, without the sign of addition, may have been the cause of the many errors.

The following items in the A-section of Part II were answered correctly by more than 90 per cent: 1, 2, 5, 7, and 8. No items in the B-section were answered by so many children; 89 per cent answered

B: 2 correctly, and 87 per cent B: 1, 8, and 13. The greater difficulty of the B-items may have been caused by the unusual Yes-No method of recording answers, a procedure less familiar to these children than the multiple-choice form used in the A-section of the test. If this explanation is valid, then the per cents of success as reported in the table are probably more nearly the true ones than the per cents corrected for chance, as discussed above.

The results of the testing in Grade IIA are here interpreted to mean that the experimental children were making sound and satisfactory progress in arithmetical learning. It is true that not all outcomes are shown to have been attained by all children, and it is even probable that no one child had attained all outcomes, but in the scheme of instruction which is evaluated there is no such expectation. The children were growing in the right direction, and with the added experiences of later schooling they should all eventually achieve all the goals as outlined for this particular half-grade.

RESULTS OF THE INDIVIDUAL TESTS

This section of the chapter reports the results of interviewing forty Grade I and sixty Grade II pupils on certain selected number combinations in addition and subtraction. The purpose of these interviews was to ascertain precisely how children actually think about number combinations (1) with which they are unfamiliar, (2) which they have been taught in the term just elapsed, and (3) which they have been taught the previous term and have used and practiced during the subsequent term. This purpose was achieved by interviewing children, the *same* children, twice during the school year—at the end of the first term and at the end of the second term.

It was expected that the interview data would serve at least two functions: first, by showing how children really go about the task of finding answers to combinations, it was hoped that facts would be available to combat the still prevalent notion that children either "know" or "do not know" the combinations; second, by comparing thought procedures used at different stages in learning them it was hoped that some evidence might be had on the nature of the process by which children eventually attain the meaningful mastery which is enjoyed by the adult.

Procedure.—The original plan, which called for at least one hundred subjects per grade, proved impracticable. There are ten subjects more or less than half this number in each grade. While these numbers of subjects (forty and sixty) may seem rather small, it should be remembered that they are samples of much larger known populations. The forty Grade I subjects represent an enrollment of 176 first-grade pupils in six classes in four states; the sixty Grade II subjects, an enrollment of 215 pupils in seven classes in four states. In each co-operating class the teacher selected his interviewees according to a plan which made his smaller group typical of the class as a whole. Thus, he chose the first name on the alphabetical roll, the fourth, the seventh, and so on, or the first, the fifth, the ninth, and so on. This arrangement tended to remove all selective factors which would have made the interviewees atypical.

Interviews were held with each child separately and were conducted by the classroom teacher, the better to assure normal conditions. A combination was presented orally to the child, and he was instructed to "think out loud." If the child hesitated and was silent, the interviewer said, "Go ahead; tell me what you are thinking." If an answer was announced without previous evidence as to the procedure used, the interviewer said, "Tell me how you got the answer. . . . How did you know that . . . is the answer?" In no case were leading questions asked; nor were suggestions given which the child could claim as describing what he had been thinking.

The combinations for the interviews were chosen in order to make possible certain comparisons which were contemplated. At the end of the first term in each grade the child was given two sets of combinations: (1) eight which he had just been taught, and (2) four with which he was unfamiliar and which he would learn in the next term. At the end of the second term the interview again involved two sets of combinations: (1) six of the eight "taught" facts in the first test (in order to see what changes in procedure had occurred with the passing of time, or, better, with more opportunity for use) and (2) seven combinations taught during the current term, four of which were the "unfamiliar" facts of the first-term testing.

In order to save time in writing and to conserve space on the interview blanks, the pupils' procedures were recorded by means of the code which is given below. Preliminary study had revealed that children tend to react in one or another of six characteristic ways in giving answers to combinations. Symbols were supplied for these ways, and these symbols were entered on the interview blanks together with the child's numerical answers. When none of the six categories seemed to fit a child's report of his procedures, interviewers were instructed to supply a complete account of what the child said. No such instances were, however, described by the teachers. The code follows:

NA: No attempt; says, "I don't know," or remains silent without offering an answer.

G: Guesses; "takes a chance"; is satisfied with incorrect answer, or attempts to correct by guessing again.

C: Counts sums or remainders by starting with 1 or ending with remainder.

PC: Partly counts; in addition, does not begin with 1, but counts to one of the numbers given; in subtraction, the procedure is the same as *C*.

S: Solves in one or another of several ways:
Reverses order of numbers (thus, changing a given $1 + 3$ to a better known $3 + 1$)
Relates the A- and S-facts; gets the answer for an A-fact by thinking of the corresponding S-fact (thus, gets $6 - 2 = 4$ from $4 + 2 = 6$)
Adds or subtracts from doubles (thus, $3 + 2 = 5$, because $2 + 2 = 4, + 1 = 5$; or $3 + 3 = 6, - 1 = 5$)
Uses some other more familiar fact in a way not described above.

MR: Meaningful recall; the child "knows" the answer, and states it at once, confidently, without use of roundabout procedures.

In order to make sure that the symbols and their use would be intelligible to the interviewers, the mimeographed interview blanks contained two hypothetical cases, with answers entered and code letters assigned thereto.

Summary of interview data, by grades.—Tables 28 and 29 summarize the results of the interviews by grades. Each table is divided horizontally into two parts, the upper for the first term, the lower for the second term. In turn, each part is further divided, for the two sets of facts included in each term test. The first complete row of Table 28 is read as follows: The combination $1 + 3$, taught in the first term of Grade I, was reacted to by the 40 subjects as follows: two made "No Attempt," 2 "Guessed," none "Counted," 1 tried "Partial Counting," 3 "Solved" the answer, and 32 recalled the answer at once, apparently under circumstances which warranted the interviewers in recording "meaningful" answers.

It is difficult to see much significance in tables as complicated as these. For this reason the facts of major importance are treated in the next three sections, each with a table or two drawn from the summary tables (Tables 28 and 29).

Thought processes or procedures on "known" combinations.—As has already been suggested, it is not uncommon to suppose that children who have been taught the combinations either "know" them or "do not know" them. If children who have studied certain combinations give the correct answers without too much hesitation, it is thought that they "know" those combinations. By "knowing" is meant that children think the answer and *only* the answer when the incomplete combination is presented orally or in written form.

That this view of "knowing" is a bit oversimplified is rather evident in Tables 28 and 29, where are exhibited the classified responses of children both to taught and to untaught combinations. If it is argued that naturally children would engage in roundabout procedures with unfamiliar combinations, then the point may be supported

TABLE 28

RESULTS OF INTERVIEWS IN GRADE I, FIRST AND SECOND TERMS; FORTY CHILDREN SELECTED FROM SIX SCHOOLS

Combination	*Procedure Employed, with Frequency*					
	No Attempt	*Guessing*	*Counting*	*Partial Counting*	*Solution*	*Meaningful Recall*
First term:						
Facts taught:						
1 + 3........	2	2	..	1	3	32
6 — 2........	2	13	..	3	4	18
3 + 2........	2	7	..	3	1	27
2 + 4........	2	14	..	..	2	22
5 — 2........	1	12	1	3	5	18
6 — 3........	4	15	..	1	5	15
1 + 5........	..	5	..	5	3	27
4 — 3........	3	17	..	2	4	14
Unfamiliar facts:						
5 + 4........	2	20	1	2	1	14
7 — 2........	5	17	1	4	2	11
7 + 2........	2	17	1	10	1	9
8 — 5........	6	24	1	5	3	1
Second Term:						
Facts retested:						
6 — 2........	..	1	..	1	5	33
3 + 2........	..	2	..	1	2	35
2 + 4........	..	1	1	1	2	35
5 — 2........	..	1	..	..	10	29
6 — 3........	..	4	..	..	8	28
1 + 5........	..	..	..	..	..	40
Facts taught:						
5 + 4........	..	..	..	1	..	39
7 — 2........	..	2	..	2	11	25
7 + 2........	..	1	..	4	..	35
8 — 5........	..	5	..	..	8	27
2 + 6........	..	3	..	4	..	33
4 + 3........	..	1	..	2	2	35
9 — 6........	..	4	..	2	8	26

TABLE 29

RESULTS OF INTERVIEWS IN GRADE II, FIRST AND SECOND TERMS; SIXTY CHILDREN SELECTED FROM SEVEN SCHOOLS

Combination	*Procedure Employed, with Frequency*					
	No Attempt	*Guessing*	*Counting*	*Partial Counting*	*Solution*	*Meaningful Recall*
First term:						
Facts taught:						
4 + 6.......	..	1	..	3	8	48
11 — 3.......	2	11	..	6	18	23
3 + 9.......	..	7	..	7	5	41
10 — 7.......	4	5	..	5	17	29
12 — 5.......	1	9	..	5	18	27
7 + 4.......	..	2	..	1	10	47
8 + 2.......	..	1	..	2	6	51
12 — 9.......	5	8	..	3	20	24
Unfamiliar facts:						
6 + 7.......	..	12	..	1	13	34
15 — 6.......	6	27	..	3	18	6
14 — 8.......	4	23	..	14	9	10
8 + 5.......	..	12	1	3	12	32
Second Term:						
Facts retested:						
11 — 3.......	..	7	..	2	8	43
3 + 9.......	..	3	..	4	7	46
12 — 5.......	1	2	..	8	11	38
7 + 4.......	..	1	1	6	6	46
8 + 2.......	..	..	..	8	1	51
12 — 9.......	..	5	..	6	17	32
Facts taught:						
6 + 7.......	..	6	..	5	3	46
15 — 6.......	1	5	..	7	15	32
14 — 8.......	2	4	1	5	12	36
8 + 5.......	..	5	2	8	2	43
17 — 8.......	..	7	..	5	12	36
5 + 9.......	..	4	2	1	8	45
16 — 7.......	1	8	..	1	12	38

on the basis of taught combinations only. For this purpose Table 30 has been prepared. In all, fifteen hundred different combinations responses are here classified, by grade, by term, and by procedure employed. The important figures appear in per cents in the last row of the table. There it is shown that less than two thirds (63.9 per cent) of the fifteen hundred responses could be classified as instances of "Meaningful Recall."

It will be noted that the combinations referred to in Table 30 are restricted to those actually taught in the term which closed just prior to the time of testing. About one seventh (14.3 per cent) of the responses were classified as "No Attempt" or "Guessing." (Prac-

TABLE 30

HOW CHILDREN THINK ABOUT THE COMBINATIONS THEY ARE SUPPOSED TO "KNOW"

Grade, and Combinations Taught	*Procedures Employed, with Frequency*						
	No Attempt	*Guessing*	*Counting*	*Partial Counting*	*Solution*	*Meaningful Recall*	*Total*
Grade I, N = 40:							
First term: 8 facts, 4 in A,* 4 in S†	16	85	1	18	27	173	320
Second term: 7 facts, 4 in A, 3 in S	0	16	0	15	29	220	280
Grade II, N = 60:							
First term: 8 facts, 4 in A, 4 in S	12	44	0	32	102	290	480
Second term: 7 facts, 3 in A, 4 in S	4	39	5	32	64	276	420
Total, both grades	32	184	6	97	222	959	1500
Percentage distribution ..	2.1	12.2	0.4	6.5	14.8	63.9	99.9

* A = addition
† S = subtraction

tically all such responses were made at the end of the first term; at the end of the second term they were negligible.) This per cent may seem to be unduly high, but one cannot tell, for there are no comparable research data available. What is certainly significant, however, is the fact that more than one fifth (21.7 per cent) of the answers were arrived at by procedures somewhat short of "Meaningful Recall" in point of maturity. The largest part of these (14.8 per cent) were obtained by "Solving" from other better known combinations, and slightly half as many by "Partial Counting."

If it is argued that these figures merely prove that these children had not yet "learned" the combinations they had been taught, it is possible to find in Tables 28 and 29 data to show what these same children did when confronted a full school term later with part of the same combinations. Briefly, the first-grade children found answers by "Meaningful Recall" for only 83.3 per cent of the facts on which they were retested, and the second-grade children, for only 71.1 per cent. Evidently they still did not "know" the facts which they had been taught; they had not yet "mastered" them.

The situation revealed by these tabulations, namely, that children employ a variety of procedures besides "Meaningful Recall" in dealing with familiar number combinations, is not peculiar to the subjects of this investigation. Brownell (5) and more recently McConnell (26) found their subjects in Grade II using just such mental processes with number combinations. The findings of the McConnell report are especially significant, since his subjects were taught by two radically different methods. The one group was asked to memorize the combinations as given, without concrete experience of any sort and without any other kind of explanation for the purpose of establishing relationships. Yet these children "Guessed," and "Counted," and "Solved." The other group of experimental subjects learned in a manner more nearly like that of the present study; and these subjects too "Guessed," and "Counted," and "Solved."

Some reader may be disposed to interpret the large use of uneconomical procedures as meaning that these children (and McConnell's as well) were not "ready" for the combinations, that they had been asked to learn the facts while they were still immature. This interpretation can be neither supported nor attacked on the basis of research data, for there are none available. No one has shown what happens when the number combinations are deferred to Grade II or IV, and investigators, like Thiele (48, 49) and Wilson (55), who report marked success in teaching the number combinations according to their programs, have reported data only from group tests, so that nothing is known about the nature of the thought processes of their subjects. It is the writer's guess, however, that in every instance facts similar to those contained in Tables 28, 29, and 30 would have been found, and will be found.

A better interpretation of the prevalence of roundabout procedures is inherent in a view of the learning process which differs from the one which heretofore has been rather generally held. According to the latter view, from which springs the idea that combinations are either "known" or "not known," mastery is attained very quickly; children are given the combinations and their answers, and they "learn" them. Drill is necessary, it is given in large amounts, and the results show "mastery," which is to say, a high accuracy percentage on group tests under stated time limits. The view which is here sponsored is that children attain "mastery" only after a period during which they deal with combinations by procedures less advanced (but to them more meaningful) than automatic responses. To illustrate this view, and to give some support to it, the facts in Tables 28 and 29 have been reclassified and are discussed in the next section.

The process of learning the combinations.—In the first-term tests, it will be recalled, four unfamiliar or untaught combinations were presented in the interviews. These same facts were taught during the second term and were included in the second-term testing. A comparison of the procedures used under the two sets of circumstances tells something about the nature of changes in thinking which followed teaching. The upper half of Table 31 contains the pertinent facts. For example, the first time the 40 Grade I subjects were interviewed on the four unfamiliar facts 15 made "No Attempt," 78

TABLE 31

NATURE OF LEARNING AS SHOWN BY CHANGES IN PROCEDURES USED WITH NUMBER COMBINATIONS FROM TERM TO TERM

Nature of Comparison	*Procedures Employed, with Frequency*						
	NA	*G*	*C*	*PC*	*S*	*MR*	*Total*
Unfamiliar facts the first term, taught the second term							
Grade I: 40 pupils, 4 facts							
First term...........	15	78	4	21	7	35	160
Second term.........	0	8	0	7	19	126	160
Grade II: 60 pupils, 4 facts							
First term...........	10	74	1	21	52	82	240
Second term.........	3	20	3	25	32	157	240
Facts taught first term and retested second term							
Grade I: 40 pupils, 6 facts*							
First term...........	11	66	1	15	20	127	240
Second term.........	0	9	1	3	27	200	240
Grade II: 60 pupils, 6 facts†							
First term...........	8	38	0	24	77	213	360
Second term.........	1	18	1	34	50	256	360

* To permit this comparison of identical combinations, two facts taught in the first term are omitted—an "easy" combination, 1 + 3, with 32 MR-responses in the first term, and 10 — 7, with 29 MR-responses.

† Two combinations taught in the first term are omitted: 4 + 6, with 48 MR-responses, and 10-7, with 29 MR-responses.

"Guessed," and so on; the corresponding figures after teaching were 0 and 8. To summarize the facts for both grades, 177 responses to the unfamiliar facts were "No Attempt" or "Guessing"; this sum represents 44 per cent of all responses. The corresponding figures for the same kinds of responses *after* learning are 31 and 8 per cent. The instances of "Partial Counting" make up 11 and 8 per cent of the responses before and after teaching; those of "Solution," 15 and 13 per cent; and those of "Meaningful Recall," 29 and 71 per cent. To say that these children were not learning because after teaching they still persisted in using indirect procedures and because they had not yet attained 100 per cent of "Meaningful Recall" is to overlook the clear evidence of learning in the changes from "No Attempt" and "Guessing." True, these children had not yet achieved mastery, but the data indicate that they were on the way. And that is enough for anyone who views learning in terms of progress toward a goal too difficult at first to be immediately attained.

The lower half of Table 31 shows how learning continues after teaching. The 40 pupils in Grade I made "No Attempts" on 11 of the 240 combinations (six per pupil) which they had been taught that term, "Guessed" the answers of 66 more, "Partially Counted" 15 more, "Solved" 20 more, and used "Meaningful Recall" on 127 more. During the second term while they were being taught new combinations they undoubtedly had occasion to use these first-term facts and also received further drill on them. At the end of the second term, "No Attempts" had dropped from 11 to 0, "Guessing" from 66 to 9, and so on.

If the figures for Grades I and II are combined, it will be noted that 123 combinations were not attempted or were guessed on the test after teaching, and 28 a term later. These per cents are, respectively, 21 and 5, of the 600 responses. The per cent changes in "Partial Counting" are from 7 to 6; in "Solution," from 16 to 13, and in "Meaningful Recall," from 57 to 76. Clearly, these children were *still* learning and were getting so much closer to the kind of mastery which is sought. The slight changes in per cents for "Partial Counting" and "Solution" do not mean that no learning was occurring at all; rather, some of the children who at first made "No Attempt" or "Guessed" were at the time of the second test in the intermediate stages of learning preliminary to final "meaningful Recall."

This view of the learning process, according to which children work their way through successive levels of thinking to eventual "Meaningful Recall," is not unreasonable. Or, it is not unreasonable unless one is content to have children learn merely the verbal formulas in which the number facts are stated. In this latter case it is difficult to account for children's use of "Counting," "Partial Counting," and "Solution": why should children who, supposedly, have heard and seen only the symbols for the numbers and the processes, and these in the most concise and direct possible arrangement —why should such children engage in roundabout procedures? The answer to this question, troublesome to those who expect children all at once to achieve mastery, is that, pretty much irrespective of the way teachers present combinations, children must learn them as they can. They can acquire a certain number of combinations as mere verbalizations, without understanding, without relating facts, by memorizing the appropriate statement for each separate fact. But facts so learned do not last long, and they are not very useful. Soon or late children discover for themselves, if they are not told by any one else, that there are ways of thinking about the combinations which make them sensible as well as easily retained and immediately applicable to arithmetical problems. One way is to count; another way is to organize groups of facts around some well-known fact ("Solution"); thus the doubles become such centers of organization.

Some programs of instruction have ignored the tendency of children to put sense into what they learn and have recommended drill from the start as the only necessary form of instruction. Other programs, like that used in this investigation, like one of McConnell's, and like Thiele's preferred method, make the most of this tendency and enlist its aid in the job of teaching the combinations. Immature procedures are illustrated and permitted at the start, and children are encouraged as rapidly as they safely may to surrender their least mature procedures for more mature procedures, which in turn are abandoned in favor of still more mature procedures. But the important fact to be noted is that children, if not at first, then later, tend to make the combinations meaningful by applying to them one or more indirect and immature procedures. They do this if they are subjected to drill as the sole teaching method, and they do it if instruction from the start emphasizes meaning. The advantage of the latter approach is that it makes the most of this tendency and at the time when it can contribute most largely.[5]

The subtraction compared with the addition combinations.—Research generally has found the subtraction combinations harder than those in addition for children to learn. This fact is confirmed in the present study, in spite of the practice of teaching the related addition and subtraction combinations together (thus $2 + 3$, with $3 + 2$, $5 - 3$, and $5 - 2$). Evidence was found (but not reported) in the results of the group tests, and it is shown in the interview data summarized in Table 32. The first-grade children employed "Meaningful Recall" 387 out of 520 possible times with the addition facts (74

[5] This view of learning in arithmetic has been stated rather fully in Chapter V of the following reference: William A. Brownell, with the assistance of Kenneth G. Kuehner and W. C. Rein, *Learning as Re-Organization: An Experimental Study in Third-Grade Arithmetic* (Duke University Research Studies in Education, No. 3; Durham, N. C.: Duke University Press, 1939). A briefer and simpler statement is to be found in: William A. Brownell, "Two Kinds of Learning in Arithmetic," *Journal of Educational Research,* XXXI (May, 1938), 656-664.

per cent); and 245 out of 480 possible times with the subtraction facts (51 per cent). The corresponding per cents for the Grade II children were 74 and 48.

TABLE 32

COMPARISON OF PROCEDURES USED WITH A (ADDITION) AND S (SUBTRACTION) COMBINATIONS IN GRADES I AND II

Grade, and Number of Combinations Taught	*Procedures Employed, with Frequency*						
	NA	*G*	*C*	*PC*	*S*	*MR*	*Total*
Grade I, 40 pupils:							
13 A-combinations......	10	73	3	34	17	387	520
12 S-combinations......	21	115	3	23	73	245	480
Grade II, 60 pupils:							
12 A-combinations......	0	54	6	49	81	530	720
13 S-combinations......	27	121	1	70	187	374	780

On the other hand, the first-grade children made "No Attempt" or "Guessed" 16 per cent of the addition combinations and 28 per cent of the subtraction combinations. The corresponding percentage figures for the Grade II children are 8 and 19. The first-grade children "Solved" 3 per cent of the addition combinations and 15 per cent of the subtraction combinations; the figures for the Grade II children are 11 per cent and 24 per cent.

An analysis of the kinds of "Solution" used in the two processes is also revealing. Three major kinds of "Solution" were identified: (a) use of the reverse fact in the same process; (b) use of a corresponding fact in the other process; (c) use of doubles. Table 33 shows a tendency for both first- and second-graders to "Solve" addition combinations more commonly than subtraction combinations by using the related fact in the same process; that is, children tended to get unknown addition answers from known addition combinations somewhat more frequently than to get unknown subtraction answers from known subtraction facts. On the other hand, they tended nearly eight times as often to get unknown subtraction answers from known addition facts as vice versa; and they obtained about twice as many unknown subtraction answers as addition answers by making use of doubles. (The data for "Solution 4" are not here treated, since the different procedures used under this heading were rather numerous, with but few instances of any one procedure.) In other words, these children not only were less familiar with the subtraction facts but also were accustomed to derive subtraction answers from the better known addition facts.

TABLE 33

EXTENT TO WHICH VARIOUS TYPES OF SOLUTION WERE USED ON A- AND S-COMBINATIONS IN GRADES I AND II

	Solution 1*		Solution 2*		Solution 3*		Solution 4*	
	A	S	A	S	A	S	A	S
Grade I, N = 40:								
First term:								
Facts taught (4 A; 4 S)...	9	0	0	11	0	7	0	0
Unfamiliar facts (2 A; 2 S)	0	0	2	5	0	0	0	0
Second term:								
Facts retested (3 A; 3 S)..	1	1	1	14	1	8	1	0
Facts taught (4 A; 3 S)...	0	4	2	22	0	1	0	0
Total, Grade I...........	10	5	5	52	1	16	1	0
Grade II, N = 60:								
First term:								
Facts taught (4 A; 4 S)...	13	3	5	60	1	1	10	9
Unfamiliar facts (2 A; 2 S)	0	7	9	18	8	0	8	2
Second term:								
Facts retested (3 A; 3 S)..	3	0	3	30	0	3	8	3
Facts taught (3 A; 4 S)...	1	0	5	43	0	3	7	5
Total, Grade II...........	17	10	22	151	9	7	33	19
Total, both grades...............	27	15	27	203	10	23	34	19

* Solution 1 = Solved by using the reverse fact in the same process; thus, answered 3 + 4 by thinking of 4 + 3 and answered 7 — 5 by thinking 7 — 2.

Solution 2 = Solved by using a related fact in the other process; thus, answered 3 + 4 by thinking 7 — 3 (or 7 — 4) and answered 7 — 5 by thinking 5 + 2 (or 2 + 5).

Solution 3 = Solved by using the doubles; thus answered 3 + 4 by solving from 3 + 3 or 4 + 4; and answered 8 — 5 by thinking 8 — 4.

Solution 4 = Solved in some other way; the category really is equivalent to "Miscellaneous."

Why subtraction facts are more difficult than addition facts cannot be explained from the interview data collected—or, for that matter, from any other research data which have been published.[6] The process of addition may really be easier for children to understand than the process of subtraction, and the effect would be to facilitate the learning of the addition facts. On the other hand, children may have more occasions to add than to subtract, and the difference in opportunity for use may account for the apparently greater ease of

[6] Knight and Behrens (24) report data which seem to be in conflict with this statement. To learn the five hardest addition facts according to their rankings, the average child required a mean of 39 practices and the eightieth percentile child, a mean of 60+. The corresponding figures for the five hardest subtraction facts were 28 and 49. To learn the five addition facts next in difficulty ranking required 37 responses of the typical child and 57 of the eightieth percentile child. These figures are to be compared with 28 and 40 for the next five most difficult subtraction facts. The smaller numbers of practices for the subtraction facts argue for greater ease of learning these facts, but it should be noted that apparently the experimental subjects studied the subtraction facts *after* they had learned those in addition and accordingly could make use of these facts in the new learning.

the addition combinations for learning. The latter explanation seems to be nearer the truth. If it is the true explanation, the implication for teaching is clear: experiences involving subtraction and the subtraction combinations should be greatly increased beyond what is now common practice.

CONCLUDING STATEMENT

Here ends the report of the original investigation to determine the results of a systematic program of arithmetic instruction in Grades I and II. The data from the group tests, summarized in pages 70-90, are treated purely in terms of accuracy of response. On this basis (which is the usual one in research of this kind) the experimental subjects revealed distinctly creditable growth toward practically all of the outcomes set for the four half-grades. The data obtained by interviewing smaller samples of the experimental subjects were treated in terms of procedures employed in thinking about the addition and subtraction facts. The purpose of this section of the chapter was primarily to reveal the nature of growth through learning.

The research analyzed in Chapter II was interpreted to imply the ability of primary children, even of children in the first term of Grade I, to learn arithmetic if arithmetical experiences are adjusted to their stages of development. The facts in this chapter test this hypothesis by showing what happened when nearly five hundred children in Grades I and II were subjected to systematic instruction: these children did make substantial progress of a measurable kind.

The purpose and the findings of Chapter III should not be misinterpreted. In the first place, the systematic program of instruction of which results have been studied is not put forward as the best or as the only possible program. On the contrary, it is regarded as *one* possible organization which may prove to be inferior to others that will be developed and tried out. It was made the basis of the present study, chiefly because it had been worked out experimentally, seemed to be in line with the powers of children in Grades I and II, and was available.

In the second place, the findings do not *prove* that systematic teaching in arithmetic should be started in Grade I. They show only what resulted when one program was undertaken and they show that children so taught actually do learn a considerable amount of arithmetic. Data were not collected, largely because they could not be, to show the pleased and intelligent way in which children went at the number tasks which were set them. The co-operating teachers were unanimous in their statement that their pupils actively enjoyed their work and gave convincing evidence of understanding and using what they learned. The success of the program means (to the writer at least) not only that children in Grades I and II *can* learn arithmetic, but that with properly graded learning activities they *will* learn arithmetic and learn it happily and satisfyingly. To some this investigation provides evidence that arithmetic should be started in the primary grades; to others, it provides no argument to refute their preference for deferring systematic teaching.

CHAPTER IV

PREVIOUSLY REPORTED RESEARCH ON THE EFFECTS OF INITIATING OR DEFERRING ARITHMETIC INSTRUCTION IN GRADES I AND II

Unity of treatment is no more possible in this chapter than it was in Chapter II. In all, forty-six different researches are to be considered, and while some of these overlap at various points, they can hardly be profitably summarized in fewer than eleven main groups. A topical review of research almost necessarily precludes a clear overall picture of the situation, but what it lacks in this respect may be balanced by the opportunity it affords for a critical appraisal of specific aspects of the situation.

The research summarized in this chapter is discussed under the following topics:

1. EVIDENCE ON THE VALUES OF SYSTEMATIC INSTRUCTION BEGINNING IN GRADE I

In 1928 Washburne (50) reported for the Committee of Seven a statistical study of the arithmetical skills of some five thousand sixth-grade pupils who had started arithmetic in different grades. Approximately one third (Group I) had begun arithmetic in Grade I, another third (Group II) in Grade II, and the rest (Group III) in Grade III. When these different groups had been equalized with respect to chronological age and intelligence, there remained for comparison 806, 939, and 724 pupils respectively. Two thirty-minute tests were administered to all subjects, and the scores for the three equated groups were tabulated according to their successes on twelve arithmetical processes. The published report contains only the average scores. Thus, Group I averaged 5.59 in column addition, Group II averaged 5.87, and Group III averaged 5.93. In advanced subtraction the average scores were, respectively, 14.40, 11.58, and 11.47.

In all but one of the twelve arithmetical processes (the exception being column addition, in which the groups were equal) Group I surpassed Group II, and Group II surpassed Group III. The Committee tried to isolate all factors, besides differences in age and intelligence, which could account for the facts found. Amount of time devoted to arithmetic, for example, was rejected after a careful comparison of time allotments for arithmetic instruction among the schools represented in the three groups. The conclusion reached was (p. 661): "We are therefore forced to the conclusion that the uniform superiority of the pupils who began arithmetic in the first grade over those who began in the second grade and third grade is due tc the fact that they had an earlier start." None of the subsequent work of the Committee of Seven has apparently uncovered any explanation other than this.

Admittedly many of the differences between the three groups in this study are small; moreover, no measures are given by which to test the reliability of the reported differences. Washburne based his conclusion upon the consistency of his findings, and this argument seems to be a sound one.

As a matter of fact, it is highly probable that the true differences between the three groups were actually larger than those reported. When one considers conditions which almost certainly would have tended to level away the differential effects of starting arithmetic at various points in the first three grades, the wonder is that numerically appreciable differences remained at all. In the first place, the groups were, of course, made up in terms of schools, not in terms of individual pupils. The shifting of pupil populations, as of pupils from Group III schools to Group I schools, and vice versa, might well have made the three groups very much alike, so much so as to wipe out potential differences in arithmetical skill and knowledge. In the second place, it is questionable whether instructional materials in 1922-25 could have capitalized on different amounts of progress made in the grades prior to Grade IV. If so, the learning advantages (if any) of Group I pupils might well have been lost when they were required to drop back to overly simple texts and learning experiences. These arguments have the effect, it will be noted, of supporting the contention of the Committee of Seven in 1928 that an early start in arithmetic pays good dividends.

Additional support for this contention is furnished in the more recent study by MacGregor (27). MacGregor administered Battery A of the Public School Achievement Test in the Fife County (Scotland) public schools to all the 5,961 pupils whose eleventh birthdays fell in the 1930-31 session. Only that part of his monograph which reports the comparative arithmetic scores of the Scottish children is of interest here. It should be borne in mind, first, that the Scottish children were relatively unacquainted with standard tests and, second, that they were penalized by several test items better suited to American than to Scottish culture. In spite of these handicaps the Scottish children surpassed the American norms by fifteen months in problem solving and by eighteen months in computation.

The significance of this study for the present purpose lies in the fact that the Scottish children began arithmetic, not at age six or later, but at age five, a circumstance which MacGregor regards as sufficient to account for the superiority of his subjects. In Scotland children enter school at age five, and they immediately begin the systematic study of arithmetic. It is to be noted further that the Scottish children not only maintained their initial advantage of a year, but that they added materially to that advantage during the six or seven years following their entrance into school. With appropriate tests, MacGregor believes, the disparity between Scottish and American children of the same age would have been even greater.

No one can say with certainty how much of the superiority of the Scottish children is to be attributed to their early start in arithmetic. It may well be that other factors than this operated in favor of the Scottish children, factors such as superior teaching, closer articulation of arithmetic instruction from grade to grade, longer periods devoted to arithmetic, and so on. Whatever be the ultimate explanation, the MacGregor research has established the ability of children even younger than those in American first grades to profit from directed experiences with number. It therefore supplements the findings of the Committee of Seven 1928 investigation just reviewed and the findings of the original investigation summarized in Chapter III, to the effect that *first-grade* children are old enough and mature enough intellectually to learn arithmetic when it is properly presented to them. Evidence of the ability of *second-grade* children to profit from systematic instruction will be summarized below, particularly in Section 4, on the learning of the simple number combinations.

2. EVIDENCE ON THE VALUES OF "SOCIAL" ARITHMETIC IN GRADE I (OR IN GRADES I AND II) AND OF DEFERRING SYSTEMATIC INSTRUCTION

Some students of arithmetic advocate no "formal" instruction before Grade III or even later. These individuals favor a kind of "social" arithmetic for the first year or two, to allow time for number concepts to develop through "maturation," through "natural" encounters with quantitative situations, or through participation in planned activities at least partly quantitative. Three research studies which seem to support this point of view have been reported by Wilson (**55**), by Benezet (**3**), and by Gunderson (**17**).

In Grades I and II Wilson had his subjects set up school banks, stores, post offices, flower shops, and the like. Money transactions and other activities involving number were developed around several such centers of interest. These experiences were supplemented by number games such as Ten Pins and Ring It, in which number serves a vital purpose. There was no drill, no memorization, no pressure, and no "annoying" check-ups. The plan was to provide children with opportunities to see number in its functional relationships and to acquire a basis for understanding and for thinking effectively in quantitative situations. In Grade III "formal" instruction was instituted, both on the basic number combinations in addition and subtraction, and in the simpler computations in these processes.

Tests given at the end of Grade III revealed a high degree of mastery of the combinations taught. In experimental City L, for example, 289 out of 475 children made perfect scores on the addition combinations; 53 more made scores of 99; 54 more, of 98, and a total of 462, scores of 90 or better. In a control school system, where the traditional program of drill on the combinations prevailed from the start of Grade I, only 48 out of 174 third-graders made perfect scores on the addition combinations; 33, scores of 99; 18, scores of 98, and a total of 172, scores of 90 or better. The situation in a third system, used also as a control, revealed corresponding figures of 48, 31, 26, and 146 for the 154 pupils involved.

Throughout Grade III one day a week was devoted to informational arithmetic, and instruction on new topics was supplemented by drill service arranged by Wilson himself. At the end of the term, a total of 1,034 pupils were given the Wilson Survey Form 2 Test in Addition and the corresponding subtraction test. The average addition score was 97.8 per cent of the possibility, with 82.7 per cent of the children making perfect scores. In subtraction, the percentage equivalent of the average score was 95.6, and 74.8 per cent made perfect scores. Wilson remarks that never before had such results been reported.

Taken literally, Wilson's claim is unassailable; no other investigator has reported results comparable with his. Such investigators as have published their data have not, as Wilson has wisely done, indicated the per cents of their pupils who obtained perfect scores and the per cents who missed one, two, or three combinations. Nevertheless, Wilson's statement is a bit misleading, since it implies that by no other program are such results obtainable. It should be noted, in the first place, that the main difference between Wilson's experimental subjects and the controls lay in the proportions scoring 98 per cent or higher. The differences between the numbers of his subjects and of his controls who scored 90 per cent or higher are not large. Thus, in City B, a control system, 99 per cent of the pupils scored 90 per cent or more; in the second control system, 95 per cent attained this score at least, while in Wilson's City L, the experimental system, the per cent was 97. It should be noted, in the second place, that the high scores of Wilson's subjects may be attributed as reasonably to extra amounts of drill given in Grade III as to the program of "social arithmetic" which preceded drill in Grades I and II.

Part of Wilson's arithmetic program is to postpone systematic instruction until mastery, 100 per cent accuracy, is attainable. Since mastery of the combinations, as mastery is defined by him, does not seem attainable in Grades I and II, the combinations should be reserved until Grade III. This conception of learning and of the arithmetic curriculum is quite unlike that outlined in pages preceding. There it was held that sound learning is gradual, that skills and concepts should be presented some time before mastery is expected or sought, and that experiences should be arranged and ordered with a view to eventual mastery, but mastery of which understanding is a large component. Accordingly, the simpler combinations are introduced as such in Grade I in meaningful activities; immature ways of dealing with the combinations are permitted at first but discouraged as children show ability to adopt and use more mature procedures. As a consequence, learning of the combinations is continuous from the time they are first presented until mastery, perhaps in Grade III, is finally achieved. It is indeed unfortunate that the writer's investigation obtained no data comparable with Wilson's on the success of his subjects in dealing with the combinations in Grade III. It is even more unfortunate that neither research report contains data on the thought processes employed by Grade III children in connection with the combinations.

In interpreting Wilson's study one should note carefully that in banishing *"formal"* arithmetic from Grades I and II Wilson did not banish *all* systematic instruction. Nor did he. He did not leave all learning to fortuitous circumstances and to the mere condition of growing older. On the contrary, his teachers laid plans, arranged quantitative situations, and unquestionably helped their pupils to identify and to react intelligently to the numerical aspects in those situations. It would, therefore, be possible to argue that Wilson's instruction was as systematic as is the traditional kind, but that it was systematic in a different way. What his teachers tried to do was to embody the arithmetical content of their grades in larger experiences, but in experiences which were selected mainly because of their suitability for the purpose of revealing number in use. If this interpretation is correct, then the caption under which the Wilson study is placed is in error, or is at least misleading. It might better have been placed under the first heading ("Evidence on the Values of Systematic Instruction Beginning in Grade I"), with due recognition of the fact that Wilson's systematic instruction was socially rather than mathematically oriented.

The second research study to be considered here is that by Benezet (3). The report of this study is most informal; exact quantitative data of a comparative sort are lacking; and, accordingly, one cannot be very confident in his evaluation of the experiment or of its significance. In the account of the study one reads that all systematic instruction was eliminated from certain schools while it was continued in the usual way in other schools. In the experimental classes arithmetic was taught only incidentally, as teachers and pupils saw quantitative elements in the school day and in the school work which, if treated mathematically, would give new insights into the situations or questions at hand. Instances of such teaching are described. Success for this program is claimed on the basis of very incomplete evidence, chiefly anecdotal in character. The experimental children were able to attack intelligently real quantitative problems utterly confusing to the control subjects who had learned their arithmetic in the traditional manner. On the basis of this evidence Benezet is of the opinion that systematic instruction in arithmetic might be deferred to Grade VI or Grade VII, and that in one year at that time children could master all the arithmetic now taught in earlier grades.

Few educational experiments have attracted the attention that Benezet's report has enjoyed. It is fortunate therefore that certain critical articles have appeared to challenge the interpretation frequently placed upon this study. Buswell,[1] after cataloging the actual arithmetical experiences of the experimental subjects as mentioned by Benezet more or less in passing, calls attention to the fact that altogether these children received a very respectable amount of arithmetic instruction in the school, that what Benezet had done was merely to eliminate from the teaching the useless and meaningless parts of the traditional subject matter. Thiele,[2] after spending some time in the experimental classes, comments on the large amount of systematic teaching he observed and the relative absence of incidental learning. Benezet's pupils, for example, were required to bring problems and their solutions from the home to the school, where the arithmetic involved was analyzed and discussed as thoroughly as in any class of the traditional kind.

If these criticisms are valid, Benezet's informal experiment turns out to be, not a test of a program of postponing systematic arithmetic, but a protest (and a valuable one) against the aimless, unproductive, unenlightened arithmetic which, it is to be hoped, will steadily decrease in favor. In other words, Benezet's research does not establish the inability of primary grade children to learn arithmetic; if it establishes anything at all, it demonstrates the infertility of a particular kind of arithmetic instruction.

[1] Guy T. Buswell, "Deferred Arithmetic," *Mathematics Teacher,* XXXI (May, 1938), 195-200.

[2] C. L. Thiele, "An Incidental or an Organized Program of Number Teaching?" *Mathematics Teacher,* XXXI (Feb., 1938), 63-67.

A more fertile kind of arithmetic instruction is described by Gunderson (17). At the time of the study her seventeen subjects were completing the first half of Grade II. During the three terms they had been in school they had been subjected to no drill or formal arithmetic; all experiences were with concrete and semiconcrete materials, and no abstract combinations as such had been introduced. It has been suggested that the arithmetic instruction (twenty minutes a day, more or less) was informal, and it was; but it was also systematic. Gunderson leaves no doubt about this: her pupils' activities were "planned so as to enrich and extend [their] experiences with number, and to make numbers meaningful to them." Attention was given to counting by 1's, 2's, 5's, and 10's, to reading and writing numbers to 100, to grouping objects and comparing groups, to "store" activities with real money, to telling time, to reading and writing the Roman numerals to XII, to simple phases of measurement, and to problems with sums and minuends to 10, which were solved with actual or substitute objects. In other words, Gunderson set herself definite arithmetical outcomes and she sought their attainment in a wholly informal but systematic way. ("Formal" and "systematic" are thus seen not to be synonymous.)

Gunderson's investigation was designed to discover what understandings and concepts her plan of instruction had yielded. She took the regular arithmetic period for several days to question the children. Her procedure may be illustrated with regard to the concept of "two." Her questions were: "Tell all you know about *two*. What is *two?* What does *two* make you think of? How can you show *two?* How can you get *two* from other numbers? Do you know any words that mean *two?* What things make you think of *two?*"

Gunderson's findings are classified in four tables: Number Concepts, Concepts of Fractions, Miscellaneous Number Concepts, and Measures. The arrays of meanings are truly impressive. Samples are: 1000 + 1000 = 2000; half of a half is ¼ pint; 5 is half of 10; 5, 10, and 15 altogether are 30; twelve 10's are 120; XXXIV means 34; CII means 102. There can be little question that Gunderson's classroom procedures had given her pupils many important ways of thinking of numbers and number relationships. Nevertheless, the data are not quantified, and for this reason one does not know how many of the children possessed each of the meanings listed. In some instances a meaning was probably known to all or most of the children; in others, to perhaps only a few. The experiment might well be repeated with larger numbers of subjects and with the attempt to ascertain the commonness of the understandings found.

3. EVIDENCE ON THE VALUES OF ABANDONING SYSTEMATIC INSTRUCTION ENTIRELY, OR AT LEAST IN THE FIRST GRADES

The earliest investigation to test the effects of abolishing systematic instruction in arithmetic in Grades I and II was reported by Ballard (1) in 1912. Since this study was made in England, where children enter school at age five, it would probably be nearer the facts to say that Ballard investigated the values of postponing systematic arithmetic until age seven, or approximately our Grade II. Since the original report of this research was not available to the writer, it has been necessary to rely upon secondary sources.

Apparently what Ballard did was to eliminate arithmetic from the first two years of schooling, to start instruction in the third year, and then to compare the arithmetical attainments of the children who had been subjected to this program with those of children taught in the usual way. The results of his comparisons led Ballard to recommend the postponement of systematic instruction to age seven, or the third year of English children's enrollment in school. His seven-year-old experimental pupils seemed to have learned as much arithmetic from one year's study as did his control subjects who had been exposed to the subject for three years. Ballard's findings, it will be noted, are in conflict with the more recent findings of MacGregor (27), which have been reviewed above. The evaluation of the Ballard study and the attempt to reconcile his findings with MacGregor's will be postponed until after the similar study by Taylor (47) has been summarized.

In 1913-15 Taylor in this country undertook a study very much like Ballard's; the chief difference lay in the fact that Taylor in his experiment postponed systematic teaching only for one year. In Grade I Taylor's subjects concentrated on learning to read. Their only arithmetical experiences, or at least the only ones reported, involved counting, and an attempt was made to develop real skill in this function. Beginning with the first term of Grade II, arithmetic was taught. It was found that at this time children could in one term learn all of the arithmetic customarily taught in the whole of Grade I and that they could in the second term learn all that was usually taught in the whole of Grade II.

Taylor's experimental subjects demonstrated great superiority over the control subjects in amount and quality of reading in Grade II, but they also demonstrated that, according to the tests used, they had suffered not at all from their fewer arithmetical experiences. At the end of the first term of Grade II control children who had had arithmetic in Grade I scored an average of 81 in an oral test and an average of 72.6 in a written test. The corresponding figures for the experimental subjects were 87.1 and 89.4. At the end of the second term the control children earned average scores of 87.2 and 69.5, respectively, on oral and written tests, and the experimental children, scores of 90.7 and 61.0.

The Ballard and the Taylor studies are not uncommonly cited as having proved the wisdom of deferring systematic arithmetic to Grade II at least, but the findings are susceptible to quite a different interpretation. This interpretation has been well stated by Buckingham.[3] In both studies the control subjects must have learned *some* arithmetic; it is hardly reasonable to suppose that they learned none at all from the opportunities which were given them. They should therefore have started the later grade with some advantage over the experimental subjects. Apparently, however, this advantage was not recognized and utilized in teaching; instead, the control subjects were in the later grade taught just about what they had already learned, and the experimental subjects were able to start with them and make satisfactory progress. Buckingham comments: "If the same energy and resourcefulness [had been] devoted to the taught group as to the untaught group during the third grade, it is probable that the two groups would [have been] wider apart at the end of the year than they were at the beginning of it."

Buckingham's explanation may serve also to account for the discrepancy between Ballard's and MacGregor's data on the value of early instruction. In the period elapsing between the two studies much has happened to improve the organization and instructional procedures of primary arithmetic. It is quite possible that the kind of primary arithmetic taught in England in 1910 *was* practically a waste of time, and equally possible that the kind taught two decades later, when MacGregor's subjects were starting school, was distinctly helpful. Five-year-old and six-year-old children can hardly be expected to have learned much arithmetic from the abstract drill of an earlier day; on the other hand, children of these ages *can* be expected

[3] B. R. Buckingham, "When to Begin the Teaching of Arithmetic," *Childhood Education,* XI (May, 1935), 339-343.

to have learned much when taught by the newer materials and methods.[4]

So much for the Ballard and Taylor studies; next to be considered are four studies sponsored by exponents of the "activity" curriculum who would banish systematic arithmetic from all grades (not merely the first one or two) in favor of purely incidental contacts with number. These studies will be reviewed separately and then criticized as a group.

The first of the four studies relating chiefly to arithmetic in the primary grades[5] was reported by Collings (**13**) in a series of tables and bar graphs. Collings's data were obtained from forty children who were taught by a "project" curriculum over a period of four years and whose arithmetic records are compared, year by year, with a group of forty other children taught by the traditional curriculum and matched with the experimental subjects on the basis of CA, and IQ, and years of schooling. The tests used were prepared by Collings himself and, for the first four grades, covered only the four operations with whole numbers. Every comparison favored the "project" children. In the figures below, the average scores of the "project" children are stated in percentage equivalents of the average scores of the controls. Thus in Grade I the "project" children's average in addition was 200 per cent of that made by the controls.

	Grade I	*Grade II*	*Grade III*	*Grade IV*
	Per cent	*Per cent*	*Per cent*	*Per cent*
Addition	200	108	110	125
Subtraction	200	134	106	104
Multiplication	...	162	108	115
Division	...	...	107	131

Hopkins's subjects (**20**) were taken from the Lincoln School of Teachers College, Columbia University. Two sets of comparisons are offered: (a) for totals of 187 to 249 pupils per grade on the "old" Stanford Achievement Test and (b) for totals of 122 to 152 per grade on the 1930 Stanford Achievement Test. Only the scores in arithmetic are pertinent to the present purpose. According to the "old" test the Grade II pupils had an average arithmetic age of 94 compared with an average CA of 91 months; in Grade II, the corre-

[4] Some of these newer materials and methods are well set forth, for example, in Kenwrick's valuable manual: Evelyn E. Kenwrick, *Number in the Nursery and Infant School* (London: Kegan Paul, Trench, Trubner and Co., Ltd., 1937). Pp. xii + 252.

[5] Other studies by Collings and Meriam, somewhat after the fashion of Benezet (**3**), report evaluations of their program for a period much longer than that here in question. These studies are omitted.

sponding figures were 110 and 104; in Grades III and IV the average arithmetic ages surpassed the average CA's by increasing amounts. In terms of MA, however, the Grade II children showed a deficiency of 14 months, and the Grade III children, a deficiency of 11 months, in arithmetic age. On the "new" tests the Grade II children had an average AQ of 91 (instead of 100) in arithmetic reasoning and an AQ of 80 in arithmetic computation. The corresponding arithmetic AQ's in Grade III were 99 and 92. From these facts Hopkins argues the adequacy of the "activity" program to teach arithmetic.

The third study in this group of four was made by Meriam (35), who employed a total of eighty Mexican children divided among four school grades but all taught by an "activity" curriculum made up predominantly of hourly periods of "play," "stories," and "handwork." No arithmetic data are reported for Grade I, but averages are given for Grades II, III, and IV as derived from the Stanford Achievement Test. In spite of a slight deficiency in average IQ Meriam's subjects reached the grade norm in Grade II and surpassed the grade norms very slightly in Grades III and IV.

Among other studies cited by Meriam as having produced findings "consistent" with his own is an earlier one made in the Lincoln School and reported very briefly indeed—as a matter of fact, in three pages (25). In this study a group of children (the number is not stated) were taught an "activity" curriculum long enough to provide scores for Grades II to VI on the Stanford Achievement Test. In October the Grade II children fell slightly short of the national norm; in May, they exceeded the norm by an even smaller amount. In Grade III the class average was practically equal to the norm in October, but somewhat lower in May. In Grade IV, the October average surpassed the norm, as did also the May average.

Interpretation of these studies is by no means easy or assured. In the first place, measures from the Stanford Achievement Test (by which most evaluations were made) are open to question as to reliability and validity for Grades II and III. In the second place, in two instances at least, the children were definitely superior in intelligence; in one study (25) the average IQ was 119; in the Hopkins study the class medians ranged between IQ's of 116 and 120. In the third place, in all probability the teachers were superior and highly motivated. Such is usually true when teachers are committed to a relatively new instructional program. In the fourth place, and most important, there is no way of knowing just what happened with respect to arithmetic in the "activity" programs adopted in the different experimental centers. The *implication* is regularly given that *no* systematic instruction was offered, but in one of the four experiments this implication is certainly in error. The teachers in the Lincoln School (25) state that for the first year and a half the arithmetic course was "largely [not completely] incidental," that "number experiences [were] frequent, but actual drill . . . rare." On the other hand, an equally unambiguous statement is made to the effect that once need for skill in addition and subtraction was felt on the part of individual children, specific drill and practice were instituted, amounting to from forty to fifty minutes per week in Grade II and to four thirty-minute periods in Grade VI. In other words, in this instance at least the "activity" curriculum was not an "activity" curriculum in the extreme sense of the word. And it may fairly be assumed that since Hopkins's study was made in the same school his experimental subjects also had the benefit of considerable (but unmentioned) systematic instruction. What may have been the case in the Collings and the Meriam studies, it is impossible to say.

This is not the place for an extended criticism of the "activity" curriculum, either in general or as it applies directly to arithmetic. This conception of the curriculum usually entrusts arithmetical learning to incidental number experiences, and the writer has elsewhere[6] offered his criticisms of the adequacy of this program. What is important to note is that these studies, purporting to represent "activity" programs, fail to establish a convincing case.

4. TEACHING THE SIMPLE NUMBER COMBINATIONS OR FACTS

The following quotations, both from writers of texts on primary arithmetic, well reveal the differences in theory which have influenced the teaching of the addition and subtraction combinations during the last fifteen years or so. From one point of view, since "the end sought in manipulating the addition and subtraction combinations is the automatic response . . . the teaching technique which most nearly eliminates, from the start, any dependence upon 'reasoning,' or counting, or any other clumsy method, is best."[7] This theory, the prevalent one a decade or more ago, is no longer subscribed to by most students of arithmetic, including the authors quoted. The sec-

[6] William A. Brownell, "Psychological Considerations in the Learning and the Teaching of Arithmetic," *Tenth Yearbook of the National Council of Teachers of Mathematics* (New York: Bureau of Publications, Teachers College, Columbia University, 1935), pp. 12-19.

[7] J. W. Studebaker, F. B. Knight, and W. C. Findley, *Standard Service Arithmetics—Course of Study and Manual for Grades One and Two* (Chicago: Scott, Foresman and Co., 1929), p. 20.

ond theory is the more popular one; it is now generally maintained that the combinations should be seen and learned "in their relationship to each other. . . . Those who object to this method on the ground that it may make permanent the use of roundabout habits of response should remember that arithmetic which is not meaningful to children is worse than no arithmetic at all and that it is shortsighted policy which causes children to memorize combinations which they do not understand and to depend upon memory in situations which they might think out for themselves."[8]

In most instances research available on the problem of this section has been concerned with whether the combinations should be taught by methods consistent with the first or the second of the theories briefly epitomized above. The studies reported below are reviewed in chronological order.

Holloway (19) taught the forty-five addition combinations (hence, the direct and reverse facts together, all 0-facts being omitted) to fifty-three girls and fifty-six boys ranging from Grade IB to Grade IIB. For each pair of facts two concrete demonstrations of meaning were specified, but reliance was placed upon drill to complete learning to the stage of mastery. A table reports separately for boys and girls (a) the number of errors made on a preliminary test, (b) the number of errors made in learning, (c) the number of errors on the final test, (d) the number of errors overcome, (e) and "coefficients of difficulty" derived from the formula (c) ÷ (e). The resemblance to the study which follows, both in technique and in point of view, is striking.

Knight and Behrens (24) obtained complete records on the learning of the basic addition and subtraction facts by twenty-five and fifteen pupils, respectively. Alternate periods of testing and drill for maintenance were given to these second-grade subjects. During both the learning and drill periods, records were kept for each presentation of each fact to each of the subjects.

Data are presented on the following factors: (a) number of responses to learn, (b) number of errors encountered in learning, (c) time of first successful response, (d) time of final learning response, and (e) time of review response. The combinations partly mastered at the time of the initial test were selected for first presentation in learning. The remaining combinations were taught in "practically random order." Whether the presentation of the com-

[8] Robert Lee Morton, *Teaching Arithmetic in the Elementary School, Volume I, Primary Grades* (New York: Silver Burdett Co., 1937), p. 95.

binations or the responses of the children were given orally or in writing is not mentioned. The data so obtained were then used to ascertain the comparative difficulty of the combinations (see Section 5, below). Since the amount of practice on each separate combination was the determining factor in advancing instruction, this investigation may be said to exemplify the drill theory of teaching.

In the Beito and Brueckner study (2) three IIB grades of approximately thirty pupils each participated in a rotated-group experiment in learning the addition facts. These investigators made use of specially prepared cards, described below in Section 8. Pupils were tested twice daily for one week on each of the cards. To obtain an expression of the amount of growth in learning, the number of errors in the final test was subtracted from those in the first test, and the remainder was divided by the total number of errors on the first test, thus yielding the per cent of possible gain. Card 2 was taught by the usual methods of the teacher, and Card 3 by the drill-card method. Card 1, taught by a combination of both methods, was learned most economically, in terms of per cent of gain. Each of these methods, however, involved repetitive practice or drill on the abstract number facts. Since this study was designed primarily to investigate phenomena of transfer in learning, it is more fully treated in the appropriate section below.

The study by Wilson (55) has already been reviewed in another connection (pp. 107-109). Wilson's experimental procedure was to postpone the teaching of the combinations until Grade III, using Grades I and II for "social arithmetic," which is to say, for experiences with social situations (school stores, banks, etc.) in which number plays a prominent part. These experiences were supplemented with games in which number serves an important function. Beginning with Grade III, drill on the combinations was instituted, with evidence of marked success: at the end of the year an unusually high proportion of the third-graders were able to give correct answers to 95 per cent and more of the combinations.

The thirteen hundred subjects of Olander's investigation (36) were taught twenty minutes daily during a seventeen-weeks' program of experimentation. Although mastery of the number combinations was achieved through drill on the particular facts, Olander did attempt to introduce "relationships" in the actual learning. He says, "some classes were given instruction in generalizing groups of combinations. For example, these children were led to recognize the law common to zero combinations, they noted that combinations appeared in reverse form, such as 6 + 7 and 7 + 6, and they observed that a combination such as 10 — 6 was intimately related to 6 + 4." Further details concerning the results obtained are reserved for Section 8 below, on "Transfer of Training in Learning Arithmetic," since this problem was Olander's chief concern.

In an effort to reveal the relative effectiveness of the "specific drill" and "general relationships" methods of teaching, McConnell (26) conducted a controlled experiment in the learning of the 100 basic addition and the 100 basic subtraction facts. These 200 facts were taught to two groups of more than 600 second-grade children by means of specially prepared learning books. The one, Book A, was organized according to the drill theory of teaching the facts; the other, Book B, according to the theory which encourages children to discover number relations themselves. Teachers in both groups were instructed to spend the first week in a review and extension of their pupils' knowledge of concrete numbers, including seeing the number of objects as grouped together. The number facts were introduced to both groups in the same order. The first fifteen addition facts were presented, then the first fifteen subtraction facts. After that, the correlative addition and subtraction facts were presented together. Thus, in each unit of instruction, four new facts were introduced. From data on matched pairs, McConnell concluded that Method A is preferable for mechanical mastery, while Method B encourages a more thoughtful and "meditative" attack. The slight sacrifice of immediacy of response by the latter method seems to be offset by greater ability to transfer learning and to manipulate the number facts in mature ways.

A somewhat different comparison was made by Breed and Ralston (4), to determine whether the addition combinations could be taught more effectively in isolation (direct method), or incidentally in connection with more complex addition (indirect method). Two groups of thirty-three pupils in each of Grades I and II were matched first on the basis of CA and IQ, and, second, on the basis of initial achievement in arithmetic. Thirty minutes per day for one semester were devoted to the teaching of addition. According to the direct method, much use was made of flash cards, games, and other drills. The second group of children, taught by the indirect method, were given practice in adding several digits and two-place numbers. When they could not recall the sum of two numbers, they were directed to find the answers on charts prepared for them and ready to hand. In general, the indirect method appeared to be more efficient as measured by tests on the simple combinations, on complex addition, and on rate of writing figures, but none of these differences were reliable.

The study reported by Thiele (49)[9] compares the effectiveness of teaching the 100 addition facts during the first half of the second grade by two methods, generalizations (G), and drill (D). According to Thiele, the problem of teaching number combinations is that of so organizing and so directing learning experiences that the exercise and development of ingenuity and resourcefulness become the chief goal of teaching, as contrasted with the more traditional method which aims to cause children to memorize number facts as so many specifics of learning. For this experiment, fifteen weeks were devoted to the teaching of the addition combinations. This was preceded by four weeks of "number readiness" instruction. For the 242 G-pupils, the combinations were organized into groups as follows: (a) adding 1 to numbers and reverses, (b) adding 2 to numbers and reverses, (c) adding 0 to numbers and reverses, (d) the doubles, (e) the near doubles, e.g., $6 + 7$, $7 + 6$, etc., (f) adding to 10 and reverses, (g) adding to 9 and reverses, and (h) miscellaneous combinations. G-subjects at no time practiced a missed combination a given number of times by writing or saying it, or drilled with flash cards. Remedial teaching was directed toward the rediscovery and the kind of practice which brings generalization (not specific combinations) into play. On the other hand, when D-pupils needed the answer for a fact which they had not as yet habituated they were immediately directed to look up the answer. For these 270 D-subjects, the difficult combinations were presented in heterogeneous order; but the additions of 1, 0, and 2 were presented as groups.

Thiele's end test was the same with respect to time, administration, and the order of the test items as the initial test, used at the outset to measure the subjects' familiarity with the 100 addition facts. Tables showing (a) gains on the initial "100 Addition Fact Test," (b) gains by intelligence letter-rating groups, and (c) gains by D- and G-groups equated on initial scores, reveal remarkably consistent results, all in favor of the G-group. For example, while the mean gain of all pupils in comparison (a) above was 16.06 number facts, the D-group showed only 65 per cent of the growth indi-

[9] Thiele's earlier investigation (48) is not here reviewed. In general, the experimental attack was like that of the later study, and the findings were similar. The earlier study, however, shows that the method of "generalization" produced superior results to the drill method in the case of the subtraction as well as of the addition combinations.

cated by the G-group. In terms of per cent of possible gain made by pupils of the same intelligence ratings, the smallest G-advantage was 10.5 per cent, obtained by the slightly below average pupils. The greatest superiority, 24.3 per cent, was shown by the brightest pupils. The mean gain in the 100 facts for all 242 subjects taught by generalizations was 63.10 facts; for the entire drill group, the average gain was 47.04 facts. Since an effort was made before the experiment proper to provide pupils of *both* groups with opportunity to understand the meaning of numbers through concrete representation, counting, etc., it would appear that any advantage in technique favored the drill group. The superiority shown by the generalization subjects, as revealed by the end test, thus becomes even more formidable.

Wheeler (**53**) reports data for an experiment involving 125 pupils in Grade IIB. His subjects were taught the addition combinations informally by the game of ADD-0, purportedly a form of free-play activity instruction. During the twenty days of the experiment, the children gained on an average of nearly two and one half combinations per day. Since the form of presentation influenced slightly the difficulty of learning, Wheeler suggests that the child should have opportunity to learn both the equation and column forms, and be taught the reverses along with the direct combinations. Wheeler's teaching methods clearly put the emphasis on memorization.

Two studies dealing with the multiplication combinations may be briefly mentioned. Brueckner (**8**) gives a few data from an investigation by Dahlgren in which 125 pupils in Grades IIB and IIIB were taught the tables of 2's, 3's, and 4's only. None of the Dahlgren subjects had ever studied these facts before. Only one order of these facts was taught, namely, the direct order, with 2, 3, or 4 as multiplier. Since the object of the study was to measure transfer, the initial and final measures included scores on the reverse combinations as well. Further information concerning this study is given below in the section on Transfer. It will suffice to say here that the pupils learned the facts, both taught and untaught, to about 90 per cent accuracy.

The second multiplication study was made in Scotland under the auspices of the Scottish Council for Research in Education (**46**, pp. 79-82, 101-111). The 100 facts were administered as a group test to 1,755 pupils in two Scottish counties who were "completing their second session of the Primary Division, that is, their average age was nine years." The purpose of this study was to determine the relative difficulty of these number facts.

5. COMPARATIVE DIFFICULTY OF THE NUMBER COMBINATIONS OR FACTS

Of the many studies to determine the relative difficulty of the addition combinations, those dealing with first- and second-grade subjects are but few in number.[10] In Grade II, investigations have been reported by Holloway, by Knight and Behrens, by Thiele, by Washburne and Vogel, and by Wheeler.

Holloway (19) measured the comparative difficulty of the number combinations in two ways: (a) by his "coefficient of difficulty" in original learning (see p. 117, above) and (b) by the losses over the vacation from spring to fall. According to his first method the eight most difficult combinations for 120 boys and girls in Grades IB to IIB inclusive were: 9 + 7 (coefficient of difficulty, 2.30), 8 + 7, 9 + 8, 9 + 6, 8 + 6, 6 + 2, 9 + 4, and 5 + 1 (coefficient, 1.56); the eleven easiest included the doubles, except 8 + 8 and 9 + 9, and besides, 8 + 2, 7 + 1, 8 + 1, 9 + 2, all with coefficients of difficulty smaller than 1.18. By Holloway's method (b), 109 children made the most mistakes, in order, on 9 + 8; 9 + 7 and 8 + 6; 7 + 5 and 8 + 7; 8 + 3; 7 + 3 and 9 + 6; 7 + 6; 9 + 4, 9 + 5, and 6 + 5. They made the fewest mistakes (four or fewer) on the doubles, 8 + 2, 5 + 1, 6 + 1, 5 + 2, 4 + 3, and other facts calling for the addition of 1 or 2. Holloway's difficulty ratings (he had similar ratings for 1,056 third-grade children) are not comparable

[10] The most frequently cited study is that made by Clapp. More recently two other studies employing the same technique have been reported by Washburn and by Murray (Scottish studies, below). These investigations regularly employed subjects in Grade III and above, and hence were removed from the first learning of the number facts. The usual procedure was to dictate the number facts or to provide mimeographed tests, and to evaluate difficulty in terms of numbers of errors. Other experimental procedures have been used also, as in certain of the references below. The bibliography is purely suggestive.

Frank L. Clapp, *The Number Combinations: Their Relative Difficulty and the Frequency of Their Appearance in Text-Books* (Bureau of Educational Research, Bulletin No. 1, Madison, Wis.: University of Wisconsin, 1924), p. 20.

S. M. Washburn, "Relative Difficulty of the Number Combinations in 1937 as Determined by Repeating Clapp's Investigation," *Pittsburgh Schools,* XIII (May, 1939), 133-140.

Studies in Arithmetic. Publications of the Scottish Council for Research in Education, XIII (London: University of London Press, Ltd., 1939), pp. 79-116. (Data on the combinations in all four processes.)

C. L. Phelps, "A Study of Errors in Tests of Adding Ability," *Elementary School Journal,* XIV (Sept., 1913), 29-39.

James H. Smith, "Arithmetical Combinations," *Elementary School Journal,* XXI (June, 1921), 762-770.

with others for the reason that he reports but a single figure for each *pair* of facts. Thus the most difficult *pair* of combinations according to the coefficient of difficulty were 9 + *7 and* 7 + 9. Other investigations have separated the figures for direct and reverse statements of combinations.

In the Knight and Behrens study (24) the difficulty encountered by twenty-five second-grade children was observed as they learned the one hundred addition combinations. These subjects, though regarded by the authors as "typical of representative second grade pupils," actually had a median IQ of 117, with a range from 88 to 148. From the initial tests the combinations partly known were selected and presented first. The remaining combinations were grouped in "practically random order," with a drift from the smaller to the larger combinations. The authors derived a "general difficulty rank" in some unspecified manner by pooling the following factors: number of responses to learn, number of errors made in learning, time of first successful response, time of final learning response, and time of review response. Throughout their study comparisons of their ratings are made with the rankings of the Clapp study, to which the reader has but recently been referred, and which was in no sense a learning study. Considerable disparity between the Knight-Behrens rankings and the Clapp rankings was found.

Thiele studied the learning of the addition combinations in two separate investigations. The later, and the more thorough, study (49) has already been reviewed in the section immediately preceding. In the report of this investigation Thiele gives data to show the rank order of difficulty of the one hundred addition combinations for both his G- (generalization) and his D- (drill) groups. Use will be made of some of these data below.

In his earlier study (48) Thiele selected approximately three hundred beginning second-grade pupils, a few more than half with intelligence ratings of "C" and below. Part of his subjects learned the combinations according to drill procedures. In the learning periods of his experimental classes the combinations were put into six groups in the manner already described (see p. 120, above). This grouping was adopted for two reasons: the basic generalizations were arrived at by children on their own initiative, and results of a previous investigation had demonstrated their worth. According to Thiele, this organization contains an intrinsic method of combination attack for pupils to use in the natural course of their learning. Ability to sense relations was checked by the use of large numbers.

Excluding the 10-facts (facts not usually taught), Thiele gives the per cent of error for each combination on a written test administered at the end of the second semester. Because of his method of grouping, certain combinations occur twice (some of those involving 1, 2, and 9 as addends), while the fact 0 + 0 does not occur at all. How much additional practice, if any, was given to the combinations found in two different sets is not known.

Washburne and Vogel's procedure (**52**) was, like Clapp's, to measure knowledge of the combinations by means of group tests without attention to the instructional methods employed. Inclusion of Grade III subjects in the testing makes their data somewhat out of line with those in the other investigations here considered, but a few of the data are nevertheless used below for comparative purposes. It should be noted that these authors rank only the twenty-five hardest and the twenty-five easiest combinations as determined by frequency of error for combined grades. No per cents of error are reported, and the middle fifty combinations can therefore be treated only as a group. Some justification for their practice in ranking only the hardest and the easiest facts is found in the unreliability of intermediate ranks, a condition to be considered at a later point.

Wheeler (**53**) determined the relative difficulty of the 100 addition combinations when taught by his game of ADD-O. He tested 125 children in Grade IIB whose median IQ was 103. Although Wheeler asserts that the repetition of the combinations and various other factors influencing the learning process were controlled during the arithmetic period by the objective nature of the game, his statement is considerably weakened by his admission that "concepts of the adding process had been taught before the experiment was begun." After all, it is hardly possible to teach these concepts without using numbers.

All of the studies mentioned above present data on the rank difficulty of all 100 addition combinations (except for Thiele's omission of 0 + 0). MacLatchy (**31**), on the other hand, selected ten combinations from the Knight-Behrens rankings and obtained a measure of the degree of familiarity which children had therewith at the time they entered school. These she ranked in order of difficulty. Her selection of combinations has been accepted as a representative sample (plus two 0-facts) in order to compare difficulty ratings of the investigations reviewed above. The data for this comparison are assembled in Table 34.

A brief examination of Table 34 reveals little agreement among the studies on the difficulty ranking of any of the twelve combinations. For example, Knight and Behrens found the combination 0 + 3 to be the least difficult; Wheeler ranks it thirty-second; Washburne and Vogel place it as the easiest of their twenty-five most difficult, while according to Thiele's first study, only one other combination was found to be harder. (Later, Thiele found 0 + 3 to be an easy combination for both the G- and D-groups.) This is an amazing range of difference in rank, and possibly represents an extreme case. However, the combinations 5 + 1 and 7 + 1 may be noted. Knight and Behrens ranked these as about equal; so did Thiele at first. Then Thiele found that 7 + 1 was about ten and twenty ranks more difficult than 5 + 1, when compared for G- and D-groups, respectively. Wheeler, however, reports that both combinations are much more difficult than found in the other studies. Similar disparity could be illustrated in most of the "difficulty rankings." For the twelve combinations appearing in Table 34, the median minimum disagreement in difficulty rating is over thirty places.

TABLE 34

A COMPARISON OF THE RANK DIFFICULTY OF TWELVE ADDITION COMBINATIONS

Combination	*Knight & Behrens (23)*	*Wheeler (53)*	*Washburne & Vogel (52)*	*Thiele (48)*†	*Thiele -G (49)*	*Thiele -D (49)*	*Maximum Range in Rank Difference*	*Minimum Difference in Rank*
0 + 0........	17	1	2	....	7	4	1-17	16
0 + 3........	1	32	76	97-98	14.5	8	1-98	96
5 + 1........	11	35.5	M*	11-15	11	12	11-35.5	24
7 + 1........	12	43	M*	11-15	22	31	11-43	31
1 + 9........	24	19.5	22	30-39	54	42	19.5-54	34
1 + 6........	37	43	17	30-39	45.5	28	17-45.5	28
8 + 2........	51	48	M*	7-10	31.5	38.5	7-51	41
5 + 2........	42	52	M*	57-67	50.5	44.5	42-67	15
4 + 4........	31	6	4	3-6	2	2	2-31	29
5 + 3........	70	50.5	M*	30-39	62.5	56	30-70	31
3 + 5........	81	54	18	40-56	81	66	18-81	63
4 + 5........	61	72	M*	18-29	71	74	18-74	45

* M refers to the middle 50 per cent; that is, these combinations would be ranked between the 26th and the 75th in difficulty, had sufficient data been given by the authors.

† Thiele's first study does not rank the combinations by difficulty. Figures in this column have been approximated from the per cents of error reported by him. For example, since six combinations were *easier* than 8 + 2, and three others had the *same* per cent of errors, the difficulty is reported here within the range 7-10.

Certain factors contributing to these differences in rankings may be considered here. (See also Washburne and Vogel, 52.) The first factor is applicable to ranks in general, namely, the fact that differences between ranks are not equal. The combination 8 + 7, for example, was "hard" for both of Thiele's G- and D-groups; it ranked with them 98 and 100, respectively. The combination 0 + 5, on the other hand, ranked 31.5 for the G-group and 8 for the D-group. These rankings make 0 + 5 "easy" for the latter group and "moderately difficult" for the former group. Yet the per cents of correct answers on these two combinations tell almost another story. For 8 + 7, these were 66.1 per cent for G and 27.7 per cent for D. For 0 + 5, the per cents correct were 93.8 and 93.0 for G and D, respectively. In the case of 0 + 5 an almost negligible difference in per cents correct means a rank difference of over twenty places. The other combination, 8 + 7, however, though "ranked" about equally for the two groups of subjects, was correctly answered by 40 per cent more of the G- than of the D-subjects.

A second factor contributing to different rankings is the effect of order of adding the figures in the combinations. The combination 5 + 3 (in column form) may be either 5 + 3 or 3 + 5 to the child, depending upon whether he adds upward or downward. The investigations of number combinations, save for those like Knight and Behrens in which this factor was controlled in learning, give no recognition to discrepancies which may have been introduced into their data by this factor. Thus, conceivably, a difficulty rank for 3 + 5 in one study ought to be compared with the rank for 5 + 3 in another study.

A third factor is related to differences in lapse of time between learning and testing. With one organization of combinations for learning 4 + 5 may be presented early, practiced for a long time, and then tested. With another organization 4 + 5 may be introduced too late to permit learning to anything like an equivalent degree of mastery. In the first investigation 4 + 5 would be "easy," in the second, "hard." In other words, different researches may have "caught" their subjects at different stages of learning with respect to the same combinations.

A fourth factor is associated with the second and third, namely, the general type of teaching which is followed. For example, some of the combinations "hard" in other studies were made "easy" in Thiele's by his practice of leading children to discover and use binding principles and generalizations. Investigations which have employed radically different instructional programs must be expected to yield markedly different difficulty rankings.

Were one to consider the influence of the four factors mentioned thus far as unimportant, there would still remain a fifth, a shortcoming of the ranking technique which is too often ignored. Reference here is to the probable error of ranking. Wheeler's data, for example, show a difference of fifteen ranks for a change in accuracy from 84.8 per cent to 82.8 per cent, a matter of only 2 percentage points. Obviously the smaller amounts within these limits by which the fifteen combinations are ranked are wholly unreliable statistically, and with full propriety the differences might have been disregarded and all the combinations affected might have been assigned the same rank. This, it will be remembered, is precisely what Washburne and Vogel did for the fifty combinations at the middle of their accuracy distribution.

As for the subtraction combinations, only two studies need to be mentioned, and the difficulty rankings assigned therein to the one hundred subtraction facts are subject to the same differences and the same explanations of the differences as apply in the case of the addition facts. Knight and Behrens (24) determined the relative difficulty of the subtraction combinations from fifteen bright second-graders by methods comparable to those described in their addition investigation. The Scottish Studies in Arithmetic (see footnote, p. 122) reports a very thorough study of the subtraction facts (and multiplication facts), but the subjects employed averaged eight and nine years of age, and were in grades which are equivalent to American third and fourth grades. For this reason no data are reported at this point.

To conclude this section on the difficulty of the number combinations, research has, in the opinion of the writer, been pretty largely fruitless. Certainly findings and rankings differ very greatly, and if one were to try to decide upon *the* comparative difficulty of the facts one would hardly know how or where to start. It may be assumed that all difficulty rankings are authentic for the conditions under which they were obtained and for the techniques by which they were determined. And this is precisely why research to ascertain *the* comparative difficulty of the combinations has been unprofitable—and must continue to be unprofitable. There is no such thing as "intrinsic" difficulty in the number facts; their difficulty is relative, contingent upon many conditions, chief of which is method of teaching, or stated differently, the number, order, and nature of learning experiences on the part of pupils.

6. READING NUMBERS

Evidence on children's ability to learn to read numbers in Grade I by the use of a free-play activity game and by the ordinary method of instruction (or lack of it) is available from a study reported by Wheeler and Wheeler (**54**). This is as yet the only investigation reported in this field. Early in the school year 157 children in Grade I and 103 children in Grade II were tested individually for immediate recognition of the numerals from 1 to 100. Following this testing the game of Count-O was introduced in Grade I and was played twenty minutes a day for five consecutive days. The game of Count-O is similar to the games of Bingo, Lotto, and the like. Two sets of flashcards are used, one for the numbers 1-50, and the other for the numbers 51-100. Following the five-day practice period the recognition test was readministered.

The results of the initial recognition test showed that the children in Grade I knew a median of 9.21 numbers and that the children entering Grade II knew a median of 96.51 numbers. The difference of 87.30 between the two grade levels may be taken as a rough indication of the numerals learned in the first grade by the customary plan of instruction (which the authors do not describe). The range of numbers known in Grade II was from 23 to 100; over 75 per cent knew 87 or more numbers.

Of the 157 cases in Grade I who were given practice in reading the numbers, 31 who knew originally more than 90 numbers were not used in the final tabulation. Twelve other cases were dropped because of absences. The 114 cases for whom results are given showed an increase of 53.71 in the median number of numbers known, from 6.29 on the original testing to 60.00 after the five-day period of practice. The range of numbers known on the second testing was from 2 to 100; 75 per cent knew 18 or more. The per cent of children knowing each of the digits on the second test ranged from 67 for 9 to 99 for 8; 8 was known by more children than was 6 or 7; the differences for the other numbers were small. The "teens" decade was the most difficult decade to learn as evidenced by the number of children knowing each decade on the second testing. The other decades came in the order: 80's, 40's, 60's, 90's, 70's, 50's, 30's, and 20's. The correlation between total of numbers known and mental age was .51 for the initial test and .43 for the second test.

7. PSYCHOLOGICAL DEVELOPMENT OF NUMBER CONCEPTS AND SKILLS

Under this heading belong several studies which have already been reviewed in other connections. For example, Polkinghorne's investigation (40) (see pp. 46-47, preceding) reveals the manner in which children, quite apart from teaching, extend and enrich old meanings and develop new meanings for fractions. Thus the unit fraction is understood in its various applications sooner than other proper fractions, and proper fractions sooner than improper fractions. Moreover, within the area of the unit fraction and other proper fractions ability to use fractions with single objects is developed before ability to use them with groups of objects.

Likewise, Woody's extensive inventory (56), also summarized in Chapter II, shows the increase in extent and depth of meaning for a number of arithmetical skills and concepts on the part of children from the kindergarten through Grade IIA. Here again learning took place without direct school instruction. The nature and the rate of growth in the various skills and concepts cannot however be ascertained from Woody's report, since but few of his data for separate half-grades are in print.

In this same group of researches on the psychological development of number concepts and skills elsewhere treated in this monograph are Russell's study (42) of ability to discriminate between groups of objects of different sizes (pp. 56-57), Court's[11] and Drummond's[12] biographical accounts of quantitative development in the case of individual children (p. 55), and the learning of the addition and subtraction combinations in the original research reported in Chapter III, pages 92-102 particularly.

In this section space is given primarily to the development (a) of ability to count and (b) of number concepts and of ways of thinking of numbers in combination.

(a) Counting (Enumeration).—By far the most extensive and the most thorough study of counting has been made by Judd (22), whose subjects ranged from Grade I pupils to adults and whose counting situations involved lights, sounds, and tactual stimulations. Even among adults large differences in counting were found, not only

[11] Sophie Ravitch Altshiller Court, "Numbers, Space, and Time in the First Five Years of a Child's Life," *Pedagogical Seminary,* XXVII (March, 1920), 71-89. Also, "Self-Taught Arithmetic from the Age of Five to the Age of Eight," *Pedagogical Seminary,* XXX (March, 1923), 51-68.

[12] Margaret Drummond, *Five Years or Thereabouts* (London: Edward Arnold and Company. 1921).

among the various kinds of things counted (lights, sounds, etc.) but also within any one kind. The process is shown to be a very complicated pattern of behavior involving the co-ordination of several different kinds of response and their adjustment to a series of independent external events.

For the study of counting in childhood Judd selected 120 pupils from Grades I to VI inclusive, taking from each grade twenty pupils who were competent and twenty who were relatively incompetent in arithmetic. Data are presented to show the increase of ability to count from grade to grade, as measured by success (1) in counting different kinds of sensory objects (2) under different rates of presentation. Described also are the changes in the behavior which accompanied counting. The first-graders indulged in many irrelevant movements ("irrelevant" from the standpoint of the adult observer, of course), such as hand movements, lip movements, head movements, and even grosser bodily movements. Gradually these unnecessary movements are sloughed off with increase in skill in counting until the economical performance of the expert counter is attained.

Judd interprets his findings, both the measures of success and failure in counting and the occurrence of irrelevant movements just mentioned, to show the difficulty children face in mastering a skill which by common consent is easy to acquire. Psychologically considered, counting is no simple form of behavior, and it is small wonder that children require time to develop the skill and then to carry their ability to mature levels of proficiency and precision.

(b) Number concepts.—The development of number concepts has been studied by observing pupils' reaction both to concrete groups or number pictures and to abstract numbers. Studies based upon concrete groups and pictures are far more numerous. Several have already been summarized in Chapter II,[13] but with the exception of the researches by Douglass (14), McLaughlin (33), and Russell (42) the developmental aspects of number apprehension have not been emphasized. To be reviewed in this chapter are the investigations of Howell (21) and Brownell (5), the latter of whom also studied genetically the development of ability to deal with abstract numbers in combinations.

[13] To this group of studies may be added that by Yocum (59) which however was not available to the writer. According to Buckingham and MacLatchy (12), 70 per cent of Yocum's one hundred subjects, all school entrants, were able to "recognize" a group of two objects, and 12 per cent, a group of six objects. Just what is meant by "recognize" is not clear. Actually, these children may have but counted or enumerated the objects, in which case the data might have been reported under that topic in Chapter II.

Howell's research,[14] or that part which is relevant to the purpose of this monograph, consisted in exposing number pictures for the numbers 3 to 12 to more than five hundred children in Grades IB through VIIIB. The average number of pupils present for each testing period was about forty-five per half-grade (both half-grades were used from Grade IB through Grade IVA, B-grades only from Grade V through Grade VIII). In the major part of the study ability to apprehend groups numbering three to twelve objects was studied. These groups all dealt with the form known as "quadratic," which may be illustrated in the case of two numbers: 7 = :: :· and 12 = :: :: :: In all, forty-five cards were shown—one card for 3, two for 4, four for 5, four for 6, five for 7, six for 8, seven for 9, eight for 10, four for 11, and four for 12. These pictures, carefully prepared to remove all extraneous factors, such as blending, confusion from too great closeness of groups, and the like, were exposed in random order for five seconds each. Pupils entered their estimates on special blanks, which facilitated the tabulation of errors.

So few errors were made on 3 and 4 that most of the figures for these numbers are omitted. Growth in ability to apprehend the number groups is shown by the following per cents of error: Grade IB, 6.0; Grade IA, 2.3; Grade IIB, 1.4; Grade IIA, 1.2; Grade IIIB, 1.0; Grade IIIA, 1.1. Grades above IIIA made error records of 0.4 per cent or less. So far as the difficulty of the separate number pictures is concerned, the per cents of error for 352 pupils in Grades IB to IIIA were as follows: 5, 1.1; 6, 1.0; 7, 2.5; 8, 0.8; 9, 1.9; 10, 1.6; 11, 3.8; 12, 1.6. The order of difficulty from hardest to easiest is therefore 11, 7, 9, 10, 12, 5 and 6, 8.

The records made in Grade IB yielded a difficulty order very similar to that for the six half-grades combined: 11, 7, 9, 10, 12, 5, 6, 8, though, as would be expected, the numbers of errors made were much larger. The order of difficulty in Grade IA was 11, 12 and 7,

[14] Prefacing the report of his own work, Howell gives a most complete and scholarly analysis of arithmetic research prior to 1914. Discussed are "Genetic Studies" (primitive men, children, prodigies, and number forms), "Psychological Studies" (perception, counting, the fundamental processes, and reasoning), "Statistical Studies" (efficiency, ideation, transfer, and hygiene), and "Didactical Studies of Apprehension." It is the latter group with which his own work is most closely affiliated. None of the material covered by Howell is here canvassed, partly because not much of the research was quantitative, partly because the quantitative research employed few or no primary-grade children, and partly because the research was done with foreign subjects. Prominent among the names of those who investigated the apprehension of number pictures are Beetz, Born, Lay, and Walsemann, all Germans. No better source than Howell can be found for a survey of the early research.

9, 10 and 5, 6, 8. Results secured in the other half-grades are also treated in the report.

In later experimentation Howell introduced certain changes and extended his study in various ways. Thus, for the uniform five-second exposure he substituted a procedure in which for each picture the length of the exposure in seconds was equal to the number of dots exposed. Comparisons too were made between records in apprehending the number pictures and teachers' marks on arithmetic and estimates of general intelligence. In the lowest grades some agreement of a positive sort was found. No differences in comparative variability or accuracy of apprehension were noted as between boys and girls.

Howell's principal conclusions relate to the comparative difficulty of the numbers for apprehension, from which he deduces an order of conceptual development. In general, the odd numbers were harder than the even ones, and it was Howell's belief that his data establish a correspondingly later time of development for the concepts of these numbers. More particularly, "upon the assumption . . . that the earliest appearance of mastery of a number in any grade is, at least, a sign of the *possibility* of its mastery by the children of that grade" (p. 229), Howell gives the following "tentative order of sure apprehension" and so, the order and time of development of the corresponding number concepts:

6........	Grade IB,	approximate age 6½-7½
5........	Grade IIB,	approximate age 7 -8
8........	Grade IIA,	approximate age 7½-8½
10........	Grade IIA,	approximate age 8 -9
12........	Grade IIIB,	approximate age 8½-9½

Age and grade for these numbers were determined by using an arbitrary standard of 0.7 per cent of errors. When the same standard is applied to the other numbers in the series to 12, the numbers 9 and 7 are placed in Grade IVA, and 9 in Grade VB.

It will be noted that Howell used number pictures of only one kind, the quadratic, in which the basic unit is a group of 4. If the reader will make for himself quadratic number pictures for 4, 5, 6, 7, 8, 9, 10, 11, and 12, he will observe that the pictures for 4, for 8, and for 12 are wholly "regular" in the sense that the basic unit is not complicated by the addition of incomplete units. The pictures for 6 and 10 are slightly less "regular," for besides the basic unit there are additional groups of 2 (thus, $10 = 4 + 4 + 2$). By contrast, the pictures for 7 and 11 are very "irregular" and that for 9 is "irregular" to a smaller degree. Howell's findings with respect to odd and even numbers and with respect to the difficulty of particular numbers from 3 to 12 may have been conditioned by the degree of "regularness" of the corresponding number pictures. To the extent that number pictures are "regular" the child equipped with the ability to deal effectively with 4 or 2 as a unit might be aided in their apprehension; to the extent that pictures are "irregular" ability to recognize and use groups of 2 and 4 might be of less use. "Regularness" of the number pictures might be an advantage even for the counter, that is, the individual who counts each separate dot in each picture. For him the groups might serve as units to be dealt with one at a time; they might aid him, in other words, to keep his place as he counts all the dots exposed.

On the hypothesis that Howell's results (and so, his conclusions) were influenced by the particular grouping used, Brownell (5) repeated Howell's experiment and supplemented his quadratic number pictures with several other kinds: the diamond (basic unit, 4 ·⁚·); the domino (basic unit, 5 ⁚·⁚); the triangular (basic unit, 3 ∴); the odd (basic unit, 6 ⁚·⁚·); and the linear (basic unit, 1 : . . . . . .). Cards in these different patterns were made for the numbers 3 to 12, forty-five cards for each pattern, and were exposed in random order for five seconds each to a total of 1,858 pupils in Grades IB to VIIA. When the results for the different patterns were combined, the evidence showed that, as a group, the odd numbers were no more difficult for apprehension than were the even numbers and that, "regularness" of pattern being eliminated from consideration, ease of apprehension decreased (or difficulty increased) from the number 3 to the number 12.

With only 51 subjects in Grade IA, 42 in Grade IIA, and 47 in Grade IIIA, Brownell found a positive relationship between number picture scores and MA of .55, and between the first of these factors and IQ of .33. These measures were obtained when the 140 pupils were considered as a single group. Within the grades, and with CA held constant, the coefficients between MA and the number apprehension scores were .33 in Grade IA, .28 in Grade IIA, and .31 in Grade IIIA. These relationships though positive are comparatively small; apparently the ability in question is not largely dependent on mental "maturation" in general. More important factors in the development of the ability were found in later parts of the study.

Howell had reported only the gross results obtained from his group testing, and his inferences as to the nature of development of number concepts were correspondingly limited. Brownell extended

Howell's study by securing information on the way in which children in the various grades actually found the total numbers of dots exposed in the pictures. Part of this information was obtained by analyzing the original group test data, part by repeating the number picture tests with a shorter time interval (two and a half seconds), and part by questioning individual children. In Grade I the counting of separate dots one by one was employed most commonly; comparatively little use was made of more mature procedures, such as counting by 2's or 4's or adding together subgroups, as 4 and 3 in the quadratic picture for 7. Immature procedures were abandoned by increasingly larger proportions of the pupils from grade to grade, but were still employed by some children long after school instruction had supposedly made available to them processes of a higher or more mature level.

Next, Brownell gave group tests of the simple addition facts in Grade II and of three-digit column addition in Grade III. Children were selected for interviews on the basis of their scores on these group tests, some who had been most successful, some who had made average scores, and some who had done most poorly. Teachers were also asked to send for interviews the three pupils whom they rated highest and the three whom they rated lowest in arithmetic in general.

The selected pupils were then subjected to individual tests with number pictures, with the simple addition combinations, and (in Grade III) with examples in column addition of the kind in the group test. In this way it was possible to compare processes used both with concrete and with abstract numbers. By and large, fairly close agreement was found in the kinds of processes used with the two types of number. Children who counted to apprehend the number pictures tended to count in dealing with abstract numbers. Children who used more mature methods (that is, "abstract" methods) with the concrete numbers used similar methods with abstract numbers. Illustrations of the kinds of procedures employed with abstract number combinations have already been given in Chapter III, pages 92-102.

These facts were interpreted to mean that the way a child thinks of numbers, whether concrete or abstract, is at least as important an index of his level of conceptual development as is the evidence furnished by the accuracy of his answers and the speed with which he obtains them. Failure of children to use mature procedures even after they had been taught them was taken to mean lack of understanding and, in many instances, the mere memorization of verbal statements which for all practical purposes were senseless to the children who had mastered them. Attention was called to what seemed to be a serious weakness in the teaching of primary number, namely, failure to carry learning with concrete numbers to mature levels of thinking which could then be transferred directly and meaningfully to abstract numbers.

8. TRANSFER OF TRAINING IN LEARNING ARITHMETIC

Studies of transfer have related (a) to the learning of the basic number combinations and (b) to the learning of simple addition and subtraction operations. Examples of (a) are the already mentioned studies by Olander (36) and by Beito and Brueckner (2). The only conspicuous example of (b) was made by Overman (38, 39).

(a) Transfer with number combinations.—Beito and Brueckner (2) used three IIB classes numbering thirty-two, thirty-three, and twenty-eight pupils, to whom they taught thirty-six simple addition facts in the direct order, that is, with the larger addend stated first (e.g., $6 + 3 = 9$). These facts were divided into three groups of twelve facts each, roughly equivalent in difficulty, and were printed on cards. On one side of the cards the answers were given; on the other, the answers were omitted. A card was given out on Monday and a test administered to cover the facts thereon, together with their reverses. During the week the facts on the card were studied, tests being given at the beginning of each period of study and at the end, but only on the facts in the direct order. On Friday the testing covered both the direct forms, which had been studied, and the unstudied reverse forms. Three methods of teaching were tried: (1) the teachers' usual methods, (2) these methods, plus use of the drill and test cards, and (3) use of the drill and test cards alone.

Transfer was measured by comparing scores on the reverse (untaught) facts made at the beginning of each week and at the end. In Class M, for example, thirty-nine direct and forty-five reverse facts were missed on Monday, and 0 direct and 0 reverse facts on Friday. Instruction on the direct facts yielded 100 per cent accuracy, while transfer produced 100 per cent accuracy as well. These records were made on one of the three sets of facts. On the second set the children in Class M learned 96.6 per cent of the direct facts unknown on Monday, and acquired control of the reverse facts by transfer to the extent of 94.4 per cent. On the third card instruction resulted in 90.0 per cent accuracy, and transfer (reverse facts) resulted in 95.2 per cent. For Class M as a whole, the 138 direct facts missed on the three cards in the first test were 95.5 per cent learned during the week, and the 141 unknown reverse facts were acquired by transfer to the extent of 96.6 per cent. In Class L instruction produced 71.5 per cent accuracy on the thirty-six direct facts, and transfer produced 75.1 per cent accuracy on the thirty-six reverse facts. In Class G, the corresponding per cents were 78.2 and 82.5. In other words, these investigators report extraordinary amounts of transfer, often exceeding even the amounts of gain from teaching. This finding held also for certain minor comparisons. Transfer exceeded the effects of teaching in two of the three methods of teaching compared, namely, (2) and (3) as described above. Data reported for a group of sixteen pupils with IQ's above 120, a group of twenty-seven pupils with IQ's between 100 and 120, and a group of nine with IQ's below 90 showed greater transfer for the less capable children. This result, which is counter to the usual finding (see Overman below, for example), could be explained by the small number of subjects used, particularly in the really low brackets of intelligence. However, checking of the table reveals a computational error which sets aside this conclusion in its entirety.

The Beito and Brueckner study then shows that transfer can play a large part in learning the combinations. The nature of this transfer is not however considered in the report. The inference is that children transferred their "knowledge" of the direct facts to the unlearned reverses. More reasonable hypotheses are that the children either (1) generalized to the point where they discovered the relation between direct and reverse combinations or (2) actually reversed the untaught reverse facts to the learned order (thus, changed $3 + 6$ to $6 + 3$) and treated them in the direct form. These hypotheses might well be checked by further research.[15]

Olander's subjects (36) were a total of approximately thirteen hundred IIB pupils. As in Beito and Brueckner's study, transfer from taught to untaught combinations was the center of interest.

[15] Two other facts about Beito and Brueckner's investigation should be noted. One is the extraordinarily large number of the addition combinations "known" before they were taught. On the pretests the ninety-three children missed only 1,059 of the 6,696 facts on which they were tested (a total of 72 per cent). The per cent of accuracy was therefore 84. The other noteworthy fact is that on the end tests the 93 subjects scored a per cent accuracy of 97 (211 errors out of a possible 6,696). In other words, these second-grade children within a period of three weeks developed a surprising degree of control over the combinations, both taught and untaught—further evidence that second-grade children are psychologically "ready" for the number facts, whether they are required to learn them or not.

Olander taught the 100 addition and the 100 subtraction combinations to one part of his subjects, and only 55 addition and 55 subtraction combinations to the rest of the children. Teaching and practice were continued for seventeen weeks, after which tests on the whole 200 combinations were given to all pupils. Olander found that the ability gained by children who had studied a total of only 110 combinations (55 in each process) transferred almost completely to the 90 which they had not studied. In other words, these children made scores as good (or nearly so) as did the children who had studied separately the 200 combinations. There was almost perfect transfer in both addition and subtraction, though slightly less in the latter process.

Another problem investigated at the same time by Olander was the effect of *teaching* for transfer. Three minutes a day were taken in some of his classes to encourage children to "generalize," that is, to see relationships between facts. Among the generalizations taught were: the principles covering the 0-combinations, the relation between direct and reverse facts (Beito and Brueckner had not explicitly called attention to this relation), and the relation between addition facts and their corresponding facts in subtraction. Others of his subjects, denied this special instruction, spent the full period of study on the combinations as such. On the end tests no reliable difference was found between the groups in their knowledge of the combinations. Apparently *without* external aid in developing these generalizations, his subjects were able to arrive at the generalizations themselves—(1) either this, or else, as the author suggests, (2) not enough time was given to generalization, (3) or his "generalization" subjects were really too immature to profit from the kind of instruction given them. The related studies by Thiele (48, 49) and by Overman (38) seem to indicate that the first explanation is the correct one. After all, Olander studied transfer within the limits of a very narrow function.

Thiele's investigations (48, 49) bear upon the problem of transfer, in as much as reliance was placed upon the ability of children themselves to formulate generalizations to cut down the amount of specific drill and practice required for learning. These studies are described in some detail in pages 120-121 preceding, and hence need not be summarized again at this point. It will suffice to say that in both studies Thiele shows convincingly the value of "binding principles" in learning the number facts.

Transfer has also been studied as it operates in the learning of the multiplication combinations. Brueckner (8) reports an investigation by Dahlgren which in technique resembles very closely the Beito and Brueckner study (2) in the case of the addition combinations. One hundred twenty-five pupils in Grades IIB and IIIB, none of whom had previously studied the multiplication facts, were practiced on the direct order for the tables of 2's, 3's, and 4's (2 x n, 3 x n, etc.), and gains in knowledge of the untaught reverse order facts were used as measures of transfer. In the tables of 2's direct instruction produced 97.3 per cent of improvement on the direct facts, a reduction from 867 errors on the initial test to 15 on the end test; transfer accounted for 89.7 per cent gain on the reverse facts in the same table—a reduction from 847 initial to 87 final errors. In the case of the table of 3's, direct instruction yielded improvement to the extent of 94.0 per cent, and transfer to the extent of 85.9 per cent. The corresponding per cents for the table of 4's were 88.1 and 85.4, respectively.

Besides these four studies (Beito and Brueckner, Olander, Thiele and Dahlgren) which reveal immediately and directly the influence of transfer in learning the combinations, there are other studies in which similar evidence of transfer is indirectly furnished. For example, Knight and Behrens (24) in analyzing the comparative difficulty rankings of the addition and subtraction facts point to correlations of .61 between pairs of related addition facts, of .42 between pairs of related subtraction facts, and of .64 between addition and the corresponding subtraction facts. The inference is that their subjects found in the relationships between facts powerful aids in learning.

(b) Learning simple addition and subtraction operations.—Overman's investigation (38) is an important contribution, not only to transfer as transfer applies to arithmetic, but to transfer as a topic in general psychology. Few experiments on transfer reveal so much insight into the problem of transfer and the learning of arithmetic. For this reason a rather full table has been compiled to report the major findings.

The subjects were second-grade pupils drawn from forty classes, thirty-eight of them in Toledo, Ohio. Previous to the start of the experiment these pupils had been taught the addition and subtraction skills recorded in the first section of Table 35. In other words, they had been taught to add two, three, and four digits and to subtract one digit from another (thus, the addition and subtraction facts, plus column addition involving three and four digits with sums to 9). The

TABLE 35

PERCENTAGE SCORES ON FIRST AND LAST TESTS, TOGETHER WITH PER CENTS OF POSSIBLE GAIN ACTUALLY MADE (Adapted from Overman*)

Type of Example	*Percentage Scores*		*Per cent of Possible Gain Actually Made*	*Facts as to Teaching and Testing*
	Test 1	*Last Test*		
1 + 1†	95.0	98.0	60.0	All taught before beginning of experiment
1 + 1 + 1‡	87.2	96.8	75.0	
1 + 1 + 1 + 1	82.5	93.7	64.0	
1 — 1	86.1	94.2	58.3	
2 + 2§	43.2	96.5	93.8	Taught day 1; practiced days 4, 5, 9, 13
2 + 2 (dictated)	51.1	96.4	92.7	
2 + 2 + 2	37.4	93.1	89.8	Taught day 8; practiced days 9 and 13
2 + 2 + 2 (dictated)	45.8	95.3	91.0	
2 + 2 + 1‖	19.9	93.3	91.6	Taught day 12; practiced day 13
2 + 2 + 2 + 2	35.3	87.7	81.0	None of the skills in the last section of the table were taught during the experiment.
3 + 3	36.3	93.5	89.8	
3 + 3 (dictated)	35.5	85.7	77.8	
3 + 3 + 3	30.0	88.4	83.4	
3 + 3 + 3 + 3	27.1	82.6	76.1	
2 — 2	43.7	87.3	77.4	
2 — 2 (dictated)	38.2	84.8	78.6	
3 — 3	35.7	84.4	75.7	
2 + 2 + 2	17.0	92.9	90.4	
1 + 2 + 2	13.6	76.8	73.1	
1 + 2 + 1	10.0	71.4	63.2	
3 + 2	3.3	56.2	54.7	
2 + 3	4.2	47.1	44.8	
3 + 1	7.8	64.7	61.7	
1 + 3	3.6	55.8	54.2	
2 + 3 + 1	2.5	46.0	44.6	
2 — 1	14.4	63.4	57.2	
3 — 2	2.1	47.3	46.2	
3 — 1	6.8	57.3	54.1	
Carrying in addition	1.3	4.1	2.8	

* Adapted from Table 8 (38, p. 55) and Table 1 (39, p. 178). Overman does not give all of the computations here reported.
† 1 + 1 means the addition of two digits.
‡ 1 + 1 + 1 means the addition of three digits.
§ 2 + 2 means the addition of two two-place numbers.
‖ 2 + 2 + 1 means the addition of two two-place numbers and a digit.

experiment proper lasted fifteen days, during which the three skills in the second section of the table were taught. None of the other skills in the table were taught. The chronology of the experiment was as follows: first and second days, testing; third day, teaching of addition of two two-place numbers; fourth and fifth days, practice on this new skill; sixth and seventh days, retesting on all of the types described in the table, whether they had been taught or not; eighth day, teaching of addition of three two-place numbers; ninth day, practice on the same, as well as other types taught; tenth and eleventh days, retesting on all types; twelfth day, teaching of additions like 23 + 13 + 2; thirteenth day, practice on this skill and on all others taught; fourteenth and fifteenth days, final retesting.

Table 35 shows a gain of 93.8 per cent of the possible gain in the addition of the two two-place numbers, of 89.8 per cent in the addition of three such numbers, of 81.0 per cent (a bit lower in the table) in the addition of four such numbers, this skill never having been taught, etc., etc. The figures in the last column of the third large section of the table are those of chief interest. It will be noted that in several instances transfer produced almost as much gain in ability as did direct instruction (the second section of the table). Overman's analysis of his data shows that transfer occurred to skills most closely resembling those taught. Even so, the transfer amounted to from 46.2 per cent to 78.6 per cent in the case of subtraction skills.

The facts thus far presented do not differentiate between methods of teaching. Part of Overman's purpose was to discover the degree to which transfer is affected by unlike methods of teaching. One group of children were shown how to perform the operations without any other kind of explanation (Method A). Another group were helped to formulate nonmathematical generalizations to cover the procedures employed (Method B). A third group were led to consider the reasons and mathematical principles underlying procedures (Method C, the method of "rationalization"). The fourth group had the benefit both of generalization, as in Method B, and of rationalization, as in Method C (Method D). One hundred twelve "quartets" of children taught by the four methods were matched with respect to sex, MA, teachers' estimates of general ability, and scores on the preliminary test.

> The four methods . . . were found to produce practically equal amounts of transfer to those types of examples that did not involve the difficulty of placing numbers having different numbers of digits. The relation of these types to the types taught was apparently so close that the transfer took place without conscious generalization, or the pupils were able to make this generalization without help. In the case of examples involving the placement of numbers having different numbers of digits, the generalization alone increased the amount of transfer by 45.1%,

the rationalization alone increased it by 15.5%, and the two combined increased it by 36.9% (39, p. 179).

To these conclusions may be added several others of some importance. (1) There was evidence that the pupils taught by Method D were toward the end of the experiment increasing the amounts of transfer more rapidly than any of the other three groups. Perhaps the more meaningful approach took longer to develop, but in the long run might easily justify the slower progress at first. (2) There were no reliable differences between the sexes with respect to amount of transfer. (3) In general there was a positive relation between transfer and MA. The degree of this relationship varied with the method of teaching. Thus Method D seemed most effective for the children of highest MA. (4) The largest amount of transfer took place immediately after children had been taught the most nearly related type of operation.

Conclusions with regard to transfer.—The effects of research on transfer, both those here reviewed and others,[16] have had far-reaching influence on the teaching of arithmetic. The findings that positive transfer is not only possible but actual, that it occurs in learning to some extent whether the method of teaching encourages it or not, that improvement in teaching procedures can increase the amount of transfer, that at times (usually between functions that are obviously closely related) positive transfer may produce favorable results almost as large as direct teaching—findings of this kind have removed the last argument in favor of the teaching of "specifics" which was advocated some fifteen and more years ago. The early psychological experiments having been interpreted as establishing negligible gains from transfer, pedagogical practice was to determine each separate skill and concept which must be known and then to teach those items in relative isolation. The new data on transfer warrant the expectation of large positive effects from transfer when materials and methods are appropriately designed.

9. GRADE PLACEMENT OF TOPICS

Various bases may be adopted for assigning particular arithmetic topics to this grade or that. One basis is illustrated at the end of Chapter II, where inferences from the number knowledge of children entering Grade I were used to make up sets of concepts and skills apparently requiring different amounts of instruction and perhaps

[16] For example, F. B. Knight and A. O. H. Setzafandt, "Transfer within a Narrow Mental Function," *Elementary School Journal,* XXIV (June, 1924), 780-787.

different grade allocations. This procedure, namely, inference from facts obtained prior to instruction, assumes that systematic teaching will start when children seem to be ready to profit therefrom. Another basis is illustrated in Chapter III, where the results of testing after actual teaching show the degree to which children have profited from instruction. Here again the assumption is that children should be subjected to instruction when they are intellectually capable of learning.

This assumption, that children should be taught when they can learn, is not however approved by all students of education and of arithmetic. Many believe that by deferring systematic instruction important gains will be made: children, being older and more mature, can learn more easily in later grades what they would learn uneconomically before that time. Those who hold to this view do not then assign topics to the grade at which children *can* learn them; rather, they place the topics a half-grade, or a grade, or more than a grade higher than that point. This is what is done by the Committee of Seven. Their research on the addition and subtraction combinations, first published in 1930 (**51**), gives a mental age standard of six years five months (early in Grade I) as the minimum required for learning the addition facts with sums to 10, a mental age standard of seven years four months as the minimum for the rest of the addition facts, a minimum of seven years in mental age for the fifty easier subtraction facts, and of eight years three months for the fifty harder subtraction facts. A year later[17] while the same minimum MA standards were adhered to, "optimum" standards of seven years four months, seven years eleven months, eight years three months, and eight years eleven months (respectively) were proposed for these groups of combinations. In other words, apparently without new research data the Committee raised the MA standards by from seven to twenty months of mental age, and would thereby defer instruction by from one term to three terms. The Committee of Seven has held consistently to these revised standards in its more recent pronouncements.[18]

One important condition governing the Committee of Seven's

[17] Carleton W. Washburne, "Mental Age and the Arithmetic Curriculum: A Summary of the Committee of Seven Grade Placement Investigations," *Journal of Educational Research*, XXIII (March, 1931), 210-231.

[18] Raymond W. Osborne and Harry O. Gillet, "Mental Age is Important Factor in Teaching Arithmetic," *The Nation's Schools*, XII (July, 1933), 19-24. See also Carleton W. Washburne, "The Work of the Committee of Seven on Grade-Placement in Arithmetic," *Thirty-eighth Yearbook of the National Society for the Study of Education* (Bloomington, Ill.: The Public School Publishing Company, 1939), Pt. I, chap. xvi, pp. 299-324.

mental age standards is not generally recognized; indeed, it was barely mentioned by the Committee itself prior to 1936. This condition is that its standards apply to the *completion* of topics, and not to their introduction. Applied to the number combinations, this means that *mastery* should not be expected before the stage of general development stated by the Committee, though instruction prior to that time may be begun. In the case of the number combinations, however, the Committee favors postponing even a start on systematic instruction until Grade II or even Grade III.

The work of the Committee and its proposals for grade placement have been vigorously criticized and as vigorously defended. References given below will guide the reader who is interested in the controversies which have arisen.[19] These references also contain citations to all the numerous reports made by various members of the Committee of Seven, setting forth the general experimental procedure employed and specific recommendations as to placement of topics.[20]

[19] Louis E. Raths, "Grade-Placement of Addition and Subtraction of Fractions," *Educational Research Bulletin,* XI (June 20, 1932), 29-38. Also, "The Last Word: A Reply to Mr. Washburne's Rebuttal," and "Once Again: A Reply to Mr. Washburne's Criticism," *Educational Research Bulletin,* XI (Nov. 23, 1932), 401-405 and 409-410.

Carleton Washburne, "Arithmetic Grade-Placement Investigations of the Committee of Seven: A Reply to Louis E. Raths"; also (with William H. Voas) "Rebuttal," *Educational Research Bulletin,* XI (Nov. 23, 1932), 396-410 and 405-409. Also, "Reply to Brownell's Critique of the Committee of Seven's Investigations," *Elementary School Journal,* XXXIX (Feb., 1939), 417-430.

William A. Brownell, "A Critique of the Committee of Seven's Investigations on the Grade Placement of Arithmetic Topics," *Elementary School Journal,* XXXVIII (March, 1938), 495-508.

[20] "Arithmetic readiness" has been the subject of a rather considerable body of theoretical discussion. Besides the articles of the Committee of Seven and the other articles mentioned above, the references below may be studied with profit.

William A. Brownell, "Readiness and the Arithmetic Curriculum," *Elementary School Journal,* XXXVIII (Jan., 1938), 344-354.

Leo J. Brueckner, "Deferred Arithmetic," *Mathematics Teacher,* XXXI (May, 1938), 195-200.

B. R. Buckingham, "When to Begin the Teaching of Arithmetic," *Childhood Education,* XI (May, 1935), 339-343.

Guy T. Buswell, "Deferred Arithmetic," *Mathematics Teacher,* XXXI (May, 1938), 195-200.

John W. Dickey, "Readiness for Arithmetic," *Elementary School Journal,* XL (April, 1940), 592-598.

F. B. Knight, "Observations upon Beginning Arithmetic," *Childhood Education,* IX (Feb., 1933), 234-240.

Ben A. Sueltz, "Arithmetic Readiness and Curriculum Construction," *Mathematics Teacher,* XXX (Oct., 1937), 290-292.

C. L. Thiele, "An Incidental or an Organized Program of Number Teaching?" *Mathematics Teacher,* XXXI (Feb., 1938), 63-67.

10. CHILDREN'S USES OF, AND NEEDS FOR, NUMBER IN THE PRIMARY GRADES[21]

Four types of study have been made of children's uses of, and needs for, number. Studies have been made (1) by tabulating the incidental situations arising in the classroom; (2) by interviewing children as to their out-of-school activities and tabulating the situations involving number reported; (3) by submitting a questionnaire to parents asking them to report in what quantitative activities their children engage; and (4) by tabulating the nature and amount of number found in various readers.

Reid (41) reports a study of the social arithmetic situations which arose in a Grade I class which had no formal arithmetic program. During the first three months of the school year a stenographic record was made of the arithmetic situations occurring during a schedule of observations which included a cross section of all the subjects taught in that grade. An analysis of the stenographic records showed that the frequency of occurrence of numerals decreased consistently from 299 mentions of "one" to only one or two mentions of numbers between "thirty" and "seventy-eight." There were 110 mentions of "five," 96 mentions of "ten," and 14 mentions of "twenty-five." The frequency of mention of the ordinals dropped from 130 for "first" to only one mention of ordinals from "sixth" to "twenty-eighth." There were 31 mentions of "second" and 16 mentions of "third." Rational counting was used 190 times. Forty-two situations involved addition, and 18 involved subtraction. In a tabulation of vocabulary used, quantitative terms were first in frequency, followed by matching terms, ordinals, and fractional concepts. It is to be noted that Reid's figures refer not to the frequencies for *all* the three months of the study, but to the restricted schedule of observations.

Smith (44) asked one hundred regular classroom teachers to

Clifford Woody, "A General Educator Looks at Arithmetic Readiness," *Mathematics Teacher*, XXX (Nov., 1937), 314-331.

Also should be mentioned the chapter by Sueltz in the forthcoming yearbook (1941) of the *National Council of Teachers of Mathematics*.

[21] Too late to be included in this summary the following three articles came to the writer's attention. Each article reports briefly a study relating to children's uses of number, though the data are by no means limited to uses in Grades I and II. The three articles appear in the *1940 Yearbook of the California Elementary Principals' Association*, and the titles are:

Clark N. Robinson, "Children's Arithmetic Needs Arising in the Home Environment," 54-59.

Mary Marjorie Culver, "A Study of Children's Interest in Arithmetic," 60-70.

Roy DeVerl Willey, "Social Situations Which Lead Elementary School Pupils to Natural Arithmetic," 71-79.

select five children from their class and to interview these five children each morning for twenty-five consecutive school days. The pupils were directed to tell everything they had done from the time they left school in the afternoon until they returned in the morning. In case a child did not make a sufficiently detailed report the teacher questioned him as to the nature of his activities. The following table (Table 36) shows the relative frequency of occurrence of the situations involving number as found from the interviews.

TABLE 36

Relative Frequency with Which Situations Involving Number Occurred (From Smith)

Activity	*Per cent*
Transactions carried on in stores	30.0
Games	18.0
Reading Roman numerals on the clock	14.0
Reading Arabic numerals in finding pages in a book	13.0
Dividing food with playmates and pets (fractions)	6.0
Depositing and withdrawing money from a toy bank	5.0
Playing store	3.0
Measuring distance	2.2
Using calendars	2.0
Running errands	1.2
Setting the table	1.2
Buying and selling tickets	1.1
Acting as newsboy	1.0
Eight other activities	1.3

Woody (**56, 57**) reports that 18 per cent of the parents of children in Grade IA of a school system responded to an item in a questionnaire by saying that they had their children go to the store, set the table, or engage in other activities which involved number. Approximately 50 per cent said that their children played such games as Dominos, Ring Toss, Horse Shoes, Jacks, and the like, which require the use of number.

Gunderson (**16**) made an analysis of ten sets of primary readers to determine the extent to which the reading textbooks in Grades I and II contribute to the arithmetical vocabulary which a child is expected to have upon entering Grade III. The table (Table 37), which follows, shows the topics treated, the number of items within each topic and the per cent of the total number of items for each topic, and the number of mentions of the items within a topic and the per cent of total number of mentions.

Several limitations of these studies of children's uses of, and needs for, number are readily apparent. The interview and questionnaire technique are not likely to reveal nearly all children's ex-

TABLE 37

RESULTS OF THE ANALYSIS OF TEN SERIES OF PRIMARY READERS (AFTER GUNDERSON)

Topic	*Different Items*		*Mentions*	
	Number	*Per cent*	*Number*	*Per cent*
Terms referring to size	18	4.3	5,113	22.3
Terms referring to quantity	30	7.3	3,619	15.8
Terms referring to time	113	27.2	3,104	13.6
Terms referring to location	34	8.2	2,760	12.0
Numbers as words	19	4.6	2,142	9.4
Arabic numerals	42	10.1	1,886	8.2
Miscellaneous concepts	21	5.0	1,540	6.7
Money	32	7.7	1,071	4.7
Miscellaneous terms	25	6.0	641	2.8
Comparison	39	9.4	558	2.4
Ordinals	4	1.0	181	0.8
Groupings	16	3.8	126	0.6
Measures	16	3.8	100	0.4
Roman numerals	7	1.7	75	0.3

periences which involve number. Should the child being interviewed or the person responding to the questionnaire be able to recall *all* the activities engaged in (which is extremely unlikely), even so the full scope of number uses is not likely to be brought to light. This is especially true in the case of the interview technique, since the interviewer is required to ferret out number uses from a child's description of a total activity.

Textbook analysis shows not only the number knowledge which may be required in general reading, but also the number terms and concepts which are deliberately introduced for the purpose of teaching the child those terms and concepts. Likewise, tabulations of number situations arising in the classroom show both the incidental use of number and the use of number directly or indirectly introduced by the teacher for purpose of instruction.

Perhaps the most serious limitation of such studies is the fact that they report only the need for, and use of, *number*. Quantitative thinking apart from the use of number is neglected except in the case of vocabulary and interview studies where a few nonnumerical mathematical ideas are reported. Whereas the studies reviewed above should seem to indicate that children have relatively few and relatively simple needs for number knowledge, the needs for number *understanding* would be many more, were it possible to tabulate the occurrences of quantitative thinking.

No study (except Gunderson, **17**, summary on p. 111) has yet been reported to show children's need for, and use of, number after they have been given arithmetic instruction in the primary grades. All the studies reviewed involved children who had received no number instruction. It is entirely possible that children who have more number knowledge and understanding than is usual in primary grades in which number experiences are incidental would show much more use both of number and of inexact mathematical ideas. To a very large extent, need and use are dependent upon knowledge; the greater the knowledge, the greater the potential need and use.

11. SPECIFIC TECHNIQUES; MINOR PROBLEMS

In this section will be considered the research on a group of pedagogical techniques or teaching procedures. The amount of research on any one technique (except the first) is rather limited, so that none of the subsections are very long.

(a) Drill.—Frequently throughout this monograph the word *drill* (repetitive practice) has been mentioned as a common teaching procedure. Research not here summarized[22] had by 1930 pretty well established the value of drill, and teaching practice had in many instances come very close to making it the sole instructional technique in the primary grades. More recent research, particularly that on the learning of the number combinations, has shown the greater worth of other instructional procedures, at least at certain stages in the learning process. See, for example, the studies by Olander and Thiele reviewed above (p. 118 and pp. 120-121).

In this section two other investigations are reviewed, the first clearly appropriate to the scope of the monograph, the second less clearly so, since it was made in Grade III. The latter, however, is included because the findings are as significant for Grades I and II as for Grade III.

Sauble (43) investigated the value of ten minutes of extra drill on the combinations in Grades IIB and IIA. Her experimental and control classes were matched as to mental ability, age, race, nationality, teachers, materials, and methods. Gains from the extra drill

[22] This research has dealt almost exclusively with arithmetic above the primary grades. The one major investigation most nearly related to primary arithmetic was conducted by Sister Immaculata. In her study she was able to verify, for Grades III and IV, many of the distinguishing characteristics of effective drill which had been noted for the intermediate and higher grades. Thus short periods of drill are to be preferred to long periods; distributed practice is better than concentrated practice; written drills lead to longer retention than oral drills. See Sister Mary Immaculata, *Permanence of Improvement and the Distribution of Learning in Addition and Subtraction* (*Catholic University of America Educational Research Bulletin,* Vol. V, Nos. 8 and 9; Washington, D. C.: Catholic Education Press, 1930).

in the experimental classes were not proportionate to the additional time requirement, though the over-age pupils in Grade IIB seemed to profit as shown by improved achievement. This finding, like the findings in the next study, emphasizes the limitations of drill at certain stages in learning.

The Brownell and Chazel research (6) employed as subjects sixty-three third-grade children in a single system who had been taught the 200 addition and subtraction facts by drill procedures in Grade II. At the start of Grade III these children were given a timed group test on the 100 addition facts (Test A). On the basis of the accuracy scores on this test thirty-two children were selected for interviews: the nine making the poorest scores, thirteen making average scores, and the ten making the highest scores. The interviews were designed to ascertain how these thirty-two children thought out or obtained their answers to sixteen addition combinations, the ten hardest on Test A and six of average difficulty.

During the next school month five minutes a day were devoted to drill on the addition combinations. At the end of this time the timed group test on the addition facts (Test B) was repeated, and the thirty-two selected children were given a second interview (Interview II) on the same sixteen facts of the first interview. Following this by a month, during which no particular drill was given on the addition facts, Test C and Interview III were administered.

The results of the group tests showed the usually found improvement in efficiency: on the average, Test A required 17 minutes, and 11 errors were made; Test B required 11 minutes, and 4 errors were made; Test C required 7 minutes, and the number of errors remained at 4. Briefly, then, the drill in Grade III and the later incidental use of the addition facts both increased speed and decreased mistakes in furnishing answers.

The interview data, however, revealed a different story: the months of drill in Grades I and II, the later month of drill in Grade III, and the incidental use of the number facts for a month in the same grade did little to develop mature methods of thinking about the facts. Twenty-three per cent of the 512 performances in Interview I (32 children, 16 facts for each) involved "counting." On Interview II, after a month of drill, "counting" still accounted for 17.4 per cent of the answers. On Interview III, the per cent was 19.3. "Indirect solution" was used for 14.1 per cent of the facts on Interview I, for 16 per cent on Interview II, and for 13 per cent on Interview III. "Guessing" declined from 24 per cent through 18 per cent to 15 per cent. "Immediate recall" (supposedly of a meaningful kind) increased only from 40 per cent, through 49 per cent (after drill), to 53 per cent.

These data are interpreted to show the dangers of premature drill, that is, drill on verbal statements or formulations before children have reached mature procedures in thinking about the facts. ("Premature" does *not* imply that Grade III is too soon for drill on the facts; Grade II, or even Grade I, is not too soon if children's thought procedures on particular facts have developed to the point where their procedures on those facts should be habituated). Children who after two years of drill in Grades I and II "counted" to get answers for the facts continued in general to "count" after a month more of drill in Grade III. Children who at the start of Grade III tended to "solve" the facts continued to "solve" them after a month's drill. These findings should be compared with the discussions of learning the combinations at earlier points in this monograph (see pp. 92-102).

(b) Use of number patterns and other perceptual aids.—As reported by Howell (21), the value of number pictures for teaching and testing the meanings of numbers was early investigated in Germany and later in this country by himself. Freeman[23] also suggested their usefulness in this connection.

McNamara under Knight's direction (23) investigated the comparative merits of (1) pictured groupings of dots, splints, and dolls (2) arranged in linear, diamond, and domino patterns. The number pictures were exposed by means of a Whipple tachistoscope to a total of 100 children, 32 in Grade II, 33 in Grade III and 35 in Grade IV. The relative worth of the type-patterns was determined from measures of success. There was little to choose between the objects (dots, splints, and dolls) as such, though a slight difference was found in favor of dots. As to patterns, the results were more clear-cut: the domino arrangement was superior for every number between 6 and 12 (the full range), except for the number 8. It is accordingly recommended that domino number pictures of dots be used in the classroom, extra diamond pictures for the number 8 being included.

In a second study under Knight's direction (23) Harter developed a series of forty-eight pictures by means of which all of the addition and subtraction combinations could be represented. For

[23] Frank N. Freeman, "Grouped Objects as a Concrete Basis for the Number Ideas," *Elementary School Journal,* XII (March, 1912), 306-314.

example, one picture shows nine toy autos leaving a garage in groups of four and five, thus making possible the identification of the four combinations $4 + 5 = 9$, $5 + 4 = 9$, $9 - 4 = 5$, and $9 - 5 = 4$. Use of these cards is proposed as a means of teaching children to discover and understand the basic number facts.

Another kind of perceptual aid, the lantern slide, was investigated by Zyve (**60**). Results of a control-group experiment involving 37 second-grade and 39 third-grade children seemed to indicate superiority for lantern-slide presentation of the number combinations over blackboard presentation, the more traditional procedure. But few quantitative data are reported, and, as indicated, the numbers of subjects used were small. On this account further research is needed to test the investigator's conclusion that about two days' work with lantern slides are as effective as three days' work with blackboard presentation. The mere novelty of the new procedure, besides other factors mentioned, might easily account for the differential results.

(c) Adding upward versus adding downward.—Two investigations of the relative value of teaching pupils to add upward or to add downward have been conducted on the primary level. The first of these, by Buckingham (**11**), was carried on in seven centers and used pupils from Grade I or Grade II depending upon the place in the curriculum where column addition was introduced. The classes were divided into two equal groups on the basis of a preliminary test. One half of the classes were taught to add upward and the other one half were taught to add downward. The time devoted to addition (twenty minutes a day), the elimination of related topics, amount of drill, and so on, were controlled by carefully prepared instructions to the teachers. At the close of the period of instruction (the periods were not the same in length for different centers) the classes were tested on column addition. The results showed that in six of the seven centers the method of downward addition was the more accurate. Since the columns in the test were short, Buckingham suggests that with longer columns there should have been a more decided advantage in favor of the downward method, and a still greater advantage, had the examples involved carrying.

The second investigation, conducted in Scotland under the sponsorship of the Scottish Council for Research in Education (**46**), makes a somewhat different approach to the study of the value of the two methods. Ten classes of Primary I pupils (average age of eight years) who were taught the method of downward addition first were compared with ten other classes who were taught first the upward method. These pupils were tested on their ability to add both upward and downward and were compared on speed and accuracy of computation. The classes which had first been taught the downward method were superior in adding both up and down in point of average time required on the tests. The classes which had first been taught the upward method were superior in accuracy. Both groups added upward more rapidly than they added downward. They were about equally accurate adding either way. The differences in all cases were exceedingly small.

The answer to the question of which method to teach is probably not forthcoming from these investigations but rather from other considerations. The necessity for checking computation and the probability that errors in adding often are repeated when the check is performed in the same direction as in the original computation practically require that both methods be taught. The fact that small differences are obtained when children are taught only one of the methods seems to indicate at the present time that the method which should be taught first may be decided also on the basis of considerations other than speed and accuracy.

(d) Subtractive versus additive subtraction.—While numerous studies have been made of the values of various methods to be used in subtracting, only two of these relate to the primary grades. These have compared the merits of the subtractive and additive approach to subtraction. In the example 12 — 5 (in vertical form) one may say, "5 from 12 is ?" or one may say, "5 and ? are 12." The first of these statements involves the subtractive idea, the second involves the additive idea. The subtractive approach is the older method and is based on logical considerations. The newer additive approach may have two possible advantages over the subtractive. First, it may, by teaching only one process, avoid the necessity of teaching separately both the addition and the subtraction facts. Second, it may increase speed and accuracy in computation as subtraction forms are converted into addition forms.

In an investigation of the relative merits of the two methods Mead and Sears (34) used two second-grade classes of nearly equal ability, as determined by a short test on the addition facts which had been previously learned. Over a period of four months one class was taught the subtraction facts by the additive method, the other by the subtractive method. Besides the initial addition test, five tests in subtraction (four of them on subtraction facts) were given during the course of the investigation. Each test lasted only one minute. Between the third and fourth subtraction tests ten minutes of daily practice on the addition facts was introduced.

The medians on all tests except the fourth in subtraction favored the additive group. For this reason, the findings at first glance would seem to support the additive method. But the results are by no means convincing. In the first place, if one takes into account the initial superiority of one point for the additive group, assumes the various tests to be equivalent, and subtracts 1.0 from all differences, the results favor the subtractive method in three out of five comparisons; and this is the way the authors interpret their data. In the second place, the differences found are probably unreliable. Since the tests required only one minute to administer, and since the number of cases is small, the probable error of measurement is undoubtedly quite large in comparison with the small differences.

The most interesting result of this investigation is the fact that the additive group lost on the fourth subtraction test. The authors attribute this falling off to the fact that the addition practice introduced before this test caused interference. Since the major argument in favor of the additive method is that it makes use of addition facts already known, this interference implies that the argument is incorrect.

Less ambiguous data on the comparative values of the two methods are reported by Buckingham (10). A total of 110 pupils in seven centers were paired on the basis of intelligence into comparable groups in each center. One group was taught the additive method, and the other the take-away method. The results of the investigation favored the take-away method in six of the seven comparisons made. None of the differences are statistically reliable. However, had it been possible to combine the data from the various centers, the difference between the two groups undoubtedly would have been a reliable one.

(e) Teaching the related combinations together or separately.— The basic facts of addition and subtraction may be organized for instruction in many different ways. In the first place, the addition facts as a group may be taught (1) before or (2) with the subtraction facts. In the second place, each fact may be taught (3) separate from all its related facts, or (4) the direct and the reverse facts within a process (e.g., $3 + 5 = 8$ and $5 + 3 = 8$; $8 - 5 = 3$ and $8 - 3 = 5$) may be paired and taught together, or (5) the related facts in both processes may be taught together as a "whole family" or a "whole story" (e.g., the four facts above). In the third

place, whether facts are taught through their relationships or independently of these relationships, the order of teaching may vary. Thus, (6) facts or groups of facts may be taught in some assumed order of difficulty (see section 5 above, pp. 122 ff.) ; or (7) they may be presented at random; or (8) they may be grouped around generalizations (as was done by Thiele, **48, 49**) ; or (9) they may be organized around the successive numbers (thus, all facts, or parts of the facts, for 3 before the facts for 4, those for 4 before those for 5, and so on). When the various possible combinations of these varying approaches are worked out, they become numerous indeed.

Curiously enough, there is exceedingly little research bearing directly upon the comparative merits of different programs for organizing the number combinations. In the days when "specific" teaching held sway (say, prior to 1930), the general practice probably was to present facts in random or difficulty order in such a way that no use could be made of relationships. More recently, practice has veered to one of the several organizations in which related facts are taught together. Thus McConnell (**26**), Olander (**36**), and Thiele (**48, 49**), the latter two through developing generalizations, taught addition and subtraction facts together. Even so, the research support for this practice was meager (see Buckingham, **9**, below). Moreover, the order of presenting groups of combinations probably differed materially from study to study, and again there was little research, except that on the supposed comparative difficulty of the combinations, to support the order adopted. Thiele, it will be recalled, even went directly against the findings in these studies of difficulty by arranging the combinations around generalizations.

None of the investigations mentioned above, with which should be included the original study reported in Chapter III, can supply an answer to the best way to organize combinations for learning. In Chapter III data are furnished only for the organization adopted; no comparable data are furnished for other organizations. McConnell's research, as well as Olander's and Thiele's, involved many factors other than those of order and organization of the facts for presentation; hence no answer to the question is forthcoming from these sources.

But one research was found in which an attempt was made to test one organization against another. In this investigation by Buckingham (**9**) a total of 129 pairs of second-grade children were taught a specified list of thirty-eight addition and thirty-eight subtraction combinations. One member of each pair learned the facts separately;

the other member learned the correlative addition and subtraction facts together. In all, the experiment involved seven "centers," or co-operating schools. No rigid time schedule was adhered to, but rather each teacher was advised to stop her experiment as soon as she had reason to believe that her pupils had mastered the seventy-six facts. Then tests were given. Tabulation of results revealed that the children taught the facts together excelled those who were taught them separately in six of the seven "centers." Buckingham argues from the consistency of these findings, the differences between classes being small in each center, that the "together" method is to be preferred. Conflicting results in the one "center" are attributed to differences in teaching material and to the final test used.

As noted above, the trend seems to be in the direction of greater use of relationships in teaching the number combinations. This trend has not followed, however, from arithmetic research directly upon the problem. Rather, it seems to reflect the new emphasis put upon meaning throughout the subject of arithmetic and the newer points of view in psychology which stress understanding as an important factor in economical learning.

(f) Games and devices.—Wilson (55) made use of number games as one means of providing his subjects with useful practice on the number combinations. He is careful to explain that these games required the use of number in a truly functional manner, as in Ring Toss, Ten Pins, and the like. In such games successes are numerically described, and there is a reason for operations with these numbers in order to secure totals and to find who wins. These games are to be contrasted with the artificial kind in which drill is merely sugar-coated.

Wheeler (53, 54) used his own games, ADD-O to teach the combinations and COUNT-O to teach the reading of the numbers to 100.

The only experimental study, so far as could be ascertained, in which an attempt was made to evaluate the worth of games for primary arithmetic is that by Steinway (45). Every fifth day during the course of the experiment a new number game was introduced to her first-grade subjects. Practice followed for four days, to make sure that the children understood the game and could play it correctly. Then the children were allowed to select any number game which they had previously learned.

A homemade game of Lotto proved to be simple to learn. Water Wheel and Roly-Poly were found to occasion much difficulty in scoring. Parchesi was simple after Lotto had been learned, and was very popular. According to interest span, the number of pupils playing the game, and the number of times the game was played Lotto was most interesting. The report contains a few quantitative data to show that the games alone produced higher scores on an arithmetic test (the nature of which is not described) than did a combination of games and formal drill.

It may not be inappropriate to suggest that games should be selected for instructional purposes on the basis of several criteria, and not the usual one or two. Interest is of course important; Wilson has emphasized that the games should put number to functional uses; in general, games which call for activity on the part of many children, instead of a few, are to be preferred. But there is another criterion, namely, that particular games are serviceable to differing degrees at different stages in learning. The game of Parchesi, for example, is especially good for teaching number concepts, since the *content* of the numbers used is perceptually evident: to advance five spaces, children must count out five spaces. The game of Lotto, even if played with small numbers, is of value only in teaching children to read numbers: the numbers as used reveal nothing as to their meaning. Games involving purchases may or may not teach the meaning of numbers: if coins are used, children may learn only that a purchase with *this coin* brings change of *that coin;* that is, there may be no five-ness in the nickel and no ten-ness in the dime. If, on the other hand, the children know number meanings, Playing Store may be good practice on the combinations. ADD-O may be recommended for drill on number combinations *only* after children understand numbers and the meanings of the combinations used in the game. The point made here is that number games are not to be classified either as good or bad; rather, they are good or bad in terms of what they can do for the learner at the particular stage of his development.

(g) Permanence of learning.—Data on the permanence of learning are limited to the amount of forgetting over the period of the summer vacation, admittedly a poor criterion, but the only one available in the research on primary arithmetic.

Holloway (19) tested his subjects at the end of the school year and again at the start of the new term in the fall. His results relevant to the purpose of this monograph deal with memory of the addition combinations. His 516 Grade III pupils made 491 mistakes on the addition facts in the spring and 614 mistakes in the fall. In other words, they made 25 per cent more mistakes after the vacation than before the vacation. The corresponding per cent figures for the twenty Grade II subjects and for the twenty-two Grade I pupils are 241 and 163, respectively. Clearly, the Grade III pupils had carried learning to the point where it could resist the effects of lapse of practice without much loss. This was not true in the case of the pupils in Grades I and II. The unexpected larger per cent of loss for the Grade II pupils as against the Grade I pupils may have been due to poorer quality of pupil material; Hollaway's data convey no explanation.

Brueckner (7) made a similar investigation with second-grade pupils only. For his subjects there were practically no vacation losses in the case of the addition facts, but there was a considerable decrease in knowledge of the subtraction facts, perhaps because the latter had been learned later in the grade and therefore had not been carried to the point in learning necessary for a high degree of retention.

Osburn and Foltz (37), using subjects from Grades II to VIII inclusive, found that, in general, vacation losses amounted to an average of one twelfth to three fifths of the amount learned in a year. The test used was the Wisconsin Inventory Test, and the data are therefore crude test scores derived from an instrument designed to survey arithmetic abilities over a wide range of grades. The greatest losses, in terms of topics, were in those which had been most recently taught. Brighter children, as would be expected, showed smaller losses than less capable children and made speedier recovery under a program of remedial instruction.

CONCLUSION

This chapter is brought to a close with a series of brief statements as conclusions. The shorter the conclusion, however, the more dogmatic it must appear, and the more prominent the peculiar predispositions of the interpreter of the relevant research. On this account the reader is advised to be on the alert against bias on the part of the writer.

1. The evidence is that an early start in arithmetic is profitable, provided that it is of the right kind and that advantage is taken in later grades of the gains produced in the first grades. The criterion for evaluation here is success in learning purely mathematical skills and concepts, admittedly an incomplete criterion. If gains of this kind are offset (a) by unfortunate personal maladjustment on the part of pupils or (b) by uneconomical efforts on the part of teachers, no research has yet demonstrated the fact.

2. "Social" arithmetic, which is to say, the systematic and careful provision of number experiences centered around significant applications of arithmetic, produces good results in Grades I and II. However, the extent of the arithmetical gains reported in one study (55) and attributed to "social" arithmetic may equally as well have been produced by subsequent drill of an abstract type. To a less certain degree the same possibility holds for the second study (17) considered under the heading of "social" arithmetic.

3. The supposed abandonment of systematic instruction in Grades I and II in favor of an "activity" or a "project" curriculum seems to have produced no harmful consequences with respect to arithmetic. Still, this conclusion is to be challenged on grounds which are partly experimental and partly theoretical. (a) The subjects in some, and the teachers probably in all, these experiments were of a superior type; (b) the instruments used for measurement are of questionable value in the early grades; (c) the suspicion is warranted that practice and drill (and perhaps developmental instruction as well) were by no means eliminated (20, 25). If the "activity" and the "project" approaches produce happier children and more economical teaching, the evidence is still to be supplied. On theoretical grounds it is to be doubted whether rich concepts of numbers and high degrees of skill can be developed through incidental contacts with number. Such contacts are too few, too unpredictable, too poorly ordered, however great the vitality of such experiences in the attainment of the larger, essentially nonarithmetical goals.

4. Various procedures have been employed to teach the number combinations. Some of these (4, 53) have relied largely upon drill; others (26, 36, 48, 49) have emphasized relationships between the direct and reverse facts within the same process and between related facts in the two processes of addition and subtraction. The latter procedures are preferable if meaning is regarded as important (see especially 26); drill procedures in this case should be withheld until after meaning has been assured.

5. The research on the comparative difficulty of the combinations has already been described as futile. This statement, however, is valid only when attempts have been made to determine the comparative difficulty of the facts regardless of organizations thereof, methods of teaching, types of drill employed, and the like. For any one set of circumstances there probably are ascertainable differences in difficulty, but information on this point is of value to teachers only when they accept the special combinations of these factors which

operate in the research which produces the difficulty rankings. As groups, the doubles in addition and subtraction and the combinations in which 1 is added or subtracted seem to be easy. Outside of these combinations rather large dissimilarities are found in the rankings of different investigators.

6. Children can be taught to read the numbers to 100 in Grade I. No one has yet determined the degree to which the meanings of the larger numbers can be taught thus early; but if not much can be done to develop meanings, the reading skill is largely superficial. Number concepts under these circumstances are but slightly quantitative, though they have their value in such tasks as locating house numbers, finding pages in books, and the like.

7. The development of arithmetical concepts and skills takes time. Each concept, each skill, is a complex of many experiences, and fullness of meaning is dependent upon diversity of experience. The common practice of providing many experiences of the same kind is not conducive to meaningful development. Expert skills and adult concepts seem to result from the organization of experience at successively higher levels. Teachers must be willing at first to accept immature and uneconomical procedures, which, however, have the virtue of "making sense" to children. The function of instruction is to direct children, or to lead them, to discover and adopt higher and higher, or more and more mature, procedures as rapidly as they are ready for them.

8. Transfer may be utilized to great advantage in teaching arithmetic. There is no longer any question about the fact of positive transfer (if ever really there was any such question); but the extent of this transfer is largely determined (a) by the closeness or remoteness of the new to the old and (b) by the way in which children are instructed. The second of these factors is of slight consequence when the new and the old are very close together, but it becomes increasingly important as the distance between them enlarges.

9. The allocation of arithmetical topics to Grades I and II (and to any other grade, for that matter) is either a simple or a complicated problem, depending upon whether one adopts a single basis or a variety of bases. Chapter III shows what *can* be done with respect to several concepts and skills if reasonable outcomes are set and if means are found (learning activities, experiences of various kinds) which are adapted to their attainment. Students of arithmetic who do not accept the criterion of successful learning as sufficient are

inclined to postpone such topics as the addition combinations to grades beyond Grade II.

10. Children in Grades I and II have many uses for number, as shown by their reading needs, their life in the school, and their extraschool experiences. The extent to which they employ number is of course conditioned by the amount of arithmetic they know. The relationship is not however necessarily close, for if children are taught arithmetical skills and concepts which they do not understand, they can hardly be expected to use them.

11. *a*. Drill is to be recommended when children have attained the understanding and the maturity of thought processes which ought to be habituated. Given too soon, drill merely fixes upon children empty verbal formulas and mechanical skills. *b*. Number pictures and other perceptual aids are of value in helping children to understand what they learn. Research (23, 60) has supplied certain approved aids of this kind. *c*. The evidence seems to support the practice of adding downward, if a choice between adding upward and downward must be made. However, since checking is frequently advisable and is more effective if done in the reverse direction from the initial adding, children should be taught to add in both directions. *d*. The substraction facts and the process of subtraction should be taught as subtraction and not as addition. That is, children should say for $9 - 7 = ?$ (in vertical form), not "Seven and what are nine?" but "Seven from nine are what?" (or any of the other subtraction language forms). *e*. There is not much research to prove the point, but psychological theory supports what research there is: the related combinations should be taught together rather than separately (thus, $6 + 3 = 9$ with $3 + 6 = 9$, and with $9 - 6 = 3$ and $9 - 3 = 6$). *f*. Games need to be chosen carefully in terms of what they will do to quantitative development; when properly selected in the light of this and other criteria, games are of aid in teaching arithmetic. *g*. Permanence of learning, as measured by vacation losses, varies directly with the degree to which learning has been carried. That is to say, well-learned number combinations resist disuse much better than do combinations which are but slightly known.

CHAPTER V

A POSITION WITH RESPECT TO ARITHMETIC IN GRADES I AND II

The collection of research data is one matter; the interpretation of research data is quite another matter. In both activities, it is true, one's philosophy of education in general, one's conception of arithmetic, and one's view of the learning process—all of these are called into play. Thus the investigator tends to select his problem, to organize his experimental procedures, and to plan his measures in the light of theoretical considerations. But these theoretical considerations have no less weight when one critically appraises research findings. The truth of this statement has been amply demonstrated to the reader who has studied the monograph up to this point. In not a few instances the writer has opposed his own interpretation of certain data to that of the investigator and has sometimes suggested different significance and implications from the investigator's. Few such dissimilarities of opinion would have resulted had the writer been content merely to review findings as reported. To have done so, however, would have been to expose the reader to all sorts of inconsistencies and discrepancies and to have given him but a series of disjointed and often irreconcilable "facts."

THE THREE CRUCIAL QUESTIONS

Some degree of interpretation, then, seemed essential. Up to this point, however, interpretation has been pretty much restricted to particular issues and to particular studies. As yet no interpretation has been given to research findings as a whole. Nevertheless, it is to the whole body of research evidence, and not to separate studies, that one must look for help in answering what in Chapter I were called the three crucial questions with regard to primary number.

To the first of these questions, "*Can* primary-grade children learn arithmetic?" research gives the answer, "Yes." The evidence is of two kinds. The first kind of evidence is inferential: children, when they enter Grade I, already posses (that is, have learned) much arithmetic, and the assumption is that, properly taught, they can learn much more in the primary grades. This evidence comes from the extensive inventories, new and old, which have been conducted by personal interviews and by group testing and which are summarized at length in Chapter II. The second kind of evidence is more direct: research shows unmistakably that if primary-grade children are taught arithmetic, they will learn it. The evidence for this conclusion is contained in the studies reviewed in Chapter IV and, in greater detail, in Chapter III. The only occasion for dispute with respect to this conclusion lies in the meaning assigned "learn." If "learn" is made to mean "attain mastery in," the conclusion does not hold. If, however, the word means "make substantial progress toward the outcomes set," then the conclusion is valid. It is in the latter sense that the word is here used. The former connotation involving "mastery" seems to the writer to be quite inapplicable when primary pupils are under discussion. "Mastery," as has been repeatedly pointed out in this monograph, is best viewed as the end-product of long periods of experience and practice.

The second crucial question is not to be answered so definitely, "*Should* primary pupils be asked to learn arithmetic?" In so far as school practice is determinable from the ability of children measurably to profit from teaching, the answer here is "Yes." But ability to learn is admittedly only one of the factors which affect policy. Other factors, sociological, philosophical, and psychological, are also involved. Indeed, according to some educational theories ability to learn is of slight importance as compared with "natural interests and needs." According to other theories, perhaps more "traditional," ability to learn is more important, especially when the thing to be learned is of demonstrable social significance. The answer to this second question is, then, largely a matter of theory and opinion. To the writer the facts argue for systematic arithmetic starting in Grade I. The supporting reasons must be withheld for the moment.

The third crucial question is, "Granted that primary pupils *can* and *should* learn arithmetic, how should the program be organized?" There is no final answer to this question. Research has not yet tested enough programs and collected enough facts. The writer's conception of an appropriate program has been outlined in Chapter III and will be further explained in the paragraphs to follow. But it would be the height of folly to suppose that this particular program as it stands is the final answer to the need. Instead, as teachers and research workers continue to study the way children learn and to locate and remove the sources of their difficulty, we may confidently expect steadily improved programs.

There should be no occasion for surprise in the writer's frank statement that he favors systematic arithmetic from the outset of schooling. The reader must long since have detected this belief and have noted instances where interpretation was governed by it. Now, the person who subscribes to this view is put at once on the defensive: he may be accused of being a "subject matter specialist," more interested in arithmetic than in children; he may be charged with "forcing" children to learn things they are not "ready" to learn. It may even be thought that he is indirectly (or directly) supporting the hammer-and-tongs tactics of an earlier decade.

The basic fallacy in these charges lies in failure to recognize that there are different kinds of arithmetic and different kinds of systematic instruction. The "drill" program of arithmetic, described briefly in Chapter I, *is* formal and *is* open to the charges listed above, for it calls for a type of learning of which primary-grade children at the outset are incapable. But to speak in behalf of systematic instruction does not commit one to this program of arithmetic; there is another kind of arithmetic against which the objections named cannot properly be leveled.

MEANING AND SIGNIFICANCE IN ARITHMETIC

Programs of primary arithmetic were at one time evaluated solely according to a criterion of so-called efficiency. If pupils subjected to a program tested up to set norms or standards of speed and accuracy in computation and in the conventional kind of problem solving, then that program was acceptable. Now it is rather generally conceded that this criterion is both narrow and incomplete. It is narrow because success in the skills measured by tests is no real guarantee of quantitative efficiency in the affairs of life. And it is incomplete because it does not recognize the importance of other elements in learning, namely, (1) meaning and (2) significance,[1] two terms which are commonly but erroneously taken to be synonymous. An object or idea or skill is meaningful to the degree that it is understood. An object or idea or skill is significant to the degree that the values are known and to the degree that it is used.

The typical driver of an automobile knows how to use the gear-shift of his car without knowing very much about the principles which govern or explain its operation. He knows when to shift gears and how to do so, and what the effects of the gear shifting are. In a

[1] William A. Brownell, "Trends in Primary Arithmetic," *Childhood Education*, XIII (May, 1937), 419-421.
B. R. Buckingham, "Significance, Meaning, Insight—These Three," *Mathematics Teacher*, XXXI (Jan., 1938), 24-30.

word, he knows the *significance* of the gear apparatus. This he has learned by actually employing the gear shift in appropriate ways and at appropriate times. But this kind of activity teaches him very little about the mechanism itself. To acquire information of this kind an entirely different sort of activity is needed. To learn the *meaning* of the gear apparatus he must study that apparatus at first hand. He must break it down, examine the parts separately, reassemble them and observe their interrelationships, and interpret what he sees in the light of the relevant principles of mechanics.

This analysis in the field of automobile mechanics holds also in the case of arithmetic. If the pupil is to appreciate the significance of number he must engage in appropriate learning activities. That is, he must have many experiences in the actual use of the arithmetic he learns. If the pupil is to possess the meanings of arithmetic, he must also engage in appropriate learning activities, but they will not be like those which yield significance. Use need not reveal the meaning of number any more than it reveals the meaning of gear apparatus. Mathematical meanings result only from a grasp upon mathematical principles. A program of arithmetic is "significant," then, when pupils know when and how to use number and when they have well-developed habits of using the arithmetic they know. It is "meaningful" when it makes arithmetic a sensible, intelligible system of quantitative thinking.

The distinction between meaning and significance, readily understandable in the case of arithmetic in the intermediate and higher grades, is no less important for arithmetic in the primary grades. At whatever point arithmetic instruction is begun, whether in Grade I, or Grade II, or later, children should see both sense (meaning) and value and usefulness (significance) in the number they encounter. If arithmetic experiences cannot be found to satisfy both of these requirements in Grade I, then arithmetic should be deferred. If, on the other hand, arithmetic experiences which meet these requirements can be found for pupils in Grade I, then arithmetic may be started at this point.

EVALUATING PROGRAMS OF PRIMARY ARITHMETIC

Seldom indeed are programs of primary arithmetic evaluated from the standpoints both of meaning and of significance. As has already been stated, the sole criterion customarily employed in research studies has been efficiency in the narrow sense. It has been only in theoretical evaluations that considerations other than efficiency have been invoked. In such instances the criteria have been subjective (though none the less worth while) and have usually related either to dangers to child personality or to excessive and wasteful effort on the part of teachers and pupils alike. Application of these criteria has uniformly led to condemnation of primary number programs based largely upon drill with abstract numbers.

Those who would abolish this kind of arithmetic have tended to divide among three groups. One group would abandon all attempts to provide number experiences for children, trusting entirely to incidental quantitative contacts for such learning as is necessary (Program (1), p. 6). A second group would make provision for number experiences by setting up social situations in which number is imbedded (Program (2), p. 6). Each of these two groups defends its recommendations on the ground that its program alone enables children to approach number *meaningfully.* Both groups are, in the opinion of the writer, wrong. Their programs present number *significantly,* and to this extent are indeed admirable; but they must perforce neglect meaning. Children engaged in large units of activity which are only in part quantitative are hardly in the mood to isolate and to study the number elements in their intrinsic relationships. To do so interferes with the attainment of the goal. Yet, this isolation and study of number elements are essential if meanings are to be acquired. Informal "social" encounters with number reveal the value and usefulness of number, but, no matter how frequent, they cannot develop meanings of a high order.

It was suggested that opponents of drill programs tend to fall into three groups, two of which have been mentioned. The third group consists of those who strive to make number meaningful without sacrificing significance. Like the advocates of Program (1), those who sponsor this program (Program (4), p. 7) would utilize all incidental occurrences of number; like the advocate of Program (2), they would plan other experiences with number in "social" settings—all of this to guarantee significance. But, unlike either group, they would provide for meaning with equal care. That is to say, in addition to informal number contacts they would have children study number as number, grasp the mathematical principles which govern relationships, understand the number system, formulate binding generalizations to aid organization for learning, and so on.

Efficiency, the goal of Program (3), (p. 7), is not regarded in Program (4) as an end immediately to be achieved (nor is it in Programs (1) and (2), for that matter). The necessity for ultimate efficiency is fully recognized, and steps are taken to assure it. The procedure, however, is not drill (on which in the last analysis Programs (1) and (2) must rely), but the development of understanding. Without understanding (meaning) true efficiency is impossible, and what passes for efficiency proves to be a bag of mechanical tricks which are not susceptible to effective use and which are soon mislaid.

ARGUMENTS AGAINST AND FOR SYSTEMATIC INSTRUCTION

But can such a program be safely instituted? Will it not, in the first place, lead to unhappiness, to anxiety neuroses, to feelings of insecurity and inferiority on the part of the children? And, in the second place, what is the hurry? Will not economy be best served if systematic instruction is postponed until children are "mature" enough, are "ready" for it?

Effects on personality.—The answer to the first objection will not be satisfactory to those who are already convinced that all systematic instruction in arithmetic (or anything else) must necessarily be "formal," unpleasant to children, and stifling to their creative impulses. The answer is that within the limits of systematic instruction there is ample opportunity for children to discover and create (though the teacher knows ahead of time what they must inevitably discover and create), to engage cheerfully in co-operative undertakings, and to progress each at his own rate. "Systematic instruction" does not imply rigid systems of uniform standards, tension-developing periods of comparison and testing, competition of an unwholesome kind, pressures to achieve beyond ability to achieve. If these undesirable conditions appear in connection with systematic instruction, the reason is that they have been grafted upon it. The writer and probably most readers must have observed first- and second-grade classrooms where children were happily and intelligently studying arithmetic, quite unaware of the dangers to which they were purportedly exposed.

For some reason one seldom hears the other side of this issue. It seems to be assumed that peril lies alone in the learning of arithmetic. Yet, there are perils no less numerous and no less serious in *not* learning arithmetic. In a culture which is steadily becoming more quantitative, only the person who possesses in arithmetic an effective way of dealing confidently and exactly with the quantitative can co-operate well with his fellows, can feel secure, and can live happily. One has but to observe the self-conscious and confused behavior of the adult with his feelings of inferiority in matters arithmetical, to recognize the truth of these statements. Nor are the advantages of sound arithmetical equipment confined to adult life. The child has his own quantitative environment, and sound arithmetical equipment enables him as well as the adult to avoid bewilderment and to solve his number problems with greater effectiveness and ease of mind.[2]

The policy of postponement.—But what is the hurry? Why not wait? The policy of postponement carries a spurious appeal. It seems to spring from a concern for childhood which is quite lacking in those who prefer an earlier start. The picture is one of sympathetic understanding as contrasted with unfeeling compulsion, of loving kindness as contrasted with ruthless intimidation and forcing.

Three objections to the policy of postponement and the "stepped-up curriculum" may be mentioned. In the first place, mere postponement does not solve the problem. In the second place readjustment of materials and methods of instruction to the capacities and interests of primary-grade children makes for economy of learning in the long run. In the third place, as has already been suggested, arithmetical experiences in the primary grades make immediate contributions to richer and fuller living and so justify themselves quite apart from more remote benefits. Each of these objections will be considered below.

The policy of postponement is based upon the assumption that by deferring systematic instruction a year or two we shall have pupils who are "readier" to learn. The factor which supposedly creates this greater "readiness" is the increased maturity gained by an extra year or so of growth and "maturation." No advocate of this policy has as yet explained just how increase in age produces this effect, and it must be inferred that the effect is the result of biological "ripening," of incidental experience, or of both.

How does the program of deferring arithmetic actually work out? Unfortunately, all the evidence that can be adduced up to now is anecdotal. The advocates of the program insist upon the basis of informal experience (and therefore anecdotal evidence) that the "stepped-up curriculum" has solved their problems. It may not therefore be inappropriate to report two anecdotes of a contrary kind, but anecdotes which in both instances involve dozens of school systems and thousands of school children.

In accordance with the policy of postponing instruction the course

[2] The reader is referred to the chapter by Guy T. Buswell in the forthcoming (1941) yearbook of the National Council of Teachers of Mathematics, for an excellent statement of this point of view.

of study in one state virtually abandons all arithmetic in Grades I and II; but, influenced by tradition, it still requires that in Grade III teachers prepare pupils for Grade IV as they have always done. The assumption is that children in Grade III, being more "mature" and "readier" for arithmetic, can in one year learn all that their older brothers and sisters had learned in three. The results are little short of disastrous. Third-grade teachers rather generally report that their load is too heavy, that they cannot "cover the ground" assigned them, and that they are sending pupils into Grade IV less well prepared than ever before.

To the exponent of the policy of postponement there is no mystery here: this course of study is not sufficiently "stepped up"; part of the Grade III arithmetic should be deferred to Grade IV, part of that in Grade IV should be deferred to Grade V, and so on. What are the consequences of this plan? The writer is acquainted with the situation in one large area where the policy of postponement has been carried to the extremes of its advocates. Now, after several years of experience with this plan, it is discovered that the problems are by no means solved: children in the intermediate and higher grades are *still* encountering difficulty in arithmetic. The process of "stepping-up" the courses can hardly be carried further. Administrators and research workers in this area have lately been forced to re-examine their program, and they have recently (though not publicly) found the source of the trouble in the primary grades. Abandonment of instruction, save for the use of incidental number situations and occasional units of "social" arithmetic, has not had the anticipated effect, and children come into Grades III and IV more "unready" than they ever were before.

We shall some day come to appraise the policy of postponement for what it is, namely, a futile flight from the facts. Children are not through biological heredity equipped with arithmetic "instincts" or "capacities" which "mature" and "ripen" apart from experience. Children do not merely grow into "readiness" by accumulating birthdays. On the contrary, children are "ready" for any phase of arithmetic when they have had the experiences which fit them to learn what is new in that phase, and not before.

This last statement leads directly to the second objection to the policy of postponement. This policy oversimplifies the process by which children acquire mathematical meanings and insight. If the studies in the psychology of quantitative development reveal anything, they reveal the complexity and the slowness of this development. Children approach mature forms of quantitative thinking by reorganizing at steadily higher levels their ways of dealing with number situations. They do not attain mature procedures all at once; the addition of successive birthdays does not magically move them to higher processes. It is only by dealing with number as number that these new processes can be discovered. This means that children need an abundance of diversified experiences with number, begun early and continued long.

This view of the learning process is not intended to justify indiscriminate "experiencing," or the application of pressure. What it calls for is a wise selection and ordering of learning activities suited to the interests and abilities of children, but none the less designed to carry them forward in understanding and in the effective use of what they learn. Postponement, offered as an aid to children and as a means of slackening the pace of learning for their benefit, has in many instances proved to be a real disservice to them. The problem of the arithmetic teacher is not to wait for "readiness" but to create it.

One program (but only *one* program) designed to accomplish this purpose has been outlined in Chapter III, and the results of the program have been set forth in objective fashion. If this program was successful, it provided primary children with concepts, understandings, and insights whose real contribution was by no means limited to the arithmetic of the primary grades. Instead (it is hoped) these children started on the road of arithmetical learning equipped with basic understandings, sensitive to the logical nature of number and number relationships, expecting to use their intelligence, and not merely their memory, in further learning, and emotionally disposed to accept the challenges in this continued learning.

This is not to say that the effects of systematic instruction in the primary grades are all withheld to later years. As a matter of fact, this objection may be more validly raised to the policy of postponement. This latter policy robs primary children of opportunities for living richly and fully in the present. The child who can count (and children even in schools where the "stepped-up curriculum" prevails can count) is better able to adjust to his quantitative environment than is the child who cannot count. But he is less well equipped to deal with quantitative problems than is the child who can think of numbers as concrete groups. And the latter child is less well equipped for enriched living than is the child who can deal more effectively with number situations by means of his knowledge of abstract number and number combinations.

Nor is this the whole story. As has been stated, the primary child who knows little about number is limited in his practical adjustments to the quantitative problems *which he apprehends.* But the quantitative problems which one apprehends are directly related to the quantitative equipment which one possesses for *seeing* problems. One of the important immediate effects of number knowledge is that it makes one sensitive to the quantitative all about one. Added number knowledge increases one's quantitative sensitivity, develops new needs and uses for number, and thus directly enriches life. A program of systematic instruction in primary arithmetic can serve these ends for children in Grades I and II, ends which cannot be served by postponing instruction to Grade III or later.

The policy of starting arithmetic in Grade I, then, has other support than that of tradition. If we want primary pupils to live fully and richly as an end in itself and if we want them to ground their later learning of arithmetic on meanings and basic understandings, systematic instruction in Grades I and II is justified. More than that —it is demanded.

CODE OF TOPICS, TOGETHER WITH CLASSIFICATION OF STUDIES USED THEREWITH

TOPICS, WITH BIBLIOGRAPHICAL REFERENCES BY NUMBER

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