

DETERMINATION OF VERTICAL TURBULENT DIFFUSIVITIES OF HEAT  
IN A NORTH FLORIDA LAKE

By

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To my parents  
Louis Steinberg and Bette King Steinberg  
who have given to me,  
directly and indirectly,  
in ways I know  
and in ways I will never comprehend,  
much of the ability necessary to accomplish  
what is represented here.

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Abstract of Dissertation Presented to the Graduate Council  
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Knowing or predicting the distribution of water quality parameters in water bodies is essential to efficient management of water as a valuable natural resource. Theoretical concepts which describe the transport of substances throughout the water have been developed, and the effects of turbulence have been accounted for by the semiempirical approach of relating turbulent diffusive flux to the average gradient of concentration through coefficients of diffusivity. Because of the significance of vertical temperature distributions in lakes and reservoirs, this research has focused on evaluating vertical diffusivities of heat in a deep Florida lake.

A complex data acquisition system was designed, built, and installed at Lake Mize, Florida, to gather field data to be used to evaluate diffusivities. The investigation involved measuring both the actual turbulent motion responsible for the transport, and the change in heat content as a result of the motion. Emphasis was also given to making

observations rapidly enough to enable the study of lake motion due to convective cooling.

The results indicate that motion in the lake was at most times too low to be detected by the velocity measuring equipment used. Other results point out the need for closely spaced temperature measurements near the surface in order to adequately define the convective cooling process. Detailed analysis of convective cooling revealed that it may be described by the semiempirical concepts provided that suitably short averaging intervals are used. Finally, diffusivities found in Lake Mize are much smaller in the upper layers than those observed in other, larger lakes, but in the lower strata values are more nearly equivalent, and in most cases, two orders of magnitude greater than values for molecular motion.

CHAPTER I  
INTRODUCTION

The problem of determining the distribution of a substance in a water body is often encountered as an aspect of managing the water as a resource. The ever-increasing need for water for industrial, recreational, and potable uses, among others, requires that the management of it be given more attention. For example, these needs may dictate that available water be reused several times and often essentially shared by successive users as it progresses through the hydrologic cycle. Thus, one important facet of water resources management is determining the fate of substances introduced into receiving waters as discharges from some user. Another important aspect is to understand and predict the behavior of naturally occurring physical phenomena in fresh water and marine environments. Both man-caused and intrinsic variations in physical, chemical, and biological water quality are important because of the effects they may have on other uses.

In order to obtain better knowledge of variations in water quality, it is necessary to better understand the hydrodynamics of receiving water bodies. This is because both artificial and naturally occurring substances are transported by the movement of the water. By defining this motion, a description of the levels of substances existing at

various times and places can be obtained. Knowing or predicting these levels is valuable for purposes of planning uses of waterways, administering regulations aimed at maintaining minimum standards of water quality, and otherwise managing the resource in a way which provides optimum usefulness.

Most water motion existing in nature is turbulent, that is, much of the movement is characterized by randomness which is usually described in terms of temporal and spatial averages. To describe this turbulent flow, equations of motion, continuity, and conservation of mass, among others, may be used to express the existing hydrodynamic phenomena. The equations are used to develop a model for making predictions of concentrations, and included in the model are usually coefficients which relate to the turbulent nature of the water motion. The lack of information regarding these coefficients is usually a serious impediment to the application of the hydrodynamic concepts to engineering and management problems. In their most common form, the coefficients are called diffusion coefficients or merely diffusivities and may be functions of direction and location and the type of transported quantity (i.e., heat, mass, or momentum). The focus of this study is to present a better understanding of diffusivities, especially diffusivities of heat in the vertical direction.

In particular, transport of heat in lakes and reservoirs is a subject which has received much emphasis for a variety of reasons. The intrinsic relationship between temperature and density makes the vertical distribution of temperature important because it reveals the structure of existing thermal stratification. In reservoirs the

vertical distribution and transport of heat affect the withdrawal operations of the impoundment due to considerations of thermal water quality downstream. Another example of the importance of evaluating heat transport is the need to solve problems of recirculation of heated discharges. Because of the widespread use of lake and reservoir waters for cooling in many industrial processes, especially power production, this matter has received much attention.

The particular problem of transport in relatively deep, quiescent water bodies like most lakes and many reservoirs is one which has received much attention and is continuing to receive attention. This is primarily due to the occurrence of stratified conditions which develop in lakes and reservoirs annually. In many cases the stratification exists with such stability that vertical transport throughout the entire depth is greatly restricted, and the effect on water quality is significant. Because mixing between upper layers (epilimnion) and lower layers (hypolimnion) is restricted, undesirable accumulations of chemical and biological substances can take place in the hypolimnion, and effluents discharged into the stratified waters will move to depths of equivalent density and tend to concentrate there. An increased knowledge of the magnitudes and variability with time and depth of the diffusion coefficients will provide improved application of the turbulent transport concepts.

This work is a report of a study of vertical turbulent transport of heat in a small, deep sinkhole lake in North Central Florida. Specifically, mathematical methods are presented which express the transport but require the evaluation of diffusion coefficients.

The usefulness and propriety of the concept of diffusion coefficients are considered. In addition, special attention is directed to the phenomenon of convective mixing due to surface cooling in an effort to learn whether or not the notion of diffusivities is compatible with this type of turbulent water motion. Because the usefulness of the transport equations is often limited by the lack of knowledge about the diffusivities, the evaluation of these by two conceptually different methods is described and demonstrated. One method entails the assessment of the effects of vertical turbulence by monitoring the changes in heat content of the lake, and the other method seeks to measure the turbulent motion in the lake by directly monitoring vertical water velocity. The study requires a large amount of field data collection, and the measurement of velocity requires the use of a relatively new type of velocimeter because of the very low levels of motion encountered. A complex and highly automated data acquisition system which is capable of fulfilling the rigorous data requirements is developed and discussed. A key feature of the system is its ability to record data that can be read directly into a digital computer for subsequent analysis.

The results of the analysis of the data collected at the lake intermittently over a period of eight months are given. Use of diffusivities during periods of convective mixing is shown to be hazardous because of the nebulous definition of the temperature gradient under these conditions. Included are computations showing the magnitudes and variability of the diffusivities over both short and long time intervals and at various depths; however, the extreme

quiescence of the experimental lake makes extrapolation to other conditions difficult. Suggestions for future research and guidelines for necessary data acquisition are provided.

CHAPTER II  
TURBULENT TRANSPORT CONCEPTS

Introduction

The general direction of this study is presented in this chapter. Included is a discussion of the various types of studies in which turbulent transport has been modeled using solutions to fundamental equations. After a development of the general equation and accompanying theory for defining transport due to turbulent diffusion, the presentation is narrowed to consideration of the transport of a specific quantity, heat, in lakes. Summaries of causes of lake motion and schemes for evaluating turbulent diffusivities are given along with a review of earlier efforts to evaluate the diffusivities. Specific problems encountered in these previous efforts are recounted. The latter portions of the chapter present arguments for accepting the usefulness of semiempirical descriptions of turbulent transport. The final section discusses the efforts of this study to support those arguments.

Transport in Receiving Waters

Knowing the distribution of substances or heat in a water body can be of great importance. The concentrations of various water quality parameters throughout a region of the water body may be due to a nearby municipal or industrial discharge or due to normal physical conditions

which cause spatial variations. Regardless, it is often necessary from a management point of view to be able to know or predict the levels which exist or might exist at various locations and times. Historically, much attention has been given to making such determinations, and in most instances use has been made of hydrodynamic relationships which describe the transport of substances in the water in terms of the dynamic and physical characteristics of the liquid and the substances being transported. Glover (1964) described the dispersion of sediment in a flowing stream using solutions to the convective diffusion equations for turbulent flow. The same type of mathematical approach has been used to predict distributions of nutrients in lakes and reservoirs (Chen, 1970), and Munk (1966) studied the vertical variations of temperature and salinity in the oceans. Studies of oxygen content in natural waters have been concerned mostly with reaeration processes in rivers and streams, and vertical variations in oxygen levels have been disregarded or found to be negligible in all but a few studies. The work of Eloubaidy and Plate (1972) is one exception. Numerous investigations have been directed toward finding specific point concentrations of various pollutants which have been introduced into receiving waterways. Attention has been given to estuaries (Pyatt, 1964), lakes (Liggett and Lee, 1971), rivers (Cleary and Adrian, 1973), and reservoirs (Morris and Thackston, 1969).

In addition to studies of substance concentration, a myriad of attention has been given to spatial and temporal distributions of heat in natural and impounded waters. The study by Dake and Harleman (1966) focused on the temperature distributions in lakes and the work of Huber (Huber and Harleman, 1968) involved predicting temperatures in reservoirs.

In most of these efforts the essential approach was to use advective diffusive principles of transport which were modified and in most cases, simplified to accommodate the particular situation. Some of the works cited above adopt and use mathematical expressions which include prior assumptions regarding the behavior of turbulent diffusion terms without calling attention to the significance of those assumptions. Others of the works take care to stress the acceptance of certain assumptions and nonetheless are forced to accept the inadequacies they offer. While for a given analysis any of the components of the transport equations may present evaluation problems, the diffusive terms are invariably a source of uncertainty and difficulty.

Further discussion of the use of the turbulent advective-diffusion equation should be preceded by a development and presentation of the equation itself. The transport of mass, momentum, or heat in a water body can be described mathematically by considering the fluxes of any of these quantities into and out of an elemental volume of water. The ensuing derivation may be found in any of several published works on transport processes and is offered here for the sake of completeness.

#### Advective-Diffusion Equation

Consider the elemental volume shown in Figure 2-1. Through each face of the volume passes a quantity of a substance of concentration,  $s$ . If the effects of molecular diffusion are assumed negligible relative to the effects of turbulence, the net flux of material into the volume in the vertical direction, for instance, is

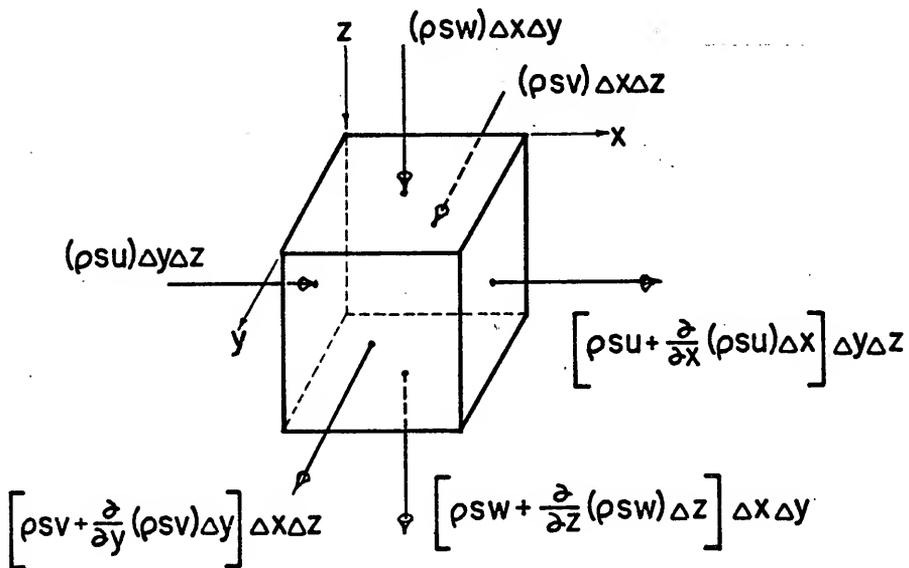


FIGURE 2-1. FLUXES OF SUBSTANCE OF CONCENTRATIONS THROUGH THE FACES OF AN ELEMENTAL VOLUME

$$\begin{aligned} \text{Net flux (z)} &= (\rho s w) \Delta x \Delta y + \left[ \rho s w - \frac{\partial}{\partial z}(\rho s w) \Delta z \right] \Delta x \Delta y \\ &+ r_s \Delta x \Delta y \Delta z \end{aligned} \quad (2-1)$$

where  $\rho$  = density of water (mass - volume<sup>-1</sup>)  
 $s$  = concentration (mass substance - mass solution<sup>-1</sup>)  
 $w$  = instantaneous vertical water velocity  
(distance - time<sup>-1</sup>)  
 $x, y, z$  = orthogonal distances  
 $r_s$  = net rate of production of  $s$  inside the volume  
(mass substance - volume<sup>-1</sup> - time<sup>-1</sup>)

After simplifying, equation 2-1 becomes

$$\text{Net flux (z)} = - \frac{\partial}{\partial z}(\rho s w) \Delta V + r_s \Delta V \quad (2-2)$$

where  $\Delta V$  = element volume =  $\Delta x \Delta y \Delta z$

Given that the velocities in the  $x$  and  $y$  directions are  $u$  and  $v$ , respectively, the fluxes in these directions can be similarly expressed so that

$$\text{Total flux} = \left[ - \frac{\partial}{\partial z}(\rho s w) - \frac{\partial}{\partial x}(\rho s u) - \frac{\partial}{\partial y}(\rho s v) \right] \Delta V + r_s \Delta V \quad (2-3)$$

Now, the total flux must also equal the time rate of change of mass of substance in the volume, therefore

$$\text{Total flux} = \frac{\partial}{\partial t}(\rho s \Delta V) \quad (2-4)$$

where  $t$  = time

For incompressible fluids changes in density and volume are negligible so that when equations 2-3 and 2-4 are equated, the result is

$$\frac{\partial(s)}{\partial t} = - \frac{\partial}{\partial z}(s w) - \frac{\partial}{\partial x}(s u) - \frac{\partial}{\partial y}(s v) + \frac{r_s}{\rho} \quad (2-5)$$

The instantaneous values can be expressed in terms of time-averaged means plus an instantaneous fluctuations from the mean, as

$$s = \bar{s} + s', \quad w = \bar{w} + w', \quad u = \bar{u} + u', \quad v = \bar{v} + v' \quad (2-6)$$

where the overbar denotes mean values and the prime denotes fluctuation values. Average values over a time interval T are defined by

$$\bar{A} = \frac{1}{T} \int_t^{t+T} A dt \quad (2-7)$$

The total transport over the time interval is found by averaging equation 2-5 over the interval, which results in the time-average of each term in that expression. Taking the vertical flux term for example and recalling the meaning of the overbar

$$\overline{\frac{\partial}{\partial z}(sw)} = \overline{\frac{\partial}{\partial z}(\bar{s} + s')(\bar{w} + w')} = \overline{\frac{\partial}{\partial z}(\bar{s}\bar{w} + s'w' + s'\bar{w} + \bar{s}w')} \quad (2-8)$$

Because the coordinate is not a function of time, the averaging can be performed on the quantities before differentiation. Also, equation 2-7 implies that the averaging may be done on each term separately, and furthermore, that  $s'\bar{w} = \bar{s}w' = 0$  and  $\overline{sw} = \bar{s}\bar{w}$ . Therefore, equation 2-8 becomes

$$\overline{\frac{\partial}{\partial z}(sw)} = \frac{\partial}{\partial z}(\bar{s}\bar{w} + \overline{s'w'}) = \frac{\partial}{\partial z}(\bar{s}\bar{w}) + \frac{\partial}{\partial z}(\overline{s'w'}) \quad (2-9)$$

Notice that the term involving the product of two fluctuations does not vanish. This is because a positive vertical velocity fluctuation must transport a small amount of substance downward. Depending on the local gradient of the substance, a small increase or decrease of substance concentration will occur, and hence, a positive or negative fluctuation of s will be detected. A reverse, upward velocity fluctuation would cause

a reverse substance fluctuation provided the average gradient were the same. Therefore, the product of the two fluctuations will always have the same algebraic sign regardless of the direction of water motion and unless there is no concentration gradient will always be nonzero. The flux represented by the term  $\overline{s'w'}$  is known as the turbulent diffusive flux. It is so described because it accounts for the transport due to the irregular variations from mean values.

Applying a similar averaging process to other terms in equation 2-5 using equation 2-6 yields

$$\begin{aligned} \frac{\partial(\bar{s})}{\partial t} = & -\frac{\partial}{\partial z}(\overline{sw}) - \frac{\partial}{\partial z}(\overline{s'w'}) - \frac{\partial}{\partial x}(\overline{su}) - \frac{\partial}{\partial x}(\overline{s'u'}) \\ & - \frac{\partial}{\partial y}(\overline{sv}) - \frac{\partial}{\partial y}(\overline{s'v'}) \end{aligned} \quad (2-10)$$

Substituting the quantities of equation 2-6 into the continuity equation and time averaging produces the continuity equation for turbulent flow as

$$\frac{\partial(\bar{u})}{\partial x} + \frac{\partial(\bar{v})}{\partial y} + \frac{\partial(\bar{w})}{\partial z} = 0 \quad (2-11)$$

By substituting this into equation 2-10, the result is

$$\begin{aligned} \frac{\partial(\bar{s})}{\partial t} = & -\bar{w}\frac{\partial(\bar{s})}{\partial z} - \bar{u}\frac{\partial(\bar{s})}{\partial x} - \bar{v}\frac{\partial(\bar{s})}{\partial y} - \frac{\partial}{\partial z}(\overline{s'w'}) \\ & - \frac{\partial}{\partial x}(\overline{s'u'}) - \frac{\partial}{\partial y}(\overline{s'v'}) + \frac{r}{\rho} \end{aligned} \quad (2-12)$$

which is the advective-diffusion equation for turbulent flow. It states that the time rate of change of average concentration over some interval at a point is due to a flux of material advected by average velocities and diffused by fluctuations of velocity in each of these directions plus any production of the substance which occurs during the interval.

Introduction of Diffusion Coefficients

As noted above the terms containing the time average of the product of fluctuation quantities accounts for transport due to turbulent diffusion and are called the turbulent fluxes. The usefulness of equation 2-12 is limited because the fluctuations are difficult to evaluate; however, conventionally the flux terms are replaced by the relationship

$$\begin{aligned}\overline{s'w'} &= -K_{sz}(\partial\bar{s}/\partial z) \\ \overline{s'u'} &= -K_{sx}(\partial\bar{s}/\partial x) \\ \overline{s'v'} &= -K_{sy}(\partial\bar{s}/\partial y)\end{aligned}\tag{2-13}$$

where the K values are known as coefficients of turbulent diffusion or turbulent diffusivities. Generally,  $K_{sz} \neq K_{sx} \neq K_{sy}$ , and they will only be equal in the special case of isotropic turbulence. Analogous diffusivities exist for the cases of heat, mass, or momentum transport, and they are identified accordingly. By relating the turbulent flux to mean concentration (or velocity or temperature) gradient, the solution of equation 2-12 is facilitated; yet the theoretical basis for such action is questionable. This should be restated more specifically: to substitute equation 2-13 into equation 2-12 does not inherently pose any theoretical questions because the unknown fluctuation product is merely replaced by a different unknown K, but at the same time no progress is made toward improving the usefulness of the equation. If on the other hand, the substitution is made and some specific assumption is made regarding the variation of diffusivity with coordinate distance (e.g.,  $K = \text{constant}$ ), then the theoretical structure becomes much less rigorous.

The need for more useful forms of equation 2-12 have stimulated and perpetuated the existence of, the less rigorous use of equation 2-13. Originally, Boussinesq developed the coefficient of eddy viscosity to express turbulent momentum transport as an analogy to the Fickian equation for molecular transport (p. 25, Hinze, 1959); however, the theoretical basis of the molecular process does not exist for the turbulent process because the nature of the turbulence is a property of the flow field and not the transported quantity. A semiempirical basis for the form of equation 2-13 was presented by Prandtl as his mixing length theory (p. 277, Hinze, 1959). A review of his salient points will be presented at this time.

The one-dimensional transport in the x direction of a substance of concentration s taking place in the presence of a gradient of average concentration  $\bar{s}$  is shown in Figure 2-2. The transport through a y - z plane at x = 0 does not result from the effects of the gradient but from the effects of turbulence. During a time interval, t, any elemental volume of fluid may pass through the plane of area  $\Delta y \Delta z$  but the average distance a volume will travel is  $h_t$  before the initial character of the volume is lost. The average rate of transport in the positive x direction is given by

$$F_s = \frac{1}{\Delta y \Delta z} \frac{1}{t} \left[ \int_{\Delta y} \int_{\Delta z} \int_{-x_t}^0 \rho s dx dz dy - \int_{\Delta y} \int_{\Delta z} \int_0^{x_t} \rho s dx dz dy \right] \quad (2-14)$$

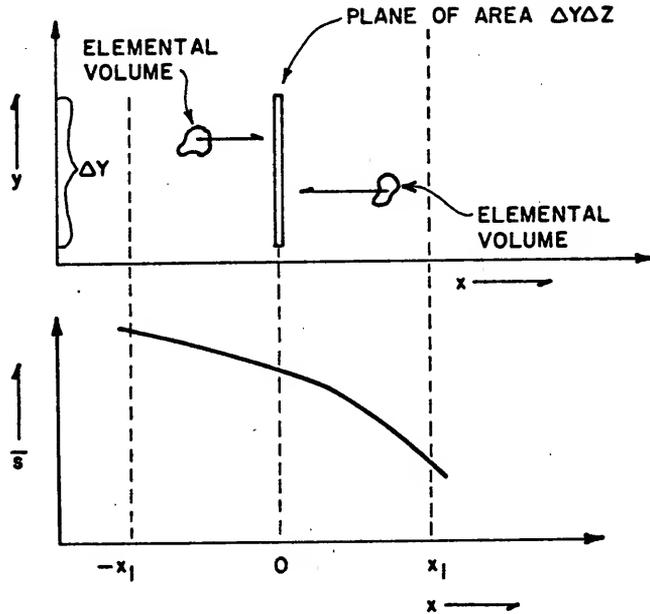


FIGURE 2-2. TURBULENT TRANSPORT OF SUBSTANCE THROUGH A X-Y PLANE

Expanding the concentration in a Taylor series about  $x = 0$  gives

$$s(x) = s(0) + x \left( \frac{\partial(s)}{\partial x} \right)_{x=0} + 1/2x^2 \left( \frac{\partial^2(s)}{\partial x^2} \right)_{x=0} + \dots \quad (2-15)$$

Prandtl assumed the distance  $x$  to be small enough to cause the higher order terms to be negligible, so after dropping all non-linear terms, and integrating equation 2-14

$$F_s = - \frac{1}{t} \rho x_l^2 \left( \frac{\partial(s)}{\partial x} \right)_{x=0} \quad (2-16)$$

By substituting

$$K = \frac{x_l^2}{t} \quad (2-17)$$

into equation 2-16, the resultant direct proportionality between flux of substance through a plane at  $x = 0$  to the gradient at  $x = 0$  is realized, and the similarity to equation 2-13 is apparent.

Other semiempirical formulations have been offered. Two of the most notable of these are by Taylor and von Karman and are discussed by Monin and Yaglom (1971) who note that attempts to find improved forms of these relationships were in progress as recently as the time of their writing. They also indicate the suspect nature of semiempirical attempts at simplifying transport equations and offer the possibility that radical new approaches to the question may provide more nearly correct solutions. On the other hand, the statistical solutions to transport problems are usually very unwieldy, and the concept of diffusion coefficients continues to be considered. This is supported by Okubo (1962) who notes the usefulness of the semiempirical approach in relation to the more theoretically supported turbulence doctrines.

The theoretical concepts of turbulent transport summarized by equation 2-12 and the semiempirical concepts of turbulent diffusion given by equation 2-13 have been presented and discussed. These equations form the basis for the development of methods for predicting the spatial and temporal distribution of concentrations of substance in water bodies. Having established this basis, it is now possible to direct attention toward specific areas of application. In particular, the transport of heat by turbulent diffusion in the vertical direction in lakes will be considered.

#### Heat Transport in Lakes

The focus of this study is on the transport of heat in natural lakes. The thermal structure of these water bodies may be studied using the turbulent transport concepts presented in the preceding discussions. As indicated then, the conveyance of heat is analogous to the conveyance of substance. To adapt equation 2-12 for use in the study of thermal advection and diffusion it is only necessary to substitute the scalar quantity, heat concentration, for the scalar, substance concentration. Also, analogous expressions to those of equation 2-13 may be written. The resultant equations will contain the turbulent diffusivities of heat in each of the three directions. It should be noted that since temperature is the physical indicator of heat content, it is the quantity most often used in equations. The substitution is a simple one because the relationship between heat and temperature is given by

$$H = c\theta$$

(2-18)

where  $H$  = heat concentration (energy - mass solution<sup>-1</sup>)  
 $c$  = specific heat of water (energy - mass solution<sup>-1</sup> -  
 temperature degree<sup>-1</sup>)  
 $\theta$  = temperature

The value of  $c$  varies only slightly over ambient temperatures and is considered constant. Its value is one calorie per gram mass per degree centigrade. Care must be taken to express the production term in the proper units depending on the transport quantity used.

The use of the equation to describe heat transport in lakes is presented after a discussion of the factors affecting motion in lakes is given.

#### Motion in Lakes

Water movement in natural lakes is nearly always turbulent (Hutchinson, 1957). The turbulent and advective transport in lakes may be caused by several factors. In larger lakes the dominant factor is induced movement from wind. Energy is transferred from the air to water by the shear at the interface. Surface currents caused by wind action transport water to the leeward shore where it piles up, and soon a return current begins to flow due to the tilted water surface. The extent of the motion varies according to individual situations. In the case of stratified lakes there may exist additional induced currents at interfaces between layers of different densities. The shear profile existing at the surface will increase the levels of turbulence in the water and thereby contribute to increased vertical transport. Also, the formation of surface waves adds to the turbulent character of the near surface zone. The

degree of water movement caused is directly related to the magnitude of wind velocity at the water surface and the length of fetch; therefore, the surface area and surrounding topography are significant factors as regards wind effects.

Another factor affecting lake motion is convective mixing. Surface heat exchange at various times may cause the water to cool and become more dense than water below. The gravitational instability will soon result in the sinking of the cooler water to a level of equal density. The downward displacement necessarily causes an upward movement of other water parcels, and if the surface cooling continues, pronounced mixing currents will form. In most lower latitudes where significant solar heating occurs during daytime, the convective process will occur nightly to some extent all year. In other climates this phenomenon may happen on a diurnal basis during only brief periods, and other lakes may only experience significant convection currents on a seasonal basis. The converse is not nearly so common, yet the situation does exist in which bottom waters are warmed by sediments, especially in shallow regions, causing convective mixing.

Seiches are another type of lake motion, however, because seiches are caused by the relaxation of a wind-tilted water surface or atmospheric pressure variations, they are phenomena existing in large lakes only. Internal seiches or waves are similarly stimulated and likewise are encountered only on large water bodies.

Most lakes exhibit some density change with depth during most or all of the year. The result is a dampening of vertical motion relative to horizontal motion. The effects of stratification on the vertical structure

of lakes and other water bodies are usually considered in terms of the Richardson number

$$R_i = \frac{g}{\rho} \frac{\frac{\partial \rho}{\partial z}}{\frac{\partial \bar{U}}{\partial z}^2} \quad (2-19)$$

where  $\bar{U}$  is average horizontal velocity and the other terms are as previously used. The number represents the ratio of the rate of increase in potential energy due to buoyancy to the rate of turbulence generation by energy transfer from the mean flow. It is a measure of the stability of the lakes. Higher values of  $R_i$  indicate that vertical mixing is being inhibited because the energy necessary to overcome stratification is not available from the mean flow. Use of the Richardson number for evaluating turbulent motion in lakes is difficult because of problems of defining vertical change in average velocity, and in smaller lakes which are characterized by very low velocities, only crude evaluations can be made which are not very useful.

Although the Richardson number has been used in some cases of atmospheric turbulence to predict the behavior of vertical diffusivities, the most workable approach in lakes is to consider the concepts of turbulent diffusion presented in equations 2-12 and 2-13. From these expressions, methods may be developed which will yield values for the diffusivities without the need to know velocity profiles.

#### Evaluation of Diffusivities

Various methods may be used to define the behavior and magnitudes of the diffusivities of heat to be used in the transport equations. One such approach is to assume the diffusion coefficients are constant which permits

an exact solution to the transport expressions. Measured data can then be used to find the values of the K's that produce the optimum agreement with data. The insufficiencies of this method are apparent, since observations clearly indicate that the levels of vertical turbulent mixing vary with depth. Dutton and Bryson (1962) used this method to describe the vertical temperature structure of Lake Mendota. When their constant value of diffusivity failed to produce satisfactory results, they divided the lake into two layers roughly corresponding to the epilimnion and hypolimnion and calculated separate diffusion coefficients.

A second and somewhat similar method involves assuming an alternative manner of variation of the diffusivity with depth (e.g., exponentially decreasing). This also permits a solution of the transport equation, either exact or numerical, to be used in conjunction with measured data in an effort to determine magnitudes of coefficients by fitting predicted values to measured ones. McEwen's method (p. 468, Hutchinson) is of this type and has been used extensively to evaluate hypolimnic diffusivities. A more detailed discussion of this approach will be given in Chapter III.

Another method of diffusivity evaluation entails finding the quotient of the heat flux and the heat gradient at a particular depth. This may be accomplished in different ways. For instance, the expression for vertical heat flux analogous to equation 2-13 is

$$(\overline{w'\theta'})_d = -K_d(\partial\theta/\partial z)_d \quad (2-20)$$

where  $\theta$  = water temperature  
 $K_d$  = diffusivity of heat  
 $d$  = depth  
 $z$  = vertical distance

Therefore, if the product of the fluctuations can be determined and the gradient of temperature also determined, then the diffusivity of heat can be found. This method may be labeled a first principles type of evaluation because the flux is considered in terms of the turbulent water motion actually effecting the heat transport.

Another option is to use the one dimensional form of the advective-diffusion equation for heat transport into which equation 2-20 has been substituted. Care must be taken to include proper production terms. By integrating the equation over the vertical distance between depth,  $d$ , and the bottom, an expression results which may be used to evaluate diffusivity at  $d$  over a specific time interval using vertical temperature data taken over the interval. The principle is to assess the turbulent phenomena in the water by ascertaining the amount of transport of heat they caused. This method is often referred to as the flux-gradient method and will be discussed further in Chapter III.

Finally, it may be noted that diffusivities may be evaluated by monitoring the variations in content of any of a number of other substances in the water body and then applying an analysis procedure similar to the one just discussed. The diffusivities of substance and of heat may then be related by the fact that they are approximately equal (Hinze, 1959). Accordingly, the diffusivities for substance transport may be obtained from studies of thermal transport. That the two diffusivities are nearly equal can be seen by comparing the turbulent Schmidt and turbulent Prandtl numbers. The turbulent Schmidt number is the ratio of the diffusivity of momentum (eddy viscosity) to the diffusivity of mass substance, and the turbulent Prandtl number is the ratio of eddy viscosity to the diffusivity

of heat. Empirically, the two numbers have been evaluated, and both have been found to have values of about 0.7. Therefore, the diffusivities of mass and heat are known to be approximately equal.

#### Results of Previous Studies

The evaluation of diffusivities has been the goal of many research efforts because as already discussed the values are necessary to the application of the advective-diffusion equation for predictions of transport. In most of these studies, heat or some substance, either introduced into the water or naturally occurring, was used as a traceable quantity. The volume of these various investigations prevents a comprehensive review of them here. Most deal with horizontal transport, and many tracer studies regardless of title are in fact efforts in dispersion measurement. As far as finding reported values of vertical diffusivities as computed from considerations of the conservation of either mass or heat, the number of reports is meager. Bowden (1964) presents results of deep sea studies which include values of from 1 to 2000 square centimeters per second (9 to 17000 square meters per day) and show much variability with depth. Ryan and Harleman (1973) compile results of similar studies in lakes and reservoirs. They list several cases with values ranging from 0.3 to 13 square feet per day (0.03 to 1.17 square meters per day) which do not apply to epilimnic regions. Morris and Thackston (1969) used dye as a tracer in reservoir studies and found that the vertical diffusion coefficients varied with time and depth. They obtained values of from 0.5 to 6.3 square meters per day. In a study of the effects of thermal discharges on receiving waters Sundaram et al. (1969) used both the conservation of energy and McEwen's

method and found values in the lower lake strata of 18 square feet per day (1.6 square meters per day). Morris and Thackston (1969) summarize the results of several studies performed by Orlob and Selna (e.g., Orlob and Selna, 1970) who used a modification of the conservation of heat approach. The values reported are grouped according to depth and vary from 0.81 to 9.2 square centimeters per second (7 to 80 square meters per day) at the surface, 0.02 to 0.17 square centimeters per second (0.2 to 1.4 square meters per day) at the thermocline, and 1.2 to 1.7 square centimeters per day (10 to 15 square meters per day) at selected hypolimnic locations. The values reported vary greatly. Part of the reason for this variation is that the values apply to different lake depths. Epilimnic values are much higher than values in lower strata. Also, it is interesting to note that despite the smaller hypolimnic values reported, all of the diffusivities are at least two orders of magnitude greater than the molecular diffusivity of heat in water, 0.012 square meters per day.

Regarding investigations of vertical turbulent transport by measuring velocity fluctuations, the comment by Wiseman (1969, p. 8) is apropos:

In the existent literature one finds not only a lack of turbulent spectra, but especially a dearth of information concerning vertical motions in natural bodies of water.

Bowden and Fairbairn (1956) used a custom-made electromagnetic flow meter, Wiseman (1969) used a custom-made doppler-shift meter, and Grant et al. (1962) used a hot-film anemometer to measure shear stress in estuaries under conditions of brisk currents; however, they reported only momentum fluxes and no diffusivities.

### The Problem of Convective Mixing

Earlier a discussion of various phenomena causing turbulent motion in lakes included mixing due to convective cooling currents. In addition to wind these currents account for much of the motion in the epilimnion, and nearly always, this near-surface movement is far greater in magnitude than the movement occurring below the thermocline in stratified waters. The lack of knowledge regarding the behavior and vertical variations of turbulent transport mechanisms has caused much uncertainty in studies of predicting lake temperatures. Orlob and Selna (1970) developed a methodology for matching observed and predicted temperatures in reservoirs by using values of effective vertical diffusivities of heat substituted into a predictive model based on conservation of heat principles. The usefulness of their model has been questioned because the diffusivities were originally calculated from the observed data using an inverse solution of the model. This type of complication prompted Dake and Harleman (1966) and later Huber and Harleman (1968) to describe the thermal transport in lakes and reservoirs alternatively. They have reasoned that the stability conditions existing in lakes and reservoirs due to density gradients are strong enough to inhibit virtually all turbulent transport except for the regions influenced by convective mixing. Their methodology entailed specifying a very small, constant value of diffusivity throughout the lower strata. In some cases, the value of molecular diffusivity of heat (0.012 square meters per day) was used. The upper strata were characterized as completely mixed and isothermal. Their procedure for defining the single surficial temperature and the depth to which it extended used thermal energy budgets to predict a temperature profile. Whenever an unstable near-surface profile was

predicted, a unique average temperature was calculated which indicated an amount of energy equivalent to the predicted condition over a particular depth. The depth was specifically determined by the depth existing in the stable portion of the predicted profile at which the unique mixed temperature occurred.

The results obtained by comparing predicted temperatures to observed values were good in both studies, and the method was deemed satisfactory. However, Huber's results mainly showed that vertical advection was dominant in the reservoir, and this condition is not applicable to most lakes. In the lake study by Dake, much of the success of the model application depends on the assessment of the vertical distribution of absorbed solar radiation. This is a highly variable phenomenon as Dake points out and may affect significantly the predictive performance of the model when applied to various lakes, especially ones with high or variable turbidities.

These authors as well as others (Sundaram et al., 1969 and Powell and Jassby, 1974) have all questioned the ability of the concept of turbulent diffusivities to adequately describe the process of convective mixing. Although not stated explicitly in any of these investigations, much of the problem with describing convective cooling using turbulent transport equations is as follows. The normal annual thermal cycle of lakes is reviewed by Hutchinson (1957), among others. When climatic conditions cause more heat to leave the water than enters, the lake cools. This process is usually a diurnal one wherein most of the cooling occurs at night and most of the heating is due to daytime solar radiation. If during the cooling portion of the cycle, the thermal profile of the lake is examined over a period of days or weeks, the loss of heat is apparent. Yet, in many

cases the shapes of the profiles observed will be very similar, and temperature structure will be seemingly stable. This is very likely to happen if, for instance, noontime temperature measurements are made days or weeks apart. Such behavior cannot be readily explained by conservation of heat methods because the loss of heat observed is incompatible with the stable gradients (i.e., a loss of heat indicating surface cooling and profile slope indicating a downward heat transport). Two possibilities exist. Either the semiempirical approach to turbulent transport problems is unsatisfactory or the method of application is in error. The process of convective mixing seems to have a specific length scale associated with it. The Dake and Huber models both compute a characteristic depth over which the mixing extends. Furthermore, the nature of the process suggests the formation of eddy-type currents. It, therefore, seems appropriate to describe convective mixing in the semiempirical manner. On the other hand, there may be some basis for assuming that the usual methods of applying conservation of heat approaches to conditions of mixing are in error. This stems from the fact that the mixing is a diurnal process and cannot be considered properly without giving attention to this fact. Specifically, the aspect of the length of the time interval used to make calculations must be considered because there may be marked influences of this interval on the values of diffusivities calculated. This line of thought will be developed in the following section.

#### Averaging

Equation 2-7 indicated the averaging method used to determine mean quantities; nevertheless, the subject of proper averaging requires some

additional commentary. Although equation 2-7 was presented without discussion, certain facts are implied by its form. The equation states that the average value of a quantity, say concentration, in a turbulent flow is found by time averaging over some interval  $T$ . This is true only as a result of assuming ergodic behavior of the quantity. Ergodicity implies that the mean value of any individual sample taken over a finite time interval will be approximately the value obtained by taking an ensemble average (i.e., the average of different samples taken at the same time). Such behavior is in certain cases strictly provable, and in other cases, safely hypothesized (i.e., the ergodic hypothesis) (Monin and Yaglom, 1971).

In addition to this question of theoretical propriety is the practical matter of what averaging intervals to use. Generally the guidelines for selecting the proper interval entail using a time span which is somewhat longer than the largest frequencies of turbulent motion being studied and short enough so that the mean does not vary during it. Conceptually these requirements present no problems; yet in practice they may. In studies of large scale, naturally occurring motions, it may be much harder to establish which are the largest scales of motion existing than it is to do so in a laboratory study. Large eddies with frequencies lower than  $2\pi/T$  will be excluded from analysis using  $T$  as the averaging time (Okubo, 1962). Furthermore, as Bowden (1964) notes, the method of getting mean values depends on the scale of movement occurring and also the particular aspect of that movement being considered. Conversely, selecting and applying a certain averaging interval to a set of measurements may have an effect on the nature and quality of results obtained, which is the premise presented at the conclusion of the preceding section.

The emphasis of this discussion so far has related to averaging procedures for records of fluctuating turbulent quantities versus time; however, certain parallels may be drawn between computation of diffusivities by the first principles approach and by integration of the conservation of heat (flux-gradient) methods. In particular, if the averaging interval used on a time record of a quantity is long relative to a specific period of fluctuation of the quantity, then the effects of the shorter scale variations are diminished. If the magnitude of the shorter scale fluctuations is less than that of the longer scale variation, the effects may be diminished to the point of not being detected at all. Satisfactory description of the transport by calculating and using diffusivities depends greatly on using an averaging interval pertinent to the turbulent motion. As regards diurnally variable convective mixing, the use of a one-day averaging time to determine diffusivities by equation 2-13 will yield the same inappropriate results as would be found using the flux-gradient method calculated over a twenty-four-hour period. For example, should the turbulent motion in a lake over the period of one day be caused exclusively by convective mixing, then the velocities occurring in the lake would fluctuate (about a zero mean) during the period of cooling. During the remainder of the day no velocity would exist. Now the temperature in the lake at a depth affected by the convective currents would also fluctuate during the mixing period and coincidentally cool. Considering the daily period, the cooling during the mixing and heating during the day (regardless of periods of no change should they exist) would produce a daily mean temperature. If the daily mean of velocity (zero) were used, even the convective velocities, regardless of size, would be calculated as

instantaneous fluctuations. On the contrary, if the daily mean of temperature were used, the entire period of cooling would be represented by only negative fluctuations. The results of the subsequent cross-correlating (i.e.,  $\overline{w'\theta'}$ ) over the averaging period or even a portion of the day, would not reflect the shorter scale convective process suitably. Analogously, using a daily time step to calculate diffusivities in the lake by the flux-gradient approach would not reflect the shorter scale convective process suitably. Therefore, using a daily time step to calculate diffusivities in the lake by the flux-gradient approach would yield poor results due to the insensitivity of the method to the shorter scale motions.

#### Goals of This Study

The preceding discussion of averaging requirements tends to support the suggestion that the inability of the semiempirical theory to satisfactorily describe convective mixing is due to improper use of the method. It is a goal of this research to determine whether or not the transport in a lake can be adequately described by the semiempirical theory of turbulent transport. Emphasis will be given to the convective mixing phenomenon. Specifically, the vertical transport of heat will be monitored, and from it the vertical diffusivities of heat will be determined using the flux-gradient method. At the same time, the fundamental soundness of the semiempirical theory will also be tested using the more difficult but more direct first-principles approach of measuring the turbulent water motion in the lake. As indicated by equation 2-20 time-averaging the product of fluctuations of vertical water velocity and temperature at a given depth will

produce the heat flux term needed to find  $K$ . The gradient term at the same depth can be obtained from the flux-gradient study.

Additionally, the computation of the diffusivities by two different methods could yield some worthwhile insight into the relative merits or deficiencies of each. Furthermore the study will seek to examine the behavior of the diffusivities as a function of various vertical positions in the lake. Not only will diurnal variations of the diffusion coefficients be evaluated as the convective mixing process is studied, but also longer term variations which may occur will be documented and analyzed.

A final objective involves the effort to establish to what extent the vertical diffusivity information obtained can be related to the environmental conditions which exist coincidentally. Such relationships might prove most beneficial to the application of transport theory to practical problems, especially problems of predicting pollutant transport. The ascertainment of the diffusivities by separate methods will provide the many values of diffusivities necessary to accomplish this goal.

CHAPTER III  
METHODOLOGY OF DIFFUSIVITY EVALUATION

Introduction

The overall goals of this research were stated in Chapter II. Specific use is to be made of the flux-gradient method and the correlation of fluctuations method to evaluate diffusivities of heat in a lake. To perform the planned analysis, values of diffusivities are needed at various depths, several times per day, and over a span of several months. This indicates that a large volume of data must be gathered and subsequently analyzed.

This chapter presents the conceptual basis for the flux-gradient method which is followed by discussions pertaining to the use of the concepts. Included in the description is a scheme for approximating the measured temperature profiles with analytical expressions so that the requisite calculations for the flux-gradient analysis need not be performed manually. The feasibility of diffusivity computation by this approach is determined by whether or not such a scheme is workable. Therefore, an account of a trial of the methodology on Cayuga Lake data is given. The result of the test is that the procedures developed work well, and that when applied to data gathered for this study, they should enable many accurate computations of diffusion coefficients using a digital computer.

Following the discussion of the flux-gradient method is a presentation of the correlation of fluctuations method for diffusivity evaluation. The

measurement of water velocities is essential to this approach and the entire following chapter is devoted to the subject of making such measurements.

### Flux-Gradient Method

The basis for the flux-gradient method is the one-dimensional form of the advective-diffusion equation for the turbulent transport of heat. This expression may be obtained by first modifying equation 2-12 to describe the transport of heat using equation 2-18 to express heat concentration as temperature, then substituting equation 2-20 so that the turbulent fluxes are represented as the product of diffusion coefficients and gradients, and finally by amending the three-dimensional expression to a one-dimensional form. This is done by assuming the lake is horizontally homogeneous which is valid if the isotherms in the lake are horizontal. This condition is often not met in large lakes which may be affected by seiches; however, for small lakes studies (e.g., Smith and Bella, 1973) this has been found to be a reasonable assumption. Also the production term must be included to account for the absorption of incident solar radiation which varies vertically. The result of these manipulations is

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z} \left( -K_d \left( \frac{\partial \theta}{\partial z} \right)_d \right) - \frac{1}{\rho c} \frac{\partial R_d}{\partial z} \quad (3-1)$$

where  $\theta$  = temperature

$t$  = time

$z$  = vertical distance

$K_d$  = vertical turbulent diffusivity of heat at depth  $d$   
(distance<sup>2</sup> - time<sup>-1</sup>)

$\rho$  = density of water (mass - volume<sup>-1</sup>)

$c$  = specific heat of water (energy - mass<sup>-1</sup> -  
degree temperature<sup>-1</sup>)

$R_d$  = flux of absorbed solar radiation at  $d$   
(energy - distance<sup>-2</sup> - time<sup>-1</sup>)

This equation describes in differential form the vertical transport of heat in a water column in which no horizontal transport takes place. Furthermore, the only sources or sinks for energy are boundary fluxes of heat and solar radiation, and turbulent transport is assumed proportional to the gradient of the average heat content.

An explicit expression for the diffusion coefficient is obtained by integrating equation 3-1 from depth  $z_1$  to the bottom depth  $z = h$ ,

$$\int_{z_1}^h \frac{\partial \theta_d}{\partial t} dz = \int_{z_1}^h \frac{\partial}{\partial z} \left( K_d \left( \frac{\partial \bar{\theta}}{\partial z} \right)_d \right) dz - \int_{z_1}^h \frac{1}{\rho c} \frac{\partial R_d}{\partial z} dz \quad (3-2)$$

which gives

$$\int_{z_1}^h \frac{\partial \theta_d}{\partial t} dz = K_d \left( \frac{\partial \bar{\theta}}{\partial z} \right)_d \Big|_{z_1}^h - \frac{R_d}{\rho c} \Big|_{z_1}^h \quad (3-3)$$

Assuming that there is no heat transport or solar flux through the bottom gives,

$$\int_{z_1}^h \frac{\partial \theta_d}{\partial z} dz = K_{z_1} \left( \frac{\partial \bar{\theta}}{\partial z} \right)_{z_1} - \frac{R_{z_1}}{\rho c} \quad (3-4)$$

Solving equation 3-4 for  $K_{z_1}$  yields

$$K_{z_1} = \left[ \frac{R_{z_1}}{\rho c} - \int_{z_1}^h \frac{\partial \theta_d}{\partial t} dz \right] \left/ \left( \frac{\partial \bar{\theta}}{\partial z} \right)_{z_1} \right. \quad (3-5)$$

Because the limits of the integral,  $z_1$  and  $h$ , are not functions of time equation 2-5 may also be written

$$K_{z_1} = \frac{\frac{R_{z_1}}{\rho c} - \frac{\partial}{\partial t} \int_{z_1}^h \theta_d dz}{\left( \frac{\partial \bar{\theta}}{\partial z} \right)_{z_1}} \quad (3-6)$$

The first term in the numerator of the right-hand sides of equations 3-5 and 3-6 represents the energy content due to absorbed solar radiation. The other numerator term represents the rate of change, or flux, of the total heat stored in the water column below  $z_1$ , and the denominator is the gradient of the mean concentration of heat at  $z_1$ , hence, the nomenclature: flux-gradient method.

#### Evaluation of Heat Flux and Gradient Terms

The evaluation of the heat flux term in equation 3-6 may be attempted by either analytical, numerical, or graphical methods, or combinations of these. If analytical expressions are known for temperature as a function of depth at two times, then

$$\int_{z_1}^h \theta_d dz$$

may be solved analytically or numerically. Alternatively, plotting  $\theta$  versus  $z$  and determining the area under the curve between  $z_1$  and  $h$  can be done. Analytical or numerical integration is usually easy when digital computers are used, while plotting points, drawing curves, and measuring areas are usually time consuming. However, arriving at satisfactory analytical expressions for  $\theta = f(z)$  may not be feasible, and the graphical approach may be required. Dividing the integral quantity by the time increment,  $\Delta t$ , which is determined by the times the two temperature profiles were measured, yields the flux term.

Another alternative when  $\theta = f(z)$  is not known is to evaluate

$$\int_{z_1}^h (\partial\theta_d/\partial t) dz$$

as is indicated by equation 3-5. Using temperature versus depth data at two times, it is possible to approximate  $\partial\theta_d/\partial t$  at various depths by computing  $\Delta\theta_d/\Delta t$  at depths between  $z_1$  and  $h$ . The flux is computed by evaluating the integral by analytical, numerical, or graphical means. Both of these methods will be described in greater detail below as they are applied to data from Cayuga Lake.

Determining the gradient value  $\partial\bar{\theta}/\partial z$  is more difficult. Whereas knowing the amount of stored heat at the end points of a given time span is sufficient for quantifying heat flux, the gradient of the average temperature over the same time span may or may not be accurately found by considering only the end points. If the variation of the gradient with time is not linear, significant errors may result unless additional temperatures measured during the original time span are used to improve

the accuracy of the averaging process. Computation of the gradient should be made using data taken at periods over which the changes in temperatures are known to be linear, and the likelihood of error should be recognized when a longer time span is used.

Evaluation of the gradient term entails computing the time average of the temperatures at various depths, thereby determining an average profile. If an analytical expression can be found for the average profile, differentiation gives the gradient at any  $z_1$ . Otherwise, plotting the profile and graphically determining the slope at  $z = z_1$  or adopting some numerical technique such as straight line interpolation between data points may be used.

#### Evaluation of Solar Flux Term

Of the solar radiation striking the water surface, the longer wave portion is absorbed at the surface while the shorter wave portion is absorbed within the water body in a manner which decreases exponentially with depth. The degree and extent of subsurface penetration depends on wavelength and the physical and chemical quality of the water. Dake and Harleman (1966) described this twofold behavior with the expression

$$R_z = R_0 (1 - \beta) e^{-\eta z} \quad (3-7)$$

where  $R_z$  = solar flux at depth  $z$  (energy - area<sup>-1</sup> - time<sup>-1</sup>)  
 $R_0$  = solar flux at surface (energy - area<sup>-1</sup> - time<sup>-1</sup>)  
 $\beta$  = fraction of  $R_0$  absorbed at the surface  
 (dimensionless)  
 $\eta$  = extinction coefficient (depth<sup>-1</sup>)

They note that values of  $\beta$  and  $\eta$  vary greatly depending on the water quality and suggest that the coefficients be determined for a given lake using solar absorption data applicable to specific conditions.

The average solar flux over the time span  $\Delta t$  must be used, and as noted earlier regarding the suitability of the use of end-point values to compute averages, the variation of solar flux should be linear during  $\Delta t$ ; otherwise interim calculations should be made.

#### McEwen's Method

McEwen (1929) offered an approach for calculating diffusion coefficients in the hypolimnion of lakes which has received much attention. Hutchinson (1957) presented a description of this scheme which is often called McEwen's Method, and Powell and Jassby (1974) have restated its development while including solar radiation terms which are usually neglected. In many stratified lakes the thermal profile exhibits an exponential form in the region of the lower metalimnion and upper hypolimnion. The metalimnion is the zone of lake depth wherein the temperature variation with depth is greatest; it separates the epilimnion and hypolimnion. This region of exponential temperature decrease is known as the clinolimnion, and McEwen's Method pertains to diffusivities in it.

Let the depths  $z_1$  and  $z_2$  indicate the extent of the clinolimnion. The exponential form of the temperature profile for  $z_1 \leq z \leq z_2$  may be expressed as

$$\theta_d = \beta_1 + \beta_2 e^{-\alpha z} \quad (3-8)$$

where  $\theta_d$  = temperature at depth d (degrees)  
 $\beta_1, \beta_2$  = constants (degrees)  
 $\alpha$  = constant (depth<sup>-1</sup>)

The one-dimensional conservation of energy equation (equation 3-1) derived earlier as the basis for the flux-gradient method also forms the basis for this approach. Assuming the diffusion coefficient to be constant in the clinolimnion, this equation becomes

$$\frac{\partial \theta_d}{\partial t} = K \left( \frac{\partial^2 \theta}{\partial z^2} \right)_d - \frac{1}{\rho c} \frac{\partial R_d}{\partial z} \quad (3-9)$$

where each term is as previously defined except that since K does not vary with depth, the subscript denoting its value at a specific depth is superfluous and is not used. The assumption that K is constant is made so that some value for it may ultimately be found. The validity of the assumption is checked by a parallel line criterion that is discussed below. Differentiating equation 3-8 to get  $\partial^2 \theta_d / \partial z^2$  and substituting into equation 3-9 and then rearranging yields

$$\frac{\partial \theta_d}{\partial t} - \frac{1}{\rho c} \frac{\partial R_d}{\partial z} = K(\beta_2 \alpha^2 e^{-\alpha z}) \quad (3-10)$$

Taking the natural logarithms of equation 3-8 and 3-10, respectively, gives

$$\ln(\theta_d - \beta_1) = \ln \beta_2 - \alpha z \quad (3-11)$$

and

$$\ln \left( \frac{\partial \theta_d}{\partial t} - \frac{1}{\rho c} \frac{\partial R_d}{\partial z} \right) = \ln(K \beta_2 \alpha^2) - \alpha z \quad (3-12)$$

McEwen concluded that if plots of  $\ln(\theta_d - \beta_1)$  versus  $z$  and  $\ln[\partial\theta_d/\partial t - (1/\rho c)\partial R_d/\partial z]$  versus  $z$  produced straight, parallel lines for a given set of temperature data, then the assumption of constant diffusivity was justified, and the value of diffusivity could be obtained by finding the slopes and intercepts of the two plots. Powell and Jassby demonstrated mathematically that this hypothesis was incorrect. They showed that permitting diffusivity to vary with depth produced a more complex expression but one which was still resolvable to a form that gave straight and parallel plots. The result of their analysis was that the diffusivity could have the form

$$K_z = r_1 + r_2 e^{\alpha z} \quad (3-13)$$

where  $r_1, r_2 = \text{constants (distance}^2 - \text{time}^{-1})$

The crux of this argument is that assuming  $K$  constant defines  $r_2$  as zero, which is not generally justifiable.

Powell and Jassby point out that in some lakes using  $r_2$  equal to zero is appropriate. In other lakes this assumption cannot be made and applying McEwen's Method leads to erroneous conclusions because the value of  $r_2$  remains to be evaluated. They also use some values of diffusivities found by the flux-gradient method to lead to values of  $r_2$  and show that about fifty percent of the magnitude of  $K$  is due to the  $r_2 \exp(\alpha z)$  term. Powell and Jassby do not offer a new method of finding diffusivities, but they do show conclusively that in many cases using constant hypolimnetic diffusivities is inappropriate.

Analytical Representation of Temperature Profiles

As already discussed one alternative for evaluating the heat flux and heat gradient terms in equation 3-6 is to integrate and differentiate, respectively, an analytical expression which defines lake temperature as a function of depth. Analytical analysis utilizing digital computers seems preferable to other methods requiring manual operations, especially when there are many diffusivities being calculated. The crux of this approach is to be able to represent the measured temperature profile by an analytical expression because once such an expression is found, integration and differentiation may be performed handily. There are at least two techniques which might be used to approximate measured temperatures analytically: a polynomial fit and a least squares fit. Both are types of statistical regression.

Initially, a polynomial fit of the temperature data was tried. This involved the multiple regression of successive powers of depth,  $z$ , on to temperature,  $T$ , thusly,

$$T' = \sum_{i=1}^n a_i z^{i-1} \quad (3-14)$$

where the prime denotes a predicted value and the  $a$ 's are regression coefficients. Digital computer programs prepared as part of the Scientific Package by the International Business Machines Corporation (Anon., 1968) were used as the basis for the computations. The results of the effort were unsatisfactory because the fits obtained did not approximate the measured values well at all depths. Also, the predicted profiles in some cases assumed erroneous shapes in regions between data points.

Upon the suggestion of Professor Robert Dean (1974), a new approach involving the nonlinear least squares fit of a series of terms was tried. Computer programs included in the UCLA (University of California at Los Angeles) Biomedical Computer Programs Library (Dixon, 1973) served as the basis of the analysis, although some custom modifications were made. A function of the form

$$T' = \sum_{i=1}^n b_i \cos[(i-1)\pi z/h] \quad (3-15)$$

where  $h$  is the depth of the lake bottom, was subjected to least squares analysis to determine the values of  $b_i$  giving the best fit to the measured temperatures. Problems arose because this function yielded zero slopes (i.e.,  $\partial T'/\partial z = 0$ ) at both water surface and bottom, so a modification was made which extended the primary period slightly to permit nonzero slopes at the water surface. This was accomplished by substituting  $z - h'/h'$  for  $z/h$  where  $h' = h + c$  and  $c = \text{constant}$  (meters). The results seemed improved, and further modification was made by making  $c$  one of the regression constants. The effect of this was not helpful because the regression procedure would not converge to satisfactory values of  $c$  consistently.

It was then decided to revert the cosine expression to its initial form and amend the entire function to be fit by adding an exponential term which was weighted such that it only had an effect near the surface. The shape of the temperature profiles suggested an exponential term; furthermore, during periods of surface cooling, the profile shape suggested a skewed function of the general form

$$T' = z^\lambda e^{-z} \quad (3-16)$$

where  $\lambda = \text{constant}$

A more general expression for the upper layer

$$T' = b_1 z^{b_2} e^{-b_3 z} \quad (3-17)$$

was combined with the Fourier-type terms and used as the function to be fit by the least squares scheme. The coefficient  $b_3$  could not be determined consistently by the scheme and was removed as a regression coefficient; however, it was kept as a constant. Its primary effect was to regulate the extent of depth over which the entire term is influential, and tests made using both integer and noninteger values for it indicated that best overall performance of the curve fit was realized when the value six (6) was used. Therefore, the ultimate form of the nonlinear function to be fit by the least squares scheme became

$$T' = b_1 z^{b_2} e^{-6z} + \sum_{i=3}^n b_i \cos[(i-3)\pi z/h] \quad (3-18)$$

Other slight modifications were tried but none noticeably improved the predictive performance of equation 3-18.

Throughout the process of selecting the optimum function to be used, the quality or goodness of fit produced by any one function was evaluated by both analytical and observational criteria. Estimates of error, residuals, and correlation levels were provided as part of the calculation of the regression coefficients; however, these parameters only indicated the relative ability of the particular coefficients to predict temperatures at the data points used in the analysis. Consequently, each set of

coefficients was used to calculate temperatures at short intervals throughout the water column. Therefore, erroneous oscillations and other forms of erratic behavior could be detected. The criteria for satisfactory temperature profile prediction were that the measured temperatures should be approximated to within 0.1 degree centigrade, that no extrinsic behavior exist in the predicted profile, and that the performance of the regression scheme be repeatable and not erratic.

Temperature versus depth data for three days, one each from October, November, and December, 1973, were used to test the various prediction functions. Both daytime and nighttime values were used to insure a variety of near surface profile shapes. The ultimate choice (as given by equation 3-18) yielded results which most closely met the criteria described above. For the October, 1973, profiles the predicted temperatures differed from those observed by as much as 0.2 degrees centigrade at some of the points. The procedure was deemed a qualified success.

#### Cayuga Lake

It was desirable to better evaluate the merits and suitability of using a nonlinear least squares fit of equation 3-18 to analytically define measured temperature profiles. One possibility was to apply the regression procedure and subsequent flux and gradient computations to data from which calculations of diffusivities by the flux gradient method had already been made by manual means. The recent article by Powell and Jassby (1974) appeared to provide such an opportunity. Included in the article is an analysis of data taken at Cayuga Lake,

New York which was originally examined by Sundaram et al. (1969).

Data for periods in 1950 and 1968 were considered.

Although the original data for 1968 were not tabulated by Sundaram et al., they were presented in graphical form (Figure 49a, p. 277). The 1950 data which were tabulated in another publication were averaged and perhaps otherwise manipulated by Sundaram before being used by Sundaram et al. Therefore, only the graphs of lake temperatures for the weeks of August 14-20 and August 21-27, 1968, were used to tabulate temperature profiles to which least squares approximations could be made. It should be noted that the resolution of these plots is poor due to small size and reproducing effects, and that temperatures read from them may disagree with the data used to construct the plots originally. Nevertheless, the value obtained should indicate the ultimate utility, or lack of same, of the procedures devised for this study.

Before a comparison of results can be made, it is necessary to consider in greater detail the alternative methods for calculating flux-gradient quantities. As discussed earlier in this chapter, the flux term may be considered in two different ways as given by equations 3-5 and 3-6, respectively. If the form indicated in equation 3-6 is used, temperature data taken at times  $t_1$  and  $t_2$  are used to approximate  $\partial\theta/\partial t$  by figuring  $\Delta\theta/\Delta t$  ( $\Delta t = t_2 - t_1$ ) at various depths. It is then necessary to integrate this newly created set of data below the depth being considered,  $z_1$ . The form of the flux indicated by equation 3-6 requires that the temperature data taken at  $t_1$  be integrated below  $z$ , and the data for  $t_2$  be treated likewise. The flux is obtained

by dividing the difference between the two integrals by  $\Delta t$ . Either method should yield a proper result; however, one or the other may be preferable when questions of implementation are considered.

Evaluation of the gradient (at  $z_1$ ) may be made by either one of two similar methods, assuming the gradient changes linearly during  $\Delta t$ . If this is the case, then the average of the slopes of the two temperature profiles taken at  $t_1$  and  $t_2$  should yield the desired quantity. Alternatively, the two sets of temperature data may be averaged to form a new set of data the slope of which may be used to provide values of the gradient.

Sundaram et al. calculated diffusivities of heat by both the flux-gradient method and McEwen's Method. The emphasis of his work is on the latter which may explain why the data are presented in a manner which makes flux-gradient calculations difficult. It should in fact be noted that their computations of diffusion coefficients by the flux-gradient method are in error. The text (p. 117) refers the reader to Figure 54 (p. 284) which shows plots of  $\Delta\theta/\Delta t$ ,  $\int(\Delta\theta/\Delta t)dz$ , and  $K$  (diffusivity). The values of  $K$  shown (and also discussed in the text) appear to have been calculated from the quotient of the other two sets of data in the figure, i.e.,  $\int(\Delta\theta/\Delta t)/(\Delta\theta/\Delta t)$ , when in fact  $K$  should be given as the quotient of the quantity  $\int(\Delta\theta/\Delta t)dz$  and  $\partial\theta/\partial z$  (not  $\partial\theta/\partial t$ ). Fortunately, Powell and Jassby independently compute diffusivities using Sundaram's data, and their values appear to be more precise.

As part of the presentation of the results of the use of McEwen's method, plots are given (Figure 56a, p. 286) by Sundaram et al. which are easy to discern and which provide data suitable for flux-gradient

calculations. Powell and Jassby tabulate in their Table 3 (p. 196) these data, which include  $\theta - \beta_1$  (cf., equations 3-8 and 3-11) and  $\Delta\theta/\Delta t$  at 1.52 meter (5 foot) intervals to a depth of 27.4 meters. These data are presented in Table 3-1. Also in Table 3-1 are computed values for other quantities required to calculate flux-gradient diffusivities. Powell and Jassby do not elaborate on their methods of obtaining these other quantities; however, after examining their computed values of diffusivities, it seems that the gradients are obtained for a specific depth by taking differences between values on either side of the point in question. Therefore, gradients calculated in this manner have been included under the heading  $\Delta\theta/\Delta z$  of Table 3-1. For example, the gradient  $\Delta\theta/\Delta z$ , at  $z = 7.62$  meters is given by  $(13.3 - 12.2)$  degrees centigrade divided by  $(6.10 - 9.14)$  meters =  $-0.36$  degrees centigrade per meter. While this method may be employed easily, it may not be accurate; therefore, a representation of the temperature profile as given by  $(\theta - \beta_1)$  was plotted and slopes evaluated by visually constructing tangents to the profile curve at the specified points. This information is listed in Table 3-1 in the column headed  $(\partial\theta/\partial z)_{\text{slope}}$  so that comparisons may be made.

Again, because Powell and Jassby do not describe their methods for evaluating the flux term, it was assumed they used graphical techniques. At any rate the data for  $\partial\theta/\partial t$  were plotted versus  $z$ , and the integrals were evaluated by measuring areas on the plot. The results of this procedure are given in Table 3-1 also. Some question remains about the Powell and Jassby technique because they do not show calculations of diffusivities (see Table 3-3) at those depths where values of  $\Delta\theta/\Delta t$  are

TABLE 3-1. FLUXES AND GRADIENTS IN CAYUGA LAKE 8/14/68 - 8/27/68  
DETERMINED GRAPHICALLY

(1)	(2)	(3)	(4)	(5)	(6)
z m	$\theta_z - \beta_1^*$ °C	$\Delta\theta/\Delta t^*$ °C/day	$\Delta\theta/\Delta z$ °C/m	$\int_z^h (\Delta\theta/\Delta t) dz$ °C-m/day	$(\partial\theta/\partial z)_{\text{slope}}$ °C/m
0.0	14.1	0.366	---	6.96	- 0.67
1.52	13.5	0.293	- 0.10	6.51	- 0.05
3.05	13.8	---	- 0.27	5.96	- 0.21
4.57	12.7	0.352	- 0.17	5.55	- 0.48
6.10	13.3	---	+ 0.10	4.98	+ 0.25
7.62	13.0	0.382	- 0.36	4.36	- 0.34
9.14	12.2	---	- 0.85	3.73	- 0.31
10.70	10.4	0.539	- 0.75	2.96	- 0.62
12.20	9.9	---	- 0.92	2.22	- 0.58
13.70	7.6	0.311	- 1.36	1.65	- 0.93
15.20	5.7	0.259	- 1.02	1.19	- 0.89
16.80	4.5	0.173	- 0.75	0.85	- 0.76
18.30	3.4	0.138	- 0.55	0.62	- 0.65
21.30	2.0	0.075	- 0.42	0.32	- 0.37
22.90	1.5	0.065	- 0.26	0.20	- 0.22
24.40	1.2	0.045	- 0.20	0.19	- 0.17
27.40	0.6	0.025	- 0.19	0.02	- 0.21
30.5**	0.0	0.0	---	0.0	- 0.10

\*From Sundaram et al. (1974, p. 196).

\*\*Values at z = 30.5 meters added by extrapolation.

TABLE 3-2. FLUXES AND GRADIENTS IN CAYUGA LAKE 8/14/68 - 8/27/68  
FROM LEAST SQUARES FITS

(1) z m	At $t_1$ (8/14-8/20)			At $t_2$ (8/21-8/27)		
	(2) $\theta^*$ °C	(3) $\partial\theta/\partial z$ °C/m	(4) $f\theta dz$ °C-m	(5) $\theta^*$ °C	(6) $\partial\theta/\partial z$ °C/m	(7) $f\theta dz$ °C-m
0.0	23.1		446.92	23.7	- 108.00	477.55
1.52		- 0.30	411.66		- 0.32	442.82
3.05	22.4	- 0.46	377.04	22.2	- 0.21	408.83
4.57		- 0.42	342.73		+ 0.04	374.57
6.10	21.2	- 0.38	310.13	22.2	+ 0.10	341.02
7.62		- 0.56	278.46		- 0.21	307.32
9.15	19.4	- 0.96	248.14	21.5	- 0.66	274.14
10.70		- 1.30	219.32		- 0.94	241.72
12.20	15.6	- 1.23	194.17	18.7	- 1.00	212.20
13.70		- 0.75	171.79		- 1.03	155.01
15.20	13.4	- 0.25	151.15	15.6	- 1.04	160.22
16.80		- 0.18	130.46		- 1.03	137.19
18.30	12.5	- 0.56	111.02	12.5	- 0.74	117.25
21.30	10.0	- 0.88	76.10	11.2	- 0.34	81.08
22.90		- 0.46	62.02		- 0.56	64.74
24.40	8.4	- 0.11	49.05	9.4	- 0.74	49.69
27.40	8.1	- 0.36	23.11	8.1	- 0.20	23.11
30.50	7.2	- 0.0	0.0	7.1	2.0	0.0

\*From graphs given by Sundaram *et al.* (1964, Figure 49a, p. 277).

TABLE 3-3. DIFFUSIVITIES IN CAYUGA LAKE 8/14/68 - 8/27/68

(1)	(2)	(3)	(4)	(5)
z	$\frac{f(\Delta\theta/\Delta t) dz}{\Delta\theta/\Delta z}$	$K_z^*$	$\frac{f(\Delta\theta/\Delta t) dz}{(\partial\theta/\partial z)_{\text{slope}}}$	$\frac{\Delta f/\Delta t}{\partial\theta/\partial z}$
m	m <sup>2</sup> /day	m <sup>2</sup> /day	m <sup>2</sup> /day	m <sup>2</sup> /day
0.0	---	---	10.4	---
1.52	65.0	68.0	130.0	14.0
3.05	22.0	---	28.0	13.0
4.57	33.0	32.0	12.0	24.0
6.10	- 50.0	---	- 20.0	31.0
7.62	12.0	14.0	13.0	11.0
9.14	4.4	---	4.6	4.5
10.70	3.9	4.3	4.7	2.9
12.20	2.4	---	3.8	2.3
13.70	1.2	1.4	1.8	2.1
15.20	1.2	1.5	1.3	1.9
16.80	1.1	1.6	1.1	1.6
18.30	1.1	1.6	0.95	1.4
21.30	0.76	1.6	0.87	1.2
22.90	0.76	1.9	0.90	0.76
24.40	0.95	2.3	1.1	0.21
27.40	1.1	---	0.09	0.0
30.50	---	---	---	---

\*After Powell and Jassby (1974, p. 196).

missing (see Table 3-1). Since a plot of the data available should permit evaluation of the integral quantity at any desired point (provided a smooth curve may be drawn), and gradient values,  $\Delta\theta/\Delta z$ , are easily determined at every depth, it is hard to understand why some diffusion coefficient values are missing.

Table 3-3 lists values of diffusivities of heat at 1.52 meter (5 foot) intervals. Column 3 shows the results of Powell and Jassby's analysis. Column 2 shows the quotient of the pertinent quantities from Table 3-1, as indicated by the column heading. The close agreement of these two columns suggests that, in fact, Powell and Jassby performed their analysis identically. Column 4 also is calculated directly from entries in Table 3-1. A comparison of these values against the first two sets reveals the effects of more precisely evaluating gradients as the slopes of a plotted profile. Also, of interest is the negative value of the diffusion coefficient at 6.10 meters which was among the values omitted by Powell and Jassby. Such a value should not exist, and discussion of such occurrences will be presented in a later section.

The final column of Table 3-3 lists diffusivities as computed by the least squares fit of the temperature data taken from plots in Sundaram et al. These data and intermediate quantities are shown in Table 3-2. By comparing items in both Table 3-1 and Table 3-2 it can be seen that the information is not completely compatible. Values of  $\theta$  at  $t_1$  and  $\theta$  at  $t_2$  when averaged do not differ from  $\theta - \beta_1$  by some constant amount (i.e.,  $\beta_1$ ) and values of  $\theta$  at  $t_1$  minus  $\theta$  at  $t_2$  divided by  $\Delta t$  equals 7 days do not correspond to  $\Delta\theta/\Delta t$ . These differences may be caused by either the inability to accurately read the plots from which the data

were taken or an error in the reporting or calculation of  $\theta - \beta_1$  and  $\Delta\theta/\Delta t$ . Despite the discrepancies, the data were nonetheless subjected to analysis by the least squares fit method. Each set of temperature data was approximated by the function given in equation 3-18. The analytical expression containing the resultant regression coefficient was then used to evaluate the integral,  $\int \theta dz$ , at various depths by a numerical procedure. The mathematical derivative was computed analytically. Integral and derivative quantities computed at  $t_1$  and  $t_2$  are shown in Table 3-2. The difference between the two integrals,  $\Delta J$ , was divided by  $\Delta t = 7$  days to obtain the average gradient. The quotient is presented in Table 3-3 in column 5.

The values tabulated generally agree with those calculated manually by the alternative approach. The value for the surface is not shown. The entries in Table 3-2 could be used to arrive at a diffusivity; however, the quantities in Table 3-2 are meaningless because the function as given in equation 3-18 is not usable at  $z = 0$ . The negative values encountered at 6.10 meters are not predicted by this method. Even though the slopes at  $t_2$  in that region (see Table 3-2) are positive, which would yield negative diffusivities, the corresponding slopes at  $t_1$  are sufficiently negative to cause the average slope to be negative, thereby predicting diffusivities that seem satisfactory. As mentioned earlier the question of negative diffusivities will be considered below.

The overall agreement between the diffusivities calculated manually and calculated by analytically representing temperature profiles lends credibility to the latter approach. Furthermore, the closeness of fit

of the calculated temperatures to curves drawn by eye to the observed data indicates the satisfactory performance of the nonlinear least squares approach. The observed and calculated points are shown in Figure 3-1 for both the week of August 14 and the week of August 20, 1968. The diffusivities shown in column 5 of Table 3-3 were determined using the analytical expressions and using the digital computer for all calculations, including finding the analytical expressions. These results are also plotted in Figure 3-2. The plot indicates a marked variation of diffusivity over the depth of the lake and although values below ten meters are much lower than the maximum, there are definite variations throughout the lower depths.

The preceding discussion indicates the manner in which the flux-gradient method can yield values of diffusivities of heat at various depths in a lake using two observed temperature profiles. Different interpretations of the flux quantities and gradient quantities can produce different values of diffusivities as was shown. Also, pointed out was the effort required to make determinations of diffusivities by graphical methods. The use of a computer-oriented method requiring significantly less manual effort was demonstrated, and the results compared to those obtained graphically. The generally satisfactory behavior of the method and apparent suitability of its results suggest the overall usefulness it can have in applications employing large amounts of temperature profile data to calculate many diffusivities.

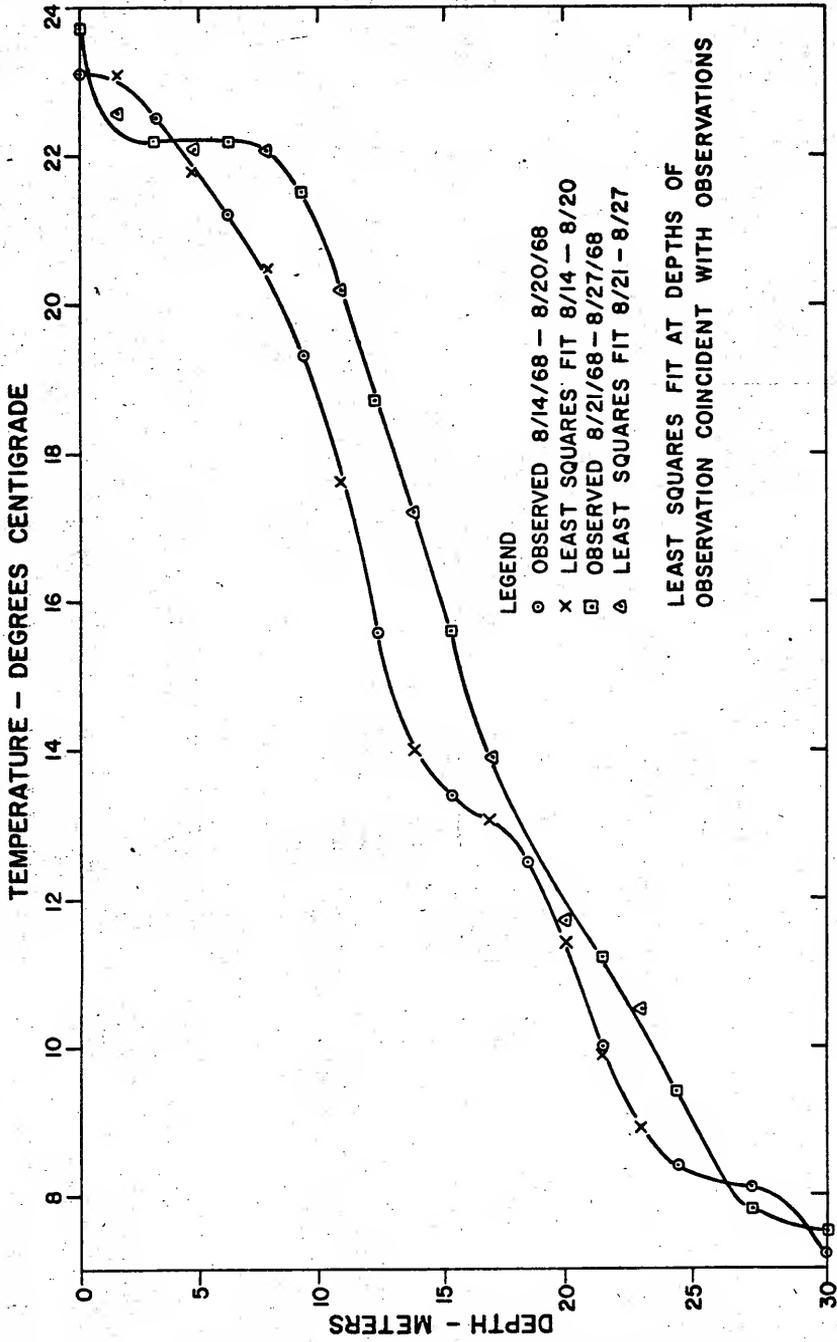


FIGURE 3-1. OBSERVED AND CALCULATED TEMPERATURES IN CAYUGA LAKE

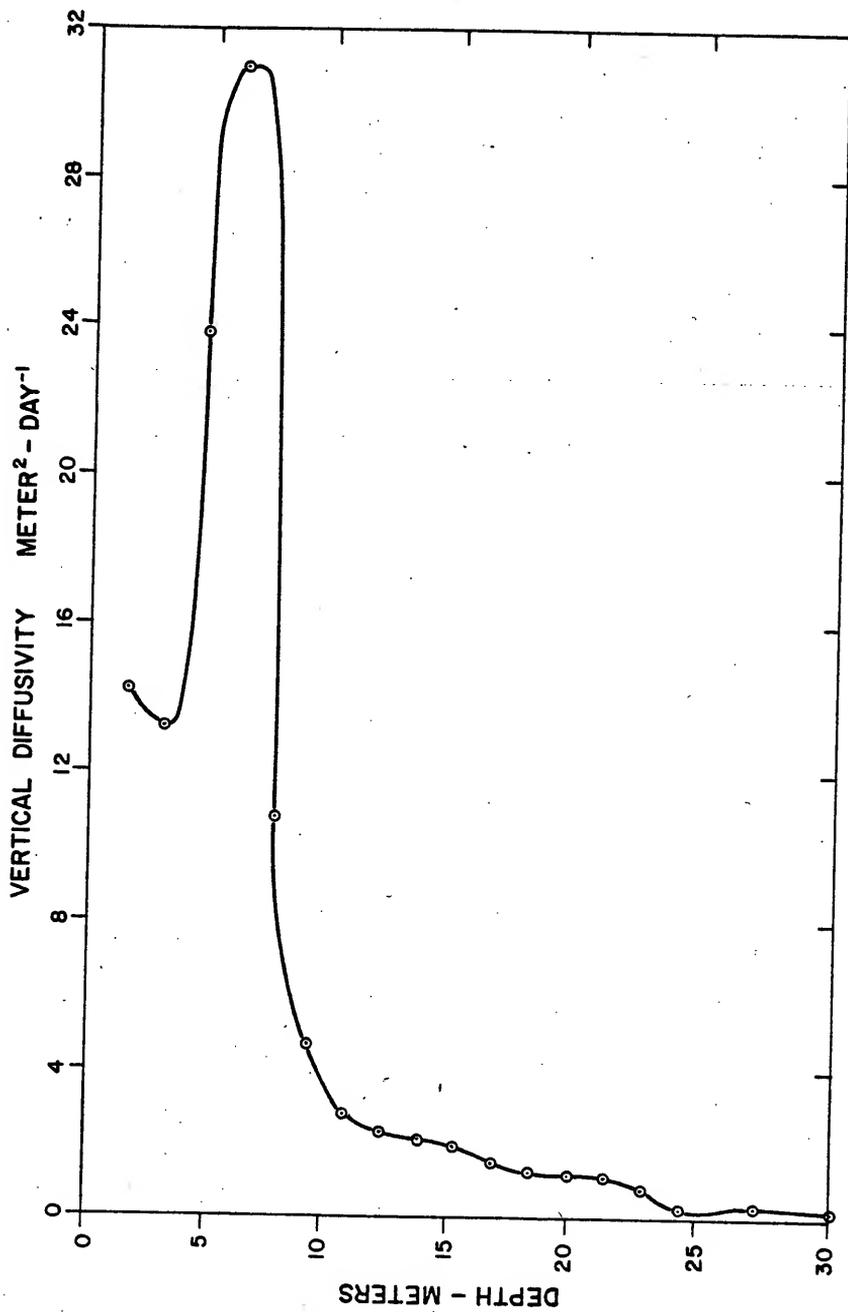


FIGURE 3-2. DIFFUSIVITIES IN CAYUGA LAKE

Correlation of Fluctuations Method

In Chapter II the correlation of fluctuations of velocity and temperature was shown to be equal to the turbulent heat flux. The time-averaged term in the fundamental convective-diffusion equation, equation 2-12, was related to the diffusivity by equation 2-20. By rearranging equation 2-20, the expression for defining vertical diffusivity of heat by the first-principles approach results as

$$K_d = \frac{(\overline{w'\theta'})_d}{-\left(\frac{\partial \bar{\theta}}{\partial z}\right)_d} \quad (3-19)$$

where the terms are as defined in equation 2-20. Over a given time interval, T, the individual records of w and  $\theta$  are averaged using

$$\bar{w} = \frac{1}{T} \int_t^{t+T} w dt, \quad \bar{\theta} = \frac{1}{T} \int_t^{t+T} \theta dt \quad (3-20)$$

Then the records are used again to determine  $w'$  and  $\theta'$  by

$$w' = w - \bar{w}, \quad \theta' = \theta - \bar{\theta} \quad (3-21)$$

from which the product  $w'\theta'$  is calculated. The flux is the time-average of the product over the same integral. Thusly,

$$\overline{w'\theta'} = \frac{1}{T} \int_t^{t+T} w'\theta' dt \quad (3-22)$$

The gradient term is determined by the slope of the average temperature profile at depth d during the same time interval. By computing the quotient of the two terms the average diffusivity over the time interval at the depth of the measurements is found.

The most important aspect of this first-principles approach is the determination of the fluctuation terms, especially the vertical water velocity fluctuations. The successful application of the method to the problem of evaluating diffusion coefficients depends on the ability to detect the turbulent water motion. In the following chapter a detailed survey of turbulence measuring equipment is given in an effort to determine the manner in which the correlation of fluctuations method may be used in this study.

CHAPTER IV  
SELECTION OF A TURBULENCE MEASURING DEVICE

Introduction

As discussed earlier, the evaluation of turbulent diffusion coefficients describing the transport of mass or heat may be accomplished by either a first-principles approach or the flux-gradient method. The former requires the measurement of turbulence-caused fluctuations of both a directional water velocity component and the quantity being studied. The latter requires the measurement of spatial and temporal concentrations of the quantity being studied. Depending on the method used the evaluation of turbulent transport of heat or mass may or may not require the detection of water velocity; however, sensing water velocity is an important aspect of most studies of turbulence, or more specifically, turbulent intensity. So, although relatively few studies have been made of correlations between measured fluctuations of velocity components and transportable quantities, many evaluations have been made of velocimeters from the standpoint of suitability for use in studies of turbulent intensity.

The criteria applicable to the selection of a velocimeter used in a first-principles approach to transport measurement are similar to those required in a study of intensity. Many of these criteria,

in a general sense, are pertinent to the selection of any device to be used in a scientific investigation.

This chapter presents an overview of the factors which must be considered when selecting turbulence measuring equipment, a survey of the various types of velocimeters available with comments about the potential usefulness of each in turbulence measurement and lastly, a discussion of the specific requirements for velocity sensing in the study and the ways that the device ultimately chosen satisfies those requirements.

#### Equipment Requirements

The criteria which must be satisfied when selecting a device for measuring turbulent water velocity fluctuations are similar to those criteria pertinent to the selection of most devices used in scientific investigations. Consideration must be given to instrument performance in the areas of range, sensitivity, stability, ruggedness, power requirements, noise generation, calibration requirements, and frequency response, among others. Turbulent velocity fluctuations may be of a small magnitude when compared to the average velocity, and velocimeters used in such studies must have sufficient range to measure the mean and be sensitive enough to detect the fluctuations. The frequency of the fluctuations may vary from near zero hertz to several thousand hertz. Frequency response limits indicate the maximum frequency at which the device can accurately detect the existing velocity. Stability is a parameter

which may be better considered in three categories: lack of drift, retention of calibration, and the ability to be unaffected by changes in the water other than velocities. Drift, short term deviation, must not occur to a significant degree during the length of time of continuous measurements. The same may be said for a permanent alteration of the device calibration, and if water quality changes during measurement, the device must not respond to the change. The nature of the turbulence will dictate the length of time stability must persist.

#### Hot-wire, Hot-film Anemometers

Hot-wire anemometers have been used for many years as turbulence measuring tools. A short wire of platinum or similar metal is heated above ambient temperature by an electrical current, and subsequently cooled by fluid flowing across it. The amount of cooling can be related to the velocity. The hot-wire probe is more useful in gas flows than in liquid flows because the greater electrical conductivities usually experienced in liquids interfere with normal operation. This problem stimulated the development of hot-film probes for use in liquids. Hot-films are essentially hot-wires coated with an electrically insulating substance such as quartz or, more recently, Teflon. The application of the film is done in such a manner as to preserve as much of the thermal response of the probe as possible. Also, increased structural strength resulting from the film coating makes them preferable for use in water where more impact

force is exerted on the probe.

Hot-wire and hot-film probes are used in two basic electrical models: constant current and constant temperature. The constant current method maintains a uniform current through the probe, and changes in velocity cause changes in wire temperature which cause changes in wire resistance. Therefore, the voltage drop through the probe is an indication of the velocity across the probe. An electronic feedback network helps constant temperature anemometers to maintain the probe at a uniform temperature. Hence, the amount of current through the wire is indicative of level of velocity passing it. Problems of electronic stability within the feedback circuits inhibited the early usefulness of constant temperature anemometers; however, now sufficient electronic capability exists, and the constant temperature method is prevalent among commercial anemometer manufacturers.

Because turbulence measurements in water are nearly always made using hot-film probes, the remainder of this discussion will focus on anemometry using them. Hot-film probes are available in various geometric shapes and configurations. The shape of the probe may make it more or less suitable for a particular application. Furthermore, probes and associated electronics are now available which sense water velocity in the three component directions. Thermal response and electronic response of modern anemometer systems are such that very high frequency turbulence can be accurately sensed. Also, electronic instruments are available which perform correlation and averaging operations as the velocity measurements are being made.

Although, hot-films have been used successfully in many studies of turbulent flow, they do have some drawbacks. The physical size of the probes is very small; however, the somewhat larger size of the probe supporting apparatus restricts the use of them in some instances. The use of hot-films in natural water causes probe deterioration due to accumulations of dirt, biological growth, gas bubbles, and scale. The result of this degradation is, among other things, loss of calibration and thermal response (see Morrow and Kline, 1971). Another problem arises from the heating of the probe. The quality of output signal is proportional to the amount over ambient the probe is heated, and in some cases this heating may in fact inject significant amounts of heat into the water being studied.

#### Laser-Doppler Velocimeters

Laser-Doppler anemometry is one of the newest velocity measuring techniques and has only recently been applied to studies of turbulence. A beam of light is focused on a small volume of fluid in which particles reflect and scatter the beam. If the particles are moving, then they reflect the light with an apparent difference in wavelength, the Doppler effect. An indication of the particle velocity is obtained by determining the frequency shift. This is done by heterodyning (i.e., producing a new frequency by adding or subtracting, the shifted and the unshifted signal). In some cases, two signals shifted in a controlled manner are heterodyned. Lasers provide

illumination sources with very narrow bandwidths, which permit measurement of even relatively low velocities. If the positions of the laser and the scattered beam sensor are fixed, the velocity detected represents a specific direction of particle motion. Arranging three mutually perpendicular sensors to detect the scattered light enables a determination of the three velocity components of the particle (Fridman, Huffaker, and Kinnard, 1968).

The most significant advantage of laser-Doppler anemometry is realized in studies of fluid flow through transparent ducts or pipes. In these cases the velocity sensing equipment need not be inserted into the flow, and no alteration or other disturbance of the flow should occur. This is especially useful for investigating fluid motion very near conduit walls. Investigations of flow in opaque conduits or natural waters would require that much of the laser-Doppler equipment be enclosed in a submersible package which would make satisfactory measurements unlikely.

Many problems encountered in the formative stages of laser-Doppler velocity measurement have been resolved or are being resolved. Developments in electronic capability, beam splitting techniques, and heterodyning procedures are providing better equipment performance than in the past. Indications are that this method of anemometry will permit much progress in laboratory studies of turbulence.

#### Rotating Element Velocity Meters

Velocity meters which use rotating elements such as propellers,

turbines, and cup assemblies are very common. Propellers and turbines are similar to each other; however, propellers usually have fewer blades and propeller blades usually have more curvature. Turbines are used to measure high velocities in pipes, and lower pipe velocities and open channel (free-surface) currents are measured by propeller-type meters. Cup-type meters are used for free surface flow. Propeller and cup-type velocimeters sense water motion in a plane parallel to the propeller axis or perpendicular to the cup axis of rotation. However, the response of each to the direction of velocity within the plane being considered is different. The propeller is affected only by flow components parallel to its axis while the cup is affected by all velocity components in the plane of its rotation. For these reasons the cup type meter is often preferred for use in studies of total planar velocity, and propeller type meters are required when it is desirable to distinguish various directional components of velocity.

There are several techniques used to indicate the rate of revolutions of the propeller or cup element. The most common is pulse generation by means of electrical contact closure. The pulses are counted as clicks heard through ear phones or as digital signals which trigger electronic counters. Other meters operate by sensing changes in electrical resistance or direct current voltages generated at different rotational speeds. The design of these meters has been refined to the point that they are highly reliable and rugged, and they are used extensively in both field and laboratory studies. They have been used mostly in studies of flow or current measurement and similar applications requiring detection of average velocities.

Attention has been given to the effects of turbulent water velocities on propeller meters, and besides improving the reliability of average flow data obtained using them, the better understanding of propeller response to turbulence has permitted them to be used in studies of turbulent flow structure. (Plate and Bennett, 1969).

Propellers do not foul or degrade or otherwise lose calibration when used in natural waters. Furthermore, they resist damage due to impact from foreign matter. Using two meters positioned at right angles enables resolution of separate velocity components from simultaneous measurements. Advanced designs using lightweight materials and low friction pulse generation techniques allow quick, accurate response to even turbulent velocity fluctuations. However, the limit of this response due to inertia effects of even improved designs precludes the use of propellers in investigations of very low velocities. The lowest threshold velocity of meters currently being used is in the range of one-half centimeter per second. Slower water motions are not detected. The physical size of the propeller also limits the minimum eddy size which it can detect.

#### Acoustic or Ultrasonic Devices

Methods have been developed and improved which detect fluid motion using acoustic signals whereby the behavior of ultrasonic waves emitted into the flow is monitored and processed into velocity information. Acoustic devices function according to either one of two basic principles: time of travel or Doppler shift. Measuring

water velocity by measuring the Doppler shift of high-frequency signals reflected from particles in the flow is the same principle of operation used by laser velocimeters. The chief difference is the wavelength of the emitting source. Time of travel devices sense the incremental propagation of the emitted signal caused by the fluid motion. In other words, the ultimate velocity of the acoustic signal is the vector sum of its generated velocity plus the spatially integrated velocity of the fluid in the region through which the signal passes. Indication of water velocity is obtained by discerning that portion of the received signal altered by the water motion.

Various techniques may be employed to detect the alteration of emitted waves. The simplest in concept is to produce a pulse or wave form and transmit it toward a receiver unit which senses it. By accurately measuring the time of travel of the pulse and subtracting it from the expected time of travel in still water, calculated from a knowledge of the speed of sound for ambient water conditions and sender-receiver separation distance, the water velocity along the sender-receiver path may be determined. However, the necessity of knowing the ambient speed of sound inhibits the usefulness of the technique, and it may be averted by adopting a dual path procedure. If the times of travel of two signals traveling the same path but in opposite directions are measured, then the difference between the two should indicate twice the water velocity only, due to the canceling out of the speed of sound component common to each. A modification of this elementary concept is the sing-around circuit

which operates by emitting an initial signal that is received by a secondary unit and upon reception triggers the emission of a return signal which, when detected by the primary unit, triggers yet another signal. The process continues for a selected time or number of rounds. It uses the dual path concept and is essentially a summation of individual measurements. This design yields improved resolution or sensitivity while indicating an average velocity over the repetition period. Another modification of the time of travel method is the differential time circuit which also uses a dual path instrument arrangement. Two signals are originated at each end of the path coincidentally. The reception of one signal at the other end triggers the start of a timing circuit which is stopped when triggered by the reception of the slower signal. Knowing the difference in time and which signal arrived first permits computation of water speed and direction along the instrument path.

Acoustic methods are desirable because they may be non-intrusive; hence they do not disturb the water at the point of measurement. They do not foul or degrade rapidly when used in "dirty" waters, and they are rugged. They have given very satisfactory results when used as flow quantity meters because they intrinsically average flow between sensors. Stream flows have been measured using time-of-travel acoustic systems installed on opposite banks. Doppler shift systems may be the more satisfactory in studies of turbulence which examine local flows. It is possible to construct a Doppler shift device using properly oriented sensors which will detect water velocity in three directional components. This meter, used in a study of estuarine

turbulence was calibrated over a wide velocity range of from about one-half centimeter per second to thirty meters per second (Wiseman, 1969). Problems with acoustic velocimeters are caused by electronic noise, sensing of spurious waves, inadequate alignment of emitter and receiver units, and interference from foreign matter in the water.

#### Lagrangian Methods

An alternate method to measuring water velocity past a fixed sensor is to mark water parcels and follow the movement of the markers. Such studies of the Lagrangian nature of turbulence are carried out by a wide range of methods among which are flow visualization techniques, tracer tests, and drogue studies. Flow visualization methods are often used in laboratory investigations of turbulence. They involve making a succession of photographs, which record the positions of fluid markers at the time of exposure. Markers must be detectable by the photographic equipment and possess physical properties which permit them to behave as suitable indicators of fluid behavior. Types of markers include small spheres, hydrogen bubbles, and small volumes of dyes, among others. Merritt and Rudinger (1973) studied flume turbulence by using a solution of water and pH sensitive dye. A pulse of a current through a thin metallic wire positioned in the flow created a surplus of hydrogen ions at the wire surface which changed the pH and consequently the color of a thin line of water particles. The fate of these particles

was then recorded photographically and subsequently analyzed.

The travel of water parcels may also be monitored by tracer tests. Tracers have been used in both laboratory and field studies of turbulence. They may occasionally be detected visually, but more often, the nature of the tracer is such that it may be sensed in invisible concentrations by equipment designed for such a purpose. Fluorescent dyes, radioactive substances, and salt solutions, among others, have been used extensively in investigations.

Field studies of larger scale motions have been performed using drogues or similar devices which are transported by water currents. Various schemes have been employed to reduce unwanted interferences, for example those of wind. The primary problem in such studies is that drogues which can be located and monitored successfully are often affected by wind or buoyancy or inertia and do not accurately indicate water motion.

Studies of turbulence by Lagrangian means require constant monitoring throughout the course of the experiment. Observations of the entire flow field must be made often enough to detect the effects of the smallest eddies being studied and over sufficient total times to satisfy statistical requirements. Laboratory studies of small scale eddies may last only several seconds while field studies of larger scale motions may take days or weeks. Improvements in photographic techniques, electronic drogue tracking capabilities, and tracer detection equipment have made the performance of Lagrangian studies easier; however, they are still difficult and laborious to carry out because they are not adaptable to high levels of automation.

### Electromagnetic Flowmeters

The movement of charged particles through a magnetic field induces currents. This was discovered by Faraday in 1831 with ordinary solid electrical conductors. The same principle can be used to measure fluid flows, and since all particles have at least atomic level electrical charges, they are affected by magnetic fields regardless of whether or not the fluid is an electrically conductive medium. Electromagnetic flow meters have been used primarily to measure pipe flow (Grossman, et al., 1958), and blood flow in humans and animals, but instruments for measuring local velocity have also been developed (Bowden and Fairbairn, 1956).

There are two methods of electromagnetic flow meter operation. The first senses the induced voltage generated by the flow of charged particles through a magnetic field. The second senses an induced magnetic field generated by a conducting fluid flowing through an established magnetic field. The second method is often used to measure flow in electrically conductive media because it need not be in electrical contact with the media. The induced voltage method requires electrical contact between the sensing probes and the flow. Cushing (1958) showed by theoretical arguments and experimental results that the sensitivity of induction flow meters is not affected by variations in water conductivity as long as the conductivity is above a threshold value of about  $10^{-5}$  mho's per meter. The vector nature of the induced voltage field enables the determination of velocity direction, and instruments exist which can measure at least

two velocity components simultaneously.

In theory, velocities down to zero can be detected; however, electronic noise in the signal processing portion of electromagnetic meters prevents detection of motion below some threshold level. Also, some error in measurement is caused by a "transformer effect" voltage which is due to the use of alternating current to create the magnetic field. Electronic circuitry is capable of diminishing the resultant variations in zero baseline due to this "transformer effect", but some drift still occurs. Boundary-layer effects at the probe-water surface cause minor errors in the response of the instrument; however, the probe is affected by water motion over a distance equal to two or three times the probe radius and the boundary-layer is small in thickness relative to the overall distance of influence. The device has no moving parts, is rugged, and does not lose calibration quickly because of fouling.

#### Other Devices

Several other types of devices have been developed for measuring turbulent flow. Some are altogether different from those already discussed, and some are similar to or use similar principles to those mentioned previously. Several kinds of pressure transducers have been devised to gage instantaneous changes in water velocity. The dynamic pressure at the probe is often sensed by electronic means; for instance, Ippen and Raichlen (1957) used a diaphragm which moved slightly due to pressure change. The diaphragm also served as a

variable capacitance in an electrical circuit, and the response of the circuit indicated flow velocity. Force transducers have also been used to measure water velocities. The force of the fluid impinging on a small sensor surface is converted to an electrical signal which is calibrated to velocity (Earle et al., 1970; Siddon, 1971). Pressure and force transducers cannot measure very low water velocities, and the use of them is limited to velocities above 1 centimeter per second.

Electro-kinetic transducers have also been used in studies of turbulence (Binder, 1967). This type of sensor is unusual because it is not energized. Fluctuations in water velocity cause a thin, small wire probe to emit detectable electrical impulses. However, the wire does not respond to laminar flow and can only be calibrated dynamically, which makes it useful only in studies of the relative intensities of turbulence. Other studies have been performed using thermal methods which impart small quantities of heat to the flow and then attempt to sense the heat nearby. Similarly a device known as the Deep Water Isotropic Current Analyzer (DWICA) has been developed for use in investigations of reservoir transport and turbulence. It omits a small quantity of radioactive material near the center of a ring of sensors. Flow direction and magnitude are determined by which sensor detects the radioactivity and the time lag between emission and detection.

### Price Comparisons

No information on prices of the various types of devices has been given, although this is an important consideration when selecting suitable instrumentation. Some of the reasons for not stating prices follow. Many devices reported in the literature are custom-made, and often cost information is not given. Even when costs are noted, they may only reflect a fraction of the total resources necessary to produce the equipment. Furthermore, many research systems are integrations of commercially available devices and custom-made ones. Also, quoting prices for commercially available equipment is of itself difficult because optional or auxiliary equipment is usually available. The desirability and, more importantly, the necessity of any or all of these supplemental devices varies greatly according to the specific application.

### Requirements of the Lake Mize Study

As discussed in Chapter VI, Lake Mize, near the University of Florida campus, was selected as the site for the field study of vertical turbulent transport. The nature of the lake together with the overall objectives of the study imposed a set of rather rigorous constraints on the selection of a velocity measuring device. The primary requirement was for the instrument to be able to measure water velocities in two or three directional components at a point in the lake. It had to be suited to field use, although battery operation was not necessary because of the 110 vac supply at the lake.

The scheme of the field study was to take measurements over a long period of time, so stability of calibration was essential. The time scales of turbulence were suspected to be such that stability over at least a period of one day was necessary while the maximum frequency response needed would be 1 hertz. Furthermore, an acceptable flow meter had to be unaffected by changes in chemical and physical water quality such as ion concentration, temperature, and concentrations of dissolved substances, since significant variations occurred with depth in the lake and with time. Probably the most stringent criterion was that of sensitivity. Although definite values of water velocities were not known, it was known that the rate of movement would be very low. A target value of 0.3 cm/sec (0.01 ft/sec) was established. This value represented a compromise between the desire to sense large scale water motion and the awareness that ultra-low velocity measurement was not feasible at the time. For the approximate lake depth of 25 meters, a 0.3 cm/sec velocity was indicative of vertical time scales on the order of 1 day. Another requirement involved instrument output. Because the entire field study would be automated, the device had to indicate water velocity in analog form. Manual operation was unacceptable. It was felt that the design and fabrication of an adequate velocimeter was beyond the capabilities of the personnel and physical resources available on hand. Funds for the purchase of a suitable instrument were limited to well under ten thousand dollars, which was the total amount allotted for all equipment expenses of this study.

### Selection of the Electromagnetic Flow Meter

Of all the types of devices available for use in turbulence measurement only one, the electromagnetic flow meter, seemed capable of satisfying the criteria established above. Laser-Doppler meters were unacceptable for field work. No Lagrangian technique could be used for a continuous, highly automated study. Pressure and force sensors and propellers could not respond to low enough water velocities. Hot-film probes would foul too quickly. The acoustic Doppler meters seemed to exhibit promising characteristics, but they were not available commercially, and similarly dual-path acoustic devices were only supplied in massive units for measuring streamflow. The electromagnetic meters were previously available only for pipe flow measurements; however, it was learned that a meter for local velocity measurements in natural waters was newly available. The meter measured two perpendicular flow components and seemed to meet all the other criteria established. The lower limit of its sensitivity was 0.3 cm/sec; however, drift and noise limits were on the same order. Despite these possible shortcomings, it was decided that the electromagnetic meter was acceptable and possessed superior overall capabilities to any other device being considered. Its cost of less than five thousand dollars was also acceptable.

CHAPTER V  
DESIGN OF THE DATA ACQUISITION SYSTEM

Introduction

Because the ultimate objectives of this study involved the collection of large amounts of data over a period of at least one year, there existed a need for acquiring this information in an automated fashion. This chapter describes the projected requirements for a data acquisition system capable of satisfying these needs. The design of such a system was made by incorporating the data needs for each phase of analysis planned. The result was a system capable of performing a combination of many types of in situ measurements and recording them in a compatible fashion on a single punched paper tape recorder. Brief descriptions of the principal components of the system are given in this chapter, and a discussion of the physical installation of these components at the field research site is presented in the following chapter. Also included in this chapter is a description of the calibration procedures used on each of the sensors in the data acquisition system.

Data Requirements for Flux-Gradient Method

Evaluation of heat flux and gradient quantities is done by using temperature profile data taken at two separate times. Temperature at

various lake depths must be known, and the times at which the temperatures occur must also be known.

The number of locations within the water column at which temperatures are recorded and the frequency at which the observations are made which will provide sufficient information for flux-gradient analysis depend on the individual situation. For instance, the lake temperature may be nearly constant over several meters of depth in certain regions (e.g., the hypolimnion) and demonstrate large spatial gradients at other levels (e.g., the epilimnion).

Furthermore, the zones of highest gradient may change with time. Similarly, changes in temperature at a point may occur regularly over a long time span or may become irregular relatively quickly. The adequacy of using a given vertical spacing or time interval is indicated by whether or not actual temperatures occurring between measurements may be inferred from the data collected.

Since the field experiment also involved measuring water velocities, the use of a single temperature sensor mounted on a moving support was not feasible. Therefore, a survey of water column temperature required multiple sensors at fixed positions. Consideration of temperature profiles taken by Nordlie (1972) at Lake Mize indicated vertical spacing of less than one meter near the surface and about one or more meters in the lower region would be advisable. Diurnal changes in temperature were foreseen so the sampling frequency was specified as at least six times per day.

The required precision of temperature measurements was established

as 0.1 degree centigrade. While resolution to minute fractions of a degree would have been desirable, preliminary study indicated a practical limit of resolution of commercially available temperature sensors of about 0.1 degree centigrade.

#### Data Requirements for Correlation of Fluctuations Method

Evaluation of heat flux by the correlation of fluctuations method entails the measurement of the vertical component of water velocity (i.e., magnitude and direction) and coincident water temperature at the place of velocity detection. The gradient evaluation requires knowledge of the change in temperature with depth at the depth of the velocity measurement. Although discussed in detail in the preceding chapter an additional comment regarding velocity measurement is in order here. Of particular import is the ability to sense water velocity in only one direction and at low levels. A magnitude of water velocity was not known a priori; however, the small size and sheltered nature of Lake Mize suggested that very low levels of water speeds might be expected. By estimating distance scales of motion in the lake to be from less than one to about ten meters and time scales of from several seconds to several minutes, a crude estimation of a minimum velocity of 0.003 meters per second (0.01 ft/sec) was made.

The requirement for temperature measurement was established as 0.1 degree centigrade due to prior knowledge of equipment limitations. Since evaluation of the gradient was to be accomplished as part of

the flux-gradient method analysis, it was felt that sufficient gradient data would exist from those measurements, and no new data would need be collected.

As noted above, the time scales of the turbulent motion were estimated to be as low as several seconds. Therefore, a maximum sampling frequency of one second was specified for the velocity-temperature correlation data.

It was also desired to evaluate diffusivities by the correlation of fluctuations method at various depths throughout the water column. To do this the velocity probe and its companion temperature sensor had to be capable of being positioned at any depth. Of course, once at a specified depth the apparatus had to be rigid so that even slight water movement might be detected.

#### Other Data Requirements

Other data needs were those related to environmental-meteorological conditions. Wind speed measurements at three different heights were planned, and wind direction was also specified. Incident short-wave solar radiation, relative humidity, and air temperature complete the list of required parameters. Since these parameters all change daily and most exhibit marked diurnal fluctuation, a sampling frequency of several times per day was deemed desirable. After the study began, it became apparent that instantaneous measurements of these variables were subject to great error. A discrete observation of wind speed, for instance, could not accurately be applied to a lengthy time

interval as the average value over the interval. Therefore, during the field study the requirements for wind speed and solar radiation were changed such that summed or integrated values of each were needed. Instead of wind speed in meters per second, wind in total meters past the sensor during the interval since being last recorded was specified. Similarly, total langley's of solar radiation were called for. The variations in air temperature, relative humidity, and wind direction were such that readings taken half-hourly (as will be discussed later) sufficed.

Wind velocity up to about five meters per second was judged suitable. The capability to measure air temperature from zero to forty degrees centigrade and relative humidity from twenty to one hundred percent was specified. 1.2 to 1.5 langley's per minute was estimated as the maximum levels of solar radiation it would be necessary to measure.

#### Aggregate Requirements

Before considering other factors associated with the plan of the data acquisition system, a recapitulation is in order. Two groupings of data needs existed due primarily to different requirements in frequency of collection. One was velocity-temperature correlation data sampled as often as once per second. As the study developed, this group became known as Group A. The other, a larger group of data signals, monitored the environmental conditions at a rate as slow as several times per day. This latter group, eventually labeled

Group B, included water temperatures at a series of depths, air temperature, solar radiation, wind speed at a series of heights above the water surface, wind direction, and relative humidity. Finally, although not stated earlier, a precise knowledge of the time of sampling of each parameter had to be available.

#### Data Recording Requirements

The amount of data to be collected was forecast to be very large because not only was the overall length of the field study projected at one year but also the number of separate data sources was great and the frequency of sampling high. The need to employ special procedures to accommodate this volume of information was evident. Certainly manual data collection was not feasible. A high level of automation was required. Due to the number of inputs (parameters), some type of multiplexing or switching was indicated, because a different recording device for each signal would necessitate the use of well over ten units. The satisfactory recorder must then be able to accept data from several sources either in parallel or serially and also be able to operate accurately at the rate of at least one measurement per second. Finally, the form in which the data were recorded should permit easy access for subsequent analysis; in other words, converting the data to a form which could be read into digital computers should be a process requiring as little effort as possible.

### Additional Data Acquisition Needs

The major requirements of the data gathering equipment have already been stipulated or alluded to; however, some factors should be discussed further.

The speed of operation of the system had to be at least one data item per second to accommodate the velocity-temperature correlation information. Yet, since the actual time scales of motion in the lake were not known and could perhaps be very much slower, say, on the order of minutes, the ability to select from a range of sample rates was desired. This sample rate flexibility could permit optimum system operation by enabling the selection of the slowest sample rate providing adequate data and thereby reducing the volume of data collected.

The data collecting equipment had to be suitable for use in the field. Furthermore, the ability to function on, in, or very near the water was requisite of most of the system components.

There were few requirements on the various sensors to be used other than that they be capable of measuring the levels of the respective parameter of each which were specified earlier. It was necessary, however, that each of the types of sensors be compatible with the overall data gathering effort.

The budget for the procurement of field equipment was ten thousand dollars. This figure was less than requested and considered to be a significant constraint on the ultimate comprehensiveness of the data acquisition system.

### Commercial Equipment

As a general rule, equipment needs can most efficiently be met by purchasing commercially available equipment whenever possible. The necessity to use custom-made devices arises whenever there is no satisfactory equipment on the market or when there is not enough money to afford equipment which is otherwise available. A survey of manufactured data acquisition equipment was made. This included a study of sensors capable of measuring each of the physical parameters required and also the available alternatives for gathering and recording these measurements. In summary, as could well be expected, no single device capable of performing all of the stipulated tasks was found.

As described in detail in the preceeding chapter, the search for a suitable velocity meter culminated in the purchase of an instrument which detected velocities in two perpendicular directions and the output of which was an analog (voltage) signal at a level of one volt out per foot per second of water speed. Other instruments were found which could measure each of the other parameters; however, several drawbacks were encountered. Among these was the fact that the total cost of all of these exceeded available funds. Additionally, the aggregate of instruments would occupy much space if they all were operated at the same location and, besides, there was much duplication of duty because most were designed to function independently.

The data assembling and recording phases of the acquisition system could also be performed by commercially available equipment.

Disadvantages encountered included very high costs, incompatibilities with certain types of sensors, and unworkable requirements of power and operation. No system was found which offered satisfactory performance yet was affordable. It should be noted that the high costs usually were associated with instruments which provided greater sampling speeds or capacities than needed for this project.

As could well be expected, considering the range of performance requirements associated with this field study, very little of the necessary equipment could be purchased already assembled. Most of the sensors were available commercially, but even they had to be acquired in a form which would permit the incorporation of them into a mostly custom-fabricated instrument package.

#### Design of Data Acquisition System

The preceding sections have discussed in detail requirements of the data acquisition system. What follows is a description of the system as it was ultimately fabricated within the constraints of available resources (especially money).

To stay within the budget of ten thousand dollars it was necessary to amend the goals of the project by limiting its scope or modifying technique, or find alternate sources of supply of equipment, or make better use of material already on hand. The result was an optimization procedure which combined all of these alternatives to various extents. While every step in the decision making process cannot be explained, suffice it to say that many

characteristics of the completed system were dictated by the availability of instruments at low or no costs, even when preferences would have been otherwise. However, in most cases, the requirements outlined earlier were met or exceeded.

The system consisted of several stages. One stage was the sensors which sensed the level of a physical parameter (e.g., temperature, wind speed) and reflected that level as an electrical signal (e.g., resistance, current). Another stage of the system received the sensor outputs and conditioned them, individually, to yield compatible voltage signals. The voltages from the various conditioners were multiplexed or switched one at a time into the recording device. The multiplexing and recording stages were supported and controlled by a stage which contained all the digital logic circuitry. This digital stage was also the source of the system timing.

As a rule the sensors were manufactured devices and the signals from each were conditioned by custom-made circuits. An exception was the water velocity meter which sensed and conditioned its own signal so that the output was a voltage directly related to water speed and direction. The only other system component not custom-made was the paper tape punch which included an analog-to-digital converter.

The requirements for a suitable recording device outlined earlier indicated the use of telemetry, magnetic tape recording, or punched paper tape recording. Other alternatives such as photographic methods and manual recording seemed infeasible, and even strip-chart recording

was thought to be acceptable only as a last resort. The degree of complexity and the high cost of installing and operating a telemetry system eliminated that option. Magnetic tape recording satisfied all of the stipulations; however, its cost was high and could only be justified for data collection at much higher speeds (thousands of items per second). Also, magnetic tape was thought to be susceptible to extraneous electronic noise, especially when used at slow rates. The capabilities of punched paper tape seemed matched to the needs of this study. Data could be recorded sequentially at rates up to tens of items per second, and the stored information could be read directly into digital computers for subsequent use. No cumbersome data handling was required.

It was determined that the local computing facility had the capability of reading the punched paper tape, so the decision to procure such a device was made providing costs were within budgetary limitations. Fortuitously it was learned that an apparently suitable piece of equipment might be obtained gratis from a nearby governmental agency which no longer used it. Ultimately this tape punch was acquired and after testing proved most satisfactory; therefore, the remainder of the data acquisition system was planned such that optimum use could be made of it.

The recorder was manufactured by Towson Laboratories (Model DR-100P-35) and contained an analog-to-digital converter as part of the unit. Zero to ten volt signals of either polarity were stored in digital form by punches in a one-inch wide paper tape. The device

punched whenever a pulse command from the digital control stage was received.

The nerve center of the system was the digital control stage. This stage used integrated circuits of transistor-transistor logic (TTL) type to generate pulses which controlled all the system functions. Precise timing was achieved by using a sixty cycle per second signal from the alternating current power line as a source for a digital timing chain. The chain was composed of several serial stages each dividing the frequency into it to produce a lower frequency. Any one of several time chain outputs could be selected as the time control for the velocity-temperature correlation data, Group A. By actuation from shore, any of a range of sample frequencies from one sample per second to one per three minutes, could be selected.

A one pulse per minute signal was used to control and activate the Group B sensors. Ultimately, there were twenty thermistor probes which sensed water and air temperatures, a pyranometer for measuring solar radiation, three cup anemometers detecting wind speed, a wind direction vane, a polymer-type sensor for relative humidity, a voltage signal indicating depth of the Group A probes, another voltage signal indicating the rate of Group A sampling, and two signals indicating power supply levels in the system. These thirty parameters were sampled at the rate of one per minute making a complete scan every half-hour.

The timing chain controlled the switching and recording of Group A and Group B so that they were sampled in an integrated, coincident manner. The design of the chain was such that the Group B signal

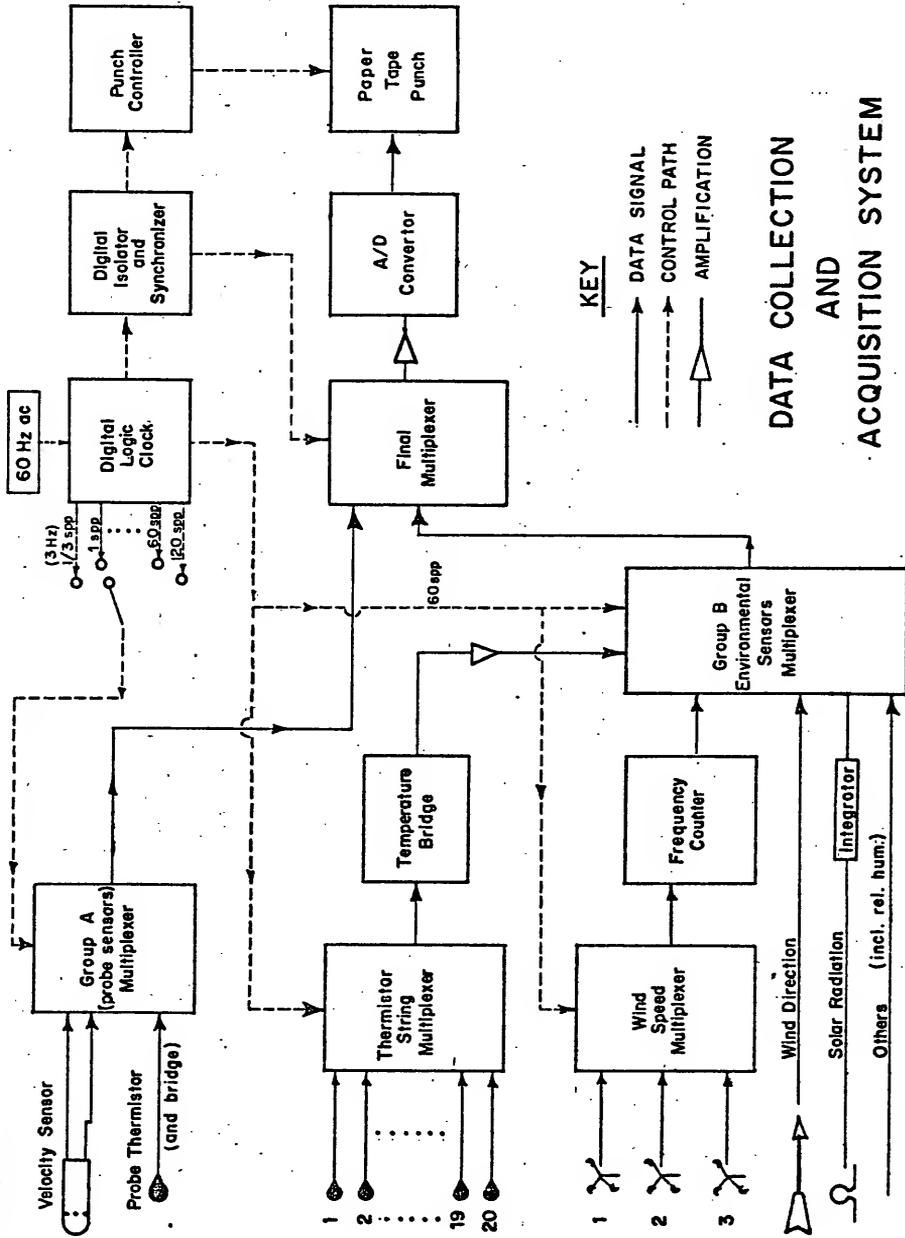
could be interspaced between Group A recordings even at the fastest Group A sampling rate.

The switching in the system was done by solid state FET (Field-effect transistor) switches. The analog signals were amplified using integrated-circuit operational amplifiers. The amplifiers were of a common style (741) but specially selected for low levels of signal drift due to ambient temperature changes.

Figure 5-1 gives a conceptual diagram of the system in its ultimate form. The various analog and digital control phases are as indicated in the preceding discussion. No further description of the electronic control circuitry will be presented, but the sensors used to detect parameter values will be discussed in greater detail. Brief descriptions and identifications of each of the sensor types used and the circuits used to condition the sensor outputs are given. Following that is a summary of calibration procedures.

#### Velocity Measurement

The velocity meter purchased was made by the Engineering-Physics Company, Rockville, Maryland. The unit selected was Model EMCM-3BX (Serial number 620). The meter has been discussed earlier and is not described in detail here. It produces a voltage output of one volt per foot per second water velocity in either of two perpendicular directions. The algebraic sign of the output voltage indicated the orientation of the movement in either direction.



**DATA COLLECTION AND ACQUISITION SYSTEM**

FIGURE 5-1. CONCEPTUAL DIAGRAM OF THE DATA ACQUISITION SYSTEM USED AT LAKE MIZE

### Temperature Measurement

The choice for temperature detection was between using thermistors or thermocouples. Thermistors were picked because the voltage outputs of thermocouples was lower than that which could be tolerated due to electronic noise associated with boosting the signal. Originally thermistors manufactured by the Yellow Springs Instrument Company (Part No. 44018) were selected because they offered linear change in resistance with change in temperature. Each thermistor was connected to a multiconductor cable, and the joints sealed with potting resin. One by one the joints leaked, and new probe making techniques were attempted. Before a satisfactory technique was developed nearly all of the thermistors deteriorated to the point of uselessness. New thermistors were bought from Fenwal Electronics, Inc. (Part No. GA51J1). They were conventional non-linear devices but were ordered because of lower cost. The revised probes were constructed by connecting the thermistor to two-conductor, vinyl-covered wires (thereby using one wire per probe) and covering the thermistor and joint with a short (2-3-inch) piece of copper tubing which had been crimped and soldered at one end. The region of juncture between the tubing and wire covering was sealed by wrapping the area with overlapping turns of plastic electrical tape and then repeating with another wrapping of tape. These probes performed satisfactorily even though with time some failed for various reasons.

The probes were initially spaced at intervals of two feet (0.610 meters) from lake surface to bottom, but when the first cable

was replaced, the spacing was changed such that probes were located at the following depths: surface, 0.15, 0.31, 0.46, 0.61, 1.22, 1.83, 2.44, 3.05, 3.66, 4.27, 4.88, 6.10, 7.32, 8.54, 9.76, 10.98 meters.

The thermistors were switched one at a time into a resistance bridge circuit which translated thermistor resistances into voltages. The bridge was adjusted such that a temperature range of eight to thirty-eight degrees centigrade could be measured. The probe thermistor of Group A was used with a separate bridge of similar construction.

#### Solar Radiation

Incident solar radiation was detected by a pyranometer made by the Eppley Laboratory, Inc. (Model Number 8-48, Serial Number 10,000). It measured total radiation both direct and diffuse. Similar units have been used by many researchers and enjoy a good reputation. For this reason and the fact that the device was already on hand, it was selected for use. Originally, the output of the pyranometer, which is a voltage proportional to radiation flux (i.e., about 8 millivolts out per 1 langley per minute), was sampled discretely once every thirty minutes. Later in the project, an integrator was designed and constructed which summed the voltage out over a thirty-minute period, and the summed value was recorded. The integrator was then reset to zero to begin the next half-hour cycle.

The unusual design of the integrator merits a brief description.

The millivolt signal from the pyranometer was amplified and then input to another amplifier which summed the signal in. The integrator was adjusted so that up to 46 langley's could be measured in thirty minutes. The output of the summing device was referenced against a fixed voltage. Whenever the summed voltage reached the reference level, a pulse was created and the summing device reset. The pulses were counted, and the final count at the end of thirty minutes represented the amount of total radiation for that time period. The final stage of the integrator converted the pulse count back to a voltage signal so that it could be processed with the other analog signals in the system.

#### Cup Anemometers

Wind speed was measured by cup anemometers made by the Beckman and Whitley Company (Model 1564B, Serial numbers 119, 120, and 122) using their standard cup assembly (Model 170-41). The units were available gratis and since the performance of them was very good, they were chosen. The output of the anemometers was electrical pulses with a frequency directly related to wind speed. At the start of the study one discrete sample every thirty minutes was taken. The frequency signal from the instrument was fed to a circuit which converted frequency to voltage. Later the circuits were modified to sum the frequency thereby summing the wind past the sensor. The integrated value was recorded every thirty minutes and the integrator reset to zero.

The integrator was adjusted such that up to 9000 meters of wind past the sensor could be measured each half-hour. This corresponds to a constant 5 meter per second wind over the entire time span.

#### Wind Vane

The wind direction sensor was also manufactured by Beckman and Whitley (Model M1571, Serial number 174). The shaft to which the vane was attached was connected to a continuous turn variable resistor. The orientation of the vane was indicated by a specific resistance. One volt was applied to the device, and the output fraction of that volt indicated the relative position around a circle of the vane with zero volts corresponding to zero degrees and 1 volt corresponding to 359 degrees. No additional conditioning of the signal was necessary.

#### Relative Humidity Sensor

The sensor used to detect relative humidity was manufactured by Phys-Chemical Research Corporation (Model PCRC-11). A polymer-type coating on the sensor changed electrical resistance with changes in humidity. This type of sensor seemed preferable to types using hygroscopic salts and requiring recharge. Furthermore, its cost was low, and it was available in a form easily usable with the rest of the system. An alternating current bridge was built which translated sensor resistance to a voltage. The voltage was rectified and adjusted such that zero to one hundred percent relative humidities could be recorded.

### Velocimeter Calibration

The calibration of the electromagnetic velocity meter was performed by the manufacturer before it was shipped; however, because very low water velocities were anticipated, it was decided to verify the response of the instrument. Since no suitable calibration facility existed nearby, a procedure was devised using available apparatus. The screw-feed mechanism on a metal-working lathe was selected as the conveyance of the probe. A small, portable trough measuring about 15 centimeters wide by 2 meters long by 25 centimeters deep was positioned parallel to the lathe and filled with water. The velocity probe was connected to the lathe carriage in such a manner as to cause the probe to move through the water as the carriage moved along the lathe bed. The screw-feed device was controlled by a variable speed motor and a gear assembly which permitted a continuous range of speed and alternate directions of travel. The average velocity of the probe was calculated by recording elapsed time and distance of travel. The alignment of the probe to the direction of travel was done by eye. Meter output was recorded on a chart recorder.

The lathe speed was adjusted to provide a probe velocity of about 0.3 centimeters per second, and the first run was made. After the trough had stilled, the probe was moved in the opposite direction with the motor speed unaltered. Then the motor speed was increased slightly, and the procedure was repeated. Velocities up to 1.3 centimeters per second were created before the induced movement

in the trough began interfering with the test.

Good agreement between measured and calculated velocities was found. A plot of the data (Figure 5-2) shows good linearity and the 45-degree lines drawn indicate satisfactory scale adjustment. The data in Figure 5-2 were also substituted into a linear regression algorithm which yielded the following equations relating calculated velocity,  $V_{cal}$ , to measured velocity,  $V_m$ , and values of correlation coefficients:

$$\text{Channel I, } V_{cal} = 0.974 V_m - 0.0592, R = 0.998$$

$$\text{Channel II, } V_{cal} = 1.01 V_m + 0.0488, R = 0.996$$

As can be seen, the slopes are very near 1 (45 degrees), and the linearity or goodness of fit is extremely high. It was concluded that the nominal calibration of the velocity meter was accurate.

The stability of the electromagnetic velocimeter was also examined. The probe was placed in a small volume of water, covered and allowed to stand in a laboratory in which room temperature varied only slightly. Continuous strip chart recordings were made for several days. A similar test was made with the instrument located at the field site but with the probe in a bucket of lake water. Both tests indicated that the meter output was stable as long as the temperature remained constant. If the electronic signal processing part of the meter was heated or cooled (a heat gun and bags of ice were used in the laboratory), a zero drift resulted. The maximum value of this offset was equivalent to  $\pm 0.15$  centimeters per second.

Electronic noise was also studied during the test just described.

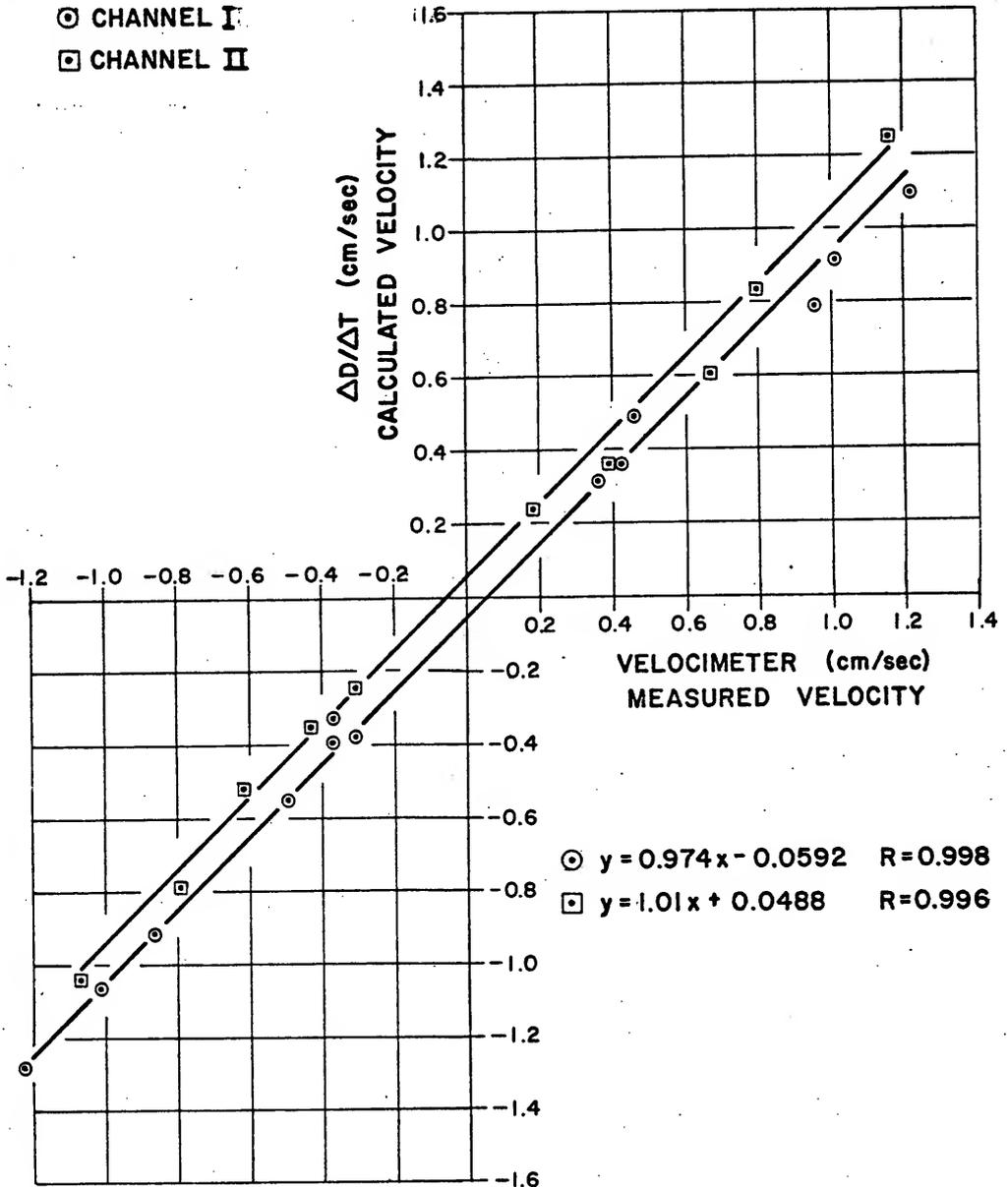


FIGURE 5-2. VELOCIMETER CALIBRATION

Output voltage variations for a constant velocity (zero) were found to be dependent on the time constant of the instrument. The highest time constant of 2 seconds was chosen in order to minimize noise. The peak-to-peak noise for this time constant was equivalent to 0.3 centimeters per second.

#### Wind Anemometers and Direction Indicator

All efforts to calibrate the Beckman and Whitley cup anemometers were unsuccessful. The anemometers generate electronic pulses as the cup rotates and the speed of rotation is indicated by the frequency of pulses. The calibration of the custom-made electric circuit to count the pulses was performed using electronic pulse generators and oscilloscopes and presented no problem. The calibration of the anemometers entailed relating wind speed to output frequency and proved to be more difficult. A search for a nearby wind tunnel or similar apparatus yielded only one such installation. Unfortunately, the lowest velocities attainable in the test section of the tunnel were about 10 meters per second. The range of wind speed desired was from the device threshold of 0.3 meters per second to 10 meters per second. Although the velocity in the test section was too high, it was decided to try to use another area in the wind tunnel with a much larger cross-sectional area. Continuity principles would enable calculation of the relation between test section velocities and those at the anemometer location. Such velocities should have been low enough to provide useable results.

Regrettably, the layout of the tunnel was such that the only portion of enlarged cross-section which was even minimumly suitable was very near the test section. Non-uniformity of velocity throughout the cross-section in this region was suspected; however, a series of tests was arranged and conducted. As feared, the output signals from the three wind speed sensors varied unpredictably, and no correlation between velocity in the tunnel and the sensor outputs was found.

Another attempt was made to calibrate the anemometers by comparing the outputs of them to another type of anemometer on hand. There was no calibration information available about the second type sensor other than that supplied by the manufacturer. However, should the two types of instruments indicate the same velocities using the manufacturer's calibration for each, then the inference could be made that such calibrations were acceptable. The respective sensors were compared under ambient wind conditions, but the results were inconsistent, and no conclusion could be made except that at least one was not as specified by its maker.

No other satisfactory calibration technique was devised, and ultimately it was decided to accept the calibration provided by the instrument manufacturer.

The wind direction indicator, also made by Beckman and Whitley, was calibrated easily. The device contains a continuous potentiometer that is linked to the dynamic wind vane. The orientation of the vane determines a specific resistance. Calibration of the sensor consisted of attaching a straight, rigid wire about 15 centimeters

long to the spindle which normally held the vane. The wire handily simulated vane position. With the instrument body stationary, the wire was rotated through a complete circle and a check of resistance versus angular position was made. The indicator responded satisfactorily showing good linearity and a very small "dead" region.

#### Solar Pyranometer

The Eppley pyranometer is accurately calibrated at the Eppley laboratory using a group of reference standards. No additional checks were made before the field study, but after completion of the field study the pyranometer was checked against another unit in use nearby. The outputs of the two indicated the same values of solar radiation.

#### Relative Humidity

The relative humidity probe purchased from the Phys-Chemical Research Corporation senses changes in relative humidity by changing electrical resistance. An alternating current bridge circuit was designed and built to detect resistance changes of the humidity element. Probe and bridge were calibrated as a unit. The probe was exposed to a range of relative humidities produced by a custom-made apparatus used on another research project. Basically, the apparatus worked by mixing carefully measured amounts of dry, zero humidity, air and saturated, 100 percent humidity, air together. Elaborate temperature control and other refinements permitted a high

degree of confidence that the nominal humidities were actually attained.

Hysteresis effects were noted in the response of the probe. The output voltage varied slightly for a given relative humidity depending on whether current humidity was attained as a result of a decrease from a higher value or increase from a lower value. These effects were observed to be most pronounced near 50 percent humidity. A plot of the calibration data is given in Figure 5-3. When hysteresis effects caused two values to exist for a single humidity, the two were averaged to provide the values of voltage plotted.

The data shown in Figure 5-3 are for 25°Centigrade. Temperature variations were not investigated during the calibration procedure, and the manufacturer's temperature correction factor was used to modify the relationship derived from the constant temperature test. According to the manufacturer 0.36 percent relative humidity must be added to the humidity value determined by the calibration equation per degree centigrade over 25°Centigrade, and the same amount subtracted per degree below 25°Centigrade.

The line shown in Figure 5-3 was drawn as an eyeball fit of the data points. Predicted values from the equation of that line were used to calculate a correlation coefficient. The goodness of fit is high, as indicated by  $R = 0.9994$ . For this computation, the point at zero relative humidity was ignored because it was deemed in error.

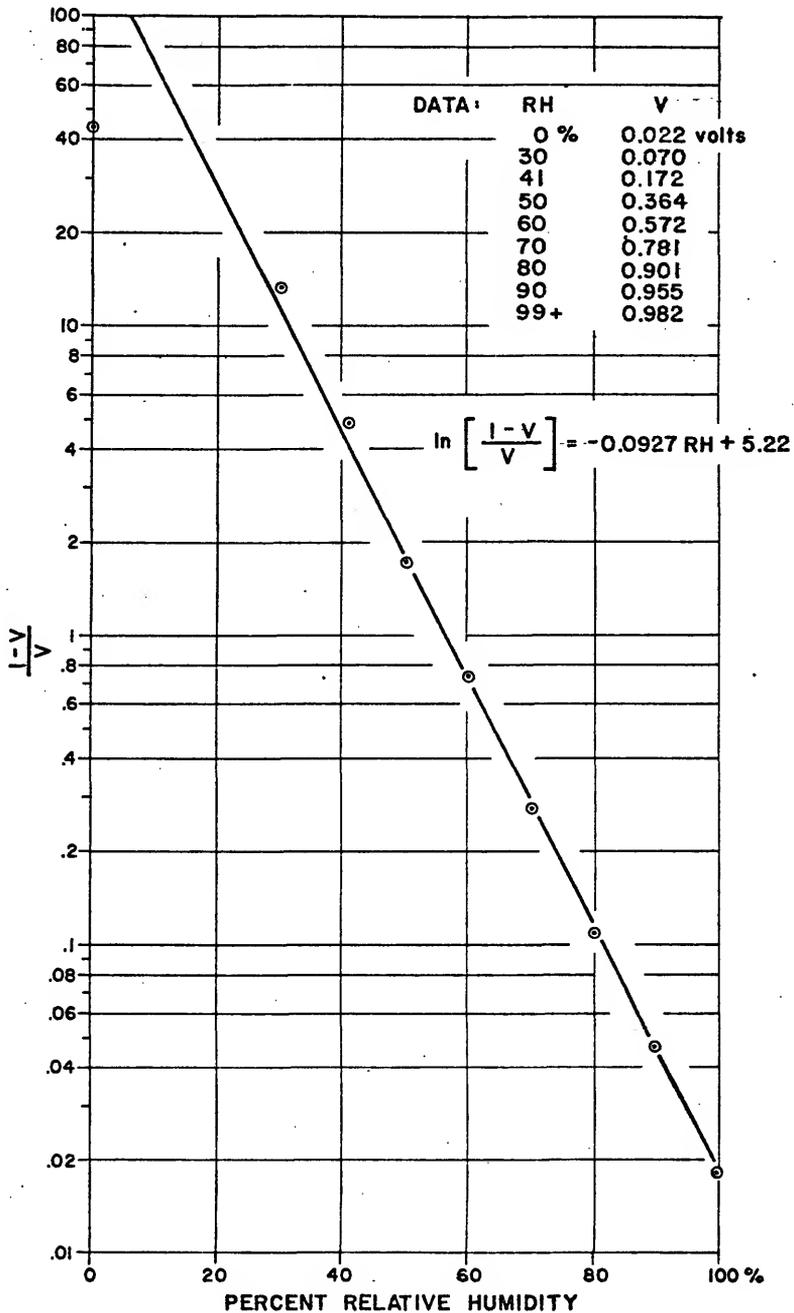


FIGURE 5-3. RELATIVE HUMIDITY CALIBRATION

### Thermistors

Calibration of the thermistor probes was a straightforward procedure. The probe strings were connected to an electric bridge circuit designed to detect the resistance of the thermistors. The circuit includes a switching network consisting of solid state switches which exhibit small but finite resistances; so the calibration was performed using the entire circuit. The probes were immersed in a water bath monitored by a precise laboratory-grade mercury thermometer. The temperature was varied from zero to forty degrees centigrade, and the voltage output of the bridge was recorded as each probe was switched into the circuit. Inherent differences among the thermistors caused a linear offset of voltage for each. The relationship between temperature and bridge voltage for a specific thermistor was determined, and each offset was related to that calibration. A separate calibration equation was determined for the thermistor mounted at the velocity probe using identical methodology.

A theoretical form of the calibration equation was developed by combining analytical expressions for thermistor resistance as a function of temperature and bridge output voltage as a function of its fixed and thermistor resistances. Although somewhat complex the resultant expression enabled a very satisfactory calibration equation to be determined. The plot of the temperature and calculated voltage function values are shown in Figure 5-4. The line from which the constants in the equation are determined is an eyeball fit

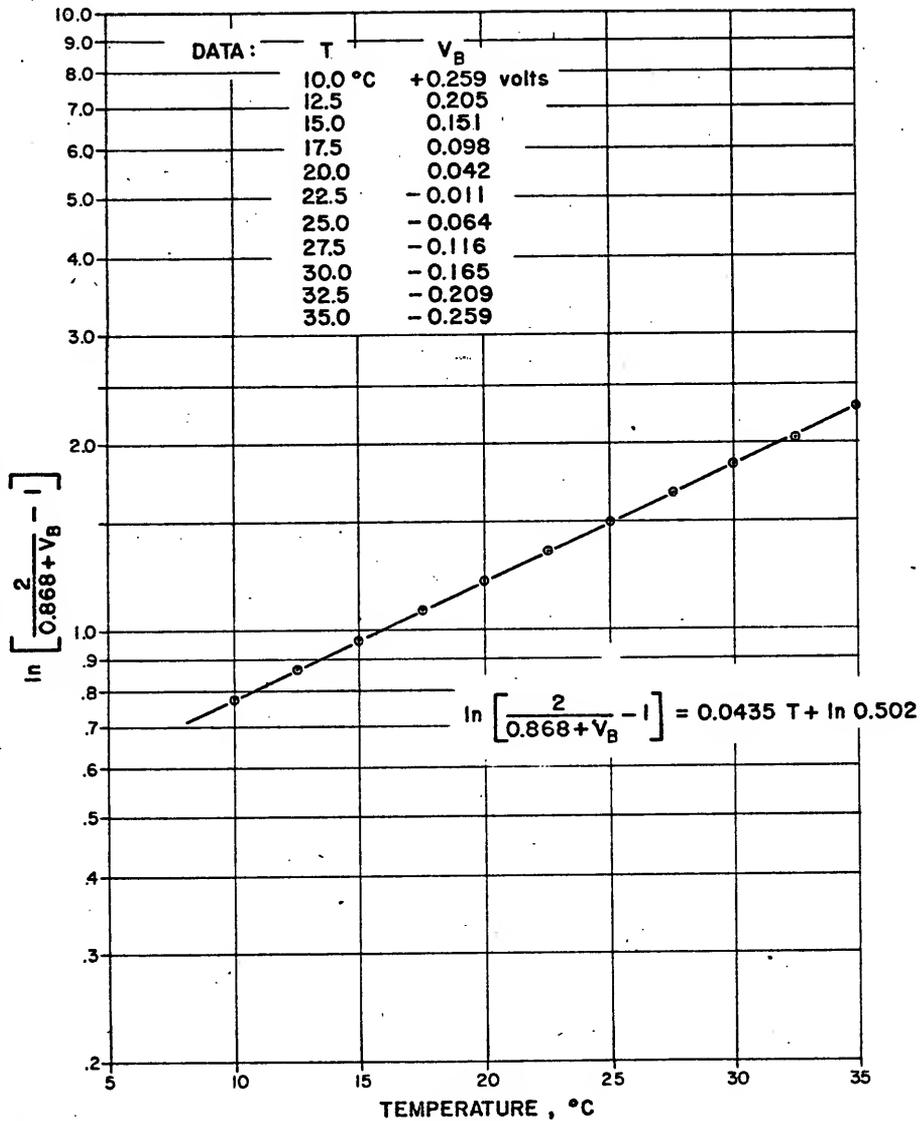


FIGURE 5-4. THERMISTOR CALIBRATION

of the points. As can be seen, excellent fit resulted.

#### Punch Paper Tape Recorder

The paper tape recorder and its integral analog-to-digital converter were tested both in the laboratory and at the field study site. The recorder zero was adjusted so that equal voltage inputs with opposite signs produced identical punches except for the opposite polarity. The scale span of the instrument was adjusted to nominal values. The linearity of the recorder was checked over its entire operating range by comparing punch response to various voltage inputs. The instrument performed satisfactorily.

Stability was checked by feeding known voltages into the tape punch repetitively over a two-day period. This test was performed twice, once in the laboratory and once at the field site. No significant zero drift was observed in either case.

## CHAPTER VI

### ORGANIZATION OF THE FIELD STUDY AND DATA REDUCTION TECHNIQUES

#### Introduction

This chapter contains a description of the data gathering and reduction processes. The subsequent analysis of the data acquired is the topic of the following chapter. In this chapter the organization and conduct of the field research is explained. This includes information on the selection of the field site, the physical installation of the instruments and other equipment, and the manner in which the equipment was operated. Also given is an account of the procedures used to assemble the raw data into a form appropriate for further analysis.

#### Selection of Lake Mize as the Research Site

Lake Mize in the Austin Cary Forest near Gainesville was chosen as the location for the field research project. The lake was desirable because of its location, depth, bottom slope, power availability, and security. The lake is located about fifteen miles northeast of the University of Florida campus and requires less than thirty minutes driving time to reach. It is a sinkhole lake about twenty-five meters in depth which is much deeper than

most Florida lakes. This factor insured a stratified condition during at least part of the year. The bottom slopes are relatively steep, and therefore, the distance from shore to a point several meters deep is not great. A preliminary study revealed that water about ten meters deep at this latitude would thermally stratify (Shannon and Brezonik, 1972), and this depth in Lake Mize exists approximately one hundred feet from shore. This was important because electrical cables had to be run between instruments and shore. Depths and bottom slopes can be seen in the bathymetric map of Lake Mize shown in Figure 6-1.

Electrical power is available at the lake within a distance of one hundred feet from the edge of the water. The instrumentation necessary to carry out the proposed experiments required more current than could be supplied by battery, and on-site generation was undesirable, so the presence of nearby power was an important consideration. Also, of importance was the fact that access to the lake was restricted. The Austin Cary Forest is University of Florida property, and there was little chance of research equipment being exposed to mischief.

Another positive factor associated with the lake is its lack of outlets. No net vertical velocities should exist in it.

Lake Mize also had disadvantages the most serious of which was its small surface area of approximately 1.6 acres (8600 square meters). Also, the lake is surrounded by tall trees which restrict wind effects on the water. These two factors suggested limited

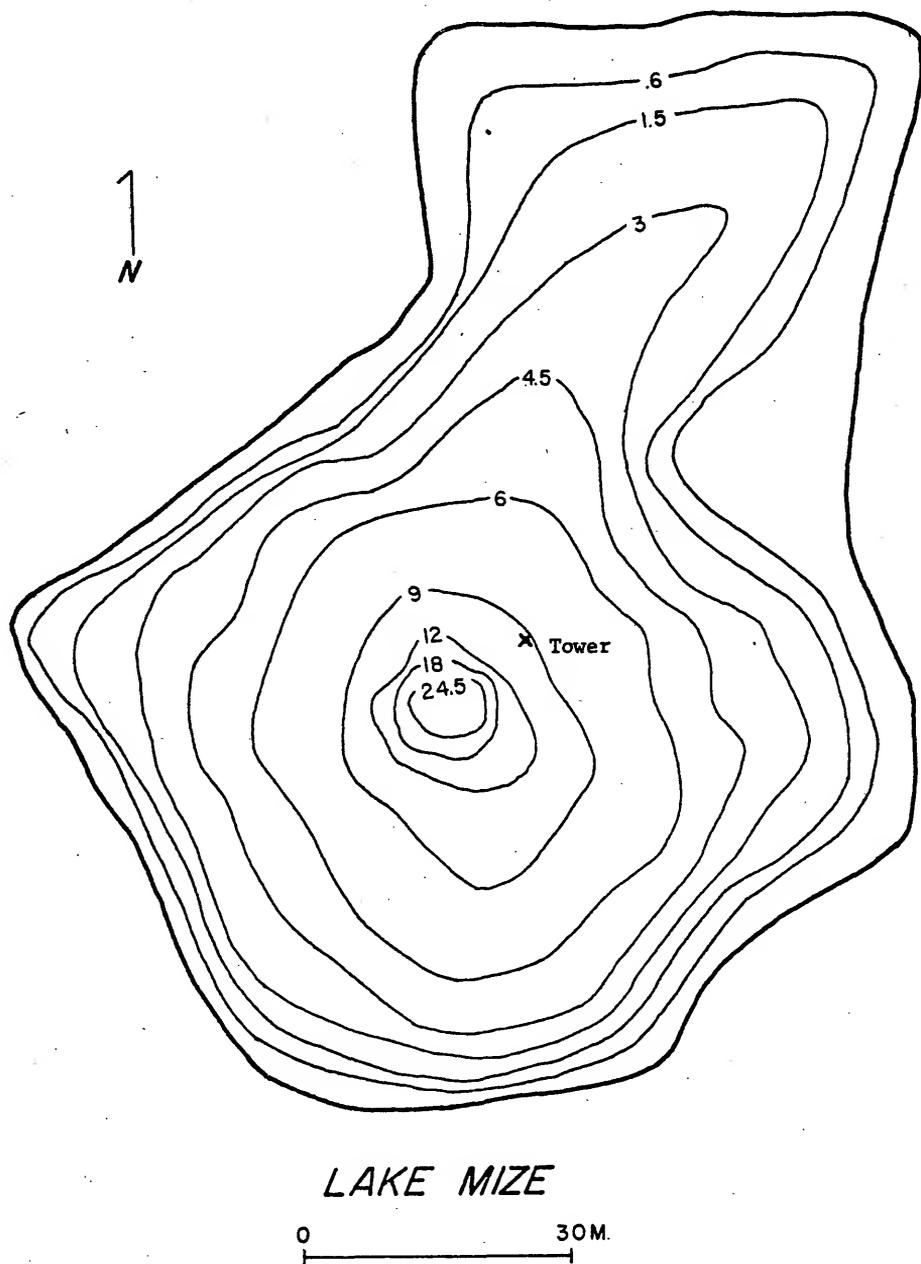


Figure 6-1. Bathymetric map of Lake Mize, Florida. Contours in meters.

circulation in the lake due to winds. Although the levels of mixing were certain to be less than those in a larger, less sheltered water body, the advantages seemed to outweigh this, and it was decided to select Lake Mize for the field study.

### The Instrument Installation

The data acquisition system was installed at Lake Mize in the late spring of 1973. The criteria for the performance of the system and the ways in which those criteria were satisfied were discussed in the preceding chapter. The actual fabrication of the equipment and its manner of installation will be described here. The major portion of the equipment was located on a tower placed in about ten meters of water. The position of the tower is indicated in Figure 6-1, and Figure 6-2 is a photograph of the tower as seen from the lake shore. The tower was of the type often used to support communications antennae. It was composed of interlocking ten-foot-long sections of triangular cross-section about one foot on a side and made of aluminum. Two rails were added to each section to serve as guides for casters which were mounted on a frame assembly. Thus the frame could be rolled from the top to the bottom of the tower. In order to diminish any interfering effects of the tower, the velocity probe and its companion thermistor were mounted at the end of a five-foot length of channel section positioned horizontally and attached to the moving frame. The frame, channel, and mounted probes are shown in Figure 6-3. Most of the frame is out of the water. Figure 6-4



FIGURE 6-2. PHOTOGRAPH OF FIELD RESEARCH TOWERS

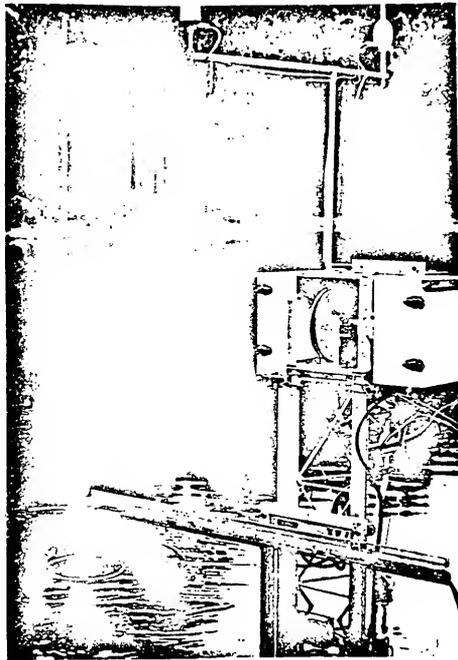


FIGURE 6-3. PHOTOGRAPH OF FIELD RESEARCH SENSORS

shows the velocity and temperature probes in greater detail. The dark velocity probe is about two centimeters in diameter. The temperature probe is mounted in the same horizontal plane, and about thirty centimeters separate the tips of the two probes.

In Figure 6-3 an aluminum wheel (between the two white boxes) can be seen. The cable which was used to raise and lower the probe assembly ran over the wheel and caused the wheel to turn whenever the probe was moved. A variable resistor attached to the shaft of the wheel changed resistance as the wheel turned. In this manner an electrical signal indicative of probe depth could be produced. Behind the wheel and not visible in the photograph was the electric motor and winch which when activated would move the probe assembly to a different depth.

Figure 6-5 is another photograph of the instrumented tower with the velocity and thermistor probes positioned just below the water surface. The string of thermistors for sensing temperatures at various depths was attached to the side of the tower away from the camera. In Figure 6-6 two of the fabricated thermistor sensors are displayed. The arrangement of the cup anemometers and wind vane can be seen in Figure 6-5. At the time the photographs were taken the pyranometer was not in place. When installed it was located directly above the wheel described earlier. The relative humidity and air temperature sensors were mounted in a shaded area below the bottoms of the white boxes. The white boxes contained electronic components which formed the major portion of the data

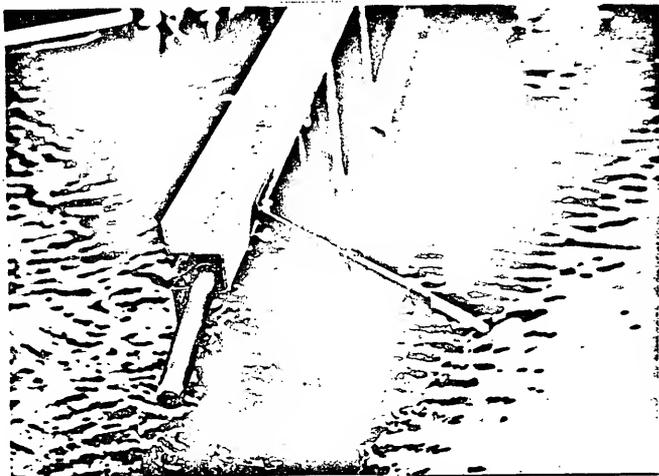


FIGURE 6-4. PHOTOGRAPH OF VELOCITY PROBE

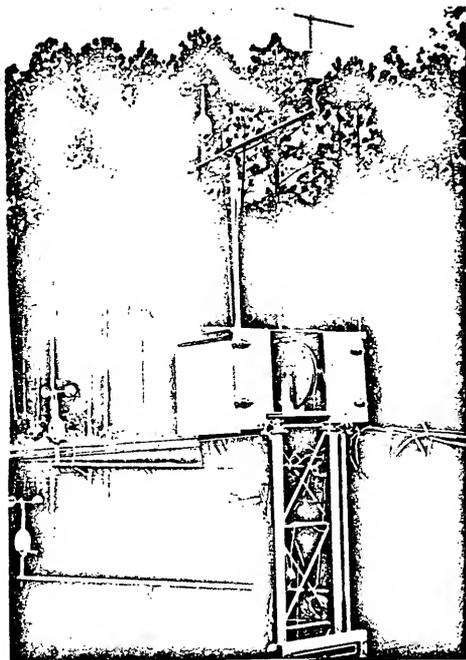


FIGURE 6-5. PHOTOGRAPH OF FIELD RESEARCH SENSORS



FIGURE 6-6. PHOTOGRAPH OF THERMISTOR STRING

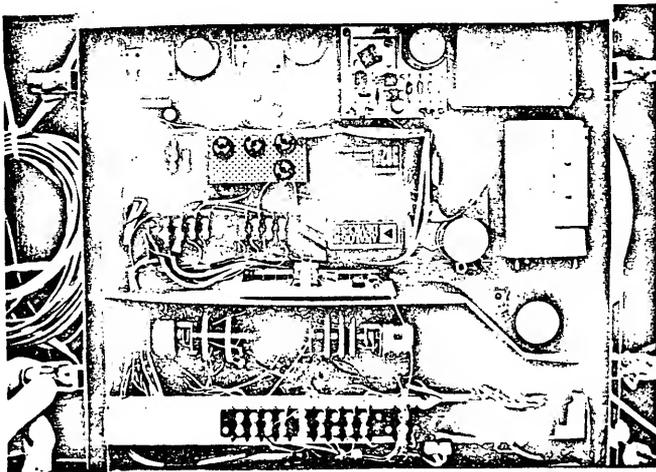


FIGURE 6-7. PHOTOGRAPH OF POWER SUPPLY CABINET

acquisition system. Figure 6-7 is a photograph of the equipment inside one of the boxes. This box contained both alternating and direct current power supplies which were vital to the operation of the system. In the lower portion of the box are relays and stepping switches which when activated from shore adjusted the operation of the system as desired (e.g., select sample rates, move the probe, etc.). Cables from shore delivered line power and control pulses to the box. The internal elements of the other cabinet are shown in Figure 6-8. The removal of one circuit board from a chassis containing approximately fifteen such boards is demonstrated. Each board performs either a digital control or signal conditioning function. Each type of sensor at the tower was connected to its respective board. The final data signal ready to be recorded and a pulse signal instructing the recorder to punch were transmitted back to shore via cable.

On shore, the recorder and other auxiliary equipment were housed in a wooden cabinet which is shown in Figure 6-9. The paper tape punch is the instrument at the lower left.

#### Data System Operating Procedure

Once the data acquisition equipment was installed at Lake Mize, the system was made operational. The plan for data gathering involved the continuous operation of the system over the period of one year. Since the equipment had been designed to operate automatically, the only attention it would need would be periodic collecting of the paper

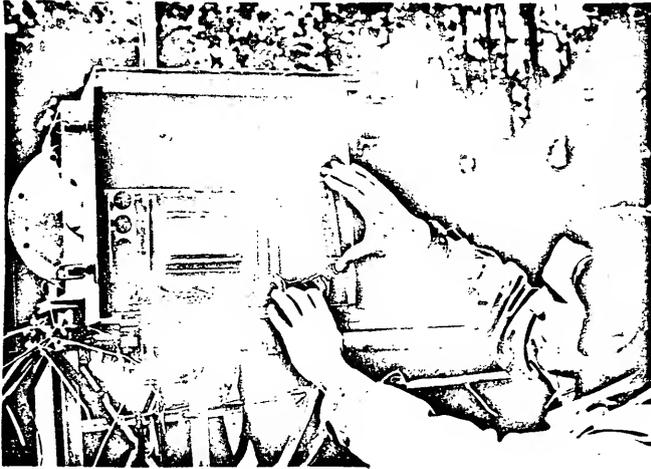


FIGURE 6-8. PHOTOGRAPH OF SYSTEM CIRCUIT CABINET

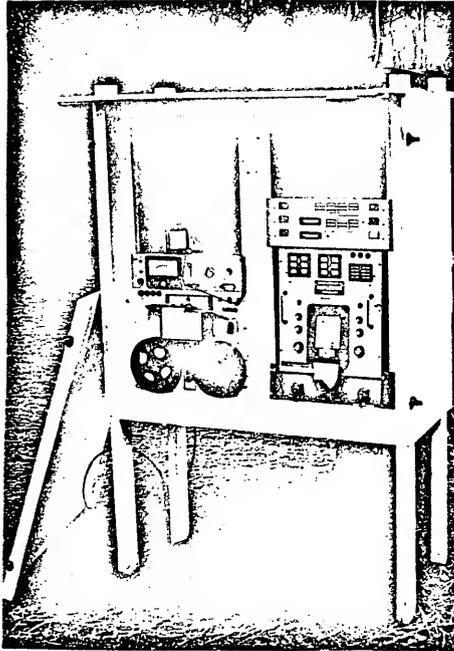


FIGURE 6-9. PHOTOGRAPH OF RECORDER CABINET

tapes. Therefore, it was planned that at least partial analysis of the data could begin as soon as a small amount had been collected. It was thought that preliminary results could be used to provide information on continuing operation procedures. Unfortunately, the system did not function in as trouble-free a manner as was planned, and much of the time during that period of data collection was spent repairing and modifying the installation.

Enough early information was obtained to indicate that movement in the lake was slow and mostly near the surface. A Group A sample rate of one minute was chosen as appropriate. Maintaining the system involved so much effort that further evaluations were not made, and the one-minute interval was used throughout the study.

The equipment was actually operated as early as August, 1973, but various problems prevented full scale operation until late October. Data collection continued throughout the fall; however, deterioration of sensors caused several short periods of down time and continued degeneration finally forced a stoppage at the end of December. Between then and the middle of March, 1974, the system was completely overhauled and partially revised. Data collection resumed in March and continued into July. This period was also marked by intervals of inoperation due to maintenance problems. It should be noted that nearly all of the functional problems encountered over the entire data gathering period (October through July) were caused by sensor failures. Malfunctions of the custom-made thermistor

probes caused the most trouble, and failures of the velocity meter were also prevalent. The performance of the digital logic controls and most of the signal conditioners was laudable throughout the data taking period.

Positioning the probe at various depths was part of the data collection plan; however, throughout the fall it remained at or near the surface most of the time, because such low velocities were being measured even there. During the spring series of collection the probe was positioned at various depths. The site was visited every two or three days so that system performance could be checked. At these times the accumulated data were removed, a new tape begun, and the probe moved to a new level.

#### Data Reduction

The data stored on punched paper tape were collected from the field research site every two or three days. Such a segment of data was termed a run. During the data collection period well over one hundred runs were made. Because the local digital computing facility did not have an on-line paper tape reader, an off-line device was used to transfer information from the paper tape to magnetic tape which could be read directly into the computer. The transfers were made one run at a time onto a small magnetic tape, and when four or five runs had been transferred, they were submitted to the digital computer. Initially, the data were checked for sequencing errors. The mode of recording the data in the field included a constant data value as one of the Group B parameters. The checking program

scanned the entire run to make sure this value occurred at the proper place in every round of Group B (a thirty-minute recording period).

If the appropriate value was not found, the program provided a diagnostic report of the tape status which aided the locating of sequencing errors on the paper tape. The paper tape was then examined and the cause of the problem detected so that corrective measures might be taken. After taking the requisite action, that run was transferred to magnetic tape again and re-checked. Once runs were certified as satisfactory in magnetic form, they were copied onto a permanent magnetic tape. After all of the runs had been added to this permanent tape, another copy was created with the runs in chronological order.

To this point the data still existed in raw form; that is, the magnetic symbols on the tape were merely digitally coded representations of the voltage levels originally seen and recorded by the paper tape punch, in the same sequence as originally recorded, and still arranged according to runs. The next data reduction process involved the creation of a final data tape in a form which facilitated further analysis. Creating this tape entailed several data manipulating operations. One operation was the conversion of the raw data to the values of the parameters each entry stood for. Supplementary to this endeavor was the sorting of the sequential values and the assigning of each to that parameter it represented. Also, since each round of Group B parameters consisted of thirty values with one taken each successive minute, these values were

linearly adjusted to account for the different times of measurement. All values were referenced to the time of measurement of parameter number one. This meant a value of parameter fifteen, taken fourteen minutes later, was averaged with the value of parameter fifteen recorded during the last half-hour round, taken sixteen minutes before the beginning of the current round. The values were weighted according to position in the round, thus

$$P_i' = P_i - (P_i - PP_i) \cdot (1/30) \quad (6-1)$$

where

$P_i'$  = time-adjusted value of the parameter in the i-th position in the thirty-minute round

$P_i$  = value of the i-th parameter measured during the round being considered

$PP_i$  = value of the i-th parameter in the preceding round (i.e., thirty-minutes before  $P_i$ )

Yet another facet of this phase in the data reduction process was to fill in short gaps in the data that occurred between the end of one run and the start of the next. This was only done to the Group B data and then only when the period of missing data was less than ten hours. Linear interpolation between the end points of the gap was used to create data for each half-hour period missing. The changes in these environmental parameters were small over the short time spans and the interpolation should not have introduced significant errors. For gaps longer than ten hours an obviously erroneous value was assigned to each data storage location when the data were transferred to the final stored form on still another magnetic tape. By

using this procedure, a data storage location was created for every parameter for every half-hour period of each of the two data collection terms, and subsequent use of the tape was made easier because of this consistent format. Gaps in the Group A data could not be filled because it was only useful as instantaneous information; therefore, every gap was filled with obviously erroneous values as the final storage tape was created.

The supplementing of the data as just described applied to those periods between runs, (i.e., no data existed). There were also some periods during runs when one or more meteorological sensors were inoperative even though the remainder of the system was working. Supplying suitable data for these instances was accomplished by using hourly surface weather observations taken at the Gainesville, Florida, airport located five miles south of Lake Mize and by using analytical expressions for calculating incident short wave radiation. In both cases measurements made at Lake Mize at other times were used to calibrate the application of the airport data.

The final magnetic tape volume consisted of four sections of Lake Mize data. For the two terms of data collection, October through December, 1973, and March through July, 1974, there were separate sections for the Group A and Group B data. The data were stored in segments of one day's values which contained forty-eight consecutive half-hour groupings. Analysis could be performed by first accessing the data according to day of the year and then locating a specific round for that day. The Group B data were

time-adjusted to remove the effects of the values being taken at one-minute intervals over a half-hour time period, and all the data were stored as values of the parameter they represented, e.g., velocity in meters per second, temperature in degrees centigrade, etc. Information regarding the format of the data as they are stored on magnetic tape and a description of the methods to be used to access it are given in Appendix A.

CHAPTER VII  
RESULTS OF ANALYSIS

Introduction

The first sections of this chapter include the results of the flux-gradient analysis of diurnal heat transport. A study of convective cooling is presented initially. This is done by examining the temperature profiles taken every half-hour during a fall night. This detailed knowledge of the thermal structure is used to permit calculations of diffusivities throughout an entire daily cycle and the results are quite satisfactory. Attention is then given to the correlation of fluctuations scheme. Irregularities detected during the early stages of analysis are pointed out and followed by a presentation of the methods used to effectively seek out and explain the sources of the questionable behavior. The planned comparisons of the performance of the two methods are shown and the results are discussed. The remaining objectives of this study are accomplished in the final section of the chapter. The variations of diffusivities over long intervals of fall and spring data collection are presented, and the variations of diffusivities with depth are also reported.

### Flux-Gradient Analysis

The development of the procedures planned for evaluating diffusivities by the flux-gradient method was detailed in Chapter III. Also, a test of the procedures using data taken at Cayuga Lake was described. The field data from Lake Mize were stored on magnetic tape such that a temperature profile measured at the lake at any of forty-eight times each day could be used for flux-gradient analysis. Data for two such profiles were to be accessed by the computer and subjected to the non-linear least squares algorithm so that analytical expressions for temperature as a function of depth could be determined and then used to calculate heat content and gradients for each time. The use of digital computers would enable many such calculations to be made handily, and consequently several aspects of diffusion coefficient behavior could be examined. Variations of diffusivities with depth, variations of diffusivities with time, and comparison of diffusivities calculated by both the flux-gradient and correlation of fluctuations methods were planned.

### Inadequacy of the Flux-Gradient as Typically Applied

In Chapter II it was noted that the flux-gradient method is typically not used to define diffusivities during that portion of the yearly thermal cycle when lakes are cooling. Also, the basis for this was explained in general terms. The data collected at

Lake Mize during the fall of 1973 may be used to examine the problem in detail. Figure 7-1 gives temperature profiles measured at the lake one week apart near the end of October, 1973. Also indicated on the figure are temperatures predicted at selected depths by the regression procedure, and as can be seen, the predicted temperatures closely approximate the curves fit by eye to the measured data points. Although unsatisfactory performance was anticipated, the methods developed for computer analysis of diffusivities were used to evaluate the average diffusion coefficients during the week shown, and some of the results were in fact unrealistic. Values below four meters were satisfactory, but upper values were negative. The cause of this occurrence can be understood by considering the profiles in Figure 7-1. In the upper regions there is a pronounced loss of heat during the week interval; however, the slopes of the curves are negative at every point. The negative gradients indicate the transport of heat downward, and even though the lower waters did warm somewhat, a comparison of the increase in area below about four meters versus the decrease in area above indicates a net loss of heat. The values of solar radiation for both days were nearly equal so subtracting radiation effects does not affect the situation demonstrated. The inappropriate results indicate the deficiency in the flux-gradient method when a one-week averaging interval is used. The inability of the method to satisfactorily describe the heat loss from the lake is caused by the non-linear behavior in the changes of the thermal structure between the times

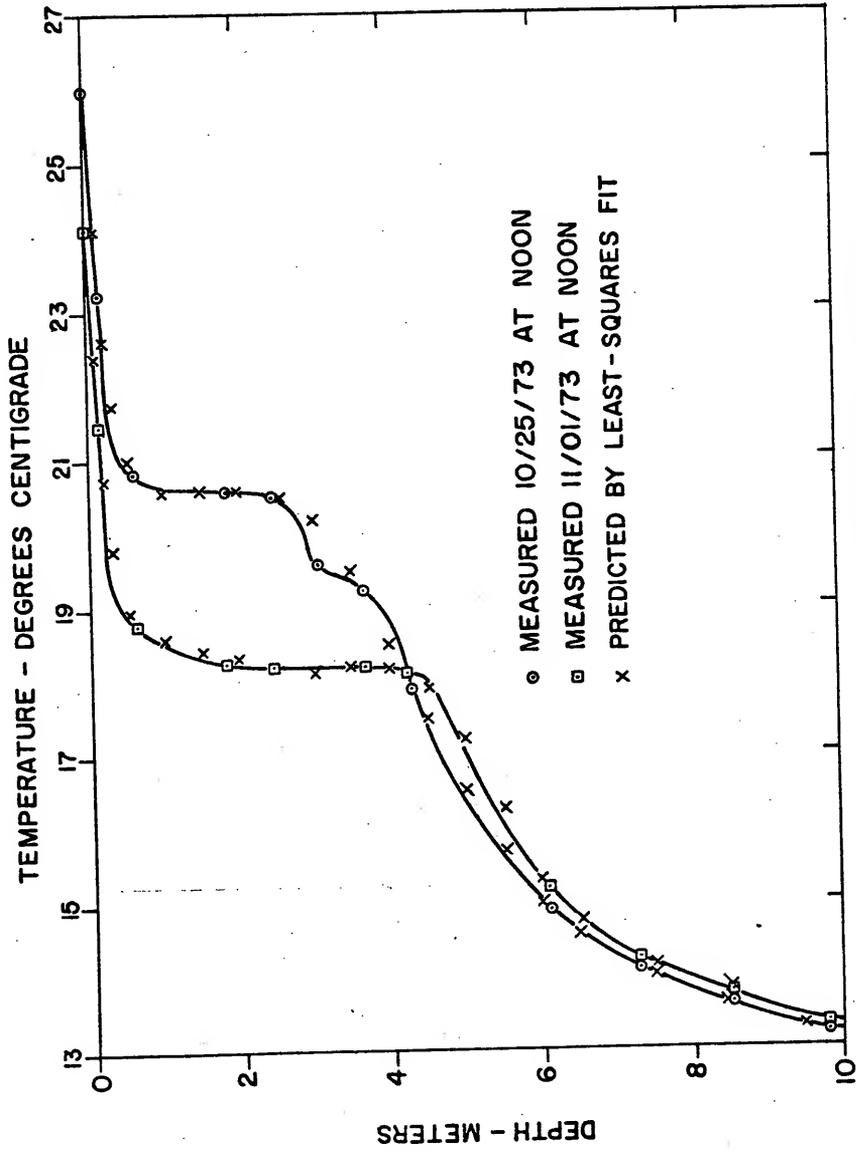


FIGURE 7-1. TEMPERATURE PROFILES AT LAKE MIZE

the two profiles were measured. The suitability of using the end point values of a given time span for making calculations was discussed as part of the presentation of the flux-gradient method, and there it was pointed out that the calculation of the gradient term should be made using data taken often enough to reflect the changes in temperature profiles. The average values of temperature gradients do not accurately reflect the mean values of the gradient during the period. In other words the averaging interval selected is not appropriate to the scale of the turbulent transport process occurring.

#### Detailed Examination of Convective Mixing

As already stated it is the contention of this work that the typical inability of the semiempirical concepts of turbulent diffusion coefficients to describe convective mixing is due to the use of improper averaging times. To support this contention a detailed examination of overnight cooling in Lake Mize is presented. Figure 7-2 shows temperature profiles measured at the lake during the night of October 25 and the early morning of October 26, 1973. The profiles are identified by numbers which represent the half-hour period (round) during which they were recorded, (i.e., 45 stands for hour 22.5 or 10:30 pm). As evening progresses the effect of convective cooling is documented by each successive profile. The earliest time depicted in Figure 7-2 is round 30, or three o'clock in the afternoon, when the surface temperature reaches 25.7 degrees

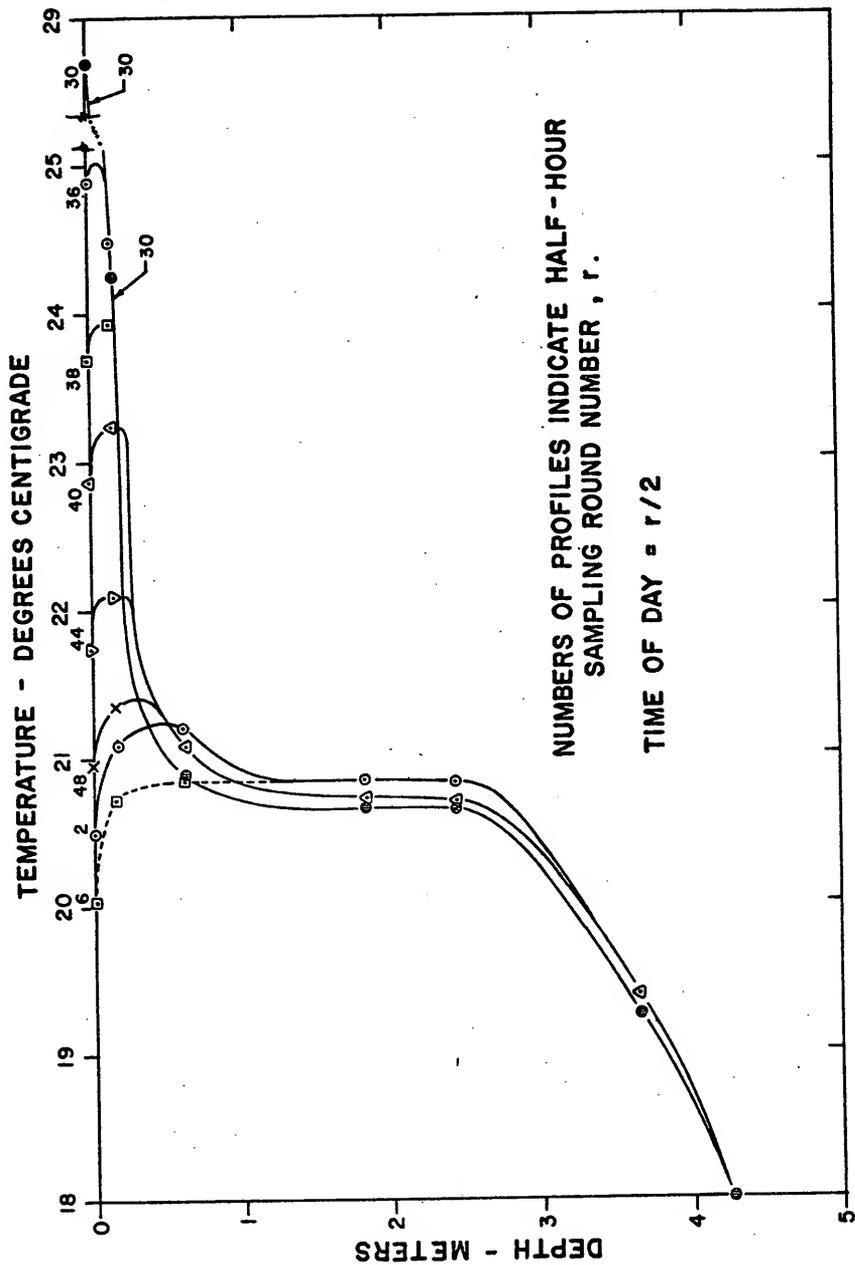


FIGURE 7-2. TEMPERATURE PROFILES DURING A PERIOD OF CONVECTIVE COOLING AT LAKE MIZE, OCTOBER 25-26, 1973

centigrade, and subsequent surface cooling can be noted. Also apparent is the continued heating of the subsurficial waters even as the surface is losing heat. This lower heating continues until after midnight.

The data points measured in the lake are indicated on the graph and the profile lines have been fit by eye to these points. As may be readily noted, the lines have been drawn using some degree of creative license which stems from postulating convective cooling mechanisms. During the period represented it is assumed that the effects of mixing are felt at progressively lower levels as cooling proceeds. At any given time there is a point to which the influence of the process has extended and beyond which no effects are noted. This then makes the definition of the subsurface heat gain important because the portions of the warmest temperature profiles below the surface will define the shape of later profiles below the point of influence of convective mixing. This mechanism is depicted in Figure 7-2. For example, the profile for round 38 is shown intersecting the profile for round 30, and since the temperature measured at all depths below the intersection depth are the same for both times, the round 38 profile is assumed coincident to the one for round 30 below the intersection. The limiting profile warmed slightly by the time round 40 was recorded, and the profile for rounds 40, 44 and 48 are shown meeting it. It can be seen that each successive intersection occurs at a greater depth. The profile for round 2 can be seen joining what seems to be the warmest lower profile, and the line for round 6 meeting the same curve at a

lower depth. Although the continued transport of heat downward causes the base or limiting profile to change slightly during the cooling phenomena, the scheme of the process as postulated can be discerned. In summary, a gradual erosion of the warmest profile takes place as the cooling process advances the extent of depth affected. It is thought that if more closely spaced data points were available, they would confirm the shapes of the profiles as speculated above. However, since this additional definition is not at hand, the profiles (as proposed) can only be drawn by eye giving consideration to both the temperature data recorded and the position of the warmest antecedent profile. For this reason, the procedure of approximating the profiles with least-squares curves or straight-line interpolations will not yield satisfactory results without a greater number of data points.

A better understanding of the cooling mechanisms may be gained by considering the process in greater detail. Therefore, some of the profiles shown in Figure 7-2 and intermediate profiles have been plotted on an expanded scale and are shown in Figure 7-3. Every profile recorded from midnight through 8:00 a.m. is presented, as well as three taken before midnight. The greater resolution of Figure 7-3 permits a closer examination of the diurnal phenomena. The progressive erosion of the limiting profile can be seen occurring through round 6 in an orderly fashion. However, the data for round 7 indicate an unusual profile shape. Despite this and some subsequent irregularities, the continued eroding of the limiting profile is evident through round 16. Apparently, the extent of effect is about

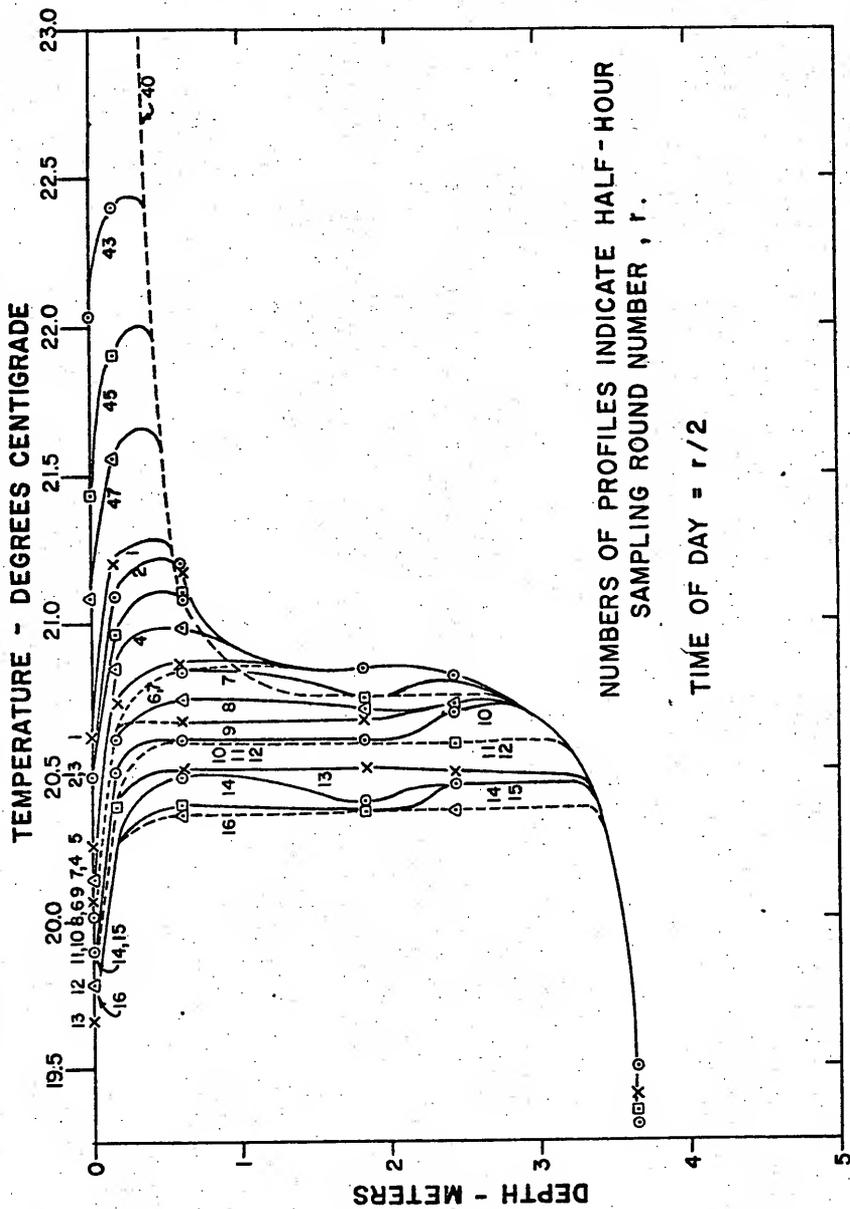


FIGURE 7-3. DETAILED TEMPERATURE PROFILES DURING A PERIOD OF CONVECTIVE COOLING AT LAKE MIZE, OCTOBER 25-26, 1973

3.4 meters. Later profiles were not plotted because they indicate the reversal of the cooling process, and the points would overlay those points shown. Suffice it to say that round 16 is the last round during which convective cooling occurred. By plotting data for consecutive rounds as is done in Figure 7-3, a better insight into the shape of each profile is gained, and the fit of the profile curves to the data as shown seems apropos.

Further examination of Figure 7-3 yields more insight about the convective mixing taking place. It is important to remember the way the data were recorded at the lake. The thermistors were scanned at the rate of one per minute. Represented in Figure 7-3 are values for thermistors 1, 2, 5, 7, 8 and 9 (thermistors 3, 4 and 6 had ceased functioning). Even though during data reduction procedures the data for each round were adjusted to account for this lag (i.e., thermistor 5 being sampled three minutes after thermistor 2), the adjustments were linear interpolations. Non-linear changes in temperature during the interim between recordings may have affected the data. The time lag may account for the vertical nature of the profiles from about 0.5 to 3.0 meters of depth. Had the thermistors been scanned nearly instantaneously slightly cooler measurements should have been noted with decreasing depth; however, the convective circulation probably altered the thermal regime between the times during which measurements were made.

Since each of the profiles shown in Figure 7-3 was recorded at equally-spaced time increments, the irregular spacing and shapes of the profiles indicates the likelihood that convective mixing

is intermittent in nature. While the loss of heat by evaporation and conduction at the water surface is an on-going process, the submerging of cooled water and the consequent rising of warmer water does not seem to proceed at a uniform rate. On the contrary, the varied spacing and shapes of consecutive profiles suggest that a certain inertia must be overcome before a mixing event happens. It appears that once a threshold level of instability is exceeded, transport takes place followed by a return to relative quiescence.

Of particular interest are the profiles for rounds 7 and 14. As can be seen the temperatures of each at  $z = 1.83$  meters is cooler than the temperatures at  $z = 0.61$  meters, yet by the time the following rounds are recorded even the upper temperatures are cooler than the lower ones at rounds 7 and 14, and a return to the expected profile shape soon occurs. What appears to have happened is that an eddy has been detected in action. The timing of the data recording was such that one of the discrete exchanges of surface and lower waters has begun during the two-minute interim between temperature measurement at the two depths. What has been monitored is a warmer upper temperature before the discrete mixing begins and a cooler lower temperature measured later and indicating the presence of fresh cooler water from above which has just arrived at the lower position. It seems that the periodicity of these discrete mixing events is such that they can only be detected by the thirty-minute sampling period every 3.5 hours.

With the exception of these atypical profiles just discussed, it seems that the surface cooling processes in the lake take place

in a manner describable by the flux-gradient method. Examination of Figure 7-3 reveals that between consecutive profiles, the transport of heat is in the direction of negative gradient. Therefore, it can be concluded that proper definition of the profile shapes will yield flux-gradient quantities which will not give negative values of diffusion coefficients.

#### Manual Calculations of Diurnal Values of Diffusivities

As was noted above the curve fitting techniques could not satisfactorily be applied to the Lake Mize data during periods of convective cooling during the fall. The reason for this is that the particular data points available do not provide enough information about the temperature changes with depth. The malfunctions of thermistors 3, 4, and 6 located at 0.3, 0.45, and 1.22 meters, respectively, proved costly because the absence of those measurements has severely restricted the application of the special methods developed to provide extensive diffusivity calculations. Therefore, the only technique available for flux-gradient calculations during the fall when convective mixing caused unwieldy profile shapes was that of plotting the profiles and manually determining areas and slopes. Because of the laborious nature of such a process, the number of calculations of diffusion coefficients had to be greatly reduced. However, the manual method was applied to the twenty-four hour period covered by the profiles graphed in Figures 7-2 through 7-5. Figures 7-2 and 7-3 have already been described. Figure 7-4

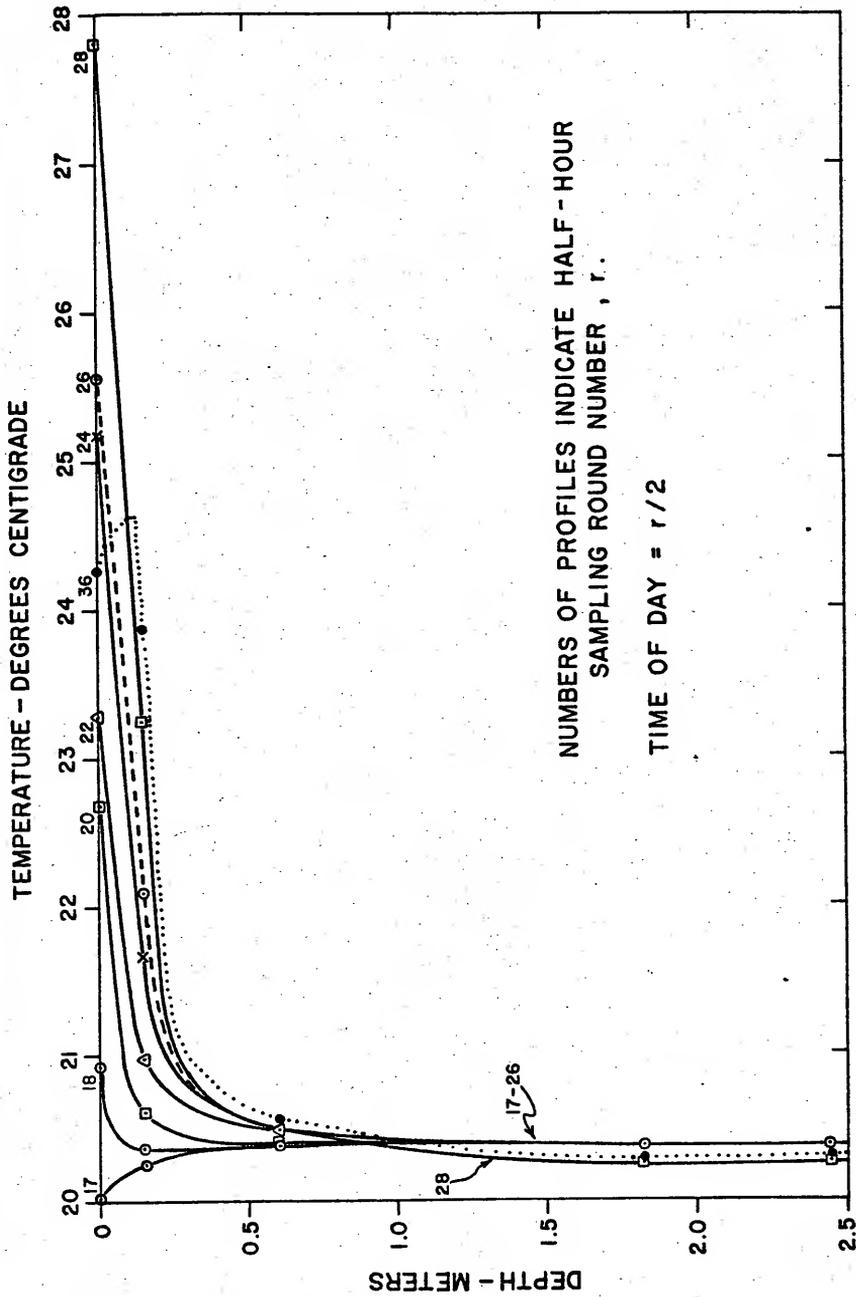


FIGURE 7-4. MORNING AND AFTERNOON TEMPERATURE PROFILES IN LAKE MIZE, OCTOBER 26, 1973

illustrates the thermal behavior of the upper portion of the lake throughout the daylight hours on October 26, 1973, beginning with round 17 and continuing through round 36. Only the upper portion is shown because changes in the lower strata were too small to be detected. Figure 7-5 shows temperatures over the entire water depth at the beginning and end of the period being considered. As can be seen the one-day change in heat content of the lake as shown in Figure 7-5 is very similar to the one-week changes shown in Figure 7-1.

In order to illustrate the diurnal variations in rates of heat transport, diffusivities of heat have been calculated for various times between 1800 hours on October 25 and 26, respectively. Only one epilimnic point, 0.5 meters, was examined due to the extensive nature of the computations required in that region. Three hypolimnic points, 4, 6, and 8 meters, were considered, but the 0.5-meter depth was chosen because the thermistor probes at 0.15 meters and 0.61 meters yielded sufficient data to define the temperature profile in that region well. The results of the diurnal analysis for the 0.5-meter depth are presented in Table 7-1. No diffusivity was calculated for the daylight hours between rounds 17 and 36. As can be seen in Figure 7-4 there is a slight gain in heat content below the 0.5-meter depth; however, when the values of solar radiation were subtracted, the result was zero flux.

The values of diffusivity given in Table 7-1 vary considerably throughout the days. Generally, the higher values occur at night during the period of convective cooling and mixing. However, even

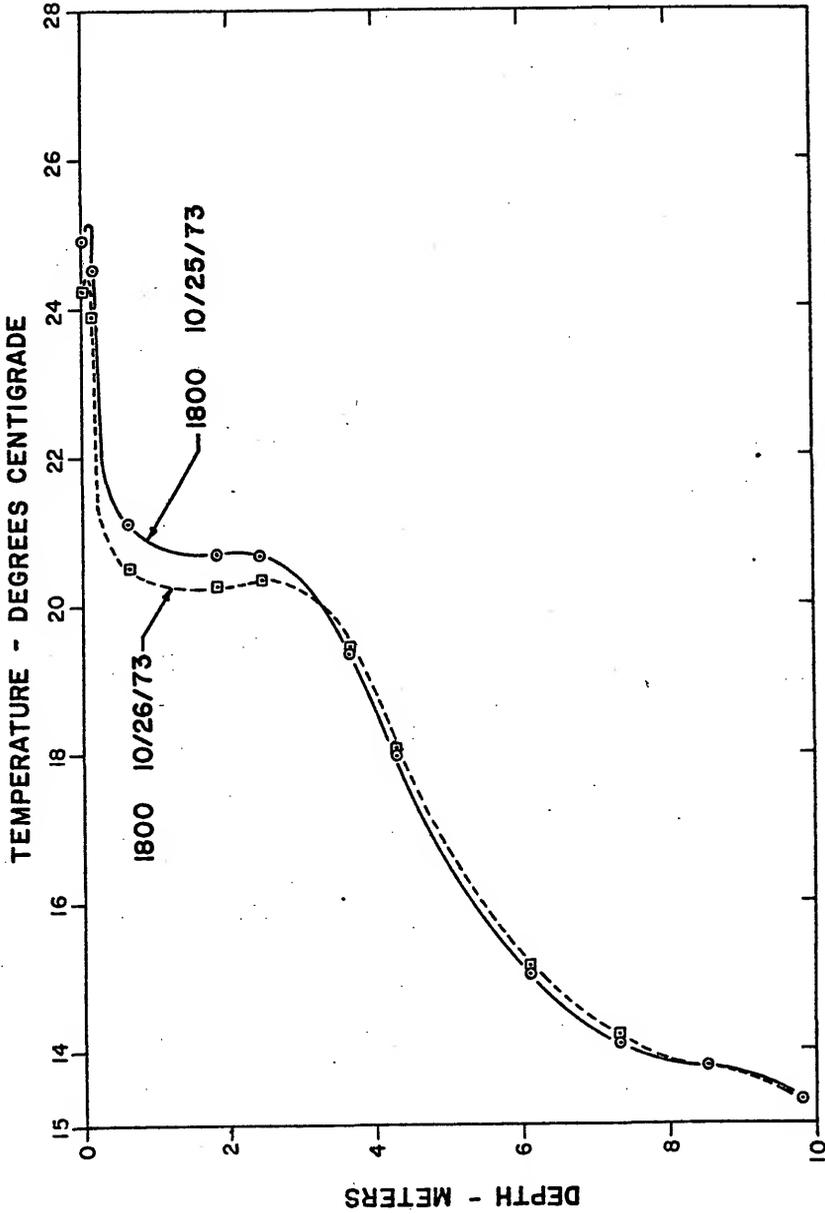


FIGURE 7-5. TEMPERATURE PROFILES IN LAKE MIZE FOR OCTOBER 25 AND 26.

TABLE 7-1. DIURNAL VALUES OF DIFFUSIVITIES AT 0.5 METERS IN LAKE MIZE,  
1800 OCTOBER 25 TO 1800 OCTOBER 26, 1973

Between Rounds	Time Interval days	Difference in Area °C-m	Average Slope of Profiles °C-m	Diffusivity m <sup>2</sup> /day
36 - 38	0.042	0.0	---	---
38 - 40	0.042	0.390	- 1.400	6.60
40 - 47	0.166	0.0	---	---
47 - 1	0.042	0.200	- 8.300	0.57
1 - 2	0.021	- 0.004	0.008	25.00
2 - 3	0.021	- 0.013	0.056	11.00
3 - 4	0.021	- 0.026	0.081	15.00
4 - 5	0.021	- 0.057	0.099	28.00
5 - 6	0.021	- 0.008	0.110	3.50
6 - 7	0.021	- 0.076	0.180	20.00
7 - 8	0.021	- 0.162	0.180	43.00
8 - 9	0.021	- 0.103	0.060	83.00
9 - 10	0.021	- 0.148	0.004	1800.00
10 - 11	0.021	- 0.107	0.031	170.00
11 - 13	0.042	- 0.271	0.078	185.00
13 - 14	0.021	- 0.156	0.100	75.00
14 - 15	0.021	- 0.112	0.130	41.00
15 - 16	0.021	- 0.133	0.150	43.00
16 - 36	0.418	---	all negative	---

within this period there seems to be much variation of values. It can be noted from the quantities given in the table that the factor which appears to cause the greatest part of the variations in diffusivity values is the value of average slope. Table 7-2 gives diffusivities found in the hypolimnion. While lower than epilimnic values, they are at least ten times molecular levels.

Although during periods of rapid convective cooling in the lake diffusion coefficients had to be calculated by manual graphical methods, other periods were suitable for analysis by the more convenient computer-oriented approach. Useful results could be also obtained during the fall at the lower depths, because the lower profiles could be approximated well with the analytical techniques. The results of these analyses are presented later in this chapter.

#### Correlation of Fluctuations

Diffusivities of heat have been evaluated using heat flux values calculated by the correlation of fluctuations method discussed in Chapter II. This analysis was performed on the Group A data taken at Lake Mize during both fall and spring. What follows is a discussion of the analysis procedures used and a presentation of the results obtained.

The evaluation of the heat flux entailed the computation of the time average of the product of temperature and vertical water velocity fluctuations as given by the expression

$$H_{cf} = \rho c \frac{\sum_{i=1}^n \theta_i' w_i'}{n} \quad (7-1)$$

TABLE 7-2. HYPOLIMNIC DIFFUSIVITIES IN LAKE MIZE,  
1800 OCTOBER 25 TO 1800 OCTOBER 26, 1973  
TIME INTERVAL = 1 DAY

Depth	Difference in Area	Average Slope of Profiles	Diffusivity
m	°C-m	°C-m	m <sup>2</sup> /day
4	0.747	- 2.20	0.34
6	0.308	- 1.10	0.28
8	0.031	- 0.24	0.13

- where  $H_{cf}$  = heat flux as given by the correlation of fluctuations method (energy-distance (vertical) - time<sup>-1</sup>)
- $\theta_1'$  = temperature fluctuation
- $w_1'$  = vertical velocity fluctuation (distance - time<sup>-1</sup>)
- $n$  = number of pairs of temperature-velocity data
- $\rho$  = density of water, assumed constant at 1 gram - centimeter<sup>-3</sup>
- $c$  = specific heat of water, assumed constant at 1 calorie - degree centigrade<sup>-1</sup>-gram<sup>-1</sup>

The values of  $\theta_1'$  and  $w_1'$  were determined by the relationships

$$\theta_1' = \theta_1 - \bar{\theta}, w_1' = w_1 - \bar{w} \quad (7-2)$$

The overbar denotes time averages which were calculated using

$$\bar{\theta} = \sum_{i=1}^n \frac{\theta_i}{n}, \bar{w} = \sum_{i=1}^n \frac{w_i}{n} \quad (7-3)$$

where  $\theta_i, w_i$  = the discrete values of temperature and velocity, respectively, measured coincidentally in the field.

Because the sample rate for the Group A data taken at Lake Mize was one per minute, the period over which temperature-velocity data are averaged using equation 7-3 is equal to "n" minutes. Therefore, the quantity computed using equation 7-1 is the average heat flux over "n" minutes.

Making these calculations from the Group A Lake Mize data was a straightforward process using a digital computer. The data were stored in sequential form with coincident measurements easily accessible. The procedures for making the heat flux calculations involved specifying a beginning time and an averaging time interval. The corresponding segment of stored data was then located and used by the computer to first compute average values and then re-used

to find fluctuation quantities which were then multiplied together and time-averaged.

The aspect of the analysis procedure which posed one of the biggest problems was that of deciding what averaging interval to use. The stipulations regarding proper averaging intervals were that the interval used should be long with respect to the period of turbulent fluctuation phenomena and short enough to span a period of constant mean values. Because one-minute intervals were used to collect the data, and since the data were stored in blocks of thirty measurements each, the minimum length of the averaging intervals was selected as thirty minutes. The behavior of the temperature variations in the lake, at least for the epilimnion and metalimnion, was known to be diurnal, so the mean of the temperature data varied approximately sinusoidally with a period of twenty-four hours. Using half-hour intervals seemed appropriate because the diurnal effects should have been insignificant during any one interval.

Once the heat flux was calculated, the average diffusivity over the averaging period used was computed using gradient values derived from the coincident temperature profiles. The slope of the profile at the depth of the velocity probe was used as the gradient, and the diffusion coefficient,  $K_d'$ , for that depth was given by

$$K_d = \frac{H_{cf}}{(\partial\theta/\partial z)_d} \quad (7-4)$$

where the subscript d denotes the depth at which the temperature and

velocity measurements were made.

Figures 7-6, 7-7, 7-8, and 7-9 are plots of heat fluxes computed by correlating velocity and temperature fluctuations using an averaging time of thirty minutes. The computations for every half hour from midnight to midnight are shown. In Figures 7-6 and 7-7 fluxes for three days during the fall series of data are shown. Throughout the fall the probes were near the surface, and in the upper waters nightly convective cooling should have taken place. The other plots are of spring data with the probe near the surface in Figure 7-8 and at 4.8 and 3.2 meters in Figure 7-9. Note that the heat flux scales in the plots of the fall data differ from the scale of the other graphs and also that there is a gap in the data occurring about midday in Figure 7-6 because the field equipment was temporarily down. Comparison of the various plots indicates higher levels of fluxes in the fall than in the spring, and a diminishing of flux level with increased depth. Another observation is that there seems to be the greatest range of values occurring around noon of each day.

Earlier in this chapter the diurnal thermal behavior of Lake Mize was discussed in detail. Then it was shown that temperatures in the upper layers of the lake increase markedly throughout the morning, begin cooling in the middle of the afternoon, and continue to cool and mix throughout the night. Verification of this pattern is not found from the calculated quantities depicted in Figures 7-6 and 7-7. Consistently positive fluxes, indicating downward transport of heat during daylight hours, and negative fluxes, indicating

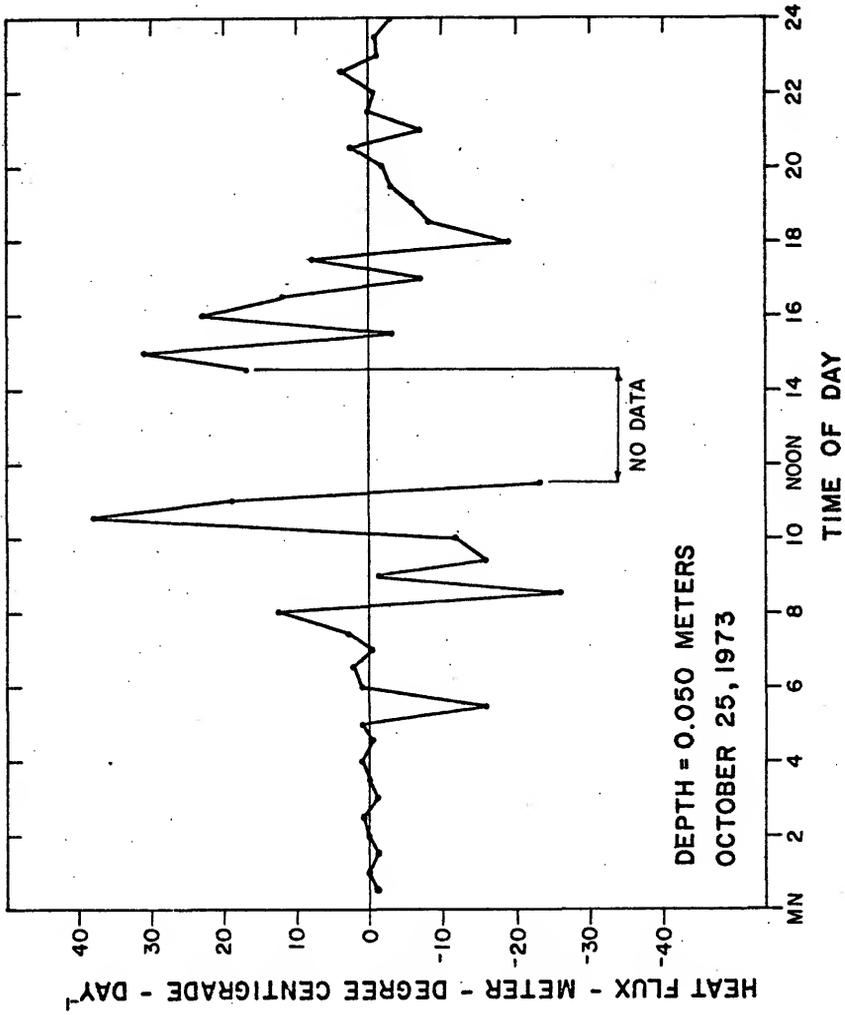


FIGURE 7-6. . . HEAT FLUXES FROM CORRELATIONS OF FLUCTUATIONS

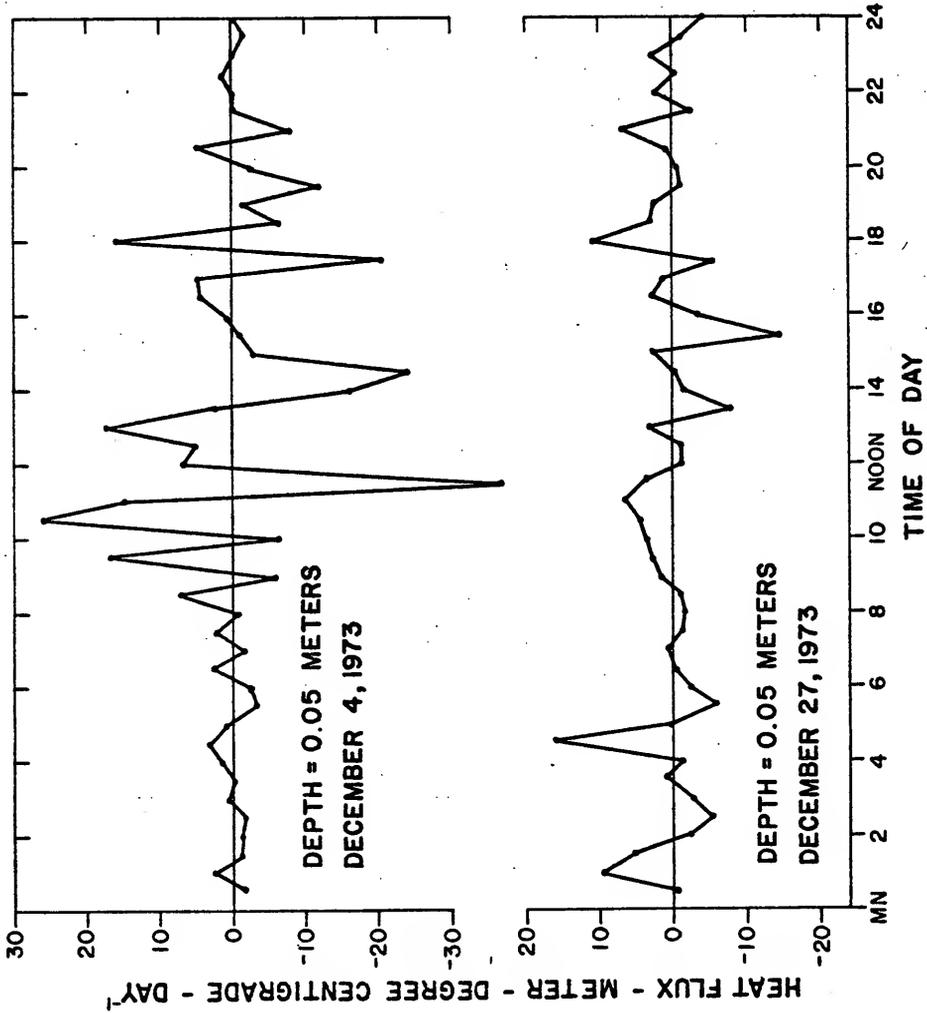


FIGURE 7-7. HEAT FLUXES FROM CORRELATIONS OF FLUCTUATIONS

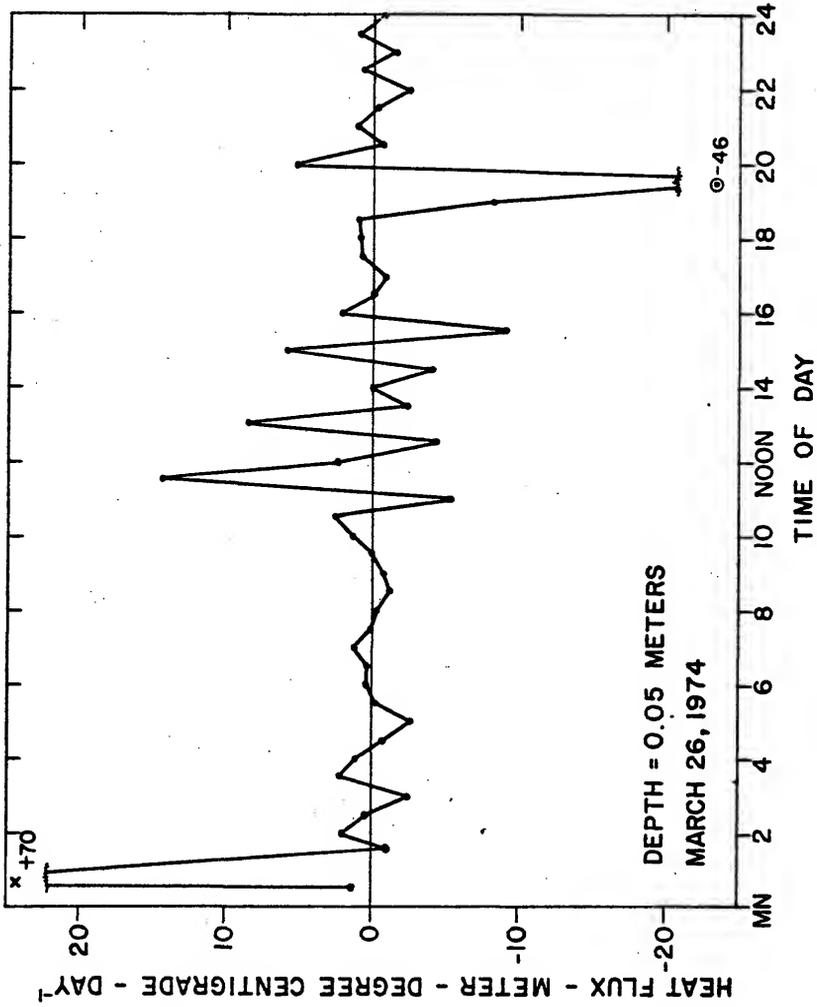


FIGURE 7-8. HEAT FLUXES FROM CORRELATIONS OF FLUCTUATIONS

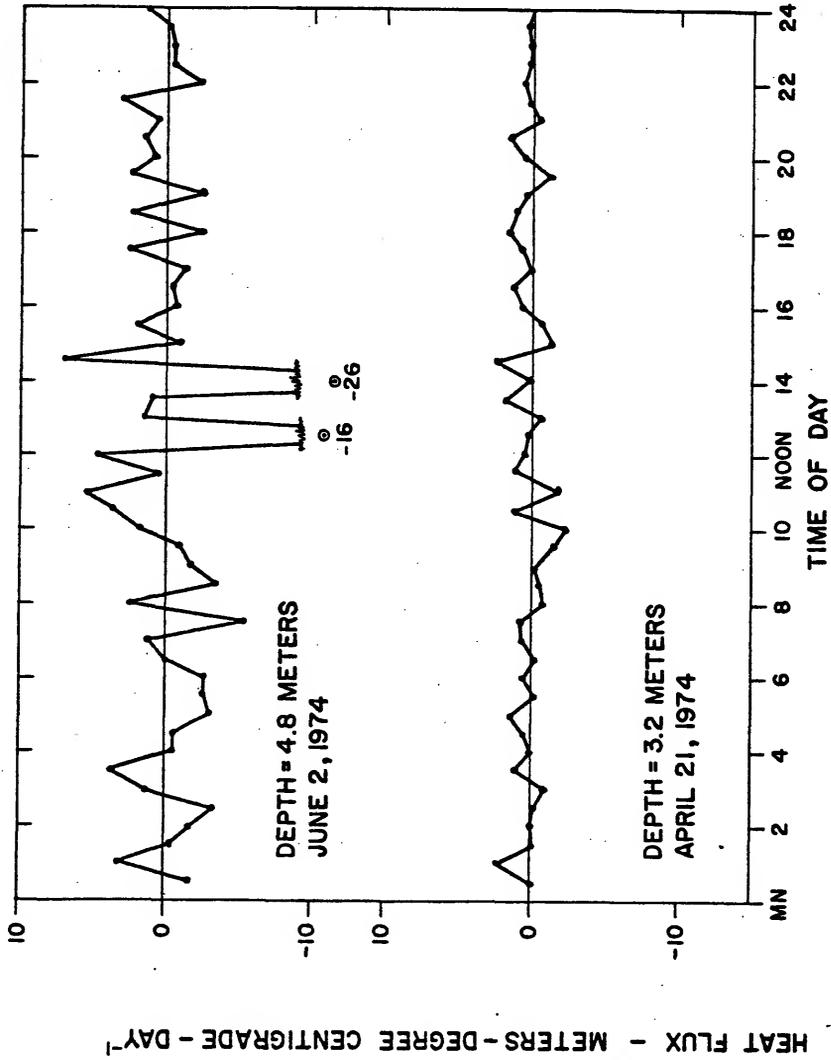


FIGURE 7-9. HEAT FLUXES FOR CORRELATIONS OF FLUCTUATIONS

upward transport during night hours are not evident. Instead, each plot reveals continuous positive to negative variation in the fluxes with the largest amplitudes existing during midday and smallest amplitudes existing at night. This variability in the fluxes made evaluating diffusion coefficients difficult. As indicated in Table 7-1 gradients determined every half-hour do not alternate between positive and negative values frequently during the daily cycle. Therefore, if the quotient of the heat flux and profile slope terms are used to calculate diffusivities as indicated by equation 7-4, many of the values will be inappropriately negative. An examination of temperature profile data for the spring days depicted in Figures 7-8 and 7-9 showed the same thing. For instance, at the depths shown in Figure 7-9 the gradient remained negative throughout the day, yet the fluxes vary to both positive and negative values which would certainly yield negative diffusivities. It seemed that meaningless results were being obtained. Along with the fluxes standard deviations of velocity and temperature had been calculated, and these were very low. This suggested that either the thirty-minute averaging interval was inappropriate or that the data themselves were inadequate. Therefore, it was decided to examine directly the velocity and temperature data (one value of each recorded every minute) in an effort to identify the causes of the inconsistencies cited above and then determine whether or not the data could be used for diffusivity evaluations.

Measurements made at the rate of one per minute yielded 1440 data values per day per parameter, so the presentation of graphs

covering long time periods is infeasible. However, three short time intervals at different portions of a single day have been detailed in the plots shown in Figure 7-10. The day was chosen at random and the hours selected such that they indicate diverse diurnal thermal conditions in an effort to present an impression of the general behavior of the temperatures and velocities during a fall day. Each interval is two hours long. The upper pair of plots span the hours 0030 to 0230, the middle pair 0700 to 0900, and the lower pair 1200 to 1400. Much more of the data were examined using graphs made on a digital computer printer and requiring much less manual effort to produce. A survey was made to determine whether or not the use of the thirty minute averaging interval was justified. As can be seen in Figure 7-10 the mean of the velocity data varies only slightly during the times shown, and therefore, thirty minutes is not too long an averaging period. The temperature data do not uniformly support or contradict this because the temperature behavior changes during the day. Most of the time the mean varies only slightly during a thirty-minute interval, but as can be seen in the lower, midday plot, there are both short and long period fluctuations which may not be satisfactorily described using half-hour averaging times. The longer periods appear to be about thirty to forty minutes long and do not appear to be correlated with the velocity data, at least over the two-hour interval shown. The shorter periods are about two to five minutes long, and a relationship to the fluctuations in the velocity record seems to exist.

Each of the intervals was analyzed in greater detail to

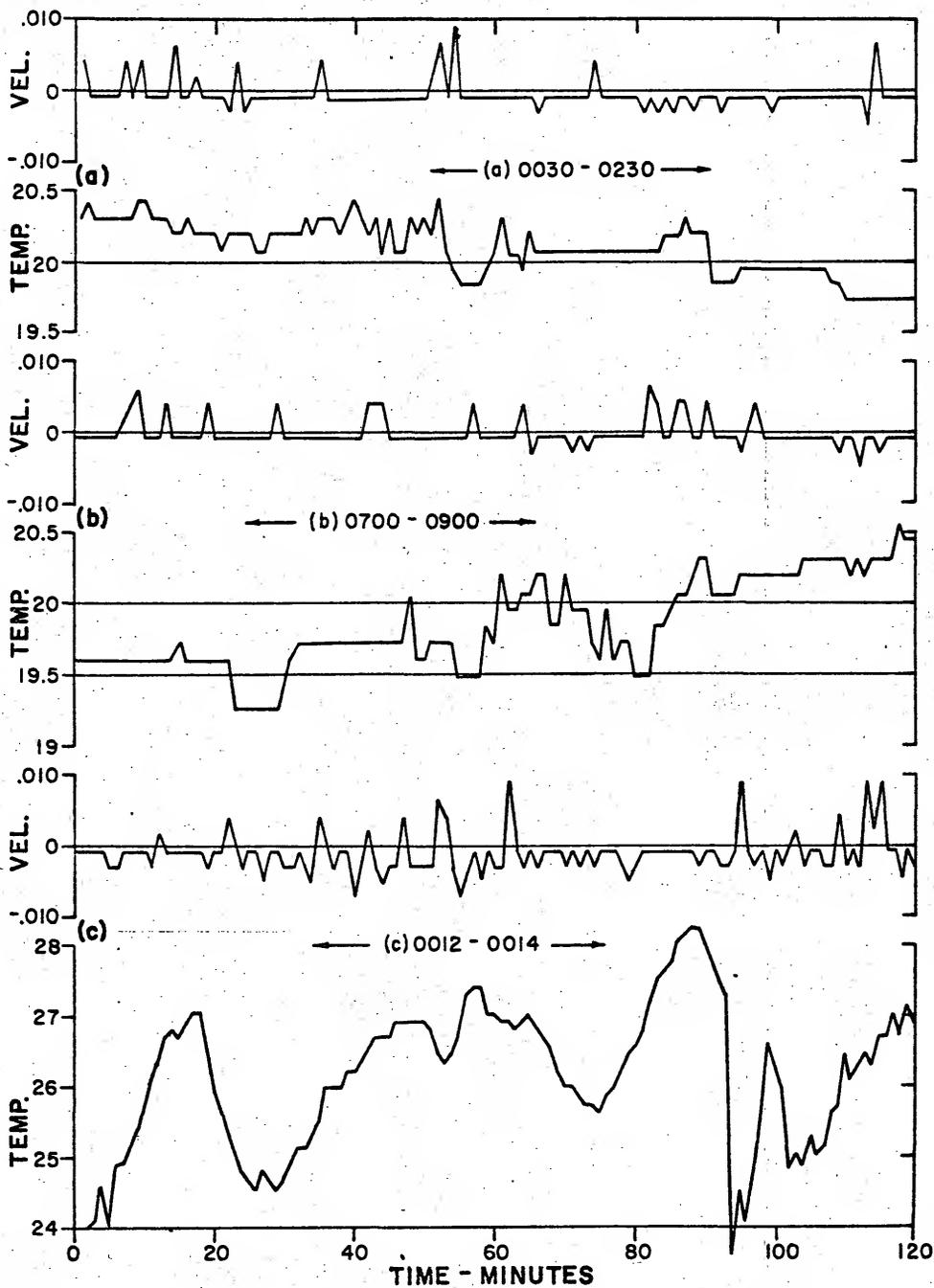


FIGURE 7-10. WATER VELOCITY AND TEMPERATURE, OCTOBER 27, 1973.

determine the effect of using different averaging times. Averages were computed over time spans of five, six, ten, thirty and sixty minutes, and subsequently fluctuations and heat fluxes were computed for each span. The results of these computations are presented in Figure 7-11, which is similar in form to Figure 7-10. The upper plot shows fluxes over the two-hour span of from 0030 to 0230 hours. The middle and lower plots are from 0700 to 0900 and 1200 to 1400 hours, respectively. Shown on each plot are the values of flux for each of the averaging intervals cited above. Flux values are plotted in the center of the time interval over which they apply, and straight line segments connect the values for the intervals of five, six and ten minutes. The results are interesting but inconclusive. As can be seen, at any one time the magnitude of the velocity-temperature cross-correlations varies greatly depending on the averaging time used. Generally, the lowest flux values are associated with the longer averaging intervals because local peak values are diminished when averaged with adjacent periods of very low flux.

Whether or not the turbulent transport processes in the lake are best defined by any one of the time intervals chosen is impossible to determine because supporting data such as complete temperature-depth profiles are available only as often as every thirty minutes. By checking the Group B profile data for the thirty minute rounds coincident with the times represented in Figures 7-10 and 7-11, the slopes of the profiles for each round at the depth of the velocity probe, 0.05 meters, were determined. For times covered by the upper

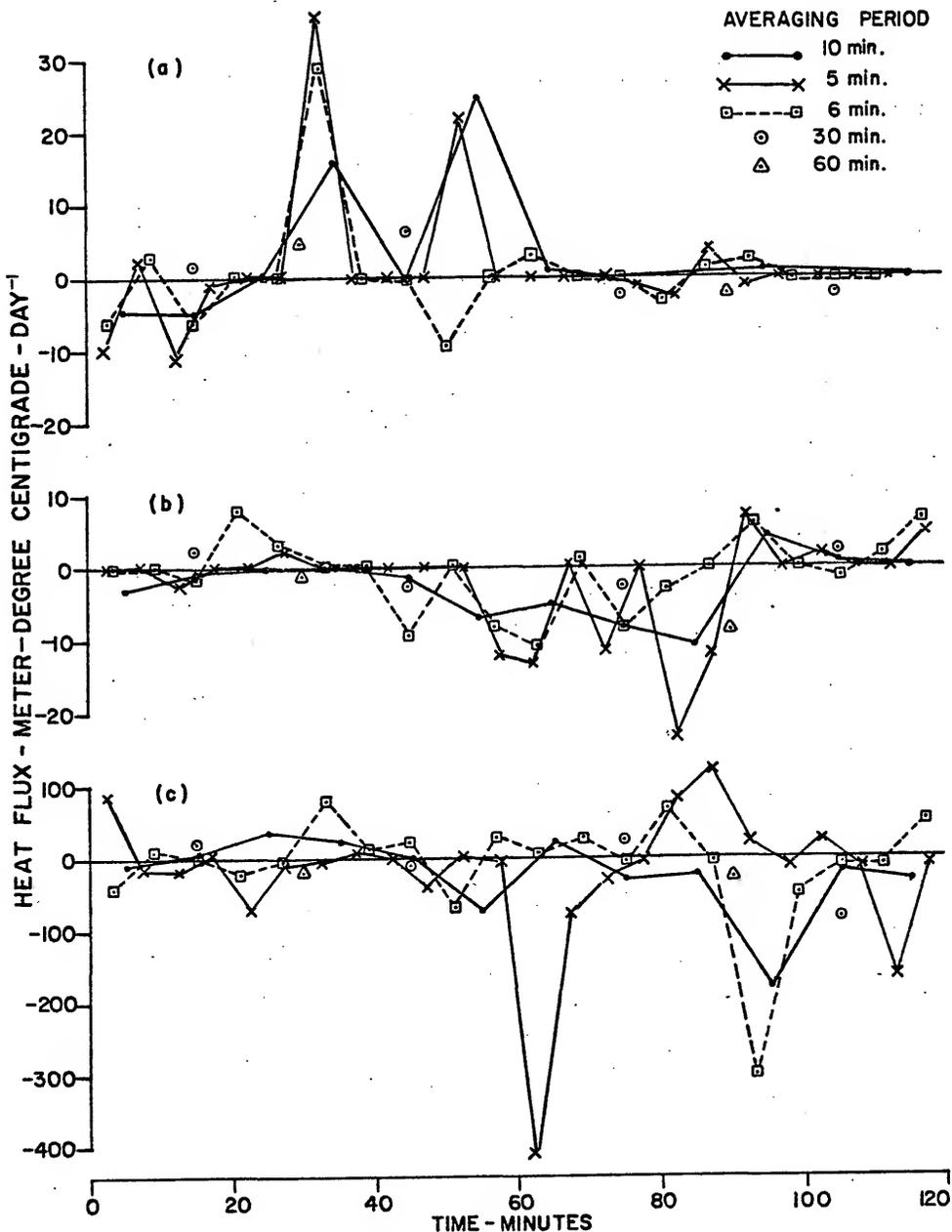


FIGURE 7-11. HEAT FLUXES CALCULATED USING VARIOUS AVERAGING PERIODS, OCTOBER 27, 1973, A) 0300-0230, B) 0700-0900, C) 1200-1400 HOURS

and middle plots, positive slopes, indicating upward heat transport, were noted while for the midday period shown in the lower plot, negative slopes were found. None of the sets of flux computations shown in the upper two plots appear to demonstrate upward heat transport (negative fluxes) better than the half-hour computations. In the top graph the inappropriately positive, thirty-minute flux values calculated for the first two half-hour periods are shown to be heavily influenced by two large short-term positive values. These may or may not accurately reflect the occurrence of brief but intense periods of downward heat transport even though the time was generally one of convective cooling. To substantiate such an occurrence additional average temperature profile data are needed. The possibility also exists that the velocity and temperature data are spurious due to the low levels being encountered and the limitations of the sensors.

The magnitude of the shorter-term oscillations shown in the bottom graph in Figure 7-10 and the corresponding fluxes shown in Figure 7-11 suggest that brief periods of intense transport contrary to general trends do actually occur. This midday period is characterized by marked variations in solar intensity, and during intervals of increased cloudiness lasting several minutes, it seems plausible that surface cooling may cause a reversal of the temperature gradient very near the surface. Supporting the probability of this occurring is the fact that often daytime winds are higher which increases rates of evaporation and reduces the likelihood of the persistence of unstable surface conditions

(i.e., thermal inversions).

In order to better assess the effects of the various averaging times the values of flux presented in Figure 7-11 were averaged over thirty-minute periods. The results are shown in Table 7-3 along with values of the gradient of temperature as given by Group B temperature versus depth data. No single set of calculations appears superior, especially in terms of agreement with the gradients (i.e., proper agreement implies opposite algebraic signs). However, the question remains of whether or not the thirty-minute slope values adequately reflect gradient behavior during the interval.

The lack of a definite conclusion regarding the velocity and temperature data for heat flux evaluation prompted continued examination. As noted above the possibility existed that the data were spurious because of limitations of the sensors causing inadequate performances under the conditions occurring in the lake. Because the signal from the velocimeter indicated such low velocities which were often at the lower end of the sensitivity capability of the device, there existed the possibility that most of the signal output from the device was noise. It was then decided to evaluate the performance of the velocity meter by determining what, if anything, could be related to the occurrence of the water movement that it detected. A visual comparison of water velocity and wind speed data indicated the possibility of some relationship. It seemed that in several instances periods of wind were accompanied by increased water velocity and periods of lull corresponded to times of little detectable water velocity. Therefore, all of the fall water velocity

TABLE 7-3. AVERAGE HEAT FLUXES IN DEGREES CENTIGRADE-METER PER DAY  
FOR THREE TWO-HOUR INTERVALS ON OCTOBER 27, 1973

Time of Day hour	Averaging Interval minutes					Gradient °C/m
	5	10	15	30	60	
0030-0100	- 3.5	- 2.0	- 3.0	2.0	4.0	3.0
0100-0130	9.7	4.0	14.0	6.3		3.0
0130-0200	0.2	0.3	0.5	- 1.5	- 3.0	1.0
0200-0230	0.4	0.5	0.3	- 1.7		3.0
0700-0730	0.0	2.0	- 2.0	2.0	- 1.0	3.0
0730-0800	- 2.0	- 3.5	- 3.0	- 2.0		3.0
0800-0830	- 10.0	- 4.2	- 8.0	- 2.0	- 9.0	1.0
0830-0900	2.5	2.8	2.0	2.0		-0.1
1200-1230	- 4.5	- 15.0	10.0	18.0	- 20.0	- 22.0
1230-1300	- 6.0	13.0	- 21.0	- 16.0		- 32.0
1300-1330	- 56.0	15.0	- 5.0	23.0	- 30.0	- 37.0
1330-1400	- 25.0	- 65.0	- 77.0	- 87.0		- 24.0

and wind speed measurements were subjected to a correlation analysis in order to determine the formal statistical relationship between the two parameters. The linear relationship between water speed and each of the three wind speed values, as well as an average of the three wind speeds, was considered. The resulting correlation coefficients were all very low (i.e., less than 0.2), and although it could be shown that such a low value was statistically significant, the highest correlation found was associated with a wind speed sensor that yielded a defective (i.e., zero) signal during most of the data collection period, and this suggested that no worthwhile correlation should be claimed. No other factor affecting water movement was discerned.

Because attempts to relate measured water velocity to coincident physical conditions failed, and oftentimes velocity levels peaked during the hottest part of the day, implying the possibility of increased electronic noise due to heating of the equipment, the reliability of the velocimeter output was questionable. On the other hand, the nature of the velocity values recorded indicated that in fact water movement was being sensed. Small drifts in zero offset over short time intervals were sometimes noted (at times other than those plotted in Figure 7-10), yet, even accounting for drift, values of velocity existed which were greater in absolute value than what could be attributed to random signal noise. This was based on results of calibration tests, which showed that the zero signal noise in the instrument was equivalent to  $\pm 0.0015$  meters per second. Many of the values indicated in Figure 7-10

fall further than that from the apparent zero value of  $-0.0006$  meters per second. Furthermore, it is reasonable to believe that during daytime, as noted earlier, much could be happening in the lake, at least in the near surface region, to cause some or all of the variations in flux detected. For instance, surface heat exchange processes other than absorption of solar radiation were in progress. Periods of higher winds increasing evaporation, varying air temperatures altering conduction, and varying cloudiness affecting incident solar flux could all cause short-term variations in near-surface heat flux. Although these factors are reduced or nonexistent at night, the larger magnitude velocities detected could actually have occurred.

In summary, the seemingly inappropriate heat flux values calculated by correlated fluctuations of temperature and velocity using thirty-minute averaging times cannot be absolutely explained by considering the individual velocity and temperature measurements. Some of the variations computed seem likely to actually have occurred, and more frequent gradient data are required before more insight into this matter can be gained. There were times when the specific nature of either the temperature or velocity record or both caused better correlation performance to be attained by using different ones of several alternative averaging times. Yet, no time yielded results consistently superior to results gotten using an averaging interval of thirty minutes. As for the question of whether or not the equipment was sensitive enough to detect the levels and changes of velocity and temperature and

therefore indicate turbulent transport in the lake, only partial resolution has been attained. Some water motion was detected by the velocimeter, yet much of the signal from it is near its threshold of sensitivity and may be virtually all noise.

#### Comparison of Flux-Gradient and Correlation of Fluctuations Results

The results of applying the flux-gradient method to a diurnal period to study convective mixing and the correlation of fluctuations method to a similar period have been presented. In both instances problems were encountered which made the evaluation of diffusivities difficult, and furthermore the usefulness of the Group A data is suspect. By comparing the results of the two methods, some insight should be gained as to whether or not turbulent heat transport in the lake can be described by the correlation of velocity and temperature data recorded there.

The problems of satisfactorily evaluating the gradient term so that subsequent calculations of diffusivities can be made need not impede the comparison of the two methods. Comparison of the heat flux alone as calculated by each approach should serve as a basis for judging the suitability of each. By eliminating the need for determination of the profile slopes (i.e., the gradient), the major obstacle associated with using the computer to analyze a multitude of profiles is averted. This is because a check of the integral quantities computed from the analytical approximations to the temperature-depth profiles against carefully measured areas

of several plotted profiles showed that the analytical integration method performed well.

It was decided to establish the level of heat content in the lake at any time as the basis for the comparison analysis. This was done principally as a result of evolution from some preliminary data analysis, and since the flux is merely the time rate of change of the heat content, heat content was deemed suitable to use. The time average of the cross-correlations,  $H_{cf}$ , was simply integrated (summed) over time to yield a quantity identical to the level of stored heat as given by the integration of the temperature profiles after subtracting absorbed radiation. In analytical form the two quantities were given by

$$\frac{1}{\rho c} \sum_{i=0}^n (H_{cf})_i \Delta t + C = \int_d^h \theta dz - R_d \quad (7-5)$$

$t=i\Delta t$

where  $(H_{cf})_i$  = heat flux as given by equation 7-1 at depth  $d$  during the  $i$ -th interval

$\Delta t$  = length of time interval

$d$  = depth at which velocity temperature is measured

$h$  = depth of lake bottom

$\theta$  = temperature

$z$  = vertical distance

$R_d$  = total solar energy during  $\Delta t$  per unit area at depth  $d$

$C$  = constant of integration =  $\int_d^h \theta dz - R_d$  at  $t=0$

In all cases  $\Delta t$  was used as thirty-minutes (0.0208 days), and  $n$  varied depending on the length of time of continuous data record.

Figures 7-12 through 7-19 show values of heat content in Lake Mize as calculated by each method. In each figure the upper of the set of graphs presents the results of summing the heat flux as calculated by correlating fluctuations, which corresponds to the left-hand side of equation 7-5. The value of C in all instances has been arbitrarily set equal to zero; it could have just as easily been set equal to the initial value of heat content in the lower plot. However, since the scales of both plots are identical, behavior of the curves can easily be compared regardless of the specific axis designations. The lower plot shows the integral of the temperature profiles as given by the right-hand side of equation 7-5. The values plotted are for every two hours throughout the day. To determine the heat content, computations were performed using every fourth half-hour round of Group B data, while the summed quantities were determined by computing and summing the heat flux using half-hour averaging and then plotting only every fourth sum. In other words, thirty-minute and not two-hour averaging intervals were used.

The various figures present a range of times and depths for inspection. Each plot covers a minimum of a two-day continuous interval, and intervals were selected at various times ranging from late October, 1973, to July, 1974. Also, results of analysis are given for periods during which the velocity probe was positioned at various depths. Examination of the plots reveals very little similarity between the results of the contrasting methods of calculation. Furthermore, there seems to be no discernible pattern or regularity to

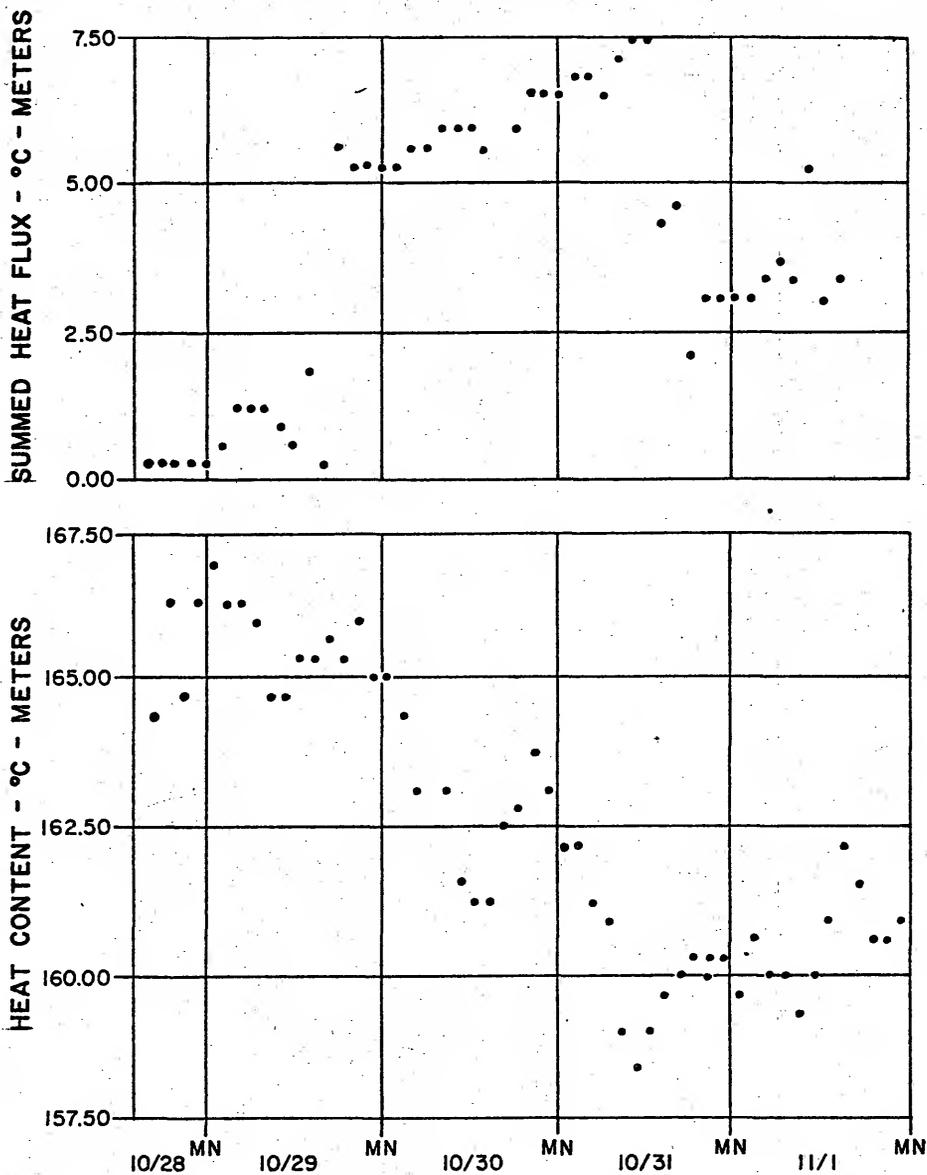


FIGURE 7-12. HEAT CONTENT BELOW 0.05 METERS DURING OCTOBER AND NOVEMBER, 1973

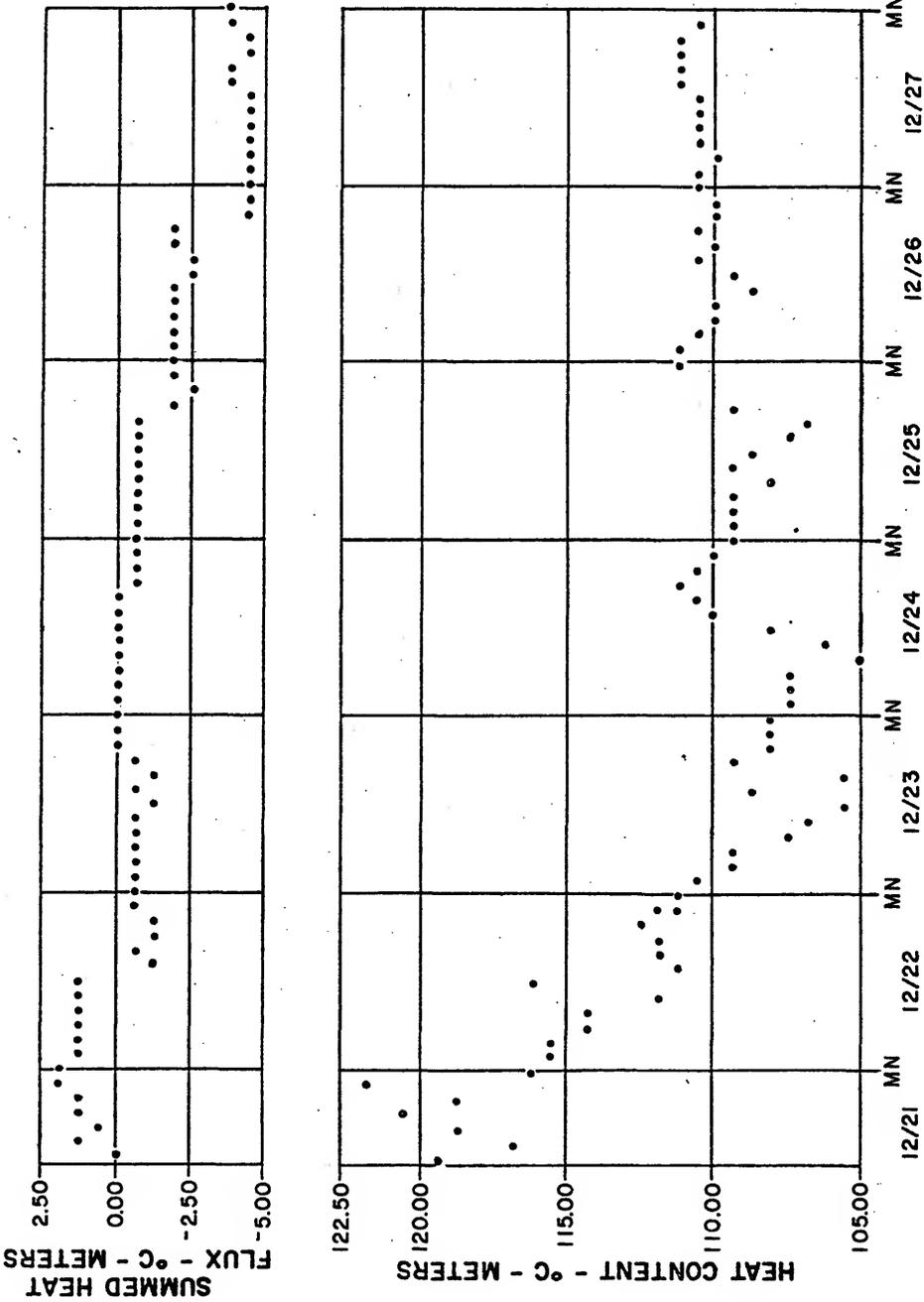


FIGURE 7-13. HEAT CONTENT BELOW 0.05 METERS DURING DECEMBER, 1973

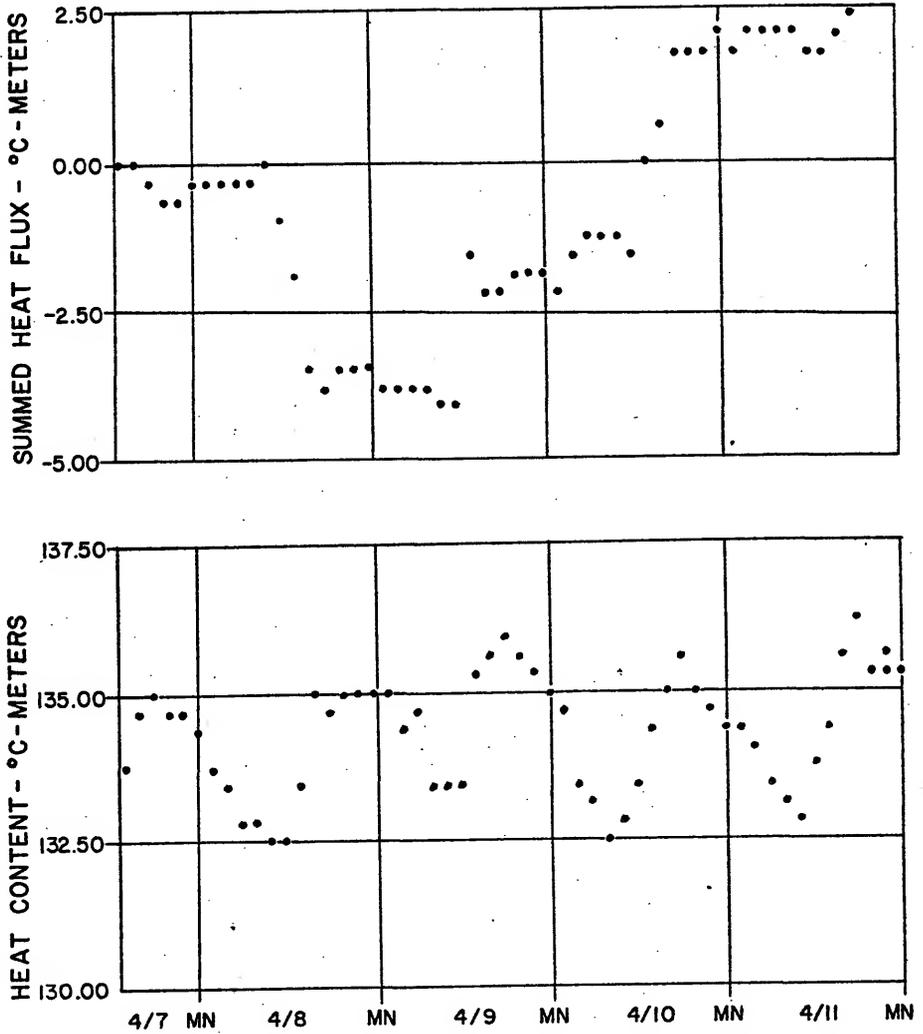
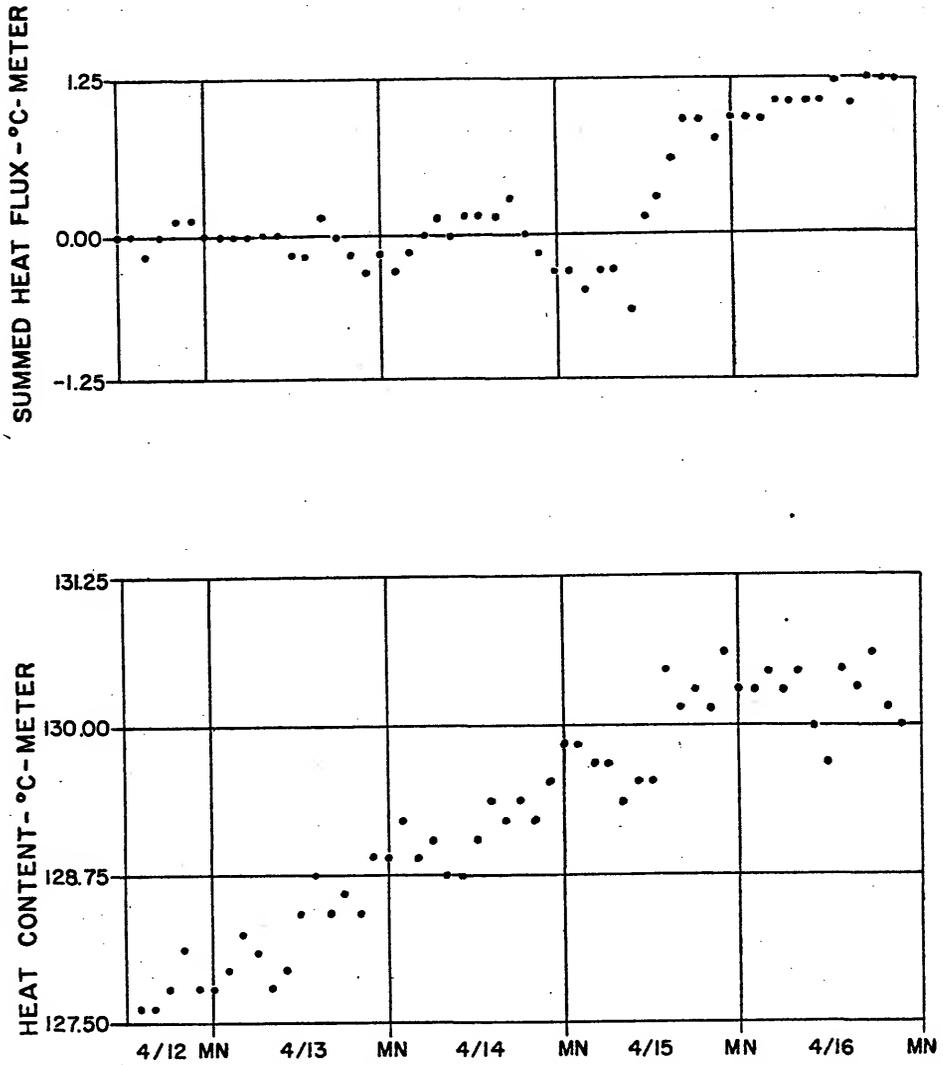


FIGURE 7-14. HEAT CONTENT BELOW 0.05 METERS DURING APRIL, 1974





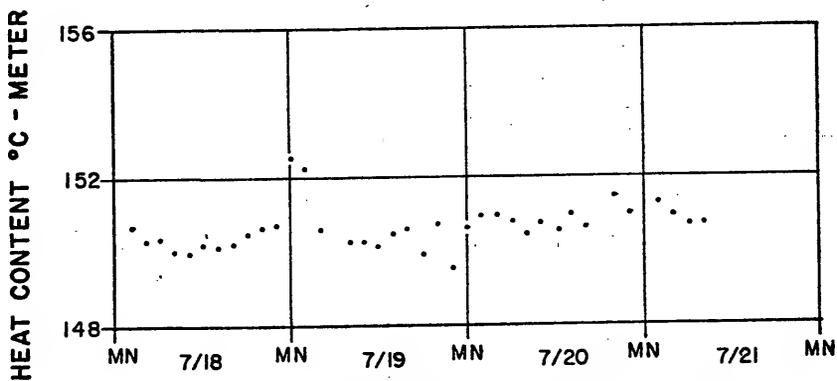
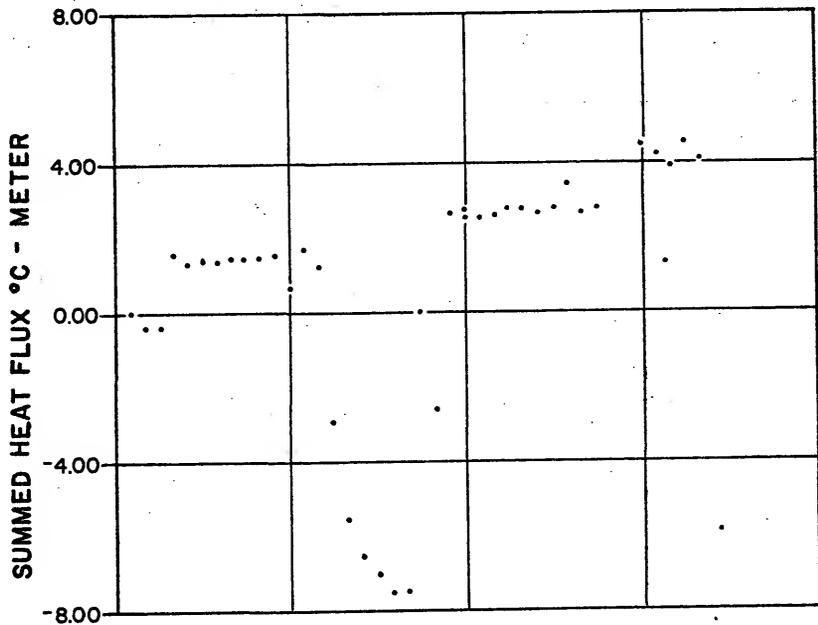


FIGURE 7-17. HEAT CONTENT BELOW 0.9 METERS DURING JULY, 1974

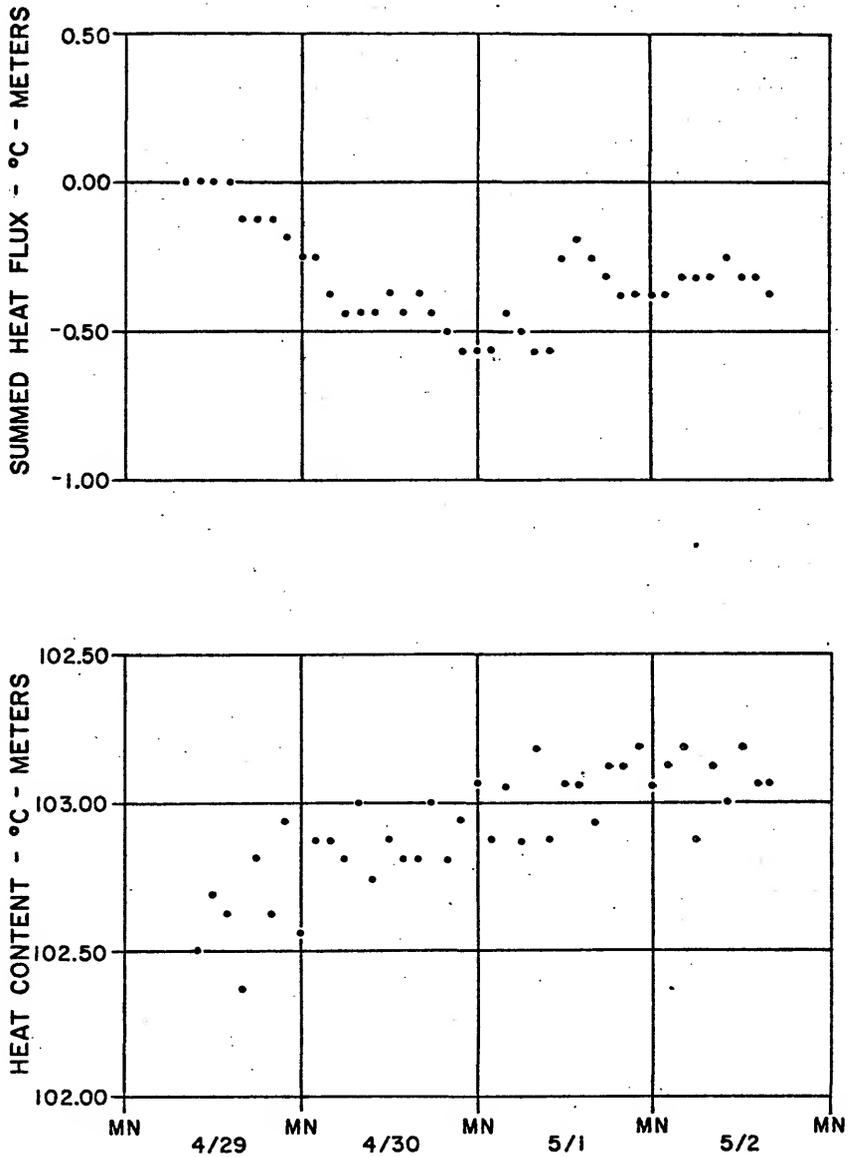


FIGURE 7-18. HEAT CONTENT BELOW 1.9 METERS DURING APRIL AND MAY, 1974

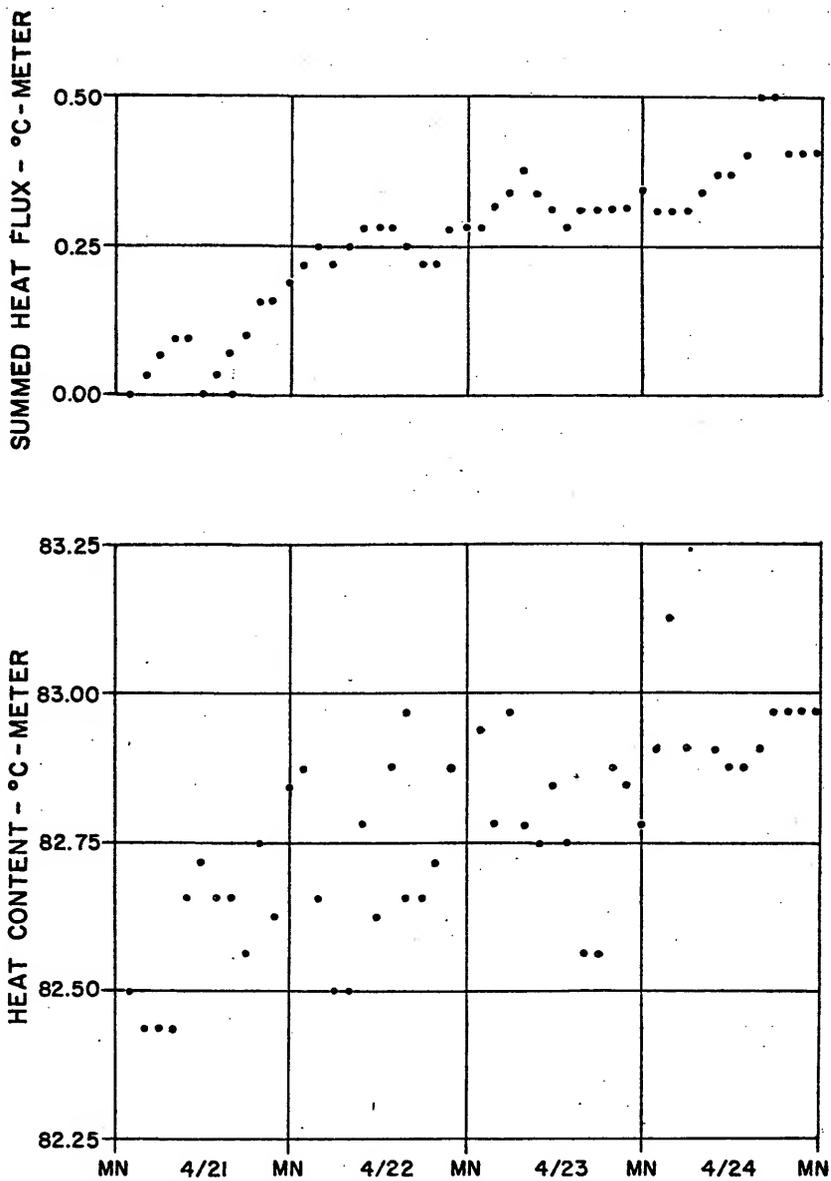


FIGURE 7-19. HEAT CONTENT BELOW 3.2 METERS DURING APRIL, 1974

the manner in which the curves disagree. At times the general trend of the two coincides; however, instances of opposite trends can be seen, and at other times there seems to be absolutely no relationship between the two curves.

It should be noted that the effect of incident solar radiation can be detected in many of the graphs. Obvious diurnal variations occur in the calculations of integrated temperature profiles even though the effect of solar radiation is supposed to be accounted for by subtracting  $R_d$ , as is indicated in equation 7-5. This points out the lack of accuracy for the near surface zone of the analytical expression (equation 3-7) for calculating the attenuation of absorbed radiation. Even though the coefficients were determined by using Lake Mize temperature changes and measured radiation values during the morning hours of calm days, the variations in water quality and the great rate of change of absorption in the top few centimeters makes an accurate, universal predictive expression most difficult to formulate. However, regardless of the tainting of the results by solar flux, the comparison of nightly values which are not affected by incident radiation still yields virtually no agreement between the two methods of computation. Also, the problem of solar effects is averted in those cases when the probe was not near the surface, yet again poor coordination is found.

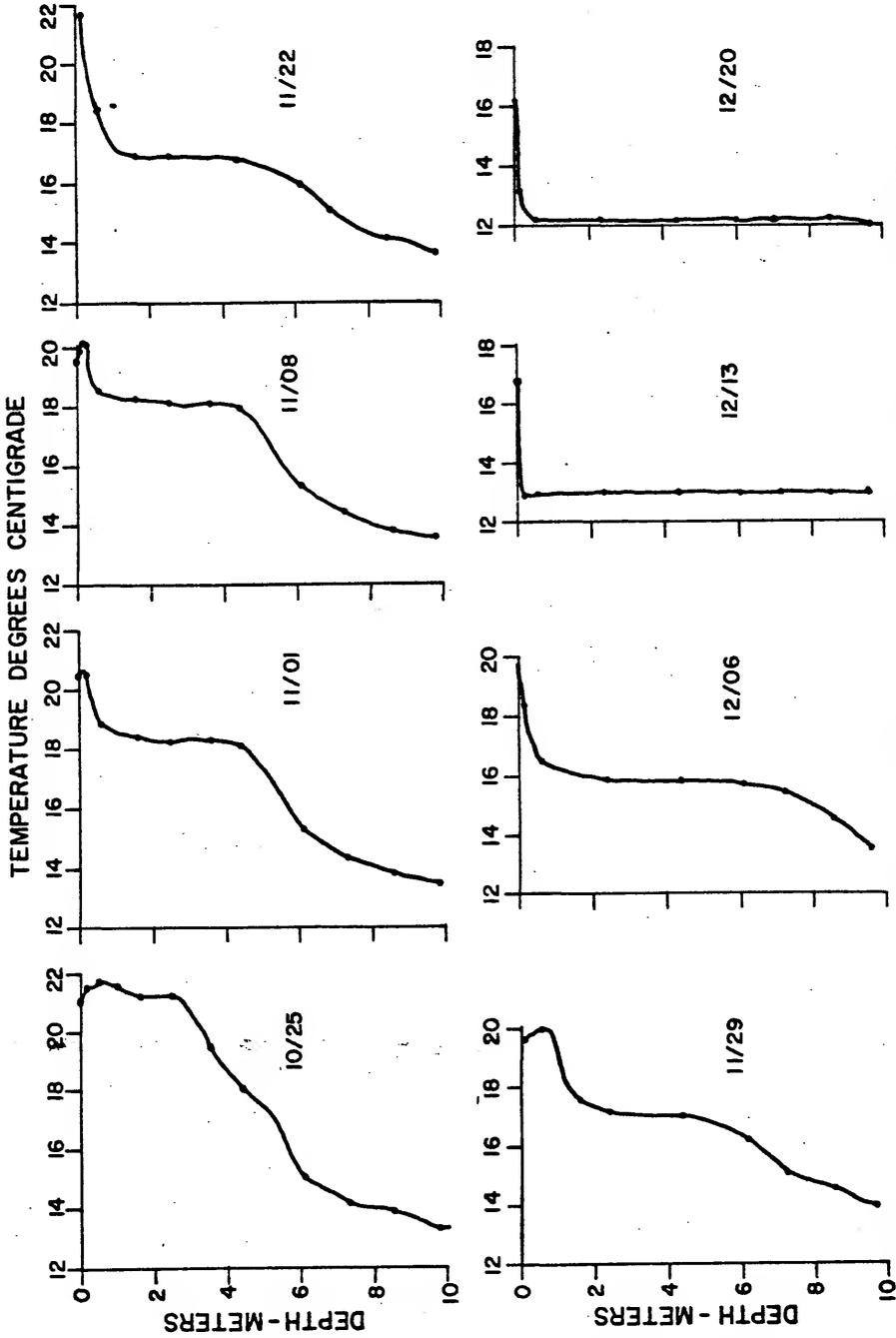
Since the integration of temperature-depth profiles is a good indication of the amount of stored heat below a given horizontal plane at any time, and since continuous periods of data exist during which heat content changes are large enough to be detected

easily with the thermistors used, the results plotted in the lower graphs in Figures 7-12 through 7-19 can generally be relied upon. The failure of the results of correlating fluctuations of velocity and temperature to agree with these results, indicates that the suspicions discussed earlier in this chapter regarding the usability of the velocity temperature data are valid. The sporadic agreement of the results of the two methods suggests that the measured values of one or both of the cross-correlated parameters are contaminated by noise.

#### Variations of Diffusivities with Time and Depth

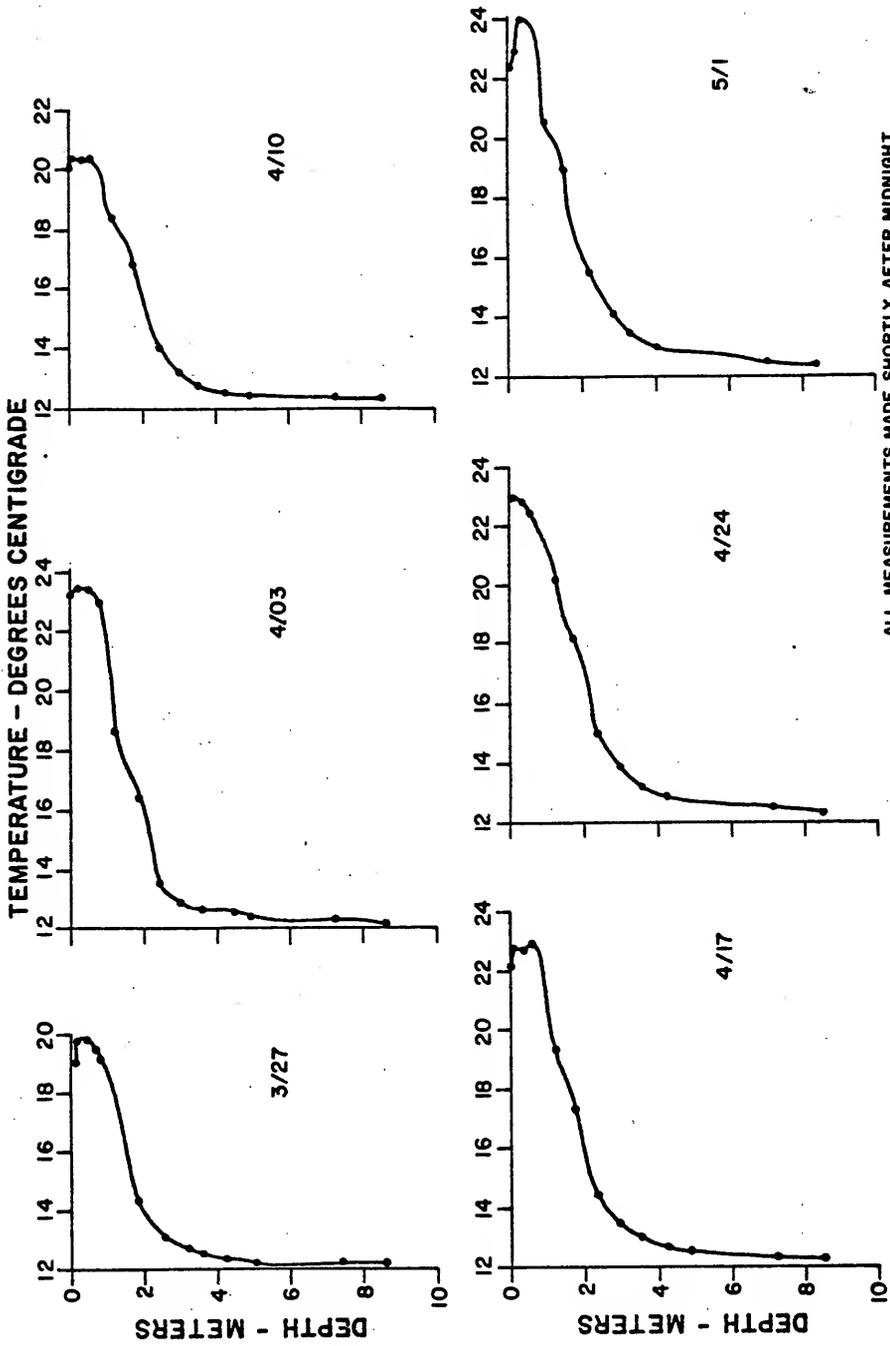
As already discussed inadequacies in the temperature data limited the use of the computer-oriented methods which were designed to easily yield a myriad of analysis results. Specifically, during periods of convective cooling, calculations have been only partially useable because negative diffusivities often resulted. Nevertheless, a variety of calculations have been made and will be presented at this time.

Figure 7-20 shows weekly temperature profiles as measured at midnight at Lake Mize during the fall data collection period. The nighttime values were used to reduce the effects of absorbed solar radiation. Figures 7-21, 7-22, and 7-23 show weekly profiles during the spring. Certain weeks are not represented because the field equipment was inoperative at various times, and in order to preserve an order to the analysis missing weeks were simply skipped. In no case is the interval greater than two weeks. Each of the times displayed was used as end points for an interval over which diffusivities were computed. The flux-gradient method was used and the calculations



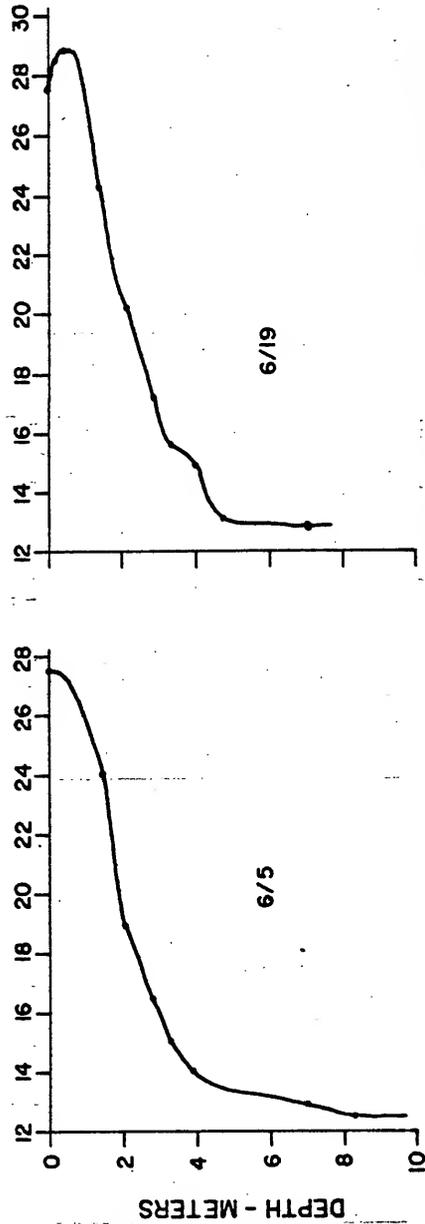
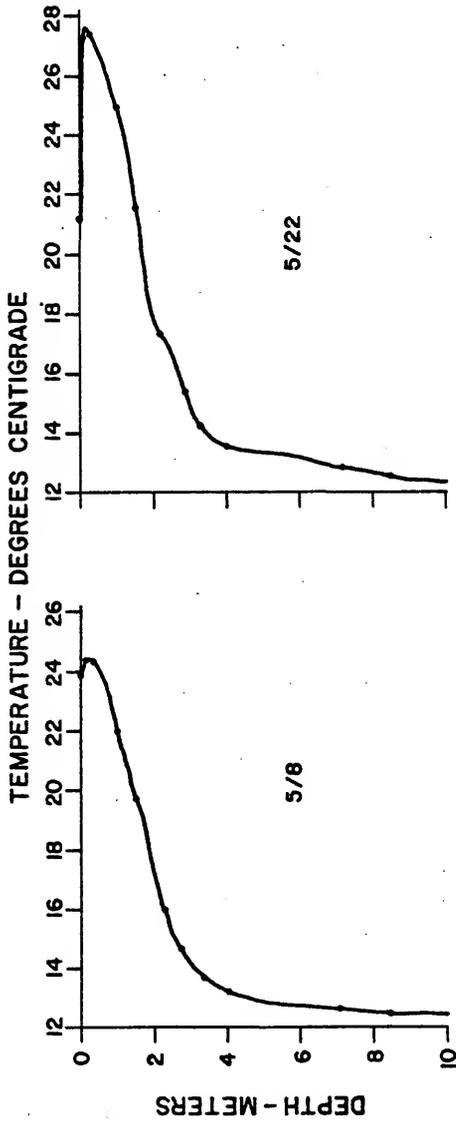
ABSCISSAS ARE ALL TEMPERATURE IN DEGREES CENTIGRADE. ALL MEASUREMENTS MADE SHORTLY AFTER MIDNIGHT.

FIGURE 7-20. WEEKLY TEMPERATURE PROFILES THROUGHOUT THE PERIOD OF FALL DATA COLLECTION



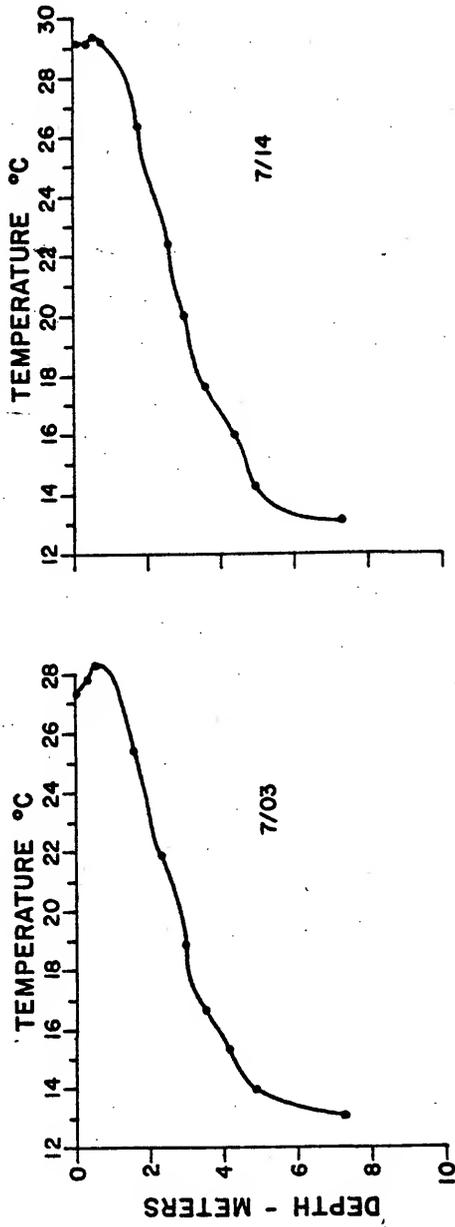
ALL MEASUREMENTS MADE SHORTLY AFTER MIDNIGHT

FIGURE 7-21. WEEKLY TEMPERATURE PROFILES DURING PERIOD OF SPRING DATA COLLECTION



ALL MEASUREMENTS MADE SHORTLY AFTER MIDNIGHT.

FIGURE 7-22. WEEKLY TEMPERATURE PROFILES DURING PERIOD OF SPRING DATA COLLECTION



ALL MEASUREMENTS MADE SHORTLY AFTER MIDNIGHT

FIGURE 7-23. WEEKLY TEMPERATURE PROFILES DURING PERIOD OF SPRING DATA COLLECTION

were made at several depths from surface to 9.5 meters. Temperature data during the fall were approximated by the nonlinear least squares technique, as described in detail earlier, and the spring profiles were approximated by straight lines between data points because this method yielded better results when used with the spring thermistor spacing.

The results were analyzed and reduced for presentation. Figures 7-24, 7-25, and 7-26 show the variations of diffusivities over time. Depths of 1, 2, 4, 6, and 8.5 meters were selected for presentation for the fall results, and 0.5 meters was added for the spring. The inability to suitably analyze much of the fall data is evident in Figure 7-24. Only scattered data were obtained, and no diffusivities at all were gotten after the first week in December. During the first part of the period the two lowest depths represented provide some useful results. The initial higher levels of diffusivities are followed by a decrease, and a subsequent increase is again followed by a decrease. The coincident occurrences in the lake temperature structure can be seen in Figure 7-20. The type of pronounced changes apparent between 10/25 and 11/01 are virtually nonexistent during the succeeding interval. Upper cooling and lower heating resume thereafter but by 12/06 lower heating has ceased as the isothermal condition begins to develop. Of interest is the indication that the times of greatest downward heat transport in the lower layers are also the times of most rapid cooling in the upper strata. The possibility exists that the upper turbulence is causing increased turbulence in the lower waters.

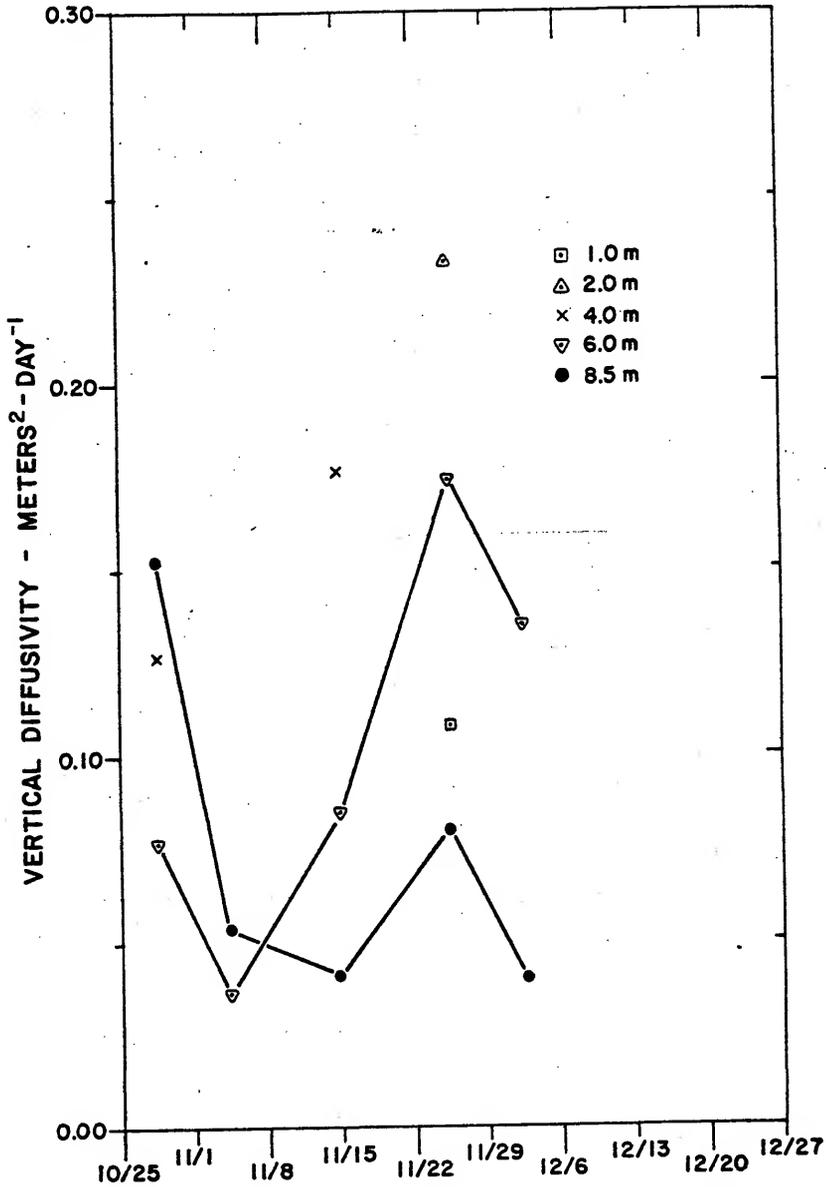


FIGURE 7-24. VARIATION OF DIFFUSIVITIES THROUGHOUT PERIOD OF FALL DATA

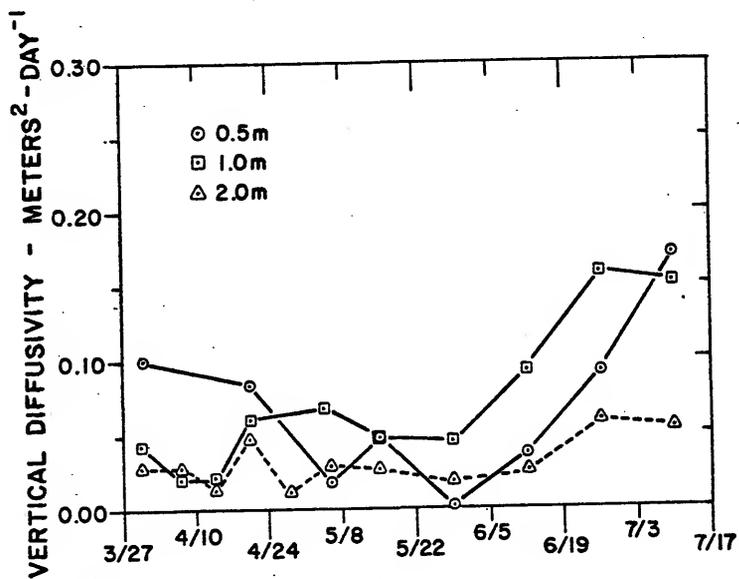


FIGURE 7-25. VARIATION OF DIFFUSIVITIES THROUGHOUT PERIOD OF SPRING DATA

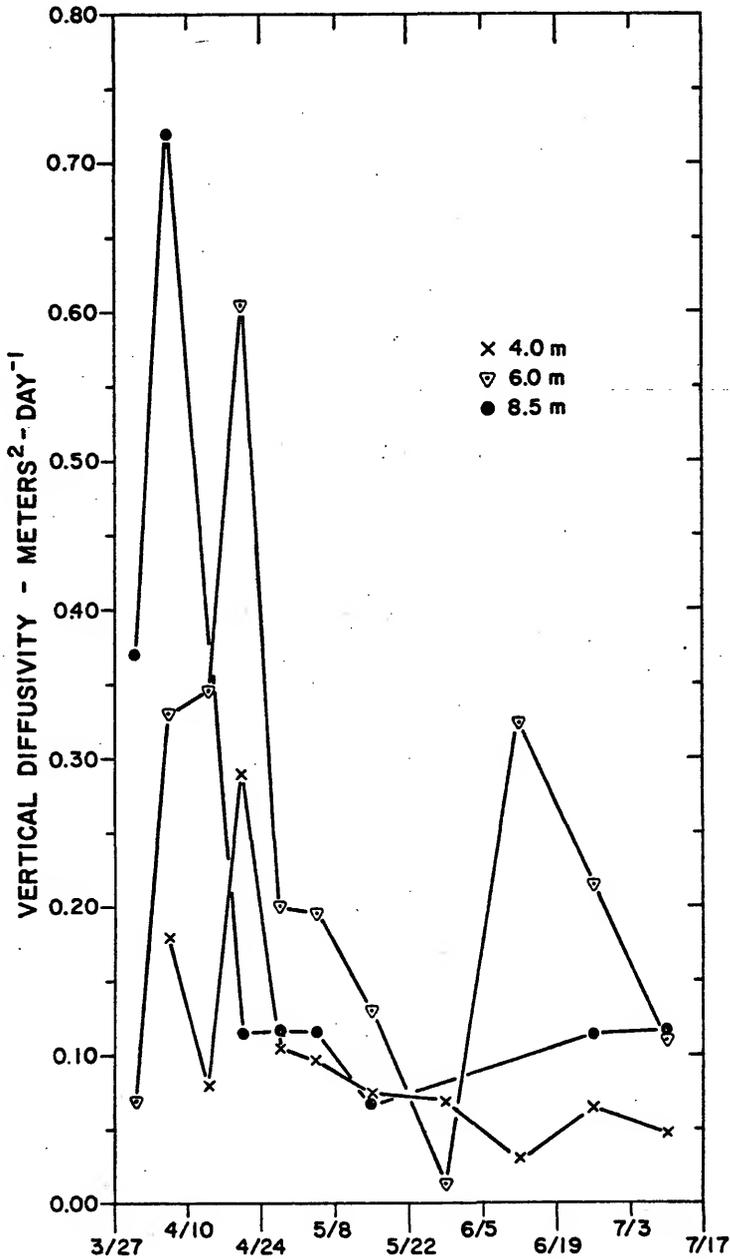


FIGURE 7-26. VARIATION OF DIFFUSIVITIES THROUGHOUT PERIOD OF SPRING DATA

In figures 7-25 and 7-26 the behavior of the diffusion coefficients during a period of spring data gathering is shown. The scales of both sets of axes are the same so that comparison is facilitated. Early in the period there is a general trend toward increasing diffusivity with increasing depth, while by the middle of June that trend has reversed. This process of restratification following the isothermal condition reached in December is well underway by 3/27, although the lower strata remain near the isothermal temperature of 12 degrees centigrade. The early weeks are characterized by a steadily warming upper layer, and the lower layers are also warming. The absolute amount of temperature increase in the lower waters (4 to 10 meters) is not great, yet the very low gradients cause very high diffusivity values to result from even small fluxes. As the warming increases, the degree of stratification expectedly increases. This increased stability is reflected in the sharp declines noted in the values of the lower-level diffusivities. Examination of the temperature profiles shown in Figures 7-21, 7-22, and 7-23 points out the nature of the warming cycle. The increases in temperature are concentrated near the surface through April, general warming occurs throughout May over much of the depth, and by June increased depression of the thermocline can be seen occurring at an accelerated rate while lower temperatures have stabilized. The epilimnion has become more nearly isothermal and more distinct from the hypolimnion. The increases in the upper diffusivities shown in Figure 7-25 demonstrate the transport near the surface.

Because so few useable results were obtained from the analysis of the fall profiles, no detailed examination of the variation of diffusivity with depth was possible. The spring data analysis did produce useable results, and these are plotted in Figures 7-27 and 7-28. The plots indicate the marked variations of diffusivities with depth and also present in another form much of what has already been noted. While the preceding discussion considered only five or six depths, Figures 7-27 and 7-28 were constructed using diffusion coefficient calculations at twenty depths. As can be seen in the plots high values of diffusivity generally occur very near the surface. Otherwise generally very low and unchanging values exist in the upper four meters. This characteristic is altered intermittently until early June when the levels of diffusivity begin to vary much more. Also evident is the persistence of a peak value near five meters which reaches a maximum in early April and then declines gradually until mid-June when it increases somewhat. This five-meter depth seems to be about the lower extent of the region of maximum heat gain, although by looking at the temperature profiles, it can be seen that the five-meter depth is a part of the hypolimnic region. Again it is felt that the relatively high levels of mixing occurring just above induce water turbulence by means of vertical shear yielding high diffusivities due to the low gradient. The much steeper gradients above reduce the value of diffusivities calculated at those depths. The data used to make the calculations of diffusion coefficients just presented are included in Appendix B. Also, in Appendix B are values of some of the intermediate quantities calculated as part of the analysis.

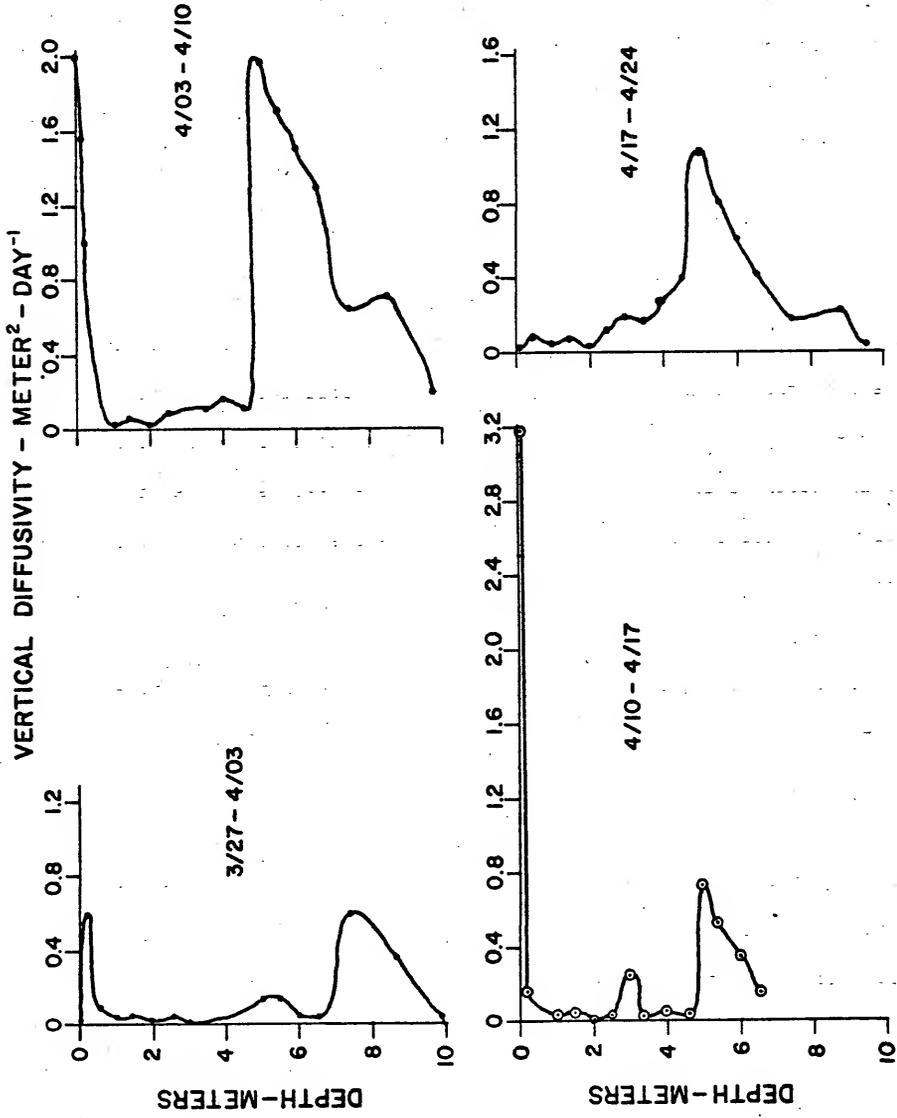


FIGURE 7-27. VARIATIONS OF DIFFUSIVITIES WITH DEPTH AS COMPUTED WEEKLY DURING SPRING

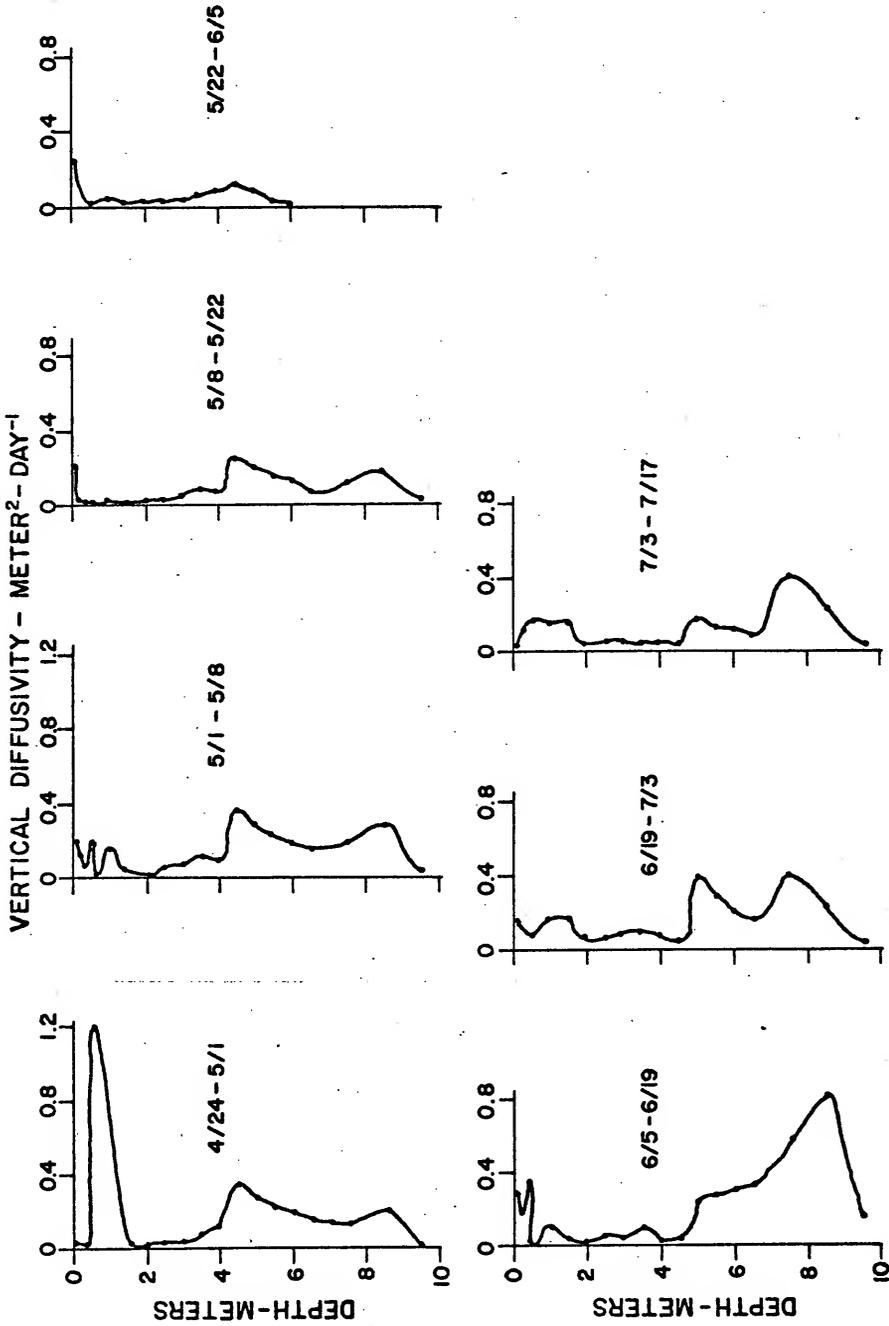


FIGURE 7-28. VARIATIONS OF DIFFUSIVITIES WITH DEPTH AS COMPUTED WEEKLY DURING SPRING

Comparisons with Other Lakes

Figure 7-29 shows plots of diffusivity versus depth for two lakes as computed by Orlob and Selna (1970). The results shown are for Castle Lake and Lake Tahoe. Although not stated, a check with other figures in the same work indicated that the Lake Tahoe calculations were made from May or June data, and no time was attributable to the Castle Lake data. These results and the results of the Cayuga Lake computations given in Figure 3-2 may be compared to the results of this study. The shapes of curves are similar but the levels of diffusivity found in Lake Mize are much lower than those found in the other lakes. Maximum values of from about 20 to 60 square meters per day in the other lakes are compared to the overall maximum of 3.2 square meters per day occurring near the surface during the 4/10 to 4/17 interval.

On the other hand, the hypolimnic values at Lake Mize are closer to the values calculated for the hypolimnions of the other lakes, and in some cases they are even greater. The peaks of diffusivity in the lower portions of the profiles for Lake Mize are also evident in the other profiles, but they are much more pronounced in Castle Lake and Lake Tahoe. The Cayuga Lake results show a more uniform decrease of heat transport with depth. This may be due to the fact that the Cayuga Lake data were taken in August, several months later in the year than the Lake Mize and Lake Tahoe data. The more stable conditions which might be expected in late summer may account for the absence of well-defined peaks.

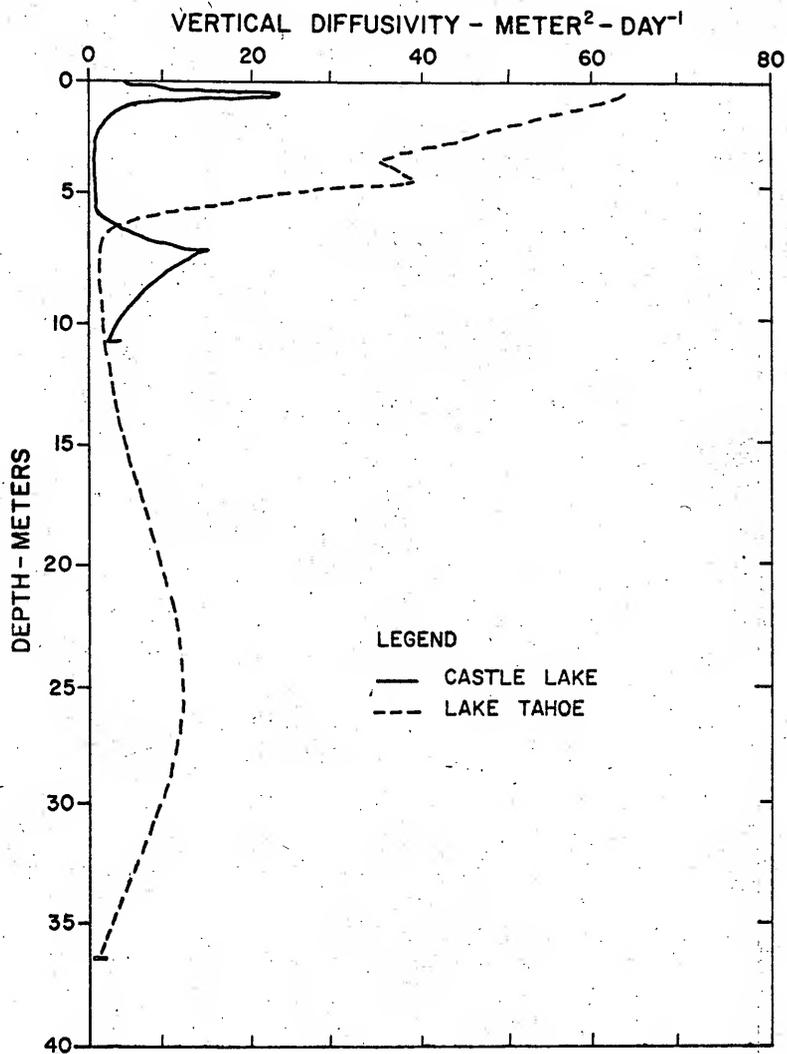


FIGURE 7-29. DIFFUSIVITIES MEASURED BY ORLOB AND SELNA (1970, p. 396)

As indicated in the preceding discussion there is general similarity between the diffusivities calculated for Lake Mize and those applying to other lakes. This suggests that to some extent the results obtained in this study are applicable to other water bodies. This is not unreasonable because while the horizontal scales of the very small Lake Mize and other much larger lakes differ, the vertical scales are comparable. Because of the low winds detected at Lake Mize, it is known that the principal mechanism of epilimnic motion is convective currents caused by surface cooling. The other lakes are probably mixed in the upper layers by both wind and convective currents, and hence this marked difference in epilimnic diffusivities.

Regarding the causes of hypolimnic turbulence, it has been suggested in this work that induced currents from upper layers may be the major factor causing transport. It is clear that regardless of mechanism, the level of motion in lower strata is orders of magnitude less than levels of motion above. The low values are common to most other studies also, so that even in larger lakes in other climates, there may be similar types of motion, and possible differences are diminished to a great extent because of the highly stable conditions which are nearly universal. In the final analysis, however, it is not possible nor prudent to attempt unqualified generalizations of the results obtained here to other situations.

## CHAPTER VIII

### SUMMARY, CONCLUSIONS, AND SUGGESTIONS FOR FUTURE RESEARCH

#### Summary

The theoretical concepts describing the transport of substances in natural waters due to turbulent motion have been developed and modified by semiempirical methods to obtain a more useful form. The modification involved the introduction of diffusivities which relate turbulent flux to average gradient of substance. Because of the significance of vertical temperature distributions in lakes and reservoirs, this research has focused on the problem of evaluating vertical diffusivities of heat in a deep Florida lake. The methodology of determining diffusivities by two methods has been developed wherein both the turbulent motion and the resultant transport due to the turbulent motion have been studied.

A detailed survey of equipment and techniques available for detecting turbulent water motion has been made, and a velocity meter was selected which most closely met the requirement of this work. The velocity meter was integrated into a mostly custom-made data acquisition system designed to provide the data necessary to evaluate diffusivities. The data acquisition equipment was operated at Lake Mize intermittently over an eight-month period, and the large amounts of data obtained were reduced to proper form for analysis using

digital computers. The analytical techniques developed for data reduction and diffusivity evaluation have been presented.

The analysis was adversely affected by certain deficiencies in the data, and the extent of planned analysis was curtailed, chiefly due to the inability of the velocity meter to detect the low levels of velocities in the lake. However, results were obtained which provide better insight into the behavior of the diffusivities during periods of convective mixing over daily time intervals and behavior of the diffusivities over longer time intervals when convective cooling did not predominate.

#### Conclusions

Several conclusions may be drawn from the results of this investigation: due to the low surface winds and the small size of the lake, the principal mechanism for near-surface mixing is convective currents caused by surface cooling. The vertical water velocities associated with this convective mixing were very small and generally not detectable by the velocity measuring device which was capable of sensing velocities as low as 0.003 meters per second but with high noise influence at such levels. The basis for the rejection of the velocity information was excessive contamination due to electronic noise, and this was indicated by the lack of agreement between heat fluxes determined using the velocities and using the flux-gradient method. A detailed examination of the effects of time-averaging was made by comparing values of heat flux as given

by cross-correlating temperature and velocity records. It was shown that the use of different averaging intervals could have pronounced effects on the value of heat flux used, and furthermore, that the proper averaging interval was difficult to ascertain.

An in-depth analysis of the process of convective mixing occurring during a fall day and night was performed using temperature versus depth data taken every thirty minutes. It was learned that to fit accurately analytical expressions to the measured data requires closely spaced temperature readings in the near-surface region because of the continual eroding of the stable, pre-cooling thermal structure of the lake. Manual analysis of the half-hour data showed that inappropriate results usually obtained by applying flux-gradient analysis procedures to the lake using daily time increments could be averted by using shorter time increments which accurately reflected the time scales of the convective mixing process. Also, it was pointed out that the convective mixing process seems to be intermittent in nature, which suggests that abrupt termination of unstable conditions occurs intermittently.

Levels of diffusivities for Lake Mize calculated weekly using the flux-gradient method were found to range from less than 0.01 to about 3.2 square meters per day. Epilimnic values are smaller than those occurring in the epilimnia of other, much larger lakes. The small, sheltered nature of Lake Mize seems responsible, and indicates that wind and possibly, wave effects may account for the two and three orders of magnitude difference between Lake Mize and the other lakes. Levels of hypolimnic diffusivities were more nearly equivalent.

This suggests that the mechanisms responsible for turbulent transport in both the small and large lakes may be similar. Induced motion from upper waters is felt to cause most turbulent transport in the hypolimnia.

Also, concluded and heretofore unmentioned, is that the type of data acquisition system designed and built for this research is a viable method of gathering data. The majority of problems encountered during the operation of the system were with sensors, and these would be the easiest to avoid given an ample budget. The degree of automation attained, the volume of data recorded, and the comparative ease with which data were made ready for analysis are important factors which should aid future studies.

#### Suggestions for Future Research

The experience with and knowledge of recent advances in solid state electronics indicates that the velocity meter used for this research could easily be built using less noisy, more stable components now available. The improvement in performance would probably permit reliable detection of even the low velocities existing at Lake Mize. It is felt that greater insight into turbulent motion in natural waters is still needed, and that measurement of velocities will be the best way to gain that insight. Therefore, improved versions of the velocity meter used in this study as well as those using other principles of operation should be sought and used to successfully attain the full objectives of this work.

The questions concerning the adequacy of semiempirical concepts to describe turbulent transport are far from being resolved. Although the results of this study tend to support the notion of relating transport to average gradient, much more work must be done. One principal aspect of future efforts needs to be the consideration of the appropriateness of various averaging intervals, because of the influence of them on the calculations of gradients and coincident fluxes.

Other efforts should be made toward relating levels of transport, and therefore diffusivities, to the meteorological conditions existing during specific time intervals. Better insight into possible correlations between diffusivity behavior and levels of environmental parameters should be most useful to engineering problems relating to transport and receiving water quality.

**APPENDIX A**

**FORMAT OF STORED DATA**

All of the data recorded at Lake Mize are stored on a single volume of magnetic tape. The information on the tape is stored in the units of the parameter associated with each piece of data (e.g., velocity in meters per second). The name of the tape volume is STEIN, and it contains four separate files. File 1 contains Group B data for the fall data collection interval; File 2 contains Group A data for the fall; File 3 contains Group B data for the spring data collection interval; and File 4 contains Group A data for the spring.

The data are all arranged by days and are stored and accessed as records of one day's data. Within a record are several pieces of information pertaining to the entire day, followed by data taken sequentially for each half hour starting with the interval recorded between 0030 to 0100 hours, denoted as round 1, and ending with the interval recorded between 2400 to 0030 hours, denoted as round 48. On Files 1 and 3 the variables on a single record exist in the following order: day of year, identifying numbers of bad thermistors, elevations of each thermistor, the elevation of the water surface at the end of the day, the height of the wind speed sensors, the readings of the thermistors (including defective ones), air temperature, wind speed at three heights, wind direction, solar radiation, and relative humidity. Seventeen thermistor values are followed by one air temperature, followed by three wind speeds, etc. all for round one. Values for rounds two through forty-eight follow accordingly.

Files 2 and 4 contain the following on each day's record: day of year, depth of the probe at the end of the day, sample interval

between readings of the same variable, elevation of the water surface at the end of the day, horizontal water velocity, vertical velocity, and temperature at the velocity probe. The velocities and temperature are recorded at thirty values of each for round one followed by data for rounds two through forty-eight.

The data are all stored in unformatted style, and therefore it is only necessary to read a record knowing the proper sequence of information stored therein. All variables are real Fortran type except for day and defective thermistor identifiers which are integer variables. STEIN is a standard label tape. The recording format used is variable, blocked, spanned (VBS), and the blockage used is 4744 bytes. The data set name for all files is BERG. The days of fall data collection are 298 (October 25, 1973) through 367 (January 2, 1974), and spring data exists for day 85 (March 26, 1974) through 202 (July 21, 1974). During these periods there are many periods of no data due to equipment failure. When no data exist for a given time, each parameter in that round is assigned the value -1000, which makes identification of missing data easier.

Table A-1 lists all the parameters stored and the respective units of each. Table A-2 presents examples of data definition statements to be used when accessing the data. Also, in Table A-2 are examples of Fortran statements which have been used during data analysis. The Fortran variable names are specifically defined in Table A-1.

TABLE A-1. UNITS OF STORED VARIABLES.

Variable	Fortran Name Used in Table A-2	Number	Units or Range
Day of Year	ITIME	1	1-365
Defective Thermistor Numbers	NOTEMP	10	0-17
Thermistor Elevation	THRMEL	17	Meters
Surface Elevation	SURELE	1	Meters
Height of Wind Sensors	WSHT	3	Meters
Water Temperature	TEMP	17	Degrees Centigrade
Air Temperature	AIRTEM	1	Degrees Centigrade
Wind Speed	WS	3	Meters per Second* Meters per 30-Minutes**
Wind Direction	WD	1	Degrees Clockwise From North
Solar Radiation	SR	1	Langleys per 30-Minutes
Relative Humidity	RH	1	Percent
Probe Depth	PRBDEP	1	Meters
Sample Interval	SAMPER	1	Seconds
Horizontal Velo- city	VH	30	Meters per Second
Vertical Velocity	VV	30	Meters per Second
Probe Temperature	TP	30	Degrees Centigrade

\*Fall Data (File 1)

\*\*Spring Data (File 3)

TABLE A-2. ACCESSING LAKE MIZE DATA FROM STEIN

---

Example of Data Definition Statement Parameters and Subparameters Needed to Access STEIN

```
UNIT = (TAPE9,,DEFER)
VOL  = SER = STEIN
LABEL = (xxx,SL)*
DISP = NEW
DCB  = (RECFM=VBS,BLKSIZE=4744)
DSN  = BERG
```

\*The Value of xxx Used in the File Number Desired (e.g., 002 for File 2)

Example of Fortran Statement Used to Access Group A Data Stored on Files 2 and 4

```
READ(uuu)**
1  ITIME, PRBDEP,SAMPER,SURELE,((VH(J,JJ),J=1,30),
2  (VV(JJ,JJ),J=1,30),(TP(J,JJ),J=1,30),JJ=1,48)
```

Example of Fortran Statement Used to Access Group B Data Stored on Files 1 and 3

```
READ(uuu)**
1  ITIME,(NOTEMP(J),J=1,10),(THRMEL(J),J=1,17),
2  SURELE,(WSHT(J),J=1,3),((TEMP(J,JJ),J=1,17),
3  AIRTEMP(JJ),(WS(J,JJ),J=1,3),WD(JJ),SR(JJ),
4  RH(JJ),JJ=1,48)
```

\*\*The Value of uuu Is the Logical Unit Number Specified on the Data Definition in Addition to the Parameters Described Above

---

APPENDIX B

DATA FOR DIFFUSIVITY CALCULATIONS

This appendix presents the temperature data plotted in Figures 7-20 through 7-23, and some of the intermediate values used in the determination of the diffusivities are shown in Figures 7-24 through 7-28.

Listed in Table B-1 are the values of measured temperatures recorded at Lake Mize during the fall period of data collection. Table B-2 gives the temperatures measured during the spring. The depths corresponding to each measurement are also shown. All temperatures are in degrees centigrade, all depths are meters below water surface, and all the measurements were made at midnight.

Also shown in Tables B-1 and B-2 are the totals of the incident solar radiation measurements for the entire day. As discussed in the text the radiation values were used with equation 3-7 to evaluate the solar flux at any depth. The weekly diffusivity computations were made using average daily values of solar flux, and the coefficients used in equation 3-7 were 0.2 and 2.0 for  $\beta$  and  $\eta$ , respectively.

Tables B-3 through B-6 give the calculated temperatures as predicted by the analytical expressions determined for each measured profile, the areas under the predicted curves determined analytically, and the slopes of the curves as determined analytically. These are intermediate values pertinent to the flux-gradient calculations. They are presented here to demonstrate the relative ability of the analytical expressions to approximate heat storage levels and profile slopes at various depths.





TABLE B-3. CALCULATED TEMPERATURES, HEAT CONTENTS, AND GRADIENTS DURING FALL 1973

Key: D = Depth (Meters), T = Temperature (Degrees Centigrade)  
 HC = Heat Content (Degrees Centigrade-Meter), G = Gradient (Degrees Centigrade per Meter)

D	T	HC	G	Date:	T	HC	G	Date:	T	HC	G
000	53	165	2	10/25	41	162	5	11/1	20	161	-1
000	50	160	0	10/25	38	158	-5	11/1	19	157	-1
000	47	159	0	10/25	35	153	1	11/1	18	153	-1
000	44	158	0	10/25	32	144	0	11/1	17	143	0
000	41	157	0	10/25	29	141	0	11/1	16	143	0
000	38	153	0	10/25	26	135	0	11/1	15	134	0
000	35	148	0	10/25	23	125	0	11/1	14	125	0
000	32	147	0	10/25	20	119	0	11/1	13	117	0
000	29	147	0	10/25	17	108	0	11/1	12	108	0
000	26	140	0	10/25	14	99	0	11/1	11	98	0
000	23	132	0	10/25	11	89	0	11/1	10	89	0
000	20	125	0	10/25	8	81	0	11/1	9	80	0
000	17	117	0	10/25	5	72	0	11/1	8	71	0
000	14	109	0	10/25	2	65	0	11/1	7	65	0
000	11	101	0	10/25	0	57	0	11/1	6	57	0
000	8	94	0	10/25	0	49	0	11/1	5	49	0
000	5	87	0	10/25	0	41	0	11/1	4	41	0
000	2	80	0	10/25	0	33	0	11/1	3	33	0
000	0	72	0	10/25	0	25	0	11/1	2	25	0
000	0	64	0	10/25	0	17	0	11/1	1	17	0
000	0	56	0	10/25	0	9	0	11/1	0	9	0
000	0	48	0	10/25	0	1	0	11/1	0	1	0
000	0	40	0	10/25	0	0	0	11/1	0	0	0
000	0	32	0	10/25	0	0	0	11/1	0	0	0
000	0	24	0	10/25	0	0	0	11/1	0	0	0
000	0	16	0	10/25	0	0	0	11/1	0	0	0
000	0	8	0	10/25	0	0	0	11/1	0	0	0
000	0	0	0	10/25	0	0	0	11/1	0	0	0

D	T	HC	G	Date:	T	HC	G	Date:	T	HC	G
000	55	197	7	11/22	89	197	7	12/6	20	197	7
000	52	193	6	11/22	86	193	6	12/6	19	193	6
000	49	189	5	11/22	83	189	5	12/6	18	189	5
000	46	185	4	11/22	80	185	4	12/6	17	185	4
000	43	181	3	11/22	77	181	3	12/6	16	181	3
000	40	177	2	11/22	74	177	2	12/6	15	177	2
000	37	173	1	11/22	71	173	1	12/6	14	173	1
000	34	169	0	11/22	68	169	0	12/6	13	169	0
000	31	165	0	11/22	65	165	0	12/6	12	165	0
000	28	161	0	11/22	62	161	0	12/6	11	161	0
000	25	157	0	11/22	59	157	0	12/6	10	157	0
000	22	153	0	11/22	56	153	0	12/6	9	153	0
000	19	149	0	11/22	53	149	0	12/6	8	149	0
000	16	145	0	11/22	50	145	0	12/6	7	145	0
000	13	141	0	11/22	47	141	0	12/6	6	141	0
000	10	137	0	11/22	44	137	0	12/6	5	137	0
000	7	133	0	11/22	41	133	0	12/6	4	133	0
000	4	129	0	11/22	38	129	0	12/6	3	129	0
000	1	125	0	11/22	35	125	0	12/6	2	125	0
000	0	121	0	11/22	32	121	0	12/6	1	121	0
000	0	117	0	11/22	29	117	0	12/6	0	117	0
000	0	113	0	11/22	26	113	0	12/6	0	113	0
000	0	109	0	11/22	23	109	0	12/6	0	109	0
000	0	105	0	11/22	20	105	0	12/6	0	105	0
000	0	101	0	11/22	17	101	0	12/6	0	101	0
000	0	97	0	11/22	14	97	0	12/6	0	97	0
000	0	93	0	11/22	11	93	0	12/6	0	93	0
000	0	89	0	11/22	8	89	0	12/6	0	89	0
000	0	85	0	11/22	5	85	0	12/6	0	85	0
000	0	81	0	11/22	2	81	0	12/6	0	81	0
000	0	77	0	11/22	0	77	0	12/6	0	77	0
000	0	73	0	11/22	0	73	0	12/6	0	73	0
000	0	69	0	11/22	0	69	0	12/6	0	69	0
000	0	65	0	11/22	0	65	0	12/6	0	65	0
000	0	61	0	11/22	0	61	0	12/6	0	61	0
000	0	57	0	11/22	0	57	0	12/6	0	57	0
000	0	53	0	11/22	0	53	0	12/6	0	53	0
000	0	49	0	11/22	0	49	0	12/6	0	49	0
000	0	45	0	11/22	0	45	0	12/6	0	45	0
000	0	41	0	11/22	0	41	0	12/6	0	41	0
000	0	37	0	11/22	0	37	0	12/6	0	37	0
000	0	33	0	11/22	0	33	0	12/6	0	33	0
000	0	29	0	11/22	0	29	0	12/6	0	29	0
000	0	25	0	11/22	0	25	0	12/6	0	25	0
000	0	21	0	11/22	0	21	0	12/6	0	21	0
000	0	17	0	11/22	0	17	0	12/6	0	17	0
000	0	13	0	11/22	0	13	0	12/6	0	13	0
000	0	9	0	11/22	0	9	0	12/6	0	9	0

TABLE B-4. CALCULATED TEMPERATURES, HEAT CONTENTS, AND GRADIENTS DURING FALL, 1973

Key: D = Depth (Meters), T = Temperature (Degrees Centigrade)

HC = Heat Content (Degrees Centigrade-Meter), G = Gradient (Degrees Centigrade per Meter)

D	T	HC	G	T	HC	G	Date:	T	HC	G	Date:	T	HC	G
0.25	14.66	126.47	**24	14.86	118.45	**58	12/13	19.86	111.18	**44	12/27	19.86	111.18	**44
0.50	14.95	123.35	-0.70	14.80	115.72	-1.59	12/13	14.31	107.86	-0.89	12/27	14.31	107.86	-0.89
1.00	11.47	115.05	0.48	11.97	113.26	0.13	12/13	12.48	109.17	0.03	12/27	12.48	109.17	0.03
1.50	12.72	108.92	0.34	12.02	110.39	0.09	12/13	11.33	93.69	0.13	12/27	11.33	93.69	0.13
2.00	13.04	102.35	0.13	12.06	95.27	0.06	12/13	11.23	82.40	0.03	12/27	11.23	82.40	0.03
3.00	13.07	89.80	0.04	12.07	89.24	0.01	12/13	11.18	76.95	0.00	12/27	11.18	76.95	0.00
4.00	13.02	86.30	-0.12	12.05	77.20	-0.02	12/13	11.08	71.77	-0.01	12/27	11.08	71.77	-0.01
5.00	13.93	79.37	0.10	12.05	71.15	0.04	12/13	11.00	60.21	0.00	12/27	11.00	60.21	0.00
6.00	13.93	63.30	-0.05	12.05	65.13	-0.01	12/13	11.18	54.63	0.00	12/27	11.18	54.63	0.00
7.00	13.96	56.41	0.05	12.05	53.11	0.03	12/13	11.09	43.04	0.00	12/27	11.09	43.04	0.00
8.00	13.97	53.99	0.17	12.08	47.96	0.08	12/13	11.11	37.86	0.00	12/27	11.11	37.86	0.00
9.00	13.89	30.99	-0.06	12.10	26.81	-0.02	12/13	11.11	26.60	-0.01	12/27	11.11	26.60	-0.01
10.00	13.84	1.55	0.02	12.10	1.81	0.02	12/13	11.11	1.44	0.01	12/27	11.11	1.44	0.01





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## BIOGRAPHICAL SKETCH

The author was born in Midland, Texas, on May 20, 1945 to Louis and Bette King Steinberg. He was raised in Nashville, Tennessee, where he attended Burton Elementary and Hillsboro High Schools. He matriculated at Vanderbilt University, Nashville, Tennessee, where he majored in civil engineering and received the Bachelor of Civil Engineering degree in June, 1967. He entered graduate school at Vanderbilt University in September, 1967 and received the Master of Water Resources Engineering degree in 1969. He began work toward the doctorate degree at the University of Florida, Department of Environmental Engineering, in 1968, and attained the Doctor of Philosophy degree in 1975.

He married the former Emily Elaine Gill in Nashville in 1968. He is a member of Woodmont Baptist Church in Nashville and an active participant in the programs of the Southwest Methodist Church, Gainesville, Florida. He is a member and active alumnus of Phi Kappa Psi Social Fraternity, and in 1966 he was elected to membership in Tau Beta Pi National Engineering Honorary Fraternity.

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation of the degree of Doctor of Philosophy.



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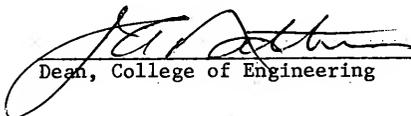


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