NEUTRON SPECTRUM MEASUREMENTS IN HETEROGENEOUS MEDIA

By
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NEUTRON SPECTRUM MEASUREMENTS IN HETEROGENEOUS MEDIA

By

Sagid Salah

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Major Department: Nuclear Engineering

In this investigation neutron spectra were measured both integrally and differentially and then compared with theoretical calculations. Measurements were made in highly absorbing media in the University of Florida Training Reactor (UFTR) thermal column, in the UFTR core and in the subcritical assembly sitting on top of the graphite pedestal of the UFTR. The subcritical assembly contained some typical D₂O-moderated natural uranium lattices.

Precise knowledge of the neutron spectrum in a heterogeneous medium makes it possible to optimize the neutron economy. This is true because fuel elements in a reactor have varying cross sections for different energy neutrons. By proper choice of the fuel-to-moderator ratio
and the geometry and composition of fuel elements, optimum use of neutrons can be attained for maximum fuel burn-up with the highest conversion ratio.

In the thermal column of the UFTR, thermal neutron spectra were measured integrally using activation detectors such as Au	extsuperscript{197}, Dy	extsuperscript{164}, Lu	extsuperscript{175}, Lu	extsuperscript{176} and Pu	extsuperscript{239}. The detectors Lu	extsuperscript{176} and Pu	extsuperscript{239} have low-lying absorption resonances, whereas Au	extsuperscript{197} and Lu	extsuperscript{175} have a 1/v absorption cross section for neutrons below 1.0 ev. The detector Dy	extsuperscript{164} has an absorption cross section that is steeper than 1/v. The measurements were made in a stainless steel rod, a natural uranium slab and in an Al can filled with borated water and results were interpreted using Westcott's formulation.

In a natural uranium slab the effective neutron temperature change from the center to the outside boundary of the slab was obtained by measuring the activity ratios of Lu	extsuperscript{176} and Lu	extsuperscript{175}. In a stainless steel rod, the spacial effective neutron temperature change was measured using Pu	extsuperscript{239} and Lu	extsuperscript{176} activation detectors and then the ratios of activities to the activities of 1/v absorbers were obtained.

The same type of measurements were made in an Al can filled with different concentrations B	extsubscript{2}O	extsubscript{3}. These results were compared with THERMOS calculations; however, due to the anisotropy of the experimental source conditions, agreement was poor.
The effective neutron temperature and the epithermal index, $r\sqrt{\frac{T}{T_0}}$, were measured by using activation detectors in the UFTR core between the two fuel boxes in the North-South center line. The result of the effective neutron temperature measurement was compared with the open-beam differential spectrum and found to be in good agreement.

A crystal diffraction spectrometer for differential spectrum measurements in the subcritical assembly was used. For these measurements, the total efficiency of the machine for different Bragg's angle was first obtained by taking a neutron beam with known spectrum out of the subcritical tank. The tank, 24 in. in diameter and 60 in. long, was filled with D$_2$O and rested on top of the graphite pedestal of the UFTR. The neutron spectrum out of this medium was measured at two different temperatures. The ratios of the two spectra were plotted with the ratios of the calculations and found to be in good agreement.

The neutron spectrum out of the subcritical assembly was measured for two different lattice pitches using Mark I and Mark V-B natural U fuel elements in D$_2$O. A neutron beam was extracted from the cell boundary and from the center of the cell for each of the three lattice arrangements. The results were compared with the theoretical spectra calculated with the THERMOS code and found to be in excellent agreement.

Integral measurements were also made in the subcritical using Lu$^{176}$ and Au$^{197}$ detectors. The experimental
activation measurements were compared with the THERMØS calculations and were also found to be in excellent agreement.

Finally, the effective neutron temperature change in a unit cell for four U-D₂O lattice arrangements were measured differentially and integrally and compared with the theoretical calculations. The results were in good agreement.
CHAPTER I

DESCRIPTION OF THE PROBLEM

Neutron spectrum measurements in heterogeneous media are important for several reasons: (1) for evaluation of the thermal utilization factor $f$ and the thermal reproduction factor $\eta$; (2) for the calculation of reactor fuel cycles; (3) for comparison with multigroup calculations of the neutron spectrum; and (4) for the prediction of dose rates in a sample, which depends on the neutron energy distribution.

Precise knowledge of the neutron spectrum in a heterogeneous medium makes it possible to optimize the neutron economy. This is true because fuel elements in a reactor have varying cross sections for different energy neutrons. By proper choice of the fuel-to-moderator ratio and the geometry and composition of fuel elements, optimum use of neutrons can be attained for maximum fuel burn-up with the highest conversion ratio. In many cases, by controlling the leakage spectrum from a reactor core to a desired form, the maximum conversion ratio in the blanket material
will be realized. This follows because $^{238}\text{U}$ and $^{232}\text{Th}$, which are the usual fertile blanket material for nuclear reactors, have high resonances for neutron capture at 6.8 ev and 22-24 ev, respectively. In order to predict the spacial and spectral distribution of neutrons theoretically, the behavior of neutrons in the heterogeneous media must be well understood.

In this investigation, the neutron spectrum was measured both integrally (1) and differentially (2), and then compared with theoretical calculations (3, 4, 5).

Measurements were carried out in a highly absorbing medium in the University of Florida Training Reactor (UFTR) thermal column, in the UFTR core, and in the subcritical assembly sitting on top of the graphite pedestal of the UFTR. The subcritical assembly contained some typical D$_2$O-moderated natural uranium lattices.

Underlined numbers in parentheses refer to the List of References.
CHAPTER II

MEASUREMENT OF NEUTRON SPECTRA

Introduction

As early as 1936, neutron diffraction by crystalline material was suggested by Elsasser (6) and experimentally demonstrated by Halban and Preiswerk (7) and by Mitchell and Powers (8). Using Ra-Be sources, these experiments proved that neutrons were diffracted, but they were by no means able to obtain monochromatic neutrons due to the weakness of the sources.

Soon after the development of the nuclear reactor, neutrons were available in great quantity, and neutron diffraction spectrometers were built at Argonne (9, 10), Oak Ridge (11), Chalk River, Canada (12), Harwell, England (13) and elsewhere. Also, at the same time, time-of-flight techniques were developed (14, 15). About the same time integral methods of neutron spectrum analysis were described by Westcott (16), Campbell, et al. (17) and Bigham and Tunnicliffe (18). Trice (19) and Nisle (20) also shed light on this subject.
An early theoretical treatment of the spectral hardening was carried out by Plass (21), who assumed neutrons of each energy diffused independently without energy interchange. Later models were developed to calculate neutron spectra by the use of free hydrogen and bound hydrogen scattering kernels (22, 23).

One of the better methods for calculating the space and energy dependent flux in a unit cell was developed by Honeck (24, 25) for H\textsubscript{2}O and D\textsubscript{2}O-moderated systems; it is described in Appendix II. Recently at KAPL, Goldman (26) constructed a scattering law based on Nelkin's model, and agreement with the experimental results using (CH\textsubscript{2})\textsubscript{n} is excellent. At General Atomics, Young (27) has measured neutron spectra using a pulsed high-current electron linear accelerator in H\textsubscript{2}O, CH\textsubscript{2}, and C\textsubscript{6}H\textsubscript{6} moderators poisoned with B, Sm, Er Gd and also D\textsubscript{2}O (28). Measured spectra were compared with DSN (5) transport-theory calculations utilizing the bound hydrogen scattering model for water. Earlier, Beyster (29) measured neutron spectra in pure and poisoned H\textsubscript{2}O, (CH\textsubscript{2})\textsubscript{n}, and ZrH using the pulsing technique and compared it with DSN calculations.
Integral Method

The measurement of neutron energy spectra using the time-of-flight technique or a crystal spectrometer requires the extraction of a neutron beam from a reactor lattice, which is difficult to do for more than a few representative positions. However, by using activation detectors, spectrum changes throughout the cell may be mapped out (i.e., good spacial resolution is obtained with minimal cell perturbation).

The usual experimental technique for observing the spectral change in a unit cell of a reactor involves mapping the activity ratio of resonance and 1/ν cross section detectors. For this type of measurement, various types of detectors are summarized in Table 2.1. In Figure 2.1 the cross sections of the 1/ν absorber Lu\(^{175}\), and the resonance absorber Lu\(^{176}\)(30) are shown. Also included in the figure are the cross sections for Dy\(^{164}\)(31) and Pu\(^{239}\)(32). A Maxwellian neutron distribution in the thermal energy region with an average temperature of 293°K is also shown in the same figure.

By inspection of Figure 2.1, it is seen that, as the Maxwellian distribution shifts to higher energies, the ratio of the activities of Lu\(^{176}\) and Lu\(^{175}\) will increase since the reaction rate is proportional to the integral of the neutron flux times the absorption cross section (33).
Figure 2.1. Activation Cross Sections
<table>
<thead>
<tr>
<th>Detector</th>
<th>$^{176}$Lu</th>
<th>$^{175}$Lu</th>
<th>$^{164}$Dy</th>
<th>$^{197}$Au</th>
<th>$^{115}$In</th>
<th>$^{239}$Pu</th>
<th>$^{55}$Mn</th>
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<tr>
<td>Activity</td>
<td>$^{177}$Lu</td>
<td>$^{176}$Lu</td>
<td>$^{165}$Dy</td>
<td>$^{198}$Au</td>
<td>$^{116}$In</td>
<td>Fission Product</td>
<td>Mn$^{56}$</td>
</tr>
<tr>
<td>Half-life</td>
<td>6.74d</td>
<td>3.69h</td>
<td>2.33h</td>
<td>2.69d</td>
<td>54 m</td>
<td>1h</td>
<td>2.58h</td>
</tr>
<tr>
<td>Cross section</td>
<td>35b*</td>
<td>4,000b*</td>
<td>2,600b</td>
<td>96b</td>
<td>190b</td>
<td>742b</td>
<td>13.4b</td>
</tr>
<tr>
<td>(0.025 ev)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Resonance cross</td>
<td>13,600b</td>
<td>---</td>
<td>1,000b</td>
<td>30,000b</td>
<td>30,000b</td>
<td>5,900b</td>
<td>2,000b</td>
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<td></td>
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<td></td>
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<td>Resonance energy (ev)</td>
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<td>4.9</td>
<td>1.45</td>
<td>0.296</td>
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<td>Wire diameter (cm)</td>
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<td>0.005</td>
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<td>0.001</td>
<td>0.002</td>
<td>0.030</td>
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<td>Gamma energy (Mev)</td>
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<td>0.412</td>
<td>1.2</td>
<td>---</td>
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<td>0.095</td>
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<td>0.095</td>
<td>0.412</td>
<td>1.2</td>
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<td>2.06</td>
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</table>

* Activation cross section.
In well-moderated media with low absorption, (e.g., D$_2$O, H$_2$O, C, Be) far away from neutron sources and boundaries, the spectrum approaches a Maxwellian distribution with the average temperature corresponding to the temperature of the medium. When these well-thermalized neutrons enter a highly absorbing medium (e.g., a fuel element which has a 1/v absorption cross section), lower energy neutrons are preferentially absorbed, resulting in a "hardening" of the energy spectrum within the fuel.

The activation results were interpreted by using the Westcott formulation (34). This method applies to well-moderated systems such as the UFTR. In a reactor spectrum, effective cross sections are given as:

$$\tilde{\sigma} = \sigma_0 (g + rs)$$  \hspace{1cm} (2.1)

where

- $\tilde{\sigma}$ = effective cross section
- $\sigma_0$ = cross section at $v = 2200$ m/sec
- $r$ = relative intensity of the slowing down spectrum (or epithermal index)
- $g, s$ = functions of the neutron temperature, $T$

The values of $g$ and $s$ are tabulated (35) for various nuclides. For 1/v absorbers $g = 1$ and $s = 0$. Using this notation, the Cd ratio for a very thin detector is given as:
\[ R_{cd} = \frac{g + rs}{rs + \frac{gr\sqrt{T}}{K T_o}} \] (2.2)

where \( \frac{1}{K} \) is approximately the density fraction of epithermal neutrons transmitted by the Cd (34). The factor \( \frac{1}{K} \) varies with the thickness of the Cd filter and is tabulated in the literature (34). For \( \frac{1}{v} \) absorbers \( g = 1 \) and equation (2.2) becomes

\[ R_{cd} = \frac{1 + rs}{rs + \frac{r\sqrt{T}}{K T_o}} \] (2.3)

Rearranging this equation gives:

\[ r\sqrt{\frac{T}{T_o}} = \frac{1}{s(T_o/T)^{\frac{1}{2}} (R_{cd} - 1) + R_{cd}/K} \] (2.4)

Therefore, by using very thin detectors, the factor \( r\sqrt{\frac{T}{T_o}} \) can be measured.

For detectors with appreciable self absorption, equation (2.4) becomes (18)
\[ r \sqrt{\frac{T}{T_0}} = \frac{G_{th}(1 - R_{cd})^h}{(FR_{cd} - 1) \frac{G_r s_o}{g} + R_{cd} (1/K - W)} \]  

(2.5)

where

\[ s_o = s \sqrt{T_o/T} \]

\[ G_{th} = \text{thermal neutron self shielding factor,} \]

\[ G_r = \text{resonance self shielding factor,} \]

\[ F = \text{Cd transmission for resonance flux,} \]

\[ W = \text{fraction of resonance activation below Cd cutoff,} \]

\[ h = \text{thermal transmission of Cd filter.} \]

In order to measure the effective temperature of the thermal neutrons, the activity ratios of Lu$^{177}$ and Lu$^{176m}$ were measured. The activity ratios of these isotopes at position x with respect to a standard position s is expressed as

\[ R = \frac{\left[ \frac{A^{177}}{A^{176m}} \right]_x}{\left[ \frac{A^{177}}{A^{176m}} \right]_s} = \frac{\left[ \frac{(g + rs)^{176}}{(g + rs)^{175}} \right]_x}{\left[ \frac{(g + rs)^{176}}{(g + rs)^{175}} \right]_s} = \frac{Y_x^*}{Y_s} \]  

(2.6)

* Equation (2.6) is for very dilute detectors, where resonance and thermal self absorption is negligible.
By assuming \( G_{th} = 1 \) and \( G_r = 1 \), and solving for \( Y_x \),
the following relationship is obtained:

\[
Y_x = \frac{\left( (g + rs G_r)^{176} \right)^x}{\left( (g + rs G_r)^{175} \right)^x} \quad (2.7)
\]

For \( r = 0 \) (i.e., no epithermal neutrons) equation (2.7) reduces to

\[
Y_x = [g^{176}]^x = R Y_s = R[g_{176}]_s \quad (2.8)
\]

The effective neutron temperature for \( g_{176} \) (temperature index of Lu\(^{176}\)) is obtained from Figure 2.2 (36).

If \( r = 0 \) the \( \left( (g + rs)^{176} \right)^x \) must be evaluated.

For Lu\(^{175}\), \( g = 1 \) and \( r\sqrt{T/T_o} \) is a measurable value with dilute foils.

\[
\left[ s\sqrt{T_o/T} G_r \right]^{175}_x = \left[ s_0 G_r \right]^{175}_x
\]

** This equation takes into account the self-shielding of resonance neutrons.
Figure 2.2. Calibration Curve for $g_{176}$
is also determined experimentally. Then using the values of $g_{176}$ and $s_4$ given by Westcott (35), corrected values of $g_{176}$ and $s_4$ are obtained by trial and error. The effective neutron temperature is found from the value of $g_{176}$ (see Figure 2.2).

An alternative method of evaluating $r\sqrt{T/T_o}$ is to irradiate two detectors and express their activation ratios as follows:

$$R' = \left[\frac{A^a}{A^b}\right]_x \frac{\left[\frac{gG_{th} + rs_{G_r}}{gG_{th} + rs_{G_r}}\right]^a}{\left[\frac{gG_{th} + rs_{G_r}}{gG_{th} + rs_{G_r}}\right]^b} = \frac{x_x}{x_s}$$

or rearranging

$$X_x = R' X_s = \frac{[gG_{th} + r\sqrt{T/T_o} s_o G_r]^a}{[gG_{th} + r\sqrt{T/T_o} s_o G_r]^b}$$

$X_s$ is determined from the Cd ratio of one of the detectors. Solving for $r\sqrt{T/T_o}$ in equation (2.10), the following is obtained:
Differential Method

Time-of-Flight

The time-of-flight method utilizes a mechanical rotor with narrow slits called a chopper (37) or a pulsed accelerator to produce bursts of neutrons with a time duration depending on the speed of the rotor or accelerator pulsing system. The bursts of neutrons travel through an evacuated tube to a bank of detectors. The energies of neutrons are determined by electronically measuring the time it takes for neutrons to travel from the source to the detectors.

Usually choppers are classified as slow and fast. Slow choppers give bursts of about 20 \( \mu \) sec. duration whereas the burst of a fast chopper is on the order of 1 \( \mu \) sec. Unlike crystal spectrometers which produce monoenergetic neutrons, choppers select certain velocity neutrons out of heterogeneous beams.
The energy resolution of a chopper is described by the equation (38)

\[
\frac{\Delta E}{E} = -0.0279 \frac{\Delta t}{m} E + 2 \frac{\Delta m}{m}
\]

(2.12)

where

\[\Delta E = \text{uncertainty in energy,}\]
\[E = \text{energy of the neutron,}\]
\[\Delta t = \text{uncertainty in flight time per meter,}\]
\[m = \text{flight path, meters,}\]
\[\Delta m = \text{uncertainty in flight path.}\]

For a fast chopper, the second term is negligible and the resolution reduces to the first term. Better energy resolution is obtained with longer flight paths. A list of resolutions together with flight paths for different choppers is listed by Anderson (38). Multi-channel analysis determines the intensities of neutrons in different energy intervals.

Slow choppers can be constructed much more easily than the fast choppers. This is due to the fact that for the former, chopping is accomplished with thin layers of Cd, whereas it takes many inches of plastic and steel for fast choppers (39).
An alternate method for measuring neutron energy spectra (40) is the time-of-flight method with a pulsed source. The advantages of this method are: (1) the fuel elements get low irradiation, (2) the rotor cut-off function does not have to be determined, and (3) by the use of the chopper and pulsed source, the neutron spectrum as a function of slowing down time can be determined.

Crystal Spectrometer

The use of crystal diffraction spectrometers for detecting monochromatic neutrons has been investigated by many authors (9, 10, 41-47). Diffraction is a scattering process. A regularly arranged series of atoms of a crystal will scatter the neutron waves in all directions, but only in particular directions will scattered waves be in phase and reinforce each other to form a diffracted beam. The atoms of a crystal are arranged periodically in parallel planes, so that in general scattered waves are out of phase except in a few directions where reinforcement takes place.

The diffraction process applies to X-rays as well as to neutrons. The fundamental difference between the two is that X-rays are scattered by the orbiting electrons, where as the neutrons are scattered by the nuclei of the atom. For this reason, the X-ray
scattering amplitude is proportional to the atomic number of the element, while the neutron scattering amplitude shows a relatively small variation with atomic number.

When neutrons from a collimator impinge on a crystal plane of a single crystal at an angle, \( \theta \), only neutrons of one energy are diffracted in the direction of the angle 2\( \theta \). Therefore, the crystal and the detector must maintain an angle ratio of one-half in order to obtain monoenergetic neutrons.

The diffraction of the neutron beam depends on the lattice spacing in the crystal. This space varies depending on the crystal material and on the crystal axis along which it is cut. The coherent scattering of neutrons from the nuclei of atoms in a single crystal to produce a monochromatic neutron beam is governed by the familiar Bragg's relation:

\[
\text{n} \lambda = 2d \sin \theta (2.13)
\]

where

- \( d \) = lattice spacing,
- \( n \) = order,
- \( \lambda \) = wave length,
- \( \theta \) = glancing angle.

In energy terms, this relationship becomes:
\[ E(\text{ev}) = \frac{k\pi^2}{\sin^2 \theta} \quad (2.14) \]

where \( k \) is a constant for each lattice spacing.

Table 2.2 gives a list of crystals with \( k \) and the detectable energy at 1°.

**TABLE 2.2**

**TABLE OF \( k \) AND ENERGY FOR DIFFERENT CRYSTALS AT 1°**

<table>
<thead>
<tr>
<th>Crystals</th>
<th>( k \times 10^3 ) (ev)</th>
<th>Energy (ev)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cu (111)</td>
<td>4.71</td>
<td>15.40</td>
</tr>
<tr>
<td>NaCl (200)</td>
<td>2.58</td>
<td>8.42</td>
</tr>
<tr>
<td>Al (111)</td>
<td>3.76</td>
<td>12.28</td>
</tr>
<tr>
<td>Ge (111)</td>
<td>1.92</td>
<td>6.27</td>
</tr>
<tr>
<td>LiF (111)</td>
<td>3.81</td>
<td>12.25</td>
</tr>
</tbody>
</table>

From equation (2.14) it can be seen that the higher order interference will be due to neutrons with energy \( n^2 \) times the lowest order. Also, there is interference due to the mosaic spread of the crystal.
Mosaic Structure

In order to know the energy of diffracted neutrons at different angles, it is necessary to know the distance between the atomic planes of the crystal. The distance depends on the direction in which the crystal is cut and also on the crystal material. If the crystal were perfect, neutrons of a single energy would be diffracted at any given angle. However, all real single crystals have a mosaic structure, that is, they possess structural imperfections which break up the arrangement of atoms into a number of small blocks, each slightly disoriented from one another. This mosaic structure causes the diffracted neutrons to have a spread of energies at any angle.

Higher Order Diffraction

Also, according to equation (2.14), there are higher order neutrons diffracted at each angle. The ratio of the second order to first order neutrons detected in the beam can be written as (48):

\[ f(E) = \phi(E) \epsilon(E) \delta(E) \sum_{n=2} w(E) \sum_{n=2} \rho(E) \]  

(2.15)

where
\( \phi(E) = \frac{\phi(4E)}{\phi(E)} \)  
\( \phi(E) \) is assumed to be \( 1/E \) for energy greater than 0.4 ev

\( \varepsilon(E) = \frac{\varepsilon(4E)}{\varepsilon(E)} \)  
\( \varepsilon(E) \) is the ratio of the detector efficiency for second and first order neutrons

\[ \delta(E)_{n=2} = \frac{\Delta E(n=2, 4E)}{\Delta E(n=1, E)} = \frac{1}{2} \left( \frac{(4E)^{3/2}}{(E)^{3/2}} \right) \frac{\cos[\theta(4E, n=2)] \Delta \theta(4E)}{\cos[\theta(E, n=1)] \Delta \theta(E)} \]

(2.16)

where

\[ w(E)_{n=2} = \frac{w(4E), n=2}{w(E, n=1)} = 1 \]  
\( w(E) \) is the fraction of reflected beam transmitted by collimating system

\[ \rho(E)_{n=2} = \frac{r(4E)_{n=2}}{r(E)_{n=1}} \]  
\( r(E) \) is reflectivity of the crystal plane used

In equation (2.16) \( \cos[\theta(4E, n=2)] = \cos[\theta(E, n=1)] \),

and let \( \alpha = \frac{\Delta \theta(4E)}{\Delta \theta(E)} \), which reduces the energy band transmitted to the detector to

\[ \sigma(E)_{n=2} = \sigma_o \alpha \]

(2.17)

where \( \sigma_o = 4 \).
The reflectivity of a crystal for neutrons is given by equation (11) of Holm (49) for the Laue Case

\[ r(E) = \exp(-2M) \frac{\pi}{2} \eta e^{-A} \sum_{j=1}^{\infty} \gamma \tan^2 \theta (-1)^{j+1} \]  

(2.18)

where

\[ \exp(-2M) = \text{Debye-Waller temperature correction factor} \]

\[ \eta = \"mosaic spread\" \text{ of the crystal} \]

\[ A = \text{product of the linear absorption coefficient and the path length in crystal} \]

\[ \gamma = \frac{8d^3tN^2F^2}{(2\pi)^{1/2} \eta n^3} \]

\[ t = \text{thickness of crystal} \]

\[ N = \text{reciprocal of the unit cell} \]

\[ F = \text{crystal structure factor} \]

The Debye-Waller temperature correction factor accounts for the reduction in intensity of the Bragg reflection due to the thermal motion of the atoms in the lattice, where

\[ M = \frac{6h^2}{mk(H)} \left( \frac{n}{2d} \right)^2 \left[ \frac{1}{4} + \left( \frac{T}{H} \right)^2 \frac{Q(H)}{T} \right] \]  

(2.19)
with

\[ Q(Z) = \int_z^x \frac{x dx}{e^x - 1} \quad (2.20) \]

Here,

\( \mathbb{H} \) = Debye or characteristic temperature of the crystal

\( h, k \) = Planck's and Boltzmann's constants

\( m \) = nuclear mass

\( T \) = temperature of the crystal in degrees Kelvin

Since the factors in equation (2.15) cannot be calculated accurately, it is easier to obtain an open beam count rate of \( E \) and \( 4E \). The ratio is:

\[ f^*(E) = \frac{\text{count (4E)}}{\text{count (E)}} = \frac{\phi(E) \epsilon(E) \sigma(E)}{\phi(E) \epsilon(E) \sigma(E)} n=1 w(E) n=1 o(E) n=1 \quad (2.21) \]

Equation (2.21) is a good approximation because the second order contamination is usually on the order of a few per cent, unless the first order energy is below the Maxwellian peak. Now, \( \sigma(E) n=1 \) is

\[ \sigma(E) n=1 = \frac{\Delta E(n=1, 4E)}{\Delta E(n=1, E)} \quad \frac{4E^{3/2} \cos[\theta(4E, n=1)] \Delta \theta(4E)}{E \cos[\theta(E, n=1)] \Delta \theta(E)} \quad (2.22) \]
where
\[ \beta = \frac{\cos[\theta(4E, n=1)]}{\cos[\theta(E, n=1)]} \]

\[ \sigma(E)_{n=1} = 2\sigma_o \alpha \beta \quad (2.23) \]

Now the \( f(E) \) term in terms of \( f^*(E) \) is expressed as:

\[ f(E) = f^*(E) \frac{r(4E)_{n=2}}{r(4E)_{n=1}} 2Bw(E)_{n=1} \quad (2.24) \]

Since \( Bw(E)_{n=1} = 1 \), then

\[ f(E) = f^*(E) \frac{r(4E)_{n=2}}{r(4E)_{n=1}} = f^*(E) R(4E) \quad (2.25) \]

Therefore, by using the calculated value of \( R(4E) \), \( f(E) \) can be estimated.

The Renninger Effect

When a neutron spectrum was measured using a crystal spectrometer and corrections were made, many large dips were found (2, 50-53) instead of the smooth spectrum characteristic of a Maxwellian distribution of neutron velocities. These fluctuations are due to
Bragg reflection of neutrons from reflecting planes other than that used to obtain the monochromatic beam. This reflection is best explained by the use of the reciprocal lattice and sphere reflection concepts (54) and the vector notation as outlined in Appendix III.

Figure 2.3 demonstrates multiple reflection by means of the sphere of reflection, where the radius of the sphere is 1/λ; it is centered on the crystal at C, and passes through the origin of the reciprocal lattice at 0. For an incident direction CO, reflection occurs in the direction CP if the reciprocal lattice vector \( \vec{r}' \), terminates at P on the sphere. Suppose a second reciprocal lattice vector \( \vec{r}_2' \) terminates at Q on the sphere, then reflection of the same wave length occurs in the direction CQ. Taking Q as the origin of the reciprocal lattice and CQ as the incident direction, further reflection corresponding to the vector \( \vec{r}_3' \) is possible. The double reflection by way of \( \vec{r}_2' \) and \( \vec{r}_3' \) gives a resultant in the same direction and at the same wave length as a reflection through \( \vec{r}_1' \). Since \( \vec{r}_1' = \vec{r}_2' + \vec{r}_3' \) the Miller indices of the planes are related by:

\[
\begin{align*}
h_1 &= h_2 + h_3, & k_1 &= k_2 + k_3, & l_1 &= l_2 + l_3
\end{align*}
\]

The double reflection can occur when any reciprocal lattice point Q is on the sphere, where Q is not necessarily
Figure 2.3. Sphere of Reflection
located coplanar with C, O, and P. Finally, it should be noted that when multiple Bragg reflection occurs for first order reflection, it also occurs for higher order reflections at the same angle of incidence.

Reflectivities

Since reflectivities for NaCl and Be have been calculated (49), a NaCl crystal cut along the (200) plane was used. The disadvantage of this crystal is that the second order contamination becomes very large at low energies (e.g., = 50 per cent at E = .025 ev). In order to reduce this second order contamination, a crystal with suppressed second order reflectivity must be used; LiF (111) is such a crystal. The suppression of second order reflection is due to the opposite sign of scattering from successive planes (12).

There are some disadvantages in using LiF crystals. These are: the lattice parameters are uncertain (55), the diffuse background scattering is fairly high, single crystals are hard to obtain and suppression of second order is not very great [F (111)/F (222) = 1.97].

Use of Ge and Si crystals, which have diamond structure, is very favorable because, for the (111) plane, the second order (222) vanishes completely. In fact,
it has been shown \(58\) that at \(0.015\) ev, the higher order contamination of a Ge (111) crystal is only 2.5 per cent. Use of a quartz filter also reduces the higher order contamination considerably \(57\).

Transmission of Reflected Beam Through Filters

Transmission of a neutron beam through a filter can be expressed as:

\[
T = T_1 X_1 + T_2 X_2 + T_3 X_3 + \cdots \tag{2.27}
\]

where

\[
T_i = \text{transmission of neutrons at energy } E_i
\]

\[
X_i = \text{fraction of neutrons at energy } E_i.
\]

The sum of the fractions of neutrons must add up to one \(\Sigma X_i = 1\). By use of absorbers such as Au, Cd, Sm, Ag, Rh, In etc., neutrons with certain energies can be suppressed. These absorbers have a large resonance and/or a \(1/v\) cross section. For instance, for a \(1/v\) absorber, suppression of the first order by 99 per cent suppresses the second order by 90 per cent. Therefore, the transmission method is a very useful one for estimating higher order contamination. The energy resolution of a crystal spectrometer neglecting the mosaic spread of the crystal is expressed as \(44\):
The actual resolution is determined by plotting the rocking curve for different angles. The rocking curve is obtained by counting diffracted neutrons with a stationary detector and by rotating or rocking the crystal to different angles. The maximum of the distribution corresponds to the position where the detector and crystal plane are aligned. The width of the peak at half-maximum is equal to $\Delta E$, the energy spread, of the system.

The main advantage of a crystal spectrometer compared to the time-of-flight method in the low energy range is that the area of the beam at the detector is quite small. A diffraction spectrometer can cover the energy range from 0.01 to 10 ev, whereas, it is difficult to cover this energy range with one chopper. The disadvantages of a crystal spectrometer are: (1) the reflectivity varies inversely as the neutron energy; (2) the contribution of the higher order diffraction increases below the Maxwellian peak, and; (3) the spectrometer detector is closer to the reactor than the time-of-flight detectors, resulting in a higher background.
CHAPTER III

DESCRIPTION OF APPARATUS

Crystal Spectrometer

Collimator

In collimating neutrons through a narrow channel, the intensity decreases very rapidly. Therefore, it is necessary to place several collimating channels side by side, where each channel has the required small angular divergence. This type of collimator is called a Sollar collimator (58).

A Sollar collimator was constructed from eight stainless steel strips 0.010 in. thick, 1.50 in. wide, and 33 in. long. Stainless steel rods one-eighth of an inch square were used for separating the strips. This collimator was then inserted into a Cd-lined slot 1 in. x 1.50 in. x 33 in. in a 3 in. diameter wooden cylinder; the collimator was held in the slot by friction. The collimator was then dipped into a boiling bath of paraffin wax, \((\text{CH}_2)_n\), dissolved in benzene, \((\text{C}_6\text{H}_6)\). This procedure gave a thin coat of \((\text{CH}_2)_n\) on the surface of the collimating plates in
order to eliminate the secondary scattering from the walls. The angular divergence of this collimator is 13.0 minutes; one section of the collimator is shown in Figure 3.1.

Neutrons entering the collimator propagate in four different ways: (1) direct transmission (2) mirror reflection (3) transmission through the collimator walls, and (4) rescattering by the walls. These effects are now considered in turn.

For direct transmission with an isotropic source on one end, consider a narrow slit with thickness T, width W, and length L, as shown in Figure 3.1.

The number of neutrons emerging from the collimator per unit width, is found from the relation (59)

\[
N(\text{neutrons/cm sec}) = \int_0^\theta \frac{1}{2\pi} \frac{T}{L^2 \sec^2 \theta} \text{d}(L \tan \theta) \times \phi_o \text{Td}(L \tan \theta)
\]

(3.1)

where

\[
\phi_o \text{Td}(L \tan \theta) = \text{source strength per unit solid angle}
\]

\[
\text{d}(L \tan \theta) = \text{line source element}
\]
Figure 3.1. Single Collimator Slit

Figure 3.2. Size of the Effective Source
\[ L^2 \sec^2 \theta = \text{distance from emitting element } d(L \tan \theta) \text{ to the receptor} \]

\[ \theta = \text{angle from the emitting element to the receptor in radians} \]

On integrating equation (3.1), the following result is obtained:

\[ N(\text{neutrons/cm sec}) = \frac{T^2}{2\pi L} \phi \theta \]

(3.2)

Dividing both sides of equation (3.2) by thickness \( T \), a relationship between the direct inlet and outlet scaler flux is obtained as follows:

\[ \phi_1 (\text{neutrons/cm}^2 \text{ sec}) = \frac{T \phi_0 \theta}{2\pi L} \]

(3.3)

The intensity of the neutron beam emerging from the collimator would be a triangular function if the collimator were perfect:

\[ I(\phi) = I_o [1 - \phi/\alpha] \]

(3.4)

where

\[ I(\phi) = \text{intensity as a function of angle } \phi \]

\[ I_o = \text{maximum intensity in the center of the collimator axis} \]
and the angle $\phi$ varies from $-\alpha$ to $+\alpha$. Due to
imperfections in the collimator and the finite trans-
mission of neutrons through the wall, this triangular
function tends to be closer to Gaussian distribution
(60). The Gaussian approximation of equation (3.4) is

$$ I(\phi) = I_0 \exp -\left(\frac{\phi}{a'}\right)^2 $$

(3.5)

where

$$ a' = \frac{a}{(\ln 2)^{1/2}}. $$

In the actual experimental set-up, the disk
source is located some distance below the collimator
inlet; therefore, an approximate relationship must be
derived between the scaler flux coming out of the
collimator and the scaler flux of the source. Again,
assume the source is isotropic and consider only those
neutrons which are emitted directly. Figure 3.2 shows
$T_s$ and $W_s$, the width and the thickness of the source
subtended by the collimator outlet which is approximately
rectangular in shape.

The expressions for $T_s$ and $W_s$ are as follows:

$$ T_s = T + 2 \left[ \frac{L_2}{L_1} \right] T $$

$$ W_s = W + 2 \left[ \frac{L_2}{L_1} \right] W $$

(3.6)
Now imagine that the collimator is of thickness $T_s$, width $W_s$ and length $(L_1 + L_2)$. Using equation (3.3), the relationship for the outlet scaler flux for this collimator is:

$$\phi'(\text{neutrons/cm}^2\text{sec}) = \frac{T_s \phi \theta'}{2L'}$$

(3.7)

Since the actual area of the outlet collimating slit is $T \cdot W$ rather than $T_s \cdot W_s$, an approximate expression for the outlet scaler flux using equation (3.5) is obtained as follows:

$$\phi(\text{neutrons/cm}^2\text{sec}) = \phi' \frac{W_s \cdot T_s}{W \cdot T} \int_{\phi_s}^{\phi_1} \frac{\exp[-(\phi/a')^2]}{\int_{-\phi_s}^{\phi_1} \exp[-(\phi/a')^2] \, d\phi} \, d\phi$$

(3.8)

where

$$\phi_1 = \arcsin \left( \frac{1}{2}(T_s + T) \right)$$

$$\phi_s = \arcsin \left( \frac{T_s}{L_s} \right)$$
The total reflection of neutrons by the wall of the collimator can distort the neutron spectrum. Such a reflection was first reported by Fermi and Zinn (61). The critical angle for such reflection is (62).

\[ \theta_c = \lambda (Na/\pi)^{1/2} \]  

(3.9)

where

- \( \lambda \) = neutron wave length
- \( N \) = number of atoms/cm\(^3\) in the scattering medium
- "a" = average coherent scattering length

Equation (3.9) shows that the critical angle is a function of neutron energy. In the relationship for the critical angle, "a" can be either positive or negative, depending on the scattering material. For instance, hydrogen has a negative scattering length, while carbon has a positive value. Experiments have been performed using neutron mirrors with various hydrocarbons (63) and it was found that \( \theta_c \) becomes zero for a ratio of H/C - 1.748, i.e., the negative scattering length of H is equal to the positive value of C. Therefore, if the collimator walls have a value of H/C larger than 1.748, total reflection cannot occur. Accordingly, the surface of the collimator walls was coated with a thin coat of paraffin which, having an
H/C ratio of approximately 2, should eliminate the total reflection of neutrons. This type of collimator has been constructed previously (64) with a thin film of polyethylene.

The probability of direct transmission through the wall of the collimator without absorption or scattering is expressed as $e^{-\Sigma_t z}$, where $\Sigma_t$ is the total cross section of the wall material and $z$ is the path length of the neutrons through the walls. Even though the absorption of steel is not high, this probability is very small since any neutrons which the steel absorbs will pass through it at very small angles. Thus, the neutron path length in steel is relatively long, and transmission through the wall is negligible compared to the direct transmission.

Calculations have been made (2) for the transmission of neutrons by rescattering in the walls of a similar collimator. For this calculation, this contribution from rescattering was found to be less than one-half per cent of the total transmitted intensity with negligible effect upon the angular distribution.

The description above states that the neutrons emerging from the collimator come predominantly from direct transmission. A calculation was made for the ratio of $\phi/\phi_0$ for the experimental set-up and was found to be equal to $7.14 \times 10^{-5}$. 
Crystals

NaCl (200)* and LiF (111)* crystals were used for these experiments. Dimensions of both crystals were 1/2 in. x 2 in. x 2 in.

Goniometer**

The goniometer consists of a sample table with a crystal holder, an arm on which the second collimator and detector shield are attached, and the gearing system. The gearing system is in a cast aluminum box, and they can be moved manually or electrically. Figure 3.3 shows the crystal spectrometer in operation on top of the University of Florida Training Reactor (UFTR). The crystal holder is made from a rectangular aluminum box with both ends open in which the crystal fits. This box is attached to the crystal table by means of a stainless steel rod 3/4 in. diameter.

The detector shield is attached to an arm with a collimator and is shown in detail in Figure 3.4. The movement of the crystal holder and the detector arm can

* These crystals were obtained from The Harshaw Chemical Company, 1945 E. 97th Street, Cleveland 6, Ohio.

** Picker Biplanar Diffractometer, Model No. 3527.
Figure 3.3. Diffraction Spectrometer in Operation
Figure 3.4. Detector Arm with Second Collimator
be performed separately or at an angle ratio of 1:2. The 1:2 movement is carried out at very high precision by means of worm gears in the machine. Two odometers are attached to the box where the angular positions of the crystal and the detector can be read to 1/100th of a degree.

**Neutron Detection System**

The detector used for these measurements is 2 1/32 in. diameter, 96 per cent B\textsuperscript{10} enriched BF\textsubscript{3} counter with a pressure of 76 cm Hg. The length of the tube is 15 3/4 in. long with 12 3/4 in. of sensitive length; it has a ceramic end window.* The center of the plateau for this detector was found to be 2800 volts using the existing cables. The length of the plateau was about 400 v. with change of slope of one per cent per 100 volts.

The efficiency of the detector for different energy neutrons was not calculated because it was determined experimentally. Theoretically, efficiency can be calculated from the following relationship (44):

\[
E = 1 - \exp\left(-\frac{N \times CE^{-1/2}}{2}\right)
\]  

(3.10)

* This detector was supplied by Reuter-Stokes Electronic Components, Inc., Model No. RSN-108S.
where

\[ N = \text{number of atoms per cm}^3\text{ in the counter} \]
\[ x = \text{length of the counter} \]
\[ C = \text{numerical constant which gives the } 1/v \text{ slope of the } ^{10}\text{Be}(n,\alpha)\text{ Li}^7 \text{ cross section} \]
\[ E = \text{energy.} \]

With the BF\textsuperscript{3} counter, a Radiation Counter Laboratory (RCL) decade scaler, preamplifier and linear amplifier (model numbers 2032, 20200 and 20100, respectively) were used. In addition, Atomic Instrument Company regulated high voltage supply, Model 319, was used.

The Subcritical Assembly

A subcritical tank made from 6061 Al, 24 in. diameter and 60 in. long and wrapped with .030 in. thick Cd, was placed on top of the graphite pedestal of the UFTR. A grid made from the same type of Al was placed in the tank, so that the fuel elements could be arranged in a hexagonal pattern with lattice pitches of 14.7 and 22 cm.

Figure 3.5 shows the location of the tank in the UFTR, and Figure 3.6 shows the top view of the lattice arrangements in the subcritical assembly.
Figure 3.5. UFTR Elevation
Figure 3.6. Top View of the Subcritical Lattice
The plane source of the neutrons entered the subcritical assembly from the bottom face of the tank. There is approximately 4 in. of moderator between the grid and the bottom of the tank.

Two types of natural uranium fuel elements were used for these experiments: Mark V-B and Mark I. Mark V-B is an annular type of element 2.684 in. o.d. and 1.970 in. i.d. and 8 9/32 in. long. Mark I is a solid fuel element one in. in diameter and 8 3/8 in. long. There were six fuel elements in each tube. The moderator-to-fuel volumetric ratio $V_m/V_f$ and pitches are listed in Table 3.1.

**TABLE 3.1**

**SUBCRITICAL LATTICE CONFIGURATIONS**

<table>
<thead>
<tr>
<th>Pitch (cm)</th>
<th>Mark V-B Rods</th>
<th>Mark I Rods</th>
<th>$V_m/V_f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>22.0</td>
<td>7</td>
<td>0</td>
<td>23.76</td>
</tr>
<tr>
<td>22.0</td>
<td>7</td>
<td>7</td>
<td>18.04</td>
</tr>
<tr>
<td>14.7</td>
<td>13</td>
<td>0</td>
<td>12.34</td>
</tr>
<tr>
<td>14.7</td>
<td>13</td>
<td>13</td>
<td>9.25</td>
</tr>
</tbody>
</table>

Some of the parameters of the subcritical assembly lattices using the BUCKSHOT Code (65) were calculated and are listed in Appendix IV. A proposal
for performing experiments in the subcritical assembly utilizing the UFTR as a plane source is outlined in Appendix E. The critical rod position of the reactor was checked after the installation of the subcritical assembly as outlined in the proposal. No coupling between the UFTR and the subcritical assembly was found.

Activation detectors were used for the integral measurements and a crystal diffraction spectrometer for differential measurements. For comparisons of experimental results with the theory, THERMOS (4) and CEPR (3) codes were used.

Two five kilowatt heaters were inserted into the tank so that the D₂O could be heated. A constant temperature in the tank was maintained by the use of a temperature control unit which utilized four thermocouples. Also, a stirrer helps to keep a uniform temperature throughout the tank. A side view of this set-up in the tank is shown in Figure 3.7.

The hydrogen content of the D₂O was determined by use of the NMR (Nuclear Magnetic Resonance) equipment in the University of Florida Chemistry Department* and found to be 0.66±0.05 per cent.

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* Thanks are due Dr. Wallace Brey of the University of Florida Chemistry Department for making this determination.
Figure 3.7. Side View of the Moderator Tank
CHAPTER IV

RESULTS OF INTEGRAL SPECTRUM MEASUREMENTS

Introduction

The first integral measurements were made in the UFTR core and thermal column where there were few fast neutrons (66). The spacial hardening of the thermal neutrons in highly absorbing media was observed. Then, the spectral hardening in a subcritical exponential facility which uses the graphite pedestal of the UFTR as a neutron source was measured spacially using differential and integral methods and results were compared with theoretical calculations (4).

For integral measurements of the neutron spectrum, various types of activation detectors were used. All of the detectors fell into two categories: either they had a $1/v$ absorption cross section or they had a large low-lying absorption resonance.
Measurements in the Thermal Column

Stainless Steel Rod

Integral measurements were made in a stainless steel rod one in. diameter and 12.5 in. long and in a uranium slab 2 in. x 2 in. x 1/2 in. Radial holes 0.050 in. diameter and 1/2 in. deep were drilled in the stainless steel pin holder 6.5 in. from the end. This was done by cutting the stainless steel rod into two pieces. These holes held wire detectors. For the uranium slab, detectors were sandwiched in between the U foils which were 2 in. x 2 in. x .005 in. Measurements were made 24 in. from the east end of the thermal column of the UFTR. At this position, the ratio of thermal to epithermal flux is approximately 100 (66). Therefore, the contribution of the epithermal neutrons is negligible and equation(2.8) is used directly to measure the change in effective neutron temperature.

For these measurements, Lu\textsuperscript{175}, Lu\textsuperscript{176}, Dy\textsuperscript{164}, Au\textsuperscript{197}, and Pu\textsuperscript{239} detectors were used. The detectors were fabricated in the form of wires 0.030 in. diameter containing 10 per cent by weight of the activant in an Al matrix, except for the Au wires, which were 0.005 in. in diameter. The first measurements were made in the

stainless steel rod using natural Lu-Al wires.* A few hours after irradiation, Lu$^{176m}$ and Lu$^{177}$ activities were counted five times using a scintillation counter.** About four days later, after the Lu$^{176m}$ activity had decayed out, five more sets of counts were taken of the Lu$^{177}$ activities. The total count rate for each time was at least 10,000 counts. The procedure for obtaining the average normalized activities of the two isotopes and their ratios is outlined in Appendix A.

Pu-Al wires were calibrated by measuring their natural gamma activities. These wires were then irradiated in a stainless steel rod and gammas from the fission fragments were counted (67). The Au$^{197}$ and Dy$^{164}$ wires were also irradiated in the stainless steel pin holder. The corrected activities from all the detectors are listed in Table 4.1. The activities of the 1/v absorbers, the Lu$^{177}$ and the Pu$^{239}$ fission products were each fitted to the Bessell function, $I_o(\lambda r)$, and are plotted on Figure 4.1. The ratios of fitted curves for (Lu$^{177}$/1/v absorber) and (Pu$^{239}$/1/v absorber)***

---

* Natural Lu contains 2.6 per cent Lu$^{176}$ and 97.4 per cent Lu$^{175}$.

** Baird-Atomic, Inc., Gamma Spectrometer, Model No. 810.

*** Activities of Pu$^{239}$ means activities of the fission fragments of Pu$^{239}$. 
Figure 4.1. Flux Traverses and Ratio of Lu$^{177}$/Lu$^{176m}$ and Pu$^{239}$/Au$^{197}$ in 1 in. Stainless Steel Rod
TABLE 4.1

Lu, Au, Dy and Pu ACTIVATIONS IN STAINLESS STEEL ROD

<table>
<thead>
<tr>
<th>Radius (cm)</th>
<th>$\text{Lu}^{176\text{m}}*$</th>
<th>$\text{Au}^{198\text{m}}*$</th>
<th>$\text{Dy}^{165\text{m}}*$</th>
<th>$\text{Lu}^{177\text{m}}*$</th>
<th>Fission Fragments From $\text{Pu}^{239}$*</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>0.254</td>
<td>1.057</td>
<td>1.018</td>
<td>1.003</td>
<td>1.021</td>
<td>1.027</td>
</tr>
<tr>
<td>0.381</td>
<td>1.032</td>
<td>1.029</td>
<td>1.020</td>
<td>1.017</td>
<td>1.040</td>
</tr>
<tr>
<td>0.508</td>
<td>1.115</td>
<td>1.089</td>
<td>1.060</td>
<td>1.022</td>
<td>1.047</td>
</tr>
<tr>
<td>0.625</td>
<td>1.117</td>
<td>1.115</td>
<td>1.075</td>
<td>1.082</td>
<td>---</td>
</tr>
<tr>
<td>0.762</td>
<td>1.156</td>
<td>1.162</td>
<td>1.070</td>
<td>1.111</td>
<td>1.114</td>
</tr>
<tr>
<td>0.889</td>
<td>1.225</td>
<td>1.216</td>
<td>1.192</td>
<td>1.178</td>
<td>1.190</td>
</tr>
<tr>
<td>1.016</td>
<td>1.288</td>
<td>1.297</td>
<td>1.258</td>
<td>1.193</td>
<td>1.261</td>
</tr>
<tr>
<td>1.143</td>
<td>1.387</td>
<td>1.385</td>
<td>1.347</td>
<td>1.284</td>
<td>1.340</td>
</tr>
</tbody>
</table>

* Estimated standard deviation is less than 1%.
are tabulated in Table 4.2 together with the change in effective neutron temperature from the outer surface to the center line of the stainless steel rod using tabulated \( g \) factors. The ratios show that the activity ratio (\( \text{Lu}^{177}/\text{L/v absorber} \)) is more sensitive to the spectrum shift in highly absorbing media than the (\( \text{Pu}^{239}/\text{L/v absorber} \)) ratio.

**Uranium Slab**

Lu wires were placed between uranium foils as shown in Figure 4.2 and the uranium foils were positioned in a slot in a graphite block. The corrected activities of \( \text{Lu}^{177} \) and \( \text{Lu}^{176m} \) together with their ratios are plotted on the bottom of Figure 4.2 and tabulated in Table 4.3. The data in this medium were interpreted in terms of effective neutron temperature using equation (2.8). Cd ratios were measured inside and outside of the U slab and it was found that the contribution of epithermal neutrons to the Lu activities was negligible. The effective neutron temperature change in the slab was found to be \( 9.4^\circ \text{C} \pm 3.4^\circ \text{C} \) using tabulated \( g \) factors.
Figure 4.2. Lu Traverses Through U-Slab
Table 4.2

Ratios of [Pu$^{239}$/l/v Absorber] and [Lu$^{177}$/l/v Absorber] Activities*

<table>
<thead>
<tr>
<th>Radius (cm)</th>
<th>(Pu$^{239}$/l/v absorber) I (.980r)/I (1.045r)$^+$</th>
<th>(Lu$^{177}$/l/v absorber) I (.880r)/I (1.045r)$^+$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>0.20</td>
<td>0.999</td>
<td>0.997</td>
</tr>
<tr>
<td>0.40</td>
<td>0.995</td>
<td>0.987</td>
</tr>
<tr>
<td>0.60</td>
<td>0.989</td>
<td>0.973</td>
</tr>
<tr>
<td>0.80</td>
<td>0.981</td>
<td>0.948</td>
</tr>
<tr>
<td>1.00</td>
<td>0.971</td>
<td>0.932</td>
</tr>
<tr>
<td>1.20</td>
<td>0.964</td>
<td>0.915</td>
</tr>
<tr>
<td>1.27</td>
<td>0.957</td>
<td>0.8977</td>
</tr>
</tbody>
</table>

* The standard deviations of these values are less than 1%.

$^+$ The change in neutron temperature using two different ratios were:

$$\Delta T_{\text{Lu}^{177}} = 28.4 \pm 3.2^\circ C$$

$$\Delta T_{\text{Pu}^{239}} = 50.0 \pm 14.4^\circ C$$
TABLE 4.3

Lu$^{177}$ AND Lu$^{176m}$ ACTIVITIES AND THEIR RATIOS IN URANIUM SLAB

<table>
<thead>
<tr>
<th>Position (cm)</th>
<th>A$^{177*}$</th>
<th>A$^{176m*}$</th>
<th>A$^{177/176m*}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.8255</td>
<td>1.1300</td>
<td>1.1820</td>
<td>0.9560</td>
</tr>
<tr>
<td>0.5715</td>
<td>1.1134</td>
<td>1.1431</td>
<td>0.9740</td>
</tr>
<tr>
<td>0.15875</td>
<td>1.0126</td>
<td>1.0369</td>
<td>0.9766</td>
</tr>
<tr>
<td>0</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
<tr>
<td>-0.15875</td>
<td>1.0092</td>
<td>1.0267</td>
<td>0.9830</td>
</tr>
<tr>
<td>-0.5715</td>
<td>1.11877</td>
<td>1.1544</td>
<td>0.9691</td>
</tr>
<tr>
<td>-0.8255</td>
<td>1.1360</td>
<td>1.1660</td>
<td>0.9743</td>
</tr>
</tbody>
</table>

* Estimated standard deviation is less than 1 per cent.

Activations in B$_2$O$_3$ Solutions

The spectral hardening in an Al can filled with aqueous solutions of B$_2$O$_3$ was measured using activation detectors. The detail of the Al can is shown in Figure 4.3.

The Al can, filled with aqueous B$_2$O$_3$ solutions, was placed 24 in. from the east end of the thermal column and irradiated for ten minutes. The flux at the center of the can was approximately $4 \times 10^9$ neutrons/cm$^2$ sec. Three different B$_2$O$_3$ solutions were used as follows: ordinary tap water, 5 grams B$_2$O$_3$/liter, and 10 grams B$_2$O$_3$/liter. The Maxwellian averaged absorption
Figure 4.3. Al Can With the Detector Holder
cross sections for these concentrations were 0.66 barns, 2.609 barns and 4.496 barns, respectively. The detectors used for these measurements were Dy\(^{164}\), Lu\(^{175}\), Lu\(^{176}\), Au\(^{197}\) and Pu\(^{239}\).

Normalized activities of the five detectors are listed in Table 4.4 and plotted in Figures 4.4-4.7. The ratios of A\(^{177}\)/A\(^{176\text{m}}\) are listed in Table 4.5 and plotted in Figure 4.8.

In order to compare the measured activities with theoretical values, calculations were made using the THERM\(\text{O}\)S Code (4). The results of the calculations gave the neutron density and flux per unit volume for thirty energy groups at different space positions and also the activities of Dy\(^{164}\), Pu\(^{239}\), Lu\(^{176}\), Au\(^{197}\) and Eu\(^{151}\). The calculated activities of 1/v absorbers were found to be almost identical to the calculated neutron flux; therefore, experimentally measured activities from the 1/v absorbers were compared with the calculated neutron fluxes. The results are plotted on Figure 4.9. There is some difference between the two which is attributed to the inability to mock-up the axial feed of the sources in the THERM\(\text{O}\)S code.* The axial feed of the sources makes it anisotropic whereas the THERM\(\text{O}\)S code assumes isotropic scattering of the sources. The theoretical flux peaks in the H\(_2\)O medium a few mm. inside the Al

---

* Kinard, F. E., Private Communication.
Figure 4.4. Normalized Lu$^{176}$m Activities in Al Can Filled With Aqueous B$_2$O$_3$ Solutions
Figure 4.5. Normalized Lu177 Activities in Al Can Filled With Aqueous B₂O₃ Solutions
Figure 4.6. Normalized Dy\textsuperscript{164} Activities in Al Can Filled With Aqueous B\textsubscript{2}O\textsubscript{3} Solutions
Figure 4.7. Normalized Fission Product Activities in Al Can Filled With Aqueous B$_2$O$_3$ Solutions
Figure 4.8. Lu$^{177}$/Lu$^{176m}$ Ratios in B$_2$O$_3$ Solutions
Figure 4.9. Comparison of Normalized THERMOS Flux
TABLE 4.4

NORMAlIZED ACTIVITIES IN AQUEOUS SOLUTIONS

<table>
<thead>
<tr>
<th>B$_2$O$_3$ Concentration (grams/liter)</th>
<th>Distance (cm)</th>
<th>Detectors</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Lu$^{175*}$</td>
</tr>
<tr>
<td>0</td>
<td>0.6</td>
<td>0.5907</td>
</tr>
<tr>
<td>0</td>
<td>1.6</td>
<td>0.6969</td>
</tr>
<tr>
<td>0</td>
<td>2.6</td>
<td>0.7814</td>
</tr>
<tr>
<td>0</td>
<td>3.6</td>
<td>0.8706</td>
</tr>
<tr>
<td>0</td>
<td>4.45</td>
<td>1.0000</td>
</tr>
<tr>
<td>5.0</td>
<td>0.6</td>
<td>0.2774</td>
</tr>
<tr>
<td>5.0</td>
<td>1.6</td>
<td>0.3288</td>
</tr>
<tr>
<td>5.0</td>
<td>2.6</td>
<td>0.4762</td>
</tr>
<tr>
<td>5.0</td>
<td>3.6</td>
<td>0.7681</td>
</tr>
<tr>
<td>5.0</td>
<td>4.45</td>
<td>1.0000</td>
</tr>
<tr>
<td>10.0</td>
<td>0.6</td>
<td>0.1598</td>
</tr>
<tr>
<td>10.0</td>
<td>1.6</td>
<td>0.2014</td>
</tr>
<tr>
<td>10.0</td>
<td>2.6</td>
<td>0.3725</td>
</tr>
<tr>
<td>10.0</td>
<td>3.6</td>
<td>0.7172</td>
</tr>
<tr>
<td>10.0</td>
<td>4.45</td>
<td>1.0000</td>
</tr>
</tbody>
</table>

* Estimated standard deviation less than 1%.
<table>
<thead>
<tr>
<th>Distance from Center (cm)</th>
<th>Light Water</th>
<th>5gm/liter*</th>
<th>10gm/liter*</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.6</td>
<td>1.0082</td>
<td>1.0630</td>
<td>1.0082</td>
</tr>
<tr>
<td>1.6</td>
<td>1.0169</td>
<td>1.0545</td>
<td>1.0169</td>
</tr>
<tr>
<td>2.6</td>
<td>1.0172</td>
<td>1.0562</td>
<td>1.0172</td>
</tr>
<tr>
<td>3.6</td>
<td>1.0028</td>
<td>1.0293</td>
<td>1.0028</td>
</tr>
<tr>
<td>4.45</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
</tbody>
</table>

* Estimated standard deviation less than 1%.
can, and this is believed to be due to the external source conditions. The ratios of $^{177}\text{Al} / \nu$ absorber for experimental and theoretical calculations are listed in Table 4.6. The theoretical spectra are softer than the experimental values and again, this is due to the THERMOS source conditions.

**TABLE 4.6**

**RATIO OF $^{177}\text{Al} / \nu$ ABSORBER IN 10 gm. $\text{B}_2\text{O}_3$/LITER**

<table>
<thead>
<tr>
<th>Distance (cm)</th>
<th>Theoretical</th>
<th>Experimental*</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.6</td>
<td>1.0469</td>
<td>1.1191</td>
</tr>
<tr>
<td>1.6</td>
<td>1.0320</td>
<td>1.1058</td>
</tr>
<tr>
<td>2.6</td>
<td>1.0256</td>
<td>1.0699</td>
</tr>
<tr>
<td>3.6</td>
<td>0.9991</td>
<td>1.0321</td>
</tr>
<tr>
<td>4.45</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
</tbody>
</table>

* Standard deviation less than 1 per cent.

In order to duplicate the source conditions imposed by the THERMOS Code, an attempt was made to perform an experiment by placing the Al can filled with borated water in the center of the UFTR. However, with only one gram of $\text{B}_2\text{O}_3$/liter, the excess reactivity of the reactor was not enough to override the poisoning effect. Therefore, the only way this experiment can be performed properly is to provide sufficient reactivity to override the $\text{B}_2\text{O}_3$ absorption.
Measurements in the UFTR Core

In order to measure the spacial dependence of the effective neutron temperature and the $r\sqrt{T/T_0}$ factor in the UFTR core, Lu wires and very thin Au foils were irradiated. The Au foils were made using Au resinate solution,* which was applied to the surface of an Al foil and then baked slowly in an electric oven for about 45 minutes until the temperature reached 900° C. The baking process volatilized the organic material in the film of resinate solution and left a thin film of Au on the surface of the Al foil. The maximum thickness of the Au film was not more than 0.5 microns. This was determined by comparing the activities of Au foils of known and unknown thicknesses in a known flux. Since the self-absorption is negligible for a Au foil with thickness of 0.5 microns (68), it was correct to assume that these foils were infinitely dilute. Activities from the impurities were found to be negligible. Accurate weighing of these thin Au foils was practically impossible; therefore, the same foils were used to measure the bare and the Cd covered activities. This process corrected for weight

* Supplied by the Hanovia Chemical and Manufacturing Company, East Newark, New Jersey.
variations. Activity from the first irradiation was subtracted from the second using the same foil for both irradiations, since the half-life of Au is well known. The Lu wires were 30 mils in diameter and contained 10 per cent by weight of Lu$_2$O$_3$. The positions where Lu wires and Au foils were irradiated is shown in Figure 4.10. One measurement was made in the center of the fuel box.

From the result of the Au foil activations, values of $r\sqrt{T/T_0}$ were calculated using equation (2.4). In this equation, $s_o$ for Au is known (69) and is approximately equal to 17.5. From these calculated values of $r\sqrt{T/T_0}$ the effective neutron temperature in the center of the UFTR core was calculated by trial and error using equation (2.7). For these calculations tabulated values of $g$ and $s_q$ were used for Lu$^{176}$. The factor $g$ for Lu$^{175}$ is approximately one and the factor $s_oG$ was determined experimentally by the use of Cd ratios of Lu$^{175}$, and by using $r\sqrt{T/T_0}$ obtained from the Au foil activation together with equation (2.5). The effective neutron temperature was calculated to be 76.0 $\pm$ 2.9° C in the center of the UFTR when compared to the reference point in the thermal column.

The numerical results for $r\sqrt{T/T_0}$, A$^{177}$, A$^{176m}$, A$^{177}$/A$^{176m}$, T$^n$, R$^{175}$, R$^{197}$ and $(sG)_o\,r$ are tabulated
Figure 4.10. Position of Activation Detectors in UFTR Core
in Table 4.7. In Figure 4.11, \( \frac{A^{177}}{A^{176}} \), \( R_{197} \) and \( r\sqrt{T/T_0} \) are plotted.

Measurements in the Subcritical Assembly

Integral measurements were made radially 12.5 in. from the bottom of the central fuel tube because the slot in the special Mark V-B fuel elements* was located at this position. At this height, the axial \( Cd \) ratio has reached a constant value indicating spectral equilibrium. An Al foil holder was attached to the tube at this height so that radial traverses could be taken to the cell boundary and beyond.

Bare and Cd covered Lu and Au foils were irradiated in four different arrangements using two lattice pitches as is listed in Table 3.1. Also, THERMOS and CEPTR calculations were made for these arrangements and all the experimental and theoretical results are plotted in Figures 4.12 to 4.19 for thermal and epithermal activities. All the experimental results were normalized at the cell boundary, and the direction of the traverse was equidistant between two fuel tubes. Au foils 0.001 in. thick and 0.25 in. diameter were used together with foils containing a 10 weight per cent

* Special unclad Mark V-B and Mark I fuel elements with slots for activation detectors were borrowed from the Savannah River Operations Office, Aiken, South Carolina.
Figure 4.11. Traverse Through UFTR Core
Figure 4.12. Au Activity Distribution for Mark V-B Natural U Fuel With 14.7 cm Pitch
Thermos Curve Normalized at Cell Edge

Subcadmium (s.d. ± 0.7%)

Epicadmium (s.d. ± 3%)

Distance From Center (cm)

Figure 4.13. Lu Activity Distribution for Mark V-B Natural U Fuel With 14.7 cm Pitch
Figure 4.14. Au Activity Distribution for Mark V-B Natural U With 22 cm Pitch
Thermos Curve Normalized at Cell Edge

Subcadmium (s.d. ± 7%)

Cell Radius

Figure 4.15. Lu Activity Distribution for Mark V-B Natural U Fuel With 22 cm Pitch
Figure 4.16. Au Activity Distribution for Mark I and Mark V-B Natural U Fuel With 14.7 cm Pitch.
Figure 4.17. Lu Activity Distribution for Mark I and Mark V-B Natural U Fuel With 14.7 cm Pitch
Figure 4.18. Au Activity Distribution for Mark I and Mark V-B Natural U with 22 cm Pitch
Figure 4.19. Lu Activity Distribution for Mark I and Mark V-B Natural U With 22 cm Pitch
<table>
<thead>
<tr>
<th>Position (cm)</th>
<th>( r\sqrt{T/T_0} )</th>
<th>A177</th>
<th>A176m</th>
<th>A177/A176m</th>
<th>( R_{197}^{\text{cd}} )</th>
<th>( R_{175}^{\text{cd}} )</th>
<th>( s_{\text{Gor}} )</th>
<th>( T_n, ^\circ C )</th>
</tr>
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<tr>
<td>0</td>
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<td>1.000</td>
<td>1.000</td>
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<td>45.76</td>
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<td>0.7115</td>
<td>0.944</td>
<td>0.754</td>
<td>1.759</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>Thermal Column</td>
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<td>0.026</td>
<td>2.504</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>20</td>
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</tbody>
</table>
dispersion of Lu₂O₃ in Al. The latter were the 0.030 in. thick and 0.25 in. diameter. Activities were corrected for the radial J_₀(Br) flux distribution in the subcritical according to the relation:

\[ \phi(R) = \frac{\phi_1(R)}{J_0(Br)} \]

where

\[ \phi_1(R) = \text{uncorrected flux} \]
\[ \phi(R) = \text{corrected flux} \]

Epithermal activities were measured using 0.030 in. Cd covers. Therefore, a Cd cut-off of 0.5 ev was used for the THERMOS calculations.

In addition, bare and Cd covered traverses were made with infinitely dilute Au foils to evaluate the epithermal index r in the unit cell for each lattice arrangement. Using the activity ratios of Lu¹⁷⁷ and Au¹⁹⁸, effective neutron temperatures at different cell positions were calculated using equation (2.10). For a reference the neutron temperature at the cell boundary was determined by a differential method in order to determine the effective temperature change in the unit cell. The results of these measurements are plotted on Figures 4.20 to 4.23. Also shown on Figures 4.24 to 4.27 are the axial thermal and epithermal fluxes with the corresponding Cd ratios.
Figure 4.20. Plot of $\frac{A^{177}}{A^{198}}$, $r\sqrt{\frac{T}{T_0}}$ and $C_{dR}$ with distance from center using Mark V-B natural U fuel with 14.7 cm pitch.
Figure 4.21. Plot of $A^{177}/A^{198}$, $r\sqrt{T/T_0}$ and $Cd_R$ With Distance From Center Using Mark V-B Natural U With 22 cm Pitch
Figure 4.22. Plot of $A_{177}/A_{198}$, $r \sqrt{T/T_O}$ and $C_d R$ with distance from center using Mark I and Mark V-B natural U fuel with 14.7 cm pitch.
Figure 4.23. Plot of $A_{177}/A_{198}$, $r\sqrt{T/T_0}$, and $Cd_R$ with distance from center using Mark I and Mark V-B natural U fuel with 22 cm pitch.
Figure 4.24. Axial Distribution of Flux in the Two Foot Tank Using Mark I and Mark V-B Natural U Fuel With 22 cm Pitch
Figure 4.25. Axial Distribution of Flux in the Two Foot Tank Using Mark V-B Natural U Fuel With 22 cm Pitch
Figure 4.26. Axial Distribution of Flux in the Two Foot Tank Using Mark V-B Natural U Fuel With 14.7 cm Pitch
Figure 4.27. Axial Distribution of Flux in the Two Foot Tank Using $D_2O$
CHAPTER V

RESULTS OF DIFFERENTIAL SPECTRUM MEASUREMENTS

Introduction

Differential spectrum measurements were made using the crystal diffraction spectrometer described previously. A beam of neutrons was extracted from the subcritical assembly for this purpose. This choice of instrument was made because the crystal spectrometer is simpler, cheaper and less space-consuming than a chopper (70). The details of the crystal spectrometer and the description of the subcritical assembly are given in Chapter III.

Analysis of the crystal spectrometer data is complicated due to the following effects: (1) secondary and multiple scattering in the walls of the collimator (2) the Renninger effect (71)* (3) imperfections of the crystal, i.e., mosaic structure (4) variations in detector efficiency, and (5) higher order contamination. In order to simplify these corrections, the instrument

* The Renninger effect is explained in detail in Chapter III.
was calibrated experimentally by using a beam of neutrons with a known energy spectrum. More detailed discussion of this method is given in the later sections of this chapter. For calibration purposes, a well-moderated neutron beam was extracted from the subcritical tank filled with D$_2$O.

Following this calibration experiment, beams from the D$_2$O-moderated subcritical were extracted from the center of the fuel element, from the fuel-D$_2$O interface and from the moderator surrounding the fuel elements. A similar work was performed by V. I. Mostovoi, et al. (72) in uranium-water and uranium monoisopropyl-diphenyl lattices using the time-of-flight technique.

For theoretical comparisons, the THERMØS code developed by Honeck (4) was used. THERMØS is a multi-group code which solves the integral form of the transport equation; it is described in Appendix B.

Alignment and Calibration

Alignment

Several different methods of aligning were tried. First, the bore-sight method of aligning the detector and the first collimator was tried. Then, an optical method was attempted. Finally, it was decided that the easiest way to align the spectrometer was to set the detector shield in the vertical position and then to
insert an Al rod 1 3/8 in. diameter through the detector shield opening with guides attached to the bottom and top of the shield. Then, a guide to the top of the collimator was attached so that when the detector hole and the collimator were aligned very accurately, the Al rod would fit into the guide on top of the collimator. The Al rod was within ±0.001 in. from being straight. In fact, a deviation of ±0.001 in. in the Al rod will cause an uncertainty in θ by ± one minute. The final method of aligning is shown in Figure 5.1.

Spectrum From the UFTR Core

In order to measure the differential neutron spectrum in the center of the UFTR core the central vertical plugs above the reactor were replaced with a collimator plug. The diffraction spectrometer was then aligned with the collimator. Rocking curves obtained with NaCl (200) and LiF (111) crystals are plotted on Figures 5.2 and 5.3. Half maximum angles, Δθ, of the NaCl and LiF were 1.7° and 0.625°, respectively.

Using the LiF crystal, the neutron spectrum coming out of the central vertical access was measured from 3° ≤ θ ≤ 27°. Data obtained from this measurement, after correcting for the background, were plotted as E versus count rate as shown in Figure 5.4. It can be seen from the figure that the results fitted a
Figure 5.1. Method of Alignment of the Diffraction Spectrometer
Figure 5.2. Rocking Curve of NaCl (200)

Figure 5.3. Rocking Curve of LiF (111)
Figure 5.4. Open Beam Neutron Spectrum From Center of UFTR Core Using LiF (111) Crystal
Maxwellian distribution very well. Therefore, the neutron temperature was calculated for the Maxwellian distribution.

According to the kinetic theory of gases (73), the average kinetic energy of thermal neutrons is given by:

\[
\frac{1}{2} \overline{m v^2} = \frac{3}{2} k T_n
\]

(5.1)

The quantity \(\overline{v^2}\) is not measurable but it is related to \(v_0\), the most probable velocity, which is measurable as shown below (74).

\[
\overline{v^2} = \frac{\int v^2 dn}{\int dn} = \sqrt{\pi} v_0^3 \int_0^\infty v^4 e^{-\frac{v^2}{v_0^2}} dv = \frac{3}{2} v_0^2
\]

(5.2)

The most probable velocity, \(v_0\), of the Maxwell distribution is obtained by differentiating the flux distribution (equation (5.3)) and setting it equal to zero.

\[
\frac{d(nv)}{dv} = \frac{4n}{\sqrt{\pi} v_0^3} v^3 e^{-\frac{v^2}{v_0^2}}
\]

(5.3)

or

\[
v_{mp} = 1.224 v_0
\]

(5.4)
where \( v_{m_p} \) corresponds to the velocity of the most probable flux or root mean square velocity. In a simpler manner, the neutron temperature of the core is related to the most probable velocity as follows:

\[
\frac{1}{2} m v_o^2 = k T_n \tag{5.5}
\]

The neutron temperature for Figure 4.9 was calculated as outlined above and was found to be 115.4 ± 6.0°C with the most probable velocity of 2531 ± 314.5 m/sec.

**Spectra From D₂O**

In an infinite moderator with vanishingly small absorption, the neutron spectrum takes the form of a Maxwellian distribution at the temperature of the media. If a poison with \( 1/v \) absorption cross section is added to a moderator, it can be shown (75) that if the Maxwellian spectrum is to be retained, the source of the neutrons must itself be a Maxwellian spectrum. In this case, the source and loss rate from each energy interval must be the same. In an actual reactor, the spectrum is distorted from Maxwellian, and this is due to the slowing down distribution of the source from fission, which does not have the Maxwell distribution.
A neutron beam has been extracted from scatterers such as H$_2$O, D$_2$O and C (76, 77) and the neutron spectrum analyzed with the time-of-flight method using a chopper and neutron pulser. The measurements confirmed that the neutron energies fit a Maxwellian distribution very well from these non-absorbing media.

In order to extract a beam of neutrons from non-absorbing media, a cylindrical tank 24 in. diameter and 60 in. high was placed on top of the graphite pedestal of the UFTR and was filled with D$_2$O. A more detailed description of the set-up is discussed in Chapter III.

The purpose of extracting a beam of neutrons out of the D$_2$O medium with variable temperature is to calibrate the overall efficiency of the neutron diffraction spectrometer using neutron spectra with fairly well-known distributions.

The count rate of a BF$_3$ counter at different Bragg angles $\theta$ for a single crystal can be expressed as:

$$C_i(\theta) = \sum_{n=1}^{n=k} T_i^T_i,n F_i, n(\theta)$$  \hspace{1cm} (5.6)
where

\[ T_i = \text{transmission of the n}^{\text{th}} \text{ order diffracted neutrons through a filter } (e^{-\Sigma t x}) \]

\[ i = \text{measurement index} \]

\[ n = \text{order of diffraction} \]

\[ C_i(\theta) = \text{count rate at the detector} \]

\[ F_{i,n}(\theta) = \phi(n^2E)\varepsilon(n^2E)S(n^2E)w(n^2E)_n r(n^2E) D(n^2E)_n \]

\[ \phi(n^2E) = \text{neutron flux distribution in the media} \]

\[ \varepsilon(n^2E) = \text{BF}_3 \text{ detector efficiency} \]

\[ S(n^2E)_n = \text{spectrometer resolution for order n} \]

\[ W(n^2E)_n = \text{fraction of reflected beam transmitted by the collimating system for order n} \]

\[ r(n^2E)_n = \text{reflectivity of the crystal plane used for order n} \]

\[ D(n^2E)_n = \text{correction factor due to the multiple diffraction for order n} \]

The evaluation of \( F_{i,n}(\theta) \) in equation (5.6) was carried out for up to two sets of \( T_i,n \)'s, by solving simultaneous equations. Since the \( \phi(n^2E) \) is known for large \( D_2^0 \) media, the combined efficiency for different energies for first order can be evaluated. Instead of evaluating the combined efficiency, ratios of \( F_{i,n}(\theta) \)
of unknown and known spectra were taken and, when multiplied by the known spectrum, gave the unknown energy spectrum:

$$\phi(n^2E)_x = \frac{[F_i,n(\theta)]_x}{[F_i,n(\theta)]_s} \phi(n^2E)_s$$

(5.7)

The neutron spectrum was extracted 7 in. from the bottom of the tank with D₂O temperatures of 25° and 75°C. The axial Cd ratio for this system is plotted on Figure 4.27. By inspection of the data it was found that the neutron temperature was 40°C higher than the moderator temperature.

The ratio of the two sets of measurements are plotted on Figure 5.5 with the theoretically calculated Maxwellian distribution ratios at two different temperatures. This was done to check if the spectrometer could see the change in neutron spectrum due to the change in medium temperature.

**Spectra from Subcritical Assembly**

For the differential spectrum measurements, the first three lattice arrangements listed in Table 3.1 were used. The first collimator described earlier was inserted vertically into the subcritical assembly first at the center and then at the boundary of the central unit cell. The LiF (111) crystal was used for these measurements.
Figure 5.5. Ratio of Maxwellian Distribution and the Experimental Spectrum at Two Different Temperatures
The experimental neutron density distributions were calculated at two locations by the use of equation (5.7) for each arrangement and are plotted on Figures 5.6 - 5.8. In Figure 5.7 the lower experimental spectrum is much softer than the THERMOS spectrum. However, the THERMOS spectrum is at the center of the Mark I fuel element, while the extracted beam of neutrons had a diameter of approximately 5 cm. Therefore, the experimental results were plotted again in Figure 5.9 with THERMOS spectrum 0.2 cm outside of the Mark I fuel element.
Figure 5.6. Experimental and THERMØS Spectra Using Mark V-B Natural U Fuel With 22 cm Pitch
Figure 5.7. Experimental and THERMOS Spectra Using Mark I and Mark V-B Natural U Fuel With 22 cm Pitch
Figure 5.8. Experimental and THERMOS Spectra Using Mark V-B Natural U Fuel With 14.7 cm Pitch
Figure 5.9. Experimental and Corrected THERMOS Spectra Using Mark I and Mark V-B Natural U Fuel With 22 cm Pitch
CHAPTER VI

RESULTS, CONCLUSIONS AND RECOMMENDATIONS

Comparisons of Experimental and Theoretical Spectra in the UFTR Thermal Column and Core

Comparisons of the theoretical activities with the experimental results in the thermal column using an Al can filled with borated water are plotted on Figure 4.9. The differences in the two fluxes were as much as 25 per cent and this is due to the THERMØS source conditions. It would be interesting to perform this experiment with isotropic source conditions in a reactor which has enough excess reactivity to override the poisoning effect of the sample.

The ratio of the activities of a resonance and a $1/v$ absorber for borated water was found to be somewhat higher than the theoretical calculations as can be seen in Table 4.6. The discrepancy is again due to the THERMØS source conditions.

No theoretical calculations were made for the natural U slab and the stainless steel rod. Again, anisotropy of the source conditions would apply here and there would probably be large discrepancies between THERMØS calculations and experimental results as with the borated water data.
From the experimental results a good comparison was made between the integral and the differential measurements in the center of the UFTR core. The effective neutron temperature determined by the differential measurement was $115\pm6^\circ C$ compared to $76\pm2.9^\circ C$ using integral measurements with Westcott's formulations (34). Thus, the difference between the two methods of measurements was $39^\circ C$. The effective temperature of the integral measurement in the UFTR core is relative to the effective temperature of the thermal column which was assumed to be equal to the temperature of the media. Johansson (76) has measured the effective neutron temperature in graphite media using the time-of-flight method and found it to be $29\pm10^\circ C$ higher than the medium temperature. If this result applies to the UFTR thermal column, then the comparison of effective temperature using the two methods will be within the experimental error.

The experimental effective neutron temperature in the UFTR was compared with theoretical calculations made by C. A. Thompson (78) who has made a multigroup-multiregion calculation for one dimension going from South to North in the reactor core. However, his calculations showed a much lower neutron temperature in the UFTR core. It is also hard to make a detailed comparison with Thompson's calculations because his energy groups in the thermal region are wide.
Comparisons of Experimental and Theoretical Spectra in the Subcritical Assembly

Intracell activation measurements were made using resonance and 1/v absorbers in each of the lattice arrangements in Table 3.1, and comparisons were made with THERMØS calculations for each detector. In general, the results are in good agreement except for a few points like the one in Figure 4.14. These few points are very probably due to inaccurate positioning of the activation detectors in the horizontal plane. This conclusion was verified by repeating some of the measurements. Also, CEPTR calculations for the flux distribution were made, but were found to be almost identical to the THERMØS flux.

For the differential measurements, experimental calibration of the equipment proved to be successful and gave excellent results, as can be seen in Figures 5.6 - 5.9. The temperature calibration curve was also satisfactory which can be seen in Figure 5.5.

The comparisons of the change in effective neutron temperature from cell boundary to cell center for three different methods (THERMØS calculation, integral and differential) are listed in Table 6.1.

The temperature change in a unit cell for the differential measurements was estimated by fitting the experimental spectrum to a Maxwellian distribution using a least square fit with the IBM 704 computer (81).
### TABLE 6.1

COMPARISONS OF THE CHANGE IN EFFECTIVE NEUTRON TEMPERATURES IN A UNIT CELL

<table>
<thead>
<tr>
<th>Pitch</th>
<th>Type of Fuel</th>
<th>$\Delta T_n$ From Cell Boundary to Center (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>THERMOS</td>
</tr>
<tr>
<td>14.7 cm</td>
<td>Mark V-B</td>
<td>51.0±12.7</td>
</tr>
<tr>
<td>22 cm</td>
<td>Mark V-B</td>
<td>55.0±19.5</td>
</tr>
<tr>
<td>22 cm</td>
<td>Mark I + Mark V-B</td>
<td>118.4±54</td>
</tr>
</tbody>
</table>
It would be of interest to extend the data to other fuel element configurations (e.g., large clusters of UO$_2$ rods) in order to test the validity of the THERMØS calculations.
APPENDIX A

METHOD OF CALCULATING $A_{177}/A_{176m}$

Natural Lu contains 2.6 per cent $Lu_{176}$ and 97.4 per cent $Lu_{175}$. Activities of irradiated natural Lu comes from $Lu_{176m}$ and $Lu_{177}$ in the first two days. After four days the activity of $Lu_{176m}$ decays out completely, and only the activity of $Lu_{177}$ is left. Normally, each foil or wire was counted ten times to get better statistics. The first five counts were made for the short and long half-life activities, and then another five counts were taken for the long life activity. All the activities were corrected back to some standard time, $t=0$, and activities normalized to standard Cs$^{137}$ source.

The average activity of $Lu_{177}$ at $t=0$ is given as:

$$A_{177}^x(t=0) = \sum_{i=1}^{5} \left[ A_{177}^x(t_i) - \text{Background} \right] e^{-\lambda_2 t_i}$$

where

$$\lambda_2 = \text{decay constant of } Lu_{177}$$

$$t_i = \text{time when activity of } Lu_{176m} \text{ and } Lu_{177} \text{ was measured}$$
x = index for the position of the detector

$\delta_x^{177}$ = standard deviation of $Lu^{177}$ activity

Now the average activity of $Lu^{176m}$ at $t=0$ is given as:

$$A^{176m}_x(t=0) =$$

$$\frac{\sum_{j=1}^{5} \left( [A^{176m}_x + 177(t_j) - A^{177}_x(t=0)] e^{-\lambda t_j} - \text{background} \right) e^{-\lambda_1 t_j}}{5}$$

$$\pm \delta_x^{176m}$$

where

$\lambda_1$ = decay constant of $Lu^{176m}$

$t_j$ = time when the activities of $Lu^{176m}$ and $Lu^{177}$ were measured

$\delta_x^{176m}$ = standard deviation of the $Lu^{176m}$ activity

All the activities were normalized to standard activities.

$$y^{177}_x(t=0) = \frac{A^{177}_x(t=0)}{A^{177}_s(t=0)} = A^{177}_x(t=0)$$
\[ y_{176m}^{(t=0)} = \frac{A_{176m}^{x(t=0)}}{A_{176m}^{x(t=0)}} = A_{176m}^{x(t=0)} \]

Since \( A_{s}^{177} = 1 \) and \( A_{s}^{176m} = 1 \). The normalized activity ratios become

\[ R_{x} = \frac{A_{177}^{x(t=0)}}{A_{176m}^{x(t=0)}} \pm \delta_{x}^{R} \]
APPENDIX B

THE THERM0S CODE

The THERM0S Code developed by H. C. Honeck of Brookhaven National Laboratory calculates the scalar neutron spectrum as a function of energy and position in a lattice cell. The code solves numerically the integral transport equation with isotropic scattering (4). One dimensional slab or cylindrical geometry can be used. Comparisons of the code calculations with integral measurements have been published (79, 80).

THERM0S code uses 30 energy groups as given in Table B.1; it uses 20 space points and mixtures of ten nuclides. The code is used for calculations of the reaction rate of \( ^{233}U, ^{235}U, ^{239}Pu, ^{164}Dy, ^{151}Eu, \) and \( ^{176}Lu \). Output of the THERM0S code is also used for calculating \( n,f \) (thermal utilization), \( \bar{E}_a \), \( fG \) (fraction of thermal neutrons captured in \( ^{238}U \)) and D.
<table>
<thead>
<tr>
<th>Group No.</th>
<th>Energy (ev)</th>
<th>Group No.</th>
<th>Energy (ev)</th>
</tr>
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<td>0.06517</td>
</tr>
<tr>
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<td>0.2346</td>
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<tr>
<td>15</td>
<td>0.05692</td>
<td>30</td>
<td>0.7067</td>
</tr>
</tbody>
</table>
APPENDIX C

THE RECIPROCAL LATTICE

Let \( \hat{\mathbf{a}}_1, \hat{\mathbf{a}}_2 \) and \( \hat{\mathbf{a}}_3 \) represent vectors of a unit cell containing no lattice points other than those lying at its corners. Then, let \( \hat{\mathbf{b}}_1, \hat{\mathbf{b}}_2 \) and \( \hat{\mathbf{b}}_3 \) represent the vectors of a reciprocal lattice. The reciprocal axis \( \hat{\mathbf{b}}_1 \) is perpendicular to the axis \( \hat{\mathbf{a}}_2 \) and \( \hat{\mathbf{a}}_3 \). Also, \( \hat{\mathbf{b}}_2 \) is perpendicular to \( \hat{\mathbf{a}}_1 \) and \( \hat{\mathbf{a}}_3 \) and \( \hat{\mathbf{b}}_3 \) is perpendicular to \( \hat{\mathbf{a}}_1 \) and \( \hat{\mathbf{a}}_2 \). These relations are summed up in the vector equations:

\[
\hat{\mathbf{b}}_1 \cdot \hat{\mathbf{a}}_2 = \hat{\mathbf{b}}_1 \cdot \hat{\mathbf{a}}_3 = \hat{\mathbf{b}}_2 \cdot \hat{\mathbf{a}}_1 = \hat{\mathbf{b}}_2 \cdot \hat{\mathbf{a}}_3 = \hat{\mathbf{b}}_3 \cdot \hat{\mathbf{a}}_1 = \hat{\mathbf{b}}_3 \cdot \hat{\mathbf{a}}_3 = 0
\]  
(C.1)

The magnitude of reciprocal vectors are fixed by relations:

\[
\hat{\mathbf{a}}_1 \cdot \hat{\mathbf{b}}_1 = \hat{\mathbf{a}}_2 \cdot \hat{\mathbf{b}}_2 = \hat{\mathbf{a}}_3 \cdot \hat{\mathbf{b}}_3 = 1
\]  
(C.2)

The length of the reciprocal lattice is given by relation:

\[
\hat{\mathbf{b}}_1 = \beta (\hat{\mathbf{a}}_2 \cdot \hat{\mathbf{a}}_3)
\]  
(C.3)
where $B$ is a constant evaluated as follows: Let $V$ be the volume of the unit cell of the lattice defined by vectors $\hat{a}_1, \hat{a}_2, \hat{a}_3$, then:

$$V = \hat{a}_1 \cdot (\hat{a}_2 \times \hat{a}_3) = \hat{a}_2 \cdot (\hat{a}_3 \times \hat{a}_1) = \hat{a}_3 \cdot (\hat{a}_1 \times \hat{a}_2)$$

(C.4)

If $\gamma$ is the angle between $\hat{a}_1$ and $\hat{a}_2$, then in Figure C.1 $|\hat{a}_1 \times \hat{a}_2| = |a| |b| \sin \gamma$ is the area of the face $0A_1A_2A_3$ so that $\hat{a}_3 \cdot (\hat{a}_1 \times \hat{a}_2) = \hat{a}_3 \cos \theta$ (area of the base of the cell) and is thus equal to the volume $V$. From equations (C.3) and (C.4), the following relationship is obtained.

$$V = \frac{(\hat{a}_1 \cdot \hat{b}_1)}{\beta} = \frac{1}{\beta}$$

(C.5)

or

$$\hat{b}_1 = (\hat{a}_2 \times \hat{a}_3), \hat{b}_2 = (\hat{a}_3 \times \hat{a}_1), \hat{b}_3 = (\hat{a}_1 \times \hat{a}_2)$$

(C.6)

Figure C.1. Unit Cell of a Lattice
The vector $\mathbf{r}(u,v,w)$ to any lattice point is given by vector equation:

$$
\mathbf{r}(u,v,w) = u\mathbf{a}_1 + v\mathbf{a}_2 + w\mathbf{a}_3
$$

(C.7)

where $u$, $v$, and $w$ are any integers, positive, negative or zero. The vector $\mathbf{r}'(h,k,l)$ to any point in reciprocal lattice is given by:

$$
\mathbf{r}'(h,k,l) = h\mathbf{b}_1 + k\mathbf{b}_2 + l\mathbf{b}_3
$$

(C.8)

where $h$, $k$, $l$ have the same properties as $u$, $v$, and $w$.

Two properties of the lattice that make it of great value in treating problems of diffraction by space lattices are:

1. The vector $\mathbf{r}'(h,k,l)$ to the point $(h,k,l)$ of the reciprocal lattice is normal to the planes $(h,k,l)$ of the crystal lattice.

2. The magnitude of the vector $\mathbf{r}'(h,k,l)$ is the reciprocal of the spacing of the planes $(h,k,l)$ of the crystal lattice.
CEPTR fluxes are used as input to the BUCKSHOT calculations. It calculates the lattice parameters of natural uranium $D_2O$-moderated cylindrical heterogeneous reactors. Some of the parameters obtained for two different pitches using Mark V-B elements are listed below.

<table>
<thead>
<tr>
<th>Pitch</th>
<th>$L^2$ (cm$^2$)</th>
<th>$M^{22}$ (cm$^2$)</th>
<th>$P$</th>
<th>$k_\infty$</th>
<th>$B_m^2$ (cm$^{-2}$)</th>
<th>$n$</th>
<th>$\tau$ (cm$^2$)</th>
<th>$f$</th>
<th>$\varepsilon$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>14.7 cm</td>
<td>41.7 cm$^2$</td>
<td>0.765</td>
<td>1.024</td>
<td>1.308x10^{-4} cm$^{-2}$</td>
<td>1.327</td>
<td>138.9 cm$^2$</td>
<td>0.981</td>
<td>1.029</td>
</tr>
<tr>
<td></td>
<td>22.0 cm</td>
<td>115.6 cm$^2$</td>
<td>0.887</td>
<td>1.181</td>
<td>7.030x10^{-4} cm$^{-2}$</td>
<td>1.327</td>
<td>127.5 cm$^2$</td>
<td>0.975</td>
<td>1.029</td>
</tr>
</tbody>
</table>
APPENDIX E

PROPOSAL TO USE UFTR AS A SOURCE FOR THE SUBCRITICAL ASSEMBLY

PROPOSAL August 3, 1964

TO: UFTR Subcommittee of the ENOF Committee

FROM: S. Salah

SUBJECT: Use of UFTR as a Source for a 24" D x 60" Subcritical Reactor

Description of the Set-up

A series of experiments is planned with a subcritical reactor sitting on top of the UFTR as shown in Figure 1. A new set of shielding blocks with heavy aggregate will be constructed which will be used as permanent shielding blocks for the top of the reactor.

When the subcritical is removed from the shield structure, it will be replaced with permanent plugs with vertical access holes. The shielding plugs will be made from heavy aggregate concrete and will be lifted out with the crane whenever necessary.

Object of the Experiment

The purpose of these experiments is to measure the differential and integral spectrum emerging from the subcritical reactor. These measurements will comprise three parts:

1. Using submerged heaters in the subcritical filled with H\textsubscript{2}O and then with D\textsubscript{2}O, a calibration experiment will be done by heating the media. Following this, the same experiments with borated H\textsubscript{2}O will be performed.
The subcritical with a lattice spacing of 5.8 in. and 8.64 in. using Mark V-B type fuel elements will be used. A beam will be extracted from the fuel element and from the D$_2$O moderator and hardening of the neutron spectrum in the fuel will be investigated using a crystal diffraction spectrometer and activation foils. The moderator-to-fuel ratios of the two lattice spacings are 12.4 and 24.8, respectively.

**Safety Measures**

The coupling between the subcritical and the critical reactor will be measured by noting the critical rod settings before and after the installation of the subcritical reactor. For the first reactor run following the installation, the usual start-up procedures for a new fuel loading will be followed.

At very low power level the flux level in the subcritical will be measured and compared with some reference point in the reactor. The area above the top of the subcritical will be monitored, and if the radiation level is not above the permissible limit, it is proposed to take the power level of the UFTR to the maximum level for the purpose of extracting a beam. A shielding block will be placed on top of the subcritical for 100 kw operation if it is found to be necessary.

Flux levels and build up of activities in the central fuel element is estimated in Appendix A. After the experiment, the lattice of the subcritical tank will be lifted out of the reactor shield and it will be stored in the water shield tank of the UFTR until the radioactivity of the fuel elements goes below the permissible limit.

A request for performing this experiment has been approved by the Atomic Energy Commission.
Appendix A

I. Calculation of the flux in the center line of the subcritical assembly.

The general time independent one-velocity age diffusion equation for multiplying media for cylindrical coordinates is expressed as

$$\frac{1}{r} \frac{\partial}{\partial r} \left[ r \frac{\partial}{\partial r} \phi(r, Z) \right] + \frac{\partial^2}{\partial Z^2} \phi(r, Z) + B_m \phi(r, Z) = 0$$  \hspace{1cm} (1)

where

$$B_m^2 = \frac{\ln k_m}{L^2 + \tau}.$$

The solution for the flux in the center line of the cylinder is given by

$$\phi(Z) = C e^{-\alpha Z} [1 - e^{-\alpha(H - Z)}] \hspace{1cm} (2)$$

where

$$Z = 0 \text{ at the reactor pedestal subcritical boundary line} \hspace{1cm}$$

$$\alpha^2 = \left( \frac{v_1}{\tau} \right)^2 + B_m^2$$

$$H = \text{extrapolated height of the subcritical assembly.}$$

Four factors of $k_m$ were calculated by using the method outlined in Murray, "Introduction to Nuclear Engineering,"
Chapter 9 and the following values were obtained:

\[ n = 1.32 \]
\[ f = 0.994 \]
\[ P = 0.876 \]
\[ \varepsilon = 1.003 \]
\[ k_{\infty} = 1.152 \]

\[ L^2 = L^2_0 (1-f) = 1.346 \times 10^4 (0.006) = 80.74 \text{ cm}^2 \]

\[ \tau = 125 \text{ cm}^2 \]

\[ B_m^2 = 6.9 \times 10^{-4} \text{ cm}^{-2} \]

which gives \( \alpha = 0.082 \text{ cm}^{-1} \)

Using Eq. (2) the flux can be calculated.

II. Dose from the irradiated fuel elements in the center of the subcritical assembly.

The total number of fission/sec in the fuel element can be calculated by evaluating the following integral:

\[
\int_{Z=0}^{Z=100} C e^{-\alpha Z} A \Sigma_f^F dZ \quad (3)
\]
where

\[ [1 - e^{-\alpha(H - Z)}] \]

term can be neglected in the above equation.

\[ C = \phi_0 \text{ evaluated at } Z = 0. \]

\[ A = \text{surface of the annular part of the fuel element.} \]

At reactor power level of 100 kw, \( \phi_0 \) is approximately \( 10^{10} \text{n/cm}^2\text{-sec} \), which was extrapolated from "Guide to Irradiations in the University of Florida Training Reactor" by A. R. Boynton.

The integral was evaluated in this power level and the number of fissions per sec was equal to \( 3.32 \times 10^{11} \). This number corresponds to 10.1 watt-sec.

Irradiation for 10 hours at this power level gives a decay rate of the fuel element of \( 3.03 \times 10^{11} \text{ Mev/sec one hour after reactor shutdown} \), which was obtained from Figure 3-12 of "Fundamental Aspects of Reactor Shielding" by Goldstein. Fifteen hours after shutdown of the reactor the activity goes down to \( 4.55 \times 10^{10} \text{ Mev/sec} \).

Since the activity of the fuel element will be hottest on the lower 20 cm of the fuel element, the dose rate of 60 cm from the lower part of the fuel element was calculated for 100 kw-10 hour operation using formula \( (4) \) out of "Reactor Shielding Design Manual" by Rockwell:

\[
D(\text{Mev/cm}^2\text{-sec}) = \frac{\text{S} \cdot R^2}{V_0} \frac{F(\theta, b_2)}{2(a + Z)}
\]

(4)

and found to be 2000 mr/hr at 15 hours after reactor shutdown, or 100 mr/hr after 26 days.
III. Total $\text{Pu}^{239}$ build-up.

$$\text{Pu}^{239} = \frac{10^{10} \times 13.61}{0.082} \times 0.1286 \times 3.6 \times 10^4$$

$$= 7.68 \times 10^{15} \text{Pu}^{239} \text{ atoms}$$

or, weight $\text{Pu}^{239} = 39 \mu \text{grams}.$

IV. Total fission product build-up.

$$\text{FP} = \frac{10^{10} \times 13.61}{0.082} \times 0.1286 \times 3.6 \times 10^4$$

where

fission/sec

$$= 3.32 \times 10^{10}$$

fissions in 10 hrs.

$$= 3.32 \times 10^{10} \times 3.6 \times 10^4 = 7.68 \times 10^{15} \text{Pu}^{239} \text{ atoms}$$

number fissions

$$= 1.55 \times 10^{16}$$
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BIOGRAPHICAL SKETCH

Sagid Salah was born in Seoul, Korea, on September 2, 1932, of Turkish parents.

He attended Seoul American School for two years, but his secondary education was interrupted by the outbreak of the Korean War in 1950, when he was captured and held prisoner in North Korea for four years.

After his release from prison camp, he immigrated to the United States and attended Gainesville High School in Gainesville, Florida, for one semester, after which he entered the University of Florida in February, 1955. On August 9, 1958, he received the Bachelor of Chemical Engineering. Immediately after his graduation, he enrolled in the Graduate School of the University of Florida, and received the degree of Master of Science in Engineering on January 30, 1960. Following this, he pursued his studies toward the Ph.D. degree in Nuclear Engineering. He has held a graduate assistantship in the Department of Nuclear Engineering for the major portion of his graduate studies.

The author became a naturalized American citizen on October 6, 1959. He is a member of Sigma Tau and the Student Branch of the American Nuclear Society.
This dissertation was prepared under the direction of the chairman of the candidate's supervisory committee and has been approved by all members of that committee. It was submitted to the Dean of the College of Engineering and to the Graduate Council, and was approved as partial fulfillment of the requirements for the degree of Doctor of Philosophy.

December 19, 1964

Dean, College of Engineering

Dean, Graduate School

Supervisory Committee:

[Signatures]