APPLICATION OF SEQUENTIAL DECISION THEORY TO VOICE COMMUNICATIONS

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A DISSERTATION PRESENTED TO THE GRADUATE COUNCIL OF
THE UNIVERSITY OF FLORIDA
IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE
DEGREE OF DOCTOR OF PHILOSOPHY

UNIVERSITY OF FLORIDA

June, 1964

ACKNOWLEDGMENT

The author wishes to acknowledge his great debt to his advisor, Professor T. S. George, whose advice and encouragement made this dissertation possible. He wishes to thank Professor W. H. Chen for his assistance throughout the period of graduate study at the University of Florida, and the other members of his committee for their aid.

The author also wishes to thank the Martin Company, Orlando, Florida, for providing the interesting work assignments which led him to return to graduate school and to pursue the research reported herein.

A special acknowledgment is due the Lockheed-Georgia Company of the Lockheed Aircraft Corporation, where the author was employed during the course of this research. Assistance was provided by the Advanced Research Organization in forms too numerous to mention, most important of which was a research climate which facilitated the completion of this dissertation.

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KEY TO SYMBOLS

Symbol	Description
a	A threshold.
$\underline{a}_1$	Input signal-to-noise ratio.
$\underline{a}_1$	A preset value of $\underline{a}$ for the sequential test.
α	Probability of false alarm in radar.
A'	An upper threshold number.
A	Amplitude of signal pulse at the receiver.
b	A threshold.
β	Probability of false dismissal in radar.
B'	A lower threshold number.
B	Amplitude of impulse at the receiver.
c	A threshold.
d	A threshold.
erf(x)	Error function, $\frac{2}{\sqrt{\pi}} \int_0^x e^{-u^2} \cdot du$.
H_o	The hypothesis of noise alone.
H_1	The hypothesis of signal and noise.
$I_o(x)$	Modified Bessel Function of first kind, zeroth order.
$\bar{K}$	Average value for the number of slots for termination.
K_1	Ratio b/A for systems of Chapter II, d/A for systems of Chapter III.
K_2	Ratio a/b for systems of Chapter II, c/d for systems of Chapter III.

Symbol	Description
K_3	Ratio b/c for systems of Chapter III.
K_4	Ratio a/b for systems of Chapter III.
λ	Average number of impulses in N-1 noise slots.
λ_m	Probability ratio for m observations.
Λ_m	Likelihood ratio at the m^{th} stage.
n'	Number of impulses in N-1 noise slots.
$n(t)$	Input noise function.
N	Number of pulse slots per speech sample frame.
p	A priori probability that H_o is true.
$p_1(R_1)$	Probability density of noise envelope.
$p_2(R_2)$	Density of signal and noise envelope.
$p_3(R_3)$	Density of impulse and noise envelope.
P(K)	Probability of termination on the K^{th} slot.
P(s)	Probability that the number of slots with envelopes in the "possible signal" zone is s, given that none are in the "signal" zone.
P(err)	Probability of error, 1-P(cor).
P_1(cor)	Probability of correct decision for QPPM, Double Threshold, Speech Statistics system.
P_2(cor)	Probability of correct decision for QPPM, Single Threshold system.
P_3(cor)	Probability of correct decision for QPPM, Double Threshold, Largest of s system.
P_4(cor)	Probability of correct decision for QPPM, Largest of N system.
P_5(cor)	Probability of correct decision for QPPM, Four Threshold, Speech Statistics system, with impulse noise.

Symbol	Description
$P_6(cor)$	Probability of correct decision for QPPM, Four Threshold, Nearest to Reference system, with impulse noise.
$P_7(cor)$	Probability of correct decision for QPPM, Single Threshold system, with impulse noise.
$P_8(cor)$	Probability of correct decision for QPPM, Largest of N system, with impulse noise.
$P_9(cor)$	Probability of correct decision for QPPM, Double Threshold, Non-Sequential system, with impulse noise.
$P_{10}(cor)$	Probability of correct decision for QPPM, Single Threshold, Largest of N system, with impulse noise.
$P_B(n)$	Probability of n impulses in N-1 noise slots.
$P_B'(n)$	Probability of n impulses in j-1 noise slots.
$P_s(j)$	Probability of speech pulse in the j^{th} slot.
$p_r(\ell)$	Probability of reference pulse in the ℓ^{th} slot.
$P_s(j/\ell)$	Conditional probability that signal pulse in j^{th} if reference is ℓ^{th} slot.
q	Number of impulses closer than signal to reference.
$Q(x,y)$	The "Q" function, $\int_y^{\infty} v \cdot e^{-\frac{v^2+x^2}{2}} \cdot I_0(xv) \cdot dv$.
R	An envelope.
$\bar{s}$	Average number of slots with "possible signal" envelope.
$s(t)$	Input signal function.
σ_n^2	Average noise power at the IF.
σ_s^2	Variance of the number of slots with "possible signal" envelopes.

Symbol	Description
Θ	Unknown parameter of a probability function.
$w_m(x,\Theta)$	Joint probability density function of $x_1, x_2, \ldots x_m$ when Θ is true.
$W_1(x)$	First probability density of the instantaneous speech amplitudes.
$W_2(x_1/x_2,\tau)$	Conditional density of speech amplitude x_1 when the value τ seconds before was x_2.

ABSTRACT

Wald's Sequential Decision Theory is applied to voice communication systems of the quantized pulse position modulation type. When interference is narrow-band Gaussian noise, two sequential systems are considered. The "Double Threshold, Speech Statistics" system divides the speech sampled frame into N discrete slots. The three decision zones for each slot are: (1) signal is present, (2) signal is possibly present, and (3) signal is not present. If the end of the frame has produced no decision for the first zone, the receiver chooses one from the group representing the second decision zone. This choice is made on the basis of the one most likely to be the signal, as indicated by speech statistics.

The "Double Threshold, Largest of S" system is the other sequential type. It is similar to the previously described technique, with the exception that when the choice must be made from the group representing the second decision zone, it is made on the basis of the largest one. When compared to the conventional, single threshold method, the sequential systems provide a reduction of about 1.5 decibels in required signal-to-noise ratio for a specified probability of error. Their performance is almost as good as a "Largest

of N" receiver, which considers all N slots and chooses the largest as signal. The sequential methods seem to represent a compromise between these two techniques, having some features of each.

When interference is narrow-band Gaussian noise, with occasional impulse-like large amplitude pulse noise, the sequential systems have four thresholds. Thus the "signal is possibly present" and the "signal is not present" decision zones consist of two separate parts for each. The two sequential types are the speech statistics and the nearest to reference systems. The reference referred to is the decision zone representing signal present. Analysis is made on the basis of a mathematical model consisting of a Poisson distribution for the number of impulses in a frame, with an average number of one, and a modified Rayleigh distribution for the envelope of the impulse-like interference, with amplitude of the pulse three times the signal pulse.

The error performance of the conventional single threshold and the "Largest of N" receivers is shown to be completely inadequate under these conditions. A conventional receiver modified to include an upper threshold for discrimination against impulses gives a reasonable performance. The sequential methods, however, give an improvement of about 2.0 decibels over this. Roughly equivalent to this improvement is that given by a "Largest of N" system which is modified to include an upper threshold for impulse removal.

The conclusion reached is that sequential techniques in the form described provide an improvement over conventional or modified conventional methods. The improvement is probably not large enough under most circumstances to warrant their use. However, specific situations may allow their beneficial application. It is felt that further study should be made of sequential techniques for other modulation methods in voice communications, possibly in other forms from those described here.

CHAPTER I

INTRODUCTION

The Beginnings of Sequential Analysis

The voice communication systems discussed in this report have resulted from efforts to apply the ideas of sequential analysis to this field of study. It is therefore appropriate to relate in this initial chapter some of the background information on sequential analysis, and to describe some earlier applications of this theory.

Whether in radar, data, or pulse-coded voice communications the basic problem is the detection of a signal in noise. When the examination of the random process is to provide a positive or negative indication as to the presence of a signal the problem may be considered a test of statistical hypotheses. That is, based on the available information a choice is to be made between hypothesis H_o: signal is not present, and H_1: signal is present. This may involve a statistical test of a mean, a signal-to-noise ratio, etc. Sequential analysis, including the sequential probability ratio test, was devised by Abraham Wald[1] for hypothesis testing. He and Wolfowitz[2] showed that this test requires on the average fewer samples to terminate than do tests of fixed sample size, for equal probabilities of error. A distinguishing feature of the test is that it is conducted in stages, with the length not specified in advance, but determined by the progress of the test. Thus the sample size is a random variable. Although the length of the test is smaller on the average than that for any other test, it may be very large for a particular trial. Stein[3] showed that not only is the expected length of a sequential test finite, however, but <u>all</u> moments of the length are finite if the samples are independent. Thus the question was resolved as to whether sequential tests terminate.

Because the length of a particular test may be longer than can be tolerated, many practical situations require that a test be terminated prematurely. At the point of truncation, some alternate criterion is adopted so that a choice of hypothesis may be made. A truncated test requires on the average a smaller number of samples, but the error performance is worse than for the untruncated sequential test. Truncation represents a compromise between the completely sequential test and the fixed-sample test.

Description of the Sequential Test

We let X_i be the random variable representing a sample of a random process for which a statistical decision problem is formulated. Sample X_i is independent of any other sample X_j for $i \neq j$. The successive samples of X are denoted by X_1, X_2, ... The joint probability density function for the samples is considered known, except for some parameter θ (or a set of parameters $\bar{\theta}$ in the general case), and is represented by $W_m(\bar{X};\theta)$. The subscript indicates that the experiment is at the m^{th} stage. Based on the sequential test a choice is to be made between two hypotheses: H_o, the value of θ is θ_o; and H_1, the value of θ is θ_1. Therefore, if hypothesis H_o is true the joint probability density function of the m samples is $W_m(\bar{X};\theta_o)$; if hypothesis H_1 is true the density is $W_m(\bar{X};\theta_1)$. If the _a priori_ probability that H_o is true is p, then the _a priori_ probability that H_1 is true is 1 - p. More generally, of course, there could be more than two hypotheses from which to choose. Here we are interested in the binary case only.

The likelihood ratio at the m^{th} stage of the experiment is defined by

$$\Lambda_m = \frac{(1 - p)W_m(\bar{X};\theta_1)}{p\, W_m(\bar{X};\theta_o)} \tag{1.1}$$

The carrying out of the sequential test requires this likelihood ratio to be computed at each stage of the experiment. Two threshold numbers $A' > 1$ and $B' < 1$ are chosen. At the m^{th} stage the decision is made to continue the test if

$$B' < \Lambda_m < A' \qquad , \text{ for } m = 1,2,\ldots,n - 1 \ . \tag{1.2}$$

The test is terminated and hypothesis H_o accepted if at the n^{th} trial

$$\Lambda_n \leqslant B' . \tag{1.3}$$

Similarly, hypothesis H_1 is accepted at the n^{th} trial and the test terminated if

$$\Lambda_n \geqslant A' . \tag{1.4}$$

Observe that if we set $B' = \frac{(1 - p)}{p} B$ and $A' = \frac{(1 - p)}{p} A$ the sequential test can be set up in terms of the conditional probabilities, independent of the <u>a priori</u> probabilities p and 1 - p. This can be seen by considering the probability ratio

$$\lambda_m = \frac{W_m(\bar{X};\theta_1)}{W_m(\bar{X};\theta_o)} \tag{1.5}$$

Note that Λ_m is $\frac{1 - p}{p}\lambda_m$. The test procedure is thus slightly modified. At the m^{th} stage the decision is made to continue the test if

$$B < \lambda_m < A \qquad , \text{ for } m = 1,2,\ldots,n - 1 . \tag{1.6}$$

The test is terminated and hypothesis H_o accepted if at the n^{th} trial

$$\lambda_n \leqslant B . \tag{1.7}$$

Similarly, hypothesis H_1 is accepted at the n^{th} trial if

$$\lambda_n \geqslant A . \tag{1.8}$$

The standout feature of the sequential test is the dividing of the decision zone into three parts (by means of the two thresholds). These are: (1) a zone of acceptance for H_o, (2) a zone of acceptance for H_1, and (3) a zone of indifference. The conventional fixed-number-of-samples test has associated with it only the two zones of acceptance and one threshold. As pointed out above, the sequential test provides a savings, on the average, in the required number of samples for given probabilities of error.

The two types of error possible are the acceptance of H_o when H_1 is true and the acceptance of H_1 when H_o is true. Wald[1] has showed that the thresholds A and B depend on these probabilities of error. The procedure followed in the conventional sequential test is to select acceptable error probabilities, thereby determining the thresholds. Of course, the smaller the error probabilities, the larger the average test length. It is emphasized at this point that a modified form of the conventional sequential test is utilized in the voice communications applications discussed in the body of this report. The several variations from the conventional will be explained in detail in the following chapters.

Radar Application

There has been much work in applying the sequential decision theory to radar systems. In the radar problem, the system decides on the presence or absence of a target in a particular region of space during a particular time period. Based on the signal received at the set, it may not be clear as to whether this is from the radar pulse returned from a target or whether it is noise alone. In this case, the radar continues to "look" until a decision can be made. Wald's theory, when considered from the point of view of detecting a radar signal in noise, offers a saving in the time required to make a decision within certain tolerable error limits as to the presence or absence of a target.

Sequential detection of signals in noise falls into the general domain of Statistical Decision Theory. Van Meter and Middleton[4] studied the general application to the reception problem of decision theory. Blasbalg,[5] Fox,[6] and Bussgang and Middleton[7] have made valuable contributions with their works involving sequential signal detection. Much of the discussion here of the radar application will follow the results of Bussgang and Middleton.[7]

For radar, the parameter for which the sequential test is performed is the input signal-to-noise ratio $\underline{a}$. Thus the two hypotheses being considered are H_o: the value of $\underline{a}$ is zero, and H_1: the value of $\underline{a}$ is greater than zero.

Acceptance of H_0 (rejection of H_1) is a decision that no target is present. This is a "dismissal." Acceptance of H_1 (rejection of H_0) is a decision that a target is present. This is an "alarm." As expected, the decision to accept H_1 is more likely to be made as $\underline{a}$ is increased. A preset signal-to-noise ratio $\underline{a}_1$ is used to set up the sequential test. The value $\underline{a}_1$ is such that for $\underline{a}$ less than $\underline{a}_1$, an acceptance of H_0 is not too objectionable. The test involves selecting tolerable values for α, the probability of a false alarm, and for β, the probability of a false dismissal. The quantity β is determined by the probability of dismissal for $\underline{a} = \underline{a}_1$. Sometimes α and β are called the probabilities of error of the first and second kind, respectively. The "strength of the test" refers to these two probabilities of error and is denoted by (α, β). The test is regarded as stronger as α, β are made smaller.

At this point we will consider the incoherent sequential detection of the radar signal in noise. Following this will be a discussion of coherent sequential detection. For incoherent detection no phase information is available at the receiver. The random variable of interest is the envelope of a narrow-band Gaussian noise plus a sine wave at the center frequency of the noise band. The appropriate probability density function for this envelope R is shown by Rice[8] to be

$$W(R;A_o) = \frac{R}{\sigma_n^2} \cdot e^{-\frac{(R^2 + A_o^2)}{2\sigma_n^2}} \cdot I_o\left(\frac{RA_o}{\sigma_n^2}\right) \text{ for } R \geq 0 \qquad (1.9)$$

where σ_n^2 is the mean-square value of noise at the i-f filter and I_o (u) is the modified Bessel function of the first kind and of order zero. A_o is the peak amplitude of the sine wave. For the envelope of noise alone ($A_o = 0$), the density function is

$$W(R;0) = \frac{R}{\sigma_n^2} \cdot e^{-\frac{R^2}{2\sigma_n^2}} \text{ for } R \geq 0 \qquad (1.10)$$

These are the Rayleigh distributions. The true signal-to-noise ratio $\underline{a}$, which is the unknown parameter of these densities, is

$$\underline{a} = \frac{A_o}{\sqrt{2}\,\sigma_n} \qquad (1.11)$$

If, for convenience, we change the random variable from $\frac{R}{\sqrt{2}\,\sigma_n}$ to X, the densities become

$$\begin{aligned} W(x;\underline{a}) &= 2x \cdot e^{-(\underline{a}^2 + x^2)} \cdot I_o(2\underline{a}x) \quad , \quad x \geq o \\ &= 0 \quad , \quad x < o \ . \end{aligned} \qquad (1.12)$$

From this the probability ratio λ_m can be found, and the test carried out in terms of the threshold numbers A and B. An alternate procedure involves the consideration of the test in terms of the logarithm of the probability ratio. For this case, the zone of indifference is bounded by log A and log B.

Under the usual condition of independent samples, the two error probabilities are

$$\alpha = \int_0^\infty dn \cdot p(n/H_1\,;\,0) \cdot \left[\int_a^b \frac{R}{\sigma_n^2} \cdot e^{-\frac{R^2}{2\sigma_n^2}} \cdot dR \right]^{n-1} \cdot \tag{1.13}$$

$$\cdot \left[\int_b^\infty \frac{R}{\sigma_n^2} \cdot e^{-\frac{R^2}{2\sigma_n^2}} \cdot dR \right]$$

$$\beta = \int_0^\infty dn \cdot p(n/H_0\,;\,\underline{a}_1) \cdot \left[\int_a^b \frac{R}{\sigma_n^2} \cdot e^{-\frac{R^2}{2\sigma_n^2} - a_1^2} \cdot I_0\left(\frac{\sqrt{2}\,R\,a_1}{\sigma_n}\right) \cdot dR \right]^{n-1} \cdot$$

$$\cdot \left[\int_0^a \frac{R}{\sigma_n^2} \cdot e^{-\frac{R^2}{2\sigma_n^2} - a_1^2} \cdot I_0\left(\frac{\sqrt{2}\,R\,a_1}{\sigma_n}\right) \cdot dR \right] \tag{1.14}$$

In the above, α and β are expressed in terms of the actual signal level thresholds a and b. Even though n, the final stage of the test, is a discrete random variable assuming integer values, it is convenient to consider it as a continuous variable. Thus $p(n/H_k;\underline{a})$ represents the conditional probability density function of n under the condition that H_k (k = 0.1) is accepted. The true value of the signal-to-noise ratio $\underline{a}$ is also required to specify this density.

Simplification of the error probability expressions yields

$$\alpha = \int_0^\infty dn \cdot p(n/H_1\,;\,0)\cdot\left[e^{-\frac{a^2}{2\sigma_n^2}} - e^{-\frac{b^2}{2\sigma_n^2}}\right]^{n-1}\cdot\; e^{-\frac{b^2}{2\sigma_n^2}} \tag{1.15}$$

$$\beta = \int_0^\infty dn \cdot p(n/H_0\,;\,\underline{d}_1)\cdot\left[Q\left(\sqrt{2}\cdot d_1\,,\,\frac{a}{\sigma_n}\right) - Q\left(\sqrt{2}\cdot d_1\,,\,\frac{b}{\sigma_n}\right)\right]^{n-1}\cdot$$

$$\cdot\left[1 - Q\left(\sqrt{2}\cdot d_1\,,\,\frac{a}{\sigma_n}\right)\right] \tag{1.16}$$

where Q(x,y) is defined by Marcum[9] as

$$Q(x,y) = \int_y^\infty v\cdot e^{-\frac{v^2+x^2}{2}}\cdot I_0(xv)\cdot dv \tag{1.17}$$

Bussgang and Middleton[7] have studied this case extensively. They show how the average number of samples required increases as the probabilities of error are lowered. The savings in average number of observations is paid for in the sample length's becoming random. The larger the variance of the sample length, the greater the savings in average length. As the variance gets smaller the savings is reduced and the sequential test approaches a non-sequential.

Similarly, Bussgang and Middleton[7] have thoroughly studied the sequential coherent detection case. Here, of course, it is assumed that the receiver has complete signal phase information. Statistically, the problem is the testing of the mean. Under H_0 the true mean is zero; for H_1 the mean is the value of the signal at the sampling time. The general technique for the test does not change. The probability ratio is constructed and tested against the two threshold numbers as before. The density functions for this case are Gaussian. Bussgang and Middleton[7] treat the general case of correlated samples, and give as an example the case with an exponential autocorrelation function.

It is interesting to note the results of the comparison between the sequential coherent and sequential non-conherent detectors. For signals smaller than the preset value $\underline{a}_1$, the coherent detector is less likely to give a false dismissal than the non-coherent. In many cases this difference is very great. For signals larger than the preset value, the coherent detector is more likely to give a false dismissal.

In the comparison of the average number of samples, it is found that in the weak signal case this average number depends on $\underline{a}$ for the coherent test and on $\underline{a}^2$ for the non-coherent. As an example of how this affects the average sample number, Bussgang and Middleton[7] plotted $\underline{a}_1{}^2\ \overline{n(a)}$ for coherent and $\underline{a}_1{}^4\ \overline{n(a)}$ for non-coherent versus $\underline{a}/\underline{a}_1$. The peaks of the two curves are the same but occur at different values of $\underline{a}/\underline{a}_1$. Since the case considered is for $\underline{a}_1$ very small, the coherent sequential detector therefore requires fewer average samples at this peak value.

For the radar case, any truncation of the test required would normally be set for some large number of samples. The result would be a test very close to the untruncated one. When a test is truncated the average number of samples required to terminate is reduced. However the strength of the test (α, β) is reduced to (α', β'). Complicated expressions relating α to α' and β to β' have been derived,[7] but they will not be repeated here.

Data Application

Data applications of the sequential theory are different in several respects from the radar case. The most conspicuous difference is the use of a feedback channel for data. The receiver employs three decision zones as before: a zone of acceptance of H_o, a zone of acceptance of H_1, and a zone of indifference. Here H_o represents the hypothesis that one of the binary data symbols was transmitted, while H_1 represents the hypothesis that the other symbol was sent.

The feedback channel is used whenever there is a reception falling in the zone of indifference. Whenever this situation occurs, the receiver notifies the transmitter over the feedback channel that it is not clear which symbol was sent. The transmitter repeats this symbol and the decision process starts over. The transmitter then either goes on to the next symbol or repeats the same symbol, depending on the decision at the receiver. With probability unity, the transmitter can eventually go on to the next symbol, although in specific instances, a long time may be required.

When compared to the method of repetition a fixed number of times, it is seen that the sequential technique provides a savings which can be realized in increased data rate or reduced necessary signal power for a given probability of error.

The receiver of the standard sequential system utilizes all past samples in deciding which signal is present and if another sample is required to give more accuracy. A modified form of this has received much emphasis in the literature. The technique referred to is the sequential test without memory, usually called null-zone detection.

In null-zone detection, the same three choices of decision are available to the receiver. However, only the most recent sample is available on which to base the decision. For this situation there is a non-zero probability that the receiver would continue asking for a repeat of a particular signal, and thus never move on to the rest of the message. Consideration of truncation becomes very important here.

Harris, Hauptschein, and Schwartz[10] have extensively studied the null-zone detection problem. Among other aspects of the problem they studied the manner in which the threshold levels should be adjusted in order to minimize communication costs. These costs depend on power, bandwidth, and transmission time. For non-truncation the uniform null (no adjustment) is optimum. For truncation, however, the levels should be adjusted for each sample so that the probability of the sample falling in the null zone is reduced over the previous sample. They show that the loss is small, however, if no adjustments are made.

In these studies the feedback channel is usually considered error-free. This is reasonable since normally only a small amount of information compared to the channel capacity is transmitted over it. The probability of error can therefore be made very small.

Boorstyn[11] has also studied null-zone detection, concentrating his efforts on the case of extremely small average number of samples; i.e., between one and two. For the most part he deals with situations in which a repeat is required only a small fraction of the time, and only one repeat is allowed. He demonstrates that even for these small average sample sizes a significant improvement is obtained.

Voice Systems

With the above providing the background a description will now be given of the voice communication systems which have been devised from consideration of the sequential techniques. Just as there are differences between the radar and data sequential systems, so are the voice systems different from them in several respects.

Search of the literature shows no previous successful efforts along these lines. The problem is of a somewhat different nature from the radar or data problems. With speech there is not enough time to allow for many samples before making a decision, as in the radar case. But, instead, a decision must be made during every speech sample interval. Similarly, no provision can be made for a feedback communications path, as in the data case, for the purpose of requesting a retransmission. (Of course, in duplex communications the listener can simply request that a word or sentence be repeated, but here a particular sample of the voice wave is being referred to in the preceding statement.)

All of the discussion here will be concerned with systems of the quantized pulse position modulation type (QPPM); i.e., information regarding the speech amplitude at the sampling instant is denoted by transmitting the pulse in a particular one of the N discrete pulse position slots in the speech sample interval. The pulse will be detected incoherently. It is believed that techniques similar to those described here can also be applied to other types of modulation, but this will be left for future study.

Two cases will be discussed. First, the interference will be considered to consist of a narrow-band Gaussian noise. A double threshold QPPM sequential system is described in Chapter II which is effective against this type of interference. Second, the interference is a narrow-band Gaussian noise and an impulsive noise. In Chapter III is described the four-threshold level QPPM sequential system which effectively combats this kind of interference.

In each case, the decision zones are of the three usual types: signal present, signal absent, and the zone of indifference. The receiver must examine the N pulse position slots sequentially during each speech sample interval, beginning with the first slot and continuing through the N^{th} slot or through the termination slot if termination occurs before the end of the frame. During the time a slot is being examined, only one sample is available from which hypothesis H_o or hypothesis H_1 is decided if possible. At this stage, the receiver is acting like the null-zone detector of the data case (the sequential test without memory).

If hypothesis H_1 is accepted for the slot under examination (the decision is made that this slot contains the signal) the test is terminated until the next speech frame begins. Acceptance of H_o for a slot does not terminate the test. This is a difference from the previous cases discussed.

The receiver has memory of those slots for which the sample falls into the zone of indifference. Use is made of this information for those frames in which no slot sample calls for acceptance of H_1. That is, at the end of the N^{th} slot, if no slot has been chosen as definitely having the signal, a choice will then be made from only those slots which were not rejected by choice of hypothesis H_0. The test is therefore truncated after N slots.

There are alternative criteria available for use in choosing the signal slot from the several "possible signal" slots. Use may be made of speech statistics for this choice, for example. Samples of the speech wave represent a Markov process. The probability density function for a sample is conditional upon the amplitudes of the preceding samples. Therefore, for example, based on the position of the pulse in the previous frame, that one of the "possible signal" slots could be chosen which represented the most probable slot. The mathematical model used for the speech marginal and conditional probability density functions is based on the experimental results of Davenport.[12] A more complete and detailed discussion of this model is given in Appendix A.

For the double threshold level situation, an alternate criterion is to make the choice from the "possible signal" slots on the basis of the one with the largest amplitude pulse.

For the four threshold level situation, a possible criterion is to make the choice on the basis of the slot whose pulse amplitude is nearest to the range of amplitudes representing acceptance of H_1. Other criteria are possible, but these are the ones considered in this study.

For all of these there is a non-zero probability that all slots in a frame represent acceptance of H_0. When this occurs the receiver chooses the center slot (the $\frac{N}{2}^{th}$) and proceeds to the next frame.

Of primary interest is how these systems compare to conventional QPPM systems. The two standard QPPM methods are the single threshold, incoherent detection type, and the largest of N type, without a threshold. In the first, the receiver selects as the signal slot the first slot whose pulse amplitude exceeds the threshold. In the second, the receiver selects as the signal slot that one whose pulse amplitude is the largest.

For comparison purposes there are many criteria possible, such as: (1) output signal-to-noise ratio, (2) bandwidth required, (3) average cost in the Bayes sense, (4) complexity, and (5) probability of correctly choosing the proper slot. For mathematical tractability, the latter one is used in this study.

In addition to the comparisons reported here, parametric studies of the systems are also given. For various input signal-to-noise ratios, the threshold settings are allowed to vary and the effect noted on the probability of correct choice. The difference between an adaptive and a non-adaptive system can also be seen. The sense in which the systems are adaptive is that the threshold settings are self-adjusting with changes in the noise level so as to be optimum.

Other quantities of interest are the mean and the variance of the number of "possible signal" slots from which a choice is to be made if no "signal" slot is available. These have a bearing on the complexity of that part of the system which performs this task. As the thresholds are varied, the report describes the variation of these quantities with particular input signal-to-noise ratios. Also of interest is the mean of the number of slots before termination of the test.

With this chapter providing the introduction, the remainder of this report will concern itself with more details of the system descriptions and with a complete discussion of the results.

CHAPTER II

CONVENTIONAL AND SEQUENTIAL SYSTEMS WITHOUT IMPULSE NOISE CONSIDERATION

Introduction

In this section is a discussion of four QPPM systems, with the interference taken to be narrow-band Gaussian noise. Two of the systems are conventional types while the other two have been devised from a consideration of sequential principles.

It should be noted that there have been made several modifications of the sequential techniques as used in radar and data for these voice applications. The radar receiver can afford a very large number of samples before choosing a hypothesis. The voice receiver, however, must make a decision no later than the end of each speech sampling frame. Therefore, for voice we are dealing with a moderate number of samples and the maximum number allowed is fixed.

Minimization of the average number of samples is not the primary consideration for voice. Instead of fixing the values of the two types of error, and thereby fixing the thresholds so as to minimize the average number of samples, we now vary the thresholds to obtain a minimum probability of error. Of course, a test with a truncation point does not have fixed thresholds for minimization of average samples. The thresholds instead should move closer together in some manner, not yet clear, as the test proceeds. Thus, fixing the errors for fixed thresholds is not appropriate.

The data application allows a feedback channel for requesting retransmission of a questionable reception. For the same reason that the voice receiver must make a decision no later than the end of each frame, no feedback channel can be used with it.

Another difference which will be made clear in the system descriptions is that the test terminates only with the acceptance of the signal hypothesis. Acceptance of the noise alone hypothesis does not stop the test.

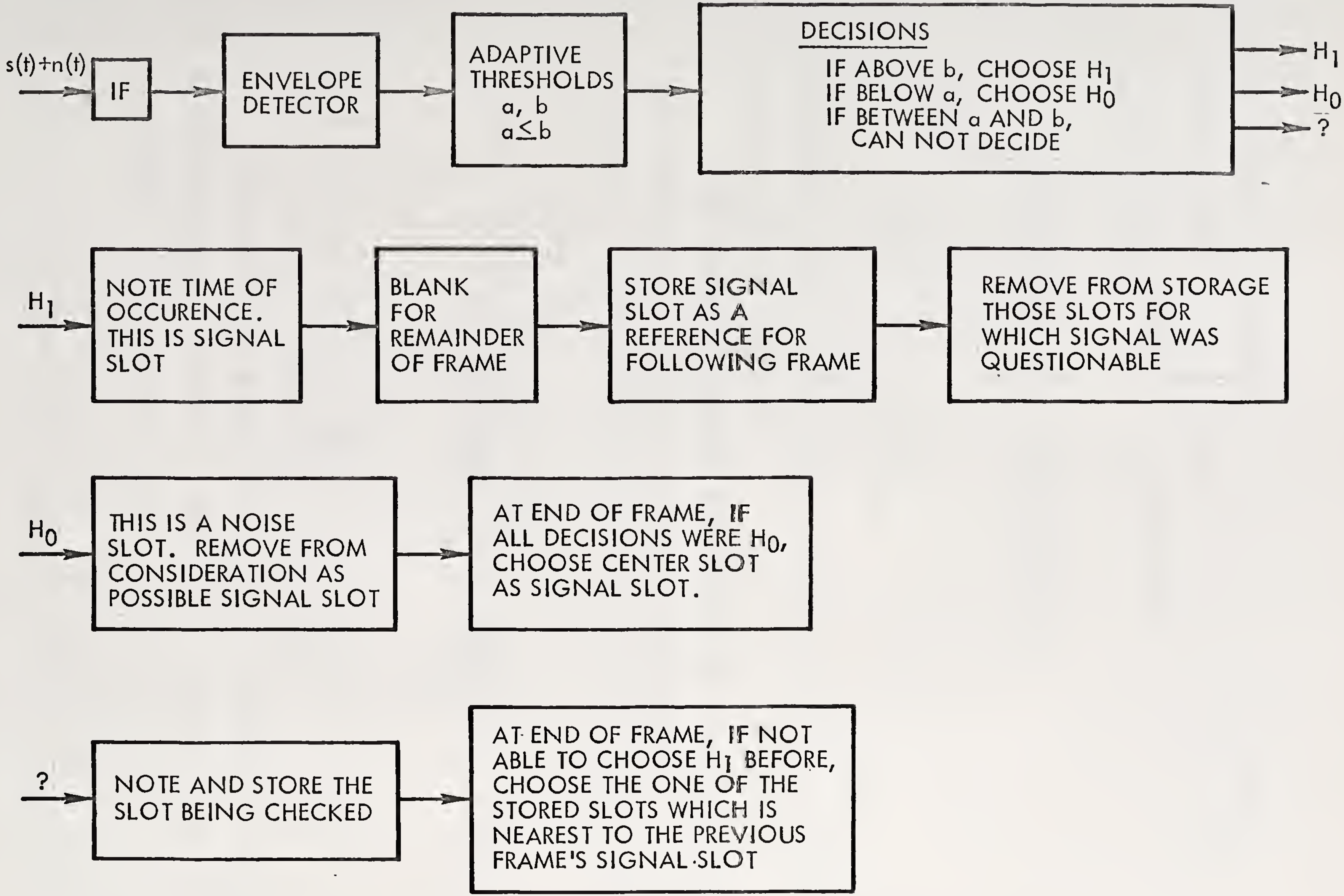
QPPM, Double Threshold, Speech Statistics

A functional diagram of this system is given in Figure 1. The signal and narrow-band Gaussian noise are input to an IF filter and then envelope detected. The optimum decision thresholds will be shown later to be adaptive ones in the sense that they change with changing noise level. This system utilizes two thresholds, a and b, with the b larger.

During each of the N pulse slots, the receiver is called upon to attempt a choice of hypothesis. If the envelope during the slot is greater than b, the signal hypothesis H_1 is chosen. Thus the receiver has selected a

Figure 1

Functional Diagram, QPPM, Double Threshold, Speech Statistics System



s(t)+n(t)
IF
ENVELOPE DETECTOR
ADAPTIVE THRESHOLDS a, b a≤b
DECISIONS
IF ABOVE b, CHOOSE H1
IF BELOW a, CHOOSE H0
IF BETWEEN a AND b, CAN NOT DECIDE
H1
H0
?
H1
NOTE TIME OF OCCURENCE. THIS IS SIGNAL SLOT
BLANK FOR REMAINDER OF FRAME
STORE SIGNAL SLOT AS A REFERENCE FOR FOLLOWING FRAME
REMOVE FROM STORAGE THOSE SLOTS FOR WHICH SIGNAL WAS QUESTIONABLE
H0
THIS IS A NOISE SLOT. REMOVE FROM CONSIDERATION AS POSSIBLE SIGNAL SLOT
AT END OF FRAME, IF ALL DECISIONS WERE H0, CHOOSE CENTER SLOT AS SIGNAL SLOT.
?
NOTE AND STORE THE SLOT BEING CHECKED
AT END OF FRAME, IF NOT ABLE TO CHOOSE H1 BEFORE, CHOOSE THE ONE OF THE STORED SLOTS WHICH IS NEAREST TO THE PREVIOUS FRAME'S SIGNAL SLOT

slot as the signal slot. The remainder of that frame may now be blanked out and possibly used for the reception of non-real-time data, for example. The receiver stores this slot as a reference for the following frame to use if necessary. All previous slots for the frame which were in the "possible signal" category are removed from storage at this time.

If the envelope is below a, the noise alone hypothesis H_0 is chosen. This does not halt the test for the frame, but removes the tested slot from consideration as a "possible signal" slot. At the end of the frame, if all slots have H_0 associated with them, the receiver selects the center slot as the desired one.

Whenever the envelope is between a and b for a slot, the receiver stores this as a "possible signal" slot, and proceeds to the next slot. If some following slot has the signal hypothesis accepted, this information is removed from storage. If, however, at the end of the frame there has been no selection of a signal slot, this slot is compared with any other stored "possible signal" slots. By using speech statistics the slot most likely to be the signal slot is then chosen. This is the slot nearest to the position occupied by the signal of the preceding frame, as stored in the receiver.

The speech probability expressions required are developed in Appendix A. The first is the a priori probability that the signal is in the j^{th} slot. This is

$$P_s(j) \cong 0.3\left[\pm e^{\pm 4.6\left\{\frac{2j}{N}-1\right\}} \mp e^{\pm 4.6\left\{\frac{2(j-1)}{N}-1\right\}}\right]$$

$$+0.1\left[\operatorname{erf}\left\{24\left(\frac{2j}{N}-1\right)\right\} - \operatorname{erf}\left\{24\left(\frac{2(j-1)}{N}\right)\right\}\right] \tag{2.1}$$

where

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-u^2}\cdot du \tag{2.2}$$

The upper sign is to be used for $1 \leq j \leq \frac{N}{2}$ and the lower sign for $\frac{N}{2} + 1 \leq j \leq N$.

The second required expression is the probability that the speech sample's amplitude corresponds to the j^{th} position, given that the previous position was the ℓ^{th}. This is given by

$$P_s(j/\ell) = \frac{\left[e^{\frac{4}{N}} - e^{-\frac{4}{N}}\right]\cdot e^{-|j-\ell|\cdot\left(\frac{8}{N}\right)}}{2 - e^{-\frac{4}{N}}\cdot\left[e^{-(\ell-1)\left(\frac{8}{N}\right)} + e^{-8\left(1-\frac{\ell}{N}\right)}\right]} \;;\; j \neq \ell$$

$$= \frac{2\left[1 - e^{-\frac{4}{N}}\right]}{2 - e^{-\frac{4}{N}}\cdot\left[e^{-(\ell-1)\left(\frac{8}{N}\right)} + e^{-8\left(1-\frac{\ell}{N}\right)}\right]} \;;\; j = \ell \tag{2.3}$$

The desired expression for the probability of correct decision during a frame is composed of three parts:

$$\begin{aligned} P_1(\text{cor}) &= P_1\,(\text{correct, signal in "signal" zone}) \\ &+ P_1\,(\text{correct, signal in "possible signal" zone}) \\ &+ P_1\,(\text{correct, signal in "noise alone" zone}) \end{aligned} \tag{2.4}$$

The probability of a correct decision, given that the signal is in the j^{th} slot and that its envelope exceeds b, is the probability that the j-1 preceding slots with noise alone do not exceed b. The probability densities of the envelopes of narrow-band Gaussian noise alone and sine wave plus noise are Rayleigh and Modified Rayleigh, respectively, as discussed in Chapter I. Therefore,

P_1 (correct, signal in "signal" zone)

$$= Q\left(\frac{A}{\sigma_n}, \frac{b}{\sigma_n}\right) \cdot \sum_{j=1}^{N} P_s(j) \cdot \left[1 - e^{-\frac{b^2}{2\sigma_n^2}}\right]^{j-1} \tag{2.5}$$

where

$$Q(\alpha, \beta) = \int_{\beta}^{\infty} x \cdot e^{-\frac{x^2+\alpha^2}{2}} \cdot I_o(\alpha x) \cdot dx \tag{2.6}$$

The second part of the expression is more complicated. Given that the signal envelope is in the "possible signal" region and that none of the "noise alone" envelopes exceed b, we can develop the probability of a correct decision with the reference slot being the ℓ^{th}. We have to consider only $\ell \leq \frac{N}{2}$ and double the resulting expression, since there is an equal contribution for $\ell > \frac{N}{2}$ due to the symmetry of the speech first probability density. Under these conditions, the probability of correct decision is the probability that none of the noise slots closer to the reference than the signal slot is, are in the "possible signal" zone. For the special case where a noise slot which is "possible signal" is the same distance from the reference as the signal slot is, the receiver chooses the one nearer to the center of the frame. The range of values for j, the signal slot, must be separated into three parts: $j \leq \ell$, $\ell + 1 \leq j \leq 2\ell$, and $2\ell + 1 \leq j \leq N$. The result is

P_1 (correct, signal in "possible signal" zone)

$$= \left[Q\left(\frac{A}{\sigma_n}, \frac{a}{\sigma_n}\right) - Q\left(\frac{A}{\sigma_n}, \frac{b}{\sigma_n}\right)\right] \cdot \left[1 - e^{-\frac{b^2}{2\sigma_n^2}}\right]^{N-1} \cdot (2) \cdot$$

$$\cdot \sum_{\ell=1}^{\frac{N}{2}} P_r(\ell) \cdot \left\{ \sum_{j=1}^{\ell} P_s(j/\ell) \cdot \left[\frac{1 - e^{-\frac{a^2}{2\sigma_n^2}}}{1 - e^{-\frac{b^2}{2\sigma_n^2}}}\right]^{2\ell - 2j} \right.$$

$$\left. + \sum_{j=\ell+1}^{2\ell} P_s(j/\ell) \cdot \left[\frac{1 - e^{-\frac{a^2}{2\sigma_n^2}}}{1 - e^{-\frac{b^2}{2\sigma_n^2}}}\right]^{2j-2\ell-1} + \sum_{j=2\ell+1}^{N} P_s(j/\ell) \cdot \left[\frac{1 - e^{-\frac{a^2}{2\sigma_n^2}}}{1 - e^{-\frac{b^2}{2\sigma_n^2}}}\right]^{j-1} \right\} \quad (2.7)$$

For the case of signal in the "noise alone" region, the probability of correct decision is the probability that all of the noise envelopes are below a and that the signal slot is the center one. Therefore,

P_1 (correct, signal in "noise alone" zone)

$$= P_s\left(\frac{N}{2}\right) \cdot \left[1 - Q\left(\frac{A}{\sigma_n}, \frac{a}{\sigma_n}\right)\right] \cdot \left[1 - e^{-\frac{a^2}{2\sigma_n^2}}\right]^{N-1} \quad (2.8)$$

Combining the three contributing parts gives the final result

$$P_1(\text{cor}) = Q\left(\frac{A}{\sigma_n},\frac{b}{\sigma_n}\right)\cdot\sum_{j=1}^{N} P_s(j)\cdot\left[1-e^{-\frac{b^2}{2\sigma_n^2}}\right]^{j-1}$$

$$+\left[Q\left(\frac{A}{\sigma_n},\frac{a}{\sigma_n}\right)-Q\left(\frac{A}{\sigma_n},\frac{b}{\sigma_n}\right)\right]\cdot\left[1-e^{-\frac{b^2}{2\sigma_n^2}}\right]^{N-1}\cdot(2)\cdot\sum_{\ell=1}^{\frac{N}{2}} P_r(\ell)\cdot$$

$$\cdot\left\{\sum_{j=1}^{\ell} P_s(j/\ell)\cdot\left[\frac{1-e^{-\frac{a^2}{2\sigma_n^2}}}{1-e^{-\frac{b^2}{2\sigma_n^2}}}\right]^{2\ell-2j}+\sum_{j=\ell+1}^{2\ell} P_s(j/\ell)\left[\frac{1-e^{-\frac{a^2}{2\sigma_n^2}}}{1-e^{-\frac{b^2}{2\sigma_n^2}}}\right]^{2j-2\ell-1}\right.$$

$$\left.+\sum_{j=2\ell+1}^{N} P_s(j/\ell)\cdot\left[\frac{1-e^{-\frac{a^2}{2\sigma_n^2}}}{1-e^{-\frac{b^2}{2\sigma_n^2}}}\right]^{j-1}\right\}$$

$$+P_s\left(\frac{N}{2}\right)\cdot\left[1-Q\left(\frac{A}{\sigma_n},\frac{a}{\sigma_n}\right)\right]\cdot\left[1-e^{-\frac{a^2}{2\sigma_n^2}}\right]^{N-1} \qquad (2.9)$$

The expression derived above for the probability of a correct decision has considered only two frames at any one time. Higher order speech statistics were not available to allow more than the preceding frame to be used for reference purposes. This would seem to have only a small effect and therefore could be safely neglected. Furthermore, the fact that the reference position is taken to be correct each time is acceptable since, for the practical situation, the probability of error for a particular frame is small.

Another quantity of some interest is the average number of slots tested before termination occurs. This is

$$\overline{K} = \sum_{K=1}^{N} K \cdot P(K) \tag{2.10}$$

where P(K) is the probability of termination on the K^{th} slot. The three cases to be considered are: signal slot is the K^{th}, signal slot is before the K^{th}, and signal slot is after the K^{th}. A termination occurs on the K^{th} slot for the first case if the signal envelope exceeds b and none of the envelopes of the preceding K-1 slots exceed b. For the second case, the signal envelope is below b, as well as the envelopes of the first K-2 noise slots. The K^{th} slot containing noise alone does exceed b. The last case requires the first K-1 slots to have noise envelopes below b, with the K^{th} above b. Special consideration must be given to the termination on the last slot. This can occur from the envelope of the last slot, whether signal or noise, exceeding b and the envelopes of the other slots being below b. It also can occur due to the test truncation requirement when there are no envelopes exceeding b during the entire frame. Consideration of these facts gives the result

$$\overline{K} = \sum_{K=1}^{N-1} K \cdot \left[1 - e^{-\frac{b^2}{2\sigma_n^2}}\right]^{K-1} \cdot \left\{ Q\left(\frac{A}{\sigma_n}, \frac{b}{\sigma_n}\right) \cdot P_s(K) \right.$$

$$+ \frac{e^{-\frac{b^2}{2\sigma_n^2}} \cdot \left[1 - Q\left(\frac{A}{\sigma_n}, \frac{b}{\sigma_n}\right)\right]}{1 - e^{-\frac{b^2}{2\sigma_n^2}}} \cdot \sum_{j=1}^{K-1} P_s(j)$$

$$+ e^{-\frac{b^2}{2\sigma_n^2}} \cdot \sum_{j=K+1}^{N} P_s(j) \Bigg\}$$

$$+ N \cdot \left[1 - e^{-\frac{b^2}{2\sigma_n^2}}\right]^{N-1} \cdot \Bigg\{ Q\left(\frac{A}{\sigma_n}, \frac{b}{\sigma_n}\right) \cdot P_s(N)$$

$$+ \frac{e^{-\frac{b^2}{2\sigma_n^2}} \cdot \left[1 - Q\left(\frac{A}{\sigma_n}, \frac{b}{\sigma_n}\right)\right] \cdot \left[1 - P_s(N)\right]}{1 - e^{-\frac{b^2}{2\sigma_n^2}}}$$

$$+ \left[1 - Q\left(\frac{A}{\sigma_n}, \frac{b}{\sigma_n}\right)\right] \Bigg\} \tag{2.11}$$

Other quantities of interest are the mean and the variance of the number of slots with envelopes in the "possible signal" zone, when no slot's envelope exceeds b during the entire frame. These are given by

$$\bar{s} = \sum_{s=1}^{N} s \cdot P(s) \tag{2.12}$$

and

$$\begin{aligned} \sigma_s^2 &= \overline{s^2} - (\bar{s})^2 \\ &= \sum_{s=1}^{N} s^2 \cdot P(s) - (\bar{s})^2 \end{aligned} \tag{2.13}$$

where P(S) is the probability that the number will be s, given that the test did not terminate due to an envelope being above b. The two cases to be considered are that the signal slot makes up one of the s, and that it does not make up one of the s slots. It is thus seen that

$$P(s) = \sum_{s=1}^{N} \left\{ \binom{N-1}{s-1} \frac{\left[Q\left(\frac{A}{\sigma_n}, \frac{a}{\sigma_n}\right) - Q\left(\frac{A}{\sigma_n}, \frac{b}{\sigma_n}\right)\right] \cdot \left[e^{-\frac{a^2}{2\sigma_n^2}} - e^{-\frac{b^2}{2\sigma_n^2}}\right]^{s-1} \cdot \left[1 - e^{-\frac{a^2}{2\sigma_n^2}}\right]^{N-s}}{\left[1 - Q\left(\frac{A}{\sigma_n}, \frac{b}{\sigma_n}\right)\right] \cdot \left[1 - e^{-\frac{b^2}{2\sigma_n^2}}\right]^{N-1}} \right.$$

$$\left. + \binom{N-1}{s} \frac{\left[1 - Q\left(\frac{A}{\sigma_n}, \frac{a}{\sigma_n}\right)\right] \cdot \left[e^{-\frac{a^2}{2\sigma_n^2}} - e^{-\frac{b^2}{2\sigma_n^2}}\right]^{s} \cdot \left[1 - e^{-\frac{a^2}{2\sigma_n^2}}\right]^{N-1-s}}{\left[1 - Q\left(\frac{A}{\sigma_n}, \frac{b}{\sigma_n}\right)\right] \cdot \left[1 - e^{-\frac{b^2}{2\sigma_n^2}}\right]^{N-1}} \right\} \tag{2.14}$$

Therefore,

$$\bar{s} = \sum_{s=1}^{N} \left\{ s \binom{N-1}{s-1} \frac{\left[Q\left(\frac{A}{\sigma_n}, \frac{a}{\sigma_n}\right) - Q\left(\frac{A}{\sigma_n}, \frac{b}{\sigma_n}\right)\right] \cdot \left[e^{-\frac{a^2}{2\sigma_n^2}} - e^{-\frac{b^2}{2\sigma_n^2}}\right]^{s-1} \cdot \left[1 - e^{-\frac{a^2}{2\sigma_n^2}}\right]^{N-s}}{\left[1 - Q\left(\frac{A}{\sigma_n}, \frac{b}{\sigma_n}\right)\right] \cdot \left[1 - e^{-\frac{b^2}{2\sigma_n^2}}\right]^{N-1}} \right.$$

$$\left. + s \binom{N-1}{s} \frac{\left[1 - Q\left(\frac{A}{\sigma_n}, \frac{a}{\sigma_n}\right)\right] \cdot \left[e^{-\frac{a^2}{2\sigma_n^2}} - e^{-\frac{b^2}{2\sigma_n^2}}\right]^{s} \cdot \left[1 - e^{-\frac{a^2}{2\sigma_n^2}}\right]^{N-1-s}}{\left[1 - Q\left(\frac{A}{\sigma_n}, \frac{b}{\sigma_n}\right)\right] \cdot \left[1 - e^{-\frac{b^2}{2\sigma_n^2}}\right]^{N-1}} \right\} \tag{2.15}$$

and

$$\sigma_S^2 = \sum_{S=1}^{N} \left\{ S^2 \binom{N-1}{S-1} \frac{\left[Q\left(\frac{A}{\sigma_n}, \frac{a}{\sigma_n}\right) - Q\left(\frac{A}{\sigma_n}, \frac{b}{\sigma_n}\right)\right] \cdot \left[e^{-\frac{a^2}{2\sigma_n^2}} - e^{-\frac{b^2}{2\sigma_n^2}}\right]^{S-1} \cdot \left[1 - e^{-\frac{a^2}{2\sigma_n^2}}\right]^{N-S}}{\left[1 - Q\left(\frac{A}{\sigma_n}, \frac{b}{\sigma_n}\right)\right] \cdot \left[1 - e^{-\frac{b^2}{2\sigma_n^2}}\right]^{N-1}} \right.$$

$$\left. + S^2 \binom{N-1}{S} \frac{\left[1 - Q\left(\frac{A}{\sigma_n}, \frac{a}{\sigma_n}\right)\right] \cdot \left[e^{-\frac{a^2}{2\sigma_n^2}} - e^{-\frac{b^2}{2\sigma_n^2}}\right]^{S} \cdot \left[1 - e^{-\frac{a^2}{2\sigma_n^2}}\right]^{N-1-S}}{\left[1 - Q\left(\frac{A}{\sigma_n}, \frac{b}{\sigma_n}\right)\right] \cdot \left[1 - e^{-\frac{b^2}{2\sigma_n^2}}\right]^{N-1}} \right\}$$

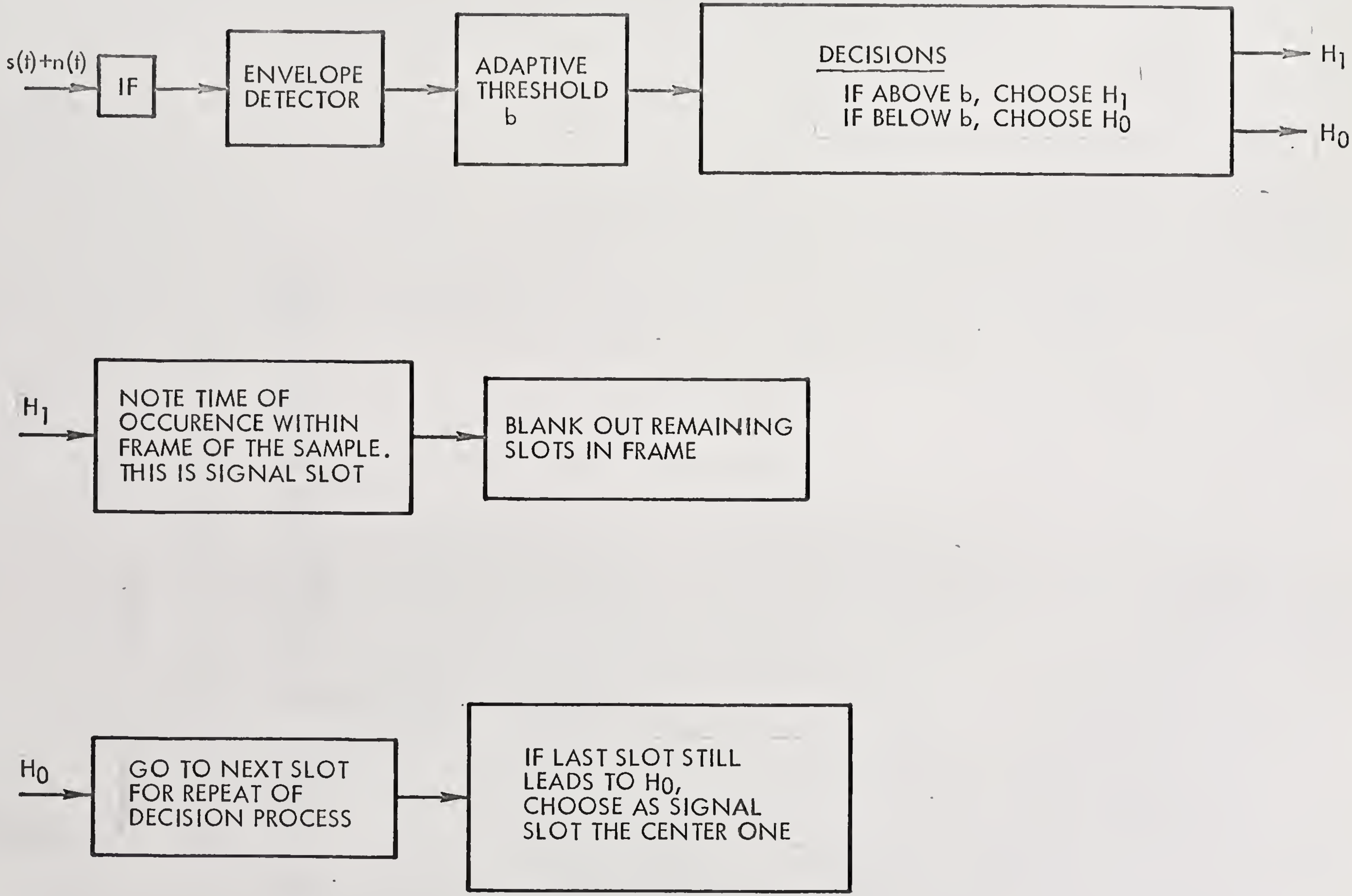
$$- (\bar{S})^2 \qquad (2.16)$$

<u>QPPM, Single Threshold, Conventional</u>

The functional diagram for this system is given in Figure 2. The output of the envelope detector is compared to a single threshold b in this case. The slot whose envelope is the first to exceed b is selected as the signal slot. That is, for each slot, either the signal hypothesis H_1 or the noise hypothesis H_0 is selected. As soon as a slot has H_1 associated with it, the test is stopped and the remainder of the frame could be blanked out. Should H_0 be selected for a slot, the receiver proceeds to examine the next one. As before, if all slots in the frame lead to choices of H_0, the receiver chooses the center slot as the signal position.

Figure 2

Functional Diagram, QPPM, Single Threshold, Conventional System



s(t)+n(t)
IF
ENVELOPE DETECTOR
ADAPTIVE THRESHOLD b
DECISIONS
IF ABOVE b, CHOOSE H1
IF BELOW b, CHOOSE H0
H1
H0
H1
NOTE TIME OF OCCURENCE WITHIN FRAME OF THE SAMPLE. THIS IS SIGNAL SLOT
BLANK OUT REMAINING SLOTS IN FRAME
H0
GO TO NEXT SLOT FOR REPEAT OF DECISION PROCESS
IF LAST SLOT STILL LEADS TO H0, CHOOSE AS SIGNAL SLOT THE CENTER ONE

The probability of correct decision within the frame is then seen to be the probability that signal envelope is above b and all preceding noise envelopes are below b, or that all envelopes are below b and signal is the center slot. Therefore,

$$P_2(\text{cor}) = \sum_{j=1}^{N} P_s(j) \cdot Q\left(\frac{A}{\sigma_n}, \frac{b}{\sigma_n}\right) \cdot \left[1 - e^{-\frac{b^2}{2\sigma_n^2}}\right]^{j-1}$$

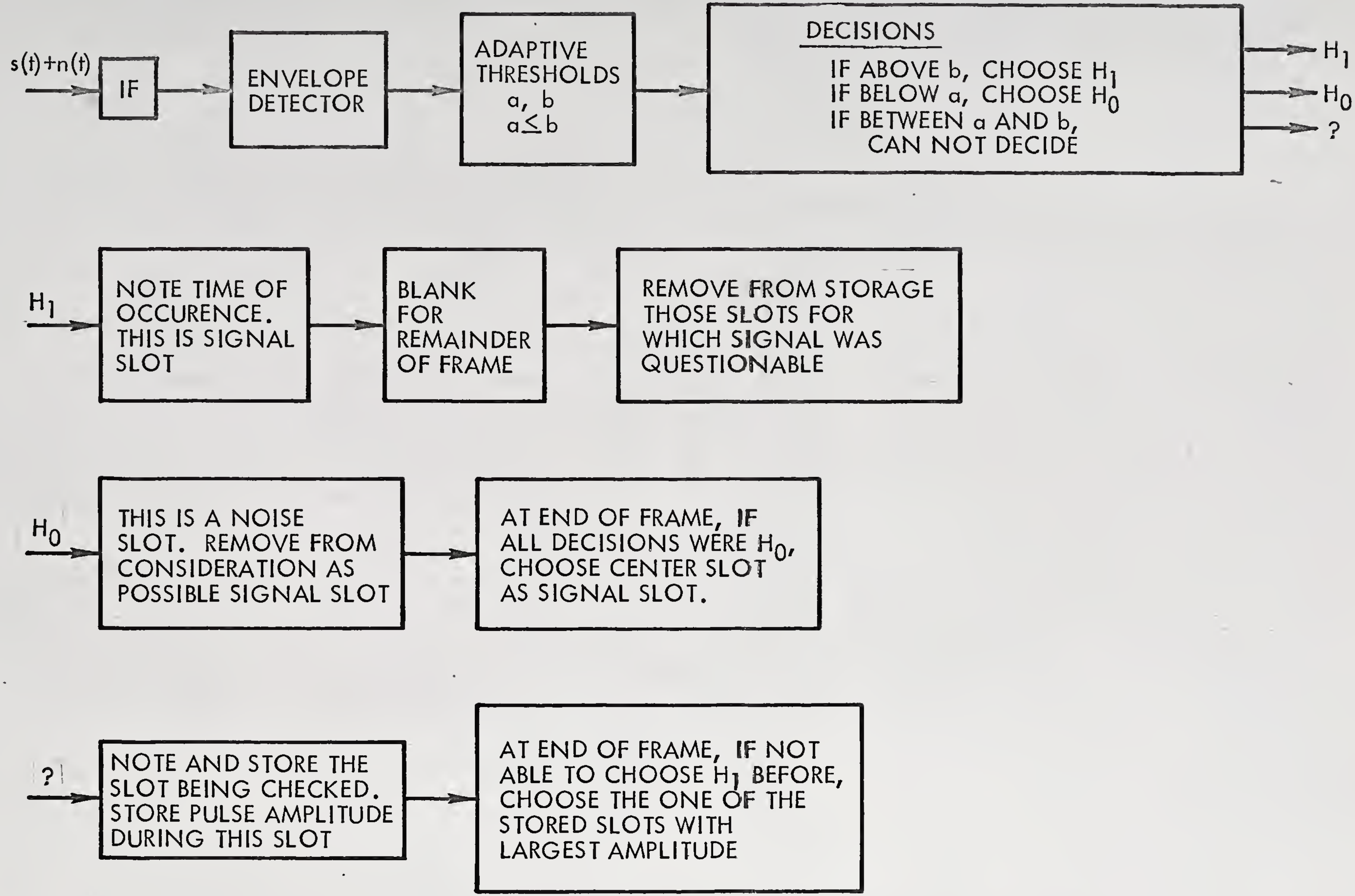
$$+ P_s\left(\frac{N}{2}\right) \cdot \left[1 - Q\left(\frac{A}{\sigma_n}, \frac{b}{\sigma_n}\right)\right] \cdot \left[1 - e^{-\frac{b^2}{2\sigma_n^2}}\right]^{N-1} \qquad (2.17)$$

QPPM, Double Threshold, Largest of s

This system is shown in Figure 3. As can be seen, it is very similar to the Speech Statistics system of Figure 1. In fact, they are identical up to the point where a hypothesis has been chosen. When H_1 is chosen for a slot, that slot is selected as the signal slot. The remainder of the frame is blanked, and all slots stored as "possible signal" slots are removed from storage. Note that it is not required that the signal slot be stored as a reference, because a new criterion is used for resolving the "possible signal" slots. Choice of H_0 for a slot removes that slot from any consideration as signal, except when all slots are H_0. In this case the center slot is used as signal. One additional storage requirement when a slot's envelope is in

Figure 3

Functional Diagram, QPPM, Double Threshold, Largest of s System


s(t)+n(t)
IF
ENVELOPE DETECTOR
ADAPTIVE THRESHOLDS a, b a≤b
DECISIONS
IF ABOVE b, CHOOSE H1
IF BELOW a, CHOOSE H0
IF BETWEEN a AND b, CAN NOT DECIDE
H1
H0
?
H1
NOTE TIME OF OCCURENCE. THIS IS SIGNAL SLOT
BLANK FOR REMAINDER OF FRAME
REMOVE FROM STORAGE THOSE SLOTS FOR WHICH SIGNAL WAS QUESTIONABLE
H0
THIS IS A NOISE SLOT. REMOVE FROM CONSIDERATION AS POSSIBLE SIGNAL SLOT
AT END OF FRAME, IF ALL DECISIONS WERE H0, CHOOSE CENTER SLOT AS SIGNAL SLOT.
?
NOTE AND STORE THE SLOT BEING CHECKED. STORE PULSE AMPLITUDE DURING THIS SLOT
AT END OF FRAME, IF NOT ABLE TO CHOOSE H1 BEFORE, CHOOSE THE ONE OF THE STORED SLOTS WITH LARGEST AMPLITUDE

the "possible signal" zone is that the envelope amplitude, as well as the slot position, is stored. At the end of the frame, if there has been no selection of H_1, the receiver selects as the signal slot the one of the s stored whose envelope is largest.

The probability of correct decision for this case is also made up of three parts.

$$\begin{aligned} P_3(\text{cor}) = \; & P_3 \text{ (correct, signal in "signal" zone)} \\ & + P_3 \text{ (correct, signal in "possible signal" zone)} \\ & + P_3 \text{ (correct, signal in "noise alone" zone)} \end{aligned} \tag{2.18}$$

The first and last of these are identical to the first and last terms of $P_1(\text{cor})$. The second probability is the probability that j of the noise envelopes, in the "possible signal" zone, are less than the signal envelope, also in the "possible signal" region, while the remaining noise envelopes are in the "noise alone" region. That is,

P_3 (correct, signal in "possible signal" zone)

$$= \sum_{j=0}^{N-1} \int_a^b p_2(R) \cdot \left[\int_a^R p_1(R_1) \cdot dR_1 \right]^j \cdot \left[\int_0^a p_1(R_2) \cdot dR_2 \right]^{N-1-j} \tag{2.19}$$

Now,

$$\int_0^a p_1(R_2) \cdot dR_2 = \int_0^a \frac{R_2}{\sigma_n^2} \cdot e^{-\frac{R_2^2}{2\sigma_n^2}} \cdot dR_2 = 1 - e^{-\frac{a^2}{2\sigma_n^2}} \tag{2.20}$$

and

$$\int_a^R P_1(R_1)\cdot dR_1 = e^{-\frac{a^2}{2\sigma_n^2}} - e^{-\frac{R^2}{2\sigma_n^2}} \tag{2.21}$$

By the binomial expansion

$$\left[e^{-\frac{a^2}{2\sigma_n^2}} - e^{-\frac{R^2}{2\sigma_n^2}}\right]^j = \sum_{\ell=0}^{j} \binom{j}{\ell}\cdot(-1)^\ell\cdot e^{-(j-\ell)\frac{a^2}{2\sigma_n^2}}\cdot e^{-\ell\frac{R^2}{2\sigma_n^2}} \tag{2.22}$$

It follows that

P_3 (correct, signal in "possible signal" zone)

$$= \sum_{j=0}^{N-1}\left[1-e^{-\frac{a^2}{2\sigma_n^2}}\right]^{N-1-j}\cdot\int_a^b \frac{R}{\sigma_n^2}\cdot e^{-\frac{R^2+A^2}{2\sigma_n^2}}\cdot I_o\left(\frac{AR}{\sigma_n^2}\right)\cdot\sum_{\ell=0}^{j}\binom{j}{\ell}\cdot(-1)^\ell\cdot e^{-(j-\ell)\frac{a^2}{2\sigma_n^2}}\cdot e^{-\ell\frac{R^2}{2\sigma_n^2}}\cdot dR$$

$$= \sum_{j=0}^{N-1}\left[1-e^{-\frac{a^2}{2\sigma_n^2}}\right]^{N-1-j}\cdot\sum_{\ell=0}^{j}\binom{j}{\ell}\cdot(-1)^\ell\cdot e^{-(j-\ell)\frac{a^2}{2\sigma_n^2}}\cdot \int_a^b \frac{R}{\sigma_n^2}\cdot e^{-\frac{R^2+A^2}{2\sigma_n^2}}\cdot I_o\left(\frac{AR}{\sigma_n^2}\right)\cdot e^{-\ell\frac{R^2}{2\sigma_n^2}}\cdot dR \tag{2.23}$$

If now we let $\sqrt{\ell+1}\cdot R$ be replaced by x, the integral becomes

$$\int_{a\sqrt{\ell+1}}^{b\sqrt{\ell+1}} \frac{x}{\sigma_n^2\sqrt{\ell+1}}\cdot e^{-\frac{x^2+A^2}{2\sigma_n^2}}\cdot I_o\left[\frac{Ax}{\sqrt{\ell+1}\cdot\sigma_n^2}\right]\cdot\frac{dx}{\sqrt{\ell+1}}$$

$$= \frac{e^{-\frac{A^2}{2\sigma_n^2}}\cdot e^{+\frac{A^2}{2(\ell+1)\sigma_n^2}}}{\ell+1}\cdot\int_{a\sqrt{\ell+1}}^{b\sqrt{\ell+1}} \frac{x}{\sigma_n^2}\cdot e^{-\frac{x^2+\frac{A^2}{\ell+1}}{2\sigma_n^2}}\cdot I_o\left(\frac{\frac{A}{\sqrt{\ell+1}}\cdot x}{\sigma_n^2}\right)\cdot dx$$

$$= \frac{e^{-\left(\frac{\ell}{\ell+1}\right)\frac{A^2}{2\sigma_n^2}}}{\ell+1}\cdot\left\{Q\left(\frac{A}{\sigma_n\sqrt{\ell+1}},\frac{a\sqrt{\ell+1}}{\sigma_n}\right)-Q\left(\frac{A}{\sigma_n\sqrt{\ell+1}},\frac{b\sqrt{\ell+1}}{\sigma_n}\right)\right\} \tag{2.24}$$

The final expression is

$$P_3(cor) = Q\left(\frac{A}{\sigma_n},\frac{b}{\sigma_n}\right)\cdot\sum_{j=1}^{N} P_s(j)\cdot\left[1-e^{-\frac{b^2}{2\sigma_n^2}}\right]^{j-1}$$

$$+\sum_{j=0}^{N-1}\left[1-e^{-\frac{a^2}{2\sigma_n^2}}\right]^{N-1-j}\cdot\sum_{\ell=0}^{j}\binom{j}{\ell}\cdot(-1)^{\ell}\cdot e^{-(j-\ell)\frac{a^2}{2\sigma_n^2}}\cdot\frac{e^{-\left(\frac{\ell}{\ell+1}\right)\frac{A^2}{2\sigma_n^2}}}{\ell+1}\cdot$$

$$\cdot\left\{Q\left(\frac{A}{\sigma_n\sqrt{\ell+1}},\frac{a\sqrt{\ell+1}}{\sigma_n}\right)-Q\left(\frac{A}{\sigma_n\sqrt{\ell+1}},\frac{b\sqrt{\ell+1}}{\sigma_n}\right)\right\}$$

$$+P_s\left(\frac{N}{2}\right)\cdot\left[1-Q\left(\frac{A}{\sigma_n},\frac{a}{\sigma_n}\right)\right]\cdot\left[1-e^{-\frac{a^2}{2\sigma_n^2}}\right]^{N-1} \qquad (2.25)$$

The appropriate expressions for this system for $\overline{K}$, $\overline{s}$, and σ_s^2 are identical to those derived previously and given in equations 2.11, 2.15, and 2.16, respectively.

QPPM, Largest of N

This system is functionally described in Figure 4. The receiver stores the N envelopes received during a frame. At the end of the frame it selects as the signal slot the one whose envelope is largest.

A correct decision is made by the receiver whenever all of the noise envelopes are below the signal envelope. This is represented by

Figure 4

Functional Diagram, QPPM, Largest of N System


s(t)+n(t)
IF
ENVELOPE DETECTOR
STORE AMPLITUDE FOR EACH OF N SLOTS
CHOOSE AS THE SIGNAL SLOT THE LARGEST OF THE N

$$P_4(\text{cor}) = \int_0^\infty P_2(R)\cdot\left[\int_0^R P_1(R_1)\cdot dR_1\right]^{N-1}\cdot dR$$

$$= \int_0^\infty \frac{R}{\sigma_n^2}\cdot e^{-\frac{R^2+A^2}{2\sigma_n^2}}\cdot I_0\left(\frac{AR}{\sigma_n^2}\right)\cdot\left[\int_0^R \frac{R_1}{\sigma_n^2}\cdot e^{-\frac{R_1^2}{2\sigma_n^2}}\cdot dR_1\right]^{N-1}\cdot dR$$

$$= \int_0^\infty \frac{R}{\sigma_n^2}\cdot e^{-\frac{R^2+A^2}{2\sigma_n^2}}\cdot I_0\left(\frac{AR}{\sigma_n^2}\right)\cdot\left[1 - e^{-\frac{R^2}{2\sigma_n^2}}\right]^{N-1}\cdot dR$$

$$= \int_0^\infty \frac{R}{\sigma_n^2}\cdot e^{-\frac{R^2+A^2}{2\sigma_n^2}}\cdot I_0\left(\frac{AR}{\sigma_n^2}\right)\cdot\sum_{K=0}^{N-1}\binom{N-1}{K}\cdot(-1)^K\cdot e^{-\frac{KR^2}{2\sigma_n^2}}\cdot dR$$

$$= \sum_{K=0}^{N-1}(-1)^K\cdot\binom{N-1}{K}\cdot\int_0^\infty \frac{R}{\sigma_n^2}\cdot e^{-\frac{R^2(K+1)+A^2}{2\sigma_n^2}}\cdot I_0\left(\frac{AR}{\sigma_n^2}\right)\cdot dR \qquad (2.26)$$

Replacing $R\sqrt{K+1}$ by x, the integral becomes

$$\int_0^\infty \frac{x}{\sigma_n^2\sqrt{K+1}} \cdot e^{-\frac{x^2+A^2}{2\sigma_n^2}} \cdot I_0\left(\frac{Ax}{\sigma_n^2\sqrt{K+1}}\right) \cdot \frac{dx}{\sqrt{K+1}}$$

$$= \frac{e^{-\frac{A^2}{2\sigma_n^2}} \cdot e^{+\frac{A^2}{2(K+1)\sigma_n^2}}}{K+1} \cdot \int_0^\infty \frac{x}{\sigma_n^2} \cdot e^{-\frac{x^2+\frac{A^2}{K+1}}{2\sigma_n^2}} \cdot I_0\left(\frac{Ax}{\sigma_n^2\sqrt{K+1}}\right) \cdot dx$$

$$= \frac{e^{-\left(\frac{K}{K+1}\right)\frac{A^2}{2\sigma_n^2}}}{K+1} \cdot Q\left(\frac{A}{\sigma_n\sqrt{K+1}}, 0\right)$$

$$= \frac{e^{-\left(\frac{K}{K+1}\right)\frac{A^2}{2\sigma_n^2}}}{K+1} \tag{2.27}$$

Thus, we have

$$P_4(\text{cor}) = \sum_{K=0}^{N-1} (-1)^K \cdot \binom{N-1}{K} \cdot \left(\frac{1}{K+1}\right) \cdot e^{-\left(\frac{K}{K+1}\right)\frac{A^2}{2\sigma_n^2}} \tag{2.28}$$

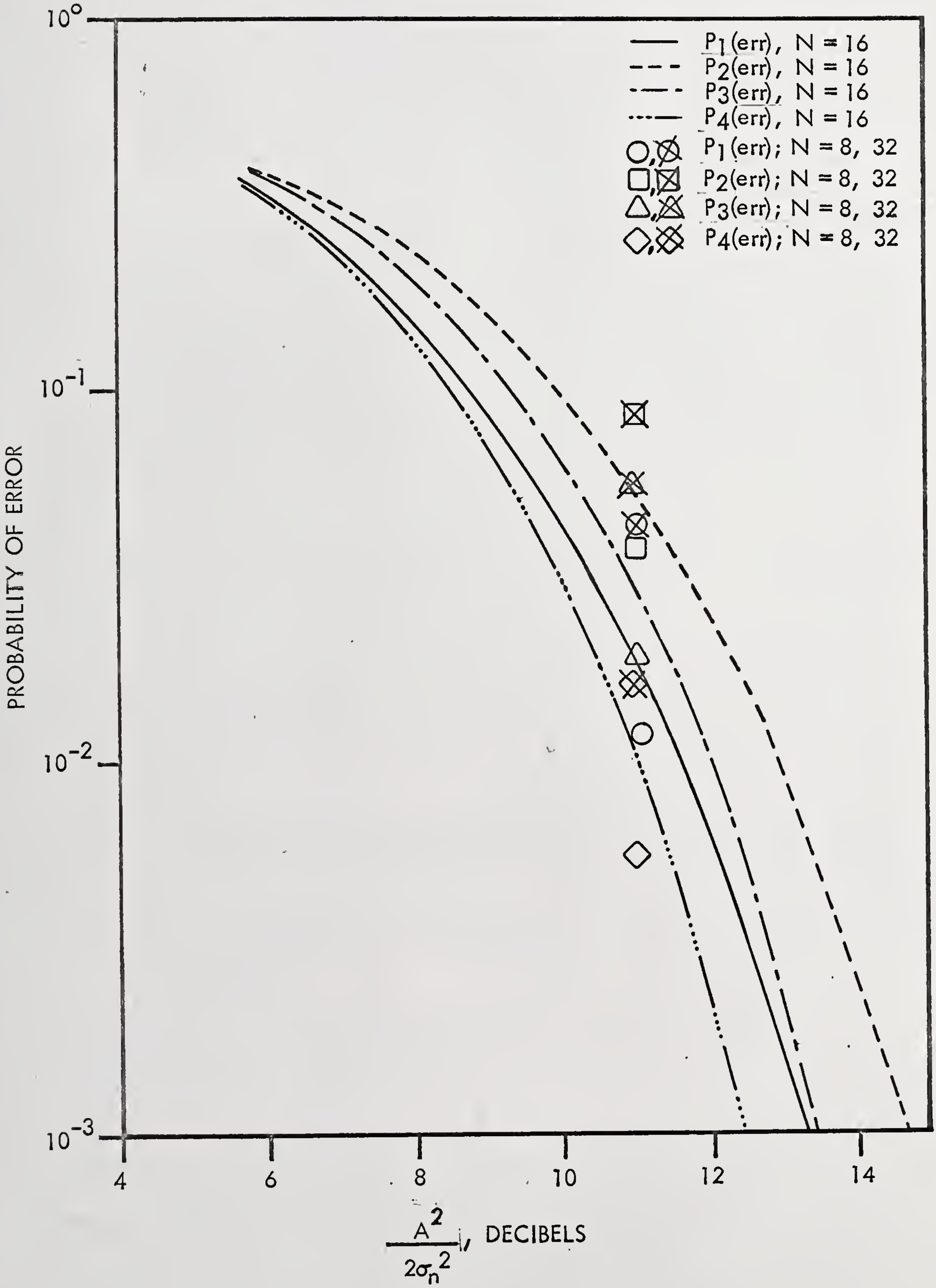
Discussion of Results

The primary result of this chapter is depicted in Figure 5. This allows a direct comparison to be made for the systems discussed. The criterion of comparison is minimum probability of error for a given signal-to-noise input ratio. Thus, the curves in this figure are based on optimum threshold settings for the given noise conditions.

It is interesting to note that the sequential system based on speech statistics is approximately 1.5 decibels better than the conventional, single threshold system, throughout the range of interest of signal-to-noise ratios. The sequential system based on a largest of s selection is about 1.0 decibel better than the conventional. Slightly better than these is the system based on selecting the largest amplitude out of N possible choices. This is a more complex system, however, and therefore has its advantage offset to some degree. It will be shown that the average value of s, with optimum thresholds, is about 1 or 2. Therefore, it would be relatively simple to select the most probable signal slot out of s under these conditions. Thus, the sequential systems offer a compromise between the simplicity of the conventional technique and the higher theoretical performance of the largest of N system. The differences noted above would appear to be insignificant under most circumstances. However, there are, foreseeably,

FIGURE 5

Probability of Error for Input Signal-to-Noise Ratios Between 4.0 and 14.0 Decibels, Without Impulse Noise



10^0
10^{-1}
10^{-2}
10^{-3}
PROBABILITY OF ERROR
P_1(err), N = 16
P_2(err), N = 16
P_3(err), N = 16
P_4(err), N = 16
P_1(err); N = 8, 32
P_2(err); N = 8, 32
P_3(err); N = 8, 32
P_4(err); N = 8, 32
4
6
8
10
12
14
$\frac{A^2}{2\sigma_n^2}$, DECIBELS

situations in which use of the sequential systems would be advantageous.

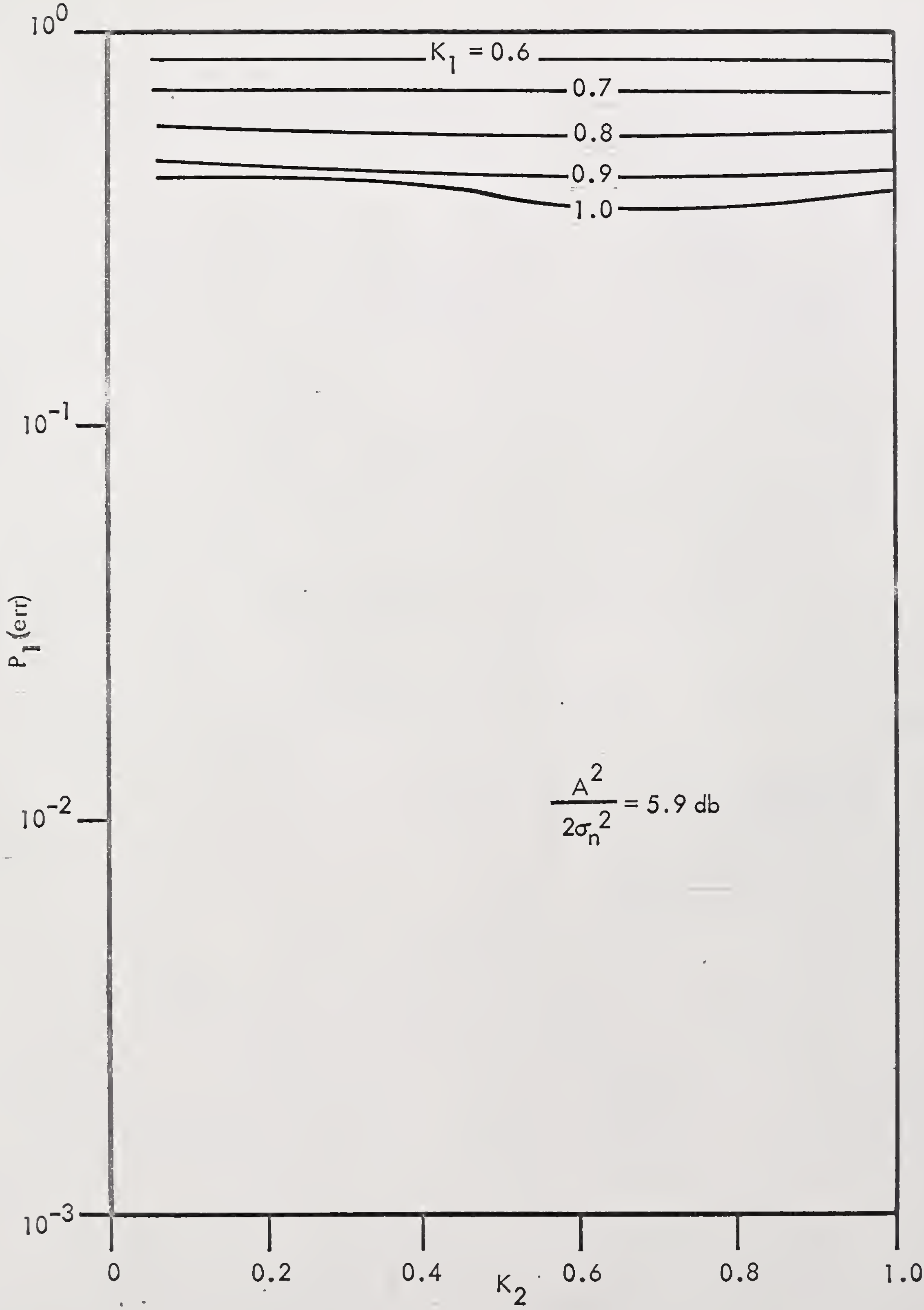
The calculations for the curves of Figure 5 were based on a value of 16 for N. To obtain an indication of what changes might result for other values, single points were obtained for each system for values of 8 and 32 for N. The relative changes between systems in the probabilities of error are about the same amount. As a result, the conclusions reached above are unchanged.

The results of a comprehensive parametric study for the speech statistics system are given in Figures 6, 7, 8, 9, and 10. These are for signal-to-noise ratios of 5.9, 9.0, 11.0, 12.1, and 13.0 decibels, respectively. Within the range of values shown, the effect on the probability of error is seen for any combination of threshold settings. The optimum thresholds can be obtained from these curves. As can be seen, the optimum thresholds are not particularly critical, in the sense that variation in the vicinity of the optimum point does not increase the probability of error by a large amount.

The variation of the optimum thresholds with signal-to-noise ratio is given in Figure 11. For the practical range of interest K_2 has the approximately constant value of 0.7, with K_1 slightly greater. For a reasonably narrow range of signal-to-noise ratios, K_1 and K_2 could be fixed

FIGURE 6

Probability of Error for Double Threshold, Speech Statistics System for K_2 Between 0 and 1.0, and for Signal-to Noise Ratio 5.9 Decibels



10^0
10^{-1}
10^{-2}
10^{-3}
$P_1(err)$
$K_1 = 0.6$
0.7
0.8
0.9
1.0
$\frac{A^2}{2\sigma_n^2} = 5.9$ db
0
0.2
0.4
0.6
0.8
1.0
K_2

FIGURE 7

Probability of Error for Double Threshold, Speech Statistics System for K_2 Between 0 and 1.0, and for Signal-to-Noise Ratio 9.0 Decibels

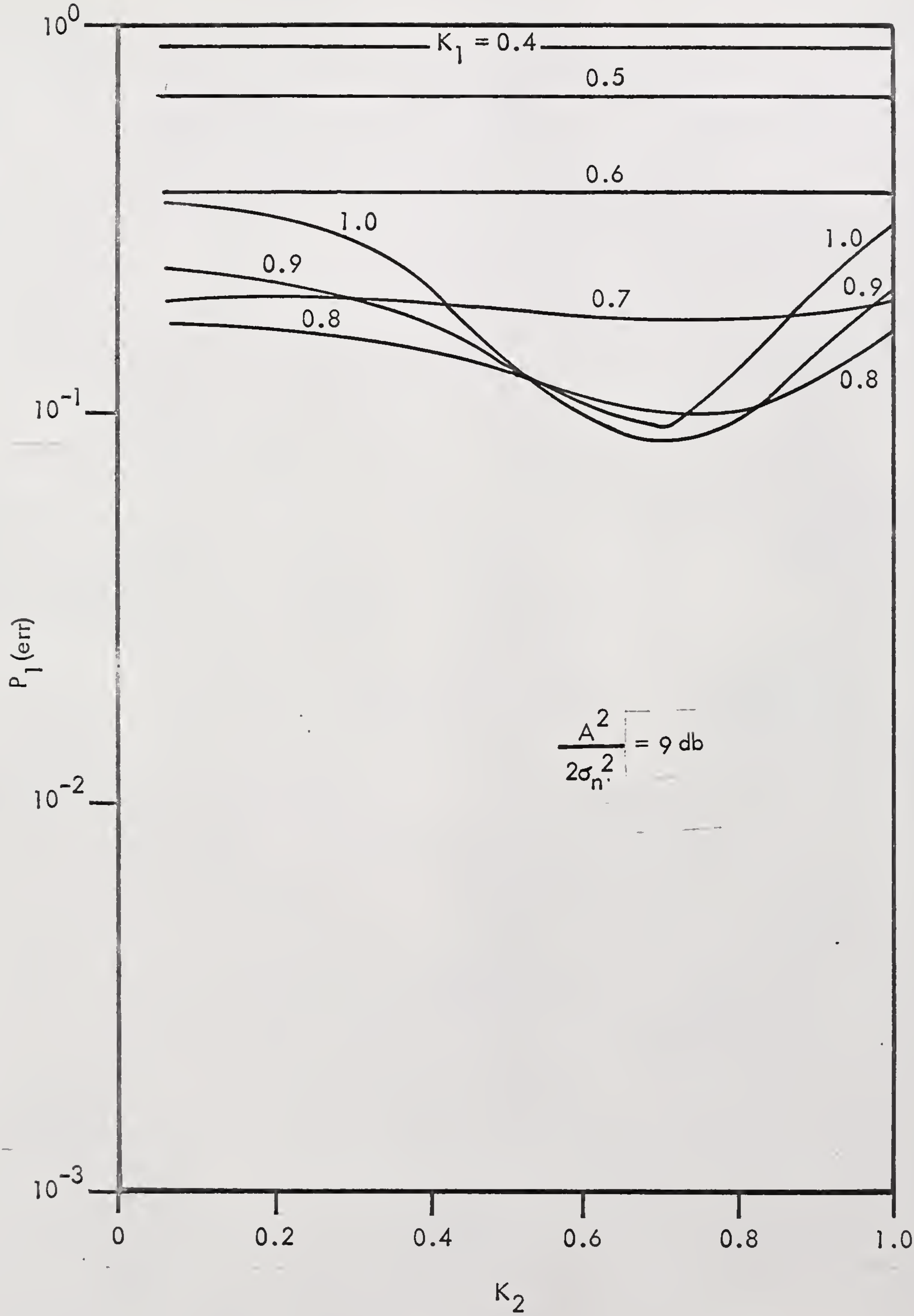
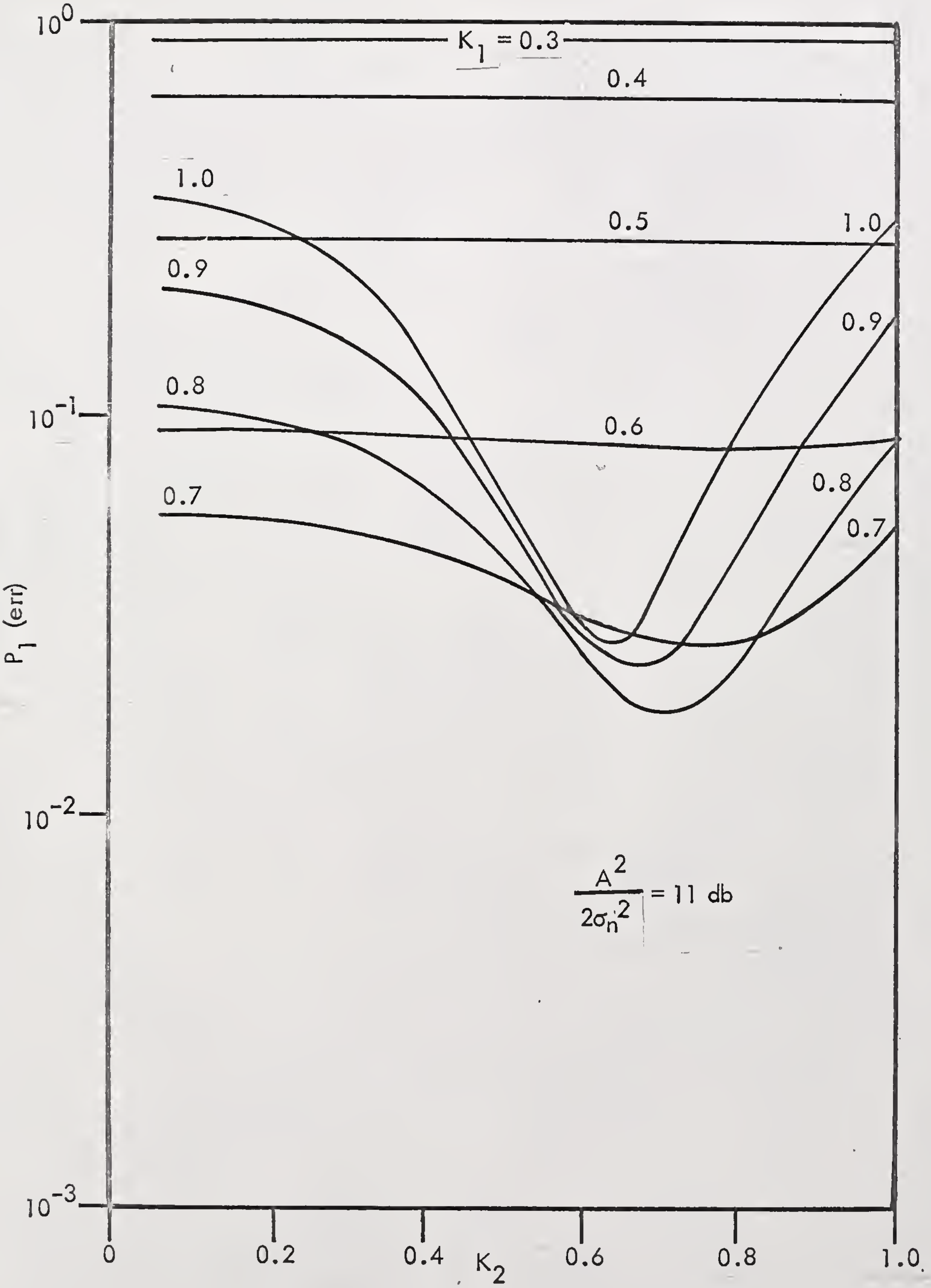

$K_1 = 0.4$
0.5
0.6
1.0
0.9
0.7
0.8
1.0
0.9
0.8
10^0
10^{-1}
10^{-2}
10^{-3}
P_1 (err)
$\frac{A^2}{2\sigma_n^2} = 9$ db
0
0.2
0.4
0.6
0.8
1.0
K_2

FIGURE 8

Probability of Error for Double Threshold, Speech Statistics System for K_2 Between 0 and 1.0, and for Signal-to-Noise Ratio 11.0 Decibels



10^0
10^{-1}
10^{-2}
10^{-3}
P_1 (err)
$K_1 = 0.3$
0.4
0.5
0.6
0.7
0.8
0.9
1.0
$\frac{A^2}{2\sigma_n^2} = 11$ db
0
0.2
0.4
0.6
0.8
1.0
K_2

FIGURE 9

Probability of Error for Double Threshold, Speech Statistics System for K_2 Between 0 and 1.0, and for Signal-to-Noise Ratio 12.1 Decibels

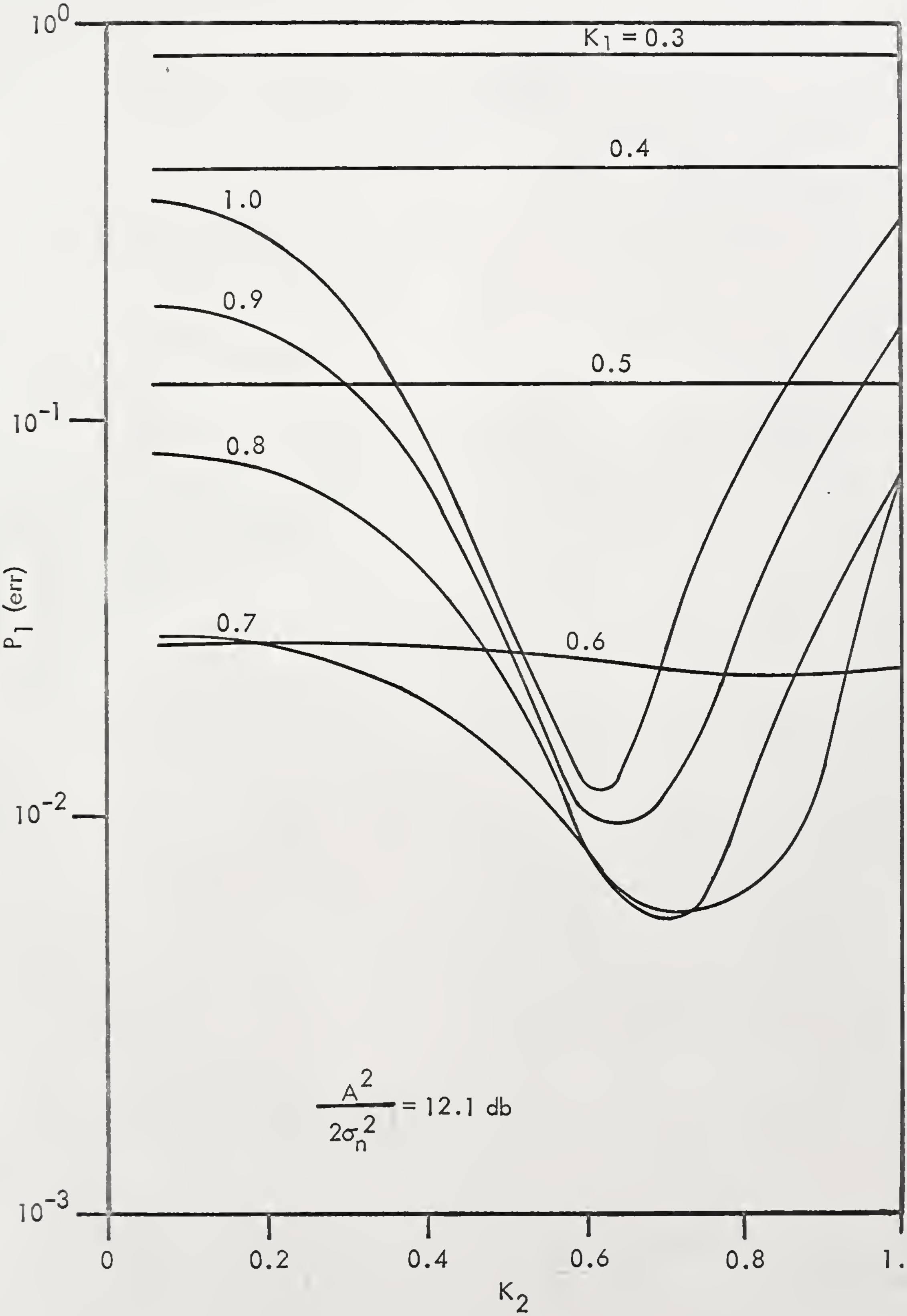
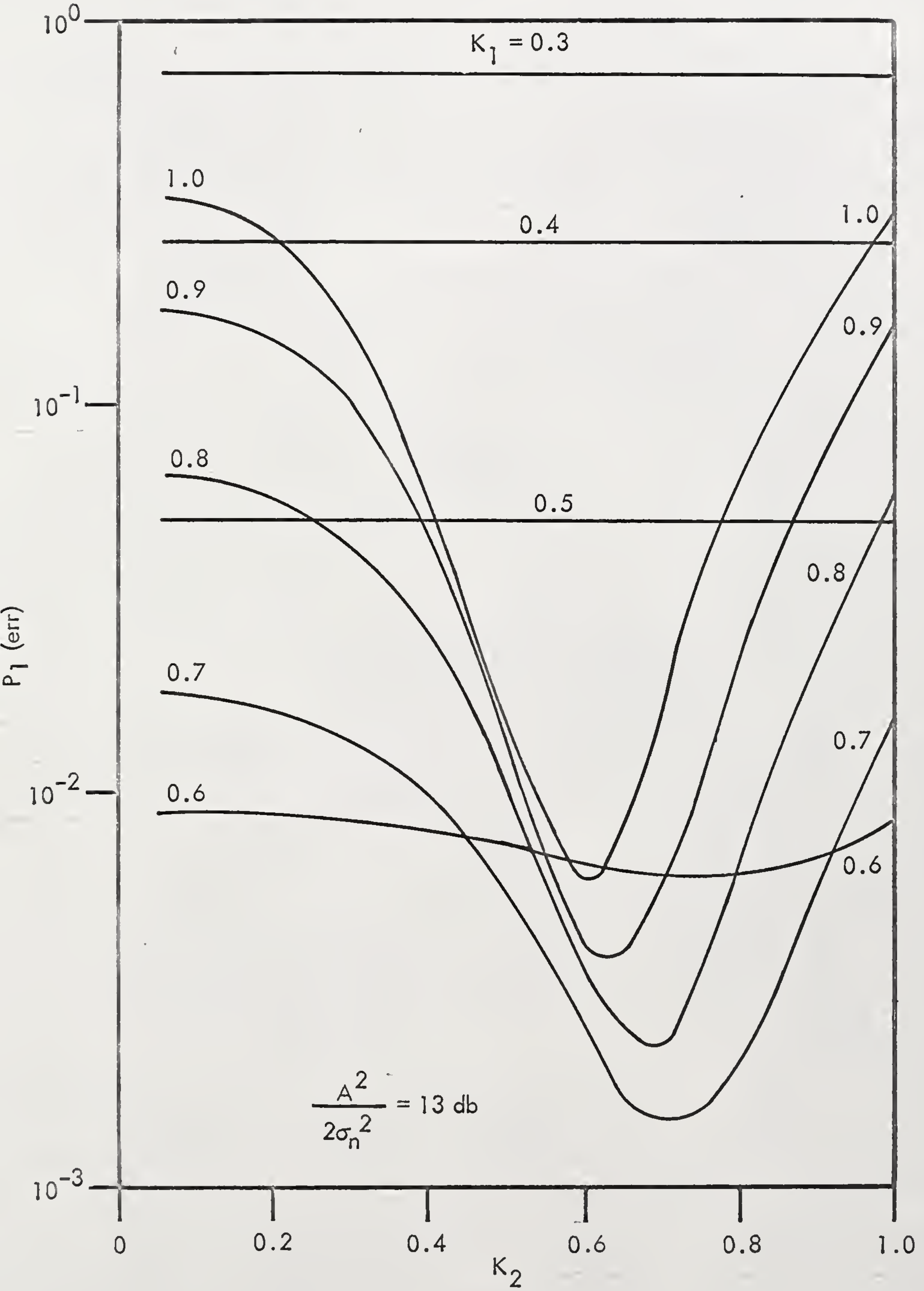

$K_1 = 0.3$
0.4
1.0
0.9
0.5
0.8
0.7
0.6
$\frac{A^2}{2\sigma_n^2} = 12.1$ db
10^0
10^{-1}
10^{-2}
10^{-3}
P_1 (err)
0
0.2
0.4
0.6
0.8
1.0
K_2

FIGURE 10

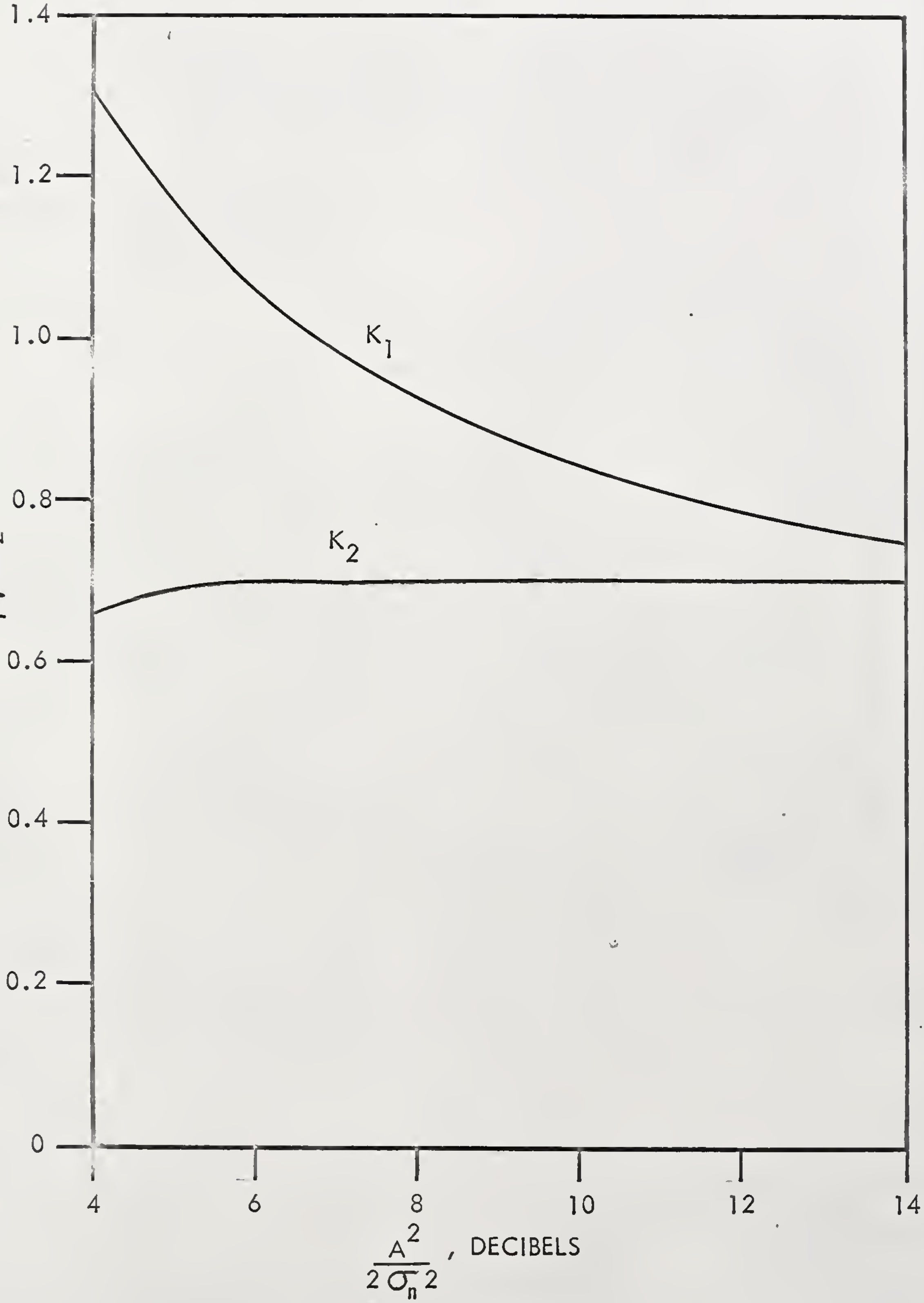
Probability of Error for Double Threshold, Speech Statistics System for K_2 Between 0 and 1.0, and for Signal-to-Noise Ratio 13.0 Decibels



10^0
10^{-1}
10^{-2}
10^{-3}
P_1 (err)
$K_1 = 0.3$
1.0
0.4
0.9
0.8
0.5
0.7
0.6
1.0
0.9
0.8
0.7
0.6
$\frac{A^2}{2\sigma_n^2} = 13$ db
0
0.2
0.4
0.6
0.8
1.0
K_2

FIGURE 11

Values of K_1 and K_2 for Double Threshold, Speech Statistics System for Signal-to-Noise Ratios Between 4.0 and 14.0 Decibels, with Minimum Probability of Error


1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
K1
K2
4
6
8
10
12
14
$\frac{A^2}{2\sigma_n^2}$, DECIBELS

beforehand. The degradation would be slight from the variable threshold situation.

The other quantities of interest for the speech statistics system are $\bar{K}$, $\bar{s}$, and σ_s^2 under optimum threshold conditions. This information is given in Figure 12. For a particular signal-to-noise ratio, the difference between the length of a frame and the value of $\bar{K}$ gives the number of slots which perhaps could be used in some other manner; for example, in the transmission of non-real time data. The value of $\bar{s}$, along with σ_s^2, gives an indication of the complexity of that part of the receiver which chooses the one of the s possible slots as the signal slot. As $\bar{s}$ gets larger, the complexity increases. As is seen in Figure 12, $\bar{s}$ is between 1 and 2 for the range of interest for the optimum settings, with small variance. The indicated complexity is not great.

The parametric study of the conventional system is depicted in Figure 13. Here, of course, only one threshold is involved. The effect on probability of error is seen for a change in the threshold, for a wide range of signal-to-noise ratios.

The variation of the optimum threshold with signal-to-noise ratio is given in Figure 14. It is very close to the variation of K_1 shown in Figure 11 for the speech statistics system.

FIGURE 12

$\overline{K}$, $\overline{s}$, and σ_s^2 for Double Threshold, Speech Statistics System for Signal-to-Noise Ratios Between 4.0 and 14.0 Decibels, with Minimum Probability of Error

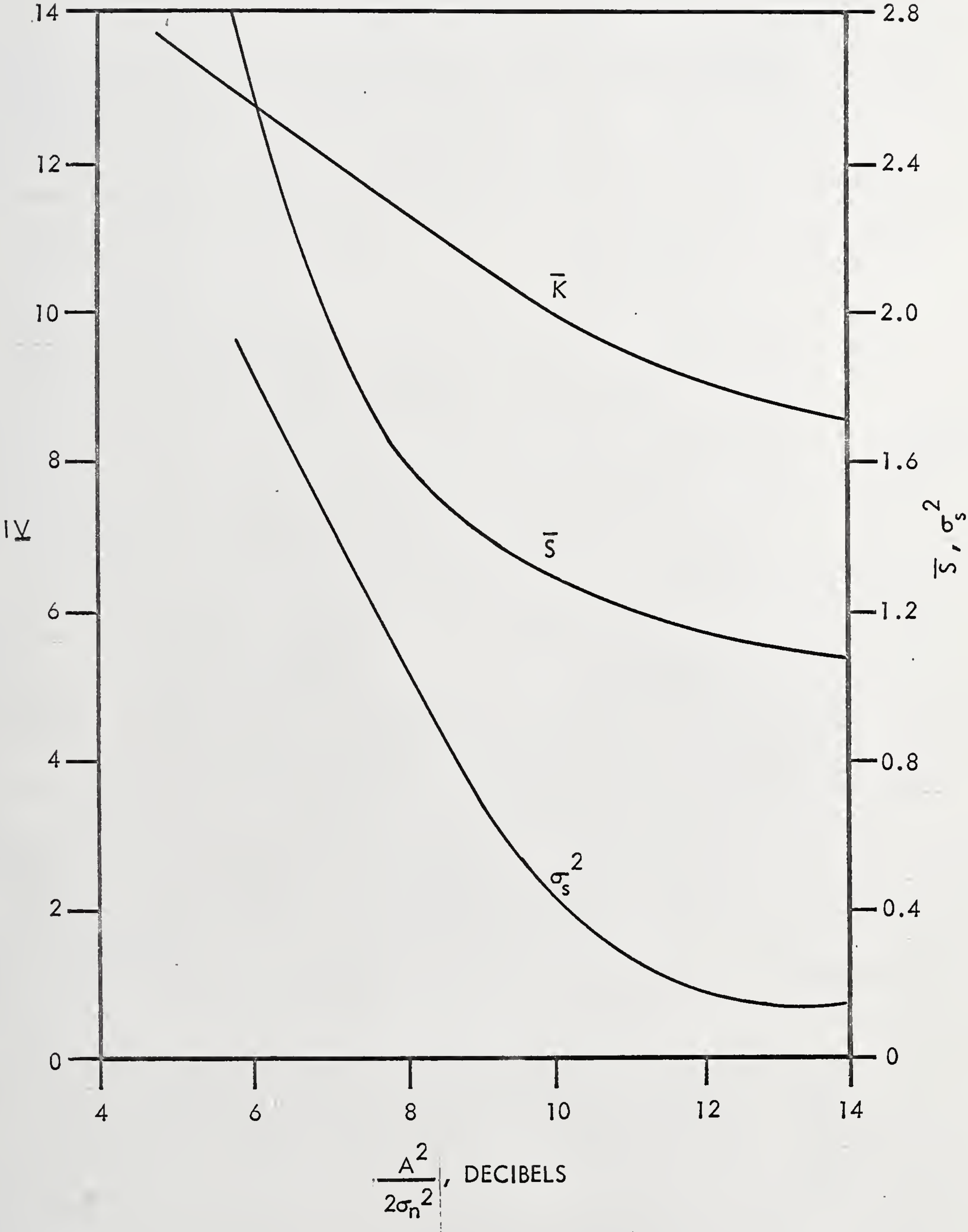

14
12
10
8
6
4
2
0
IV
2.8
2.4
2.0
1.6
1.2
0.8
0.4
0
$\bar{S}, \sigma_s^2$
$\bar{K}$
$\bar{S}$
σ_s^2
4
6
8
10
12
14
$\frac{A^2}{2\sigma_n^2}$, DECIBELS

FIGURE 13

Probability of Error for Single Threshold, Conventional System for K_1 Between 0 and 1.0, and for Signal-to-Noise Ratios Between 5.9 and 16.0 Decibels

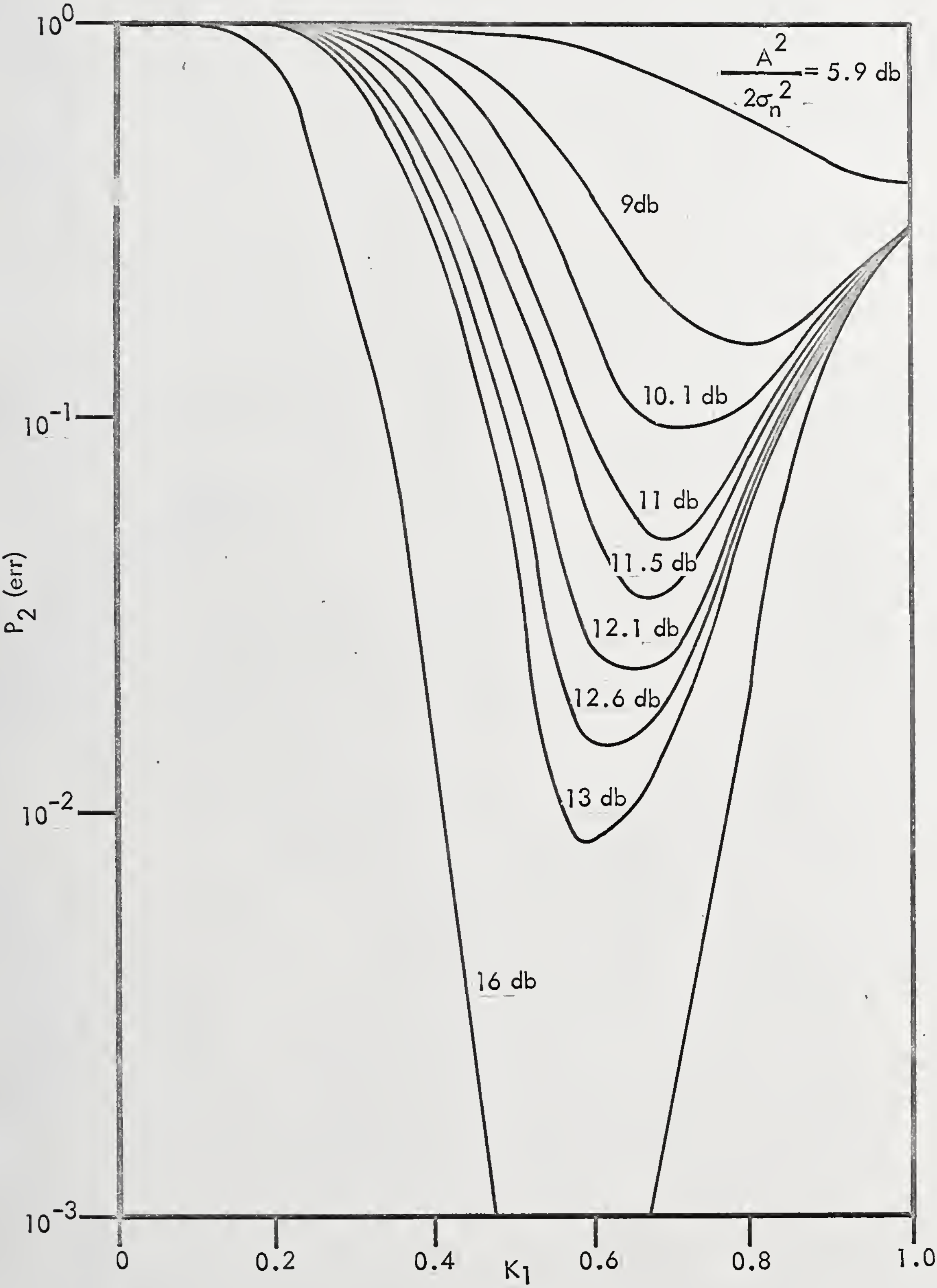
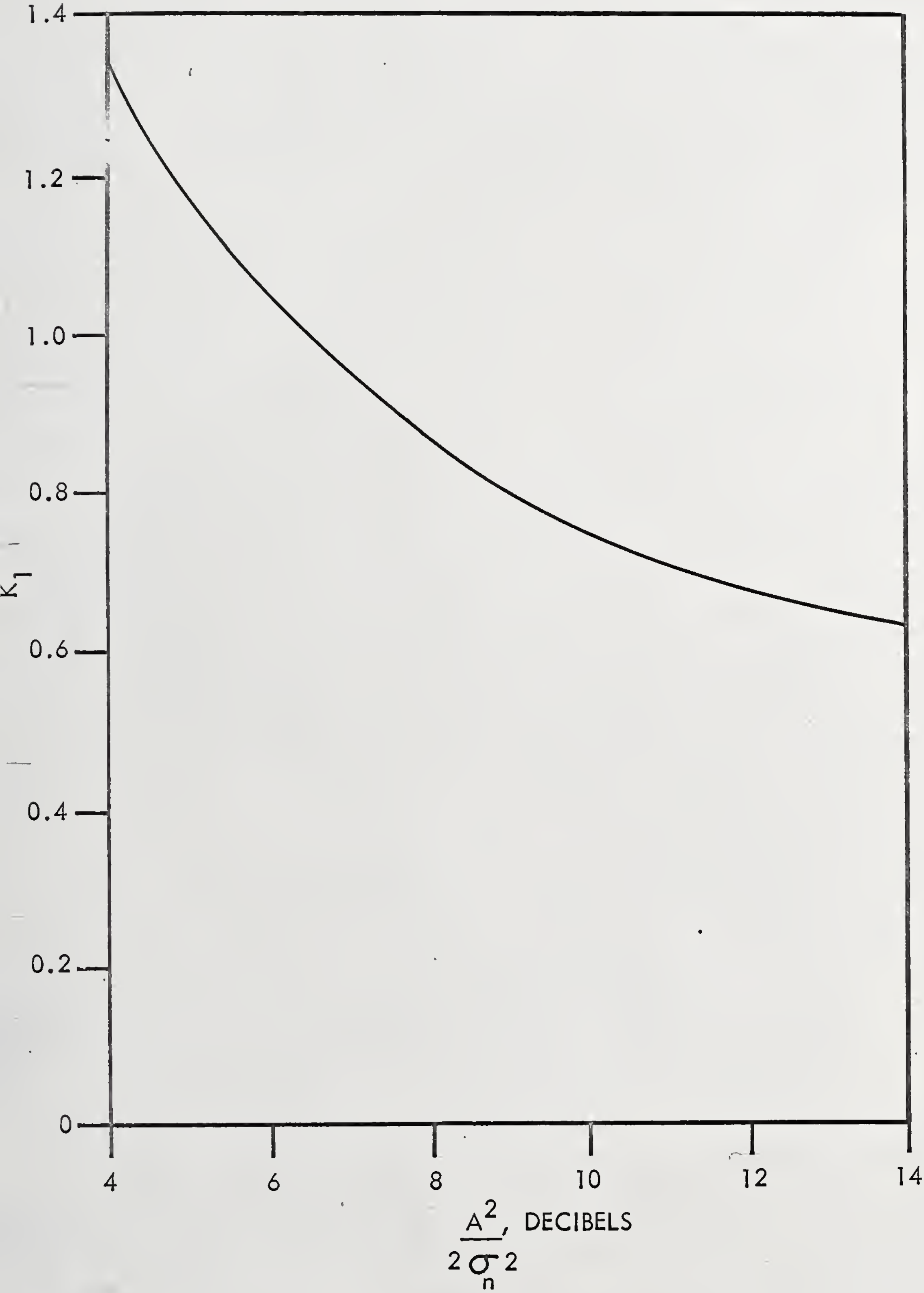

$\frac{A^2}{2\sigma_n^2}$ = 5.9 db
9db
10. 1 db
11 db
11.5 db
12.1 db
12.6 db
13 db
16 db
10^0
10^{-1}
10^{-2}
10^{-3}
P_2 (err)
0
0.2
0.4
0.6
0.8
1.0
K_1

FIGURE 14

Value of K_1 for Single Threshold, Conventional System for Signal-to-Noise Ratios Between 4.0 and 14.0 Decibels, with Minimum Probability of Error


1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
K_1
4
6
8
10
12
14
$\frac{A^2}{2\sigma_n^2}$, DECIBELS

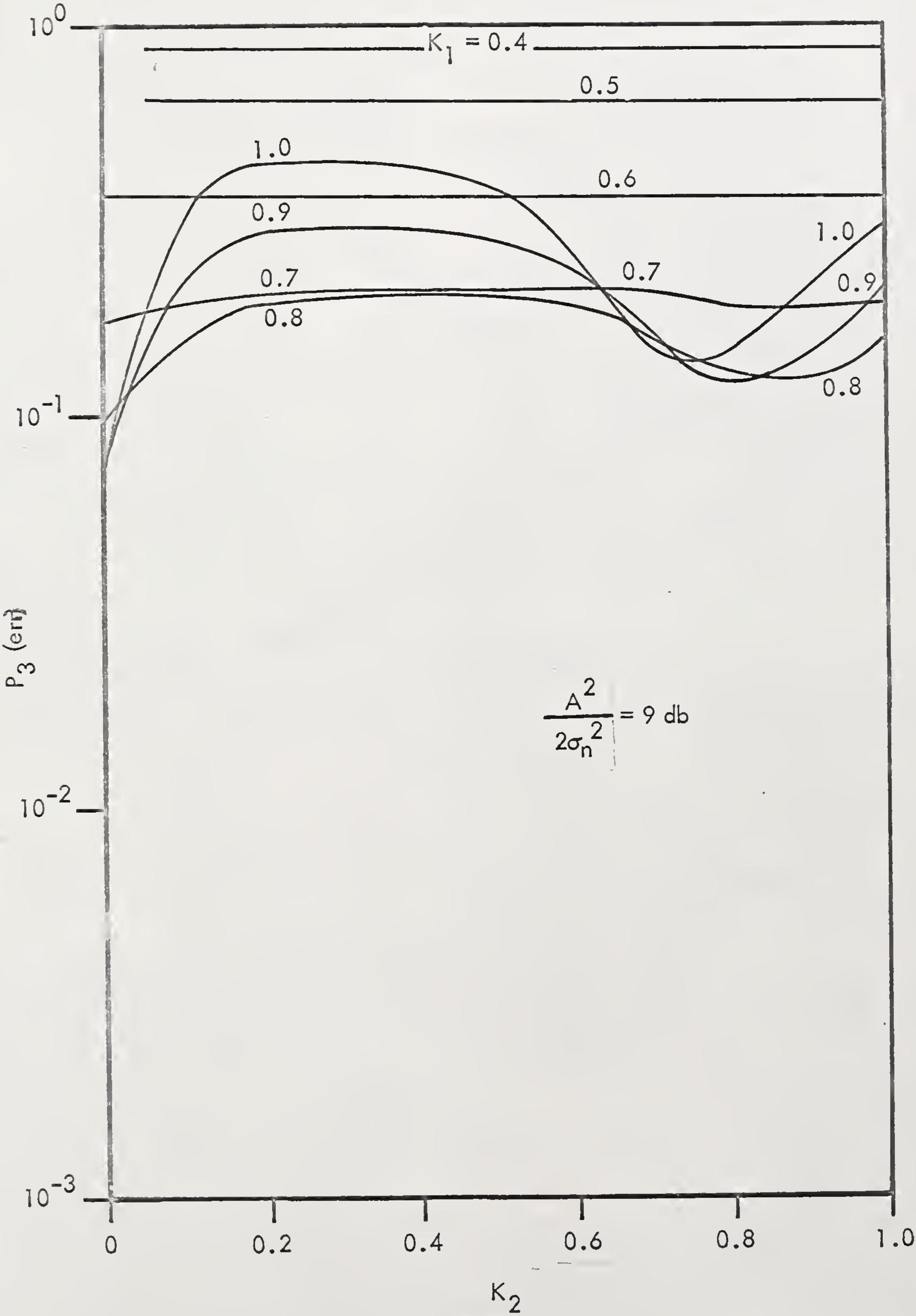
Similarly, a parametric study of the largest of s system was carried out, and the results plotted in Figures 15, 16, 17, and 18 for signal-to-noise ratios of 9.0, 11.0, 12.1, and 13.0 decibels, respectively. As before, the effect on probability of error of change in thresholds is easily seen. It is to be noted that the curves dip for values of 0 and about 0.7 for K_2. It is to be expected that probability of error would decrease as the upper threshold increases and the lower threshold approaches zero, since the largest of s system approaches the largest of N system. However, the system performance is almost as good if the optimum threshold is taken as the one for the second dip in the curve. This allows the value of $\bar{s}$ to be small, with a corresponding decrease in complexity. The variation of these threshold settings is shown in Figure 19. Again, these could be fixed beforehand for a reasonably narrow range of signal-to-noise ratios, with only small degradation in error performance.

For this system also, in Figure 20, the variations of $\bar{K}$, $\bar{s}$, and σ_s^2 are given. For the practical range of signal-to-noise ratios, the values are not significantly different from the speech statistics system.

This completes the report in this chapter of the study of the two sequential systems, the conventional system, and the largest of N system, for the case of no

FIGURE 15

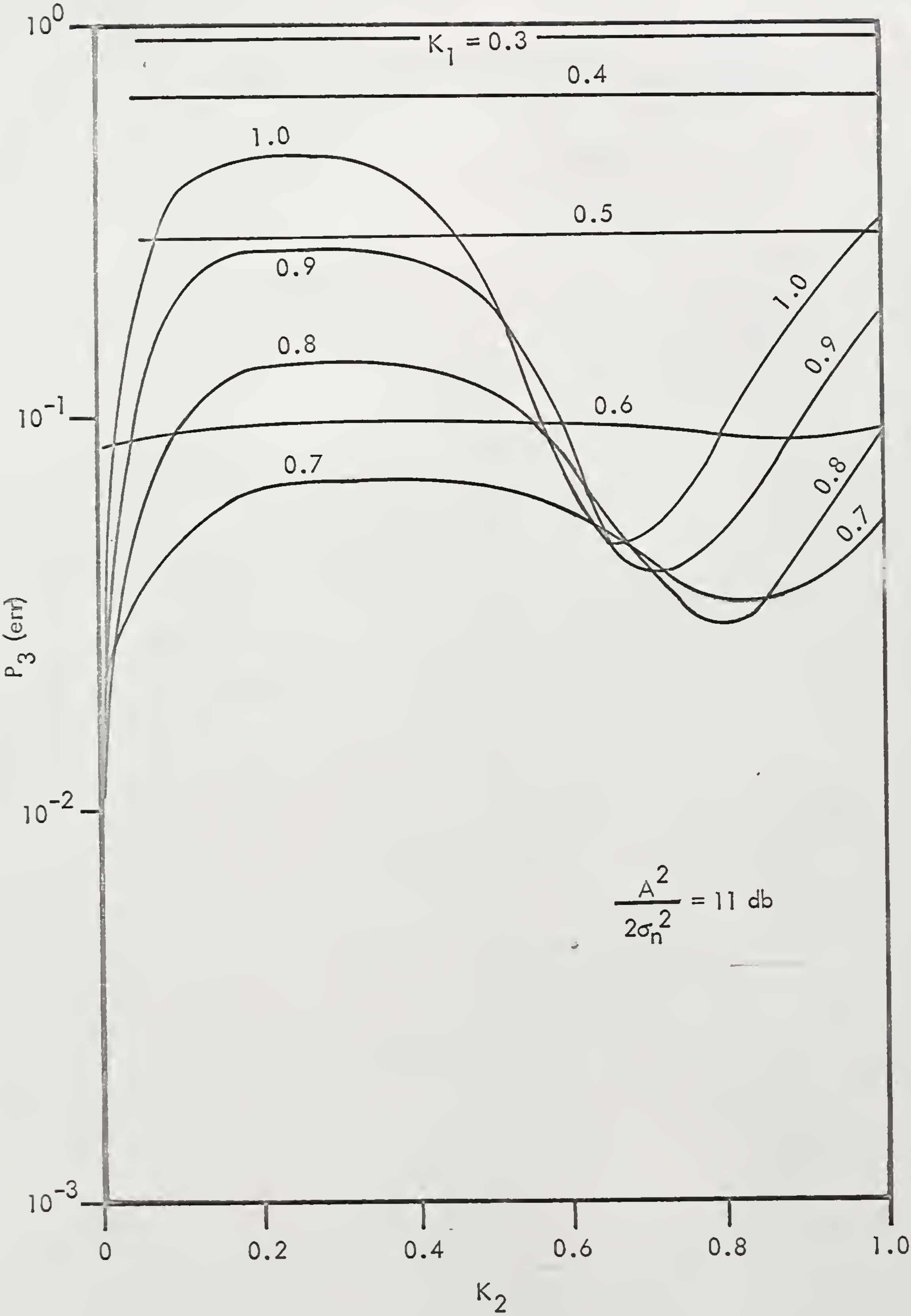
Probability of Error for Double Threshold, Largest of s System for K_2 Between 0 and 1.0, and for Signal-to-Noise Ratio 9.0 Decibels



10^0
10^{-1}
10^{-2}
10^{-3}
P_3 (err)
$K_1 = 0.4$
0.5
1.0
0.6
0.9
0.7
0.8
1.0
0.7
0.9
0.8
$\frac{A^2}{2\sigma_n^2} = 9$ db
0
0.2
0.4
0.6
0.8
1.0
K_2

FIGURE 16

Probability of Error for Double Threshold, Largest of s System for K_2 Between 0 and 1.0, and for Signal-to-Noise Ratio 11.0 Decibels



10^0
10^{-1}
10^{-2}
10^{-3}
P_3 (err)
$K_1 = 0.3$
0.4
1.0
0.5
0.9
0.8
0.6
0.7
1.0
0.9
0.8
0.7
$\frac{A^2}{2\sigma_n^2} = 11$ db
0
0.2
0.4
0.6
0.8
1.0
K_2

FIGURE 17

Probability of Error for Double Threshold, Largest of s System for K_2 Between 0 and 1.0, and for Signal-to-Noise Ratio 12.1 Decibels

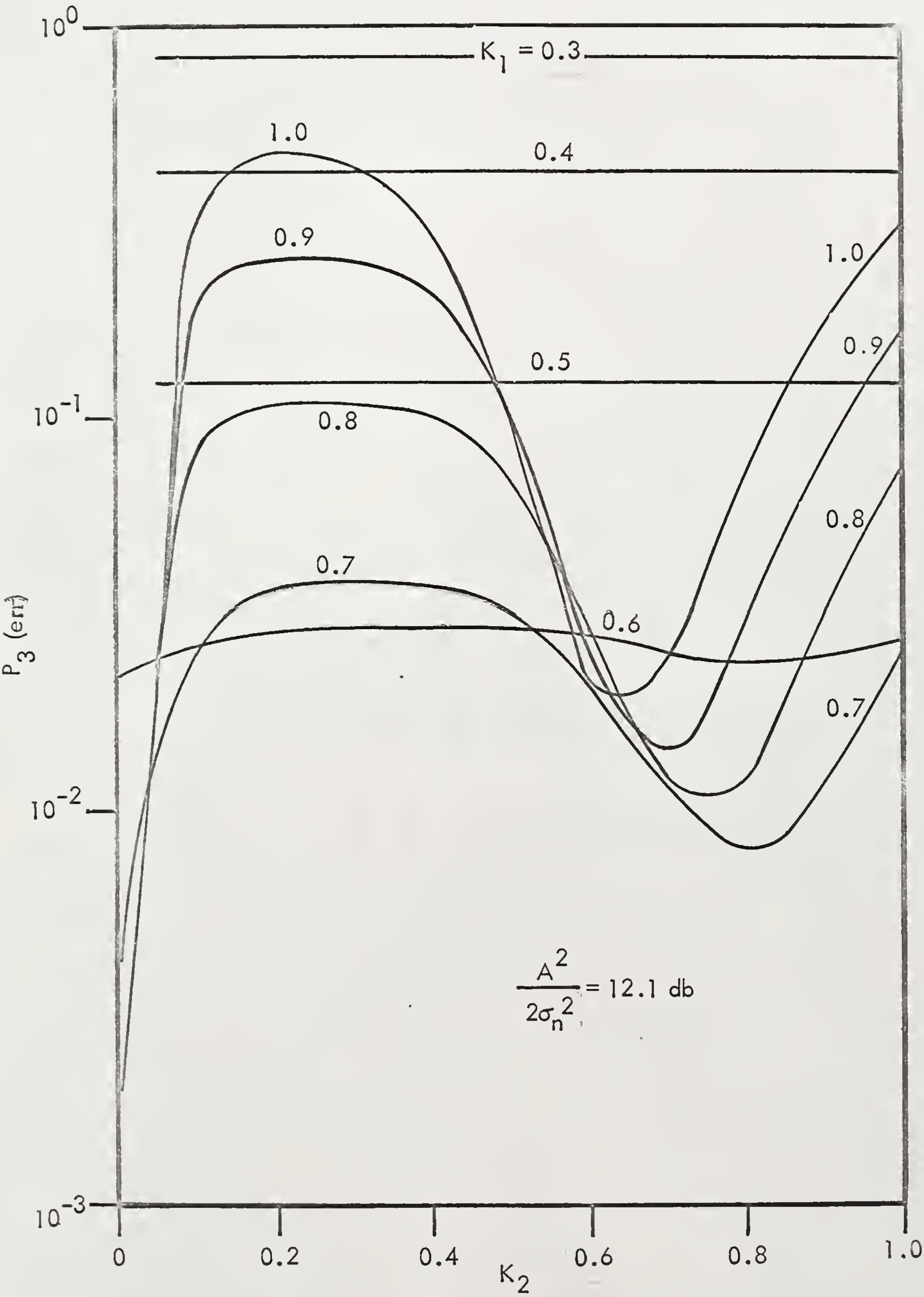

10^0
10^{-1}
10^{-2}
10^{-3}
P_3 (err)
$K_1 = 0.3$
1.0
0.4
0.9
1.0
0.5
0.9
0.8
0.8
0.7
0.6
0.7
$\frac{A^2}{2\sigma_n^2} = 12.1$ db
0
0.2
0.4
0.6
0.8
1.0
K_2

FIGURE 18

Probability of Error for Double Threshold, Largest of s System for K_2 Between 0 and 1.0, and for Signal-to-Noise Ratio 13.0 Decibels

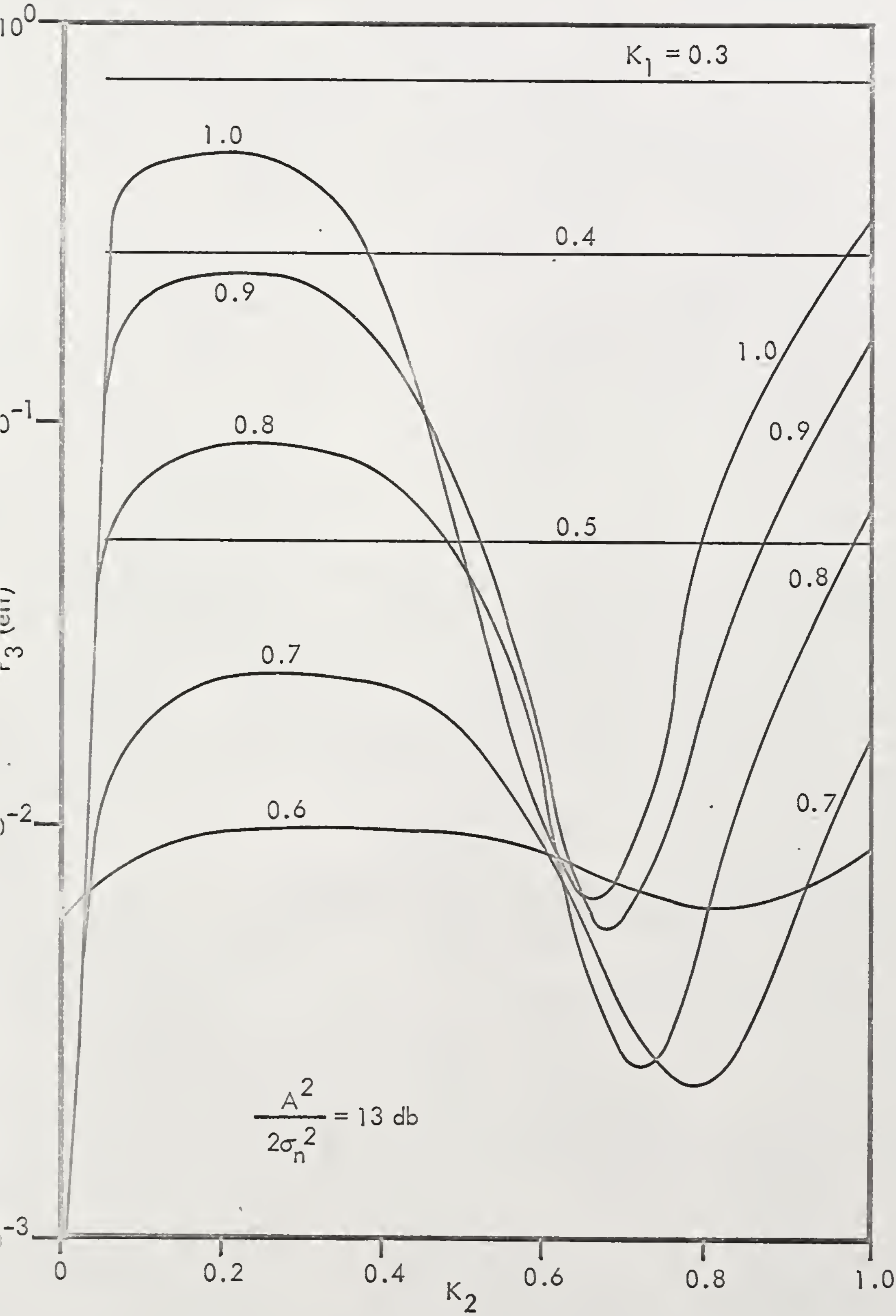

10^0
10^{-1}
10^{-2}
10^{-3}
P_3 (err)
$K_1 = 0.3$
1.0
0.4
0.9
0.8
0.5
0.7
0.6
1.0
0.9
0.8
0.7
$\frac{A^2}{2\sigma_n^2} = 13$ db
0
0.2
0.4
0.6
0.8
1.0
K_2

FIGURE 19

Values of K_1 and K_2 for Double Threshold, Largest of s System for Signal-to-Noise Ratios Between 4.0 and 14.0 Decibels, with Minimum Probability of Error

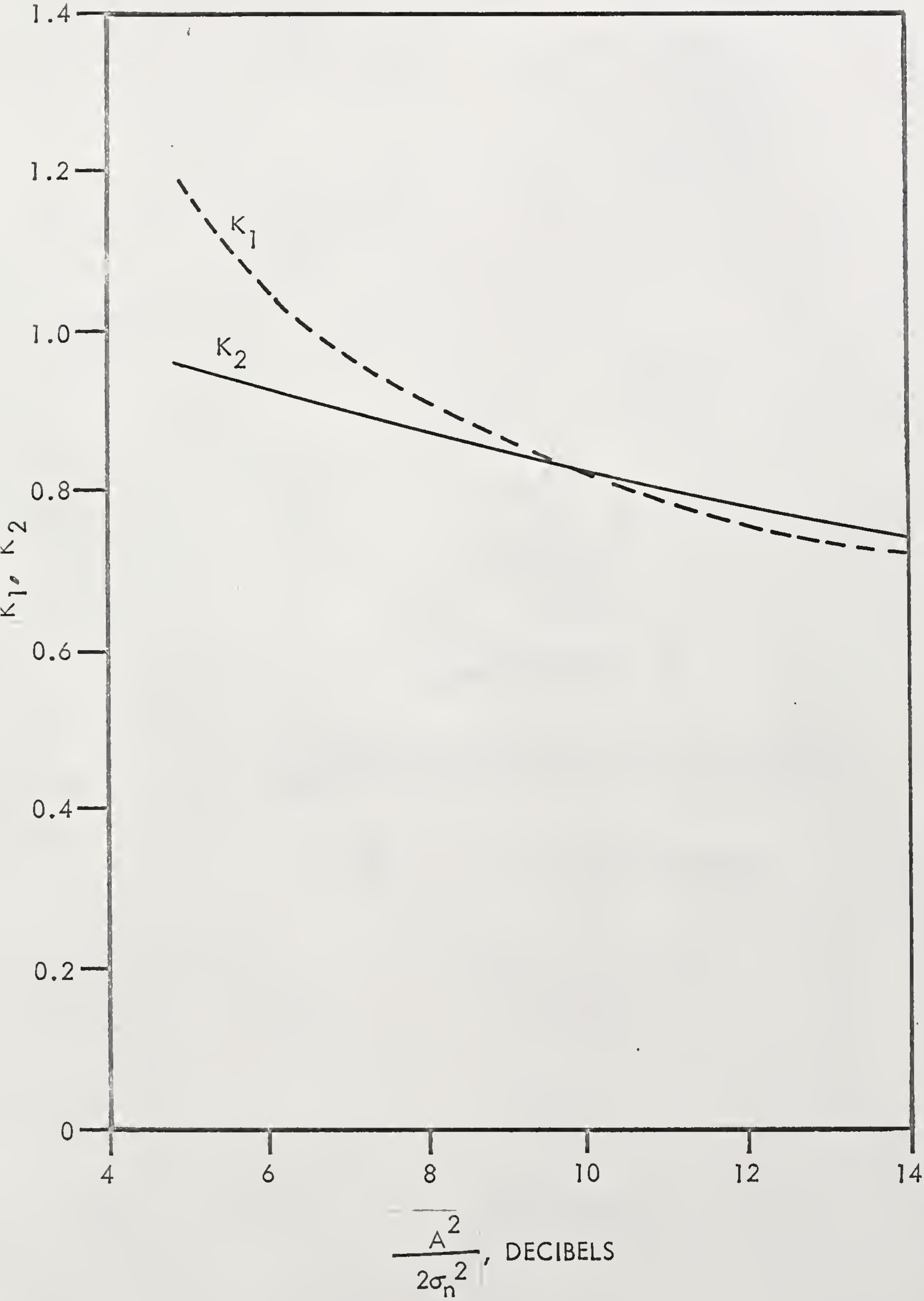
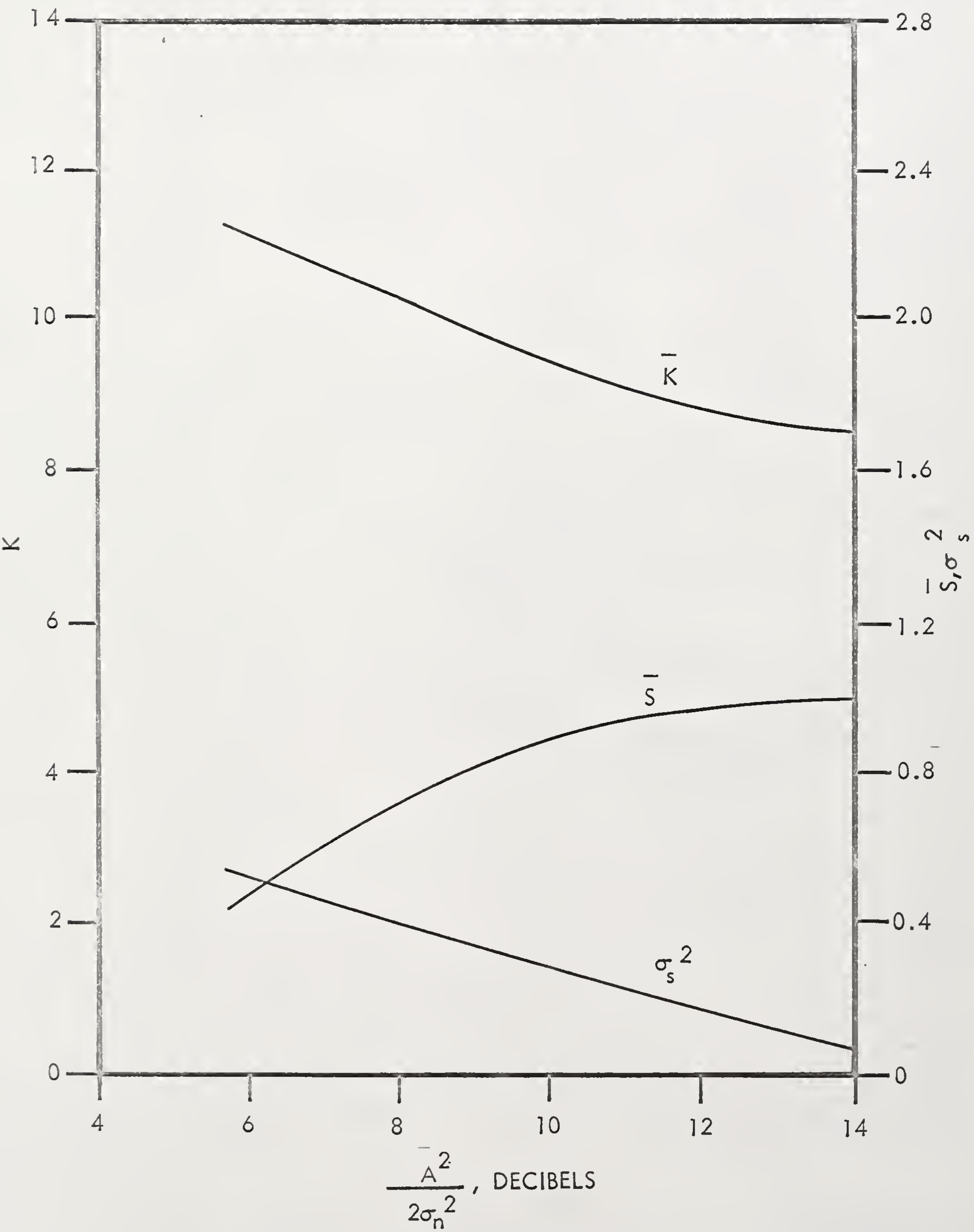

1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
K_1, K_2
K_1
K_2
4
6
8
10
12
14
$\frac{\overline{A^2}}{2\sigma_n^2}$, DECIBELS

FIGURE 20

$\bar{K}$, $\bar{s}$, and σ_s^2 for Double Threshold, Largest of s System for Signal-to-Noise Ratios Between 4.0 and 14.0 Decibels, with Minimum Probability of Error


14
12
10
8
6
4
2
0
K
2.8
2.4
2.0
1.6
1.2
0.8
0.4
0
$\bar{S}, \sigma_s^2$
$\bar{K}$
$\bar{S}$
σ_s^2
4
6
8
10
12
14
$\frac{\bar{A}^2}{2\sigma_n^2}$, DECIBELS

impulse noise. Included have been fairly comprehensive studies of the error performance of each system under various conditions. The 1.5 decibel improvement over the conventional, for the speech statistics system, would appear to be small under most circumstances. However, there are possible situations where this would be justifiably enough. The sequential systems provide other choices, other than the conventional and largest of N. They apparently offer a compromise between the simplicity of the former and the error performance of the latter.

CHAPTER III

CONVENTIONAL AND SEQUENTIAL SYSTEMS WITH IMPULSE NOISE CONSIDERATION

Introduction

The QPPM voice systems discussed in this chapter are considered to be under the influence of narrow-band Gaussian noise and an impulse-like noise. The impulse noise consists of pulses of amplitude B, greater than the signal pulse amplitude A. It arises from sources external to the receiver. The noise pulse carrier frequency is the same as the signal carrier. Of the N-1 noise slots in a frame, n will be considered to have impulses of the type described. The remainder have only narrow-band Gaussian noise. The probability distribution of n will be taken to be Poisson for purposes of this discussion. That is,

$$P_B(n) \cong \frac{e^{-\lambda} \cdot \lambda^n}{n!} \quad ; n = 0, 1, 2, \cdots N-1 \tag{3.1}$$

where λ is the average number of impulses per frame. This is felt to be representative of certain special communication situations. Descriptions of the systems considered and the derivations of the expressions for the probability of correct decision are given in the following.

QPPM, Four Threshold, Speech Statistics

This system is functionally described in Figure 21. The similarities between Figure 1 and Figure 21 should be noted. In this case, the output of the envelope detector is compared to the adaptive thresholds a, b, c, and d. Here, a is less than b, b less than c, and c less than d. If the envelope is between b and c, the receiver chooses the signal hypothesis H_1. If the envelope is below a, or above d, the noise hypothesis H_0 is selected. If between a and b, or c and d, neither hypothesis is chosen at this point. The remainder of the receiver processing is identical to the QPPM, Double Threshold, Speech Statistics system of Chapter II, shown in Figure 1. Note that the difference is the additional noise zone and the additional zone of indifference.

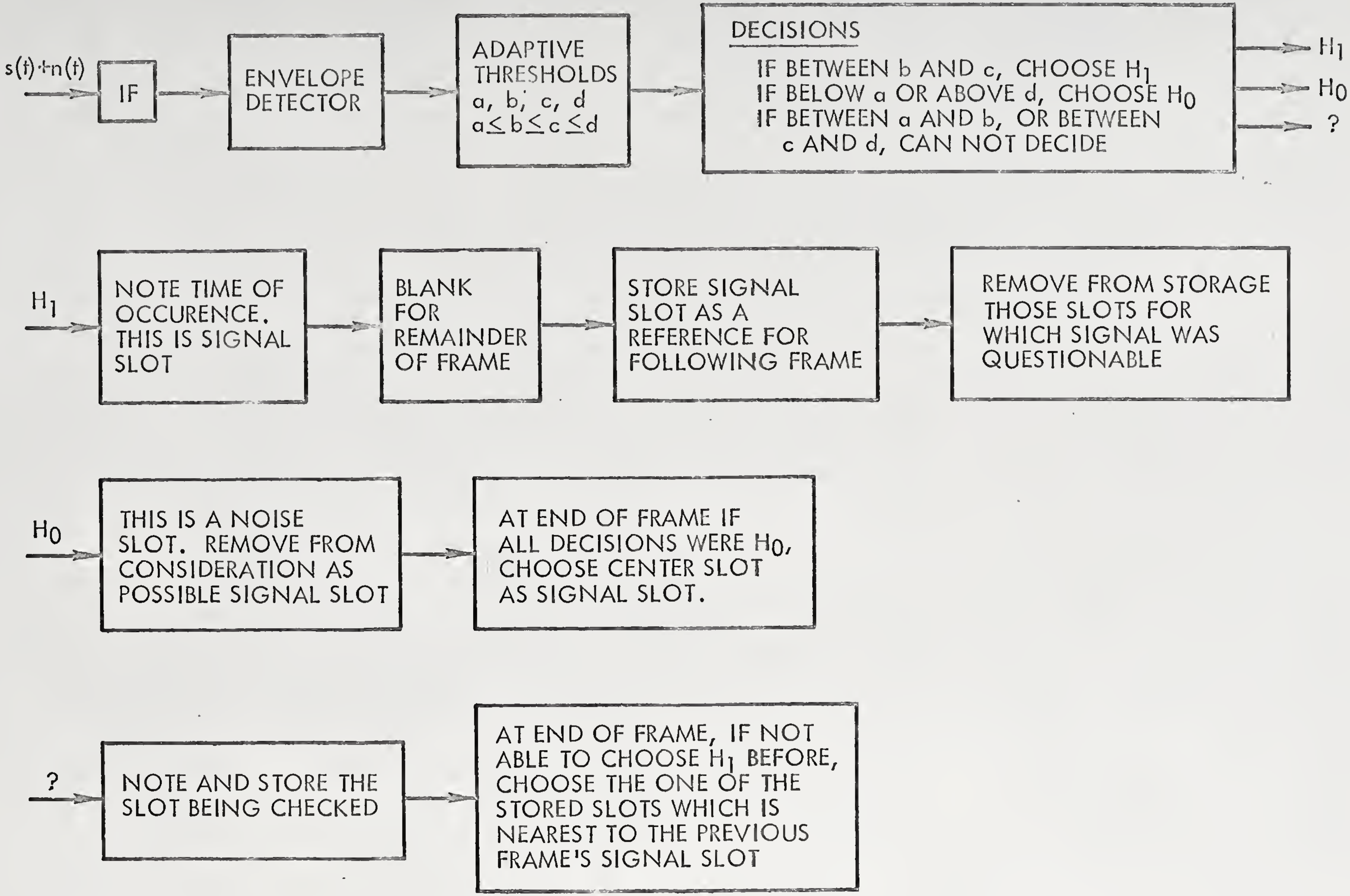
The probability of correct decision is again a sum of three contributing terms. That is,

$$\begin{aligned} P_5(\text{cor}) = {} & P_5\ (\text{correct, signal in "signal" zone}) \\ & + P_5\ (\text{correct, signal in "possible signal" zone}) \\ & + P_5\ (\text{correct, signal in "noise alone" zone}) \end{aligned} \tag{3.2}$$

If the signal, in the j^{th} slot, is in the "signal" zone, a correct decision will be made if none of the preceding j-1 noise slots has an envelope in the "signal" zone. Here we are interested in the number of noise impulses in the j-1

FIGURE 21

Functional Diagram, QPPM, Four Threshold, Speech Statistics System, with Impulse Noise


s(t)+n(t)
IF
ENVELOPE DETECTOR
ADAPTIVE THRESHOLDS a, b, c, d a≤b≤c≤d
DECISIONS
IF BETWEEN b AND c, CHOOSE H1
IF BELOW a OR ABOVE d, CHOOSE H0
IF BETWEEN a AND b, OR BETWEEN c AND d, CAN NOT DECIDE
H1
H0
?
H1
NOTE TIME OF OCCURENCE. THIS IS SIGNAL SLOT
BLANK FOR REMAINDER OF FRAME
STORE SIGNAL SLOT AS A REFERENCE FOR FOLLOWING FRAME
REMOVE FROM STORAGE THOSE SLOTS FOR WHICH SIGNAL WAS QUESTIONABLE
H0
THIS IS A NOISE SLOT. REMOVE FROM CONSIDERATION AS POSSIBLE SIGNAL SLOT
AT END OF FRAME IF ALL DECISIONS WERE H0, CHOOSE CENTER SLOT AS SIGNAL SLOT.
?
NOTE AND STORE THE SLOT BEING CHECKED
AT END OF FRAME, IF NOT ABLE TO CHOOSE H1 BEFORE, CHOOSE THE ONE OF THE STORED SLOTS WHICH IS NEAREST TO THE PREVIOUS FRAME'S SIGNAL SLOT

slots, rather than in the total N-1 noise slots. With the Poisson distribution taken here, the average number of impulses in j-1 noise slots is $\left(\frac{j-1}{N-1}\right)\lambda$. The distribution is

$$P_B'(n) = \frac{e^{-\left(\frac{j-1}{N-1}\right)\lambda} \cdot \left(\frac{j-1}{N-1}\right)^n \cdot \lambda^n}{n!} \quad ; n = 0, 1, \cdots j-1 \tag{3.3}$$

The first term can now be written

P_5 (correct, signal in "signal" zone)

$$= \left[Q\left(\tfrac{A}{\sigma_n}, \tfrac{b}{\sigma_n}\right) - Q\left(\tfrac{A}{\sigma_n}, \tfrac{c}{\sigma_n}\right)\right] \cdot \sum_{j=1}^{N} \sum_{n=0}^{j-1} P_s(j) \cdot P_B'(n) \cdot$$

$$\cdot \left[1 - e^{-\frac{b^2}{2\sigma_n^2}} + e^{-\frac{c^2}{2\sigma_n^2}}\right]^{j-1-n} \cdot \left[1 - Q\left(\tfrac{B}{\sigma_n}, \tfrac{b}{\sigma_n}\right) + Q\left(\tfrac{B}{\sigma_n}, \tfrac{c}{\sigma_n}\right)\right]^n \tag{3.4}$$

The second contributing term is much more complex. For the signal in the "possible signal" region, the reference slot being the ℓ th in the preceding frame, and the signal slot being the jth, a correct decision requires that there be no noise slot closer to the reference than the signal slot is, with envelope in the "possible signal" region. Three ranges of j must be taken:

$$j \leq \ell \;, \quad \ell + 1 \leq j \leq 2\ell, \text{ and } \quad j \geq 2\ell + 1 .$$

For j less than ℓ, there are $2\ell-2j$ slots closer to the reference than the signal slot is. For the second range of values, there are $2j-2\ell-1$ slots closer. For the last case, there are j-1 slots closer. With n impulses assumed for the N-1 total noise slots, q are assumed to be within the "closer" region. The maximum allowable value for q is determined by the number of slots in the "closer" region or by n, whichever is smaller. With n impulses in the N-1 noise slots, the approximation is made that the probability of a particular noise slot having an impulse is $\frac{n}{N-1}$. We can now write

P_5 (correct, signal in "possible signal" zone)

$$= 2\cdot\left[Q\left(\tfrac{A}{\sigma_n},\tfrac{a}{\sigma_n}\right)-Q\left(\tfrac{A}{\sigma_n},\tfrac{b}{\sigma_n}\right)+Q\left(\tfrac{A}{\sigma_n},\tfrac{c}{\sigma_n}\right)-Q\left(\tfrac{A}{\sigma_n},\tfrac{d}{\sigma_n}\right)\right]\cdot$$

$$\cdot\sum_{n=0}^{N-1}\sum_{\ell=1}^{\frac{N}{2}} P_r(\ell)\cdot P_B(n)\cdot\Bigg\{\sum_{j=1}^{\ell}\ \sum_{q=0}^{\substack{\text{SMALLER OF}\\ n,\,2\ell-2j}} P_s(j/\ell)\cdot\binom{2\ell-2j}{q}\cdot\left(\frac{n}{N-1}\right)^{q}\cdot\left(\frac{N-1-n}{N-1}\right)^{2\ell-2j-q}\cdot$$

$$\cdot\left[1-Q\left(\tfrac{B}{\sigma_n},\tfrac{a}{\sigma_n}\right)+Q\left(\tfrac{B}{\sigma_n},\tfrac{d}{\sigma_n}\right)\right]^{q}\cdot\left[1-e^{-\frac{a^2}{2\sigma_n^2}}+e^{-\frac{d^2}{2\sigma_n^2}}\right]^{2\ell-2j-q}\cdot$$

$$\cdot\left[1-Q\left(\tfrac{B}{\sigma_n},\tfrac{b}{\sigma_n}\right)+Q\left(\tfrac{B}{\sigma_n},\tfrac{c}{\sigma_n}\right)\right]^{n-q}\cdot\left[1-e^{-\frac{b^2}{2\sigma_n^2}}+e^{-\frac{c^2}{2\sigma_n^2}}\right]^{N-1-2\ell+2j-n+q}$$

$$+\sum_{j=\ell+1}^{2\ell}\ \sum_{q=0}^{\substack{\text{SMALLER OF}\\ n,\,2j-2\ell-1}} P_s(j/\ell)\cdot\binom{2j-2\ell-1}{q}\cdot\left(\frac{n}{N-1}\right)^{q}\cdot\left(\frac{N-1-n}{N-1}\right)^{2j-2\ell-1-q}\cdot$$

$$\cdot\left[1-Q\left(\tfrac{B}{\sigma_n},\tfrac{a}{\sigma_n}\right)+Q\left(\tfrac{B}{\sigma_n},\tfrac{d}{\sigma_n}\right)\right]^{q}\cdot\left[1-e^{-\frac{a^2}{2\sigma_n^2}}+e^{-\frac{d^2}{2\sigma_n^2}}\right]^{2j-2\ell-1-q}\cdot$$

$$\cdot\left[1-Q\left(\tfrac{B}{\sigma_n},\tfrac{b}{\sigma_n}\right)+Q\left(\tfrac{B}{\sigma_n},\tfrac{c}{\sigma_n}\right)\right]^{n-q}\cdot\left[1-e^{-\frac{b^2}{2\sigma_n^2}}+e^{-\frac{c^2}{2\sigma_n^2}}\right]^{N-2j+2\ell-n+q}$$

$$+\sum_{j=2\ell+1}^{N} \sum_{q=0}^{\substack{\text{SMALLER OF}\\ n,\, j-1}} P_s(j/\ell)\cdot\binom{j-1}{q}\cdot\left(\frac{n}{N-1}\right)^{q}\cdot\left(\frac{N-1-n}{N-1}\right)^{j-1-q}\cdot$$

$$\cdot\left[1-Q\left(\frac{B}{\sigma_n},\frac{a}{\sigma_n}\right)+Q\left(\frac{B}{\sigma_n},\frac{d}{\sigma_n}\right)\right]^{q}\cdot\left[1-e^{-\frac{a^2}{2\sigma_n^2}}+e^{-\frac{d^2}{2\sigma_n^2}}\right]^{j-1-q}\cdot$$

$$\left.\cdot\left[1-Q\left(\frac{B}{\sigma_n},\frac{b}{\sigma_n}\right)+Q\left(\frac{B}{\sigma_n},\frac{c}{\sigma_n}\right)\right]^{n-q}\cdot\left[1-e^{-\frac{b^2}{2\sigma_n^2}}+e^{-\frac{c^2}{2\sigma_n^2}}\right]^{N-j-n+q}\right\} \qquad (3.5)$$

If the signal is in the "noise alone" zone, the n impulses and the N-1-n noise envelopes must be in the "noise alone" zone, and the signal slot must be the center one to lead to a correct decision by the receiver. Therefore, the last term is

P_5 (correct, signal in "noise alone" zone)

$$= \left[1-Q\left(\frac{A}{\sigma_n},\frac{a}{\sigma_n}\right)+Q\left(\frac{A}{\sigma_n},\frac{d}{\sigma_n}\right)\right]\cdot P_s\left(\frac{N}{2}\right)\cdot$$

$$\cdot\sum_{n=0}^{N-1} P_B(n)\cdot\left[1-e^{-\frac{a^2}{2\sigma_n^2}}+e^{-\frac{d^2}{2\sigma_n^2}}\right]^{N-1-n}\cdot$$

$$\cdot\left[1-Q\left(\frac{B}{\sigma_n},\frac{a}{\sigma_n}\right)+Q\left(\frac{B}{\sigma_n},\frac{d}{\sigma_n}\right)\right]^{n} \qquad (3.6)$$

Summing the contributions of equations 3.4, 3.5, and 3.6 gives the final result

$$P_S(\text{cor}) = \left[Q\left(\frac{A}{\sigma_n},\frac{b}{\sigma_n}\right) - Q\left(\frac{A}{\sigma_n},\frac{c}{\sigma_n}\right)\right]\cdot \sum_{j=1}^{N}\sum_{n=0}^{j-1} P_S(j)\cdot P_B'(n)\cdot$$

$$\cdot\left[1-e^{-\frac{b^2}{2\sigma_n^2}}+e^{-\frac{c^2}{2\sigma_n^2}}\right]^{j-1-n}\cdot\left[1-Q\left(\frac{B}{\sigma_n},\frac{b}{\sigma_n}\right)+Q\left(\frac{B}{\sigma_n},\frac{c}{\sigma_n}\right)\right]^{n}$$

$$+\,2\cdot\left[Q\left(\frac{A}{\sigma_n},\frac{a}{\sigma_n}\right)-Q\left(\frac{A}{\sigma_n},\frac{b}{\sigma_n}\right)+Q\left(\frac{A}{\sigma_n},\frac{c}{\sigma_n}\right)-Q\left(\frac{A}{\sigma_n},\frac{d}{\sigma_n}\right)\right]\cdot$$

$$\cdot\sum_{n=0}^{N-1}\sum_{\ell=1}^{\frac{N}{2}} P_r(\ell)\cdot P_B(n)\cdot\left\{\sum_{j=1}^{\ell}\ \sum_{q=0}^{\substack{\text{SMALLER OF}\\ n,\,2\ell-2j}} P_S(j/\ell)\cdot\binom{2\ell-2j}{q}\cdot\left(\frac{n}{N-1}\right)^{q}\cdot\left(\frac{N-1-n}{N-1}\right)^{2\ell-2j-q}\cdot\right.$$

$$\cdot\left[1-Q\left(\frac{B}{\sigma_n},\frac{a}{\sigma_n}\right)+Q\left(\frac{B}{\sigma_n},\frac{d}{\sigma_n}\right)\right]^{q}\cdot\left[1-e^{-\frac{a^2}{2\sigma_n^2}}+e^{-\frac{d^2}{2\sigma_n^2}}\right]^{2\ell-2j-q}\cdot$$

$$\cdot\left[1-Q\left(\frac{B}{\sigma_n},\frac{b}{\sigma_n}\right)+Q\left(\frac{B}{\sigma_n},\frac{c}{\sigma_n}\right)\right]^{n-q}\cdot\left[1-e^{-\frac{b^2}{2\sigma_n^2}}+e^{-\frac{c^2}{2\sigma_n^2}}\right]^{N-1-2\ell+2j-n+q}$$

$$+\sum_{j=\ell+1}^{2\ell}\ \sum_{q=0}^{\substack{\text{SMALLER OF}\\ n,\,2j-2\ell-1}} P_S(j/\ell)\cdot\binom{2j-2\ell-1}{q}\cdot\left(\frac{n}{N-1}\right)^{q}\cdot\left(\frac{N-1-n}{N-1}\right)^{2j-2\ell-1-q}\cdot$$

$$\cdot\left[1-Q\left(\frac{B}{\sigma_n},\frac{a}{\sigma_n}\right)+Q\left(\frac{B}{\sigma_n},\frac{d}{\sigma_n}\right)\right]^{q}\cdot\left[1-e^{-\frac{a^2}{2\sigma_n^2}}+e^{-\frac{d^2}{2\sigma_n^2}}\right]^{2j-2\ell-1-q}\cdot$$

$$\cdot\left[1-Q\left(\frac{B}{\sigma_n},\frac{b}{\sigma_n}\right)+Q\left(\frac{B}{\sigma_n},\frac{c}{\sigma_n}\right)\right]^{n-q}\cdot\left[1-e^{-\frac{b^2}{2\sigma_n^2}}+e^{-\frac{c^2}{2\sigma_n^2}}\right]^{N-2j+2\ell-n+q}$$

$$+\sum_{j=2\ell+1}^{N} \sum_{q=0}^{\substack{\text{SMALLER OF}\\ n,\, j-1}} P_s(j/\ell)\cdot\binom{j-1}{q}\cdot\left(\frac{n}{N-1}\right)^{q}\cdot\left(\frac{N-1-n}{N-1}\right)^{j-1-q}\cdot$$

$$\cdot\left[1-Q\left(\frac{B}{\sigma_n},\frac{a}{\sigma_n}\right)+Q\left(\frac{B}{\sigma_n},\frac{d}{\sigma_n}\right)\right]^{q}\cdot\left[1-e^{-\frac{a^2}{2\sigma_n^2}}+e^{-\frac{d^2}{2\sigma_n^2}}\right]^{j-1-q}\cdot$$

$$\left.\cdot\left[1-Q\left(\frac{B}{\sigma_n},\frac{b}{\sigma_n}\right)+Q\left(\frac{B}{\sigma_n},\frac{c}{\sigma_n}\right)\right]^{n-q}\cdot\left[1-e^{-\frac{b^2}{2\sigma_n^2}}+e^{-\frac{c^2}{2\sigma_n^2}}\right]^{N-j-n+q}\right\}$$

$$+\left[1-Q\left(\frac{A}{\sigma_n},\frac{a}{\sigma_n}\right)+Q\left(\frac{A}{\sigma_n},\frac{d}{\sigma_n}\right)\right]\cdot P_s\left(\frac{N}{2}\right)\cdot$$

$$\cdot\sum_{n=0}^{N-1} P_B(n)\cdot\left[1-e^{-\frac{a^2}{2\sigma_n^2}}+e^{-\frac{d^2}{2\sigma_n^2}}\right]^{N-1-n}\cdot$$

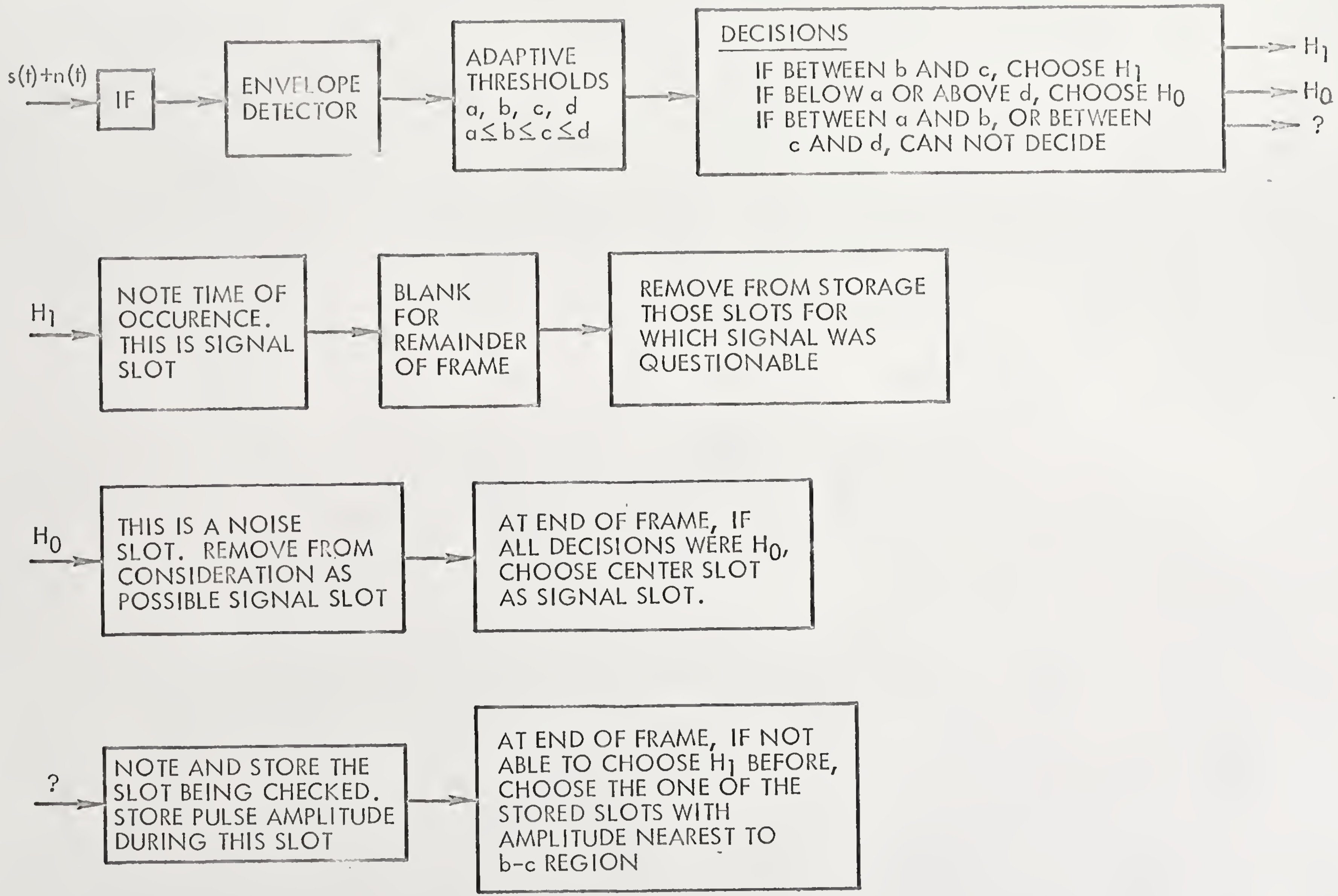
$$\cdot\left[1-Q\left(\frac{B}{\sigma_n},\frac{a}{\sigma_n}\right)+Q\left(\frac{B}{\sigma_n},\frac{d}{\sigma_n}\right)\right]^{n} \qquad (3.7)$$

QPPM, Four Threshold, Nearest to Reference

The similarities between this system, in Figure 22, and the system of Figure 3 should be noted. The four thresholds a, b, c, and d provide an additional noise zone and an additional zone of indifference. The signal hypothesis H_1 is chosen if the envelope is between b and c. The noise hypothesis H_0 is selected if the envelope is either below a or above d. Neither can be chosen if the envelope is between a and b, or between c and d. The receiver processing is identical, from this point on, to the Double Threshold, Largest of s system, with one exception. At the end of the

FIGURE 22

Functional Diagram, QPPM, Four Threshold, Nearest to Reference System, with Impulse Noise


s(t)+n(t)
IF
ENVELOPE DETECTOR
ADAPTIVE THRESHOLDS a, b, c, d a≤b≤c≤d
DECISIONS
IF BETWEEN b AND c, CHOOSE H1
IF BELOW a OR ABOVE d, CHOOSE H0
IF BETWEEN a AND b, OR BETWEEN c AND d, CAN NOT DECIDE
H1
H0
?
H1
NOTE TIME OF OCCURENCE. THIS IS SIGNAL SLOT
BLANK FOR REMAINDER OF FRAME
REMOVE FROM STORAGE THOSE SLOTS FOR WHICH SIGNAL WAS QUESTIONABLE
H0
THIS IS A NOISE SLOT. REMOVE FROM CONSIDERATION AS POSSIBLE SIGNAL SLOT
AT END OF FRAME, IF ALL DECISIONS WERE H0, CHOOSE CENTER SLOT AS SIGNAL SLOT.
?
NOTE AND STORE THE SLOT BEING CHECKED. STORE PULSE AMPLITUDE DURING THIS SLOT
AT END OF FRAME, IF NOT ABLE TO CHOOSE H1 BEFORE, CHOOSE THE ONE OF THE STORED SLOTS WITH AMPLITUDE NEAREST TO b-c REGION

frame, when the receiver has been unable to select a hypothesis, it selects as the signal slot the one whose amplitude is nearest to the signal zone, b to c. The Double Threshold system selected the one with largest amplitude.

The probability of correct decision is given by

$$
\begin{aligned}
P_6(\text{cor}) = {} & P_6 \text{ (correct, signal in "signal" zone)} \\
& + P_6 \text{ (correct, signal in "possible signal" zone)} \\
& + P_6 \text{ (correct, signal in "noise alone" zone)}
\end{aligned}
\tag{3.8}
$$

The first and last of these are identical to the corresponding terms of $P_5(\text{cor})$ and are given by equations 3.4 and 3.6, respectively. With the signal in the upper "possible signal" zone, c to d, the noise envelopes must be greater than it, or below b by at least an amount equal to the amount signal exceeds c. If this amount exceeds the difference in b and a the noise envelope only has to be below a. With the signal in the lower zone, a to b, the noise envelopes must be less than the signal, or greater than c by an amount at least equal to the amount the signal is below b. If this amount exceeds the difference in d and c, the noise envelope only has to be above d. With n of the N-1 noise slots having impulses, we have

P_6 (correct, signal in "possible signal" zone)

$$= \sum_{n=0}^{N-1} P_B(n) \cdot \Bigg\{ \int_c^d \frac{R_2}{\sigma_n^2} \cdot e^{-\frac{R_2^2+A^2}{2\sigma_n^2}} \cdot I_o\left(\frac{AR_2}{\sigma_n^2}\right) \cdot \left[1 - Q\left(\frac{B}{\sigma_n}, f\{R_2\}\right) + Q\left(\frac{B}{\sigma_n}, \frac{R_2}{\sigma_n}\right)\right]^n \cdot$$

$$\cdot \left[1 - e^{-\frac{(f\{R_2\})^2}{2}} + e^{-\frac{R_2^2}{2\sigma_n^2}}\right]^{N-1-n} \cdot dR_2$$

$$+ \int_a^b \frac{R_2}{\sigma_n^2} \cdot e^{-\frac{R_2^2+A^2}{2\sigma_n^2}} \cdot I_o\left(\frac{AR_2}{\sigma_n^2}\right) \cdot \left[1 - Q\left(\frac{B}{\sigma_n}, \frac{R_2}{\sigma_n}\right) + Q\left(\frac{B}{\sigma_n}, g\{R_2\}\right)\right]^n \cdot$$

$$\cdot \left[1 - e^{-\frac{R_2^2}{2\sigma_n^2}} + e^{-\frac{(g\{R_2\})^2}{2}}\right]^{N-1-n} \cdot dR_2 \Bigg\} \qquad (3.9)$$

where

$$f(R_2) = \frac{b+c-R_2}{\sigma_n} \quad ; \quad R_2 < b+c-a$$

$$= \frac{a}{\sigma_n} \quad ; \quad R_2 \geq b+c-a \qquad (3.10)$$

$$g(R_2) = \frac{b+c-R_2}{\sigma_n} \quad ; \quad R_2 > b+c-d$$

$$= \frac{d}{\sigma_n} \quad ; \quad R_2 \leq b+c-d \qquad (3.11)$$

Finally, the result is

$$P_6(\text{cor}) = \left[Q\left(\frac{A}{\sigma_n},\frac{b}{\sigma_n}\right)-Q\left(\frac{A}{\sigma_n},\frac{c}{\sigma_n}\right)\right]\cdot \sum_{j=1}^{N}\sum_{n=0}^{j-1} P_S(j)\cdot P_B'(n)\cdot$$

$$\cdot\left[1-e^{-\frac{b^2}{2\sigma_n^2}}+e^{-\frac{c^2}{2\sigma_n^2}}\right]^{j-1-n}\cdot\left[1-Q\left(\frac{B}{\sigma_n},\frac{b}{\sigma_n}\right)+Q\left(\frac{B}{\sigma_n},\frac{c}{\sigma_n}\right)\right]^n$$

$$+\sum_{n=0}^{N-1} P_B(n)\cdot\left\{\int_c^d \frac{R_2}{\sigma_n^2}\cdot e^{-\frac{R_2^2+A^2}{2\sigma_n^2}}\cdot I_o\left(\frac{AR_2}{\sigma_n^2}\right)\cdot\left[1-Q\left(\frac{B}{\sigma_n},f\{R_2\}\right)+Q\left(\frac{B}{\sigma_n},\frac{R_2}{\sigma_n}\right)\right]^n\cdot\right.$$

$$\cdot\left[1-e^{-\frac{(f\{R_2\})^2}{2}}+e^{-\frac{R_2^2}{2\sigma_n^2}}\right]^{N-1-n}\cdot dR_2$$

$$+\int_a^b \frac{R_2}{\sigma_n^2}\cdot e^{-\frac{R_2^2+A^2}{2\sigma_n^2}}\cdot I_o\left(\frac{AR_2}{\sigma_n^2}\right)\cdot\left[1-Q\left(\frac{B}{\sigma_n},\frac{R_2}{\sigma_n}\right)+Q\left(\frac{B}{\sigma_n},g\{R_2\}\right)\right]^n\cdot$$

$$\left.\cdot\left[1-e^{-\frac{R_2^2}{2\sigma_n^2}}+e^{-\frac{(g\{R_2\})^2}{2}}\right]^{N-1-n}\cdot dR_2\right\}$$

$$+\left[1-Q\left(\frac{A}{\sigma_n},\frac{a}{\sigma_n}\right)+Q\left(\frac{A}{\sigma_n},\frac{d}{\sigma_n}\right)\right]\cdot P_S\left(\frac{N}{2}\right)\cdot$$

$$\cdot\sum_{n=0}^{N-1} P_B(n)\cdot\left[1-Q\left(\frac{B}{\sigma_n},\frac{a}{\sigma_n}\right)+Q\left(\frac{B}{\sigma_n},\frac{d}{\sigma_n}\right)\right]^n\cdot$$

$$\cdot\left[1-e^{-\frac{a^2}{2\sigma_n^2}}+e^{-\frac{d^2}{2\sigma_n^2}}\right]^{N-1-n} \qquad (3.12)$$

QPPM, Single Threshold

It is of interest to find the probability of correct decision for this system, already discussed and described in Figure 2, when impulse noise as given above is present. This can be written down almost immediately. If the signal is in the j^{th} position and above the threshold, the preceding j-1 amplitudes must be below the threshold. A correct decision is also possible if all amplitudes are below the threshold, if the signal slot is the center one. The probability is

$$P_7(cor) = \left[Q\left(\frac{A}{\sigma_n}, \frac{b}{\sigma_n}\right)\right] \cdot \sum_{j=1}^{N} \sum_{n=0}^{j-1} P_S(j) \cdot P_B'(n) \cdot \left[1 - Q\left(\frac{B}{\sigma_n}, \frac{b}{\sigma_n}\right)\right]^n \cdot \left[1 - e^{-\frac{b^2}{2\sigma_n^2}}\right]^{j-1-n}$$

$$+ \left[1 - Q\left(\frac{A}{\sigma_n}, \frac{b}{\sigma_n}\right)\right] \cdot P_S\left(\frac{N}{2}\right) \cdot \sum_{n=0}^{N-1} P_B(n) \cdot \left[1 - Q\left(\frac{B}{\sigma_n}, \frac{b}{\sigma_n}\right)\right]^n \cdot \left[1 - e^{-\frac{b^2}{2\sigma_n^2}}\right]^{N-1-n} \qquad (3.13)$$

QPPM, Largest of N

Also of interest is the performance of this system, in Figure 4, with impulse noise. A correct decision requires that all noise envelopes be of smaller magnitude than the signal envelope. Therefore, the probability of correct decision is

$$P_8(\text{cor}) = \sum_{n=0}^{N-1} P_B(n) \cdot \int_0^{\infty} \frac{R_2}{\sigma_n^2} \cdot e^{-\frac{R_2^2+A^2}{2\sigma_n^2}} \cdot I_0\left(\frac{AR_2}{\sigma_n^2}\right) \cdot \left[1 - Q\left(\frac{B}{\sigma_n}, \frac{R_2}{\sigma_n}\right)\right]^n \cdot$$

$$\cdot \left[1 - e^{-\frac{R_2^2}{2\sigma_n^2}}\right]^{N-1-n} \cdot dR_2 \qquad (3.14)$$

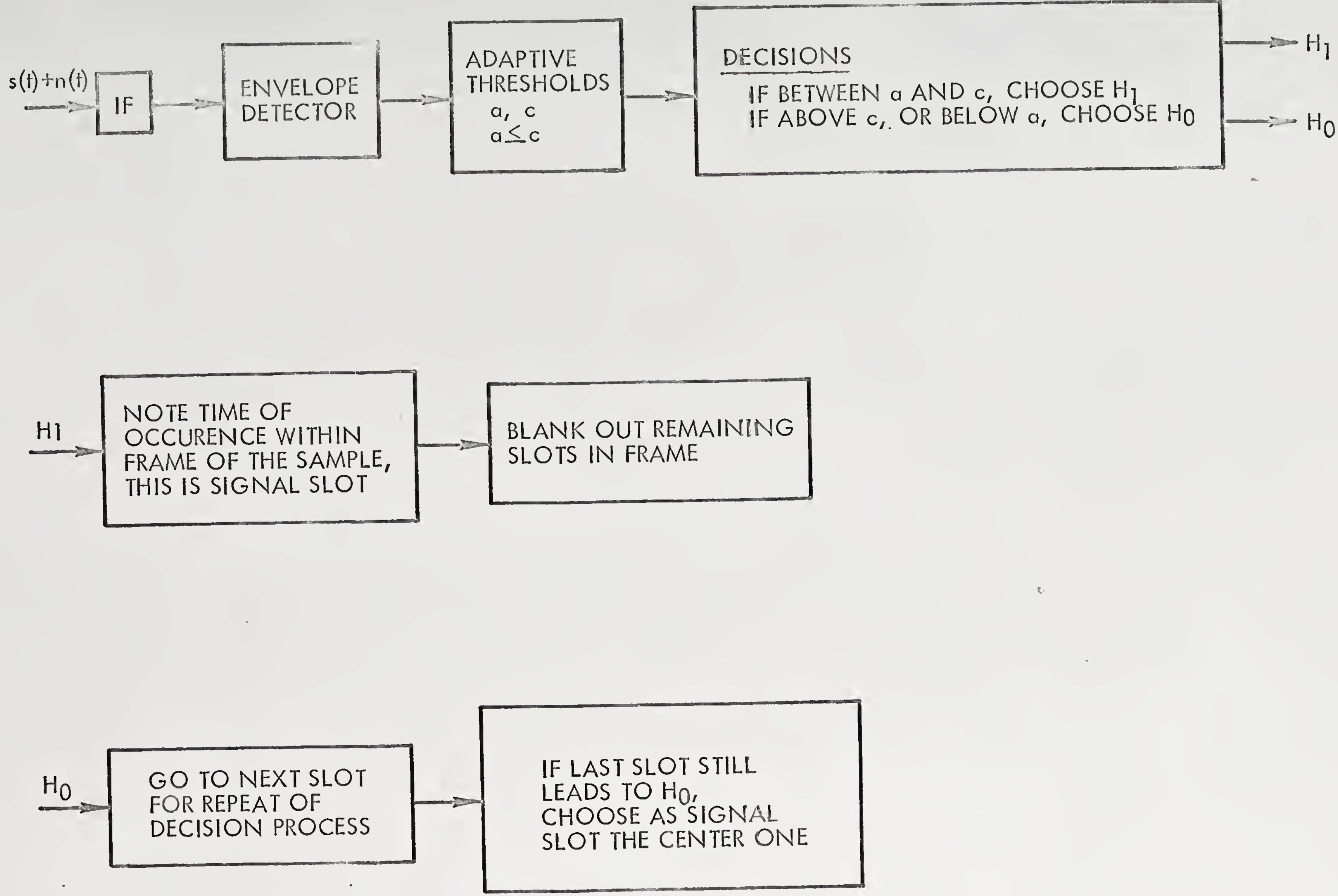
QPPM, Double Threshold, Non-Sequential

This system is identical to the Single Threshold, conventional system of Figure 2, except for the addition of an upper threshold c. It is shown in Figure 23. The signal hypothesis H_1 is chosen when the envelope is between a and c. Otherwise, the noise hypothesis H_0 is selected. The receiver processing from this stage on is the same as for the Single Threshold system.

A correct decision is made for this case when the signal amplitude, in the j^th slot, is in the "signal" zone and all preceding j-1 noise envelopes are in a "noise" zone. A correct decision is also made if the signal slot is the center one, even though all envelopes fall out of the "signal" zone. The probability of correct decision is

FIGURE 23

Functional Diagram, QPPM, Double Threshold, Non-Sequential System, with Impulse Noise


s(t)+n(t)
IF
ENVELOPE DETECTOR
ADAPTIVE THRESHOLDS a, c a≤c
DECISIONS
IF BETWEEN a AND c, CHOOSE H1
IF ABOVE c, OR BELOW a, CHOOSE H0
H1
H0
H1
NOTE TIME OF OCCURENCE WITHIN FRAME OF THE SAMPLE, THIS IS SIGNAL SLOT
BLANK OUT REMAINING SLOTS IN FRAME
H0
GO TO NEXT SLOT FOR REPEAT OF DECISION PROCESS
IF LAST SLOT STILL LEADS TO H0, CHOOSE AS SIGNAL SLOT THE CENTER ONE

$$P_q(\text{cor}) = \left[Q\left(\frac{A}{\sigma_n}, \frac{a}{\sigma_n}\right) - Q\left(\frac{A}{\sigma_n}, \frac{c}{\sigma_n}\right)\right] \cdot \sum_{j=1}^{N} \sum_{n=0}^{j-1} P_s(j) \cdot P_B'(n) \cdot$$

$$\cdot \left[1 - e^{-\frac{a^2}{2\sigma_n^2}} + e^{-\frac{c^2}{2\sigma_n^2}}\right]^{j-1-n} \cdot$$

$$\cdot \left[1 - Q\left(\frac{B}{\sigma_n}, \frac{a}{\sigma_n}\right) + Q\left(\frac{B}{\sigma_n}, \frac{c}{\sigma_n}\right)\right]^n$$

$$+ \left[1 - Q\left(\frac{A}{\sigma_n}, \frac{a}{\sigma_n}\right) + Q\left(\frac{A}{\sigma_n}, \frac{c}{\sigma_n}\right)\right] \cdot P_s\left(\frac{N}{2}\right) \cdot$$

$$\cdot \sum_{n=0}^{N-1} P_B(n) \cdot \left[1 - e^{-\frac{a^2}{2\sigma_n^2}} + e^{-\frac{c^2}{2\sigma_n^2}}\right]^{N-1-n} \cdot$$

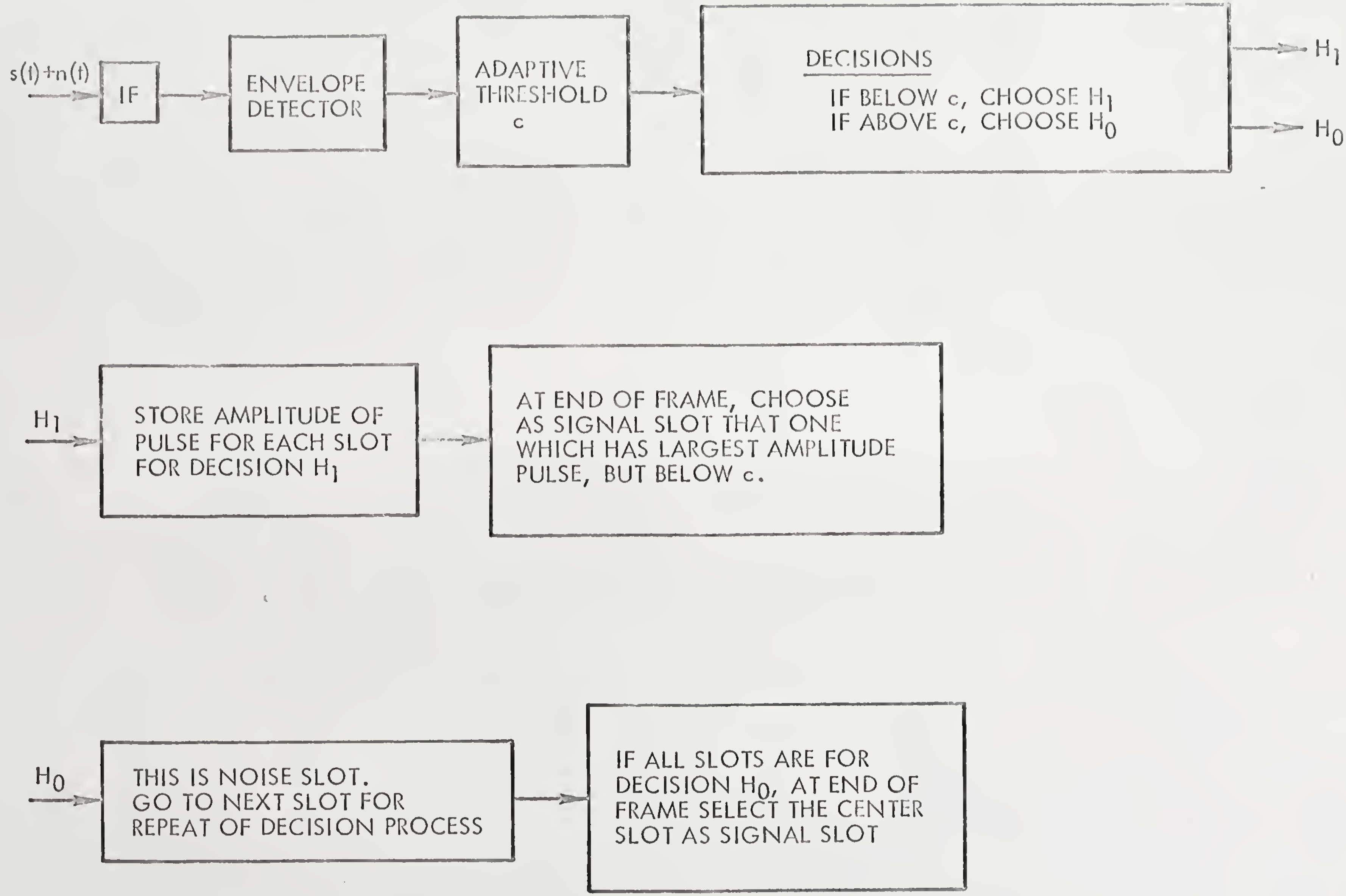
$$\cdot \left[1 - Q\left(\frac{B}{\sigma_n}, \frac{a}{\sigma_n}\right) + Q\left(\frac{B}{\sigma_n}, \frac{c}{\sigma_n}\right)\right]^n \qquad (3.15)$$

<u>QPPM, Single Threshold, Largest of N</u>

This non-sequential system is similar to the system of Figure 4. It is shown in Figure 24. There is a single threshold c. Any envelopes above c are rejected as noise envelopes and hypothesis H_0 selected. At the end of the frame, all envelopes which were below c are examined and the signal selected as the largest one. Of course, if all are above c, the receiver chooses the center slot as the signal.

FIGURE 24

Functional Diagram, QPPM, Single Threshold, Largest of N, Non-Sequential System, with Impulse Noise


s(t)+n(t)
IF
ENVELOPE DETECTOR
ADAPTIVE THRESHOLD c
DECISIONS
IF BELOW c, CHOOSE H1
IF ABOVE c, CHOOSE H0
H1
H0
H1
STORE AMPLITUDE OF PULSE FOR EACH SLOT FOR DECISION H1
AT END OF FRAME, CHOOSE AS SIGNAL SLOT THAT ONE WHICH HAS LARGEST AMPLITUDE PULSE, BUT BELOW c.
H0
THIS IS NOISE SLOT. GO TO NEXT SLOT FOR REPEAT OF DECISION PROCESS
IF ALL SLOTS ARE FOR DECISION H0, AT END OF FRAME SELECT THE CENTER SLOT AS SIGNAL SLOT

A correct decision requires the noise envelopes to be either above c or below the signal amplitude, if the signal is below c. Following the previously used procedure we can write

$$P_{10}(cor) = \sum_{n=0}^{N-1} P_B(n) \cdot \int_0^c \frac{R_2}{\sigma_n^2} \cdot e^{-\frac{R_2^2+A^2}{2\sigma_n^2}} \cdot I_o\left(\frac{AR_2}{\sigma_n^2}\right) \cdot \left[1 - Q\left(\frac{B}{\sigma_n}, \frac{R_2}{\sigma_n}\right) + Q\left(\frac{B}{\sigma_n}, \frac{c}{\sigma_n}\right)\right]^n \cdot$$

$$\cdot \left[1 - e^{-\frac{R_2^2}{2\sigma_n^2}} + e^{-\frac{c^2}{2\sigma_n^2}}\right]^{N-1-n} \cdot dR_2$$

$$+ \left[Q\left(\frac{A}{\sigma_n}, \frac{c}{\sigma_n}\right)\right] \cdot P_s\left(\frac{N}{2}\right) \cdot \sum_{n=0}^{N-1} P_B(n) \cdot \left[Q\left(\frac{B}{\sigma_n}, \frac{c}{\sigma_n}\right)\right]^n \cdot$$

$$\cdot \left[e^{-(N-1-n)\frac{c^2}{2\sigma_n^2}}\right] \qquad (3.16)$$

Discussion of Results

In order to obtain numerical results, the value of λ is taken as one, and B is taken as three times the signal pulse amplitude A. A more general study would allow λ and B to be variable, but the preliminary results desired here do not justify the great increase in mathematical complexity this would involve. It is felt that the particular case chosen for evaluation is representative of the type of problem which would arise.

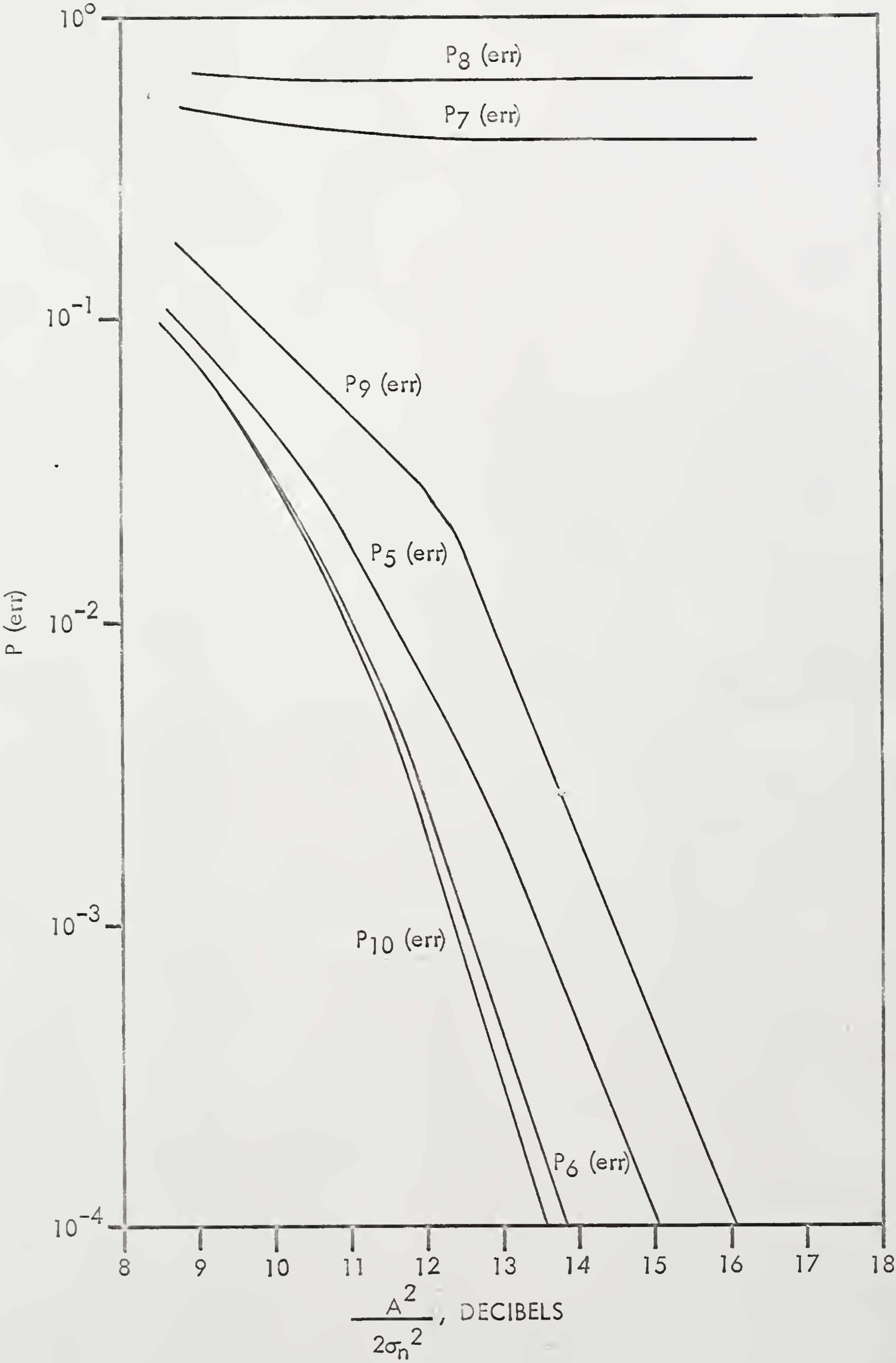
The equations $P_6(\text{cor})$, $P_8(\text{cor})$, and $P_{10}(\text{cor})$ given in 3.12, 3.14, and 3.16, respectively, involve integrals which have to be evaluated by approximation methods. Appendix B shows that if the infinite upper limit of $P_8(\text{cor})$ is replaced by a value of 13.2, the error involved is less than 10^{-5} for the range of signal-to-noise ratios considered.

The most important result of this section is depicted in Figure 25. This shows the variation of probability of error with signal-to-noise ratio for each system, with optimum threshold settings considered. Based on the mathematical model discussed above, it is seen that the conventional, single threshold system and the largest of N system have very poor performance levels. Their error probabilities, $P_7(\text{err})$ and $P_8(\text{err})$, respectively, are at very high values, and they do not improve substantially with increased signal-to-noise ratio.

The conventional system with an added upper threshold, for discrimination against the impulse noise, is somewhat better. This is represented by $P_9(\text{err})$. However, each of the sequential systems gives even better results. The four threshold, speech statistics system is about 1.5 decibels better, as given by $P_5(\text{err})$. The four threshold, nearest to reference system is about 2.0 decibels better, as given by $P_6(\text{err})$. The single threshold, largest of N

FIGURE 25

Probability of Error for Input Signal-to-Noise Ratios Between 9.0 and 16.0 Decibels, with Impulse Noise



P_8 (err)
P_7 (err)
P_9 (err)
P_5 (err)
P_{10} (err)
P_6 (err)
P (err)
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
8
9
10
11
12
13
14
15
16
17
18
$\frac{A^2}{2\sigma_n^2}$, DECIBELS

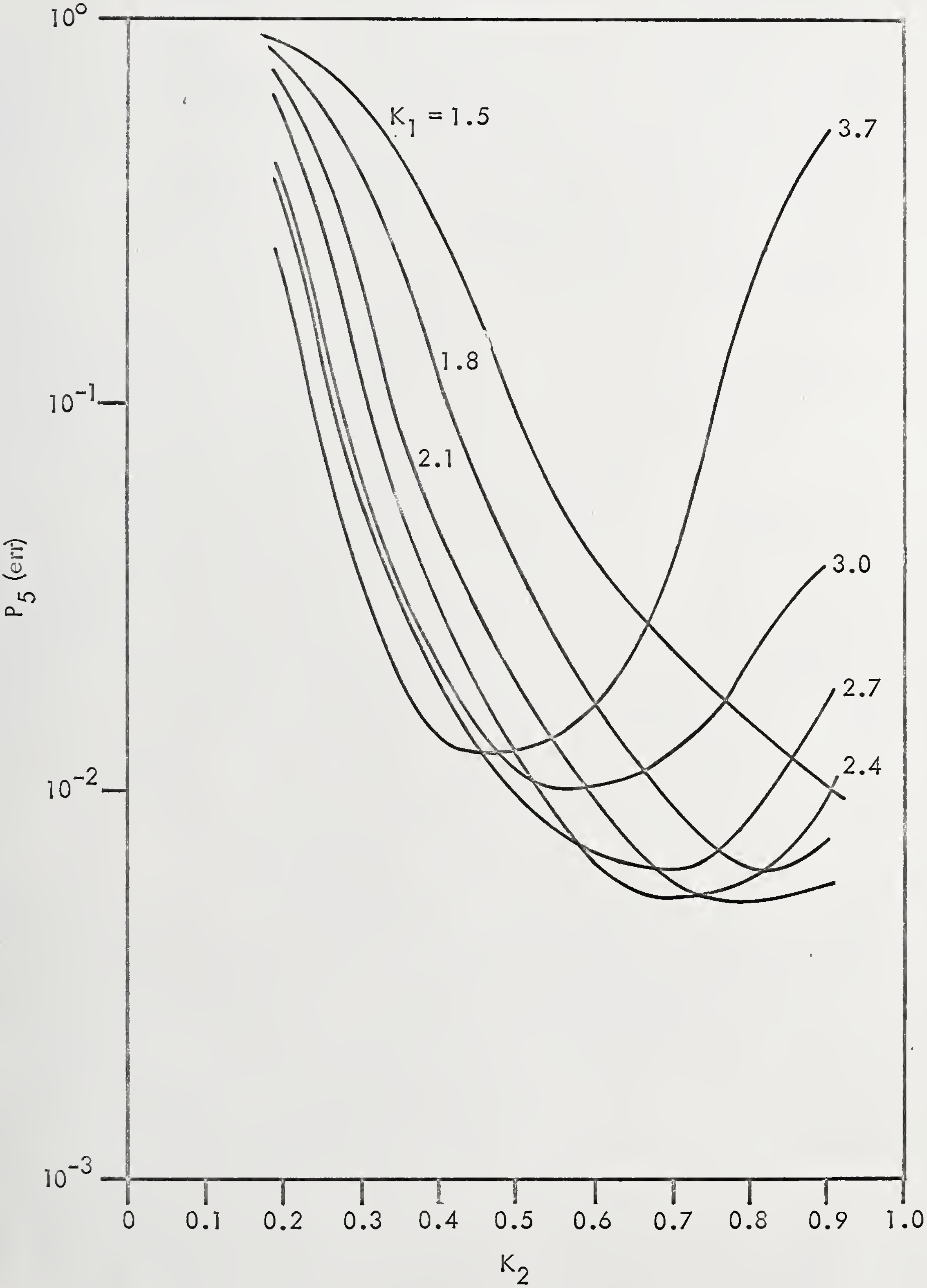
system performance is practically the same as the four threshold, nearest to reference system performance, and is slightly better. This is given by $P_{10}(err)$. However, this system is more complex in certain respects. On the average, the receiver must select one out of N-1 as the signal. For the four threshold systems, however, the selection would be made from a much smaller number. Of course, most of the time the selection process need not be resorted to for these systems.

As was the case for the systems of the preceding chapter, the improvement provided by sequential systems over the modified conventional ones is relatively small under most circumstances. In some instances, however, there may be justification for paying the cost of the increased complexity for this improvement.

A parametric study of the four threshold, speech statistics system is given in Figures 26, 27, and 28, for the case of signal-to-noise ratio of 12.1 decibels. In order to limit the required calculations to a reasonable number, the value of K_4 was taken to be 0.6 and 0.7. Based on the previous results it is felt that the optimum value of K_4 is within this range. Figure 26 shows the variation of probability of error with changes of K_1 and K_2, with K_3 and K_4 at their optimum settings. Figure 27 has K_1 and K_3 changing, with K_2 and K_4 optimum. Figure 28 has K_2 and K_3

FIGURE 26

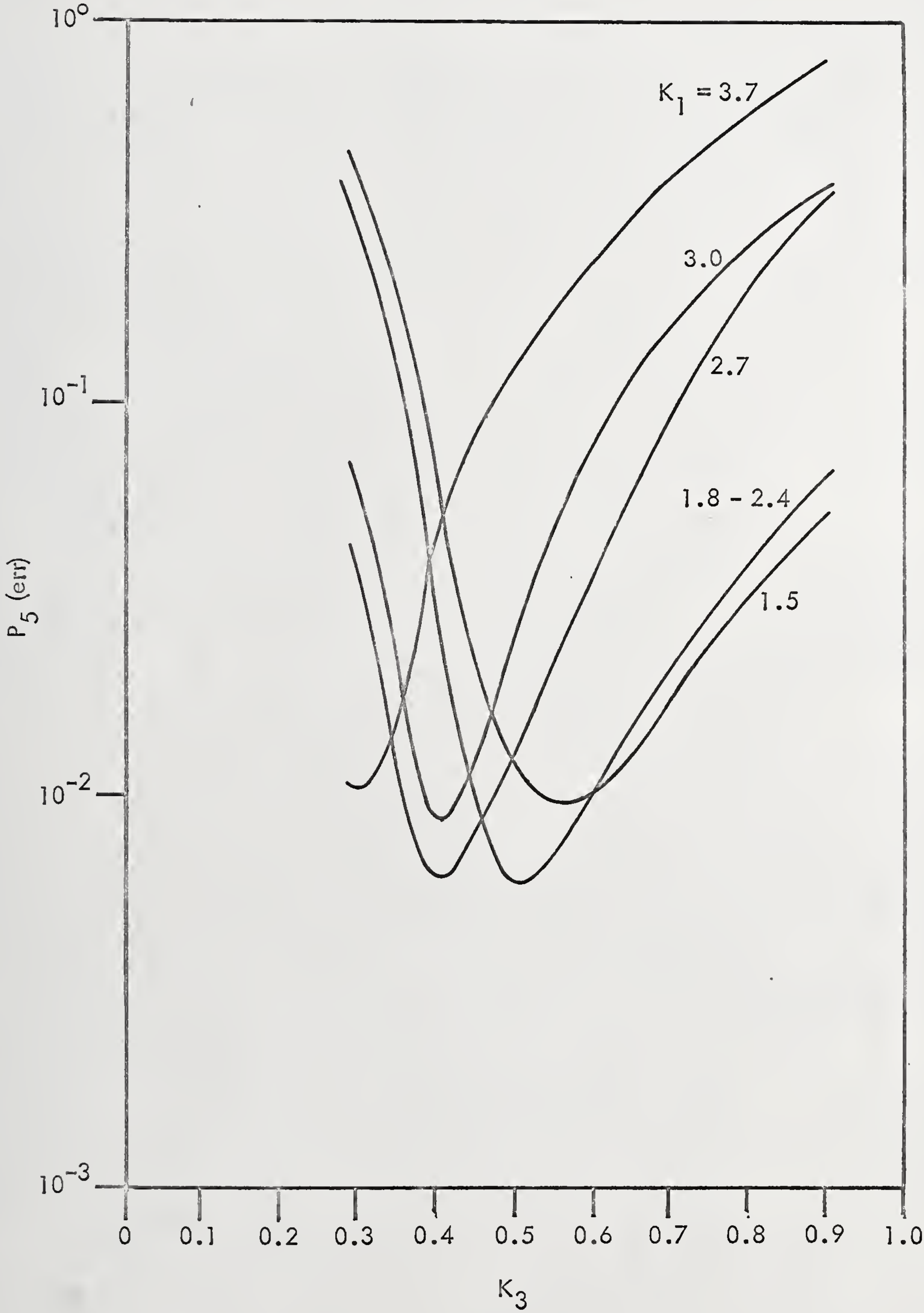
Probability of Error for Four Threshold, Speech Statistics System, with a and b at Optimum Settings, for Signal-to-Noise Ratio 12.1 Decibels



10^0
10^{-1}
10^{-2}
10^{-3}
P_5 (err)
$K_1 = 1.5$
1.8
2.1
2.4
2.7
3.0
3.7
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
K_2

FIGURE 27

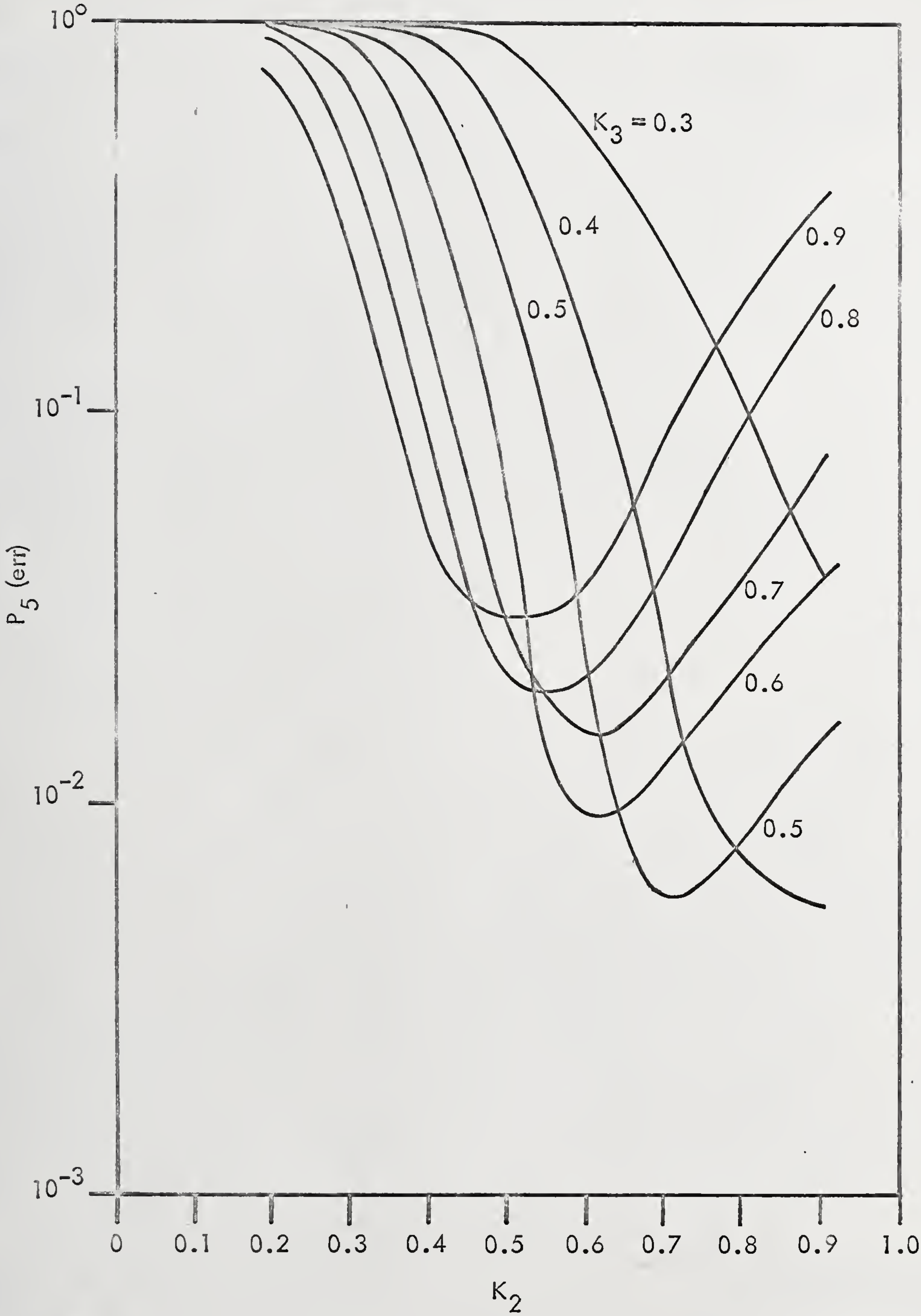
Probability of Error for Four Threshold, Speech Statistics System, with α and c at Optimum Settings, for Signal-to-Noise Ratio 12.1 Decibels



10^0
10^{-1}
10^{-2}
10^{-3}
P_5 (err)
$K_1 = 3.7$
3.0
2.7
1.8 - 2.4
1.5
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
K_3

FIGURE 28

Probability of Error for Four Threshold, Speech Statistics System, with a and d at Optimum Settings, for Signal-to-Noise Ratio 12.1 Decibels



10^{0}
10^{-1}
10^{-2}
10^{-3}
P_5 (err)
$K_3 = 0.3$
0.4
0.5
0.9
0.8
0.7
0.6
0.5
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
K_2

changing with K_1 and K_4 optimum. These three figures show the effects on error probability as thresholds are varied from their optimum settings.

In Figure 29, the optimum values of K_1, K_2, and K_3 are shown as a function of signal-to-noise ratio. They are almost constant, and could be fixed beforehand with only slight error degradation.

Similarly, for the four threshold, nearest to reference system, Figures 30, 31, and 32 provide the results of the parametric study of thresholds, and Figure 33 shows the optimum thresholds for various signal-to-noise ratios. These curves are very similar to the preceding group, and require no further comment.

Figure 34 shows an equivalent set of curves for the conventional system with a single threshold. Even with an optimum setting, as given in Figure 35, the probability of error is still very high.

For the double threshold, non-sequential case, the threshold study was performed for the cases of signal-to-noise ratios of 9.0, 11.0, 12.1, 13,0, and 16.0 decibels, depicted in Figures 36, 37, 38, 39, and 40, respectively. The minimum points can be easily located, and thus the best threshold constants determined. These follow the same pattern as those previously discussed, in that they are slowly varying with signal-to-noise ratio. Figure 41 illustrates this.

FIGURE 29

Values of K_1, K_2, K_3 for Four Threshold, Speech Statistics System for Signal-to-Noise Ratios Between 9.0 and 16.0 Decibels, with Minimum Probability of Error

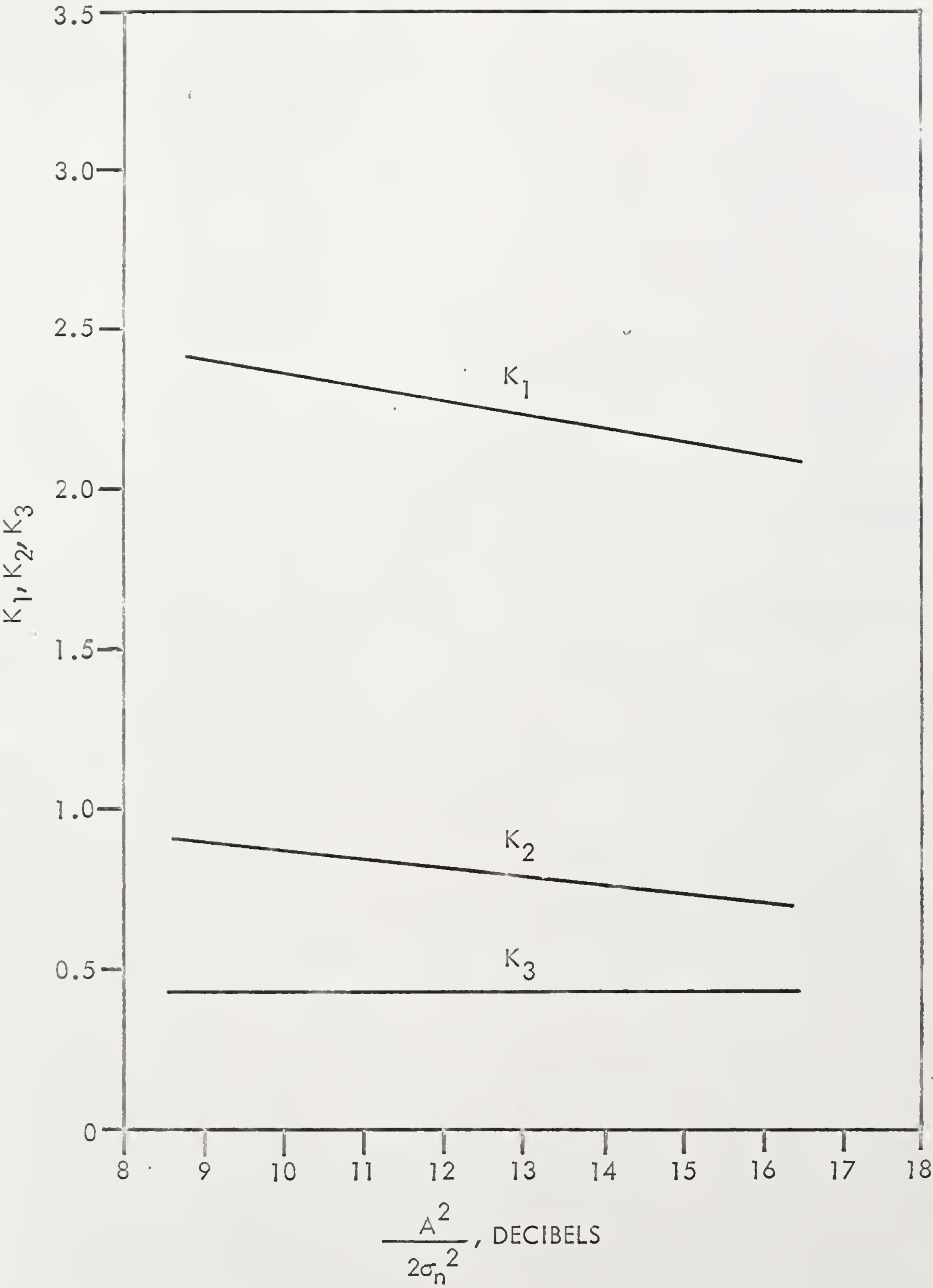

3.5
3.0
2.5
2.0
1.5
1.0
0.5
0
K_1, K_2, K_3
K_1
K_2
K_3
8
9
10
11
12
13
14
15
16
17
18
$\frac{A^2}{2\sigma_n^2}$, DECIBELS

FIGURE 30

Probability of Error for Four Threshold, Nearest to Reference System, with a and b at Optimum Settings, for Signal-to-Noise Ratio 12.1 Decibels

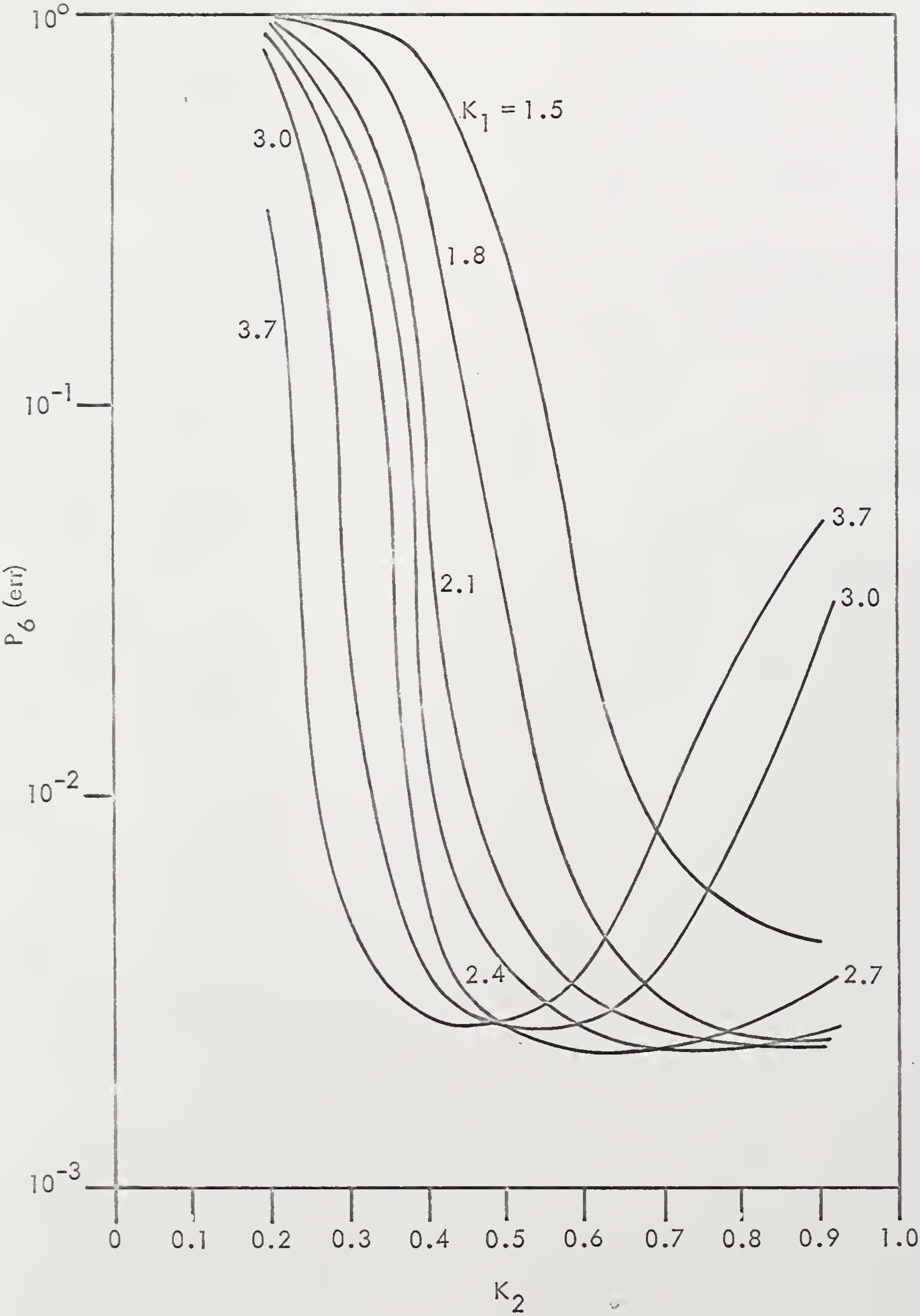

10^0
10^{-1}
10^{-2}
10^{-3}
P_6 (er)
$K_1 = 1.5$
1.8
2.1
2.4
2.7
3.0
3.7
3.0
3.7
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
K_2

FIGURE 31

Probability of Error for Four Threshold, Nearest to Reference System, with a and c at Optimum Settings, for Signal-to-Noise Ratio 12.1 Decibels

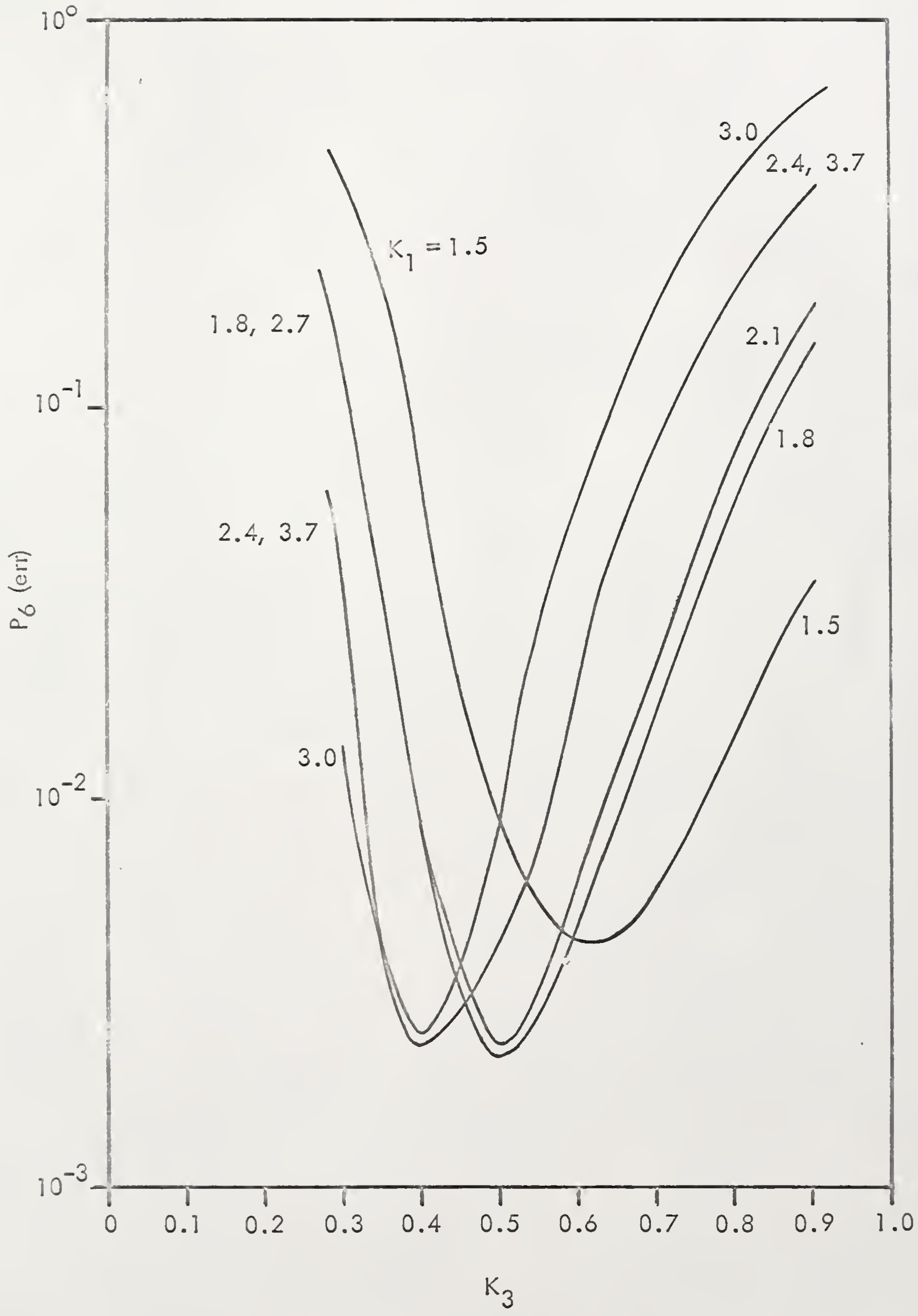
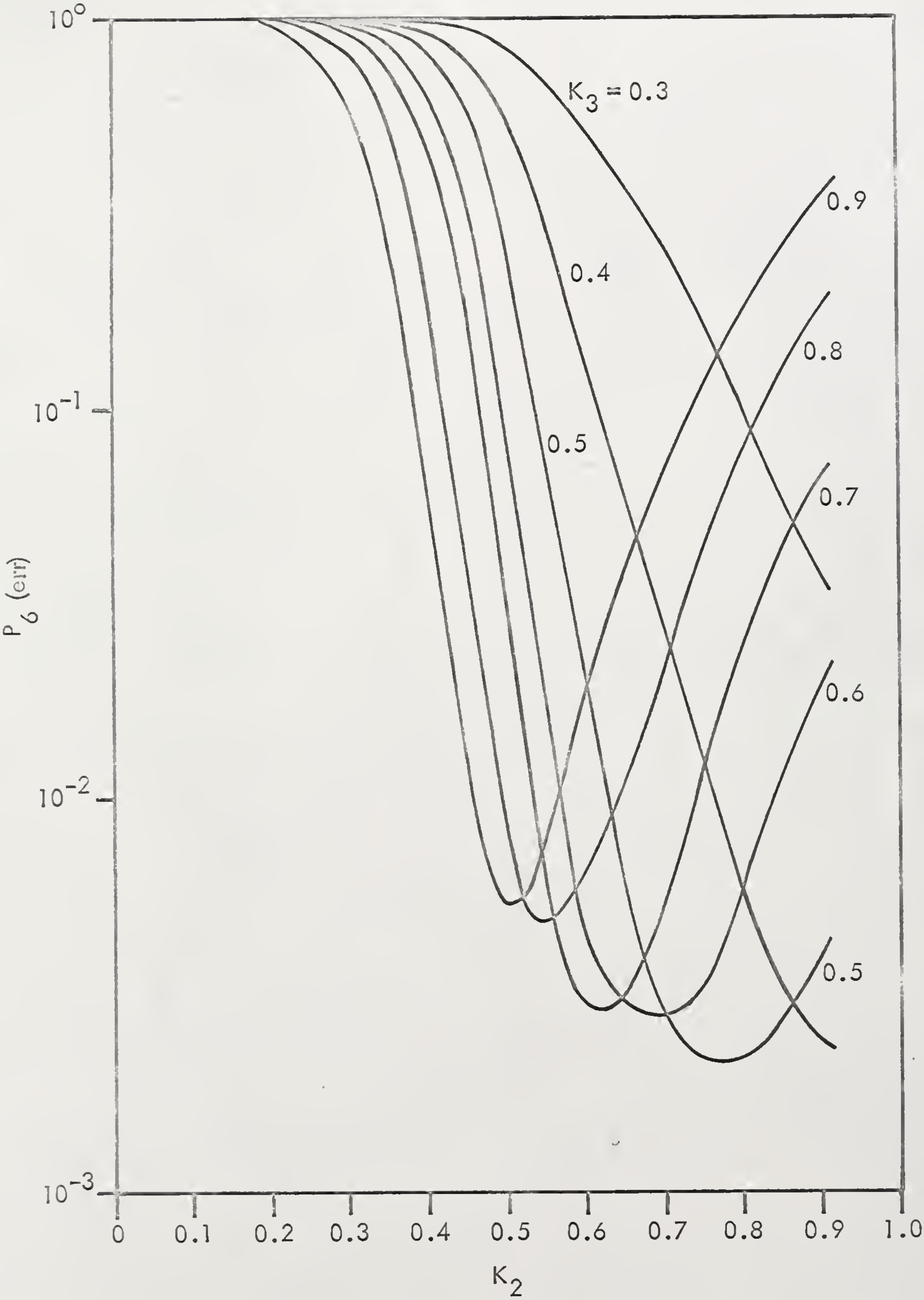

10^0
10^{-1}
10^{-2}
10^{-3}
P_6 (err)
$K_1 = 1.5$
1.8, 2.7
2.4, 3.7
3.0
3.0
2.4, 3.7
2.1
1.8
1.5
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
K_3

FIGURE 32

Probability of Error for Four Threshold, Nearest to Reference System, with a and d at Optimum Settings, for Signal-to-Noise Ratio 12.1 Decibels



10^0
10^{-1}
10^{-2}
10^{-3}
P_6 (err)
$K_3 = 0.3$
0.4
0.5
0.9
0.8
0.7
0.6
0.5
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0
K_2

FIGURE 33

Values of K_1, K_2, K_3 for Four Threshold, Nearest to Reference System for Signal-to-Noise Ratios Between 9.0 and 16.0 Decibels, with Minimum Probability of Error

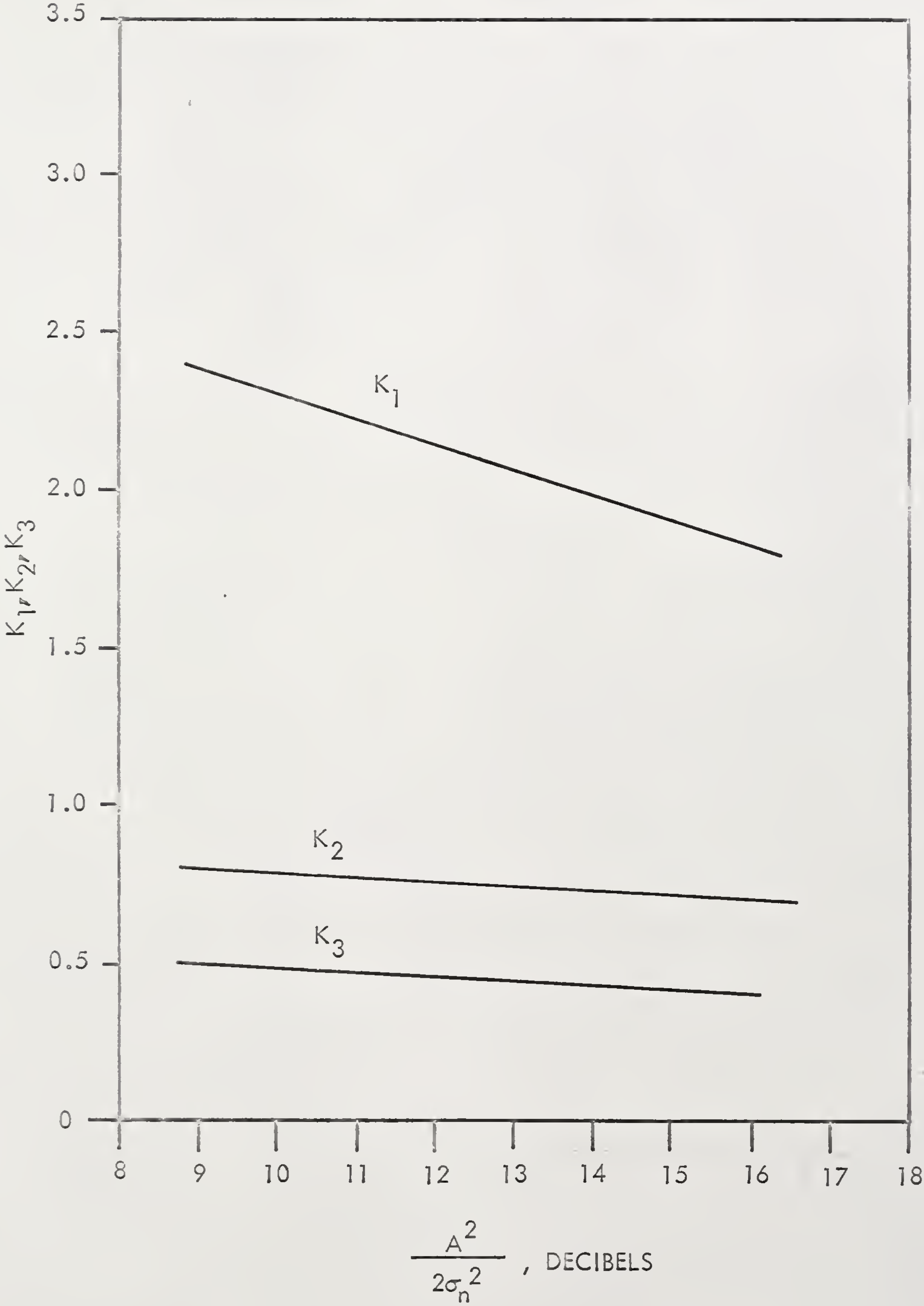

3.5
3.0
2.5
2.0
1.5
1.0
0.5
0
K_1, K_2, K_3
K_1
K_2
K_3
8
9
10
11
12
13
14
15
16
17
18
$\frac{A^2}{2\sigma_n^2}$, DECIBELS

FIGURE 34

Probability of Error for Single Threshold, Impulse Noise System for $K_1 \cdot K_2 \cdot K_3$ Between 0 and 2.0 and for Signal-to-Noise Ratios Between 9.0 and 16.0 Decibels

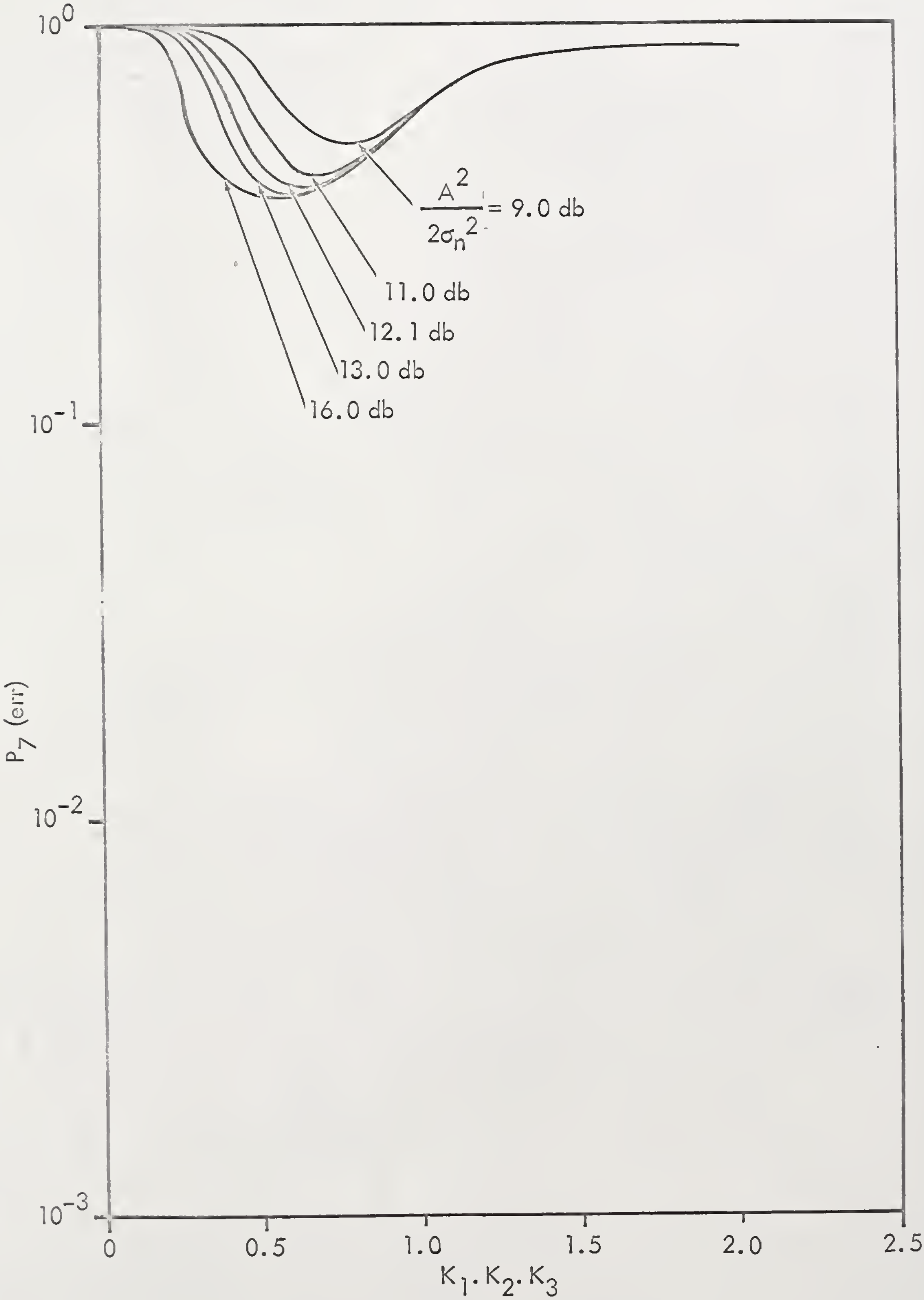

10^0
10^{-1}
10^{-2}
10^{-3}
P_7 (err)
$\frac{A^2}{2\sigma_n^2} = 9.0$ db
11.0 db
12.1 db
13.0 db
16.0 db
0
0.5
1.0
1.5
2.0
2.5
$K_1 \cdot K_2 \cdot K_3$

FIGURE 35

Value of $K_1 \cdot K_2 \cdot K_3$ for Single Threshold, Impulse Noise System for Signal-to-Noise Ratios Between 9.0 and 16.0 Decibels, and with Minimum Probability of Error

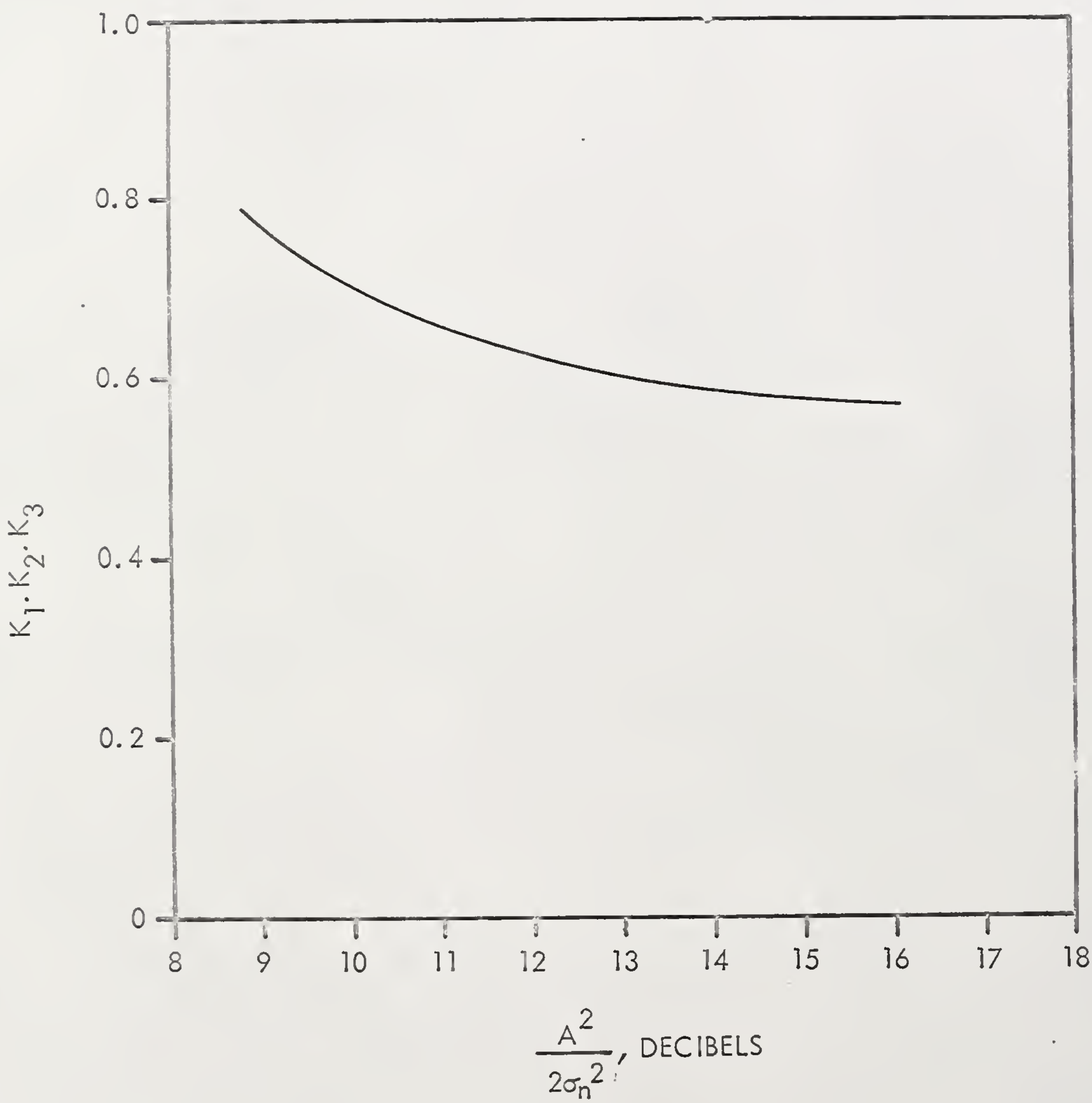
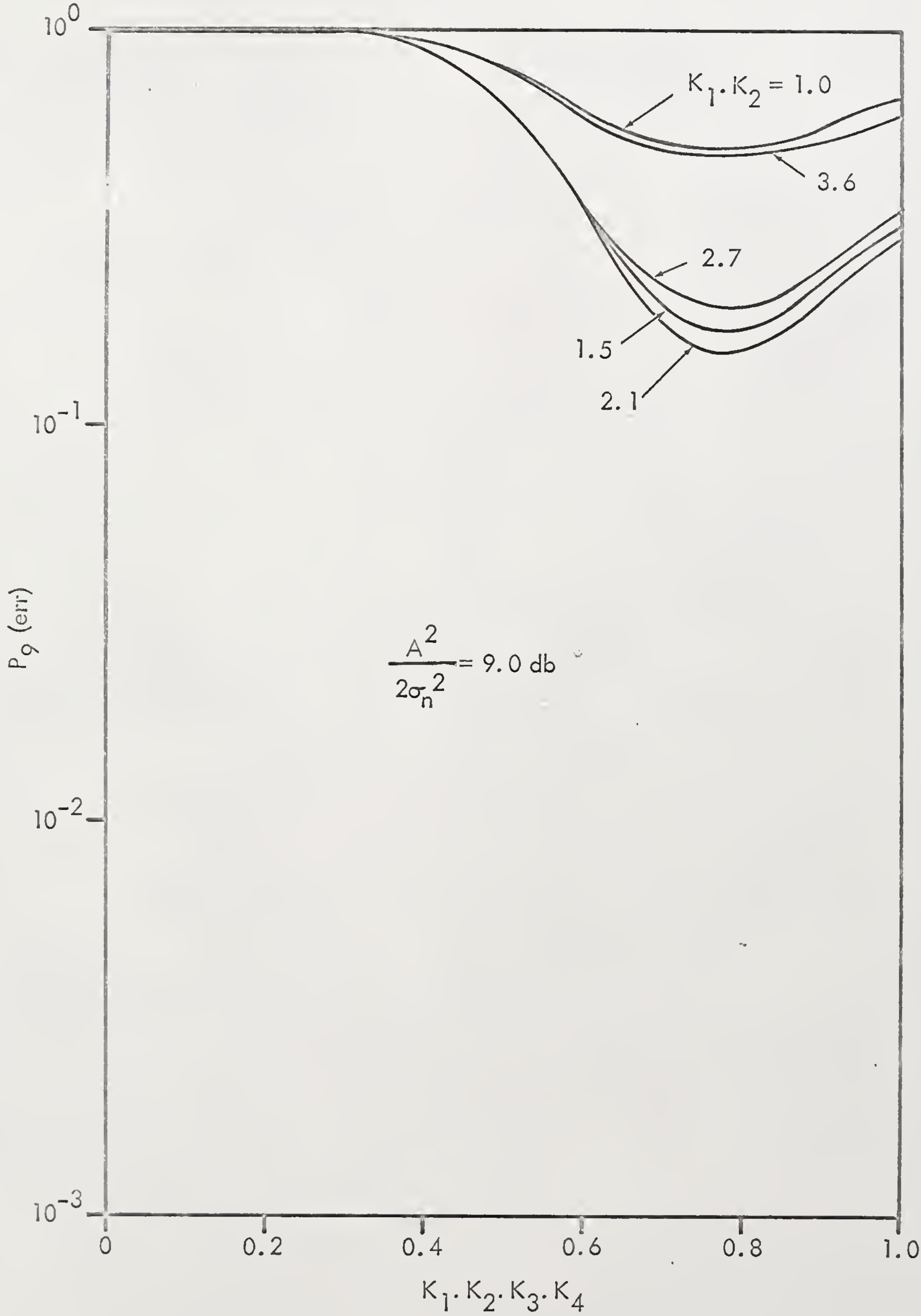

1.0
0.8
0.6
0.4
0.2
0
$K_1 \cdot K_2 \cdot K_3$
8
9
10
11
12
13
14
15
16
17
18
$\frac{A^2}{2\sigma_n^2}$, DECIBELS

FIGURE 36

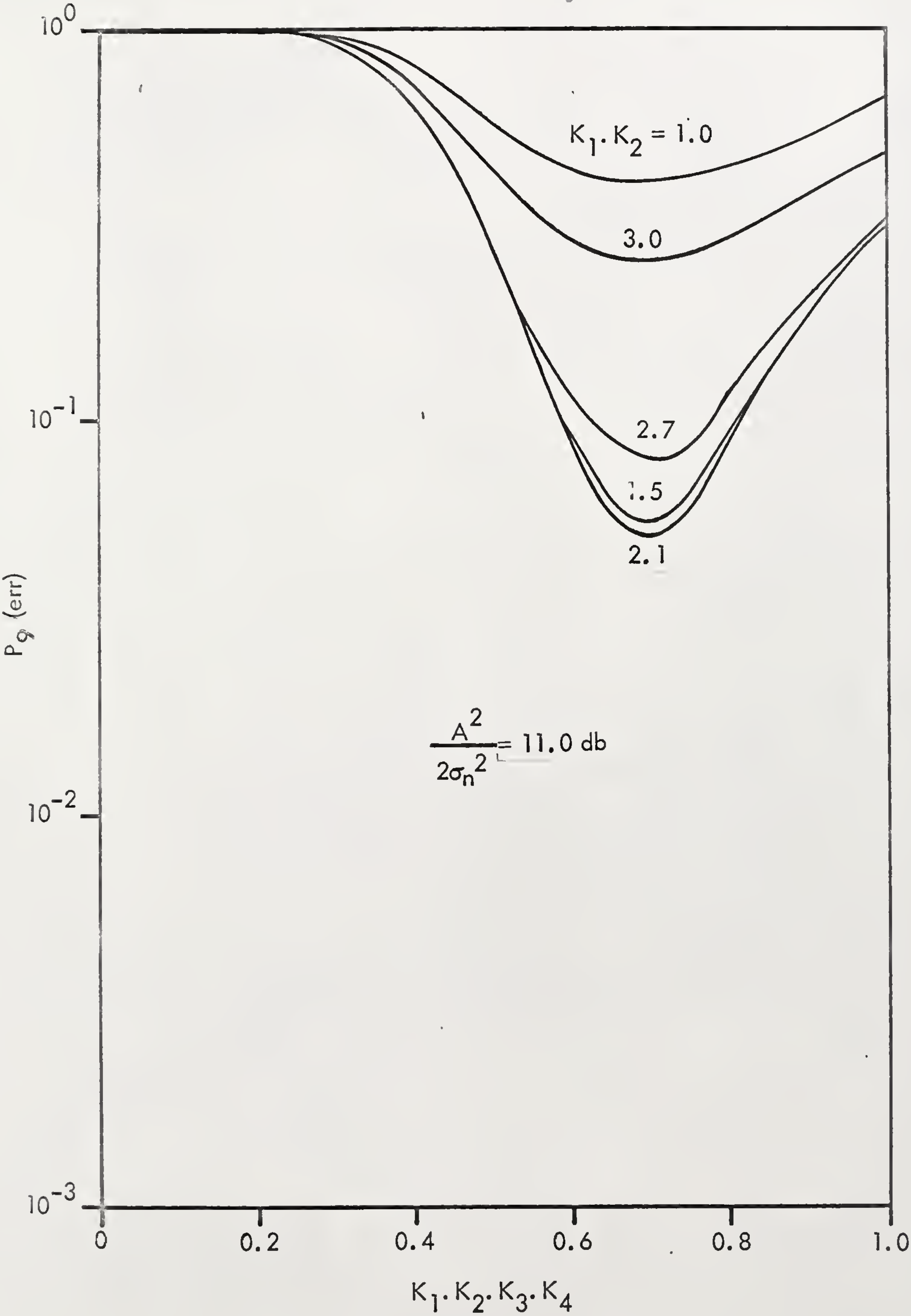
Probability of Error for Double Threshold, Impulse Noise System for $K_1 \cdot K_2 \cdot K_3 \cdot K_4$ Between 0 and 1.0 and $K_1 \cdot K_2$ Between 1.0 and 3.6, and for Signal-to-Noise Ratio 9.0 Decibels



10^0
10^{-1}
10^{-2}
10^{-3}
P_9 (err)
$K_1 \cdot K_2 = 1.0$
3.6
2.7
1.5
2.1
$\frac{A^2}{2\sigma_n^2} = 9.0$ db
0
0.2
0.4
0.6
0.8
1.0
$K_1 \cdot K_2 \cdot K_3 \cdot K_4$

FIGURE 37

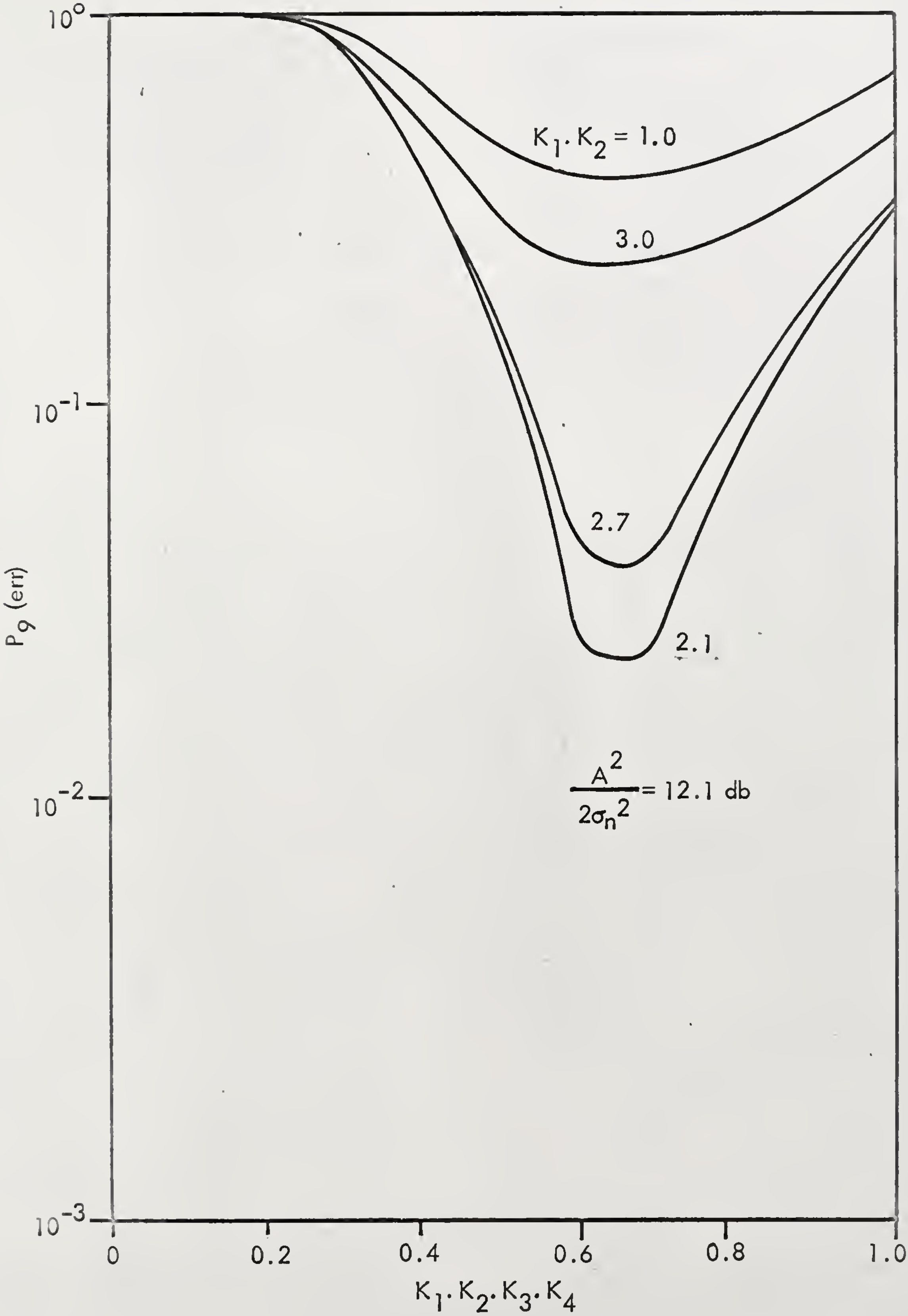
Probability of Error for Double Threshold, Impulse Noise System for $K_1 \cdot K_2 \cdot K_3 \cdot K_4$ Between 0 and 1.0, and $K_1 \cdot K_2$ Between 1.0 and 3.6, and for Signal-to-Noise Ratio 11.0 Decibels



10^0
10^{-1}
10^{-2}
10^{-3}
P_q (err)
$K_1 \cdot K_2 = 1.0$
3.0
2.7
1.5
2.1
$\frac{A^2}{2\sigma_n^2} = 11.0$ db
0
0.2
0.4
0.6
0.8
1.0
$K_1 \cdot K_2 \cdot K_3 \cdot K_4$

FIGURE 38

Probability of Error for Double Threshold, Impulse Noise System for $K_1 \cdot K_2 \cdot K_3 \cdot K_4$ Between 0 and 1.0, and $K_1 \cdot K_2$ Between 1.0 and 3.6, and for Signal-to-Noise Ratio 12.1 Decibels



10^0
10^{-1}
10^{-2}
10^{-3}
P_9 (err)
$K_1 \cdot K_2 = 1.0$
3.0
2.7
2.1
$\frac{A^2}{2\sigma_n^2} = 12.1$ db
0
0.2
0.4
0.6
0.8
1.0
$K_1 \cdot K_2 \cdot K_3 \cdot K_4$

FIGURE 39

Probability of Error for Double Threshold, Impulse Noise System for $K_1 \cdot K_2 \cdot K_3 \cdot K_4$ Between 0 and 1.0, and $K_1 \cdot K_2$ Between 1.0 and 3.6, and for Signal-to-Noise Ratio 13.0 Decibels

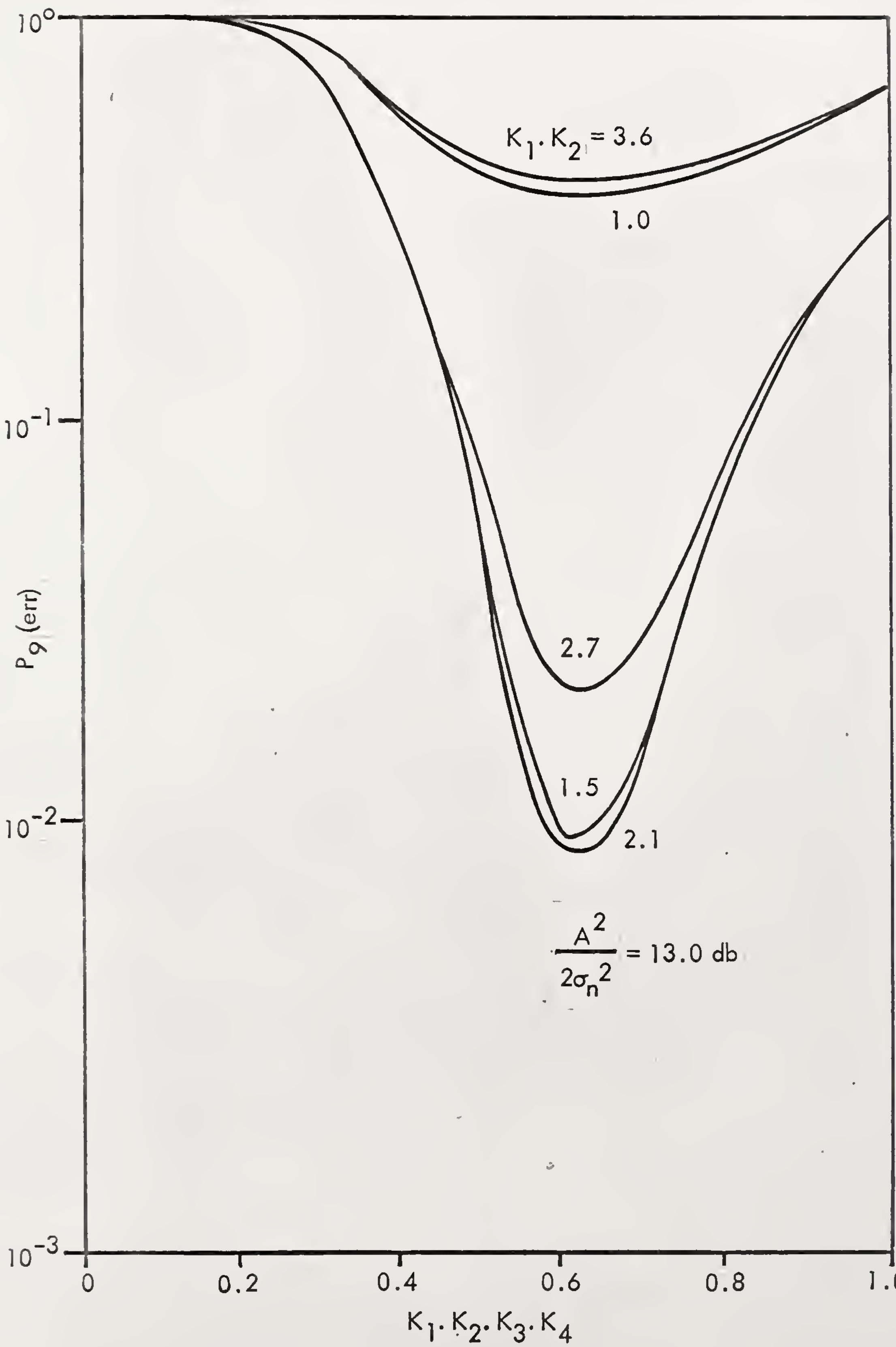

10^0
10^{-1}
10^{-2}
10^{-3}
P_9 (err)
$K_1 \cdot K_2 = 3.6$
1.0
2.7
1.5
2.1
$\frac{A^2}{2\sigma_n^2} = 13.0$ db
0
0.2
0.4
0.6
0.8
1.0
$K_1 \cdot K_2 \cdot K_3 \cdot K_4$

FIGURE 40

Probability of Error for Double Threshold, Impulse Noise System for $K_1 \cdot K_2 \cdot K_3 \cdot K_4$ Between 0 and 1.0, and $K_1 \cdot K_2$ Between 1.0 and 3.6, and for Signal-to-Noise Ratio 16.0 Decibels

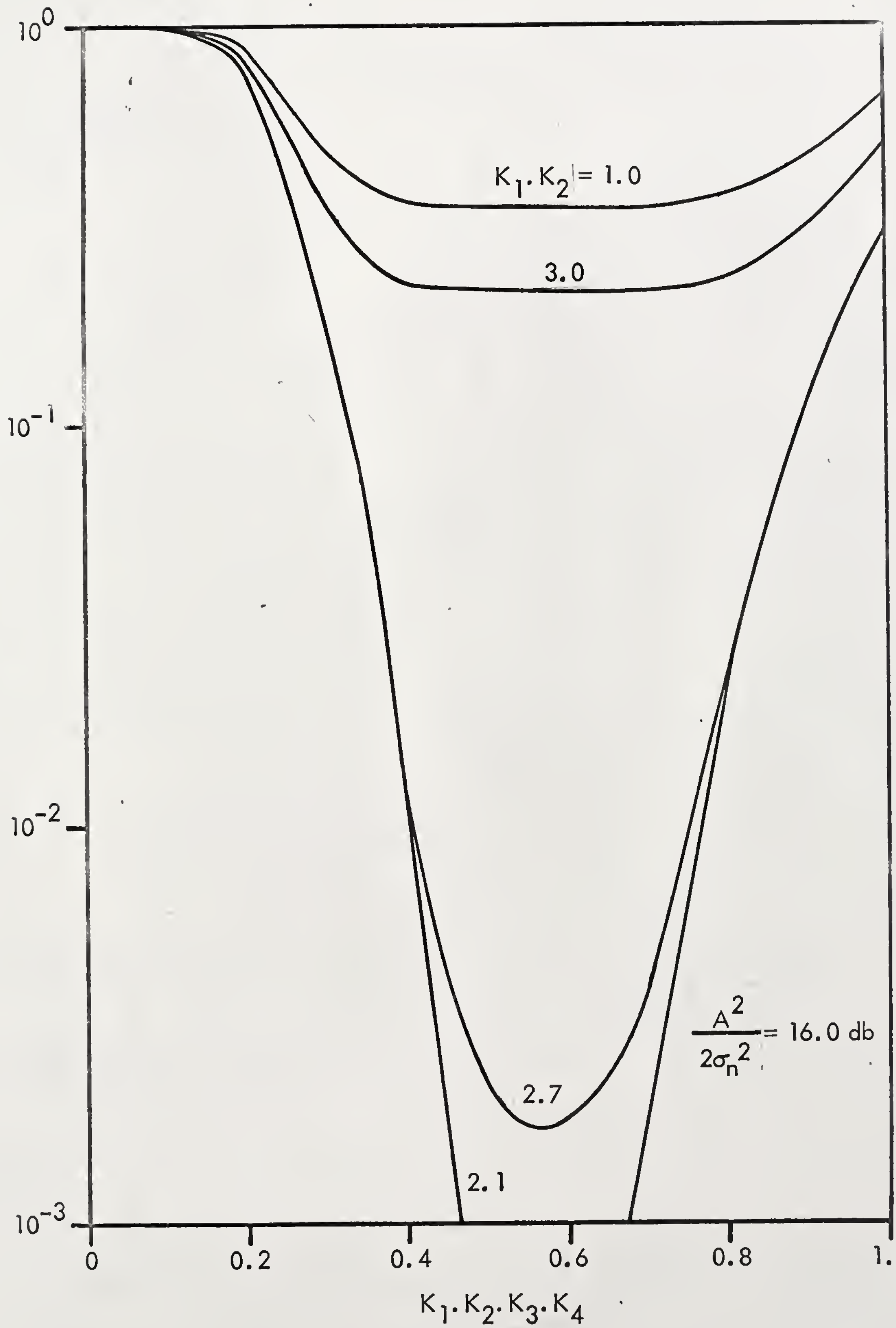
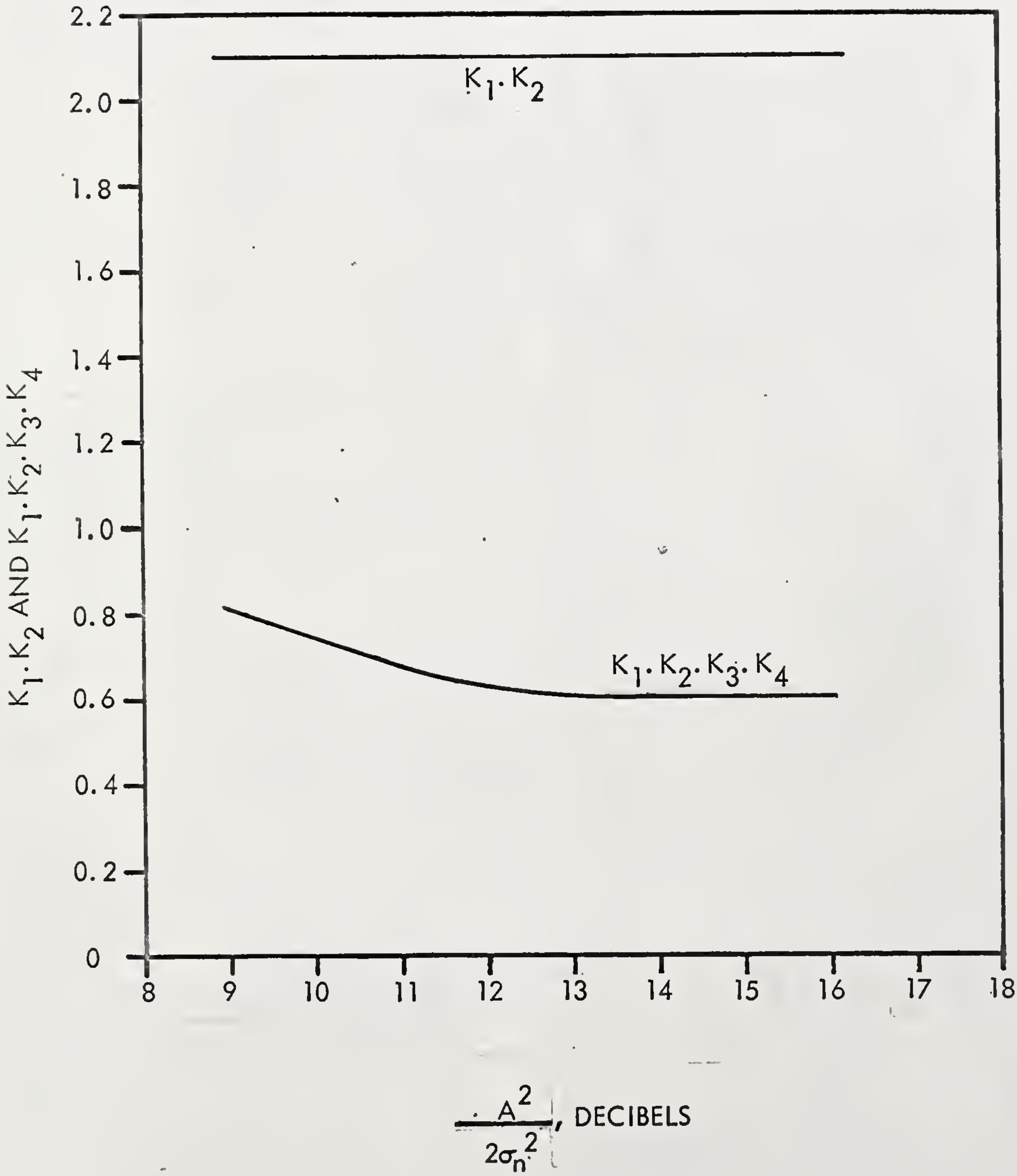

10^0
10^{-1}
10^{-2}
10^{-3}
$K_1 \cdot K_2 = 1.0$
3.0
2.7
2.1
$\frac{A^2}{2\sigma_n^2} = 16.0$ db
0
0.2
0.4
0.6
0.8
1.0
$K_1 \cdot K_2 \cdot K_3 \cdot K_4$

FIGURE 41

Values of $K_1 \cdot K_2$ and $K_1 \cdot K_2 \cdot K_3 \cdot K_4$ for Double Threshold, Impulse Noise System for Signal-to-Noise Ratios Between 9.0 and 16.0 Decibels, with Minimum Probability of Error

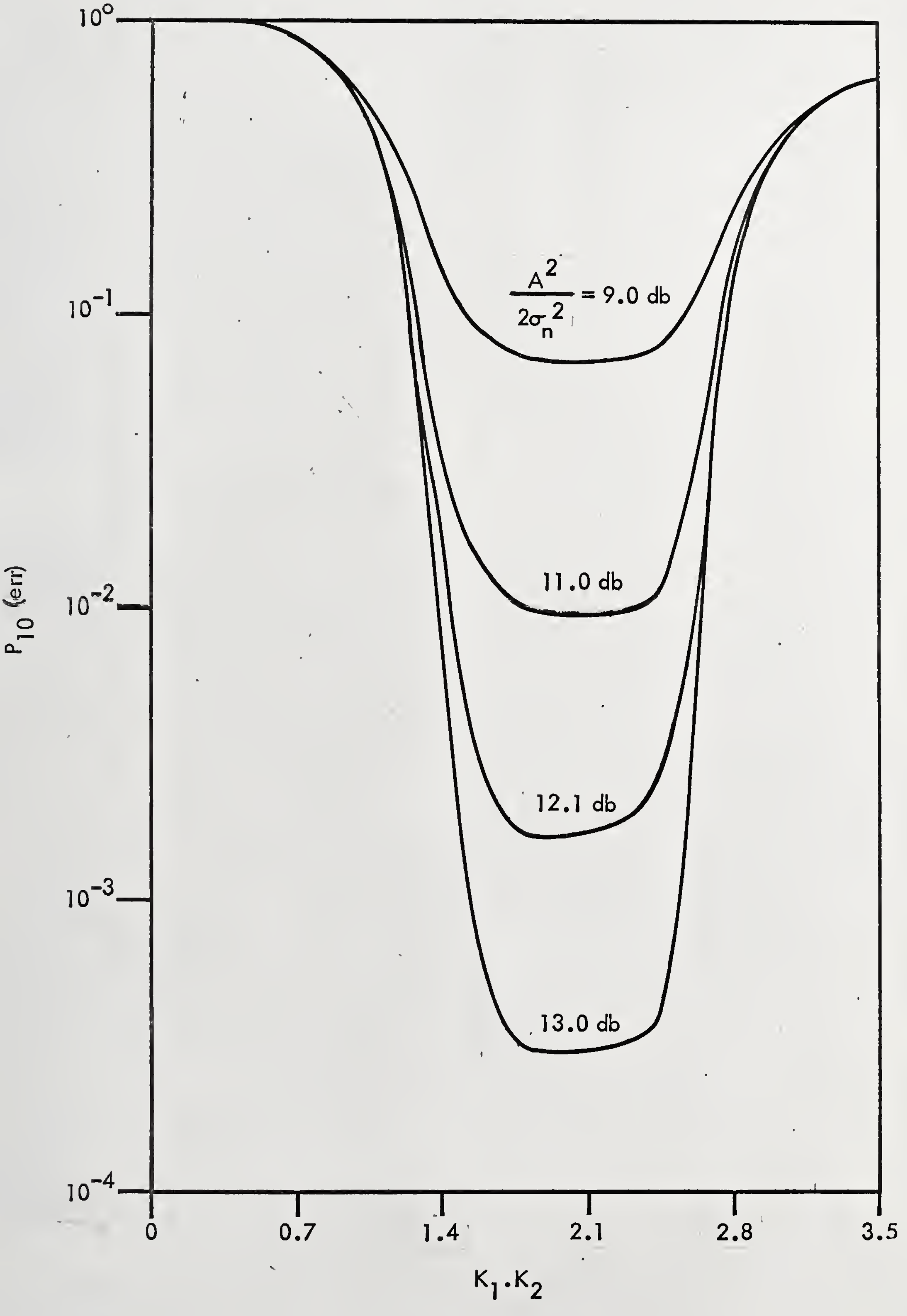

2.2
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
8
9
10
11
12
13
14
15
16
17
18
$K_1 \cdot K_2$
$K_1 \cdot K_2 \cdot K_3 \cdot K_4$
$K_1 \cdot K_2$ AND $K_1 \cdot K_2 \cdot K_3 \cdot K_4$
$\frac{A^2}{2\sigma_n^2}$, DECIBELS

The last figures represent the single threshold, largest of N system. Figure 42 illustrates clearly how drastically the probability of error depends upon the value of the threshold constant. For a fairly large range of values, the error probability is not greatly affected. However, once outside this range the change is large and rapid. Again, the minimum points occur at approximately the same threshold setting. This result is also seen in Figure 43.

The report in this chapter of the impulse-noise systems is now complete. Conventional, modified conventional, and sequential systems have been studied rather extensively from the point of view of error performance under various conditions of interference. Though the apparent gain from the sequential systems of up to 2.0 decibels is small, there may be justification under certain conditions for using these techniques. In any case, the choice is available.

FIGURE 42

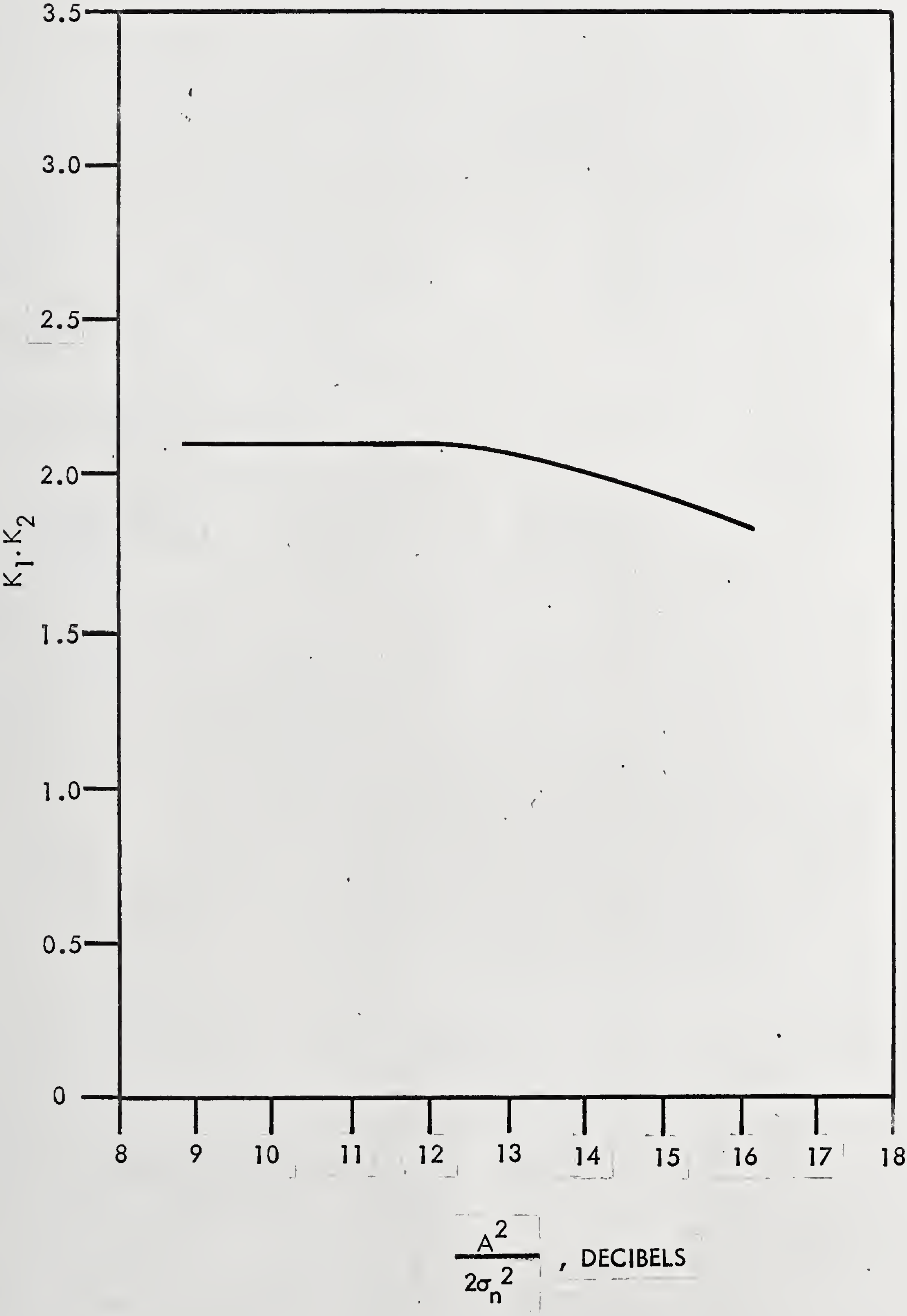
Probability of Error for Single Threshold, Largest of N System for $K_1 \cdot K_2$ Between 0.3 and 3.5, and for Signal-to-Noise Ratios Between 9.0 and 13.0 Decibels



10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
P_{10} (err)
$\frac{A^2}{2\sigma_n^2}$ = 9.0 db
11.0 db
12.1 db
13.0 db
0
0.7
1.4
2.1
2.8
3.5
$K_1 . K_2$

FIGURE 43

Value of $K_1 \cdot K_2$ for Single Threshold, Largest of N System for Signal-to-Noise Ratios Between 9.0 and 16.0 Decibels, and with Minimum Probability of Error


3.5
3.0
2.5
2.0
1.5
1.0
0.5
0
$K_1 \cdot K_2$
8
9
10
11
12
13
14
15
16
17
18
$\frac{A^2}{2\sigma_n^2}$, DECIBELS

CHAPTER IV

CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE RESEARCH

Conclusions

The studies reported here are for conventional, modified conventional, and sequential quantized pulse position modulation systems. Based on these studies, for cases with and without impulse noise, it appears that added complexity required for sequential techniques in voice applications would not in general justify their use in the form described here. Possible exceptions to this statement exist where there are special circumstances, requiring the added 1.5 to 2.0 decibels of possible improvement in signal-to-noise ratio.

More important, it is felt, is the fact that this research represents only the initial effort to apply Wald's sequential techniques to voice communications. When these techniques are applied to other modulation methods, perhaps in forms entirely different from those discussed here, the potential improvement may be high. The conclusion is reached, therefore, that more research should be performed in this area. Recommendations for future research are given in the following.

Future Research

There are many potential areas for future research in connection with sequential voice communications techniques. Some of them will be discussed in the following.

All of the work reported herein dealt with the quantized pulse position modulation method. It would appear desirable to find out if similar techniques could be used advantageously with other modulation methods, such as pulse code modulation (PCM), quantized frequency modulation (QFM), and quantized phase modulation (QPM).

The speech probability functions used here were derived from somewhat limited experimental data, considering the fact that only three talkers were involved. The use of many talkers to arrive at a set of speech statistics might lead to a change in the result. Also, a particular application, such as military, might require the use of a somewhat specialized vocabulary in obtaining the functions. This should be studied.

Several direct extensions of the work here are possible. Under some conditions it might be desirable to compare the systems on the basis of output signal-to-noise ratio, rather than probability of error. The analytical relationship between these two quantities would be of interest. Also of interest is the relationship between speech intelligibility and probability of error for each of the systems. Finding of this would involve an experimental program of fairly large proportions. The tradeoff between bandwidth and probability of error would also be of value.

The possibility of transmitting non-real time data over a portion of the frame has been mentioned previously. Much research is required before this would be possible. A comparison of the benefits, if any, of using error-correction codes with voice to the benefits of sequential techniques would be in order. The amplitude of the impulse-like noise has been considered constant here. It would be desirable to consider the effects of variable amplitude, with a suitable probability density function for the amplitude. Similarly, the threshold settings are taken as constant during a frame. Better performance might result from a setting that varied from slot to slot. It seems a difficult task to evaluate this.

The systems here have all operated on exclusive frequency channels. The same techniques have application in co-channel, frequency-time matrix coded systems. The results have not yet been evaluated for this application.

As is probably obvious from the above paragraphs, there are still many areas which need to be explored in connection with applying Wald's sequential techniques to voice communication problems. This report represents only the beginning.

APPENDICES

APPENDIX A

Speech Probability Functions

This appendix describes the mathematical model used in this report for the marginal and conditional probability functions for conversational speech. Here we are concerned with the statistics of speech in its entirety, rather than with statistics of individual speech sounds.

The experimental data which provide the basis for this model were obtained by Davenport.[12] He was concerned with $W_1(x)$, the first probability density function of the instantaneous speech amplitude, and with $W_2(x_1/x_2, \tau)$, the conditional density for amplitude x_1 when the value τ seconds before was x_2.

Considering first $W_1(x)$, it is seen that it consists of two parts. There is a spike of approximately Gaussian shape which represents the unvoiced sounds. This part of the function is significant only for small amplitudes.

Superimposed on this is a function which represents the voiced sounds. This is approximately exponential. Davenport gives as the result

$$W_1(x) \cong \frac{0.6}{\sqrt{2}\cdot\sigma_1}\cdot e^{-\frac{\sqrt{2}\,|x|}{\sigma_1}} + \frac{0.4}{\sqrt{2\pi}\cdot\sigma_2}\cdot e^{-\frac{x^2}{2\sigma_2^2}} \qquad \text{(A.1)}$$

where the random variable has been normalized so as to have a mean square value of one. The approximate values for σ_1 and σ_2 are, respectively, 1.23 and 0.118.

With this function the probability is about 0.003 that the instantaneous speech amplitude will exceed a value of four. Therefore the maximum amplitude with which we must deal is selected as four. The range of values from minus four to plus four is to be quantized into N levels, of length $\frac{8}{N}$ each. The result for the probability that the speech amplitude will be in the j^{th} position when applied in a quantized pulse position modulation system is

$$P_s(j) \cong \int_{\frac{8}{N}(j-1-\frac{N}{2})}^{\frac{8}{N}(j-\frac{N}{2})} W_1(x)\cdot dx \qquad ; \; j = 1,2,3,\text{---}\, N \tag{A.2}$$

Performing the integration leads to

$$\begin{aligned} P_s(j) \cong\; & 0.3\left[\pm e^{\pm\{j\,1.15(\frac{8}{N}) - 4.6\}} \mp e^{\pm\{(j-1)1.15(\frac{8}{N}) - 4.6\}}\right] \\ & + 0.1\left[\operatorname{erf}\left\{j\left(\tfrac{48}{N}\right) - 24\right\} - \operatorname{erf}\left\{(j-1)\left(\tfrac{48}{N}\right) - 24\right\}\right] \end{aligned} \tag{A.3}$$

Where the upper sign is to be used for $1 \leq j \leq \frac{N}{2}$ and the lower sign for $\frac{N}{2} + 1 \leq j \leq N$.

The available data for $W_2(x_1|x_2,\tau)$ are for the case of τ equal to 92 microseconds. Based on a sampling frequency of 8000 samples per second, the speech sampling period is 125 microseconds. $W_2(x_1|x_2,\tau)$ is approximately exponential for the case shown. The same shape will be taken for τ equal to 125 microseconds with the peak of the density function reduced. The peak occurs at x_1 equal to x_2. As x_2 gets further from the center position the value of the peak goes up, according to Davenport's results. This factor is taken into account by the fact that the end points of the density function must be clipped off because of the finite length of the sample interval. The required normalizing factor becomes larger as x_2 moves further from the center.

The probability that the speech sample's amplitude corresponds to the j^{th} position, given that the previous position was the ℓ^{th}, is thus given by

$$P_s(j/\ell) = \frac{\int_{x_1-\frac{4}{N}}^{x_1+\frac{4}{N}} W_2(x/x_2)\cdot dx}{\int_0^8 W_2(x/x_2)\cdot dx} \tag{A.4}$$

where x_1 and x_2 must be replaced by their corresponding values of j and ℓ. That is,

$$\begin{aligned} x_1 &= j\left(\tfrac{8}{N}\right) - \tfrac{4}{N} \\ x_2 &= \ell\left(\tfrac{8}{N}\right) - \tfrac{4}{N} \end{aligned} \tag{A.5}$$

The approximately correct expression for $W_2(x/x_2)$ is

$$W_2(x/x_2) \cong 0.5\cdot e^{-|x-x_2|} \tag{A.6}$$

Substituting these leads to

$$P_s(j/\ell) = \frac{\displaystyle\int_{(j-\ell)\frac{8}{N}-\frac{4}{N}}^{(j-\ell)\frac{8}{N}+\frac{4}{N}} e^{-|y|}\cdot dy}{\displaystyle\int_{-\ell(\frac{8}{N})+\frac{4}{N}}^{8-\ell(\frac{8}{N})+\frac{4}{N}} e^{-|y|}\cdot dy} \tag{A.7}$$

The final result of this integration is

$$P_s(j/\ell) = \frac{[e^{\frac{4}{N}} - e^{-\frac{4}{N}}]\cdot e^{-|j-\ell|\cdot(\frac{8}{N})}}{2 - e^{-\frac{4}{N}}\cdot\left[e^{-(\ell-1)(\frac{8}{N})} + e^{-8(1-\frac{\ell}{N})}\right]} \; ; \; j \neq \ell$$

$$= \frac{2\cdot[1 - e^{-\frac{4}{N}}]}{2 - e^{-\frac{4}{N}}\cdot\left[e^{-(\ell-1)(\frac{8}{N})} + e^{-8(1-\frac{\ell}{N})}\right]} \; ; \; j = \ell$$

(A.8)

APPENDIX B

Error Bound for Approximate Evaluation of the Integral in $P_8(\text{cor})$

The probability $P_8(\text{cor})$ includes the integral

$$\int_0^\infty \frac{R_2}{\sigma_n^2}\cdot e^{-\frac{R_2^2+A^2}{2\sigma_n^2}}\cdot I_0\left(\frac{AR_2}{\sigma_n^2}\right)\cdot\left[1-Q\left(\frac{B}{\sigma_n},\frac{R_2}{\sigma_n}\right)\right]^n\cdot$$

$$\cdot\left[1-e^{-\frac{R_2^2}{2\sigma_n^2}}\right]^{N-1-n}\cdot dR_2 \qquad \text{(B.1)}$$

This integral must be evaluated by approximate methods, requiring a finite upper limit of integration. The limit will be determined so that we can replace the above integral with

$$\int_0^H \frac{R_2}{\sigma_n^2}\cdot e^{-\frac{R_2^2+A^2}{2\sigma_n^2}}\cdot I_0\left(\frac{AR_2}{\sigma_n^2}\right)\cdot\left[1-Q\left(\frac{B}{\sigma_n},\frac{R_2}{\sigma_n}\right)\right]^n\cdot$$

$$\cdot\left[1-e^{-\frac{R_2^2}{2\sigma_n^2}}\right]^{N-1-n}\cdot dR_2 \qquad \text{(B.2)}$$

The error involved by this substitution is

$$\text{Err} = \int_H^\infty \frac{R_2}{\sigma_n^2}\cdot e^{-\frac{R_2^2+A^2}{2\sigma_n^2}}\cdot I_0\left(\frac{AR_2}{\sigma_n}\right)\cdot\left[1-Q\left(\frac{B}{\sigma_n},\frac{R_2}{\sigma_n}\right)\right]^n\cdot$$

$$\cdot\left[1-e^{-\frac{R_2^2}{2\sigma_n^2}}\right]^{N-1-n}\cdot dR_2 \qquad \text{(B.3)}$$

Now,

$$\left[1-Q\left(\frac{B}{\sigma_n},\frac{R_2}{\sigma_n}\right)\right]^n \leq 1 \qquad \text{(B.4)}$$

and

$$\left[1 - e^{-\frac{R_2^2}{2\sigma_n^2}}\right]^{N-1-n} \leq 1 \tag{B.5}$$

An upper bound of the error is therefore given by

$$\text{Err} \leq \int_H^\infty \frac{R_2}{\sigma_n^2} \cdot e^{-\frac{R_2^2+A^2}{2\sigma_n^2}} \cdot I_0\left(\frac{AR_2}{\sigma_n^2}\right) \cdot dR_2$$

$$= Q\left(\frac{A}{\sigma_n}, \frac{H}{\sigma_n}\right) \tag{B.6}$$

If the magnitude of the error is to be no greater than 10^{-5}, we have to solve

$$Q\left(\frac{A}{\sigma_n}, \frac{H}{\sigma_n}\right) = 0.00001 \tag{B.7}$$

As A/σ_n increases, the value of H/σ_n also increases. Taking the largest value of A/σ_n that is of interest in this report, that is

$$\frac{A}{\sigma_n} = 19 \text{ db} = 8.9 \tag{B.8}$$

we get

$$Q\left(8.9, \frac{H}{\sigma_n}\right) = 0.00001 \tag{B.9}$$

The solution yields the desired value

$$\frac{H}{\sigma_n} \cong 13.2 \tag{B.10}$$

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BIOGRAPHICAL SKETCH

Ray Howard Pettit was born May 12, 1933, at Canton, Georgia. In May, 1950, he was graduated from Canton High School. In June, 1954, he received the degree of Bachelor of Electrical Engineering from the Georgia Institute of Technology. From 1954 through 1958, he was an engineer for Westinghouse Electric Corporation. During this period, from March, 1955, until March, 1957, he was on a leave of absence while serving as an officer in the Ordnance Corps of the United States Army. In 1959 he enrolled in the Graduate School of the Georgia Institute of Technology. He earned the degree of Master of Science in Electrical Engineering, awarded in June, 1960. From January, 1960, until September, 1961, and from April, 1963, until November, 1963, he performed system design and analysis tasks for the Martin Company at Orlando, Florida. In September, 1961, he enrolled in the Graduate School of the University of Florida to work toward the degree of Doctor of Philosophy. In November, 1963, he began communications theory research for the Lockheed-Georgia Company at Marietta, Georgia. He completed work on his dissertation during this period.

Ray Howard Pettit is married to the former Sue Gatrell Hart and is the father of two children. He is a member of Eta Kappa Nu, Tau Beta Pi, and the Institute of Electrical and Electronic Engineers.

This dissertation was prepared under the direction of the chairman of the candidate's supervisory committee and has been approved by all members of that committee. It was submitted to the Dean of the College of Engineering and to the Graduate Council, and was approved as partial fulfillment of the requirements for the degree of Doctor of Philosophy.

June 18, 1964

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