MAGNETIC BRAKING DURING STAR FORMATION

By

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To Sherry

..., and my Parents

We are stardust...

- Joni Mitchell, <u>Woodstock</u>

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TABLE OF CONTENTS

PAGE

LIST OF TABLES

LIST OF FIGURES

Abstract of Dissertation Presented to the Graduate Council
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Angular momentum is *prima facie* a formidable obstacle in the theory of star formation: without rotational braking during star formation, stars would rotate with speeds close to that of light. The present investigation suggests that magnetic torques acting on a rotating, contracting, cool interstellar cloud which is permeated by a frozen-in magnetic field coupling the cloud to its surroundings, rotationally decelerate a cloud, constraining it to co-rotate with the background medium. Angular momentum is thus efficiently transferred from a collapsing cloud to its surroundings.

We examine angular momentum transfer from cool, rotating, stellar-mass condensations, collapsing isothermally on a magnetically-diluted dynamic time scale. Some mechanisms are discussed for forming gravitationally-bound protostellar condensations within cool, dense, molecular clouds. Rotation induces a toroidal magnetic field and the accompanying magnetic stresses generate a set of Alfvén waves which propagate into the background medium, thereby transporting angular momentum from a cloud to its surroundings. A modified virial approach is employed to calculate time-dependent quantities of interest at the cloud's surface in order to estimate the braking efficiency of the magnetic torques.

It is found that so long as a cloud remains magnetically coupled to its surroundings, the magnetic torques constrain a cloud to co-rotate with the background medium. Centrifugal forces are always kept well below gravity. The one single factor most important in determining the angular momentum of a protostar is the ionization rate in dense magnetic clouds: the degree of ionization controls the coupling of a cloud to the galactic magnetic field. The fractional ionization in dense magnetic clouds is therefore discussed in some detail.

The angular momentum of magnetically-braked protostars is shown to be consistent with the observed angular momenta of close binary systems and single early-type main-sequence stars. The hypothesis of magnetic braking offers support to the fission theory for the formation of close binary systems, and is able to account for the relative paucity of single stars. The calculations also suggest a common mode of formation for (close) binary and planetary systems.

This investigation shows that magnetic fields do indeed play an important, if not dominant, role during the early stages of star formation. Detailed numerical hydrodynamic collapse models have, as yet, ignored the possible effects that magnetic fields may have on the structure and evolution of a protostar. Such models are therefore highly suspect and probably not physically realistic.

SECTION I

INTRODUCTION

Angular Momentum Problem

Traditionally, magnetic fields and angular momentum have presented formidable problems to the theory of star formation (*cf*. Mestel 1965). Due to the high conductivity of the interstellar medium, the frictional coupling between plasma and neutral gas is sufficient to cause the large-scale galactic magnetic field to become 'frozen' into the fluid and dragged along with it. Accordingly, the magnetic energy density of a collapsing interstellar cloud (or fragment) increases as the cloud contracts, and the collapse is retarded and subsequent fragmentation may be prevented. Condensations in the interstellar medium will also possess angular momentum by virtue of local turbulence or galactic rotation. A simple calculation shows that a main-sequence star would rotate with an equatorial speed close to that of light if it were formed by isotropic compression from the interstellar gas, conserving angular momentum during contraction. Of course it is doubtful that stars could ever form under such conditions since centrifugal forces at the equator will increase faster than the gravitational forces, ultimately resulting in a rotational instability.

Mestel and Spitzer (1956) and Nakano and Tademaru (1972) have shown that the 'magnetic field problem' is only temporary. Ambipolar diffusion* allows the field to uncouple from the gas when the fractional ionization is reduced. Furthermore, Mouschovias (1976a,1976b) has shown that, at least for relatively low gas densities, some material may stream preferentially down the magnetic field lines, thereby increasing the ratio of gravitational to magnetic energy within a condensation.

Radio-frequency observations of molecular clouds do not show any clouds rotating much faster than the Galaxy (e.g. Heiles 1970; Heiles and Katz 1976; Bridle and Kesteven 1976; Kutner, *et al.* 1976; Loren 1977, and private communication; Lada, *et al.* 1974). Main-sequence stars are observed to rotate with equatorial velocities ranging from a few hundred kilometers per second for the early-type stars to just a few kilometers per second for stars later than spectral type F5 (Struve 1930; Abt and Hunter 1962). Evidently, nature has found a solution to the angular momentum problem.

A variety of mechanisms have been proposed to reduce the angular momentum of collapsing clouds and protostars. Hoyle (1945) and McCrea (1960, 1961) have suggested that condensation may take place in regions where the local turbulence is abnormally small. However, each object is still likely to have somewhat more angular momentum than is found in single main-sequence stars. Preferential mass flow along the rotational axis would increase the gas density at constant angular velocity. However, this process is not without its own difficulties (Mestel 1965; Spitzer 1968a). More attention has been given to the possibility of transforming the

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*Ambipolar diffusion ordinarily refers to the process of charged-particle diffusion due to a balance between a space-charge electric field and density gradients (*cf.* Krall and Trivelpiece 1973). In the astrophysical literature, ambipolar diffusion refers to the drift of a weakly ionized plasma across a magnetic field.

intrinsic (spin) angular momentum of a single massive protostar into the orbital angular momentum of a multiple star system (Larson 1972a; Black and Bodenheimer 1976). However, as Mouschovias (1977) points out, the angular momentum of such a hypothetical system is still some two orders of magnitude greater than that observed for the long-period (visual) binaries. Dicke's (1964) claim that the interior of the Sun is in rapid (differential) rotation suggests that single stars may store a large amount of angular momentum beneath their surface. Although not accounting for the possible stabilizing effect of toroidal magnetic fields, Goldreich and Schubert (1967) have shown that a necessary condition for stability in differentially rotating stars of homogeneous chemical composition is that the specific angular momentum (i.e. angular momentum per unit mass) should increase with increasing distance from the rotational axis. Thus it appears unlikely that a differentially rotating main-sequence star can have an angular momentum much in excess of a uniformly rotating star. Furthermore, convective mixing and poloidal magnetic fields redistribute angular momentum in the direction of rigid-body rotation.

It has often been suggested that the angular momentum of a contracting cloud or protostar may not be conserved. That is, angular momentum may be transferred in some manner to the surrounding interstellar material. Weizsäcker (1947) has argued that a rapidly rotating star will be rotationally decelerated as angular momentum is transferred from the star to its surroundings by turbulent viscosity. Ter Haar (1949) subsequently showed that Weizsäcker's purely hydrodynamic mechanism for angular momentum transport is probably not very efficient. Recently, Sakurai (1976) has calculated the braking torque on a Jacobian ellipsoid by a tidal acoustic wave which is generated in the surrounding medium by the rotating configuration. However, as Sakurai points out, the effectiveness of the braking for pre-main-sequence stars is uncertain because the braking time is of the same order of magnitude as the time scale of evolution.

Hydromagnetic braking appears to be more efficient in disposing of angular momentum. In an attempt to account for the sun's observed slow rotation, Alfvén (1942) suggested that the interaction of the sun's dipole magnetic field with the surrounding 'ion cloud' would produce a torque on the sun tending to brake its rotation. Ter Haar (1949) generalized this concept to include all stars magnetically coupled to HII regions. Lüst and Schlüter (1955) examined in some detail, particularly for the special case of torque-free magnetic fields, the transport of angular momentum by magnetic stresses acting on a rotating star.

Hoyle (1960) has proposed three stages of development for star formation: (1) the initial stage when a condensation is magnetically coupled to its surroundings by the frozen-in galactic magnetic field. Angular momentum is efficiently transferred from the contracting condensation to the surrounding medium with the condensation being constrained to co-rotate with the surroundings; (2) a subsequent phase when the fractional ionization becomes low enough so that the condensation uncouples from the galactic magnetic field via ambipolar diffusion, afterwhich angular momentum is effectively conserved; and (3) a recoupling with the galactic field during the final stage of slow contraction to the main sequence. Hoyle was able to explain the anomolous distribution of angular momentum within the solar system (98% of the total angular momentum of the solar system is concentrated in the planets which comprise less than 1% of the total mass of the system) as being the result of a hydromagnetic transfer of angular momentum from the primitive solar nebula to the planetary material. Hoyle's calculations were confirmed in a more quantitative fashion by Dallaporta and Secco (1975).

Following Hoyle's (1960) paper on the origin of the solar system (for a review of this and other theories of solar system formation, see Williams and Cremin 1968), it was generally believed (McNally 1965; Huang 1973, and references cited therein) that single main-sequence stars of spectral type F5 and later were likely to have planetary systems, and that their observed slow rotation was thus explained *ipso facto*. Schatzman (1962) pointed out that the transition between stars with deep envelopes in radiative equilibrium and those with well-developed subsurface hydrogen convection zones occurred among the F types. He introduced a theory in which the gas emitted by jets and flares associated with the active chromospheres of the later-type stars (those stars having subphotospheric convective zones) is magnetically constrained to co-rotate with the star out to very large distances where it carries away a large amount of angular momentum per unit mass. This theory is consistent with observational evidence. T Tauri stars undergoing pre-main-sequence contraction are ejecting matter (Herbig 1962; Kuhi 1964, 1966; see, however, Ulrich 1976). The observations of Wilson (1966) and Kraft (1967) show a connection between the rotation of stars and their age as determined by chromospheric activity (measured by the presence of H and K emission lines of CaII) which is usually associated with the hydrogen convection zone.

Dicke (1964), Brandt (1966), Modisette (1967), and Weber and Davis (1967) have calculated the solar-wind induced torque on the Sun. They conclude that the torque is sufficient to halve the sun's rotation on a cosmological time scale. Elaborating on the ideas of Schatzman, Mestel (1968) has formulated a theory of magnetic braking by a stellar wind. Using Mestel's results, Schwartz and Schubert (1969) have shown that the Sun may have lost a considerable amount of angular momentum if it passed through an active T Tauri stage. Assuming that stars in the pre-main-sequence stage are wholly convective (Hayashi 1961), Okamoto (1969, 1970) has shown that solar-type stars may lose almost all of their angular momentum via a Schatzman-type braking mechanism during pre-main-sequence contraction.

The Schatzman-type magnetic braking mechanism would apply only to those stars having appreciable subsurface convection zones and therefore enhanced surface activity (e.g. mass loss). This may account for the break in stellar rotation on the main sequence at spectral type F5, although as mentioned earlier, it may be that in some cases, angular momentum has been transferred to a surrounding planetary system. It is not clear that early-type stars ever develop a fully convective structure during pre-main-sequence contraction (Larson 1969, 1972b). Accordingly, for these stars in particular, we must examine the possibility of rotational braking during the early pre-opaque stages of star formation.

Ebert, *et al.* (1960; see also Spitzer 1968b and Rose 1973), in a pioneering study have investigated the transfer of angular momentum from a contracting interstellar cloud which is magnetically linked to the surrounding interstellar medium by the frozen-in galactic magnetic field. Kinks in the field lines introduced by the rotation of the cloud propagate into the surrounding medium in the form of magnetohydrodynamic (MHD) waves (in this case, the transverse Alfvén mode is excited), thereby rotationally decelerating the cloud. This mechanism is expected to be operative only so long as the cloud remains magnetically coupled to the background. As the collapse proceeds to higher densities, ambipolar diffusion (Mestel and Spitzer 1956; Nakano and Tademaru 1972), MHD instabilities (Mestal 1965), or perhaps intense Ohmic dissipation (Mestel and Strittmatter 1967) may act to uncouple the cloud's field from the surrounding medium. Although their results remain somewhat tentative due to the uncertainties in the formulation of the problem (e.g. assumed cylindrical symmetry), it appears that the magnetic torques may be sufficient to brake the cloud's rotation so that Hoyle's (1960) argument for efficient angular momentum transfer during the initial stage (Hoyle's stage 1) of star formation is supported. In a more detailed general analysis, Gillis *et al.* (1974) find, in one particular application of their somewhat artificial time-independent pseudo-problem, that the magnetic braking is "embarrassingly efficient" although they admit that their mathematical approximations introduce some degree of uncertainty.

Kulsrud (1971) has calculated the rate of emission of energy in the form of MHD waves (specifically, the fast magnetosonic mode) for a rotating, time-dependent, point magnetic dipole. Kulsrud has shown that stars with very large magnetic fields (e.g. magnetic A stars) and initially small rotation may be decelerated to very long periods. Indeed, this mechanism may explain the anticorrelation of rotational velocity and surface magnetic field strength observed for the magnetic stars (Landstreet *et al.* 1975; Hartoog 1977). The magnetic accretion theory of Havnes and Conti (1971) and the centrifugal wind theory of Strittmatter and Norris (1971) have also been proposed to account for the long-period Ap stars. Nakano and Tademaru (1972), Fleck (1974), and Fleck and

Hunter (1976) have adapted Kulsrud's result (even though Kulsrud's formulae are strictly applicable only to a periodically time-varying dipolar field) in order to estimate the efficiency of braking for collapsing interstellar clouds. The results of Fleck and Hunter are in good agreement with observations of molecular clouds and stellar rotation on the main sequence.

Prentice and ter Haar (1971; see also Krautschneider 1977) have suggested that a collapsing grain-cloud may lose angular momentum to the neutral gas component. However, this mechanism assumes that the grains are electrostatically neutral, and it ultimately relies on a hydromagnetic transfer of angular momentum to the outside.

Present Work

The purpose of the present investigation is to show that magnetic fields do indeed play an important, if not dominant, role during the early stages of star formation. We examine angular momentum transfer from a cool, rotating, magnetic cloud, magnetically coupled to its surroundings prior to the epoch of ambipolar diffusion, and undergoing essentially pressure-free collapse on a magnetically-diluted dynamic time scale. Rotation induces a toroidal magnetic field in the neighborhood of the cloud and the accompanying magnetic stresses produce a net torque acting on the cloud tending to keep the cloud in a state of co-rotation with its surroundings. We do not attempt a detailed solution of the coupled hydrodynamic and electrodynamic equations as to do so would require a sophisticated computer code to handle the problem numerically. Such a formidable (if not impossible) task is hardly justifiable in view of our lack of understanding of many of the details of the star formation process. Instead, we employ a modified virial approach to calculate time-dependent quantities of interest at the surface of a cloud in order to estimate the efficiency of the magnetic torques in de-spinning the cloud. We compare our results with observed properties of molecular clouds, the specific angular momenta of (close) binary systems, the angular momentum of the protosun, and with stellar rotation on the main sequence.

Uncertainties in some of the physical processes of star formation and complexities in the mathematical formulation of the problem do, of course, necessitate some degree of approximation and simplification in order that the problem remain tractable. We cannot hope to improve on the approximate nature of any theoretical study of star formation until we better understand the observations that are just now becoming available.

SECTION II

MAGNETIC BRAKING

Magnetic and Velocity Fields

It has been established (Heiles 1976, and references cited therein) that a large-scale magnetic field pervades the Galaxy. Due to the high conductivity of the interstellar medium, this field is 'frozen' into the fluid (Mestel and Spitzer 1956). Consider a uniform, spherical, interstellar cloud with radius R which is contracting isotropically.* Strict flux conservation implies that the magnetic field strength B within a radially contracting cloud increases according to

$$B = B_o(R_o/R)^2 \quad , \tag{1}$$

where the subscripts denote initial values. As a consequence of flux-freezing during an isotropic collapse, the initially uniform (galactic) field lines are drawn out from the cloud into a nearly radial structure (Mestel 1966). Accordingly, we approximate the magnetic field outside the cloud by the spherical polar coordinates $\vec{B} = (B_r, B_\theta, B_\phi)$ where

$$B_r = B_o\left(\frac{R_o^2}{r^2} + 1\right)\cos\theta \tag{2}$$

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*Observations of condensations in the interstellar medium spanning a range in mass from the massive molecular cloud complexes down to the stellar-mass Bok globules (Zuckerman and Palmer 1974, and references cited therein) indicate an approximate spherical geometry. Isotropic contraction will be partly justified and partly relaxed in a later section of this paper. Of course, the simplifying assumption that a cloud is uniform and has a well-defined boundary at R is somewhat artificial although it does simplify the calculations and it is not expected to affect the validity of the results.

$$B_\theta = -B_o\sin\theta \qquad (3)$$

$$B_\phi = 0 \quad . \qquad (4)$$

For $r \gg R_o$, the field becomes uniform and is described by the equation

$$\vec{B}_o = B_o(\cos\theta, -\sin\theta, 0) \ . \qquad (5)$$

The velocity fields outside a radially contracting, rotating cloud are given by

$$\vec{v} = (v_r, 0, v_\phi) \ , \qquad (6)$$

where

$$v_r = \dot{R}(r/R)^n \qquad (7)$$

and

$$v_\phi = \omega r\sin\theta \quad . \qquad (8)$$

In Eq. (7), $\dot{R} \equiv \frac{dR}{dt}$, the collapse velocity at the cloud surface ($r=R$), and we set the exponent $n=1$ in accordance with the findings of Gerola and Sofia (1975) and Fallon *et al.* (1977, and private communication) for the Orion A molecular cloud. However, the exact value of the exponent is somewhat uncertain (*cf.* Loren *et al.* 1973; Loren 1975, 1977; Snell and Loren 1977). In Eq. (8), $\vec{\omega}=\omega(r,t)\hat{\omega}$ is the angular velocity of the material, and we have taken the axis of rotation to be parallel to $\vec{B}_o$, i.e., $\hat{\omega}=\hat{B}_o=\hat{k}$, the unit vector along the positive z-axis. The z-axis thus becomes the axis of symmetry and derivatives with respect to the azimuthal coordinate vanish, i.e., $\frac{\partial}{\partial\phi} = 0$. Mouschovias (private communication) is investigating magnetic braking for the case of $\hat{\omega}\perp\hat{B}_o$, and believes that the braking may be *more* efficient in this case.

Paris (1971) has shown a tendency for the torques exerted by an undetached magnetic field to rotate the angular momentum vector into parallelism with the overall direction of the field. Thus, our assumption that $\hat{\omega}\cdot\hat{B}_o=1$ probably more closely approximates reality. Velocities at the cloud's surface can be found by setting r=R. The cloud is assumed to rotate rigidly at a uniform rate $\omega(r\leq R)=\omega(R)$. The magnetic viscosity due to the cloud's frozen-in magnetic field constrains the cloud to rotate uniformly as long as the travel time of an Alfvén wave through the cloud is less than the collapse time.

Torque Equation

The shear at R due to the cloud's rotation generates a toroidal field B_ϕ, and the resulting magnetic torques react on the rotation field. The magnetic stresses acting to minimize B_ϕ generate a set of Alfvén waves which propagate into the surrounding medium, thereby transporting angular momentum from the cloud to its surroundings. As pointed out by Lüst and Schlüter (1955; see also Mestel 1959), the magnetic torque exerted on currents within a given volume can be described by means of a tensor

$$D_{k\ell} = \varepsilon_{kij}x_iT_{j\ell} \tag{9}$$

analogous to the Maxwell stress tensor*

$$T_{k\ell} = \left(\frac{B^2}{8\pi}\delta_{k\ell} - \frac{B_kB_\ell}{4\pi}\right) \quad , \tag{10}$$

where ε_{kij} and $\delta_{k\ell}$ are, respectively, the Levi-Civita tensor and Kronecker delta, and x_α, α=i,j,k , denotes the Cartesian coordinates.

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*We employ the Gaussian system of electromagnetic units.

If (k,i,j) is a cyclic permutation of Eqs. (9) and (10), then

$$D_{k\ell} = x_i T_{j\ell} - x_j T_{i\ell} = (x_i \delta_{j\ell} - x_j \delta_{i\ell}) \frac{B^2}{8\pi} - (x_i B_j - x_j B_i) \frac{B_\ell}{4\pi} \quad . \tag{11}$$

The k-component of the magnetic torque density about the origin is given by

$$d_k = - \frac{\partial}{\partial x_\ell} D_{k\ell} \tag{12}$$

so that the magnetic torque acting on a volume V may be transformed into a surface integral:

$$\int d_k dV = - \int D_{k\ell} n_\ell dS \tag{13}$$

where n_ℓ is the unit normal outward from the surface element dS. If the surface S is a sphere centered on the origin, then the total outflow of angular momentum is

$$\int D_{k\ell} n_\ell dS = \frac{1}{4\pi} \int (-B_r)(x_i B_j - x_j B_i) dS \quad , \tag{14}$$

where B_r is the radial component: although the magnetic pressure $(B^2/8\pi)$ can interchange angular momentum between field streamlines, only the tension along the field lines $B_i B_j/4\pi$ contributes to the flux across the sphere S because the pressure acting normally to each surface element has zero moment about the center. The total torque can have only a z-component since our system is symmetric about the z-axis. Letting B_p denote the poloidal component of the magnetic field (i.e., $\vec{B}_p = \vec{B}_r + \vec{B}_\theta$), the total torque τ is (Lüst and Schlüter 1955)

$$\tau = \frac{1}{4\pi} \int B_\phi B_p r \sin\theta dS \quad , \tag{15}$$

where the surface element for a sphere of radius r is just

$$dS = r^2 \sin\theta d\theta d\phi \quad . \tag{16}$$

For a rotation field described by Eq. (8), it is intuitively clear that the toroidal magnetic field vanishes along the z-axis where v_ϕ is zero, and in the xy-plane where B_ϕ changes sign. Thus one can write the toroidal field as

$$B_\phi(r,\theta) = B_\phi(r)\sin\theta\cos\theta \quad . \tag{17}$$

Since the poloidal magnetic field outside a collapsing cloud has an almost purely radial structure, we set $B_p \approx B_r$ so that using Eqs. (16) and (17) in Eq. (15) and carrying out the appropriate integration, the torque at the surface of the cloud becomes

$$\tau = \frac{2}{15} R^3 B_\phi(R) B_s(R) \tag{18}$$

where $B_s(R)$ is the surface poloidal field and is given by Eq. (1).

Toroidal Magnetic Field

The time dependence of a frozen-in magnetic field is given by (*cf.* Jackson 1975)

$$\frac{\partial \vec{B}}{\partial t} = \vec{\nabla}\times(\vec{v}\times\vec{B}) \quad . \tag{19}$$

Provided that the ratio $|B_\phi/B_p|$ does not become large, the temporal behavior of the surface poloidal field should be adequately described by Eq. (1) if R=R(t) is known. For the v and B fields described by Eqs. (2), (3), and (6)-(8), the time dependence of the toroidal field, given by Eq. (19) is

$$\frac{\partial B_\phi}{\partial t} = \frac{1}{r}\left[\frac{\partial}{\partial r}\left(rv_\phi B_r - rv_r B_\phi\right) + \frac{\partial}{\partial\theta}\left(v_\phi B_\theta\right)\right] , \tag{20}$$

which becomes

$$\frac{dB_\phi}{dt} = -\frac{2\dot{R}}{R} B_\phi + B_o \sin\theta\cos\theta\left(\frac{R_o^2}{r} + r\right)\frac{\partial\omega}{\partial r} , \tag{21}$$

where we have made use of the convective derivative,

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \vec{v}\cdot\vec{\nabla} = \frac{\partial}{\partial t} + v_r\frac{\partial}{\partial r} . \tag{22}$$

In Eq. (21), the first term represents the convection of B_ϕ due to v_r while the second term shows clearly the expected dependence of dB_ϕ/dt on the shear in the azimuthal velocity field $\partial\omega/\partial r$. For $|v_r| << v_A$, where

$$v_A = \frac{B}{\sqrt{4\pi\rho}} \tag{23}$$

is the Alfvén speed in a plasma having a mass density ρ, the convection term is unimportant and the rate of growth of B_ϕ is determined soley by the rotational shear $\partial\omega/\partial r$.

We now derive an approximate expression for $\omega(r)$, and finally, $\partial\omega/\partial r$. The equation of motion in a fixed non-rotating inertial frame is

$$\rho \left.\frac{d\vec{v}}{dt}\right|_{\text{fixed}} = -\vec{\nabla}(P_g + \rho\Phi) + \vec{f}_M . \tag{24}$$

Here, P_g is the thermal gas pressure, Φ denotes the gravitational potential and

$$\vec{f}_M = \frac{1}{c}\vec{j}\times\vec{B} \tag{25}$$

is the magnetic force density, $\vec{j}$ being the electric current density, and c is the speed of light. The transformation of $\vec{v}$ between a fixed frame and a frame rotating with angular velocity $\vec{\omega}$ is given by (*cf*. Marion 1970)

$$\left.\frac{d\vec{v}}{dt}\right|_{\text{fixed}} = \left.\frac{d\vec{v}}{dt}\right|_{\text{rotating}} + \vec{\omega}\times\vec{v} \quad . \tag{26}$$

Thus, in the reference frame of our rotating cloud ($\vec{\omega}$ being the cloud's angular velocity) the equation of motion for the velocity field given by Eqs. (6)-(8) reads

$$\rho\left.\frac{d\vec{v}}{dt}\right|_{\text{rotating}} = -\vec{\nabla}(P_g+\rho\Phi)+\vec{f}_M-\frac{\dot{R}}{R}\vec{\omega}\times\vec{r}-\rho\vec{\omega}\times(\vec{\omega}\times\vec{r}) \quad . \tag{27}$$

The θ-component of this equation is

$$\rho\frac{dv_\theta}{dt} = -\nabla_\theta(P_g+\rho\Phi)+f_{M\theta}-f_{c\theta} \quad , \tag{28}$$

where $\nabla_\theta \equiv \partial/\partial\theta$, $f_{M\theta}$ is the θ-component of the magnetic force density and

$$\begin{aligned} f_{c\theta} &= \rho[\vec{\omega}\times(\vec{\omega}\times\vec{r})]_\theta \\ &= \rho\omega^2 r\sin\theta\cos\theta \end{aligned} \tag{29}$$

is the θ-component of the centrifugal force density. The first term on the right-hand-side of Eq. (28) vanishes for a spherically symmetric cloud. The left-hand-side of the equation vanishes as well since the velocity field given by Eq. (6) assumes $v_\theta=0$. Thus, Eq. (28) reduces to

$$f_{M\theta} = f_{c\theta} \quad . \tag{30}$$

Using Ampère's law

$$\vec{\nabla}\times\vec{B} = \frac{4\pi}{c}\vec{j} \tag{31}$$

in Eq. (27), the magnetic force density becomes

$$\vec{f}_M = -\frac{1}{4\pi}\vec{B}\times(\vec{\nabla}\times\vec{B}) \tag{32}$$

whence the θ-component

$$f_{M\theta} = \frac{B_r}{r}\left[\frac{\partial}{\partial r}(rB_\theta) - \frac{\partial B_r}{\partial\theta}\right] - \frac{B_\phi}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(B_\phi\sin\theta)\right] . \tag{33}$$

Combining Eqs. (29) and (33) in accordance with Eq. (30), using Eqs. (2) and (3) for B_r and B_θ, respectively, yields the following expression for ω^2:

$$\omega^2 = \frac{1}{\rho r^2}\left[B_o^2 R_o^2\left(\frac{R_o^2}{r^2}+1\right) + 2B_\phi^2(r)(\sin^2\theta - \cos^2\theta)\right] , \tag{34}$$

where we have used Eq. (17) to write out the explicit r and θ dependence of B_ϕ . The effect of the second term in brackets is to increase ω in the equatorial zones (i.e. the xy-plane) and decrease ω near the poles (i.e. along the z-axis). Averaged over a sphere of radius r which is concentric with the cloud, this term vanishes, i.e.,

$$\langle\sin^2\theta - \cos^2\theta\rangle = \frac{\int_0^\pi(\sin^2\theta-\cos^2\theta)d\theta}{\int_0^\pi d\theta} = 0 , \tag{35}$$

so that an approximate (average) angular velocity for the material surrounding the cloud is

$$\omega \simeq \frac{B_o R_o}{r^2\rho^{\frac{1}{2}}}\left(\frac{R_o^2}{r^2}+1\right)^{\frac{1}{2}} . \tag{36}$$

This procedure, which is equivalent to neglecting currents in the radial direction, is similar to that employed by Alfvén 1967; also Alfvén and Arrhenius (1976) in deriving his 'law of partial co-rotation' for a magnetized plasma.

The gas density ρ outside the cloud will be, in general, some function of r. The theoretical collapse models of Hunter (1969) and the observational findings of Loren (1977) for the Mon R2 molecular cloud suggest

$$\rho = \rho_s\left(\frac{r}{R}\right)^{-2} \tag{37}$$

where $\rho_s = 3m/4\pi R^3$ is the density at the surface of a uniform spherical cloud having a mass m. Using this result in Eq. (36) and differentiating with respect to r yields

$$\frac{\partial\omega}{\partial r} = -\left(\frac{4\pi R}{3m}\right)^{\frac{1}{2}} \frac{B_o R_o}{r^2} \left[\left(\frac{R_o^2}{r^2}+1\right)^{\frac{1}{2}} + \frac{R_o^2}{r^2}\left(\frac{R_o^2}{r^2}+1\right)^{-\frac{1}{2}}\right] . \tag{38}$$

As expected, $\frac{\partial\omega}{\partial r} < 0$. Using this expression for $\partial\omega/\partial r$ in Eq. (21), taking $\frac{R_o^2}{r^2} \gg 1$ (which should be true as the collapse proceeds and has the virtue of somewhat *underestimating* the rate of growth of B_ϕ initially), and evaluating the result at the surface of the cloud, r=R, yields

$$\frac{dB_\phi(R,\theta)}{dt} = -\frac{2\dot{R}}{R} B_\phi(R,\theta) - \left(\frac{16\pi}{3m}\right)^{\frac{1}{2}} \frac{B_o^2 R_o^4}{R^{3.5}} \sin\theta\cos\theta . \tag{39}$$

This is a linear first-order differential equation which can be cast into the form

$$\dot{B}_\phi(R,\theta) = -\frac{2\dot{R}}{R} B_\phi(R,\theta) - \frac{\text{constant}}{R^{3.5}} . \tag{40}$$

An integrating factor is R^{-2}. Making use of the free-fall collapse velocity (in a later section, we modify this equation to take into account pressure gradients within the cloud)

$$\frac{dR}{dt} = -\left[2Gm\left(\frac{1}{R} - \frac{1}{R_o}\right)\right]^{\frac{1}{2}} , \tag{41}$$

G being the Newtonian gravitational constant, and defining

$$\eta \equiv R/R_o \quad , \tag{42}$$

the solution to Eq. (40) assumes the form

$$B_\phi(R,\theta) = -\eta^2\left[B_\phi(R_o,\theta) - \left(\frac{8\pi}{3G}\right)^{\frac{1}{2}} \frac{B_o^2 R_o^2 \sin\theta\cos\theta}{m} \int_1^\eta \frac{d\eta}{\eta^5(1-\eta)^{\frac{1}{2}}}\right]. \tag{43}$$

Evaluating the integral and writing $B_\phi(R,\theta) = B_\phi(R)\sin\theta\cos\theta$ in accordance with Eq. (19) gives

$$B_\theta(R) = -\eta^2 B_\phi(R_o) - \frac{1.45\times10^{-33} B_o^2 R_o^2}{G\,(m/m_o)} \quad \frac{(1-\eta)^{\frac{1}{2}}}{4\eta^2}\,[1+\tfrac{7}{6}\eta + \tfrac{35}{24}\eta^2 + \tfrac{35}{16}\eta^3]$$
$$-\eta^2\,\frac{35}{128}\,\ln\left[\frac{1-(1-\eta)^{\frac{1}{2}}}{1+(1-\eta)^{\frac{1}{2}}}\right] \quad , \tag{44}$$

where $m_\odot = 2.0\times10^{33}$g is the mass of the Sun. Asymptotically, as $\eta\to0$, $B_\phi(R)\to\eta^{-2}$.

Rotational Deceleration

The net torque τ acting on a rotating cloud is related to the time-rate-of-change of angular momentum by

$$\tau = \frac{dJ}{dt} \quad , \tag{45}$$

where

$$J = \kappa m R^2 \omega \tag{46}$$

is the cloud's total angular momentum, κ being the gyration constant (κ=0.4 for a homogeneous uniformly rotating spherical cloud). Combining these two equations yields an expression for the rotational deceleration of the cloud:

$$\frac{d\omega}{dt} = -\frac{2\dot{R}}{R}\omega + \frac{\tau}{\kappa MR^2} \quad . \tag{47}$$

The collapse velocity at the cloud surface $\dot{R}$ is given by Eq. (41), and the magnetic torque acting on the cloud is determined from Eq. (18) using Eq. (44) to evaluate $B_\phi(R)$. Notice that for $\tau=0$, the above expression reduces to angular momentum conservation. Angular momentum is transferred from the cloud to its surrounding's so that τ is intrinsically negative and the cloud is rotationally decelerated.

A discussion of some of the relevant time scales is in order. From Eq. (47) it is apparent that braking will be efficient only if the second term on the right-hand-side dominates the first. Using Eq. (23) for the Alfvén speed, one can easily show that this is equivalent to the following condition:

$$v_A^2 \gtrsim |v_R v_\phi| \quad , \tag{48}$$

where $v_R=\dot{R}$ and $v_\phi=\omega R$ is the cloud's surface rotational velocity in the equatorial zones. The combined radial and azimuthal mass motion must not exceed the wave speed at the surface if the magnetic stresses are to transport angular momentum to the surrounding medium. A crude estimate of the power radiated away via MHD waves is given by

$$\begin{aligned} P &\simeq \frac{E_{rot}}{t_A} \\ &\simeq \frac{1}{2}\kappa MRv_A\omega^2 \quad , \end{aligned} \tag{49}$$

where

$$E_{rot} = \frac{1}{2}\kappa MR^2\omega^2 \tag{50}$$

is the cloud's rotational kinetic energy and

$$t_A \simeq R/v_A \tag{51}$$

is a measure of the characteristic hydromagnetic time scale, i.e., the travel time for an Alfvén wave traversing the cloud. (Interestingly enough, this order-of-magnitude estimate for the power-loss is, excepting for a constant of order one, just that predicted by the magnetic braking model of Ebert, *et al.* (1960)). Since $P = -\tau\omega$, Eq. (47) becomes

$$\frac{d\omega}{dt} \simeq -\frac{\omega}{R}\left(\frac{v_A}{2} + 2\dot{R}\right) \quad , \tag{52}$$

whence the condition

$$v_A \gtrsim |v_R| \tag{53}$$

in order that braking be efficient. A measure of the characteristic time scale for free-fall collapse is

$$t_f \simeq \frac{R}{v_R} \tag{54}$$

so that the condition for efficient braking becomes

$$t_A \lesssim t_f \tag{55}$$

For a marginally unstable cloud collapsing from rest, this condition is satisfied during the initial contraction stage since v_A is typically a few kilometers per second in the interstellar medium. In fact, Mouschovias (private communication) believes that the magnetic stresses acting on the surface of a contracting cloud will prevent v_R from ever exceeding v_A. It is sometimes argued that because the magnetic energy of a gravitationally-bound condensation can never exceed the gravitational energy, the travel time of Alfvén waves through the condensation is at least equal to, and may well be considerably longer than, the free-fall time which is given by

$$t_f = \left(\frac{3\pi}{32G\rho}\right)^{\frac{1}{2}} \quad . \tag{56}$$

However, this is the time required for *complete* collapse to a zero-radius singularity (*cf.* Hunter, 1962). It is more appropriate to compare time scales of interest with the 'instantaneous' dynamic time scale as given by Eq. (54). As pointed out by Mestel (1965) and Mouschovias (1976a, 1976b, 1977), the free-fall time as defined by Eq. (56) may have but an academic significance for clouds with frozen-in magnetic fields.

SECTION III

STAR FORMATION

Shock-induced Star Formation

The fact that young stars are frequently found in clusters suggests that stars are formed by a fragmentation process which occurs during the gravitational collapse of large interstellar clouds. According to the Jeans (1928) instability criterion, the minimum unstable mass is related to the gas temperature T (K) and particle density n (cm^{-3}) through the relation

$$\frac{m_J}{m_\odot} \gtrsim 10 \left(\frac{T^3}{n}\right)^{1/2} \quad . \tag{57}$$

Due to the isothermal behavior of the interstellar medium at relatively low gas densities (Gaustad 1963; Gould 1964; Hayashi and Nakano 1965; Hattori *et al.* 1969; see also Appendix A of this paper), the minimum unstable mass decreases as the collapse proceeds to higher gas densities so that a large collapsing cloud is expected to fragment into a number of smaller stellar-mass condensations.

However, rather compelling theoretical arguments and observational evidence have been presented suggesting that gravitationally-bound stellar-mass condensations (i.e. protostars) may form directly out of the interstellar medium without recourse to fragmentation. Ebert (1955) and McCrea (1957; see also the discussion in Mestel 1965) have shown that external pressures of the order 10^4 to 10^5 $cm^{-3}K$ can reduce the minimum unstable mass to stellar order. Such extreme pressure variations are known to exist in the interstellar medium (Jura 1975).

Shock waves propagating in the interstellar medium can increase the gas density up to two orders of magnitude, and thus reduce the Jeans mass by a factor of ten. Because of the cooling efficiency of the interstellar medium at low densities, the cooling time behind a shock in an HI region is typically two orders of magnitude less than the dynamic time scale (Field *et al.* 1968; Aanestad 1973), so that the shock propagates isothermally. The jump in density across an isothermal shock front is approximately (Kaplan 1966)

$$\frac{n_2}{n_1} \approx \left(\frac{v_s}{2 \text{ km s}^{-1}}\right)^2 , \tag{58}$$

where v_s is the shock velocity, (measured in km s^{-1}) and may be as large as 20 km s^{-1} for a strong shock.

Various mechanisms have been proposed for producing and maintaining interstellar shocks, and the possibility of shock-triggered star formation has been examined under a variety of physical conditions. The hydrodynamical models of Stone (1970) indicate that star formation may be enhanced by shocks generated during collisions between interstellar clouds. Indeed, Loren (1976) believes that ongoing star formation in the NGC 1333 molecular cloud is the result of such a cloud-cloud collision. Roberts (1969), Shu *et al.* (1972), and Biermann *et al.* (1972) have suggested that shock waves associated with density waves in spiral galaxies may induce the gravitational collapse of gas clouds thus leading to star formation. Giant HII regions associated with young early-type stars often line up 'like beads on a string' along the spiral arms of our Galaxy, and there is recent evidence for star formation by density wave shocks in M33 as well (Dubout-Crillon 1977).

The shock front associated with the advancing ionization front of an HII region may trigger the collapse of stellar-mass condensations (Dyson 1968). Large OB associations may be caused by a sequential burst of HII regions in a dense cloud (Elmegreen and Lada 1977), or perhaps by a supernova cascade process (Ögelman and Maran 1976). Observations of the Origem Loop supernova remnant (Berkhvijsen 1974) and the expansion of the Gum Nebula (Schwartz 1977) suggest that the strong shock from a supernova explosion may trigger star formation. Cameron and Truran (1977) explain various isotopic anomalies and traces of extinct ratioactivities in solar system material as being the result of a nearby Type II supernova that triggered the collapse of a cloud which led eventually to the formation of the solar system. The detailed two-dimensional numerical hydrodynamic calculations of Woodward (1976) demonstrate the validity of the shock-induced mechanism of star formation, particularly when the effects of self-gravitation, thermal instabilities, and dynamical instabilities of the Kelvin-Helmholtz and Rayleigh-Taylor type (*cf.* Chandrasekar 1961) which are triggered by the shock, are taken into account.

Thermal Instabilities

Thermal instabilities in the interstellar medium can result in the formation of non-gravitational condensations of higher density and lower temperature than are found in the surrounding medium (Field 1965). Basically, this is because cooling rates at low densities vary as n^2 while heating rates vary only as n. Following a thermal instability, as the density increases and the temperature drops (pressure equilibrium obtaining for short-wavelength perturbations), the critical Jeans mass given by Eq. (57) decreases rapidly. Theoretical studies by Hunter

(1966, 1969) and Stein and McCray (1972) have shown that self-gravitating, primary, stellar-mass condensations can form out of the medium *directly*, without the occurrence of fragmentation, by the two-step process of thermal instability at pressure equilibrium followed by gravitational collapse. Observationally, isolated primary condensations having stellar masses are known to exist (Aveni and Hunter 1967, 1969, 1972; Herbig 1970, 1976). Replacing the usual assumption of isothermal compression with the condition of energy balance, Kegel and Traving (1976) have generalized the Jeans criterion for gravitational instability, and they find that the minimum unstable mass is reduced by a factor $(\frac{d\ell n P_o}{d\ell n \rho_o})^{3/2}<1$, with P_o and ρ_o being the pressure and density at energy equilibrium.

Thermal-chemical instabilities may also lower the Jeans mass. The formation of hydrogen molecules on grain surfaces in interstellar clouds may result in pressure instabilities leading to the formation of protostars (Schatzman 1958; Reddish 1975). Because the cooling efficiency is greater for CO than for CII, the conversion of CII to CO during the evolution of dense interstellar clouds (*cf.* Herbst and Klemperer 1973; Allen and Robinson 1977) may lead to instabilities (Oppenheimer and Dalgarno 1975; Glassgold and Langer 1976). Generalizing Field's (1965) work to include chemical effects, Glassgold and Langer find unstable masses of stellar order. Oppenheimer (1977) has demonstrated that the interstellar gas may be unstable to the isentropic growth of linear perturbations in dense, optically-thick regions where the molecular transitions governing the cooling of the gas are thermalized, and where strong heat sources are present. Such instabilities may also lead to the formation of protostars.

The criterion for thermal instability becomes modified in the presence of magnetic fields (Field 1965). Just as in the case of shock-induced density growth in a magnetized plasma (*cf.* Kaplan 1966), magnetic pressures inhibit compression of the gas in directions perpendicular to the field lines. Even so, Mufson (1975) has shown for a wide variety of physical conditions, that the post-shocked gas is likely to become thermally unstable and that condensation modes can grow across magnetic field lines. High resolution radio observations of the supernova remnant IC 443 by Duin and van der Laan (1975) give evidence for condensation perpendicular to field lines.

Physical Conditions in Dark Clouds

Young stars (e.g. T Tauri stars, Herbig Ae and Be stars, and Herbig-Haro objects) and pre-stellar objects (e.g. IR and maser sources) are frequently associated with dense molecular clouds (*cf.* Strom *et al.* 1975). The apparent location of newly formed stars and HII regions on the *outsides* of dense, massive clouds and not at their centers (Zuckerman and Palmer 1974; Kutner *et al.* 1976; Elmegreen and Lada 1977; Vrba 1977) suggests a star-formation scenario wherein a shock-driven implosion at the boundary of a cloud initiates a thermal-gravitational instability, ultimately resulting in gravitationally-bound condensations.

Theoretical studies by Solomon and Wickramasinghe (1969) and Hollenback *et al.* (1971), and the dense cloud chemical models of Herbst and Klemperer (1973) and Allen and Robinson (1977), indicate that hydrogen is predominantly molecular in dense ($n \gtrsim 10^3$ cm^{-3}) clouds. Rocket observations by Carruthers (1970) and Copernicus satellite observations by Spitzer *et al.* (1973) support this conclusion. Other major chemical constituents include He, CO, NH_3, H, HD, OH, H_2CO, and H_2O. A representative mean molecular weight for dense cloud material would be μ=2.5. Dark clouds typically have particle densities $n = n_{H_2} \approx 10^3\ \mathrm{cm}^{-3}$ and kinetic gas temperatures $T \approx 10\mathrm{K}$ (Heiles 1969; Penzias *et al.* 1972; Zuckerman and Palmer 1974, and references cited therein, see also Appendix A of this paper for a detailed calculation of cloud thermodynamics).

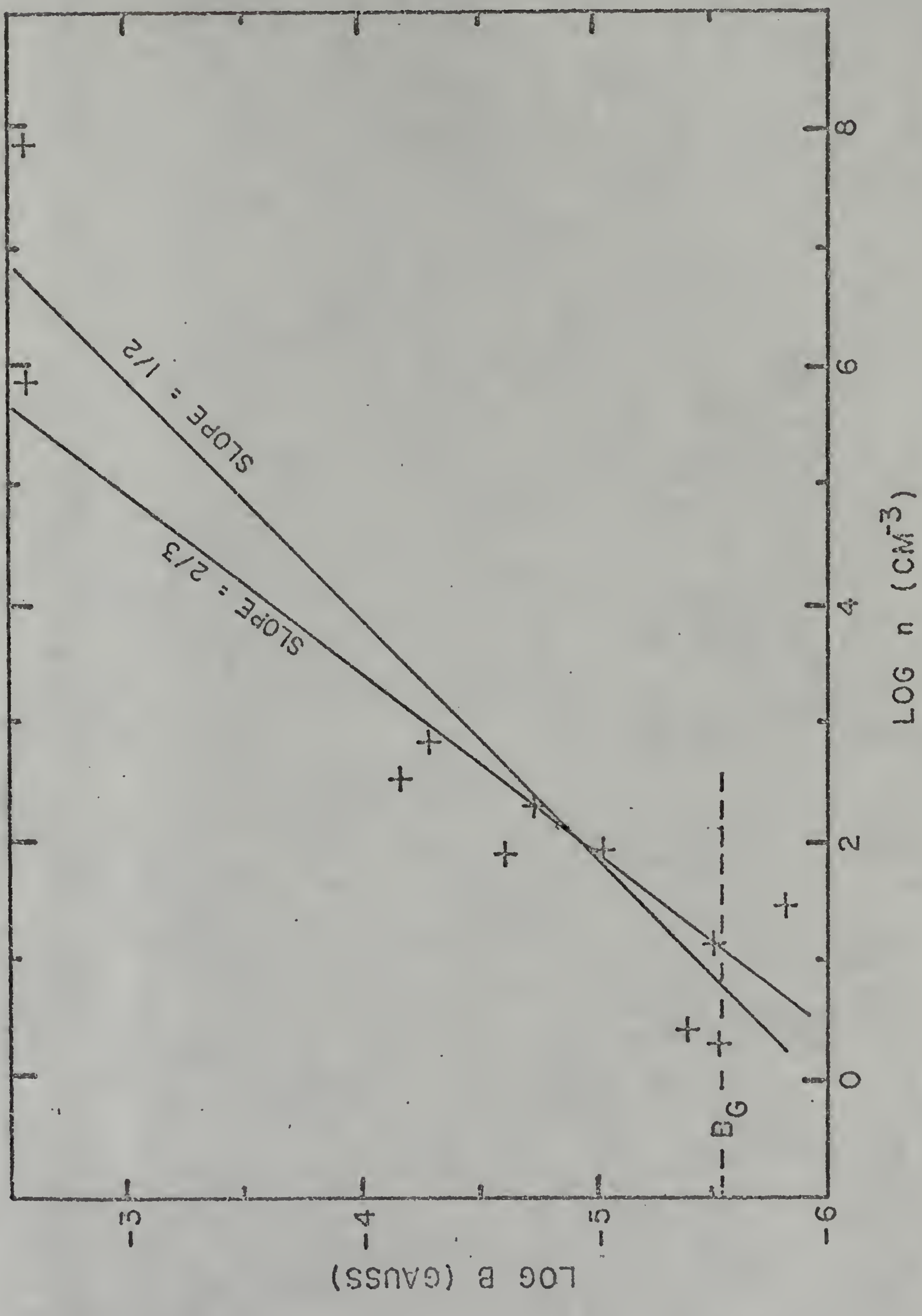
Observations do not show any interstellar clouds rotating much faster than the Galaxy (*cf.* Heiles 1970; Heiles and Katz 1976; Lada *et al.* 1974; Kutner *et al.* 1976; Bridle and Kesteven 1976). In the solar neighborhood, the Galaxy rotates with an angular velocity $\omega_G = 10^{-15}\ \mathrm{s}^{-1}$.

Observed line widths of molecular transitions originating in dense molecular clouds are almost invariably too wide to be explained by thermal motions, and they frequently imply supersonic velocities. The line widths have commonly been attributed to microturbulence (Leung and Liszt 1976) or macroturbulence (Zuckerman and Evans 1974), but difficulties with line profile interpretation (Snell and Loren 1977) and energetic difficulties associated with supersonic turbulence (Dickman 1976, and private communication) have led to the supposition that the line widths reflect systematic motions within the clouds, probably large-scale collapse (Goldreich and Kwan 1974; Scoville and Solomon 1974; Liszt *et al.* 1974; Gerola and Sofia 1975; de Jong *et al.* 1975; Snell and Loren 1977; Plambeck *et al.* 1977; Fallon *et al.* 1977). However, in favor of a turbulent origin, Arons and Max (1975) have suggested that the observed large line widths may be due to the presence of moderate-amplitude hydromagnetic waves in molecular clouds. Such waves may be generated by the magnetic braking process.

Magnetic field strengths in interstellar clouds are very uncertain. Measuring Zeeman splitting in the 21 cm line of neutral hydrogen and the 18 cm OH line, Verschuur (1970) has obtained field strengths in a number of diffuse ($n<10^3$ cm^{-3}) clouds. Clark and Johnson (1974) have suggested that the apparently anomalous broadening of millimeter-wavelength SO lines observed in Orion is caused by the Zeeman effect in 6-gauss magnetic fields. However, Zuckerman and Palmer (1975) believe that the large line widths are probably due to kinematic rather than magnetic broadening. Beichman and Chaisson (1974) find evidence from infrared polarization measurements and OH Zeeman patterns for milligauss fields in the Orion infrared nebula. Rickard *et al.* (1975) and Lo *et al.* (1975) have derived milligauss field strengths for a number of OH maser sources. However, well-known observational and theoretical problems in interpreting OH spectra in terms of Zeeman patterns (Zuckerman and Palmer 1975; Heiles 1976) make these results tentative. Magnetic field strengths obtained by Verschuur ($n<10^3$ cm^{-3}), Beichman and Chaisson ($n=10^6$ cm^{-3}), and Lo *et al.* ($n=10^8$ cm^{-3}), are plotted in Figure 1 as a function of inferred particle density in the magnetic region. We employ Zuckerman and Palmer's estimate of the gas density in the Beichman-Chaisson source, and for the density in the neighborhood of the source discussed by Lo *et al.*, we take $n=10^8$ cm^{-3} as suggested by Mouschovias (1976b).

At low densities ($n<10^2$ cm^{-3}), the magnetic field strength reflects the large-scale galactic field $B_G=3\mu$G. The constancy of the field strength for these low-density clouds suggests that material may stream preferentially along the field lines until higher gas densities are reached. Anisotropic gas flow along magnetic field lines increases the thermal gas pressure $P_g=nkT$, $k=1.38\times10^{-16}$ erg deg^{-1} being Boltzmann's constant, while holding the magnetic pressure $P_M=B^2/8\pi$ constant. Neglecting inertial forces, these two pressures will come into balance when the gas density reaches a critical value given by

Figure 1. Observed magnetic field strength B (gauss) in interstellar clouds and OH maser sources as a function of their particle density n (cm^{-3}). B_G=3 microgauss is the strength of the large-scale magnetic field of the Galaxy.


LOG B (GAUSS)
LOG n (CM^{-3})
SLOPE = 1/2
SLOPE = 2/3
B_G
-3
-4
-5
-6
0
2
4
6
8

$$n_{cr} = B^2/8\pi kT \quad , \tag{59}$$

so that for $B=B_G=3\mu G$ and $T=10K$,

$$N_{cr} = 260 \text{ cm}^{-3} \quad , \tag{60}$$

which is in good agreement with Figure 1. Indeed, the Parker (1966) instability (a magnetic Rayleigh-Taylor instability) may provide a mechanism for preferential gas flow along magnetic field lines at low densities, and the observational findings of Appenzeller (1971) and Vrba (1977) support Parker's predictions.

Assuming pressure equilibrium maintains for $n>n_{cr}$, the magnetic field strength should scale with the gas density according to Eq. (59):

$$B = (8\pi kT)^{\frac{1}{2}} n^{\frac{1}{2}} \quad , \tag{61}$$

so that

$$B \sim n^{\frac{1}{2}} \tag{62}$$

for an isothermal compression, in agreement with the detailed equilibrium models of Mouschovias (1976a, 1976b) for self-gravitating magnetic clouds. This result is to be compared with Eq. (1) which obtained for the case of isotropic contraction and strict flux-freezing:

$$B \sim n^{\frac{2}{3}} \quad , \tag{63}$$

where we have used

$$\rho = \mu m_H n = 3m/4\pi R^3 \tag{64}$$

to relate the radius of a spherical cloud to its particle density, $m_H = 1.67 \times 10^{-24}$g being the mass of the hydrogen atom. Because of the uncertainties in determining B and n for Figure 1, it is not possible to determine precisely whether a slope of 1/2 or 2/3 best fits the observations: neither is inconsistent. However, as Mouschovias (1976b) points out, a slope of 2/3 may be incompatible with certain properties (e.g. size, density, inferred magnetic fields) of maser sources. Furthermore, Scalo (1977) has shown that the heating of dense interstellar clouds by ambipolar diffusion imposes a constraint on cloud field strengths: B must not increase faster than $n^{0.55}$ so that predicted gas temperatures do not exceed those observed in dense clouds.

Initial Conditions for Collapse

From an equation of motion of the form given by Eq. (27), one can derive (*cf.* Cox and Giuli 1968) a fairly general form of the virial equation:

$$\tfrac{1}{2}\ddot{I} = 2K+3\langle\gamma-1\rangle U+M+\Omega-3P_sV \quad . \tag{65}$$

Here, $\ddot{I} = \frac{d^2I}{dt^2}$, where I is the moment of inertia of the fluid about the origin of coordinates, K, U, M, and Ω are, respectively, the total kinetic energy of mass motion, the thermal energy, magnetic energy, and gravitational energy within the volume V, P_s is the hydrostatic pressure on the surface defined by V, and γ is the ratio of specific heats (γ=7/5 for a low temperature gas of diatomic molecules). From what has been said earlier regarding rotation and turbulence, we may safely ignore the mass-motion kinetic energy term. Also, although strong surface pressures may arise from passing shock waves, it is primarily the thermal instability of the gas that drives the condensation of material to the higher densities required for eventual gravitational collapse. Accordingly, we neglect the P_s term as well.

The condition for collapse is $\ddot{I}<0$. Eq. (65) then becomes

$$|\Omega|>3\langle\gamma-1\rangle U+M \quad . \tag{66}$$

For a uniform spherical mass distribution, $\Omega = -\frac{3}{5}\frac{Gm^2}{R}$. Dividing Eq. (66) by the volume of the (spherical) cloud V, noting that for a non-relativistic gas the pressure is two-thirds the energy density, and assuming mechanical equilibrium between the thermal pressure (P_g=nkT) and magnetic pressure ($P_M=B^2/8\pi$), the condition for gravitational collapse, Eq. (66), becomes

$$n_o > 3.75\times10^5(m/m_\odot)^{-2}\ cm^{-3} \quad , \tag{67}$$

or, equivalently,

$$R_o < 6.73\times10^{16}(m/m_\odot)\ cm \quad , \tag{68}$$

where we have used Eq. (64) to eliminate n in favor of R, and the subscripts here denote initial (i.e. critical) values for collapse. The initial magnetic field strength at the cloud surface is found from Eqs. (59)-(61) to be

$$B_o = B_G(n_o/260)^{\frac{1}{2}} \quad . \tag{69}$$

Galactic rotation ($\omega_G=10^{-15}\ s^{-1}$) sets a lower limit to the angular velocity of a contracting cloud, and an upper limit is imposed by conservation of angular momentum, provided there are no external torques acting on the cloud. Since the evolution of a condensation up to the time it becomes gravitationally-bound is highly uncertain, we do not attempt to calculate the efficiency of magnetic braking during this stage. Therefore, the angular velocity of a marginally-unstable cloud cannot be determined *a priori*. It is possible that a condensation may derive its rotation from (subsonic) turbulence which may be generated by the dynamical instabilities, particularly the Kelvin-Helmholtz modes (*cf.* Woodward 1976), following the passage of a shock. Because of the strong dissipation of supersonic turbulence (Heisenburg 1948), turbulent velocities must not exceed the sound speed

$$c_o = (\gamma kT/\mu m_H)^{\frac{1}{2}} \tag{70}$$

which is about 0.3 km s^{-1} for T=10K, μ=2.5, and $\gamma = \frac{7}{5}$. If the correlation length of the turbulence is of the order of the cloud's diameter, then

$$\omega_o \simeq c_o/2R \quad . \tag{71}$$

A turbulent origin for the angular momentum of protostars has the attractive feature of explaining (1) the random orientation of rotational axes of early-type stars (Huang and Struve 1954) and field Ap stars (Abt *et al.* 1972), (2) the lack of a dependence of inclinations in visual binary systems on galactic latitude (Finsen 1933), and (3) the lack of evidence (Huang and Wade 1966) for preferred galactic distribution of orientations of orbital planes of eclipsing binaries. If stars and stellar systems acquired their angular momenta directly from galactic rotation, angular momentum vectors would generally be aligned perpendicular to the galactic plane.

Initial values of n_o, R_o, B_o, and ω_o appear in Table 1 for a range of protostellar masses from 1 $m_\odot$ up to 40 $m_\odot$. The non-integer masses correspond to main-sequence spectral types for which main-sequence

Table 1. Initial values of particle density n_o (cm^{-3}), cloud radius R_o (cm), surface magnetic field strength B_o/B_G, and angular velocity $\omega_o(s^{-1})$ for various cloud masses $m/m_\odot$ marginally unstable to gravitational collapse. Numbers in parentheses are decimal exponents.

$m/m_\odot$	Spectral type	n_o (cm^{-3})	R_o (cm)	B_o/B_G	ω_o (s^{-1})
1	G2	3.75 (5)	6.73 (16)	38.0	2.23 (-13)
1.7	F0	1.30 (5)	1.14 (17)	22.4	1.31 (-13)
2.1	A5	8.50 (4)	1.41 (17)	18.1	1.06 (-13)
3.24	A0	3.57 (4)	2.18 (17)	11.7	6.88 (-14)
6.5	B5	8.88 (3)	4.37 (17)	5.8	3.43 (-14)
10	B3	3.75 (3)	6.73 (17)	3.8	2.23 (-14)
17.8	B0	1.18 (3)	1.20 (18)	2.1	1.24 (-14)
40	O5	2.34 (2)	2.69 (18)	1	5.57 (-15)

rotation data have been accumulated. Spectral types later than early G will not be considered since these stars are expected to lose most of their primordial angular momentum during pre-main-sequence contraction and main-sequence nuclear burning (*cf*. Section I). Interestingly enough, the values of ω_o are roughly the same as those one would calculate assuming conservation of angular momentum for a condensation having initial densities of the order of a few particles per cubic centimeter, and initially rotating with the Galaxy. Clearly then, the adopted values for ω_o are probably overestimated. By adopting a possibly exaggerated ω_o, we require the magnetic braking to be correspondingly *more* efficient in decelerating a protostar. (It will turn out that the calculations are quite insensitive to a wide range of values of ω_o.) The values of n_o and ω_o for the 1 $m_\odot$ and 2.1 $m_\odot$ clouds are in agreement (fortuitously) with the initial conditions assumed by Black and Bodenheimer (1976) in their calculations of rotating protostars. For all mases, the initial ratio of gravitational to centrifugal forces at the equator, $F_g/F_c = Gm/R_o^3\omega_o^2$, is about ten. Thus, centrifugal forces are not sufficient to stabilize the initial configurations, and the assumption that $2K \ll \Omega$ appears reasonable.*

*Goldreich and Lynden-Bell (1965) and Toomre (1964) have shown by a generalization of the Jeans stability criterion to include the stabilizing effect of rotation, that whereas in the classical Jeans case with pressure effects stabilizing short-wavelength perturbations, long waves are stabilized by rotation.

SECTION IV

MAGNETIC BRAKING OF COLLAPSING PROTOSTARS

We now proceed to calculate the rotational deceleration of a magnetically braked, contracting protostar. The value of the rotational deceleration is given by

$$\frac{d\omega}{dt} = - \frac{2\dot{R}}{R} + \frac{\tau}{KMR^2} \quad . \tag{47}$$

With the initial conditions given in Table 1, $\omega = \omega(R)$ is obtained by numerical integration of Eq. (47) together with $\dot{R}$, τ, and $B_\phi(R)$ given by Eqs. (41), (18), and (44), respectively. A fourth-order Runga-Kutta integration scheme with variable step-size was employed. The accuracy of the numerical code was tested by setting $\tau=0$ in Eq. (47) and checking conservation of angular momentum ($\dot{J}=0$). The value of the cloud radius at each step in the integration was computed by a subroutine which solved Kepler's equation for a collision orbit (i.e., a degenerate ellipse with eccentricity $e=1$) by a standard iterative procedure. This was found to be easier than integrating Eq. (41) and solving the resulting transcendental equation for R at each step.

In order to keep the problem tractable, a number of simplifying, although reasonable, assumptions have been made. To simplify the geometry, the collapse is assumed to be isotropic and homologous, which implies further, that the surface poloidal magnetic field increases according to

$$B_s(R) = B_o(R_o/R)^2 \quad , \tag{72}$$

with B_o taken from Table 1. Furthermore, it is assumed that the magnetic stresses within a cloud will constrain the cloud to rotate uniformly as a rigid body. It may be argued that the assumption of homologous collapse may be somewhat artificial in view of the numerical collapse models of Larson (1969, 1972b) and Hunter (1969). However, there is little or no observational evidence for 'Larson-type' dynamical evolution (Cohen and Kuhi 1976), and Disney (1976) believes the impressed boundary conditions in the Larson approximation are probably not realistic. A nonhomologous collapse would have the effect of lowering the moment of inertia of a contracting cloud, as well as increasing the gravitational (binding) energy somewhat.

The initial value of the azimuthal magnetic field $B_\phi(R_o)$ can not be determined *a priori*. We adopt $B_\phi(R_o)=0$, with the understanding that the braking efficiency will be somewhat *underestimated* during the initial collapse phase since $\tau_o=0$.

Because the magnetic field of a cloud remains frozen-in during the collapse, the free-fall time given by Eq. (56) which is valid only for a pressure-free collapse, will underestimate the true collapse time. From the equilibrium virial theorem [$\ddot{I}=0$ in Eq. (65)], one can define an effective Newtonian gravitational constant

$$G' = G\left(\frac{|\Omega| - 2K - M - 3\langle\gamma-1\rangle U}{|\Omega|}\right) . \tag{73}$$

Since the collapse proceeds isothermally at the relatively low densities ($n \lesssim 10^8$ cm^{-3}) considered here (Gaustad 1963; Gould 1964; Hayashi and Nakano 1965; Hattori *et al.* 1969; see also Appendix A of this paper), the term $3\langle\gamma-1\rangle U$ representing thermal gas pressure is not expected to contribute significantly to Eq. (73). Furthermore, provided a cloud loses angular momentum during collapse, the term 2K can be neglected (see discussion in Section III). Thus, the collapse of a magnetic protostar proceeds on a magnetically-diluted time scale given by

$$t_c = \left(\frac{3\pi}{32G'\rho}\right)^{\frac{1}{2}} \quad , \tag{74}$$

where $G' \approx G(|\Omega|-M)/|\Omega|$. Since $|\Omega_o| \approx 2M_o$ (see Eq. (66) and discussion leading to Eq. (67)), and both $|\Omega|$ and M grow like $\sim R^{-1}$ for an isotropic collapse with flux conservation, $G' \approx G/2$.

The magnetic braking mechanism is operative only so long as a cloud remains magnetically linked to its surroundings. Mestel (1966) and Mestel and Strittmatter (1967) have argued that, as a cloud contracts, the almost oppositely directed field lines at the equatorial plane give rise to strong 'pinching' forces that dissipate flux and reconnect field lines, so that the magnetic field of a cloud is effectively detached from that of the background.* However, as Mestel and Strittmatter themselves point out, the time scale for this process is so long that this process may not be efficient. Furthermore, the equilibrium models of Mouschovias (1976b) do not show any tendency for equatorial pinching.

The expulsion of a cloud's magnetic field by ambipolar diffusion provides a more efficient mechanism for detaching the cloud's field from

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*Kulsrud's (1971) mechanism for the rotational deceleration of a rotating dipole applies in this case. Kulsrud finds $\tau \sim R^3 B_s B_o$. However, because the magnetic torques are then proprotional to the magnetic field in the surrounding medium instead of the azimuthal field, the braking efficiency is reduced considerably. Kulsrud's formulae, derived for a harmonically time-varying dipole, may not be strictly applicable to a contracting protostar, the radius of which decreases secularly with time.

that of its surroundings (Mestel and Spitzer 1956; Nakano and Tademaru 1972). The time scale for ambipolar diffusion is (Nakano and Tademaru 1972)

$$t_D = 8\pi R^2 m_{H_2} \langle\sigma v\rangle n n_e B^{-2}$$

$$= 8.26\times10^{21}\ (n_e/n) \quad , \qquad (75)$$

where we have used Eqs. (64), (67)-(69), and (72) to write the second equality, taking $\langle\sigma v\rangle = 2\times10^{-9}$ $cm^3 s^{-1}$ (Osterbrock 1961). The electron density n_e is calculated from Eq. (B7) which appears in Appendix B (see Appendix B for a discussion on the fractional ionization in dense magnetic clouds). Because the ionozation rate in dense clouds is somewhat uncertain (*cf.* Appendix B), we consider the two limiting cases: weak ionization by radioactive ^{40}K nuclei only, at a rate given by ξ_K (Eq. (B9)), and ionization by both ^{40}K and (magnetically screened) cosmic rays at a rate determined essentially by ξ_{CR} (Eq. (B19)), bearing in mind that the ^{40}K rate is probably the more realistic of the two (*cf.* Appendix B). As pointed out by Mouschovias (1977), if there is tension in the field lines, Eq. (75) overestimates the diffusion time scale. However, the field inside the cloud is assumed uniform so that t_D is probably not much less than that given here by Eq. (75).

The magnetic field becomes essentially uncoupled when $t_D \approx t_c$ (Mestel and Spitzer 1956; Nakano and Tademaru 1972). Afterwards, a cloud contracts conserving angular momentum. Thus, the angular momentum of a protostar is established at the uncoupling epoch. The integrations were therefore terminated when this condition was met. For a uniform, spherical, rigidly-rotating protostar, the angular momentum at the uncoupling epoch is simply

$$J_u = 0.4\ mR_u^2\omega_u \quad . \tag{76}$$

Results of the calculations appear in Table 2.

In all cases considered here, the magnetic torques rotationally decelerate the clouds, constraining them to co-rotate with their surroundings at an angular velocity $\omega_u = \omega_G = 10^{-15}\ s^{-1}$. Physically, the Alfvén speed just outside a cloud is always greater than the collapse velocity at the cloud surface, so that the torques are able to transmit angular momentum from a collapsing cloud on a time scale which is less than the (magnetically-diluted) free-fall time. Thus, the supposition that *magnetic braking is efficient all the way down to the breakdown of flux-freezing* (e.g. Hoyle 1960; Mouschovias 1977) appears to be vindicated.

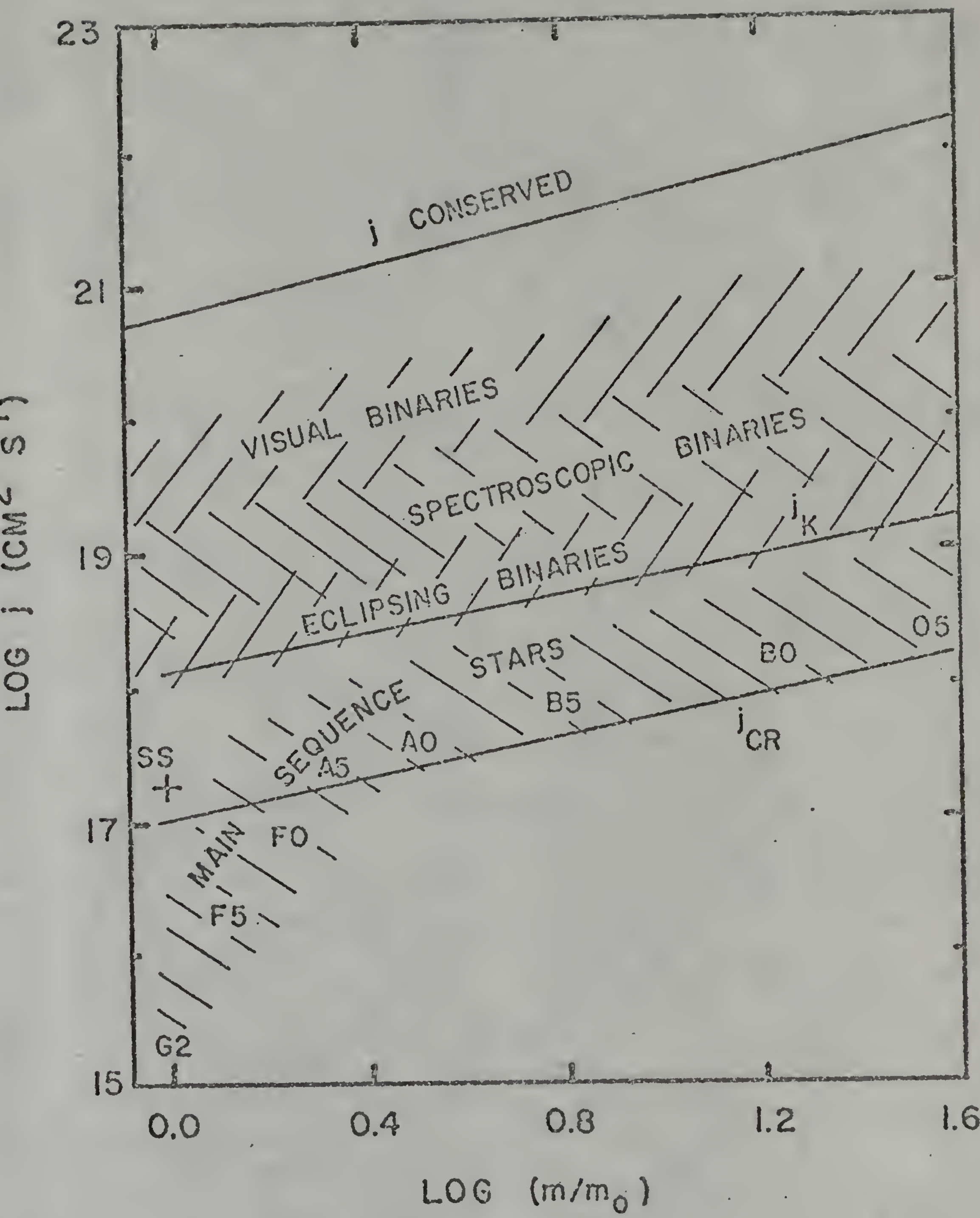
The angular momentum of an initial condensation is reduced some three or four orders of magnitude for the case of ionization by ^{40}K and cosmic rays, respectively. The angular momentum of the ^{40}K clouds at the uncoupling epoch is an order of magnitude greater than that of the cosmic-ray ionized clouds because uncoupling occurs earlier in the collapse sequence ($n_u \approx 10^6\ cm^{-3}$) for the weaker ^{40}K ionization rate. Clouds ionized primarily by cosmic rays uncouple from their surroundings later ($n_u \approx 10^7\ cm^{-3}$) since the time scale for ambipolar diffusion is longer for these clouds.

The surface magnetic field strengths at the uncoupling epoch fall nicely within the range of those observed in dense clouds (see discussion in Section III leading up to Figure 1). The ratio $|B_\phi/B_s|$ at uncoupling was found to be near unity for all clouds. This is consistent with the findings of Gillis *et al.* (1974) who predict a similar ratio in their time-independent magnetic braking model. If $|B_\phi/B_s|$ become much larger than

Table 2. Cloud radius R_u (cm), angular momentum J_u (g cm^2 s^{-1}), and poloidal surface magnetic field strength B_u (μG or mG) at the uncoupling epoch for ionization by ^{40}K only (ξ_K) and ionization by cosmic rays (ξ_{CR}). Numbers in parentheses are decimal exponents.

$m/m_\odot$	Spectral type	ξ_K			ξ_{CR}		
		R_u (cm)	J_u (g cm^2 s^{-1})	B_u (G)	R_u (cm)	J_u (g cm^2 s^{-1})	B_u (mG)
1	G2	5.90 (16)	2.78 (51)	148	1.36 (16)	1.49 (50)	2.8
1.7	F0	6.99 (16)	6.65 (51)	179	1.71 (16)	3.97 (50)	3.0
2.1	A5	7.51 (16)	9.47 (51)	192	1.86 (16)	5.80 (50)	3.1
3.24	A0	8.70 (16)	1.96 (52)	220	2.25 (16)	1.31 (51)	3.3
6.5	B5	1.09 (17)	6.13 (52)	282	3.00 (16)	4.68 (51)	3.7
10	B3	1.28 (17)	1.30 (53)	317	3.62 (16)	1.05 (52)	3.9
17.8	B0	1.51 (17)	3.24 (53)	400	4.58 (16)	2.99 (52)	4.3
40	O5	2.18 (17)	1.52 (54)	460	6.78 (16)	1.47 (53)	4.7

Figure 2. Specific angular momenta j ($cm^2\ s^{-1}$) for binary systems (visual, spectroscopic, and eclipsing), single main-sequence stars, and the solar system (SS). Also shown is the specific angular momentum predicted for the two limiting cases of ^{40}K (j_k) and cosmic-ray (j_{CR}) ionization, as well as j for the case of angular momentum conservation.



23
21
19
17
15
LOG j (CM^2 S^{-1})
j CONSERVED
VISUAL BINARIES
SPECTROSCOPIC BINARIES
j_K
ECLIPSING BINARIES
STARS
O5
B0
B5
j_{CR}
A0
A5
SEQUENCE
SS
MAIN
F0
F5
G2
0.0
0.4
0.8
1.2
1.6
LOG (m/m$_\odot$)

assuming rigid rotation, and an upper bound is estimated by adopting Bodenheimer and Ostriker's (1970) differential rotation law for the early-type stars (see Appendix C). For rapidly rotating stars, gravity darkening may lead to an underestimate of the observed equatorial velocity by as much as 40 percent (Hardorp and Strittmatter 1968), so that the transition of j between the early-type main-sequence stars and the eclipsing binaries is relatively smooth.

The decline in j for stars later than spectral type early-F ($m/m_{\odot} \lesssim 2$) is thought to be the result of angular momentum transfer during pre-main-sequence contraction (Schatzman 1962; Mestel 1968; Schwartz and Schubert 1969; Okamoto 1969, 1970) and main-sequence nuclear burning (Dicke 1964; Brandt 1966; Modisette 1967; Weber and Davis 1967), or perhaps an indication of the presence of planetary systems (Hoyle 1960; McNally 1965; Huang 1973, and references cited therein). Tarafdar and Vardya (1971) account for this discrepancy in j by presuming a rapidly rotating interior for the later-type stars and/or a slowly rotating interior for the early-type stars.

The important point illustrated by Figure 2 is that if angular momentum is conserved during star formation, then the angular momentum of a protostellar condensation, being almost two orders of magnitude greater than that of the widest separated visual binaries, will be inconsistent with the observed angular momenta of stellar systems. The two-dimensional numerical hydrodynamic models of Larson (1972a) and Black and Bodenheimer (1976) for rotating, collapsing protostars are thus highly suspect and probably not physically realistic since they assume strict angular momentum conservation throughout the collapse. Even if the toroidal figures predicted by their models (for which there is no observational evidence) are unstable to nonaxisymmetric breakup, as suggested by Wong's (1974) stability analysis of equilibrium toroids, the angular momentum of the *system* remains unchanged, and one is hard pressed to find a mechanism to dispose of angular momentum.

On the other hand, the calculations presented here for magnetic braking during star formation are consistent with the observations presented in Figure 2. Single stars are rare (Blaauw 1961; Heintz 1969; Abt and Levy 1976). It is therefore not surprising that the most likely ionization rate in dense clouds, namely, ionization by ^{40}K radioactive nuclei (*cf.* Appendix B), predicts angular momenta corresponding to that observed for close binaries, while the much less likely situation of ionization by cosmic rays accounts nicely for the angular momenta of (rare) single stars. The minimum angular momenta of single main-sequence stars is in excellent agreement with the minimum angular momentum of a contracting protostar, j_{CR} . Single main-sequence stars with rapidly rotating cores and/or large equatorial velocities apparently form in regions of lower cosmic ray flux. These are the stars in Figure 2 having $j_{CR}<j<j_K$.

Apparently, clouds having a specific angular momenta j_K become unstable and fragment into a multiple (e.g. binary) star system. After the magnetic field is expelled from a contracting cloud, the cloud continues to contract conserving angular momentum since no external (magnetic) torques act on the cloud. Eventually, as the rotational kinetic energy of the cloud increases, the ratio of centrifugal forces to gravity exceeds a critical value determined essentially by the distribution of mass and angular momentum within the configuration (see Ostriker 1970 for references), and the cloud breaks up into two or more condensations unity, there would be the danger that hydromagnetic instabilities of the twisted field might interfere with the assumed poloidal magnetic topography.

The ratio $F_g/F_c = Gm/R_u^3\omega_u^2$ indicates the important result that *centrifugal forces are kept well below gravity* throughout the collapse sequence.

SECTION V

DISCUSSION OF RESULTS: COMPARISON WITH OBSERVATIONS

> The brightest flashes in the world of thought are incomplete until they have been proven to have their counterparts in the world of fact.
>
> - John Tyndall
> (British Physicist 1820-1893)

The hypothesis of magnetic braking during star formation offers a plausible explanation for the observational fact that interstellar clouds, in general, do not rotate much faster than the Galaxy (Heiles 1970; Heiles and Katz 1976; Bridle and Kesteven 1976; Kutner *et al.* 1976; Loren 1977 and private communication; Lada *et al.* 1974). Further observational evidence for angular momentum transfer during star formation is afforded by a consideration of the angular momenta of binary systems and single main-sequence stars, stellar rotation on the main sequence, and the angular momentum of the protosun.

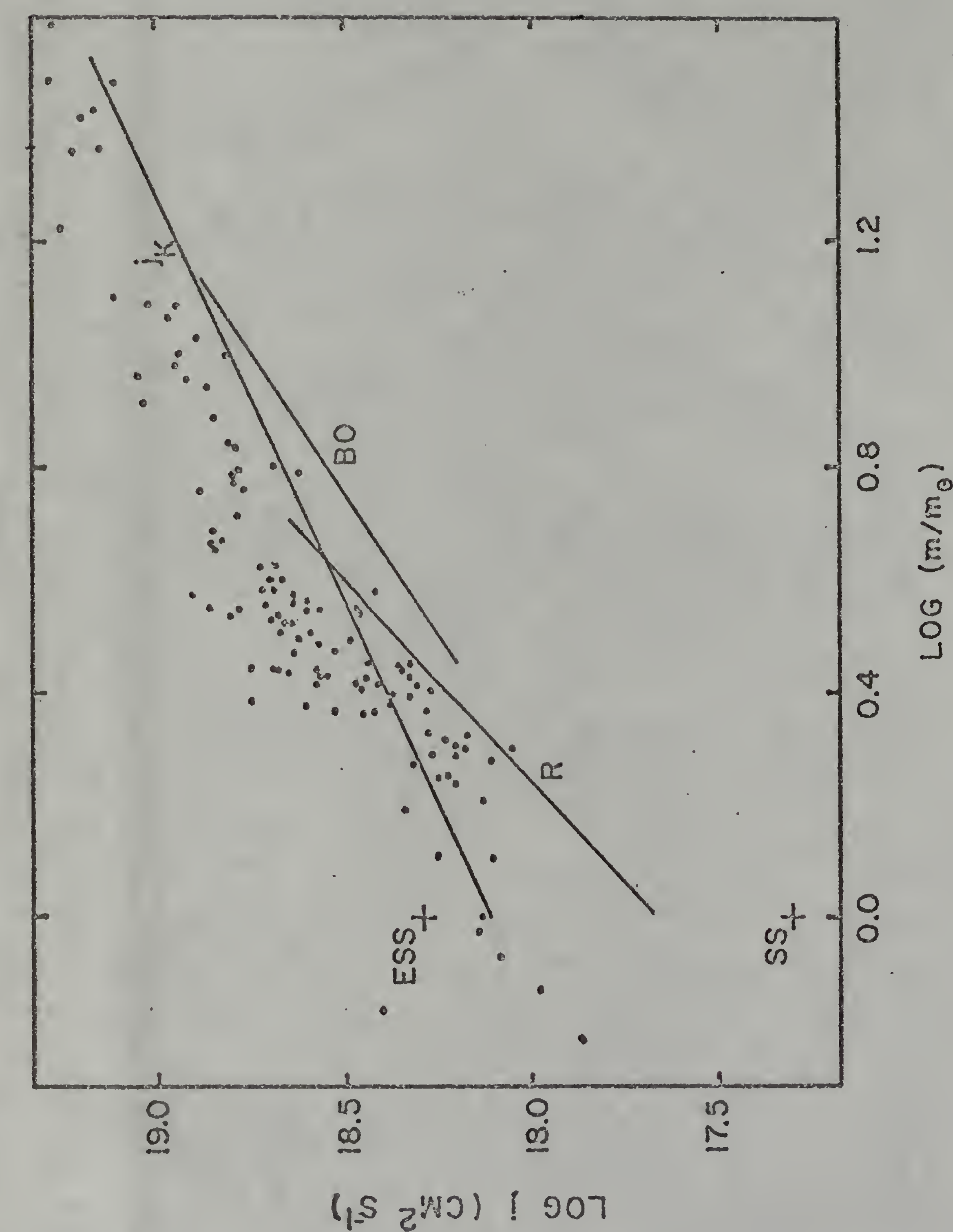
Specific Angular Momenta of Single and Binary Stars

The specific angular momenta j (angular momentum per unit mass) for binary systems (visual, spectroscopic, and eclipsing), single main-sequence stars, and the solar system (SS), is illustrated in Figure 2. In calculating j for the main-sequence stars, we have assumed that the stars have a mass distribution given by Eddington's standard model (polytropic index n = 3). Taking the observed mean equatorial rotational velocities for main-sequence stars (Allen, 1973), a lower bound for j is obtained with most of the angular momentum of the original cloud going into orbital motion of the fission fragments. The details of the 'fission' process have yet to be worked out.

That a cloud having a specific angular momentum j_K is expected to fission can be seen most easily in Figure 3. Here we plot the specific angular momenta for a number of eclipsing binary systems for which absolute orbital dimensions have been determined. Most of the data are taken from Kopal (1959). Roxburgh (1966) and Bodenheimer and Ostriker (1970) have calculated the threshold (i.e. minimum) angular momentum necessary for fission to occur; Roxburgh (R) for the W Ursae Majoris systems, and Bodenheimer and Ostriker (BO) for early-type close binary systems. Their results, reproduced in Figure 3, are shown to be in good agreement with the specific angular momenta of close binary systems, and more importantly here, indicate that our primary condensations, having a specific angular momentum j_K, will be unstable to bifurcation since j_K is greater than the threshold j for all masses.

According to our theory of magnetic braking, j_K is an upper limit to the specific angular momentum of a cloud of given mass. How then are the wide (i.e. long-period spectroscopic and visual) binaries formed? Evidently, an independent mode of formation exists for these systems. Indeed, there has been increasing evidence for two separate modes of binary formation. Contrary to the earlier suggestions of Kuiper (1955), Van Albada (1968a) finds that the division of early-type binaries into close (spectroscopic) and wide (visual) pairs is probably real and not due to the obvious selection effects. Whatever the period distribution, smooth or bimodal, Huang (private communication to H.A. Abt and S.G. Levy, 1976) feels that no single formation process will produce binaries with such a

Figure 3. Specific angular momentum ($cm^2\ s^{-1}$) for the ^{40}K ionization rate j_k, and threshold angular momentum necessary for fission, designated by R and BO. Also shown are the specific angular momenta for a number of eclipsing binary systems. The location of ESS represents the specific angular momentum of the early solar system, and SS designates its present value.



BO
R
j_K
ESS
SS
0.0
0.4
0.8
1.2
19.0
18.5
18.0
17.5
LOG (m/m$_\odot$)
LOG j (CM^2 s^{-1})

wide range of periods (1 day $\leq P \leq 10^8$ days). From a statistical analysis of the frequency of binary secondary masses, Abt and Levy (1976) conclude that there are two types of binaries: those with the shorter periods are fission systems in which a single protostar subdivided because of excessive angular momentum, whereas the longer periods represent pairs of protostars that contracted separately but as a common gravitationally-bound system. This 'neighboring-condensation' or 'early-capture' mechanism for the origin of long-period binaries is supported by the numerical calculations of Van Albada (1968b) and Arny and Weissman (1973), as well as by the findings of the recent two-body tidal capture theory of Fabian *et al.* (1975; see also Press and Teukolsky 1977).

It is not likely that a wide binary system can evolve into a close system by disposing angular momentum, thereby making unnecessary a separate formation mechanism for close binary systems. Binary stellar winds (Mestel 1968; Siscoe and Heinemann 1974) can operate only in the later-type, low-mass stars which have outer convection zones. Angular momentum loss via gravitational radiation is efficient only for low-mass systems already in near contact (Webbink 1976), and the disposal of angular momentum by mass-loss is not expected to occur until the late main-sequence evolutionary stages of already close binary systems (Webbink 1976). Furthermore, Webbink (1977) believes that most W Ursae Majoris systems have always existed in a contact state, and that fission is the only obvious formation mechanism satisfying this requirement.

Rotation of Main-Sequence Stars

The equatorial rotational velocity of a main-sequence star having a specific angular momentum j is

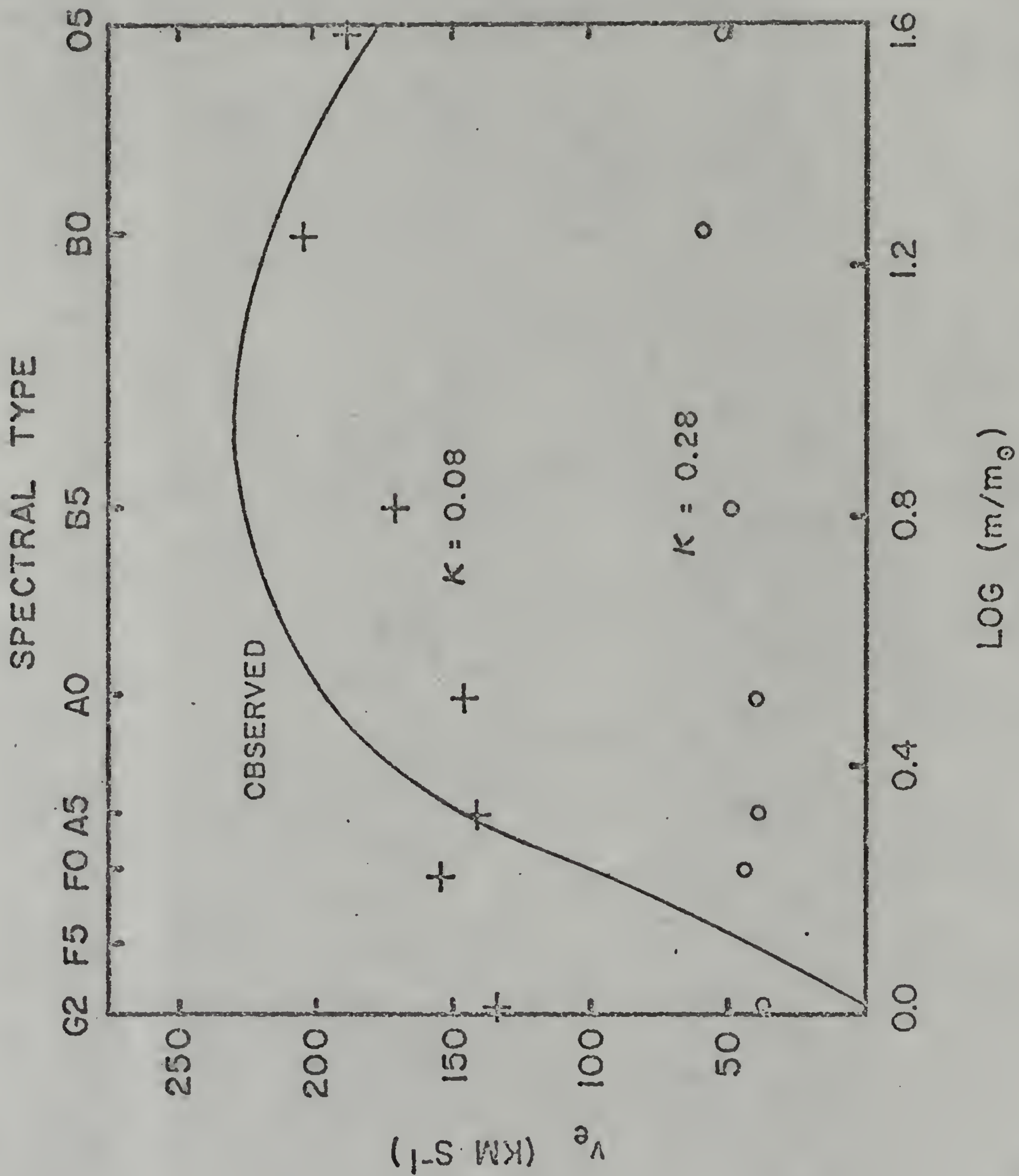
$$v_* = j/\kappa R_* \qquad (77)$$

where κ is the effective gyration constant for the star having a main-sequence radius R_*. Assuming angular momentum is conserved after the uncoupling epoch, j is given by j_K or j_{CR}. Since stars later than early-F may lose large quantities of angular momentum during pre-main-sequence contraction, we consider only the early-type ($m/m_\odot \gtrsim 2$) stars. The mass distribution within such stars is given to a good approximation by the Eddington standard model, characterized by a polytropic index n=3. For rigid rotation κ=0.08, and for the differentially rotating models of Bodenheimer and Ostriker (1970), κ=0.28 (see Appendix C for details of these calculations). Using this information together with the angular momentum data in Table 2, rotational velocities are calculated from Eq. (77) and the results appear in Table 3. Independent of the assumed rotation law, v_* for the ^{40}K ionization rate, far exceeds the critical equatorial breakup velocities for all spectral types (Slettebak 1966); *a fortiori* these stars are expected to fission into a close pair. Within the framework of our present theory of magnetic braking, single stars are believed to acquire an amount of angular momentum determined by the cosmic ray ionization rate. Figure 4 illustrates the reasonable agreement particularly for the case of uniform rotation (κ=0.08), of predicted rotational velocities with those observed for single main-sequence stars (Allen 1973). Rigid rotation may result from the actions of circulation currents, convective mixing in the core, or magnetic viscosity (Roxburgh and Strittmatter 1966), so that the case κ=0.08 may in fact represent the most plausible situation. The anomalously high rotational velocities predicted for the lower-mass stars on the assumption of angular momentum conservation during contraction to the main sequence, is evidence for further rotational braking during the later pre-main-sequence evolutionary stages of these stars.

Table 3. Predicted equatorial rotational velocities v_*(km s^{-1}) for uniformly rotating (κ=0.08) and differentially rotating (κ=0.28) main-sequence stars for the two limiting ionization rates, ξ_K and ξ_{CR}. The last column, taken from Allen (1973), gives mean values for observed stars.

$m/m_\odot$	Spectral type	ξ_K		ξ_{CR}		$\langle v_* \rangle$
		κ=0.08	κ=0.28	κ=0.08	κ=0.28	
1	G2	2500	710	134	38	2
1.7	F0	2600	740	155	44	100
2.1	A5	2300	660	143	41	150
3.24	A0	2200	630	145	41	190
6.5	B5	2200	630	170	49	230
10	B3	2200	630	180	51	---
17.8	B0	2200	630	203	58	200
40	O5	1900	540	186	53	180

Figure 4. Predicted equatorial rotational velocities v_* (km s^{-1}) for uniformly (κ=0.08) and differentially (κ=0.28) rotating main-sequence stars for the cosmic-ray ionization rate, ξ_{CR}. Also shown for comparison are mean values of v_* for observed stars.


SPECTRAL TYPE
G2 F5 F0 A5 A0 B5 B0 O5
OBSERVED
κ = 0.08
κ = 0.28
250
200
150
100
50
v_e (KM S^{-1})
0.0 0.4 0.8 1.2 1.6
LOG ($m/m_\odot$)

Angular Momentum of the Protosun

Hoyle (1960, 1963) has estimated the angular momentum of the early solar system $J_{ESS} \approx 4\times10^{51}$ g cm^2 s^{-1} by augmenting the planets up to normal solar composition. This value is in good agreement with our predicted $J_u \approx 3\times10^{51}$ g cm^2 s^{-1} (see Table 2) for a one-solar-mass cloud ionized primarily by ^{40}K.

The specific angular momentum of the early solar system (ESS) is compared with that of the present solar system (SS) in Figure 3. The fact that j_{ESS}, lying well above the threshold j necessary for fission, is comparable to the specific angular momenta of close binary systems, suggests that both (close) binary *and* planetary systems may be formed by a similar process (*cf.* van den Heuvel 1966; Fleck 1977) involving the rotational instability of a single primary condensation.* Drobyshevski (1974) has suggested a mechanism for close-binary formation wherein the convective outer layers of a rapidly rotating protostar are thrown off forming a ring in the star's equatorial plane which becomes unstable and forms a second component. This is very similar to the generally accepted Kant-Laplace nebular hypothesis for solar-system formation. The criteria for determining whether the end-product of such an instability will be planetary of stellar would be of interest. Circumstellar disks commonly

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*In the past (*cf.* Brosche 1962; McNally 1965), the angular momentum *presently* in the solar system (SS) had always been compared with that of single main-sequence stars. Such a comparison is probably fortuitous because, as can be seen from Figure 3, a condensation having a specific angular momentum j_{SS} is not expected to become rotationally unstable. One must then look for a mechanism, other than the popular nebular hypothesis wherein a planetary system forms in the equatorial plane of a rotationally unstable protostar, to account for the formation of a planetary system.

associated with Be stars may be the result of a rotational instability, particularly since these stars rotate with near-breakup velocities (Slettebak 1966).

SECTION VI

CONCLUDING REMARKS

Without rotational braking during star formation, single stars would rotate with speeds close to that of light. This statement is actually a *reductio ad absurdum*; centrifugal forces will halt the collapse perpendicular to the rotation axis long before such speeds are attained. However, even if the resulting highly flattened system fragments with most of the angular momentum going into orbital motion of the fragments about their center of mass, multiple star systems (e.g. binary stars) would have periods two to three orders of magnitude longer than the periods typically observed for even the widest pairs.

The present investigation suggests that magnetic torques acting on a rotating, contracting cloud which is permeated by a frozen-in magnetic field coupling the cloud to its surroundings, rotationally decelerate a cloud, constraining it to co-rotate with the background medium. Centrifugal forces are always kept well below gravity. The angular momentum of magnetically braked clouds is consistent with the observed angular momenta of close binary systems and single early-type main-sequence stars. The hypothesis of magnetic braking offers support to the fission theory for the formation of close binary systems, and is able to account for the relative paucity of single stars. The calculations also suggest a common mode of formation for (close) binary and planetary systems.

Throughout this investigation, some simplifying assumptions (most of which are physically justifiable) have been made in order to keep the problem tractable. In every instance, we have deliberately *underestimated* the efficiency of magnetic braking. Even so, the braking is still able to constrain the clouds to co-rotate with their surroundings; *a fortiori* a more accurate analysis will reach the same conclusion. *The one single factor most important in determining the angular momentum of a protostar is the ionization rate in dense magnetic clouds:* the degree of ionization controls the coupling of a cloud to the galactic magnetic field. For this reason, the material discussed in Appendix B concerning the fractional ionization in dense clouds is in need of further study, particularly the mechanisms for excluding cosmic rays from magnetic clouds.

Our calculations suggest that the toroidal configurations obtained in the spherical collapse models of Larson (1972a) and Black and Bodenheimer ([illegible]), wherein angular momentum is conserved throughout the collapse, may not appear until the later pre-main-sequence evolution of a protostar. Star formation is a complicated process and the (simplifying) assumption commonly made that magnetic fields (and therefore their role in magnetic braking) can be ignored may not be physically realistic. Indeed, it may be wrong to ignore effects of rotation on the evolution of a protostar. The work of Larson and Black and Bodenheimer should be re-evaluated incorporating the possible effects that magnetic fields may have on the structure and evolution of a protostar.

APPENDIX A

HEATING AND COOLING RATES IN DENSE CLOUDS

Heating

Cosmic-ray heating and compressional heat generated by the collapse are the dominant heating mechanisms in dense molecular clouds. Heating by photo-dissociation of H_2 (Stephens and Dalgarno 1973), by photo-electrons ejected from grains (Spitzer 1948), by photoionization of the gas (Takayanagi and Nishimura 1960), and by chemical reactions (Dalgarno and Oppenheimer 1974) is unimportant in dense clouds because the ultra-violet photons are mostly screened out. Heat of formation released by newly formed H_2 molecular (Spitzer and Cochran 1973) is unimportant because of the low neutral hydrogen abundance in dense clouds. Scalo (1977) has suggested that the action of ambipolar diffusion may generate an appreciable amount of heat if the magnetic field in a contracting cloud grows like $B{\sim}n^k$ where $k \geq \frac{3}{5}$. This mechanism is probably unimportant for our clouds which are characterized by $k = \frac{1}{2}$ during the initial compression stages and $k = \frac{2}{3}$ for gravitational collapse. Heating by the dissipation of (supersonic) turbulence is ignored.

The cosmic-ray heating rate per unit volume is

$$\Gamma_{CR} = \xi_{CR} n \langle E_h \rangle \quad . \tag{A1}$$

The cosmic-ray ionization rate per H_2 molecule ξ_{CR} is computed in Appendix B and is given by Eq. (B19). Glassgold and Langer (1973a) give the mean energy gain per ionization $\langle E_h \rangle$=17 eV. The heating rate by freely propagating cosmic rays (f=1 in Eq. (B19)) is then

$$\Gamma_{CR} = 2.72\times10^{-28}\ n\ \exp[-1.58\times10^{-7}(m/m_{\odot})^{\frac{1}{3}}n^{\frac{2}{3}}]\ \text{erg s}^{-1}\ \text{cm}^{-3}\ . \tag{A2}$$

A measure of the heat generated by the collapse Γ_c is the ratio of the thermal energy density of a cloud at temperature T, $\frac{3}{2}$ nkT ($k=1.38\times10^{-16}$ erg deg^{-1} is Boltzmann's constant), to its free-fall time t_f defined by Eq. (56). Thus

$$\Gamma_c \simeq 2\times10^{-31} n^{\frac{3}{2}}\ T\ \ \text{erg s}^{-1}\ \text{cm}^{-3}\ . \tag{A3}$$

Molecular Cooling

Inelastic gas-grain collisions, and rotational transitions among the more abundant molecular species, H_2, CO, and HD, will cool the gas. Other perspective molecular coolants such as H_2CO (Thaddeus 1972), HCl (Dalgarno *et al.* 1974), and CS and SiO (Goldreich and Kwan 1974) are probably not important compared to CO and HD. Atomic coolants such as CI, CII, and OI (*cf.* Penston 1970) are unimportant in dense clouds: due to the attenuation of ionizing ultraviolet radiation, carbon is mostly neutral when $n>10^4$ cm^{-3} (Werner 1970), and atomic carbon and oxygen are depleted by chemical reactions in dense clouds (*cf.* Allen and Robinson 1977). Furthermore, the cross section for collisional excitation of the CI and CII fine-structure levels by H_2 is much lower than for atomic hydrogen.

The energy radiated per unit volume per second in a transition between states u and ℓ is

$$n_u E_{u\ell} A_{u\ell} \beta_{u\ell}\quad , \tag{A4}$$

where n_u is the number of molecules cm^{-3} in state u, $E_{u\ell}$ is the energy difference between u and ℓ, $A_{u\ell}$ is the Einstein A-coefficient (i.e. the transition probability per unit time for spontaneous emission), and

$$\beta_{u\ell} = \frac{1-e^{-\tau_{\ell u}}}{\tau_{\ell u}} \tag{A5}$$

is the photon escape probability. The optical depth in a line having a rest frequency ν, and a thermal Doppler width (Mihalas 1970)

$$\Delta\nu = \left(\frac{2kT}{Am_H}\right) \frac{\nu}{c} \quad , \tag{A6}$$

is given by (Penzias 1975)

$$\tau_{\ell u} = \frac{e^2 g_u A_{u\ell} N_\ell}{8\pi g_\ell \nu^2 \Delta\nu} \left[1 - \exp\left(-\frac{h\nu}{kT}\right)\right] \quad . \tag{A7}$$

In these equations, A is the molecule's atomic weight, g_u and g_ℓ are, respectively, the statistical weights of levels u and ℓ, $N_\ell = n_\ell R/2$ is the column density of molecules in state ℓ through a mean path length R/2, R being the cloud radius, $c=3.0\times10^{10}$ cm s^{-1} is the speed of light, and $h=6.626\times10^{-27}$ erg s is Planck's constant. In Eq. (A7), the term in brackets is the correction for stimulated emission.

Under steady-state conditions, the relative population of levels is given by

$$\frac{n_u}{n_\ell} = \frac{(g_u/g_\ell)\ \exp(-E_{u\ell}/kT)}{1+(A_{u\ell}/C_{u\ell})} \quad , \tag{A8}$$

where

$$C_u = n\langle\sigma_{u\ell}v\rangle \frac{hB}{kT} \qquad \text{if} \quad \frac{hB}{kT} < 1$$

$$= n\langle\sigma_{u\ell}v\rangle \qquad \text{if} \quad \frac{hB}{kT} > 1 \tag{A9}$$

is the collisional de-excitation rate, $\sigma_{u\ell}$ being the collisional de-excitation cross section, and B is the molecule's rotational constant. The cross sections are averaged over Maxwellian velocity distributions to obtain rate constants $<\sigma v>$ for rotational excitation. For simplicity, we assume that $\sigma_{u\ell} = \sigma_{\ell u}$ ($\equiv\sigma$), and furthermore, that the rotational levels are excited (and de-excited) by collisions with H_2, the most abundant molecular species in dense clouds. Thus, the term in brackets in the denominator of Eq. (A8) is a measure of the deviation from thermal equilibrium: at higher gas densities, collisional de-excitation dominates spontaneous emission so that the levels become thermalized, and Eq. (A8) reduces to the Boltzmann distribution. When level populations become thermalized, the cooling rate which is proportional to n^2 at lower densities, goes like n [see Eq. (A4)]. The cooling efficiency is thus reduced at high densities, and the cooling is said to be 'collisionally quenched.'

The rotational levels of a diatomic molecule are indexed by the rotational angular momentum quantum number J. The level J has energy

$$E_J = hBJ(J+1) \quad , \tag{A10}$$

and the level degeneracy is

$$g_J = 2J+1 \quad . \tag{A11}$$

The energy separating levels J and J-1 (corresponding to an electric dipole transition governed by the selection rule $\Delta J=\pm1$) is

$$E_J - E_{J-1} = 2hBJ \quad . \tag{A12}$$

For thermalized levels, the population of the Jth level is, from Eq. (A8),

$$n_J = n_o(2J+1)\exp[-J(J+1)hB/kT] \quad , \tag{A13}$$

where n_o is the population of the ground state. The total number density of a particular molecular species is

$$n_T = \sum_{J=0}^{\infty} n_J$$

$$= n_o \sum_{J=0}^{\infty} (2J+1)\exp[-J(J+1)hB/kT] \quad . \tag{A14}$$

For $kT>>hB$, as is the case for all molecules considered here,

$$n_T \simeq n_o \int_0^{\infty} (2J+1)\exp[-J(J+1)hB/kT]dJ$$

$$\simeq n_o \frac{kT}{hB} \quad . \tag{A15}$$

Using this result to eliminate n_o in favor of n in Eq. (A13), we have

$$n_J \simeq n \frac{hB}{kT} (2J+1) \exp[-J(J+1)hB/kT] \quad . \tag{A16}$$

The cooling rate per unit volume for electric dipole transitions is, after Eq. (A4),

$$\Lambda = \sum_{J=1}^{\infty} n_J E_{J,J-1} A_{J,J-1} \beta_J \quad , \tag{A17}$$

with n_J, $E_{J,J-1}$ and β_J defined by Eqs. (A16), (A12), and (A5), respectively, and

$$A_{J,J-1} = \frac{512\pi^4 B^3 \mu^2 J^4}{3hc^3(2J+1)} \quad , \tag{A18}$$

where μ is the molecule's electric dipole moment.

For lines which are optically thick, cooling occurs from the surface of a cloud at a rate per unit volume given by

$$\Lambda^s = \frac{3n_J B_\nu d\nu}{Rn}(1-\beta_J) \quad , \tag{A19}$$

where n_J/n is given by Eq. (A16) and the photon spectral energy distribution for $\tau >> 1$ is given by the Planck function,

$$B_\nu d\nu = \frac{2h\nu^3}{c^2}\frac{d\nu}{e^{h\nu/kT}-1} \quad . \tag{A20}$$

For thermal line broadening, $d\nu$ is given by Eq. (A6).

Molecular Hydrogen

The H_2 molecule is a homonuclear diatomic molecule and therefore has no permanent electric dipole moment. For low temperatures ($T<100$ K), hydrogen molecules are predominantly in the ground rotational state, so that cooling occurs mainly via the electric quadrapole $J=2\rightarrow0$ transition for para-H_2. At such low temperatures, para-ortho collisional conversion is very slow. Because of the long radiative lifetimes of the excited levels, the level populations for gas densities $n \gtrsim 100$ cm^{-3}, are given by the Boltzmann equation. For the $J=2\rightarrow0$ transition, $E_{20} = 7.07\times10^{-14}$ erg so that

$$n_2 = 5n_o e^{-512/T} \quad . \tag{A21}$$

The cooling rate by H_2, assuming $n_o \simeq n_{H_2} \simeq n$ is then

$$\begin{aligned}\Lambda_{H_2} &= n_2 E_{20} A_{20} \beta_2 \\ &= 1.04\times10^{-23} n\beta_2 e^{-512/T} \text{ erg s}^{-1}\text{ cm}^{-3} \quad ,\end{aligned} \tag{A22}$$

where we have set $A_{20} = 2.947\times10^{-11}$ s^{-1} (Thaddeus 1972). The photon escape probability is related to the optical depth, as defined by Eq. (A7), through Eq. (A5). For H_2,

$$\Lambda(H_2) = 2.45\times10^{-5}T^{-\frac{1}{2}}\,(m/m_\odot)^{\frac{1}{3}}\,n^{\frac{2}{3}}\quad, \tag{A23}$$

have used $n m_H n = 3m/4\pi R^3$ to eliminate R in favor of n and m, the ... Surface cooling follows from Eq. (A19):

$$\Lambda_{H_2} = 1.\ldots\times10^{-19}T^{\frac{1}{2}}\,n^{\frac{1}{3}}\,(m/m_\odot)^{-\frac{1}{3}}\,e^{-512/T}(1-\beta_2)\ \mathrm{erg\ s^{-1}\ cm^{-3}}. \tag{A24}$$

... of the low temperatures characterizing dense clouds, H_2 cooling ... expected to be very efficient.

... Deuteride

... importance of HD as a molecular coolant in dense clouds was ... suggested by Dalgarno and Wright (1972). Due to vibronic inter- ... HD, unlike H_2, has a permanent dipole moment $\mu=5.85\times10^{-22}$ esu cm. ... K only the first two excited rotational levels need be considered. ... cooling rate per unit volume is then

$$\Lambda_{HD} = \sum_{J=1}^{2} n_J E_{J,J-1} A_{J,J-1} \beta_J\quad. \tag{A25}$$

... and Wright give $E_{21} = 3.54\times10^{-14}$ erg, $E_{10} = 1.77\times10^{-14}$ erg, ... $.41\times10^{-7}\ s^{-1}$, and $A_{10} = 2.54\times10^{-8}\ s^{-1}$. The levels are thermalized ... relatively low densities, and $n_o \simeq n_{HD}$ for T≤65 K, as can be seen from ... The HD cooling rate is then

$$\Lambda_{HD} = x(HD)n(1.35\times10^{-21}\beta_1 e^{-129/T}+1.28\times10^{-19}\beta_2 e^{-385/T})\ \mathrm{erg\ s^{-1}\ cm^{-3}}\ , \tag{A26}$$

... $x(HD)=n_{HD}/n_{H_2}$ is the fractional abundance of HD relative to H_2. ... The H:D ratio is 20,000:1. Accordingly, we take $x(HD) = 5\times10^{-5}$. ... of HD:H_2 by Spitzer *et al.* (1973), and DCN:HCN by Wilson ... (1973) suggest that x(HD) may be two orders of magnitude larger.

However, we follow the ideas of Watson (1973) and assume that these ratios reflect chemical fractionation rather than true isotopic abundances. The photon escape probabilities are determined by the optical depths which follow from Eq. (A7):

$$\tau_1(\mathrm{HD}) = 1.39x(\mathrm{HD})T^{-\frac{1}{2}}(m/m_\odot)^{\frac{1}{3}}\,n^{\frac{2}{3}}$$

$$\tau_2(\mathrm{HD}) = 2.78x(\mathrm{HD})T^{-\frac{1}{2}}(m/m_\odot)^{\frac{1}{3}}\,n^{\frac{2}{3}}\,e^{-129/T} \quad . \tag{A27}$$

The surface cooling rate for $x(\mathrm{HD}) = 5\times10^{-5}$ follows from Eq. (A19), and is given by

$$\Lambda^{s}_{\mathrm{HD}} = 3.46\times10^{-22}T^{\frac{1}{2}}n^{\frac{1}{3}}\,(m/m_\odot)^{-\frac{1}{3}}e^{-129/T}[(1-\beta_1)+79e^{-256/T}(1-\beta_2)]\ \mathrm{erg\ s^{-1}\ cm^{-3}}. \tag{A28}$$

Carbon Monoxide

The most abundant heavy molecule in dense interstellar clouds is CO. Because only 5.5 K separates the first excited rotational level from the ground state, CO is a potentially efficient coolant in the relatively cool ($T\approx10$ K) environment of molecular clouds. Indeed, Glassgold and Langer (1973b) have shown, neglecting optical depth effects, that the low temperatures [$T\approx10$ K; see Zuckerman and Palmer (1974) for references] typically observed in dense molecular clouds can be maintained by CO cooling alone.

The CO cooling rate follows from Eq. (A17):

$$\Lambda_{\mathrm{CO}} = \sum_{J=1}^{J_m} n_J\,E_{J,J-1}\,A_{J,J-1}\beta_J \quad , \tag{A29}$$

where the summation is discontinued at the first level ($J=J_m$) where the collisional de-excitation rate is less than the spontaneous transition rate. The cross section for rotational excitation of CO by H_2 is

$\sigma \simeq 10^{-15}$ cm^2 (Green and Thaddeus 1976). Because of its low dipole moment ($\mu = 1.12\times10^{-19}$ esu cm) and high abundance, the CO molecule thermalizes at low densities. Using Eqs. (A16), (A12), and (A18), Eq. (A29) becomes

$$\Lambda_{CO} = 7.42\times10^{-76} x(CO) \frac{nB^5}{T} \sum_{J=1}^{J_m} \beta_J J^5 \exp[-J(J+1)hB/kT] \text{ erg s}^{-1} \text{ cm}^{-3} . \tag{A30}$$

We adopt a fractional abundance $x(^{12}CO) = 3\times10^{-5}$ for the main isotopic species, and $x(^{13}CO) = 3.4\times10^{-7}$ for the less abundant species; the adopted isotopic ratio is the terrestrial value 89:1. For ^{12}CO, B=57,700 MHz, and for ^{13}CO, B=55,100 MHz. The optical depth in a line arising from a transition where $J\rightarrow J-1$ is, from Eq. (A7),

$$\tau_J(CO) = 7.57\times10^{-6} x(CO)\, JT^{-\frac{3}{2}} (m/m_\odot)^{\frac{1}{3}} n^{\frac{2}{3}} \exp[-J(J+1)hB/kT] \cdot$$

$$\cdot [1-\exp(-J(J+1)hB/kT)] \quad . \tag{A31}$$

Surface cooling in the optically thick lines occurs at a rate

$$\Lambda^s_{CO} = 5.62\times10^{-82} \frac{B^5 n^{\frac{1}{3}}}{T^{\frac{1}{2}} (m/m_\odot)^{\frac{1}{3}}} \sum_{J=1}^{J_m} J^4 (2J+1)(1-\beta_J) \cdot$$

$$\left[\frac{\exp[-2J(J+1)hB/kT]}{1-\exp[-J(J+1)hB/kT]}\right] \text{ erg s}^{-1} \text{ cm}^{-3} \quad . \tag{A32}$$

<u>Grain Cooling</u>

Cooling by inelastic collisions of grains with the gas particles (mostly molecular hydrogen) at temperature T occurs at a rate (Dalgarno and McCray 1972)

$$\Lambda_g = 2\times10^{-33} n^2 T^{\frac{1}{2}} (T-T_g)\Theta \text{ erg s}^{-1} \text{ cm}^{-3} , \tag{A33}$$

where T_g is the temperature of the grains, and Θ is the energy accommodation coefficient, and is a measure of the elasticity of the collision; θ is unity for a completely inelastic collision, and becomes zero for elastic collisions. We take $\Theta=1$. Because of the strong density dependence, grain cooling is expected to dominate molecular cooling at high densities.

The grain temperatures can be determined from the energy balance of a grain (Low and Lynden-Bell 1976):

$$\sigma T_g^4 Q_p(T_g) = nk(T-T_g)\left(\frac{8kT}{\pi m_{H_2}}\right)\Theta + \sigma T_b^4 Q_p(T_b) \quad , \tag{A34}$$

where $\sigma=5.67\times10^{-5}$ erg cm^{-2} deg^{-4} s^{-1} is the Stefan-Boltzmann constant, $Q_p(T)$ is the Planck mean absorption efficiency, m_{H_2} is the mass of the hydrogen molecule, and $T_b=2.7$ K is cosmic blackbody radiation temperature. The first term represents the heat loss due to radiation of a dust grain at temperature T_g. The second term is the collisional energy gain from the gas, and the last term represents heating from an isotropic blackbody radiation of temperature T_b. Eq. (A34) assumes that the cloud is shielded from the external stellar radiation field, and that there are no embedded stars within the cloud which might heat the grains to a temperature $T_g>T$, in which case the grains *heat* the gas (Leung 1976). Kellman and Gaustad (1969) give $Q_p(T) = 4.1\times10^{-7}\ T^{2.67}$ for 0.2 micron ice grains.

The optical depth through a cloud of radius R for the thermal radiation from the grains is

$$\tau_g(T_g) = Q_p(T_g)\sigma_g n_g R \quad . \tag{A35}$$

By eliminating R in favor of n and m, the mass of a uniform, spherical cloud, and by taking $\sigma_g n_g = 6\times10^{-22} n$, and $Q_p(T_g{\simeq}10){\simeq}10^{-4}$, the optical depth

$$\tau_g(T_g=10) \simeq 10^{-7}(m/m_\odot)^{\frac{1}{3}}n^{\frac{2}{3}} \quad . \tag{A36}$$

For stellar-mass clouds, $\tau_g \gtrsim 1$ for densities $n \gtrsim 10^{10}$ cm^{-3}, which is in good agreement with the results of Hattori *et al.* (1969). Since densities encountered during the initial collapse stages are much less than this, the thermal radiation from grains is assumed to pass freely out a contracting cloud.

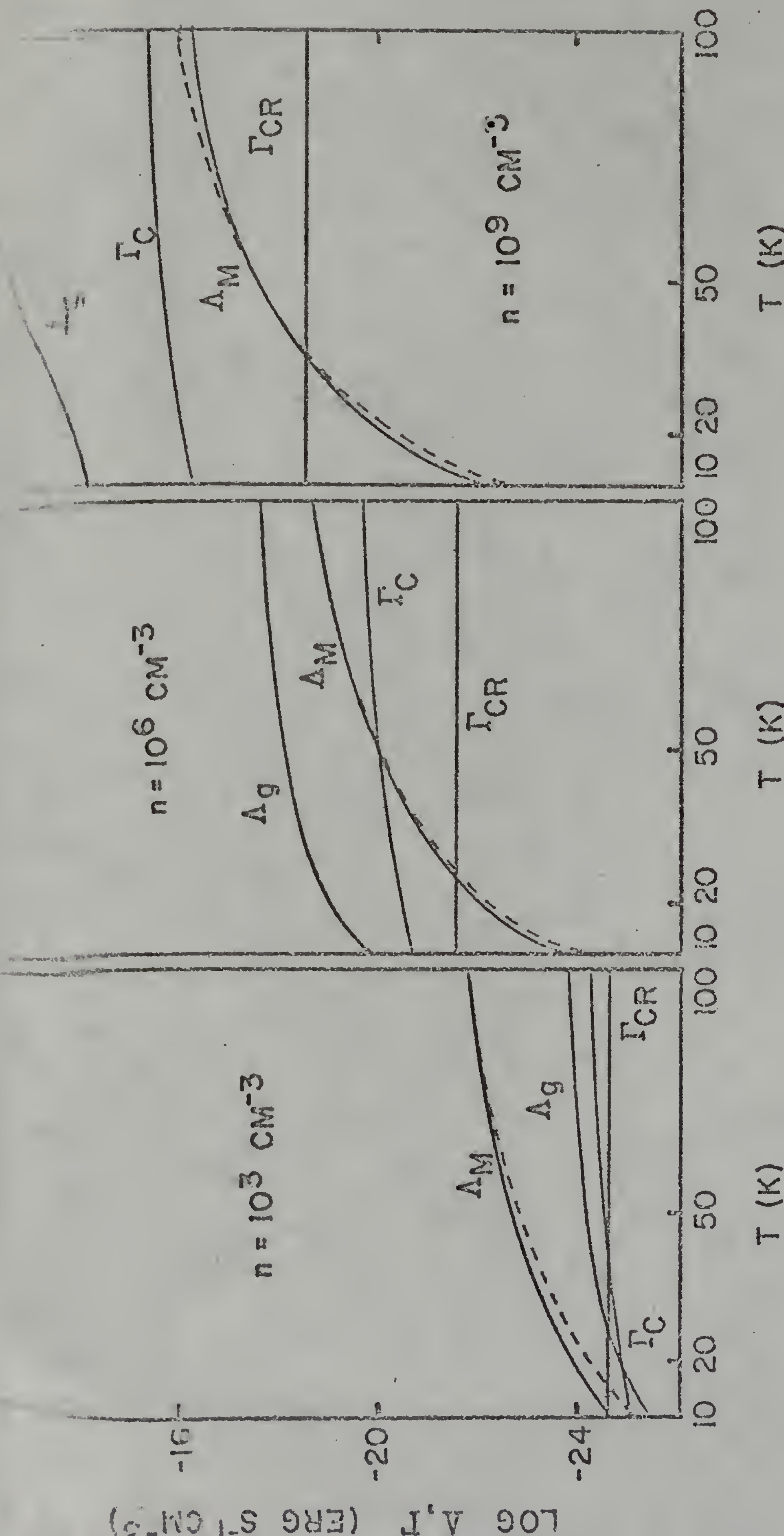
Cloud Temperature

Figure A1 illustrates the temperature and density dependence of the various heating and cooling rates described in this Appendix. The total molecular cooling rate $\Lambda_M = \Lambda_{H_2} + \Lambda_{HD} + \Lambda_{CO}$. As expected grain cooling completely dominates for $n \gtrsim 10^6$ cm^{-3}. Even if gas-grain collisions are only weakly inelastic ($\Theta \simeq 0.01$ say), grain cooling will still dominate at high densities. Grain temperatures ranged from $T_g=6$ K for $T=10$ K up to $T_g=35$ K for $T=100$ K. Gas temperatures much in excess of 20 K are rarely encountered in dense clouds; temperatures up to 100 K are considered merely to illustrate the rapid rise in the molecular cooling rates at these relatively high temperatures.

Surface cooling for all molecules never amounts to more than 10% of the total molecular cooling rate, and cooling by molecular hydrogen becomes noticeable (~10% of the total molecular cooling) only for the highest temperatures. Carbon monoxide dominates the cooling at $T=10$ K for all densities. However, except for $n=10^3$ cm^{-3} where $\Lambda_{HD} \simeq \Lambda_{CO}$, *hydrogen deuteride is by far the dominant molecular coolant for $T \gtrsim 20$ K,* its cooling rate becoming three orders of magnitude greater than CO cooling for the highest temperature.

It appears that cloud temperatures equivalent to those typically observed in dark clouds ($T \simeq 10$ K; see Zuckerman and Palmer (1974) for

Figure A1. Cosmic-ray (Γ_{CR}) and compressional (Γ_C) heating rates (erg s^{-1} cm^{-3}), and molecular (Λ_M) and grain (Λ_g) cooling rates (erg s^{-1} cm^{-3}) as a function of kinetic gas temperature T (K) for gas densities (a) $n=10^3$ (b) $n=10^6$ and (c) $n=10^9$ cm^{-3}. For Λ_M, the solid line represents the molecular cooling rate for a one-solar-mass cloud, and the broken line is the cooling rate for a 40 solar-mass cloud.



n = 10³ CM⁻³
n = 10⁶ CM⁻³
n = 10⁹ CM⁻³
Λ_M
Λ_g
Γ_C
Γ_CR
T (K)
10 20 50 100
-16
-20
-24
LOG Λ,Γ (ERG S⁻¹ CM⁻³)

references) are certainly attainable at low gas densities, and should prevail for higher densities as well, provided grain cooling operates with at least a one-percent efficiency. The rapid rise of the molecular cooling rate at higher temperatures indicates that a cloud will collapse approximately isothermally near some equilibrium ($\Gamma=\Lambda$) temperature $T_e \simeq 10$-20 K at least for densities $n<10^9$ cm^{-3}. Molecular cooling becomes even more efficient if large velocity gradients (e.g. due to cloud collapse) develop in a contracting cloud (Goldreich and Kwan 1974; de Jong *et al.* 1975). Furthermore, as is shown in Appendix B, the cosmic-ray ionization rate (and therefore, the heating rate) adopted here is probably greatly overestimated for densities $n \gtrsim 10^6$ cm^{-3}.

APPENDIX B

FRACTIONAL IONIZATION IN DENSE MAGNETIC CLOUDS

The degree of ionization in a dense interstellar cloud controls the coupling of the cloud to the galactic magnetic field. When the fractional ionization, n_e/n, is high, the time scale for ambipolar diffusion is long.

For an ionizing flux ζ, the electron density n_e in a cloud having a gas density n varies with time according to (Oppenheimer and Dalgarno [illegible]),

$$\frac{dn_e}{dt} = \zeta n - \alpha n(m^+) n_e - \alpha_r n(M^+) n_e - \alpha_g n(M^+) n \quad , \tag{B1}$$

where $n(m^+)$ and $n(M^+)$ denote the density of molecular and heavy metal ions, respectively, and α, α_r, and α_g are, respectively, the rate coefficients for dissociative recombination of molecular ions m^+, radiative recombination of heavy metal ions M^+, and recombination on grains. For simplicity, it is assumed that all rate coefficients are independent of ion type. It is easily shown that for densities [illegible] recombination on grain surfaces dominates radiative recombination so that the third term on the right-hand-side of Eq. (B1) can be neglected. In steady state equilibrium, Eq. (B1) then gives

$$n(M^+) = \frac{\zeta n - \alpha n(m^+) n_e}{\alpha_g n} \quad . \tag{B2}$$

The total ion density is

$$n_i = n(m^+) + n(M^+) \quad . \tag{B3}$$

The equilibrium rate equation for the molecular ion density gives (Oppenheimer and Dalgarno 1974)

$$n(m^+) = \frac{\xi n}{\alpha n_e + \beta n(M)} \quad , \tag{B4}$$

where n(M) is the total density of heavy neutral atoms (e.g. Mg, Ca, Na, and Fe) that undergo charge transfer with molecular ions, and β is the rate coefficient for the charge-transfer process. We neglect associative ionization (Oppenheimer and Dalgarno 1977) which is probably an insignificant source of electrons in cold interstellar clouds.

Putting Eqs. (B2) and (B4) into Eq. (B3) gives a quadratic equation for n_e which has the solution

$$n_e = -\frac{\beta n(M)}{2\alpha} + \frac{1}{2}\left\{\left[\frac{\beta n(M)}{\alpha}\right]^2 + \frac{4\xi}{\alpha}\left[\frac{\beta n(M)}{\alpha_g} + n\right]\right\}^{1/2} \quad . \tag{B5}$$

Following Oppenheimer and Dalgarno (1974), we define a depletion factor δ by the expression

$$n(M) = 4\times10^{-5}\,\delta\, n \quad . \tag{B6}$$

Observations with the Copernicus satellite (Morton *et al.* 1973; Morton 1974, 1975) suggest an average depletion factor for heavy metals, $\delta \simeq 0.1$. Oppenheimer and Dalgarno give $\beta = 10^{-9}$ cm^3 s^{-1}, $\alpha = 10^{-6}$ cm^3 s^{-1}, and $\alpha_r = 10^{-17}$ cm^3 s^{-1}. Eq. (B6) then becomes

$$n_e = 2\times10^{-9} n \quad [1 + \frac{10^{26}\xi}{n}]^{1/2} - 1 \quad , \tag{B7}$$

so that for densities $n \gg 10^{26}\xi$,

$$n_e \simeq 10^{17}\xi \quad . \tag{B8}$$

Ions in dense molecular clouds are supplied primarily by the ionization of molecular hydrogen by cosmic rays and ^{40}K radioactivity; ultraviolet radiation (Werner 1970) and X-rays (Nakano and Tademaru 1972) are screened in the peripheral regions of dense ($n>10^3$ cm^{-3}) clouds. Cameron (1962) has estimated the ionization rate by the β-decay of ^{40}K radioactive nuclei to be

$$\xi_k = 1.4\times10^{-21}\ s^{-1} \quad . \tag{B9}$$

Nakano and Tademaru (1972) have calculated the ionization rate of atomic hydrogen by cosmic rays. Adjusting their result for ionization of molecular hydrogen by taking into account the difference in ionization cross sections, $\sigma_{H_2}=1.65\sigma_H$ (Bates and Griffing 1953), the cosmic-ray ionization rate in a cloud having a mass m is given by*

$$\xi'_{CR} = 10^{-17}\ \exp[-1.54\times10^{-7}(m/m_\odot)^{\frac{1}{3}}n^{\frac{2}{3}}] \quad . \tag{B10}$$

The exponential term reflects the attenuation of cosmic rays due to their interaction with matter as they propagate through a cloud. The total ionization rate

$$\xi = \xi'_{CR} + \xi_k \quad . \tag{B11}$$

By comparing Eqs. (B9) and (B10), one can see that for stellar-mass clouds having densities $n \gtrsim 10^{10}-10^{12}$ cm^{-3}, cosmic rays are effectively screened and ions are produced primarily by *in situ* ^{40}K nuclei.

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*Brown and Marcher (1977) have shown that ionization of H and H_2 in dense clouds may be enhanced by energetic secondary electrons produced by knock-on collisions, neutron-decay reactions, and pion-decay reactions, $\pi^\pm \to \mu^\pm \to e^\pm$, following the interaction of fast (primary) cosmic rays with the material in a cloud. This effect, which may be significant only when low-energy cosmic rays are excluded from dense clouds, is difficult to estimate quantitatively, and is therefore neglected here.

Eq. (B10) may greatly overestimate the cosmic-ray ionization rate in dense magnetic clouds. If the magnetic field within a contracting cloud becomes tangled (e.g. by turbulence), cosmic rays, constrained to move along the magnetic lines of force, random walk through the cloud and must traverse more matter to reach the central regions of the cloud. Nakano and Tademaru (1972) have estimated the importance of this effect, and have concluded that for densities $n \gtrsim 10^3$ cm^{-3}, cosmic rays are effectively screened, i.e., $\xi'_{CR} < \xi_k$ so that $\xi \simeq \xi_k$.

When cosmic rays stream along magnetic fields in a collisionless plasma faster than the Alfvén velocity, they generate hydromagnetic waves which in turn scatter the cosmic rays (Wentzel 1974, and references cited therein). Indeed, Skilling and Strong (1976) have shown that the incoming cosmic-ray flux in a dense cloud may be substantially reduced by this mechanism. However, the damping effect of ion-neutral collisions may inhibit the generation of such waves by the cosmic rays themselves (Kulsrud and Pearce 1969).

As pointed out by Nakano and Tademaru (1972), cosmic rays, although not unstable to the generation of hydromagnetic waves, are strongly influenced by the presence of such waves generated by other mechanisms (e.g. by magnetic braking during cloud collapse). Cosmic rays are scattered by these waves and can not freely stream along the open magnetic field lines. Nakano and Tademaru have shown that effective screening of the cosmic rays occurs if

$$\langle \delta B^2 \rangle \gtrsim \frac{10^{10} B}{m^{\frac{2}{3}} \rho^{\frac{1}{3}}} \quad , \tag{B12}$$

where $\langle \delta B^2 \rangle / 8\pi$ is the energy density of the hydromagnetic waves having amplitude δB, B is the strength of the cloud's large-scale uniform field, and $\rho=3m/4\pi R^3$ is the mass density of the cloud material. By eliminating ρ in favor of m and R, and by defining $\eta\equiv R/R_o$ with $B=B_o\eta^{-2}$ (see Section III and IV of the text for a discussion of the rate of growth of the magnetic field in a contracting protostar), with subscripts denoting initial values, this expression becomes

$$\langle\delta B^2\rangle \gtrsim \frac{8\times10^{-24} R_o B_o}{(m/m_\odot)\eta} \quad . \tag{B13}$$

From Table 1 in the main text, we see that $R_o B_o \simeq$ constant $\simeq 9\times10^{12}$ cm gauss for all cloud masses. Thus, if

$$\langle\delta B^2\rangle > \frac{7\times10^{-11}}{(m/m_\odot)\eta} \quad , \tag{B14}$$

cosmic rays are effectively screened.

The energy lost from a magnetically braked, collapsing cloud is just the difference between the cloud's rotational kinetic energy given by angular momentum conservation, and its magnetically-braked rotational energy. For a cloud which is constrained to co-rotate with its surroundings, this energy is given by

$$E_{lost} \simeq 0.2 m\omega_o^2 R_o^4 R^{-2} \quad . \tag{B15}$$

Assuming that this energy is carried away by the hydromagnetic waves which are generated by the braking process, we can write

$$\langle\delta B^2\rangle \simeq 2\times10^{-11}\,\eta^{-5} \quad , \tag{B16}$$

where we have taken initial values from Table 1, noting that $\omega_o^2 R_o^3/(m/m_\odot) \simeq$ constant $\simeq 10^{25}$ cm^3 s^{-2}. Actually, depending on the mass of the cloud, $\langle\delta B^2\rangle$ may be an order of magnitude smaller or larger, so that Eq. (B16) represents an approximate mean value. A comparison of Eqs. (B14) and

(B16) shows that cosmic rays are effectively excluded from a contracting cloud very soon after the collapse begins.

It is for these reasons that we believe *the ionization in dense magnetic clouds is determined by the* ^{40}K *ionization rate,* i.e., $\xi=\xi_k$. This being the case, the magnetic flux linking a contracting cloud to its surroundings, uncouples at relatively low gas densities. This may explain the absence of large magnetic fields in some dust clouds (Crutcher *et al*. 1975).

Even if the cosmic rays are not magnetically scattered, there will be a reduction in the flux of cosmic rays in a magnetic cloud. The magnetic field lines in the neighborhood of a contracting cloud diverge outward from the cloud so that charged particles streaming along the field lines into the cloud will be reflected by the 'magnetic mirror effect.' Fermi (1949, 1954) proposed that such a magnetic reflection mechanism might explain the origin of the galactic cosmic rays. For slow variations of the magnetic field in time and space, the diamagnetic moment of a charged particle is an adiabatic invariant. Let θ be the angle between the direction of the line of force and the direction of motion of the spiraling particle, *viz.*, the pitch angle. Assuming an isotropic distribution of particle velocities in a region where the field strength is B, one can easily show (*cf*. Spitzer 1962) that the velocities must fall within a solid angle defined by the pitch angle such that

$$\theta = \sin^{-1}(B/B_s)^{\frac{1}{2}} \tag{B17}$$

when the field strength increases to B_s. In our case, B is taken to be the (uniform) galactic field far from a contracting cloud, and B_s is the field strength at the cloud surface. Assuming that the particle density in a given region is proportional to the size of the solid angle given by Eq. (B17), it follows (*cf.* Kaplan and Pikelner 1970) that the fractional decrease in cosmic-ray flux is

$$f = 1-\cos\theta \quad . \tag{B18}$$

The cosmic-ray ionization rate is then given by

$$\xi_{CR} = f\,\xi'_{CR} \quad , \tag{B19}$$

where ξ'_{CR} is given by Eq. (B10). Eq. (B19) provides a workable upper-limit to the ionization rate in dense magnetic clouds, the lower-limit determined by ξ_k being the most likely for reasons already discussed.

Eq. (B17) is valid only if the Larmor radius

$$r_L = \frac{3.13\times10^{12}}{B_\mu}\left[(1+\frac{E_k}{938})^2-1\right]^{1/2}\text{cm} \tag{B20}$$

of a cosmic ray having a kinetic energy E_k(MeV), and moving in a magnetic field B_μ (microgauss), is much less than the radius of the cloud. One finds r_L(2MeV)=3.4×10^{10} cm and r_L(10 GeV)=6.1×10^{12} cm, so that in fact, r_L<<R. Also, the quantity $(\frac{1}{B}\,|\frac{dB}{dt}|)^{-1}$ must be significantly greater than the Larmor period. It is easily demonstrated that this condition is equivalent to the dynamic time scale being much larger than the Larmor period, or

$$t_f \gg \frac{1.4(E_k+938)}{B_\mu} \quad \text{s} \quad . \tag{B21}$$

Since collapse times are typically 10^{13}-10^{14} s, this condition is easily satisfied. Finally, Eq. (B17) neglects collisions among the cosmic rays which have the effect of randomizing the pitch angle θ. We require that the time between collisions be much greater than the Larmor period, or equivalently, that the density of material (primarily molecular hydrogen) in a cloud satisfies:

$$n \ll \frac{5.1\times10^{-14}B_{\mu}}{\sigma}\left[\left(1+\frac{E_k}{938}\right)^2-1\right]^{\frac{1}{2}} \text{ cm}^{-3} \quad , \tag{B22}$$

where σ is the collision cross section. Low-energy cosmic rays ($E_k \simeq 2$) interact with the cloud material primarily by ionizing molecular hydrogen. Glassgold and Langer (1973a) give $\sigma(2\text{MeV})=1.89\times10^{-17}$ cm^2. High-energy cosmic rays ($E_k \gtrsim 10^3$) interact mainly by p-p scattering and pion-production reactions with a cross section $\sigma(10\text{GeV}) \simeq 2\times10^{-26}$ cm^2 (see Nakano and Tademaru 1972 for references). Thus, from Eq. (B22), $n(2\text{MeV}) \ll 4\times10^4\ B_{\mu}$ and $n(10\text{GeV}) \ll 2\times10^4\ B_{\mu}$. Since initial values for B_{μ} range from 3 up to 10^3 (see Table 1) and B_{μ} increases as $n^{2/3}$ during gravitational collapse (see Section IV of the text), these conditions are easily satisfied.

APPENDIX C

MOMENT OF INERTIA FOR DIFFERENTIALLY ROTATING MAIN-SEQUENCE STARS

The angular momentum of a rotating spherical body having a radius R, a radial density distribution $\rho(r)$, and an angular velocity field $\omega(r,\theta)$ is

$$J = \frac{8\pi}{3}\int_0^R \int_0^{\pi/2} \omega(r,\theta)\rho(r)r^4 \sin\theta d\theta dr \quad . \tag{C1}$$

Assuming $\omega(r,\theta)=\omega(r)$, this becomes

$$J = \frac{8\pi}{3}\int_0^R \omega(r)\rho(r)r^4 dr \quad . \tag{C2}$$

The actual distribution of mass throughout chemically homogeneous stars which are not completely convective often approximates that in the 'standard model' of Eddington (1926), which is just a polytrope of n=3. For a rigidly-rotating star, $\omega(r)=\text{constant}=\omega_R$, the angular velocity at the surface (r=R). The integral in Eq. (C2) is then most easily evaluated with the aid of the Emden solutions for an n=3 polytrope (see, for example, Chapter 23 of Cox and Giuli, 1968). Writing the angular momentum in terms of the moment of inertia $I=\kappa MR^2$, where κ is the gyration constant, we have

$$\begin{aligned} J &= I\omega_R \\ &= \kappa MR^2\omega_R \quad , \end{aligned} \tag{C3}$$

so that for the case of rigid rotation, we find

$$\kappa = 0.08 \quad . \tag{C4}$$

The pre-main-sequence evolutionary models of Bodenheimer and Ostriker (1970) for rapidly rotating massive stars predict a marked differential rotation, with the central angular velocity ω_c being a factor of ten greater than ω_R. Their differentially rotating configurations are stable, according to the criterion developed by Goldreich and Schubert (1967). From Figure 5 of Bodenheimer and Ostriker (1970) we approximate the angular velocity as

$$\omega(x) = 10\ \omega_R e^{-2.3x^a} \quad ,$$

where $x \equiv r/R$, and

$$\begin{aligned} a &= 1.4 \quad \text{for} \quad 0 \leq x \leq 0.3 \\ &= 1.1 \quad \text{for} \quad 0.3 \leq x \leq 1.0 \quad . \end{aligned} \tag{C5}$$

The mass distribution for a polytrope of n=3 can be approximated as

$$\rho(x) = \rho_o \bar{\rho}\ e^{-bx^c}$$

where

$$\begin{aligned} &b = 20 \text{ and } c = 2 \qquad \text{for} \quad 0 \leq x \leq 0.3 \\ &b = 11 \text{ and } c = 1.5 \quad \text{for} \quad 0.3 \leq x \leq 1.0 \quad . \end{aligned} \tag{C6}$$

Here, $\bar{\rho} = 3m/4\pi R^3$ and $\rho_o(n{=}3) = 54.18$ g cm^{-3}. Substituting the above expressions for $\omega(x)$ and $\rho(x)$ into Eq. (C2), and evaluating the integral using Simpson's Rule, we find

$$\kappa = 0.28 \quad . \tag{C7}$$

The gyration constant (and therefore, the angular momentum) of the differentially rotating configuration is thus three times that of a rigidly-rotating body.

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I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

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