IMPLICATIONS OF OPTION MARKETS: THEORY AND EVIDENCE

BY

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Many new markets for options have opened in recent times. Financial research has been interested in the experiences of option investors, and much empirical work has been done in this regard. However, in previous studies the issue of why options exist at all is rarely addressed.

The exploration of the role of options in important theoretical models of capital market equilibrium leads to the following conclusion concerning the effects of opening new option markets: If margin ceilings impose effective borrowing constraints on any investors, then the expansion of the option market will cause the equilibrium intercept of the risk-return relation to shift downward. This theory is tested in this study by an examination of a time series of returns of option-stock hedges that theoretically have zero systematic risk. The hedge returns, which empirically

did _not_ appear to evidence significant systematic risk, did behave in a manner consistent with the theory of the risk-return intercept change. However, the change in the hedge returns was not statistically significant.

CHAPTER I
INTRODUCTION

The expansion of new option trading since April, 1973, has been rapid and still continues. The purpose of this study is to examine a potential effect that the expansion may have had on the capital markets. The hypothesis is that new option trading causes a change in the intercept location of the theorized risk-return relation. The theoretical framework used for the development of the hypothesis is the mean-variance capital asset pricing model (MVCAPM).

The bases for the theory that options cause a change in the intercept of the risk-return relation are two alleged circumstances (to be analyzed later): (a) options may be employed by some traders to obtain leverage amounts that are not possible through normal margin channels; and (b) in a single-period, equilibrium-theoretic model where margin constraints are effective, the introduction of new options may cause a shift in the intercept of the risk-return relation. The first of these observations will be examined in this chapter; the second observation is considered in Chapter II. Chapter III is a discussion of other empirical work that is potentially related to this study. In Chapter IV a methodology for empirically testing the hypothesis is described in detail. The results of our own tests are presented in Chapter V. Chapter VI concludes the study by reviewing the major results and suggesting various implications and limitations. First, the nature of options is discussed.

Futures and Options

In a treatise by Bachelier (1900), translated from the French by Boness and included in Cootner's (1964) book, options are described as being futures contracts but with a limited liability feature for one of the contract parties. Trading in a futures contract is similar to spot trading in the commodity, except that in a futures contract the exchange of money for the commodity is made at a specified later date. The price at which the exchange is to be made is agreed upon when the contract originates, however, and not at the subsequent time of exchange. An option is similar to a futures contract except that one of the exchange parties has the option to cancel the future trade, and payment for this privilege is made at the time the contract originates.

In all futures contracts there is a seller, who contracts to deliver the commodity for money, and a buyer, who contracts to deliver[1] money for the commodity. If the buyer has an option to cancel the trade, then it is said that the buyer owns a call option. In this case the seller of the commodity is the writer of the call option. If the commodity seller has the option to cancel, then it is said that the commodity seller owns a put option. In this case the commodity buyer is the writer of the put.

The above description of options applies to "European" options. Alternatively a call or put option may be an "American" option. An American option is the same as a European option except in an American option the option owner may call for the exchange (still at the agreed price) anytime <u>prior</u> <u>to</u> the original settlement date. Since the settlement date is the final time at which an option owner may announce his option, this date is termed the expiration date of the option.

It is from these characteristics that the following definitions of call and put options have evolved: A call option is a contract giving its owner the right to buy a commodity at a specified price, <u>at</u> or <u>before</u> a specified time (depending on whether it is <u>European</u> or <u>American</u>); a put option is a contract giving its owner the right to sell a commodity at a specified price, at or before a specified time. In either case the right of contract is granted by the writer of the option. If an option contract is held beyond the expiration time, it becomes worthless.

In the case of either calls or puts, the option owner must pay to the writer an amount for the limited liability privilege; and in the case of an American option, the option owner must sometimes pay to the writer an additional amount for the right to activate an early exchange. Merton (1973b) has argued that in an ideal market, this additional payment for the early exercise privilege should be zero, if either (a) the underlying commodity pays no dividends prior to expiration time, or (b) the futures (settlement) price is automatically adjusted for any dividend payments. In the U.S. market for exchange listed common stock options there is no dividend protection, and since many underlying stocks do pay dividends, the early exercise privilege theoretically has some value to the owners of some options traded on the U.S. exchanges.[2]

In the case of futures contracts without options the only thing to be negotiated is the future exchange price; for options there are two amounts to be negotiated simultaneously: (a) the future exchange price, and (b) the amount that the holder of the option will give to the writer for the limited liability and early exercise features. In the present U.S. exchange listed option market, the future exchange price for stock in dollars is contract-standardized. What is negotiated is an amount called the premium. Thus, the premium is consideration for three amounts: (a) the price of the limited liability feature; (b) the price of the early exchange feature; and (c) the difference between what would have been the negotiated futures settlement price and the contract-standardized settlement price.[3]

Use of Options in the U.S.

Evolution Prior to 1973

Although put and call options originated outside of the U.S., the use of put and call options in the U.S. was begun by financier Russell Sage in 1869.[4] Sage and others were in business to lend money to brokers, who would use the borrowed funds (termed margin credit) to purchase stocks. Experience showed that the brokers could not always repay the loans when due, but it was impossible for the lenders to charge enough interest to compensate for this risk because of usury laws. Consequently, Sage devised a way to employ call and put options to conduct his business, and other lenders soon adopted Sage's method. Instead of extending margin credit to a broker, the lender would write a call option, and the broker would become the call option owner. As part of this arrangement the broker would write a put option back to the lender. Thus if the stock declined in price, the lender could exercise his put option and sell the stock to the broker at the put contract price. If the broker could not pay, then this was the lender's loss. But the lenders could charge what they wanted for the call options, and presumably they charged enough to compensate for the possibility of nonrepayment.[5]

The put and call business operated along these lines until Sage died in 1906. Sage met all of his contractual obligations, but his successors would, from time to time, dishonor the contracts if the market went against them as sellers. These reneges caused great resentment among option traders, and the business was plagued by lawsuits until the Securities and Exchange Act of 1933-1934. At that time various dealers in puts and calls formed the Put and Call Brokers and Dealers Association, Inc. This association brought two vital attributes to the business: (1) uniform contracts and (2) endorsement by New York Stock Exchange members.

The new market in options provided by the Puts and Calls Brokers and Dealers Association extended option trading to individuals as well as brokers. Individual investors were permitted to own or write puts and calls separately. However, the market involved the direct matching of owners and writers. The large transaction costs of the matching procedure for the separated contracts provided the impetus for the first option exchange in 1973.

The Post 1973 Option Market

The first U.S. option exchange created for the purpose of facilitating option trading was the Chicago Board Options Exchange (CBOE). Other option exchanges subsequently began operations following the unexpectedly huge success of the CBOE. The activities of all option exchanges are presently coordinated by the Options Clearing Corporation, which also serves as guarantor to individuals on both writing and owning sides of option contracts.

Exchange listed options are now standardized in the following respects. Exchange listed option contracts are for 100 shares. Premiums are specified in dollars per option on one share, so that a contract quote of $4 would mean that the buyer would pay a premium of $400 for the standard contract on 100 shares. The standardized option contracts which are traded on the organized exchanges are quoted by the month of expiration and by the price at which the future stock transaction will take place--the exercise price. At present, these contracts expire at 11:50 p.m. Eastern time on the Saturday following the third Friday of the expiration month. The cut-off-time for individual investors to instruct brokers concerning exercise is 5:30 p.m. Eastern time on the business day immediately preceding the expiration time.

Example: An American Telephone and Telegraph Company (AT&T) January 50 that is traded on the Chicago Board Options Exchange (CBOE) is an American call option contract to buy 100 shares of AT&T corporate common stock. The price at which the 100 shares of stock may be bought is $50 per share, and the stock may be bought any time before the January expiration time. If the price of a share of AT&T exceeds $50 just prior to the cut-off-time, then it would be profitable for the call owner to exercise the option and resell the shares, assuming no transaction costs. Net trading profit depends upon the original cost of contract (the premium).

If the price of a share of AT&T is below $50 per share near the cut-off-time, then it would make no sense to exercise the option and pay $50 per share, when the same share could be purchased for a lower price without an option. The trader in this case should let the contract expire worthless. The net loss will be the original purchase price, or premium.

An AT&T January 50 P is a put contract with the same specification, except a put contract is a contract to <u>sell</u> 100 shares. With puts, the trader should exercise if the stock price, at cut-off-time, is below the exercise price. The trader will make a net profit, if the stock price is below the exercise price by more than the original premium paid. If the stock price is above the exercise price at cut-off-time, the trader should discard the worthless put.

End of Example.

The option market is organized in such a way that individual investors can take profits without exercising their options at the expiration time; the option owner, at the expiration time, simply sells the contract back to a writer for exactly the difference between the stock price and the exercise price. This difference is the owner's gross profit or loss. Whatever is the owner's gross profit is the writer's loss, and vice versa.

Because of the efficiency of the options exchange there is usually good liquidity in the sense that an option contract, once originated, can easily be resold to another trader anytime prior to the expiration time. The consideration in any purchase-sale of an option contract, the premium, is negotiated virtually continuously according to supply and demand. Thus option traders can incur profits and losses by buying and selling option contracts over any time intervals, without ever seriously considering taking delivery of the stock.

The prices of the last contracts traded of each option type on each stock are reported daily in financial and metropolitan newspapers. Presently, transactions costs are not included in the newspaper trade quotes. Transaction costs vary with the size of the transaction in both dollar and volume terms. Brokerage commission schedules are competitive and will vary from broker to broker. Transaction costs are significant to nonexchange members who want to trade a few contracts; however, transaction costs are less significant to exchange members.[6] The consequences of present tax structures for studies of option trading are complex to analyze, because there are different tax rates for different investors. More detail on the arrangements for common stock option trading may be obtained by consulting Gastineau (1975), Golden (1975), the Chicago Board Options Exchange Prospectus (1973), and the Options Clearing Corporation Prospectus (various dates).

The Moratorium and After

Trading volume in contracts of each underlying stock and the number of underlying stocks had expanded rapidly and steadily until July 1977, when the Securities and Exchange Commission (SEC) halted the further expansion of options trading on additional underlying securities. At the time of the moratorium, call option markets on common shares of 235 different companies were open, and put option markets on 25 different companies were open. Volume of underlying shares represented by option trading continued to increase after the moratorium on expansion was imposed by the SEC.

The purpose of the moratorium was to give the SEC a chance to examine the effects of option trading and to review the trading and self-regulating practices on the exchanges. The result of the SEC study was The Report of the Special Study of the Options Markets to the Securities and Exchange Commission (hereafter called the SEC Study), which was dated December 22, 1978.

In general the SEC Study (p. v) found that "options can provide useful alternative investment strategies to those who understand the complexities and risks of options trading. But, since regulatory inadequacies in the options markets have been found, the Options Study is making specific recommendations needed to improve the regulatory framework within which listed options trading occurs and to increase the protection of public customers."

In 1980 the moratorium was lifted, and in May of 1980 the list of stocks underlying listed put options expanded from the original 25 to 105. In July, 1980, the list of new stocks underlying call and put options also began to expand again.

Exchange listed options on physical commodities, foreign currencies, or other securities (such as treasury bills, bonds, mutual funds, and futures contracts) are not presently traded in the U.S. However, options on physical commodities are traded in other countries, including gold futures options on the Winnepeg, Canada Exchange.

<u>Reasons for Listed Stock Option Usage</u>

Various reasons have been offered about why investors and speculators trade options. Uses of options are explained in passages and chapters of various investments textbooks (especially in the more descriptive textbooks) and are more thoroughly described in some of the less technical books about options.[7] Some typical rationales for the use of options by individual (non-broker) investors are developed below. The interested reader is advised to consult the sources in note 7 for more details.

Leverage. The use of borrowed funds to purchase investments is referred to as financial leverage, or more simply leverage. The use of puts and calls by brokers as an alternative to margin arrangements has already been described.

In modern markets margin credit may also be employed by individuals, who borrow the funds from brokers or banks. For individuals, the maximum amount of credit that can be employed to purchase stocks is set by Regulations T and U of the Federal Reserve. (Currently the "margin ceiling" is 50%, meaning no more than 50% of a stock's purchase price may be paid with borrowed funds. This ceiling has ranged from a low of 40% to a high of 100% in the last 40 years.) Investors who employ option contracts for leverage instead of margin credit have three distinct advantages: (1) Since an option contract premium will often be between 1% and 50% of the cost of the stock shares, option contracts represent much higher degrees of leverage than are available through the use of margin. (2) Option contracts offer a limited liability feature which prevents loss of more than the original premium paid. Such limited liability is not available in margin trading. The limited liability feature of knowing one's maximum loss in advance may be an inducement to individuals to employ the very high leverage. (3) Standard brokerage policy requires a minimum equity of

$2,000 for margin trading in common stock. Thus option trading may bring into the market some new investors, who individually may have relatively small amounts of capital.

Hedging. If a stockholder becomes uncertain about the future price volatility of a stock, or if the stockholder anticipates a price decline, writing (selling) call options would give the investor a position that is hedged against unfavorable changes in those variables. The investor can use his stock as collateral, thus temporarily changing the strategy of his position, without incurring the transaction costs of selling and buying stocks and without foregoing dividends.

Income. Although it is theoretically possible for investors to sell shares for current income purposes, this practice is not normal. The costs of transactions and the possibility of deferring taxes work to discourage share turnover. One way for stock owners to receive current income is to write call options utilizing stock shares as collateral. This technique is also useful for managers of large institutional portfolios, for whom the sale of shares is difficult without creating a significant stock price decline, due to the sheer size of the average transaction.

According to Paul Sarnoff (1968), a former options broker for many years, the scenarios above describe the primary nonbroker uses for options. Thus, in general, option buyers are thought of as speculators who employ options because there is more leverage than otherwise available in the stock market (with limited liability), and who are tolerant of risk and/or possess special information. Option writers are primarily portfolio managers who desire protection against large price drops, especially during times of income need. Sarnoff's beliefs about why options are utilized correspond to the evidence reported next.

Evidence of Reasons for Recent Option Trading

The SEC Study identified three categories of participants in the options markets: (1) public nonprofessionals, (2) professional money managers, and (3) professional traders and arbitrageurs. The SEC Study also identified the basic purposes served by the various common types of options transactions. The basic purposes are: (a) to obtain leverage, (b) to hedge positions in the underlying security, (c) to increase current income from securities holdings, (d) to arbitrage for profit, (e) to speculate or trade on perceived over-and-undervalued situations,[8] and (f) to facilitate the provision of brokerage and market-making services in the underlying stocks.

The SEC Study (pp. 106-107) describes the varying perspectives of investors as they approach the market:

> Traders, for example, attempt to capitalize on undervalued and overvalued situations by using complex mathematical models and computer techniques to detect and arbitrage against perceived illogical divergences in prices. Studies of option price patterns, however, indicate that while price divergences do occur which may provide profitable trading opportunities for professionals the divergences generally are too small for trading

> opportunities by members of the public because of transaction costs. Other, generally sophisticated, investors perceive an opportunity to adjust the risk-reward mix of their portfolio of assets in a more precise manner because of the additional combinations of risk and potential return opened up to them by the availability of exchange traded options.
>
> Risk management and risk adjusted performance have become basic criteria upon which professional managerial ability is evaluated. Most individual investors in options, however, are probably using option purchases and sales as a substitute for stock purchases and sales. Dealing in options enables them to take short-term positions in the stock, or shift out of the stock in the short-term with lower transactions costs; and, for buyers, it offers greater leverage than would be the case if they were trading directly in the underlying securities. (SEC Study, p. 107)

A survey released in 1976 and conducted by Louis Harris Associates (1976) for the American Stock Exchange listed 10 strategies that appear to be most commonly employed by investors. The 10 strategies listed by Harris are shown below:

<u>Buying</u>

1. Buying options in combination with stock ownership.
2. Buying options in combination with fixed-income securities.
3. "Pure" buying of options without underlying stock or fixed-income securities.

<u>Mixed Strategies</u>

4. Buying options against a short position in underlying stock.
5. Buying options as a hedge against a short position in securities related to the underlying security.
6. Selling options hedged against other related securities.
7. Spreading options by buying and selling different options in the same underlying securities.

Selling

8. Selling fully covered options.
9. Selling partially covered options.
10. Selling completely uncovered options.

. . .

The Harris survey found that among individual investors, the largest percentage (58%) employed the pure buying of options strategy (#3 above). Of the persons investing a total of $2,500 or less, 49% employed the pure option buying strategy. In contrast to individual investors, 79% of the institutional investors surveyed concentrated their activities in fully covered option writing strategies. Another survey, by Robbins, Stobaugh, Sterling, and Howe (1979), sponsored by the CBOE, also found that the two strategies followed most frequently by investors were the simple buying and covered writing of option contract strategies. The SEC Study (p. 116) pointed out that: "Neither survey included interviews with broker-dealers, a professional, but extremely important group, using options in their activities. Block-positioning firms, marketmakers and other broker-dealers make extensive use of options in providing dealer services to the public market."

From the foregoing discussion it appears reasonable to hypothesize that (a) margin constraints are effective in the U.S., and (b) one rationale for option owning is that options offer a viable alternative to margin trading as a means for more investors to obtain more leverage.

Overview of the Study

In the next chapter the effects of introducing options into a theoretical model of risk and return are analyzed. One assumption of the model is an effective margin constraint. It is found in the model that as more options are introduced, the equilibrium risk-return relation is altered; specifically, the expected return on securities with zero systematic risk will decline toward the riskless rate. This result is not surprising in light of the results of Sharpe (1964), Lintner (1965), Black (1972), Fama (1976), and Vasicek (1971), who originated and contributed heavily to the construction of the theoretical model. Further details are deferred to Chapter II.

In Chapter IV the methodology is developed to test the hypothetical effect of options on the risk-return relation. The reader may be interested in a brief overview of the procedure to be employed:

First are identified securities which are logical candidates for a zero systematic risk portfolio. The rate of return on a zero systematic risk securities portfolio will be referred to as $E(r_z)$. Estimates of that expected rate are denoted $\bar{r}_z$.

The time span of a time series of $\bar{r}_z$ values covers an extended period during which no new options began trading on exchanges (Period 1) and another extended period during which an ample number of new options began trading (Period 2).

During Period 1 the time series of observations for $\overline{r}_z$ may be considered as estimates of an equilibrium rate, assuming there are no changes in market equilibrium conditions. During Period 2 the time series observations of $\overline{r}_z$ must be viewed, in light of the hypothesis, as including two portions: (1) the new lower equilibrium rate of return for zero systematic risk portfolios, and (2) the return associated with the transition from the old equilibrium state to the new one, following the theorized effect of the new options.

The second portion may be significantly higher than either of the equilibrium rates as is evident from the following example: Assume that prior to the introduction of new option trading the equilibrium rate of return on a zero systematic risk asset is .08. Now suppose the introduction of new option trading causes the equilibrium rate for the zero systematic risk asset to decline to .06. In order for this to occur, the price of the zero systematic risk portfolio must increase by 33-1/3%, say from 100 to 133-1/3. The observed transition rate of return would show up in the Period 2 time series and be relatively high indeed.

This scenario establishes (qualitatively) what should be expected from the empirical analysis if the following assumptions are valid: (1) the theoretical framework employed in Chapter II is valid; (2) other factors that may affect $E(r_z)$ are properly accounted for; and (3) new option trading has a significant enough impact to be observed.

In order to accomplish the purposes of the empirical analysis, these general procedures are to be employed:

(1) The time series of observations of $\bar{r}_z$ are converted into "excess return" form by subtracting corresponding observations of r_f. This adjustment represents a method of accounting for <u>exogeneous</u> shifts in the location of the risk-free rate, <u>and</u> simultaneously of the level of $\bar{r}_z$. Such shifts could result from federal influence on interest rates, for example. Whatever the exogeneous sources of disturbance, it is the relative distance between $\bar{r}_z$ and r_f values that is being measured in this study, so the conversion to excess returns is appropriate. This excess return variable is referred to as $\bar{r}_z^e$. Thus, abnormally high observations of $\bar{r}_z^e$ are expected to be found in Period 2 relative to Period 1.

(2) In order to gauge the significance of the values of $\bar{r}_z^e$ in Period 2 relative to Period 1, the following statistical procedure is employed. The observations of $\bar{r}_z^e$ are used as dependent variable observations in a multiple regression on two variables called I_1 and I_2. The first variable, I_1, takes on the value 1 for all observation periods in Period 1 and Period 2. The second variable, I_2, takes on the value 1 for all observation periods in Period 2, but has the value 0 for all observation periods in Period 1. The resultant regression coefficient for I_1, will be the mean of the $\bar{r}_z^e$ time series during Period 1. The resultant

regression coefficient of the variable I_2 is the increase in the mean of the $\bar{r}_z^e$ values from Period 1 to Period 2. Thus, the t-statistic of the regression coefficient of I_2 can be used to judge whether the values of the observations of $\bar{r}_z^e$ are significantly higher in Period 2 than in Period 1.

(3) The possibility exists that equilibrium shifts will occur as a result of factors other than new option trading, such as changes in expected inflation or in multi-period preferences. One indication of this possibility is for the residuals in the regression to exhibit serial correlation. If this correlation occurs, then the meaningfulness of the regression is in question. A well-known statistical procedure (Cochrane-Orcutt) may be employed to counteract serial correlation in the residuals and whatever effects this correlation may have on the t-statistic of the coefficient of I_2.

The next chapter presents the theoretical basis for the hypothesis.

Notes

[1]There is only one delivery date, which is decided upon at the time the contract originates. The delivery date is also called the settlement date.

[2]Empirical examinations of option pricing models indicate that models which adjust for dividends are more valid than option models with no dividend adjustment. See, for example, Galai (1977) and Chiras (1977). Option pricing models are discussed in Chapter IV.

[3]For example, consider a call option, and let the prices of the limited liability feature and the early exercise feature be $.50 and $.25, respectively. Assume the contract standardized settlement price is $50.00, but that two traders would have preferred to negotiate a futures price of $52.00. The buyer of the call option must pay $2.00 more for the contract than he would have if the settlement price were $52.00. Thus the call contract premium should be $2.00 + .50 + .25 = 2.75 in this case.

[4]This account of option trading in the U.S. prior to 1973 is paraphrased from Sarnoff (1968).

[5]As long as risk conditions dictated that lenders required a rate of interest less than the usury law ceiling, lenders could have used either the option method or the direct lending method. In order to obtain higher "interest" compensation if the option method was chosen, lenders would build this compensation into a higher price of the call option. For example, a call option would sell for more if conditions dictated 10% interest than if conditions dictated 6%.

[6]See Phillips and Smith (1980) for a discussion of transactions costs of options exchanges.

[7]Examples of less technical investments textbooks which discuss option strategies are Johnson (1978), Mendelson and Robbins (1976), and Wright (1977). The more recent the text, the more detail about options can usually be found. Recent technical investments texts, like Sharpe (1978) and Francis (1976), usually have a formal chapter about stock options. Examples of nontechnical books about options include Clasing (1975), Cloonan (1973), Dadekian (1968), Filer (1966), and Sarnoff (1968). In addition, Malkiel and Quandt (1969) is written for a wider audience than only options experts and is detailed in its explanation of various option trading strategies.

[8]An interesting example of the use of options by brokers to arbitrage for profit and acquire capital appeared in an article of the Wall Street Journal (August 7, 1980, p. 30). Brokers went long 1000 calls and 1000 puts on the shares of Tandy Corporation. Since the prices were $9 per call and $3 per put, the "spread" was $6. Brokers are required to outlay only the amount of the spread for this position. The brokers then sold 1000 shares of the stock short at $66 to create a perfect hedge position that would return $6 per share, or $600.00. The brokers could then use the short sale proceeds to invest elsewhere. Since the short sale proceeds were $6.6 million, brokers had $6.0 million in "free" capital, upon which no interest had to be paid. In this case the brokers were exploiting a riskless arbitrage situation.

CHAPTER II
OPTIONS IN A SINGLE PERIOD CAPITAL MARKET EQUILIBRIUM FRAMEWORK

This chapter introduces options into the analysis of a single period capital market model. Equilibrium conditions are derived in the usual fashion, except that investors are permitted to hold options in their portfolios as well as stocks. A primary distinction between stocks and options in this context is that options are not issued by corporations, whereas stocks are. The aggregate market value of all option positions in the market is zero; for every investor who owns an option, there is another investor who is short an option. The aggregate market value of each stock in the market is not constrained, but must be positive, of course, to be realistic.

Three circumstances will be examined: (i) the case where no riskless security exists; (ii) the case where investors can go long or short the riskless security in any amount; (iii) the case of margin restrictions on trading the riskless security. The riskless security is defined to be a security with a fixed and known-in-advance nominal return. As with options, the aggregate holdings of the riskless security are presumed to be zero. In other words, the economy as a whole cannot have an excess of borrowing over lending or an excess of lending over borrowing.

The models to be derived are single period equilibrium models based upon the mean-variance criterion of Markowitz (1959). In addition, it will be assumed that (a) all investors have identical probability beliefs;[1] (b) all investors make their portfolio decisions at the same discrete points in time; (c) the market is perfectly void of indivisibilities, taxes, transactions costs, and monopoly influence by any investor; and (d) no consumption price inflation exists.[2] Thus, this chapter applies an elementary Sharpe-Lintner CAPM framework to a setting that includes options.

The analyses in the three sections of the chapter correspond to three distinct circumstances mentioned earlier in connection with the trading of the riskless security. In all three sections only one option is assumed at first. Generalization is subsequently made to a portfolio of options with the following result: if the riskless security is either unavailable or restricted (and if investors are constrained by the latter circumstance), then the equilibrium expected rate of return on zero systematic risk securities declines as the number of options in the market increases.

In the proofs to follow investors are assumed to employ in their portfolios any of n risky assets (stocks), the option[3] and, when specified, a riskless security. The random rates of return on the assets are denoted r_1, r_2, .., r_n, while the rates of return on the option and the riskless security are denoted by r_o (a random variable) and r_f, respectively. Similarly, the proportionate investments by an individual into each of the n risky assets are denoted by x_1, x_2, .., x_n; x_o and x_f denote the proportionate holdings of the option and the riskless security, respectively. For any investor the total of his portfolio proportions must be equal to 1, i.e.,

$$x_o + x_f + \sum_{i=1}^{n} x_i = 1. \tag{1}$$

By definition, an investor's portfolio expected rate of return is given by:

$$E(r) = \sum_{i=1}^{n} x_i E(r_i) + x_o E(r_o) + x_f r_f. \tag{2}$$

An investor's portfolio variance is given by:

$$\sigma_r^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} x_i x_j \mathrm{cov}(r_i, r_j) + x_o^2 \sigma_o^2 + 2x_o \sum_{i=1}^{n} x_i \mathrm{cov}(r_o, r_i). \tag{3}$$

The above relationships will be employed in all three of the sections to follow.

Equilibrium with One Option and No Riskless Security

In the first case to be considered, x_f is constrained to be zero, since trading in the riskless security is not permitted. Under this circumstance, the only securities available for trading are the n risky assets and the option. Substituting $x_f = 0$ into equation (1) and rearranging, we find that:

$$x_o = 1 - \sum_{i=1}^{n} x_i, \tag{4}$$

if there is no riskless security. Now equations (2) and (3) can be applied to the no riskless security case by the substitution for x_o from equation (4) to get (a superscript k has been added to denote the kth investor):

$$E_1^k = \sum_{i=1}^{n} x_i^k E(r_i) + (1 - \sum_{i=1}^{n} x_i^k) E(r_o); \tag{5}$$

and

$$(\sigma_{r_1}^k)^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} x_i^k x_j^k \mathrm{cov}(r_i, r_j) + (1 - \sum_{i=1}^{n} x_i^k)^2 \sigma_o^2$$
$$+ 2(1 - \sum_{i=1}^{n} x_i^k) \sum_{i=1}^{n} x_i^k \mathrm{cov}(r_o, r_i), \tag{6}$$

where the numerical subscripts in the terms to the left of the equal signs in equations (5) and (6) are references to the section of the chapter.

Thus the investor's portfolio problem of minimizing variance for each level of return can be solved by minimizing the following Lagrange function constructed out of equations (5) and (6):

$$L_1^k = (\sigma_{r_1}^k)^2 + \lambda^k [E_1^k(r) - \sum_{i=1}^{n} x_i^k E(r_i) - (1 - \sum_{i=1}^{n} x_i^k) E(r_o), \quad (6a)$$

where λ^k is the Lagrange multiplier for investor k.[4] Substitute equation (6) for the expression $(\sigma_{r_1}^k)^2$ in equation (6a), differentiate L_1^k with respect to each of the n portfolio weights of the n risky assets, and set the derivatives equal to zero. The result is equation system (7.1) through (7.n) below:

$$\frac{dL_1^k}{dx_1^k} = 2[\sum_{j=1}^{n} x_j^k \mathrm{cov}(r_1,r_j) - (1 - \sum_{i=1}^{n} x_i^k)\sigma_o^2$$
$$+ (1 - \sum_{i=1}^{n} x_i^k)\mathrm{cov}(r_o,r_1) - \sum_{j=1}^{n} x_j^k \mathrm{cov}(r_o,r_j)]$$
$$- \lambda^k[E(r_1) - E(r_o)] = 0; \qquad (7.1)$$

$$\frac{dL_1^k}{dx_2^k} = 2[\sum_{j=1}^{n} x_j^k \mathrm{cov}(r_2,r_j) - (1 - \sum_{i=1}^{n} x_i^k)\sigma_o^2$$
$$+ (1 - \sum_{i=1}^{n} x_i^k)\mathrm{cov}(r_o,r_2) - \sum_{j=1}^{n} x_j^k \mathrm{cov}(r_o,r_j)]$$
$$- \lambda^k[E(r_2) - E(r_o)] = 0; \qquad (7.2)$$

$$\begin{array}{ccc} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{array}$$

$$\frac{dL_1^k}{dx_n^k} = 2[\sum_{j=1}^{n} x_j^k \mathrm{cov}(r_n,r_j) - (1 - \sum_{i=1}^{n} x_i^k)\sigma_o^2$$
$$+ (1 - \sum_{i=1}^{n} x_i^k)\mathrm{cov}(r_o,r_n) - \sum_{j=1}^{n} x_j^k \mathrm{cov}(r_o,r_j)]$$
$$- \lambda^k[E(r_n) - E(r_o)] = 0. \qquad (7.n)$$

It is from equations (7.1) through (7.n) that market equilibrium conditions may be derived as follows:

It is useful to first expand equation (7.1) as (7.1a) below:

$$\begin{aligned}
&2[(x_1^k \operatorname{cov}(r_1,r_1) + x_2^k \operatorname{cov}(r_1,r_2) + \ldots + x_n^k \operatorname{cov}(r_1,r_n)) \\
&-(1 - x_1^k - x_2^k - \ldots - x_n^k)\sigma_o^{\,2} \\
&- x_1^k \operatorname{cov}(r_o,r_1) - x_2^k \operatorname{cov}(r_o,r_2) - \ldots - x_n^k \operatorname{cov}(r_o,r_n) \\
&+ (1 - x_1^k - x_2^k - \ldots - x_n^k)\operatorname{cov}(r_o,r_1)] \\
&\qquad - \lambda^k[E(r_1) - E(r_o)] \;=\; 0.
\end{aligned} \tag{7.1a}$$

Assume there are a total of P investors. Let w^k be the proportion of total market wealth represented by the kth investor's wealth. Multiply equation (7.1a) by w^k for each of the investors to get:

$$\begin{aligned}
&2w^k[x_1^k \operatorname{cov}(r_1,r_1) + x_2^k \operatorname{cov}(r_1,r_2) + \ldots + x_n^k \operatorname{cov}(r_1,r_n)] \\
&\quad -2w^k[1 - x_1^k - x_2^k - \ldots - x_n^k]\sigma_o^{\,2} \\
&-2w^k[x_1^k \operatorname{cov}(r_o,r_1) + x_2^k \operatorname{cov}(r_o,r_2) + \ldots + x_n^k \operatorname{cov}(r_o,r_n)] \\
&+ 2w^k[1 - x_1^k - x_2^k - \ldots - x_n^k]\operatorname{cov}(r_o,r_1) - w^k\lambda^k[E(r_1) - E(r_0)] \\
&\qquad = 0.
\end{aligned} \tag{7.1b}$$

Next sum the P weighted first order conditions over all investors in the market to get an aggregated version of the first first order condition:

$$
\begin{aligned}
&2\mathrm{cov}(r_1,r_1)\sum_{k=1}^{P} w^k x_1^k + 2\mathrm{cov}(r_1,r_2)\sum_{k=1}^{P} w^k x_2^k + \ldots \\
&\qquad + 2\mathrm{cov}(r_1,r_n)\sum_{k=1}^{P} w^k x_n^k \\
&-2\left[\sum_{k=1}^{P} w^k - \sum_{k=1}^{P} w^k x_1^k - \sum_{k=1}^{P} w^k x_2^k - \ldots - \sum_{k=1}^{P} w^k x_n^k\right]\sigma_o^2 \\
&-2\left[\mathrm{cov}(r_o,r_1)\sum_{k=1}^{P} w^k x_1^k + \mathrm{cov}(r_o,r_2)\sum_{k=1}^{P} w^k x_2^k + \ldots \right. \qquad (7.1c)\\
&\qquad \left. + \mathrm{cov}(r_o,r_n)\sum_{k=1}^{P} w^k x_n^k\right] \\
&+2\mathrm{cov}(r_o,r_1)\left[\sum_{k=1}^{P} w^k - \sum_{k=1}^{P} w^k x_1^k - \sum_{k=1}^{P} w^k x_2^k - \ldots - \sum_{k=1}^{P} w^k x_n^k\right] \\
&-[E(r_1) - E(r_o)]\sum_{k=1}^{P} w^k \lambda^k = 0.
\end{aligned}
$$

By definition, $\sum_{k=1}^{P} w^k = 1$. Also, $\sum_{k=1}^{P} w^k x_1^k$ is the proportionate weight of the first security in the market portfolio. Similarly, $\sum_{k=1}^{P} w^k x_2^k$ is the weight of the second security in the market portfolio; and so on for all risky assets. Let these weights be denoted $x_1^m, x_2^m, \ldots, x_n^m$. Thus, the terms in the first and third brackets of equation (7.1c) are both equal to:

$$1 - x_1^m - x_2^m - \ldots - x_n^m,$$

which is equal to zero. Make these substitutions into equation (7.1c) to obtain:

$$2\text{cov}(r_1,r_1)x_1^m + 2\text{cov}(r_1,r_2)x_2^m + \ldots + 2\text{cov}(r_1,r_n)x_n^m$$
$$-2\text{cov}(r_o,r_1)x_1^m - 2\text{cov}(r_o,r_2)x_2^m - \ldots - 2\text{cov}(r_o,r_n)x_n^m$$
$$- [E(r_1) - E(r_o)] \sum_{k=1}^{p} w^k \lambda^k = 0. \tag{7.1d}$$

Define $\lambda^m = \sum_{k=1}^{p} w^k \lambda^k$. Thus λ^m is a weighted average of the individuals' Lagrange multipliers, where the weights are proportions of aggregate wealth. By substituting λ^m into equation (7.1d) and collecting terms, we get:

$$2\sum_{j=1}^{n} x_j^m \text{cov}(r_1,r_j) - 2\sum_{j=1}^{n} x_j^m \text{cov}(r_o,r_j) - \lambda^m[E(r_1) - E(r_o)].$$
$$= 0 \tag{7.1e}$$

Now divide (7.1e) by 2 and rearrange terms to get:

$$\sum_{j=1}^{n} x_j^m \text{cov}(r_1,r_j) - \sum_{j=1}^{n} x_j^m \text{cov}(r_o,r_j) = \frac{\lambda^m}{2}[E(r_1) - E(r_o)]. \tag{7.1f}$$

Equation (7.1f) is the result of aggregating the first of the first order conditions across all investors. The other first order conditions may be similarly aggregated to obtain the following equation system:

$$\sum_{j=1}^{n} x_j^m \mathrm{cov}(r_1,r_j) - \sum_{j=1}^{n} x_j^m \mathrm{cov}(r_o,r_j) = \frac{\lambda^m}{2}[E(r_1)-E(r_o)]; \qquad (8.1)$$

$$\sum_{j=1}^{n} x_j^m \mathrm{cov}(r_2,r_j) - \sum_{j=1}^{n} x_j^m \mathrm{cov}(r_o,r_j) = \frac{\lambda^m}{2}[E(r_2)-E(r_o)]; \qquad (8.2)$$

$$\begin{matrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{matrix}$$

$$\sum_{j=1}^{n} x_j^m \mathrm{cov}(r_n,r_j) - \sum_{j=1}^{n} x_j^m \mathrm{cov}(r_o,r_j) = \frac{\lambda^m}{2}[E(r_n)-E(r_o)]. \qquad (8.n)$$

In order to complete the proof the reader should recognize the following relations:

$$\sum_{j=1}^{n} x_j^m \mathrm{cov}(r_i,r_j) = \mathrm{cov}(r_i, \sum_{j=1}^{n} x_j^m r_j) = \mathrm{cov}(r_i,r_m), \qquad (9)$$

where $r_m = x_1^m r_1 + x_2^m r_2 + \ldots + x_n^m r_n$, the return on the market portfolio (of risky assets only).

Substitute these relations into the aggregate equations (8.1) through (8.n) to obtain the following equation system:

$$\mathrm{cov}(r_1,r_m) - \mathrm{cov}(r_o,r_m) = \frac{\lambda^m}{2}[E(r_1) - E(r_o)]; \qquad (10.1)$$

$$\mathrm{cov}(r_2,r_m) - \mathrm{cov}(r_o,r_m) = \frac{\lambda^m}{2}[E(r_2) - E(r_o)]; \qquad (10.2)$$

$$\begin{matrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{matrix}$$

$$\mathrm{cov}(r_n,r_m) - \mathrm{cov}(r_o,r_m) = \frac{\lambda^m}{2}[E(r_n) - E(r_o)]. \qquad (10.n)$$

To eliminate the unknown $\lambda^m/2$ factor from the equation system, first multiply (10.1) by x_1^m, then (10.2) by x_2^m, and so forth. These multiplications result in:

$$x_1^m \mathrm{cov}(r_1, r_m) - x_1^m \mathrm{cov}(r_o, r_m) = \frac{\lambda^m}{2}[x_1^m E(r_1) - x_1^m E(r_o)]; \tag{11.1}$$

$$x_2^m \mathrm{cov}(r_2, r_m) - x_2^m \mathrm{cov}(r_o, r_m) = \frac{\lambda^m}{2}[x_2^m E(r_2) - x_2^m E(r_o)]; \tag{11.2}$$

$$\vdots \qquad \qquad \vdots \qquad \qquad \vdots$$

$$x_n^m \mathrm{cov}(r_n, r_m) - x_n^m \mathrm{cov}(r_o, r_m) = \frac{\lambda^m}{2}[x_n^m E(r_n) - x_n^m E(r_o)]; \tag{11.n}$$

Add the n equations (11.1) through (11.n) together to obtain:

$$\sum_{i=1}^{n} x_i^m \mathrm{cov}(r_i, r_m) - \mathrm{cov}(r_o, r_m) \sum_{i=1}^{n} x_i^m =$$
$$\frac{\lambda^m}{2}[\sum_{i=1}^{n} x_i^m E(r_i) - E(r_o) \sum_{i=1}^{n} x_i^m]. \tag{12}$$

Using equation (9) and the fact that $\sum_{i=1}^{n} x_i^m = 1$, equation (12) may be simplified to:

$$\mathrm{cov}(r_m, r_m) - \mathrm{cov}(r_o, r_m) = \frac{\lambda^m}{2}[E(r_m) - E(r_o)]. \tag{13}$$

Since $cov(r_m, r_m)$ in equation (13) is simply σ_m^2, the variance of the market portfolio, equation (13) may be rearranged to yield:

$$\frac{\lambda^m}{2} = \frac{\sigma_m^2 - cov(r_o, r_m)}{E(r_m) - E(r_o)} . \tag{14}$$

Now substitute the results for $\lambda^m/2$ in equation (14) into each of the equations (11.1) through (11.n). For any security, i, the market equilibrium relation for returns is:

$$cov(r_i, r_m) - cov(r_o, r_m) = \frac{\sigma_m^2 - cov(r_o, r_m)}{E(r_m) - E(r_o)}[E(r_i) - E(r_o)], \tag{15}$$

which can be rearranged into more familiar form:

$$E(r_i) = E(r_o) + \frac{cov(r_i, r_m) - cov(r_o, r_m)}{\sigma_m^2 - cov(r_o, r_m)}[E(r_m) - E(r_o)]. \tag{16}$$

Equation (16) represents the market equilibrium relation that must hold for all securities if a single option and n risky assets (but no riskless security) are available for trading by investors.

Equilibrium with One Option and Unrestricted Trading in the Riskless Security

In this section a riskless security is introduced into the framework. It is assumed that there are no constraints on the level of holding of the riskless security for any investor. Therefore, equations (1), (2) and (3) hold in full. From equation (1) it is known that for any investor:[5]

$$x_o = 1 - \sum_{i=1}^{n} x_i - x_f. \tag{17}$$

Substitute the relation above into equations (2) and (3), the expressions for an investor's portfolio mean and variance, to obtain:

$$E_2(r) = \sum_{i=1}^{n} x_i E(r_i) + (1 - \sum_{i=1}^{n} x_i - x_f)E(r_o) + x_f r_f; \tag{18}$$

$$\sigma_{r_2}^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} x_i x_j \mathrm{cov}(r_i, r_j) + (1 - \sum_{i=1}^{n} x_i - x_f)^2 \sigma_o^2 + 2(1 - \sum_{i=1}^{n} x_i - x_f) \sum_{i=1}^{n} x_i \mathrm{cov}(r_o, r_i). \tag{19}$$

Thus the investor's portfolio problem of minimizing variance for each level of return can be solved by minimizing the following Lagrange function:

$$L_2 = \sigma_{r_2}^2 + \lambda[E_2(r) - \sum_{i=1}^{n} x_i E(r_i) - (1 - \sum_{i=1}^{n} x_i - x_f)E(r_o) - x_f r_f]. \tag{20}$$

Substitute expression (19) into expression (20). Differentiating L_2 in (20) with respect to each of the portfolio weights of the first n risky securities and setting the derivatives equal to zero, we get:

$$\frac{dL_2}{dx_1} = 2[\sum_{j=1}^{n} x_j cov(r_1,r_j) - (1-\sum_{i=1}^{n} x_i - x_f)\sigma_o^2 +$$

$$(1-\sum_{i=1}^{n} x_i - x_f) cov(r_o,r_1) - \sum_{j=1}^{n} x_j cov(r_o,r_j)] \tag{21.1}$$

$$- \lambda[E(r_1) - E(r_o)] = 0;$$

$$\frac{dL_2}{dx_2} = 2[\sum_{j=1}^{n} x_j cov(r_2,r_j) - (1-\sum_{i=1}^{n} x_i - x_f)\sigma_o^2 +$$

$$(1-\sum_{i=1}^{n} x_i - x_f) cov(r_o,r_2) - \sum_{j=1}^{n} x_j cov(r_o,r_j)] \tag{21.2}$$

$$- \lambda[E(r_2) - E(r_o)] = 0;$$

$$\vdots \qquad \vdots \qquad \vdots$$

$$\frac{dL_2}{dx_n} = 2[\sum_{j=1}^{n} x_j cov(r_n,r_j) - (1-\sum_{i=1}^{n} x_i - x_f)\sigma_o^2 +$$

$$(1-\sum_{i=1}^{n} x_i - x_f) cov(r_o,r_n) - \sum_{j=1}^{n} x_j cov(r_o,r_j)] \tag{21.n}$$

$$- \lambda[E(r_n) - E(r_o)] = 0.$$

Next take the derivative of L_2 in equation (20) with respect to x_f and set equal to zero to get:

$$\frac{dL_2}{dx_f} = -2[(1-\sum_{i=1}^{n} x_i - x_f)\sigma_o^2 + \sum_{i=1}^{n} x_i \operatorname{cov}(r_o, r_i)] + \lambda[E(r_o) - r_f] = 0. \tag{22}$$

Each investor has an equation set (21.1) through (21.n) and (22). Each equation in the set may be aggregated over all investors to obtain the following aggregate demand relations:

$$\sum_{j=1}^{n} x_j^m \operatorname{cov}(r_1, r_j) - (1-\sum_{i=1}^{n} x_i^m - x_f^m)\sigma_o^2 + (1-\sum_{i=1}^{n} x_i^m - x_f^m)\operatorname{cov}(r_o, r_1) - \sum_{j=1}^{n} x_j^m \operatorname{cov}(r_o, r_j) = \frac{\lambda^m}{2}[E(r_1) - E(r_o)]; \tag{23.1}$$

$$\sum_{j=1}^{n} x_j^m \operatorname{cov}(r_2, r_j) - (1-\sum_{i=1}^{n} x_i^m - x_f^m)\sigma_o^2 + (1-\sum_{i=1}^{n} x_i^m - x_f^m)\operatorname{cov}(r_o, r_2) - \sum_{j=1}^{n} x_j^m \operatorname{cov}(r_o, r_j) = \frac{\lambda^m}{2}[E(r_2) - E(r_o)]; \tag{23.2}$$

$$\vdots$$

$$\sum_{j=1}^{n} x_j^m \operatorname{cov}(r_n, r_j) - (1-\sum_{i=1}^{n} x_i^m - x_f^m)\sigma_o^2 + (1-\sum_{i=1}^{n} x_i^m - x_f^m)\operatorname{cov}(r_o, r_n) - \sum_{j=1}^{n} x_j^m \operatorname{cov}(r_o, r_j) = \frac{\lambda^m}{2}[E(r_n) - E(r_o)], \tag{23.n}$$

where x_i^m is the weighted sum of the individual investors' proportions for the ith stock (i.e., $x_i^m = \sum_{k=1}^{P} w^k x_i^k$, where w^k is the kth investor's proportion of total market wealth). Thus x_i^m is the proportion of the wealth of the market portfolio represented by the ith asset. The term x_f^m is a similarly weighted sum of the individual investors' proportions for the riskless security.

By the use of a similar technique, equation (22) may be aggregated over all investors to obtain:

$$(1-\sum_{i=1}^{n} x_i^m - x_f^m)\sigma_o^2 + \sum_{i=1}^{n} x_i^m \mathrm{cov}(r_o,r_i) = \frac{\lambda^m}{2}[E(r_o)-r_f]. \tag{24}$$

Of course, $x_f = 1 - \sum_{i=1}^{n} x_i^m = 0$. Using that fact, along with equation (24), in simplifying equations (23.1) through (23.n) we get:

$$\sum_{j=1}^{n} x_j^m \mathrm{cov}(r_1,r_j) - \sum_{j=1}^{n} x_j^m \mathrm{cov}(r_o,r_j) = \frac{\lambda^m}{2}[E(r_1)-E(r_o)]; \tag{25.1}$$

$$\sum_{j=1}^{n} x_j^m \mathrm{cov}(r_2,r_j) - \sum_{j=1}^{n} x_j^m \mathrm{cov}(r_o,r_j) = \frac{\lambda^m}{2}[E(r_2)-E(r_o)]; \tag{25.2}$$

$$\vdots \qquad\qquad \vdots \qquad\qquad \vdots$$

$$\sum_{j=1}^{n} x_j^m \mathrm{cov}(r_n,r_j) - \sum_{j=1}^{n} x_j^m \mathrm{cov}(r_o,r_j) = \frac{\lambda^m}{2}[E(r_n)-E(r_o)]; \tag{25.n}$$

and

$$\sum_{i=1}^{n} x_i^m \text{cov}(r_o, r_i) = \frac{\lambda^m}{2}[E(r_o) - r_f]. \qquad (26)$$

Now substitute equation (26) into each of the equations of the (25.1) to (25.n) set to get:

$$\sum_{j=1}^{n} x_j^m \text{cov}(r_1, r_j) + \frac{\lambda^m}{2}[r_f - E(r_o)] = \frac{\lambda^m}{2}[E(r_1) - E(r_o)]; \qquad (27.1)$$

$$\sum_{j=1}^{n} x_j^m \text{cov}(r_2, r_j) + \frac{\lambda^m}{2}[r_f - E(r_o)] = \frac{\lambda^m}{2}[E(r_2) - E(r_o)]; \qquad (27.2)$$

$$\vdots \qquad \vdots \qquad \vdots$$

$$\sum_{j=1}^{n} x_j^m \text{cov}(r_n, r_j) + \frac{\lambda^m}{2}[r_f - E(r_o)] = \frac{\lambda^m}{2}[E(r_n) - E(r_o)]; \qquad (27.n)$$

The equation set (27.1) through (27.n) may now be easily rearranged to obtain:

$$\sum_{j=1}^{n} x_j^m \text{cov}(r_1, r_j) = \frac{\lambda^m}{2}[E(r_1) - r_f]; \qquad (28.1)$$

$$\sum_{j=1}^{n} x_j^m \text{cov}(r_2, r_j) = \frac{\lambda^m}{2}[E(r_2) - r_f]; \qquad (28.2)$$

$$\vdots \qquad \vdots$$

$$\sum_{j=1}^{n} x_j^m \text{cov}(r_n, r_j) = \frac{\lambda^m}{2}[E(r_n) - r_f]. \qquad (28.n)$$

It may be seen from the equation set (28.1) through (28.n) that no traces of the option remain in the first order conditions. Equations (28.1) through (28.n) may be solved in a manner similar to the familiar Sharpe-Lintner capital asset pricing model that has no options. To do this first recall equation (9). Thus, from equations (28.1) through (28.n):

$$\mathrm{cov}(r_1, r_m) = \frac{\lambda^m}{2}[E(r_1) - r_f]; \tag{29.1}$$

$$\mathrm{cov}(r_2, r_m) = \frac{\lambda^m}{2}[E(r_2) - r_f]; \tag{29.2}$$

$$\vdots \qquad\qquad \vdots$$

$$\mathrm{cov}(r_n, r_m) = \frac{\lambda^m}{2}[E(r_n) - r_f]. \tag{29.n}$$

Now to eliminate the $\lambda^m/2$ factor from the equation system, first multiply (29.1) by x_1^m, (29.2) by x_2^m, and so forth. These multiplications result in:

$$x_1^m\mathrm{cov}(r_1, r_m) = \frac{\lambda^m}{2}[x_1^m E(r_1) - x_1^m r_f]; \tag{30.1}$$

$$x_2^m\mathrm{cov}(r_2, r_m) = \frac{\lambda^m}{2}[x_2^m E(r_2) - x_2^m r_f]; \tag{30.2}$$

$$\vdots \qquad\qquad \vdots$$

$$x_n^m\mathrm{cov}(r_n, r_m) = \frac{\lambda^m}{2}[x_n^m E(r_n) - x_n^m r_f]. \tag{30.n}$$

Next sum equations (30.1) through (30.n) vertically to get:

$$\sum_{i=1}^{n} x_i^m \operatorname{cov}(r_i, r_m) = \frac{\lambda^m}{2}\left[\sum_{i=1}^{n} x_i^m E(r_i) - r_f \sum_{i=1}^{n} x_i^m\right]. \tag{31}$$

Using equation (9) again and noting that $\sum_{i=1}^{n} x_i^m = 1$ (since $x_f^m = 0$), equation (31) is equivalent to:

$$\operatorname{cov}(r_m, r_m) = \frac{\lambda^m}{2}[E(r_m) - r_f]. \tag{31a}$$

Note that $\operatorname{cov}(r_m, r_m)$ in equation (31a) is simply σ_m^2, the variance of the market portfolio, it follows easily that:

$$\frac{\lambda^m}{2} = \frac{\sigma_m^2}{E(r_m) - r_f}. \tag{31b}$$

Now substitute from equation (31b) into equation system (27.1) through (27.n), and after rearranging, the familiar Sharpe-Lintner capital asset pricing model results:

$$E(r_i) = r_f + \frac{\operatorname{cov}(r_i, r_m)}{\sigma_m^2}[E(r_m) - r_f]. \tag{32}$$

Thus the familiar Sharpe-Lintner model applies even in a world with an option, as long as a riskless security is available for unrestricted trading.

Equilibrium with One Option and a Margin Constraint[6] on Riskless Borrowing

In the case to be derived here x_f is constrained to be above some constant negative amount, denoted C. For example, if C is -.50, then an investor may borrow up to 1/3 of the value of his portfolio of risky securities. If C = - 1, then fully one-half of the risky asset portfolio may be financed with borrowed funds, and so on. In this case equations (18) and (19) again represent the investor's portfolio expected return and variance; they are repeated here as equations (33) and (34) below, with the margin constraint (35):

$$E_3(r) = \sum_{i=1}^{n} x_i E(r_i) + (1 - \sum_{i=1}^{n} x_i - x_f)E(r_o) + x_f r_f; \tag{33}$$

$$\sigma^2_{r_3} = \sum_{i=1}^{n}\sum_{j=1}^{n} x_i x_j \operatorname{cov}(r_i, r_j) + (1 - \sum_{i=1}^{n} x_i - x_f)^2 \sigma_o^2 + 2(1 - \sum_{i=1}^{n} x_i - x_f) \sum_{i=1}^{n} x_i \operatorname{cov}(r_o, r_i); \tag{34}$$

$$C \leq x_f. \tag{35}$$

Thus the investor's portfolio problem of minimizing variance for every level of expected return, given the margin constraint in (35), can be solved by minimizing the following Lagrange function:

$$L_3 = \sigma^2_{r_3} + \lambda[E_3(r) - \sum_{i=1}^{n} x_i E(r_i) - (1 - \sum_{i=1}^{n} x_i - x_f)E(r_o) - x_f r_f] + \lambda^1[C - x_f], \tag{36}$$

where the second Lagrange multiplier, λ^1, relates to the inequality constraint.

Now substitute the portfolio variance expression from (35) into equation (36) to get:

$$L_3 = \sum_{i=1}^{n}\sum_{j=1}^{n} x_i x_j \text{cov}(r_i, r_j) + (1 - \sum_{i=1}^{n} x_i - x_f)^2 \sigma_o^2 + 2(1 - \sum_{i=1}^{n} x_i - x_f) \sum_{i=1}^{n} x_i \text{cov}(r_o, r_i) + \lambda[E_3(r) - \sum_{i=1}^{n} x_i E(r_i) - (1 - \sum_{i=1}^{n} x_i - x_f)E(r_o) - x_f r_f] + \lambda^1[C - x_f]. \tag{37}$$

Differentiate L_3 in (37) with respect to each of the portfolio weights of the n risky assets, and set the derivatives equal to zero to obtain equations (38.1) through (38.n) below:

$$\frac{dL_3}{dx_1} = 2[\sum_{j=1}^{n} x_j \operatorname{cov}(r_1,r_j) - (1-\sum_{i=1}^{n} x_i - x_f)\sigma_o^2 + (1-\sum_{i=1}^{n} x_i - x_f)\operatorname{cov}(r_o,r_1) - \sum_{j=1}^{n} x_j \operatorname{cov}(r_o,r_j)] - \lambda[E(r_1) - E(r_o)] = 0; \tag{38.1}$$

$$\frac{dL_3}{dx_2} = 2[\sum_{j=1}^{n} x_j \operatorname{cov}(r_2,r_j) - (1-\sum_{i=1}^{n} x_i - x_f)\sigma_o^2 + (1-\sum_{i=1}^{n} x_i - x_f)\operatorname{cov}(r_o,r_2) - \sum_{j=1}^{n} x_j \operatorname{cov}(r_o,r_j)] - \lambda[E(r_2) - E(r_o)] = 0; \tag{38.2}$$

$$\vdots$$

$$\frac{dL_3}{dx_n} = 2[\sum_{j=1}^{n} x_j \operatorname{cov}(r_n,r_j) - (1-\sum_{i=1}^{n} x_i - x_f)\sigma_o^2 + (1-\sum_{i=1}^{n} x_i - x_f)\operatorname{cov}(r_o,r_n) - \sum_{j=1}^{n} x_j \operatorname{cov}(r_o,r_j)] - \lambda[E(r_n) - E(r_o)] = 0. \tag{38.n}$$

Next take the derivative of L_3 in (37) with respect to x_f and set it equal to zero:

$$\frac{dL_3}{dx_f} = -2[(1-\sum_{i=1}^{n} x_i - x_f)\sigma_o^2 + \sum_{i=1}^{n} x_i \operatorname{cov}(r_o, r_i)] + \lambda[E(r_o) - r_f] - \lambda^1 = 0. \tag{39}$$

Equation (39) is different from its counterpart equation, (22), in the second section, because of the constraint and the appearance of λ^1.

Together with the generalized Kuhn-Tucker conditions[7] and equations (40) and (41) below, equations (38.1) through (38.n) and (39) form the first order conditions to be satisfied if there is a solution to the investor's decision problem:

$$\lambda^1 (C - x_f) = 0; \tag{40}$$

$$\lambda^1 \geq 0. \tag{41}$$

From the complementary slackness condition, (equation (40)), it is obvious that either $\lambda^1 = 0$ or $x_f = C$, or both. If $\lambda^1 = 0$ and $x_f \neq C$, then the margin constraint, while it is publicized, is not effectively binding the investor's decision; if $\lambda^1 \neq 0$ and $x_f = C$, then the margin constraint is effective.

Aggregate each of conditions (38.1) through (38.n) and (39) across all investors (in a manner similar to that of the first section) to obtain equations (42.1) through (42.n) and (43) below:

$$\sum_{j=1}^{n} x_j^m \operatorname{cov}(r_1,r_j) - (1-\sum_{i=1}^{n} x_i^m - x_f^m)\sigma_o^2 + (1-\sum_{i=1}^{n} x_i^m - x_f^m)\operatorname{cov}(r_o,r_1)$$

$$- \sum_{j=1}^{n} x_j^m \operatorname{cov}(r_o,r_j) = \frac{\lambda^m}{2}[E(r_1)-E(r_o)]; \qquad (42.1)$$

$$\sum_{j=1}^{n} x_j^m \operatorname{cov}(r_2,r_j) - (1-\sum_{i=1}^{n} x_i^m - x_f^m)\sigma_o^2 + (1-\sum_{i=1}^{n} x_i^m - x_f^m)\operatorname{cov}(r_o,r_2)$$

$$- \sum_{j=1}^{n} x_j^m \operatorname{cov}(r_o,r_j) = \frac{\lambda^m}{2}[E(r_2)-E(r_o)]; \qquad (42.2)$$

$$\vdots \qquad \vdots \qquad \vdots$$

$$\sum_{j=1}^{n} x_j^m \operatorname{cov}(r_n,r_j) - (1-\sum_{i=1}^{n} x_i^m - x_f^m)\sigma_o^2 + (1-\sum_{i=1}^{n} x_i^m - x_f^m)\operatorname{cov}(r_o,r_n)$$

$$- \sum_{j=1}^{n} x_j^m \operatorname{cov}(r_o,r_j) = \frac{\lambda^m}{2}[E(r_n)-E(r_o)]; \qquad (42.n)$$

$$-(1-\sum_{i=1}^{n} x_i^m - x_f^m)\sigma_o^2 - \sum_{i=1}^{n} x_i^m \operatorname{cov}(r_o,r_i) = \frac{\lambda^m}{2}[r_f-E(r_o)] + \lambda^{1m}, \qquad (43)$$

where x_j^m and x_f^m are as before, and λ^{1m} is the weighted aggregate of all the individual investors' λ^1's divided by two, i.e., $\lambda^{1m} = \frac{1}{2}\sum_{k=1}^{p} w^k \lambda^{1k}$, where λ^{1k} is the kth investor's

λ^{1}, and w^{k} is the proportion of market wealth held by the kth investor.

If λ^{1m} is equal to zero, i.e., if the margin ceiling is not restrictive on anyone, then equation (43) reduces to equation (24). Under that circumstance the problem is no different than the case of unrestricted trading in the riskless security. If λ^{1m} is not equal to zero, then the margin constraint is binding on at least one investor. In this case the path to establishing equilibrium relations from equations (42.1) through (42.n) and (43) is different than if $\lambda^{1m} = 0$. To continue, assuming $\lambda^{1m} \neq 0$, first recognize that $x_f^m = 0$. Consequently, the aggregate amount of the riskless security is assumed to be zero in the same manner as the option.

Using the fact that $x_f^m = 0 = 1 - \sum_{i=1}^{n} x_i^m$ and using equation (9), equations (42.1) through (42.n) and (43) may be reexpressed as:

$$\mathrm{cov}(r_1, r_m) - \mathrm{cov}(r_o, r_m) = \frac{\lambda^m}{2}[E(r_1) - E(r_o)]; \quad (44.1)$$

$$\mathrm{cov}(r_2, r_m) - \mathrm{cov}(r_o, r_m) = \frac{\lambda^m}{2}[E(r_2) - E(r_o)]; \quad (44.2)$$

$$\begin{matrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{matrix}$$

$$\mathrm{cov}(r_n, r_m) - \mathrm{cov}(r_o, r_m) = \frac{\lambda^m}{2}[E(r_n) - E(r_o)]; \quad (44.n)$$

$$\mathrm{cov}(r_o, r_m) = \frac{\lambda^m}{2}[E(r_o) - r_f] - \lambda^{1m}. \quad (45)$$

Equation set (44.1) through (44.n) and (45) must hold in market equilibrium. Equation (45) is the relation between the equilibrium expected rate of return on the option and the riskless rate. Equations (44.1) through (44.n) may be used, ignoring (45), to derive the same equilibrium conditions as those in the first section of the chapter. Equation (16) would hold in the case just described, since:

$$\frac{\lambda^m}{2} = \frac{\sigma_m^2 - \mathrm{cov}(r_o, r_m)}{E(r_m) - E(r_o)} . \tag{46}$$

Substitute equations (46) and (45) into the ith equation of (44) to get another equilibrium expression for the margin ceiling case, equation (47) below:

$$\mathrm{cov}(r_i, r_m) + \lambda^{1m} = \frac{\sigma_m^2 - \mathrm{cov}(r_o, r_m)}{E(r_m) - E(r_o)} [E(r_i) - r_f] . \tag{47}$$

By rearranging equation (47) it is found that:

$$E(r_i) = r_f + \frac{\mathrm{cov}(r_i, r_m) + \lambda^{1m}}{\sigma_m^2 - \mathrm{cov}(r_o, r_m)} [E(r_m) - E(r_o)] . \tag{48}$$

Consider now equation (48) applied to a zero beta security:

$$E(r_z) = r_f + \frac{\lambda^{1m}}{\sigma_m^2 - \mathrm{cov}(r_o, r_m)} [E(r_m) - E(r_o)]. \tag{49}$$

If one assumes that options are used by many investors in lieu of margined stock, then an obvious connection exists between an increase in the number of options being traded and a decrease in λ^{1m}. As is evident from equation (49), a decrease in λ^{1m} should create a decrease in $E(r_z)$. This establishes the dissertation's hypothesis that:

> As the quantity of options being traded increases, the equilibrium expected zero beta rate of return decreases.

The model in (49) and this chapter is not sufficiently detailed in assumptions enough for one to establish a precise mathematical relation between the number of options in the option market and the aggregate margin constraint multiplier, λ^{1m}. Perhaps a direct link could be established under some assumptions involving investor heterogeneity, but this task is not undertaken at the present time.

Notes

[1]All investors are assumed to have perfect information.

[2]The no-inflation assumption is relaxed in some CAPM's (e.g., Solnik's [1978]), but such models are beyond the scope of this study.

[3]Alternatively, the option will eventually be viewed as a portfolio of options.

[4]The interpretation of λ^k is that λ^k is the amount of additional expected return the investor must get if he is to accept a small amount of additional variance in his portfolio.

[5]The superscript k has been dropped for convenience. The proofs in this section and the next are abbreviated somewhat from the detail of the previous section.

[6]Black (1972) and Vasicek (1971) previously provided results in this area. Black considered no riskless borrowing but allowed riskless lending, a circumstance that has been assumed here to be impossible. Vasicek looked at the margin constraint idea, but assumed the riskless rate to be zero.

[7]See Luenberger (1973).

CHAPTER III
REVIEW OF ASSOCIATED LITERATURE

Introduction

The literature about the effects of new option trading includes both theoretical as well as empirical papers. The theoretical contributions have been made in frameworks other than the one employed in Chapter II; however, the author is not aware of any other work that formally considers the role of any kind of futures contract, let alone options specifically, in the single-period mean-variance framework. Since the other theoretical studies employ other frameworks, a detailed review of those studies is omitted here. The interested reader is referred to the works of Hirshleifer (1975), Danthine (1978), Ross (1976), Schrems (1973), Townsend (1978), Breeden (1978), Rubinstein (1976a), Friesen (1979), and Long (1974). The roles of futures (and especially options) identified by the above theoreticians do vary depending upon which framework is employed. While the papers are interesting, they are too complex to adequately review here.

On the empirical side attention has been focused on the effects of options on the underlying securities, rather than on market-based variables. In particular, no study has been concerned with the impact of options on market factor interest rates. Despite this, several of these studies are reviewed in this chapter, since the studies do contain evidence that is interesting in light of the present topic and methodology.

There are two categories of empirical studies associated with effects of options. The first category concerns the short-run effects of option expirations. The second category concerns the effects of the advent of new option trading in the long-run as well as the short-run. Only the second category of these studies is of direct interest to review here.

Klemkosky and Maness

In a study published in 1980 by Klemkosky and Maness (K-M), the results of an extensive investigation of the impacts of new option trading on the underlying stocks were reported. The major conclusions at which K-M arrived were: (a) that the options had a negligible impact on the risk of the underlying stocks; and (b) that excess returns which had existed in underlying stocks before the commencement of new option trading had been bid out of the stock prices subsequent to the option listing.

The K-M study examined two risk measures for the underlying stocks, beta and standard deviation, and one performance measure, Jensen's (1969) alpha (α). The K-M methodology of measuring alphas, betas, and <u>changes in</u> alphas and betas is basically the same as one portion of the methodology proposed in the next chapter: the Gujarati (1970) interactive dummy variable technique applied to the excess returns version of the linear market model. In the form shown by K-M, the linear excess returns market model is given below in equation (3-1):

$$r_{it} - r_{ft} = \alpha_i + \beta_i(r_{mt} - r_{ft}) + e_{it} \quad (1)$$

where

r_{it} = the monthly holding period return, including dividends as well as price appreciation of security i in month t.

r_{ft} = the 30-day T-bill yield on a bond equivalent basis in month t.

r_{mt} = the CRSP Investment Performance Index, including dividends, in month t.

α_i = the intercept term representing Jensen's performance measure.

β_i = beta of security i, and

e_{it} = a random error term.

To the above market model, K-M applied the Gujarati interactive dummy variable technique using time series data. Thus K-M estimated the parameters in the following model:

$$r_{it} - r_{ft} = \alpha_i + \alpha_i^1 D + \beta_i (r_{mt} - r_{ft}) + \beta_i^1 (r_{mt} - r_{ft}) D + e_{it} \quad (2)$$

where

D = 0 for the period subsequent to the option listing ("post-listing")

D = 1 for the period prior to the option listing ("pre-listing").

Thus α_i and β_i are measures of the post-period alpha and beta, while $\alpha_i^1 + \alpha_i$ and $\beta_i^1 + \beta_i$ are measures of the pre-period alpha and beta.

The K-M study analyzed three "waves" of stocks that became underlying securities for options. Group 1 stocks consisted of the 32 stocks which had options listed from April 1973 through October 1973. (These were the stocks from which came the ones used as underlying stocks for this study.) The pre-listing period data for Group 1 consisted of monthly security returns for the 36 months from January 1970 through December 1972. The post-listing period was from January 1974 through December 1976. K-M dropped the 1973 period so as to avoid any problems in testing that might be associated with the effects of the announcement of option listing on the underlying securities.

Group 2 stocks consisted of the 32 securities which had options listed on the CBOE beginning December 1974 and ending June 1975. (No new options were listed between October 1973 and December 1974, but many were listed continuously after June 1975.) K-M omitted December 1974 through June 1975 from their analysis of Group 2 stocks. The pre-listing period was January 1971 through November 1974 for Group 2. The post-listing period was July 1975 through June 1978.

Group 3 consisted of the 39 stocks that had options listed on the American Stock Exchange (ASE) from January 1974 through June 1975. This time period was omitted from the analysis. The pre-ASE period was from January 1972 to December 1974, and the post-ASE period was from July 1975 to June 1978.

K-M utilized two different market indices--the CRSP equal-weighted (EW) and the CRSP market value-weighted (VW) indices. K-M noted that the 103 securities, because of their large market values, will dominate or greatly influence in the aggregate any market value-weighted index, so the authors used both indices. The empirical results of the K-M study are summarized next.

K-M observed that the performance measure (α) decreased for most securities in the post-listing period (83 out of 103 for the EW index and 68 out of 103 for the VW index). This result was also observed for the "portfolio of all underlying stocks" in each group. In all 6 cases the performance measure dropped; 5 of the 6 cases were significant. K-M also noted that the performance measure had been significantly positive in 5 of the pre-listing cases, and not significantly different from zero in any of the post-listing periods. K-M concluded that excess returns appeared to have been bid out of the underlying security returns with the advent of option trading. This conclusion is somewhat consistent with the theory and hypothesis of this dissertation; however, a major caveat for the K-M study is the Roll (1977a) critique.[1]

The changes in the betas (measures of systematic risk) for the stocks were not so consistent. Only a few stocks had significant beta changes. More stocks showed beta declines than increases. Viewing the stocks in each group as portfolios the beta changes were also not consistent: the portfolio beta for the Groups 1 and 2 stocks declined, and the Group 3 portfolio beta increased. Anyway, the changes were insignificant in all 6 cases.

K-M also observed that the coefficient of determination, R^2, did not change significantly from the pre- to the post-listing periods. The R^2 was calculated as the proportion of the total variation of stock returns explained by the linear relationship with the market portfolio.

K-M finally looked at changes in total risk (measured by variance) for the stocks and the portfolios. They found that the total risk of the Group 1 stocks went up after option listing, while the total risk of the Groups 2 and 3 stocks went down in the post-option listing period. Interestingly, the variance of the market index behaved in the same direction. The change in the variance of the market index and in a large proportion of the stocks was significant.

The changes in the variances of the portfolios were not consistent. The Group 1 portfolio variance increased, but not significantly. The Group 2 and Group 3 portfolios experienced a decline in variance; the decline was significant for Group 2, but not for Group 3. Also K-M reported the variance comparisons using "deflated" returns. Deflated returns were defined to be $(r_{it} - r_{ft})$ divided by $(r_{mt} - r_{ft})$ and were used to account directly for shifts in the market index variability of returns. When the alternative method was used the results for the individual stocks were about the same. However, the change in portfolio variance for

Group 1 became significant in the case of the EW index. The change in the variance of the Group 2 stocks switched from being significant to insignificant. The Group 3 portfolio variance changes switched from being insignificant to significant, but again only in the case of the EW index.

This concludes the review of the Klemkosky-Maness study.

Hayes and Tennenbaum

A study that was conducted by Hayes and Tennenbaum (H-T) was published in 1979, and it analyzed the impact of option trading on the volume of trading in the underlying shares. The authors' statistical tests indicated that an effect of listed options was to increase the volume of trading in the underlying shares. H-T theorized that this effect occurred because the availability of the options increased the number of ways that the underlying stock can be used in investors' portfolios.

H-T conducted 2 different types of tests. The first was a cross-sectional analysis. H-T compared a 43-company sample of optioned stocks with a control group of 21 stocks of similar size, but for which there were no options. Using a system of dummy variables for the option group and the control group, and for a pre-option trading period and a post-option trading period, H-T analyzed the percentage trading volume compared to the total NYSE volume. H-T found that the control group's trading volume was about 17% of the

NYSE total in the pre- and post-option periods. (The pre-option period was May 1972 to April 1973; the post-option period was May 1973 to September 1977.) H-T also found that the optioned stocks had a mean percentage volume of 25% in the pre-option period and that the percentage jumped to almost 34% in the post-option period.

The second analysis that H-T performed was a longitudinal analysis. The authors examined the stock volume for at least a year before and a year after options began trading on that stock. For the 43 companies, stock volume was the dependent variable in a multiple regression on the 2 independent variables: option volume and NYSE volume. In 1 version of the longitudinal test the 43 stocks' volume data were aggregated, and so was the option volume data. In the second version of the longitudinal test the stocks' volumes and option volumes were analyzed individually.

H-T found that in the first version of the longitudinal analysis there was a significant association between stock trading volume and option trading volume. The results of the second version, the longitudinal test with each individual stock, showed corroborating results.

Hayes and Tennenbaum concluded that they had provided evidence of a linkage between option trading and price "continuity" in the underlying shares. That is, H-T linked the volume increases with price continuity in the underlying stock. This is an interesting finding and appears to be evidence that options improve market efficiency, at least for the underlying stocks. Since increased volume of underlying stock trading has not been predicted by this dissertation's theory, such an increase does not contradict the hypothesis here. Increased underlying stock trading volume could (intuitively) be a manifestation of the relocating process of the risk-return intercept. Further analysis of this point here, however, is beyond the scope of this review.

Reilly and Naidu

In a paper that has been professionally presented, but not as yet published, Reilly and Naidu added their analysis of option trading impacts on underlying stock volume and volatility to the existing evidence. In addition, Reilly and Naidu (R-N) examined the impact of option trading on the market liquidity of the underlying stocks. R-N employed 2 types of measures of liquidity. The first was the bid-ask spread; the second was a version of the Amivest Liquidity Index.

The Amivest Liquidity Index attempts to relate the average dollar amount of trading to a 1% change in the price as follows:

$$\text{Amivest Index}_i = \frac{\sum_{t=1}^{n} P_t V_t}{\sum_{t=1}^{n} \left|\%\Delta P_t\right|} \tag{3}$$

where

P_t = Closing price for stock i, on day t.

V_t = Share volume of trading for stock i on day t.

$\%\Delta P_t$ = The percent change in price for stock i on day t.

The higher the dollar volume of trading is, per 1% of price change, the higher will be the liquidity of the stock, in the opinion of the users of the Amivest Index. Reilly and Naidu modified the Amivest Index so that inter-day price ranges were accounted for as follows:

$$\text{Modified Liquidity Index} = \frac{\sum_{t=1}^{n} P_t V_t}{\sum_{t=1}^{n} \left(\frac{H-L}{H+L/2}\right)_t} \tag{4}$$

where

H is the high price for the day, and

L is the low price for the day.

The R-N analysis focused on effects surrounding the listing dates of options on the CBOE and the ASE. The 5 days before and after the listing dates were excluded. Activity in the 20-day periods before and after the 5-day periods were examined. In all, 12 stocks listed on the CBOE on May 22, 1975, and May 23, 1975, and 10 stocks listed on the ASE May 30, 1975, were tested. Also, control groups of 12 and 10 randomly selected NYSE stocks were studied.

The results of the R-N tests for effects of options on the market spreads indicated that the percentage spread for the optioned stocks was lower than that for the random stocks. In addition, in going from the pre-listing period to the post-listing period, the market spread declined slightly for the optioned stocks and increased slightly for the random stocks. The change was not significant, but R-N remarked that the market for the optioned stocks was still superior in terms of liquidity to that for the random stocks.

With regard to both the Amivest Index and the Modified Liquidity Index, however, R-N observed virtually <u>no change</u> in market liquidity. Thus Reilly and Naidu observed no significant changes in any measures of liquidity.

R-N used the same data in their analysis of changes in underlying stock volatility and relative trading volume. In addition, R-N examined an aspect of price performance of the stocks in the 20-day periods on either side of the listing time. Five nonsystematic volatility measures were examined for the optioned stocks, the random stocks, and the S & P 400 Industrial Index. For all 5 measures the stock price volatility of the optioned stocks declined in the period after listing. The range of the decline was from 25% to 35%. The volatility of the random stocks declined by a smaller amount, from 3% to about 8%. Relative to the market, the volatility of the option stocks also declined in the post-listing period. Since the R-N stocks are contained in Klemkosky and Maness' Group 2 and Group 3 stocks, R-N's findings are basically consistent with those of K-M, discussed earlier.

For the volume of trading, R-N found no significant change in the relative volume of trading of either the optioned stock groups or the random stock groups. R-N concluded that there was almost no <u>short-run</u> impact on the volume of trading for the underlying stocks as a result of option listing. This conclusion is not necessarily contradictory to the Hayes-Tennenbaum report, since H-T looked at a much longer term.

Reilly and Naidu also looked at price performance from the following perspective: they measured the ratio of the average price of an optioned stock to the average price of a random stock. In the pre-listing period the price ratio was stable in the 1.73 to 1.80 range. In the post-listing period this ratio jumped to 1.98 and remained in this range for about 7 days. Then the ratio gradually declined back to the pre-listing period range. No possible reasons were offered for this finding.

Trennepohl and Dukes

An analysis of the effect of option listing on underlying stock betas was the focus of a paper, published in 1979, by Trennepohl and Dukes (T-D). T-D examined the original 32 stocks listed on the CBOE from April 1973 to October 1973. Weekly holding period returns on each of the 32 stocks and on each of 18 nonoptioned stocks (assumed to be in a control group) were examined. The nonoptioned stocks were selected as a stratified random sample, representing the same industries present in the optioned stock sample.

The betas were calculated for all of the 32 optioned and 18 nonoptioned stocks for a 2-1/2 year period from October 1970 through April 1973 ("before") and a 2-1/2 year period from October 1973 through April 1975 ("after").

The betas were calculated using the following (no excess returns) market model:

$$R_i = a_i + b_i R_M + e_i \tag{3-5}$$

where

R_i = weekly holding period returns of stock i.

a_i = Y-axis intercept.

b_i = beta of security i, i.e. $\left(\frac{\text{Cov}(R_i, R_M)}{\text{Var}(R_M)}\right)$.

R_M = the rate of return on the Standard and Poors 500 Index.

e_i = random error term.

The betas obtained from the regressions were analyzed by 3 methods. The first method was a paired differences test. The mean of all 32 optioned stock betas was 1.22 before 1973 and .873 after 1973, a mean difference of -.347. Since the t value associated with this mean difference was -3.574, T-D claimed that the observed change was significant. However, a similar result was observed in connection with the nonoptioned stocks. The average beta changed from 1.137 to .934 (a change of -.203), with a t value of 2.317. T-D observed that it appeared that the reduction in the betas had been caused by "general market influences" rather than the option trading.

However, in a t-test of the <u>difference</u> in the mean change of the betas, the authors found that the hypothesis of no mean difference change, between optioned and non-optioned stocks, could be rejected at a confidence level slightly over 89%. A nonparametric Chi-Square test of directional changes in the betas essentially confirmed the results of this t-test. Thus, it was concluded by K-D that the betas for the option stocks decreased <u>more</u> than the betas for the nonoptioned stocks, but with a statistical level of confidence that may be considered to be marginal.

Implications

There is no finding in any of the studies reviewed that is contrary to the hypothesis of this dissertation at the level of theory presented. In fact, the conclusions that excess returns have been bid out of underlying stock prices by K-M,[2] and that trading in the underlying stocks has become more continuous by H-T, tend to support the theory of this study.

Notes

[1]Roll (1977a) called into question any studies of empirical estimates of systematic risk obtained by time series regressions of returns on a market index; he showed how far off results could be if one doesn't know the "true" market portfolio.

[2]Roll's caveat notwithstanding.

CHAPTER IV
PROPOSED ANALYSIS OF RETURNS OF ZERO SYSTEMATIC RISK HEDGES

Excess Returns

The previous chapters established the general hypothesis that as more options are traded, the equilibrium expected zero systematic risk rate of return, $E(r_z)$, will theoretically decline toward the riskless rate of interest. Since additions of new options to the market have occurred over a period of time, the empirical analysis here should involve an examination of time series. Specifically, time series values of r_f and $\bar{r}_z$ are to be examined and tested during periods when new options did and did not begin trading. In a time series analysis of riskfree and zero systematic risk rate estimates, the possibility that r_f and $E(r_z)$ could change, for reasons other than new option trading, is a problem.[1] This problem may be easily overcome by focusing the analysis on excess return; an excess return is defined to be an observed return minus the corresponding time series observation for r_f. Let $\bar{r}_z^e$ denote the time series of differences between each observation for $\bar{r}_z$ and the corresponding observation for r_f.

Zero Systematic Risk Returns

Neutral Spread Returns

An obvious candidate for a zero systematic risk portfolio is a neutral option spread of the kind identified by Galai (1977). In order to simulate the performance of a neutral spread one must determine dC_1/dS and dC_2/dS, the first derivative values of the 2 call prices with respect to the stock price. The neutral spread is created by going long dC_2/dS times the first option and going short dC_1/dS times the second option.[2] The neutral spread is riskless for the instant during which it is created; neutral spreads have zero systematic risk over discrete short time intervals, assuming normally distributed underlying stock prices.[3]

In order to employ the neutral spread method for simulating $\bar{r}_z$ returns, one must know or assume a differentiable option pricing function of the underlying stock price. Fortunately, several alternative option models are suitable for usage in the method of neutral spreads. The Black-Scholes (1973) model, the most popular option model in finance research at present, is:

$$C = SN(d_1) - Xe^{-r\tau}N(d_2)$$

where

$$d_1 = \frac{\ln \frac{S}{X} + (r + \sigma^2/2)\tau}{\sigma\sqrt{\tau}} \qquad \boxed{\text{Black-Scholes Model}} \qquad (1)$$

and

$$d_2 = d_1 - \sigma\sqrt{\tau} .$$

In equation (1) C is the price of the call option; S is the price of the underlying stock; X is the option's exercise price; r is the continuously compounded riskfree interest rate, a constant over time; σ^2 is the continuous variance rate, also a constant over time; τ is the time until expiration of the option; and $N(d_.)$ is the cumulative unit normal distribution function value at $d_.$. If equation (1) is assumed as a valid call option pricing model, then dC/dS, for use in constructing neutral spreads, is $N(d_1)$.

Significantly, equation (1) holds only for options that are dividend-protected or for options whose underlying stocks pay no dividends.[4] A dividend-protected option is one whose exercise price automatically is adjusted, without loss or gain in the value of the position of the call owner, for cash dividend payments made to the holders of the underlying stock. Since U.S. exchange-listed options are not dividend-protected, and since underlying stocks commonly pay dividends, some extension of equation (1) is desirable; a popular candidate for a nondividend-protected option model is the Merton (1973b) model:

$$C = e^{-d\tau} S \cdot N(d_1') - Xe^{-r\tau} N(d_2')$$

where

$$d_1' = \frac{\ln \frac{S}{X} + (r - d + \sigma^2/2)\tau}{\sigma\sqrt{\tau}} \qquad \boxed{\text{Merton Model}} \qquad (2)$$

and

$$d_2' = d_1' - \sigma\sqrt{\tau}\ .$$

In equation (2) d is the continuously compounded dividend yield, based on the current stock price, S. The Merton model assumes d is constant and known in advance; the other variables in his model are the same as those in the Black-Scholes model. In fact, the Merton model is the same as the Black-Scholes model except for the dividend assumption. The first derivative of C with respect to S, using equation (2) is:

$$dC/dS = e^{-d\tau}N(d_1'), \qquad (2')$$

which is an input necessary to the method of neutral spreads.

The interest rate, r, in either of the equations, (1) or (2), is essentially the same concept as the riskfree rate described in Chapter II. However, as was indicated in this chapter, the r in the option models is a continuous-time instantaneous riskless rate, which is assumed to be constant over time. Of course, in reality the short term interest rate appears to be stochastic rather than constant. However, Merton (1973b) has argued that the continuously compounded equivalent to the discrete-time treasury bill rate for the next τ years is suitable as an interpretation for r in equations (1) and (2).

Four of the variables in equation (2) are directly observable; they are the stock price, S; the exercise price, X; the time to maturity of the option, τ; and the dividend yield, d. The dividend yield is not usually known in advance with certainty, but educated forecasts will very often be correct, since most companies follow "stable" dividend policies.

The variable in equation (3) which is not directly observable is σ^2. Option researchers once thought this variable could be estimated with reasonable accuracy using historical data; however, Geske (1979) has discussed and pointed out some potential inadequacies with using historical variance estimates in option models. Since some value for σ^2 must be assumed in order to calculate the derivative in equation (2'), and since historical estimates of σ^2 are potentially inadequate, the implied variance method of Latane' and Rendleman (1976), Chiras and Manaster (1978), and Trippi (1977) must be employed. The implied variance method yields a value for σ^2 by the researcher (a) observing an actual option price, (b) assuming the option model holds true, and (c) calculating the value of σ^2 that equates the option formula price to the actual option price.

Previous Empirical Results of Neutral Spreads

Empirical analyses of neutral option spread returns for daily and monthly holding periods have been reported by Galai (1977) and Chiras (1977), respectively. Galai, who used the now-suspect historical variance approach, reported the appearance of skewness in the frequency distribution of neutral spread returns (see note 3). In addition, Galai reported that the neutral spread returns had variances that were too large to permit statistical inferences to be made. Chiras avoided the historical variance problem by using the implied variance method. Chiras found, on a selected basis, some strikingly high returns; however, he did not report any analysis of the statistical properties of the neutral spread returns. In fact, neither Galai nor Chiras considered the important empirical question of whether the neutral spreads contained any systematic risk.

Neutral Hedge Returns

A second possible candidate for a model of a zero systematic risk security is very similar to a neutral option spread. However, rather than a neutral spread of 2 options, the second method involves a neutral hedge of 1 option and the underlying stock. A neutral hedge is created by going long a share of the stock, and simultaneously going short $1/\frac{dC}{dS}$ options. Neutral hedges nevertheless have zero systematic risk under the same conditions as neutral spreads.[5]

Previous Empirical Results of Neutral Hedges

Returns of neutral hedges have been examined by Black and Scholes (1972), Galai (1977), and Finnerty (1978). None of these 3 studies employed the adjusted-for-dividends model, equation (2): Black and Scholes and Finnerty used equation (1) exclusively; Galai used equation (1) primarily, and later considered the effects of dividends by a different means than equation (2). All 3 studies employed the problematic historical variance approach. Both Black and Scholes and Galai analyzed daily holding periods; they claimed to have found no evidence of significant systematic risk in their neutral hedge positions. Finnerty looked at weekly holding periods; he claimed he did find some significant systematic risk. These researchers' opinions are highly regarded, and their findings are not necessarily illogical. However, all 3 studies measured systematic risk by the commonplace method of calculating the regression coefficient in a least squares regression of hedge returns on a market index, and this method may be invalid, as Roll (1977a) has argued.[6]

Direction of the Methodology

A choice should be made between neutral spreads and neutral hedges. For this study neutral hedges are chosen, because of the report by Galai that neutral spreads had skewness and large variances. To calculate neutral hedge returns, the method of implied variance will be used; thus, the "historical variance problem" of the 3 previous studies of neutral hedges will be avoided. In addition to the improved technology in estimating σ^2, this study will utilize the adjusted-for-dividends model, equation (2); since equation (2) was not used in any of the 3 previous studies of neutral hedges, the use of equation (2) here represents an improvement over previous work. Finally, since Roll has cast doubts about the meaningfulness of the systematic risk estimates of the 3 earlier neutral hedge studies, some effort will be made in this study to assess systematic risk through an alternative method. The alternative method will be elaborated upon later in the chapter.

Two Further Considerations

1. There is a potential problem in employing neutral hedge returns to model zero systematic risk security returns for the purpose of testing the dissertation's hypothesis. The problem is that the risk-return slope may shift during the time span being studied. If the slope of the risk-return relation were to change over the time span studied, then there would be some movement of the underlying stock prices to new equilibrium levels. This movement in the underlying stock prices could cause some abnormalities in hedge returns that may obscure the direct effect of the option trading on $E(r_z)$. The abnormalities would most logically be expected to have an impact on the results if the underlying stocks used to construct the neutral hedges in this study were imbalanced by being comprised of either too many high systematic risk stocks or too many low systematic risk stocks. In order to account for this problem, an empirical analysis must be employed to check for the presence of the potential effects of a shifting risk-return slope. The procedure is described later in the chapter.

2. The other consideration in connection with neutral hedges is whether a portfolio may be viewed as the minimum variance zero systematic risk portfolio in the capital market. If investors could continuously readjust their hedge positions, a portfolio of neutral hedges would surely be the minimum variance zero systematic risk portfolio, since each of the neutral hedge returns would have no uncertainty at all. However, the problem is that investors cannot

continuously readjust hedge positions, and so a positive variance must be a property of neutral hedges held over discrete time intervals. As has been pointed out by Boyle and Emanuel (1980), it is possible to reduce the variance of individual neutral hedge positions by about 3/4 by constructing a portfolio of a large number of neutral hedges. This dramatic variance reduction is made possible by the low correlation between neutral hedges relative to the correlation between (positively correlated) underlying stock returns.[7] Thus an empirical question, that is important to this research study, arises: whether the variance of the neutral hedges can be reduced by enough through diversification to permit us to consider a portfolio of neutral hedges as having the minimum variance of all zero systematic risk portfolios.[8,9]

Selection of Time Span, Holding Period, and Data

Time Span: November 30, 1973--August 29, 1975

The empirical study should extend over a period of approximately 2 years in order that the predicted effects of new option trading be given ample time to show up. The 21-month time span from the end of November, 1973, through August, 1975, was selected. This time span is divided into 2 contiguous segments. The first segment, Period 1, is the 12-month term ending November, 1974; during this segment no options began trading on any new underlying stocks. The second segment, Period 2, is the 9-month segment beginning at the end of November, 1974; during this time options began trading on a total of 82 new underlying stocks. Table 4-1 shows the frequency distribution for the number of new underlying stocks over the months of both Period 1 and Period 2.

Holding Period Assumption: Monthly Observations

The next decision is the assumed holding period for which to simulate returns. A holding period of longer than 1 month, given the 21-month span of the study, would not be feasible, because too few time series observations would result to perform any meaningful statistical analysis. For a shorter holding period, 2 weeks or 1 week, more time series observations would lie within the time span; in addition, there should be less liability of systematic risk in the neutral hedge returns.

Although these arguments in favor of shorter holding periods are reasonable, given the time span, holding periods shorter than 1 month would entail high data gathering costs. To see why the data costs would be so high under those circumstances, consider the portion of the study concerned with the reduction of hedge variance via naive diversification. To facilitate the best possible naive diversification analysis it is necessary to use the price observations of all options quoted at any one point in time. The observation of all options at the end of each of the 21 months in the time-span will result in 2985 usable neutral hedge returns for the study. Since all of the data must be gathered

TABLE 4-1
Frequency Distribution of
New Underlying Stocks
December 1973 -- August 1975

		Month		Number of New Underlying Stocks
Period 1	1.	December	1973	0
	2.	January	1974	0
	3.	February	1974	0
	4.	March	1974	0
	5.	April	1974	0
	6.	May	1974	0
	7.	June	1974	0
	8	July	1974	0
	9.	August	1974	0
	10.	September	1974	0
	11.	October	1974	0
	12.	November	1974	0
Period 2	13.	December	1974	8
	14.	January	1975	20
	15.	February	1975	0
	16.	March	1975	6
	17.	April	1975	0
	18.	May	1975	12
	19.	June	1975	31
	20.	July	1975	5
	21.	August	1975	0
		Total Added In Period 2		82

by hand, the use of weekly or even bi-weekly data would significantly increase the data costs over those for monthly holding periods. Thus, monthly holding periods have been chosen. The problem of potential systematic risk in the hedge returns should be addressed, since some possibility exists that monthly holding periods are not "short enough" (in the Black-Scholes sense) to validate the assumption of zero systematic risk.

Description of the Data

All available options at the end of each month from November, 1973, through August, 1975, are employed in this study. For this study an option is "available," if it satisfies the following criteria: (a) The option's underlying stock must have been on the CBOE's list of underlying stocks throughout the entire 21-month time span; (b) two consecutive month-end price quotations must have been observable; (c) neither month-end price observation was allowed to be below the option's intrinsic value at that point in time, with intrinsic value defined as:

$$I = S + D - X$$

where

I = Intrinsic value

S = Stock price

D = Dividend of the stock over the life of the option

X = Exercise price of the option.

Options whose observed prices are below their intrinsic values are excluded from the study, because their price observations obviously violate the logic that riskless arbitrage opportunities have already been eliminated by professional traders.[10]

Only 32 stocks comprised the list of underlying stocks as of November 30, 1973. (This remained the entire list until new options began being added in Period 2.) The options of 2 of these stocks, Great Western Financial, and Gulf and Western, were not used in the study, due to data gathering complications in both cases. The list of the 30 underlying stocks used in the study is given in Table 4-2.

For various reasons the number of available options will vary from month to month; therefore, the number of neutral hedges from which to calculate $\bar{r}_z$ values will vary as well. This situation must and will be considered in the statistical analysis.

The option closing prices were observed for each month from the <u>Wall Street Journal</u> at the beginning of the following month. Thus, the first-of-the-month <u>Wall Street Journals</u> were consulted from December 1, 1973, through September 1, 1975. Stock prices were also observed as monthly closing quotes from the same <u>Wall Street Journals</u>. Dividends were obtained from <u>Moody's Dividend Record</u> for the period; these amounts were first converted into continuously compounded dividend yields and then employed in equation (2') for the computation of the "hedge ratios."[11]

Two proxies for the riskless rate are used in the study. For use in equation (2'), r was assumed to be the mid-point

TABLE 4-2
Underlying Stocks Used in the Study

AT&T	Poloroid
Atlantic Richfield	RCA
Bethlehem Steel	Sperry Rand
Brunswick	Texas Instruments
Eastman Kodak	Upjohn
Exxon	Weyerhauser
Ford	Xerox
INA	Avon
International Harvester	Citicorp
Kresge	IBM
Loews	ITT
McDonalds	Kerr McGee
Merck	MMM
NW Airlines	Monsanto
Pennzoil	Sears

of the two continuous rates of interest implied by the bid and ask prices of U.S. Government treasury bills maturing at the (approximate) time the option expires. For use in the time series hypothesis test, r_f is the midpoint of the 2 one-month rates of interest implied by the bid and ask prices of treasury bills with 1 month to maturity. Thus for each month t, 1 observation for r_{ft} is available. The observations for r and r_f are from the same Wall Street Journals as the option prices and stock prices.

Preliminary Procedure

For each month, t, the number of available options is necessarily the number of different neutral hedge position returns, N_t. For example, in month 1, the number of options and thus hedge returns, is 119. The number of "available" neutral hedge positions in each of the 21 months of the study is provided in Table 4-3. From Table 4-3 one can see that 1381 total observations are included in Period 1, and 1604 in Period 2, for a combined total of 2985. Since the returns in any month are representative of no more than 30 underlying stocks, there is often more than 1 hedge return for each of the underlying stocks for each month.

Neutral Hedge Returns

Consider the j^{th} neutral hedge for month t, out of the total of N_t available neutral hedges for that month. Let S_{jt} represent the price at the beginning of month t of the underlying stock of the j^{th} neutral hedge. Similarly, C_{jt}

TABLE 4-3
The Number of Options Employed in the Study

Month Number	Month		Number of Options Employed in the Study
1	December	1973	119
2	January	1974	116
3	February	1974	115
4	March	1974	157
5	April	1974	112
6	May	1974	110
7	June	1974	133
8	July	1974	97
9	August	1974	137
10	September	1974	163
11	October	1974	125
12	November	1974	197
13	December	1974	201
14	January	1975	127
15	February	1975	156
16	March	1975	182
17	April	1975	126
18	May	1975	159
19	June	1975	136
20	July	1975	136
21	August	1975	181
		Total	2,985

denotes the price at the beginning of month t of the option of the j^{th} neutral hedge of month t. The associated hedge ratio, defined from equation (2'), is $1/(e^{-d\tau} \cdot N(d_1'))$. The corresponding ending prices of the stock and the option are denoted as S_{jt+1} and C_{jt+1}, respectively. The realized return on the j^{th} neutral hedge over month t is thus given by:

$$X_{jt} = \frac{S_{jt+1} - S_{jt} - \dfrac{1}{e^{-d\tau} \cdot N(d_1')_{jt}} (C_{jt+1} - C_{jt})}{S_{jt} - \dfrac{1}{e^{-d\tau} \cdot N(d_1')_{jt}} C_{jt}} . \qquad (4)$$

The denominator in equation (4) is the amount of equity assumed to be invested in the j^{th} neutral hedge at the beginning of month t. The numerator in (4) represents the change in the dollar position of the hedge from the beginning of the month to the end of the month.

Naive Diversification

From the N_t options in period t, portfolios of neutral hedges of size 5, 10, 15, 20, 25, 30, 35, 40, 45 and 50 will be examined. Portfolios of the various sizes will be constructed for each month of the study; for any of these portfolios the neutral hedges to be included will be randomly selected (with replacement). First, the computer selects a random number representing one of the 30 stocks; next, another random number determines which of the options to choose for that stock. Then another stock is chosen randomly, and so on. This process is repeated until the desired number of neutral hedges has been randomly drawn from the N_t

available positions. The portfolio return is calculated as an arithmetic average of the returns on each of the neutral hedges assumed to be in the portfolio. Therefore, equal amounts of invested equity are simulated in each of the neutral hedges of the portfolio.

For each portfolio size for each month, 30 portfolios will be constructed according to the random selection method described above. The variance of the 30 portfolio returns will serve as an estimate of the portfolio variance of a naively-diversified portfolio of that number of neutral hedges. Although the simulated portfolios could easily contain some of the same hedge positions, the returns of each of the 30 portfolios for a given size will be assumed to be independent. Thus, the variance of the 30 portfolio returns for a given portfolio size serves as the variance estimate of a portfolio of that number of neutral hedges. The results of this naive diversification analysis are reported in the next chapter.

The Hypothesis Test

Preliminary Test Procedure

Let each of the neutral hedge returns be transformed into "excess return" form--define X^e_{jt} as

$$X^e_{jt} = X_{jt} - r_{ft} \tag{5}$$

for all j in the month, and for every month, t. For any month t, j may take on a value from 1 to N_t.

Let P1 stand for Period 1 and P2 stand for Period 2. The hypothesis test is a test for the difference between the mean of the X^e_{jt} values in P1 and the mean of the X^e_{jt} values in P2. Let U_1 stand for the mean of the X^e_{jt} in P2. To test the hypothesis, specify the following relationship:

$$X^e_{jt}\Big| P1 = U_1 + e_j \tag{6}$$

$$X^e_{jt}\Big| P2 = U_2 + e_j \tag{7}$$

where e is an error term.

In order to test the hypothesis, the mean return in <u>each month</u> t must be calculated as follows:

$$\overline{X}^e_t = \sum_{j=1}^{N_t} X^e_{jt} \div N_t \tag{8}$$

If (a) the N_t are equal for all t; (b) the standard deviations around the $\overline{X}^e_t$ are equal for all t (that is, if

$$S_t = \sqrt{\frac{\sum_{j=1}^{N_t} (X^e_{jt} - \overline{X}^e_t)^2}{N_t - 1}}$$

are equal for all t); and (c) there is no systematic risk in the neutral hedges, then the test procedure outlined in Chapter I may be employed. Thus the following regression would be performed:

$$
\begin{array}{l}
\mathrm{P1}\left\{\begin{array}{c} \\ \\ \\ \end{array}\right. \\
\mathrm{P2}\left\{\begin{array}{c} \\ \\ \\ \end{array}\right.
\end{array}
\begin{bmatrix} \bar{X}^e_1 \\ \vdots \\ \bar{X}^e_{12} \\ \hline \bar{X}^e_{13} \\ \vdots \\ \bar{X}^e_{21} \end{bmatrix}
=
\begin{bmatrix} 1 & 0 \\ \vdots & \vdots \\ \underline{1} & \underline{0} \\ I & I \\ \vdots & \vdots \\ 1 & 1 \end{bmatrix}
\begin{bmatrix} U_1 \\ U_2 - U_1 \end{bmatrix}
+
\begin{bmatrix} e_1 \\ \vdots \\ e_{12} \\ \hline e_{13} \\ \vdots \\ e_{21} \end{bmatrix}
\qquad (9)
$$

Equation (9) may also be expressed as follows:

$$
\begin{bmatrix} \bar{X}^e_t \end{bmatrix} = \begin{vmatrix} I_1 & I_2 \end{vmatrix} \begin{bmatrix} U_1 \\ U_2 - U_1 \end{bmatrix} + \begin{bmatrix} e_t \end{bmatrix} \qquad (9')
$$

where I_1 and I_2 are defined in Chapter I.

However, since N_t is <u>not</u> the same value for all t, this problem must be solved. In addition, there is potential heteroskedasticity in the $\bar{X}^e_t$ even without the problem of the varying N_t values. Both of these problems may be simultaneously overcome by converting the ordinary least squares regression in (9) into generalized least squares form. This procedure involves weighting each of the $\bar{X}^e_t$ and each of the observations of the independent regression variables. The weights, $\bar{W}_t$, are defined below:

$$
\bar{W}_t = \frac{1}{S_{\bar{x}_t}} \qquad (10)
$$

where

$$
S_{\bar{x}_t} = \frac{S_t}{\sqrt{N_t}} \quad , \qquad (11)
$$

and where

$$S_t = \sqrt{\frac{\sum_{j=1}^{N_t} (X^e_{jt} - \overline{X}^e_t)^2}{N_t - 1}} \; . \qquad (12)$$

The multiplication of the $\overline{X}^e_t$ and the independent variables vectors by the $\overline{W}_t$ values accounts statistically for the information value of each of the $\overline{X}^e_t$. The following new regression is specified:

$$\begin{bmatrix} \overline{W}_1\overline{X}^e_1 \\ \vdots \\ \overline{W}_{12}\overline{X}^e_{12} \\ \hline \overline{W}_{13}\overline{X}^e_{13} \\ \vdots \\ \overline{W}_{21}\overline{X}^e_{21} \end{bmatrix} = \begin{bmatrix} \frac{1}{S_{\overline{x}_1}} & 0 \\ \vdots & \vdots \\ \frac{1}{S_{\overline{x}_{12}}} & \\ \hline \frac{1}{S_{\overline{x}_{13}}} & \frac{1}{S_{\overline{x}_{13}}} \\ \vdots & \vdots \\ \frac{1}{S_{\overline{x}_{21}}} & \frac{1}{S_{\overline{x}_{21}}} \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 - U_1 \end{bmatrix} + \begin{bmatrix} \frac{e_1}{S_{\overline{x}_1}} \\ \vdots \\ \frac{e_{12}}{S_{\overline{x}_{12}}} \\ \hline \frac{e_{13}}{S_{\overline{x}_{13}}} \\ \vdots \\ \frac{e_{21}}{S_{\overline{x}_{21}}} \end{bmatrix} \qquad (13)$$

Another statistical problem is the potential correlation over time of the error terms. Correlation over time could be indicative that some factor other than option trading is nonrandomly affecting the equilibrium expected excess returns on zero systematic risk securities. Among the factors which could affect $\overline{r}^e_z$ returns in that manner are changes in expected inflation and changes in the interest rate term structure. Although these factors are exogeneous to the capital asset pricing model used in the theory of Chapter II, movements of these variables to new equilibrium levels could nevertheless cause trends in the $\overline{X}_j^e$ time series values. In order to overcome this potential problem, the regression in (13) above should be repeated using the Cochrane-Orcutt (1949) procedure of accounting for correlation over time.

Two Additional Procedures

Two additional problems were discussed earlier, and must now be considered. The first problem is the possibility of systematic risk appearing in the 1-month holding period returns of the neutral hedges. The second difficulty is the possibility of a change in the risk-return slope during the time period of the study. This effect could be due to the new option trading, although this idea has not been specifically hypothesized here.

As a possible way to overcome the first of these potential problems a new variable, representative of the market index, will be introduced into the regression.[12] The CRSP[13] Value-weighted index of stock returns, with dividends reinvested, will be employed as the index variable. Since new option trading could have a simultaneous effect on the behavior of the *index*, an interactive variable is also added to the regression.

Let M_t denote the value of the market index return for month t; then define the new interactive variable to be MI_t = 0 in Period 1, $MI_t = M_t$ in Period 2. As before, these 2 new independent variables must be adjusted for the impact of heteroskedasticity (differences in the $S_{\overline{X}_t}$). The full regression model is:[14]

$$\begin{bmatrix} \overline{W}_1 \overline{X}^e_1 \\ \vdots \\ \overline{W}_{12}\overline{X}^e_{12} \\ \hline \overline{W}_{13}\overline{X}^e_{13} \\ \vdots \\ \overline{W}_{21}\overline{X}^e_{21} \end{bmatrix} = \begin{bmatrix} \overline{W}_1 & 0 & \overline{W}_1 M_1 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ \overline{W}_{12} & 0 & \overline{W}_{12}M_{12} & 0 \\ \hline \overline{W}_{13} & \overline{W}_{13} & \overline{W}_{13}M_{13} & \overline{W}_{13}M_{13} \\ \vdots & \vdots & \vdots & \vdots \\ \overline{W}_{21} & \overline{W}_{21} & \overline{W}_{21}M_{21} & \overline{W}_{21}M_{21} \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 - \alpha_1 \\ \beta_1 \\ \beta_2 - \beta_1 \end{bmatrix} + \begin{bmatrix} e_1 \\ \vdots \\ e_{12} \\ \hline e_{13} \\ \vdots \\ e_{21} \end{bmatrix} \tag{14}$$

Equation (14) may be rewritten in vector form as equation (14') below:

$$\left[\overline{W}_t\right] \cdot \left[\overline{X}^e_t\right] = \left[\overline{W}_t\right] \cdot \left[I_1 \; I_2 \; M_t \; MI_t\right] \begin{bmatrix} \alpha_1 \\ \alpha_2 - \alpha_1 \\ \beta_1 \\ \beta_2 - \beta_1 \end{bmatrix} + \left[e_t\right] \tag{14'}$$

Assuming for the moment that M_t measures the true market portfolio, the coefficient of M_t in equation (14) is the systematic risk measure for the hedges for Period 1. Similarly, the coefficient for MI_t in equation (14) is the <u>change</u> in systematic risk from Period 1 to Period 2. The coefficient of I_2, $\alpha_2 - \alpha_1$ in equation (14), would no longer be interpreted as the change in the mean neutral hedge return from Period 1 to Period 2. More importantly though,

$\alpha_2 - \alpha_1$ could still be interpreted as the change in the mean of the zero systematic risk portion of the neutral hedge returns from Period 1 to Period 2.[15] In equation (14) as in equation (13), the potential serial correlation in the error terms is to be accounted for by the use of the Cochrane-Orcutt procedure.

The second problem concerns the possible impact of a change in the slope of the (single-period) risk-return relation during the time period of the study--a neutral hedge return may be affected if the underlying stock is shifting to a new equilibrium level of expected return. Thus, if the underlying stocks were, on the whole, imbalanced in terms of their systematic risk, then the $\overline{X}_t^e$ could carry effects other than changes in the risk-return intercept. Thus, if the underlying stocks consist of too many high systematic risk stocks, or too many low systematic risk stocks, and if the potential risk-return relation shift is significant enough, then the test outlined above may not capture the effects of the options on $E(r_z^e)$.

The following procedure will be employed to see if this problem is present: A set of 7 or 8 underlying stocks with the highest risk is formed; similarly, a set of 7 or 8 underlying stocks with the lowest systematic risk is assembled. For each of these 2 sets, the neutral hedge returns are examined by rerunning the basic tests previously described. If the neutral hedges of the high systematic risk stocks exhibit similar behavior to the neutral hedges of the low systematic risk stocks, then the possible shifting slope of the risk-return relation is most likely not a significant problem for this study.

The most common systematic risk measure is "beta," which is proportional to the covariance between a stock's return and the market return.[16] The betas for all of the 30 underlying stocks were gathered from Value Line Investment Survey for the periods included in this study.[17] Since Value Line updates its beta estimates every quarter, the betas on all of the underlying stocks will generally vary from quarter to quarter. However, all 8 stocks in Table 4-4a had betas among the highest 10 of the 30 underlying stocks, each quarter. Also shown are the betas for each of the 8 stocks for the beginning, middle and ending quarters of the study.

Similarly, 7 stocks were consistently in the bottom 10 of the underlying stocks, ranked by beta. These 7 stocks and their betas are shown in Table 4-4b. The empirical results of the hypothesis tests are presented in Chapter V along with the naive diversification analysis findings.

TABLE 4-4a
High Beta Stocks

	4th Quarter 1973	4th Quarter 1974	4th Quarter 1975
1. Brunswick	1.73	1.85	1.65
2. Loews	1.78	1.60	1.40
3. McDonalds	1.34	1.55	1.55
4. NW Airlines	1.71	1.70	1.55
5. Pennzoil	1.40	1.40	1.35
6. Poloroid	1.23	1.45	1.40
7. Sperry Rand	1.42	1.30	1.30
8. Texas Instruments	1.32	1.25	1.20
Average "High-Beta"	1.491	1.513	1.425
Average of all 30 stocks	1.178	1.197	1.178

TABLE 4-4b
Low Beta Stocks

	4th Quarter 1973	4th Quarter 1974	4th Quarter 1975
1. ATT	.77	.75	.75
2. Merck	1.00	1.00	1.05
3. Exxon	1.12	.85	.95
4. Bethlehem Steel	1.00	1.05	1.05
5. Minn. Mining Manuf.	1.00	1.00	1.05
6. Sears	.95	1.00	1.05
7. IBM	1.04	1.05	1.05
Average "Low Beta"	.983	.957	.993
Average of all 30 stocks	1.178	1.197	1.178

Notes

[1]A popular example of a direct exogeneous influence on r_f is federal monetary policy. Monetary policy which influences r_f could indirectly influence $E(r_z)$.

[2]See Galai (1977).

[3]Black and Scholes claimed this result for neutral hedges consisting of stock and option positions. However, the Black-Scholes argument carries over to neutral spreads. Neutral spreads and hedges are riskless (zero variance) over the instant of creation and would remain riskless if investors could continuously revise portfolio positions as stock prices change. The inability of investors to continuously rebalance is the source of the variance to neutral spreads and hedges held for discrete intervals; systematic risk is presumably not present, however.

[4]The other assumptions from which equation (1) is derived are discussed elsewhere. See, for example, Black and Scholes (1973), Cox and Ross (1976), Rubinstein (1976b), Brennan (1979), Cox, Ross, and Rubinstein (1979), and Rendleman and Bartter (1979).

[5]The conditions are small holding period intervals and normally-distributed underlying stock prices. See note 3.

[6]Black and Scholes, Galai, and Finnerty also found neutral hedge returns to be larger than treasury bill yields. Assuming the mean-variance framework is appropriate, and assuming the neutral hedges really did not have any systematic risk, the positive excess returns support the Black equilibrium model over the Sharpe-Lintner model. Thus, the positive excess neutral hedge returns would, by implication, also support the contention that, prior to the vast expanion of option trading in the late 1970's, $E(r_z)$ exceeded r_f. To some extent these results are immune to the Roll critique, since neutral hedge returns are theoretically of zero systematic risk. Instead of discussing the implications of their findings in terms of comparing the Black and Sharpe-Lintner models, the option empiricists considered the positive excess returns to be evidence of option market inefficiency. For a different point of view see Phillips and Smith (1980), who discuss the role of trading costs.

[7]Boyle and Emanuel show that, if the correlation between underlying stocks is ρ, then the correlation between two neutral hedges is ρ^2.

[8]The portfolio variance asymptote found in this study may be compared to the variance estimates of $E(r_z)$ found by other methods by researchers such as Fama and MacBeth (1974).

[9]It is assumed at this point that "empirical" neutral hedges, held for one month without rebalancing, actually have zero systematic risk. The procedure outlined here in the text suggests a possible way of gauging the amount of systematic risk in the neutral hedges. Under the procedure outlined and assuming a large number of different option hedges are available for portfolio simulation, the portfolio variance should approach an asymptote of about 1/4 of the variance of individual hedges. If portfolio variance cannot be reduced by 3/4, then some systematic risk is probably in the neutral hedge returns. If the variance, on the other hand, drops by 3/4, then little or no systematic risk is probably in the hedges. The reader should be aware that although each neutral hedge is uncorrelated with the market portfolio, each neutral hedge does have some correlation with each other neutral hedge generally. (See note 7.)

[10]Some reasons why option prices have occasionally been observed to be below intrinsic value have been pointed out by Galai (1977, p. 172).

[11]A hedge ratio is the reciprocal of dC/dS. The hedge ratio is the number of option contracts to short against 100 shares of the stock to create a neutral hedge.

[12]The results of this procedure must be viewed in the same light as the systematic risk measures obtained by Black and Scholes, Galai, and Finnerty, because of the implications of Roll's critique. However, the time series of mean neutral hedge returns _is_ theoretically uncorrelated with a market index. Whether (or not) the time series of neutral hedge returns has significant correlation with respect to the index may be information that will cause us to have less (or more) confidence that neutral hedges are really uncorrelated with the "true" market portfolio. See note 9, also, for the primary way by which the hedges will be examined for systematic risk.

[13]Center for Research in Security Prices at the University of Chicago.

[14]This model has been credited to Gujarati (1970).

[15]This method of checking for systematic risk is supplementary to the method discussed in note 9.

[16]When the mean-variance CAPM is assumed, then beta is the appropriate systematic risk measure. See Chapter II.

[17]*Value Line* betas are probably calculated with reference to a stock market index. The reader should be aware that this procedure may also be criticized by the Roll (1977a) arguments.

CHAPTER V
EMPIRICAL FINDINGS

Introduction and Summary of Results

There are basically 2 types of empirical results presented in this chapter. First, the results of the naive diversification simulation indicate that there may be no systematic risk in the neutral hedges, although this is not defended rigorously. Furthermore, there is reason to believe that if the neutral hedges have no systematic risk, then a portfolio of hedges may be considered as a possible candidate for the minimum variance zero beta portfolio. The second type of results is that of the time series regression analyses of mean neutral hedge returns. No significant change in the mean hedge returns is evident from Period 1 to Period 2. However, the observed change is positive, as was predicted.

Naive Diversification

For each month, and for each portfolio size examined within each month, 30 portfolios of randomly selected hedge positions are constructed. The returns on the 30 portfolios are used to calculate a portfolio mean and portfolio standard deviation for the given size and the given month.

For a given size the portfolio mean and standard deviation will generally be different for each of the months. For example, from the first lines of the first page of Table 5-9 (located at the end of the chapter), for a portfolio size 5, the portfolio standard deviation for month 1 (December 1973) is .033437; the portfolio standard deviation for month 2 is .024443; and so on. Table 5-9 consists of 5 pages of portfolio means and standard deviations for each month of the study. There is 1/2 page for each of the 10 different portfolio sizes examined. In all of the 21 months, when looking at the portfolio standard deviation as a function of portfolio size, the familiar asymptotic, downward-sloping pattern, found by Evans and Archer (1968) for stocks, is observed.[1] This inverse relation is evidence that there is some risk in the individual hedges that can be diversified away by naive diversification.

As a summary of the results of Table 5-9 the averages of the month-by-month standard deviations for each of the portfolio sizes are given in Table 5-1.[2]

The monthly standard deviations for hedge portfolios of size 1 are shown in the righthand column of Table 5-3. The average of these monthly single-hedge standard deviations is .057446.[3] This figure and those of Table 5-1 are graphically displayed in Figure 1.

TABLE 5-1
Summary of Results of Naive Diversification
for December 1973 -- August 1975

Number of Neutral Hedges in Portfolio	Average Monthly Standard Deviation of 30 Portfolio Returns
5	.02521
10	.01814
15	.01534
20	.01287
25	.01148
30	.01110
35	.01027
40	.00901
45	.00853
50	.00828

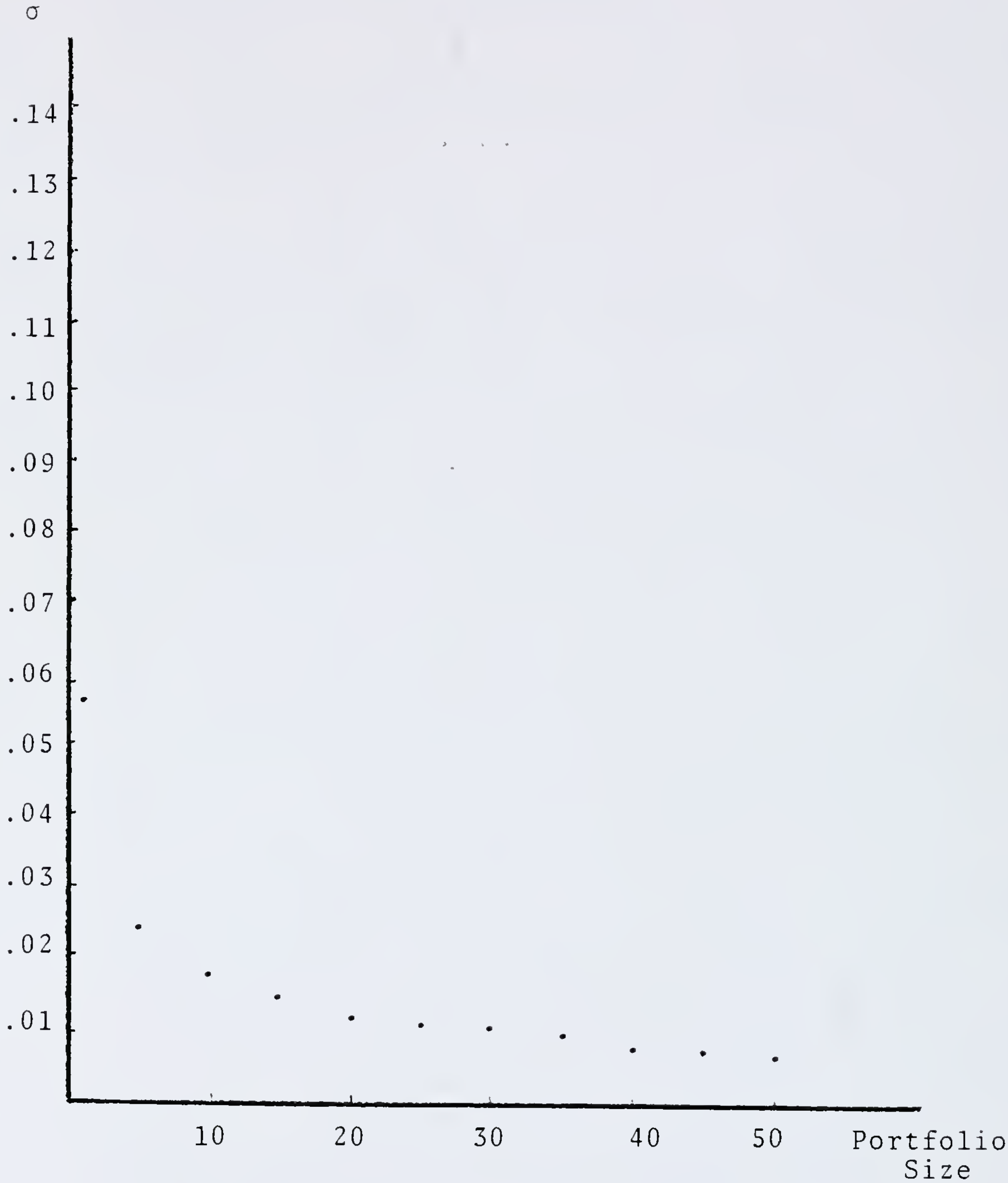


Figure 1. Summary of variance reduction of naive diversification of riskless hedges, December 1973--August 1975, 30 underlying stocks.

Systematic Risk in Hedges?

Boyle and Emanuel (1980) stated that the asymptote standard deviation of a portfolio of a large number of neutral hedges would be about 1/4 of the standard deviation of individual neutral hedges, although they claimed this figure to be the result of a "rough-guess."[4] The data here supports the Boyle and Emanuel thesis by the following reasoning:

As is evident from Table 5-2 portfolios of size 15-20 hedges yield variance of the order of 1/4 of .057, the approximate standard deviation of the individual neutral hedges. Although the portfolio standard deviations for sizes greater than 20 have been reported, it is doubtful that these numbers are extremely useful; this point follows from the fact that there are on the order of 150 hedges each month from which to select portfolios, so that portfolios of large sizes are bound to overlap to some extent in portfolio composition.

If the reader is satisfied that, for the portfolio size 15-20, the overlap of hedge positions within the 30 portfolios selected from about 150 positions is not too significant, then the Boyle and Emanuel report is essentially verified here.

The consistency with the Boyle-Emanuel prediction has the following implication for this study: Since it appears that there were enough neutral hedge positions available to force the standard deviation of a portfolio of neutral hedges close to its predicted asymptote, systematic risk was apparently absent; this finding follows, because if there had been any systematic risk, the naive diversification would not have been as effective as it was in forcing portfolio variance to its asymptotic limit. Thus on a "casual" basis there appears to be no systematic risk in the hedges.

The Minimum Variance Zero Beta Portfolio?

The final examination of the naive diversification findings is a comparison of the asymptote standard deviations with the standard deviations for the minimum variance zero beta portfolio found by other means by Fama and MacBeth (1974).

The data in Table 5-1 can now be compared to the figures below in Table 5-2, extracted from Fama-MacBeth. Table 5-2 contains the Fama-MacBeth estimates of the standard deviations of minimum variance zero beta portfolios. (Fama and MacBeth assumed monthly holding periods in their analysis.)

It is evident from the data in Tables 5-1 and 5-2 that in almost all cases the portfolio standard deviations of the hedges are lower than the ones reported by Fama and MacBeth. For all portfolio sizes 10 or greater, all of the Fama and MacBeth figures are larger than the average standard deviations calculated in this study. For portfolio size 5, the average hedge portfolio standard deviation is lower than 10 of the 12 figures reported by Fama and MacBeth. Thus, this seems to be evidence that investors could have reduced the

TABLE 5-2
Fama-MacBeth Estimates of Standard Deviations of Minimum Variance Zero Beta Portfolios

Time Period	Standard Deviation
1935 - 6/68	.0377
1946 - 6/68	.0281
1935 - 45	.0522
1946 - 55	.0256
1956 - 6/68	.0300
1935 - 40	.0636
1941 - 45	.0344
1946 - 50	.0307
1951 - 55	.0189
1956 - 60	.0202
1961 - 1/68	.0338

standard deviation of the minimum variance zero beta portfolio by employing the naive diversification strategy with neutral hedges, assuming the hedges really are of zero systematic risk.[6] This concludes the naive diversification findings.

Time Series Hypothesis Tests

Two distinct versions of the time series hypothesis test are reported. The first version assumes that the neutral hedges are zero systematic risk; equation (13) from Chapter IV is tested in the first version. The second version of the test assumes that if there is any systematic risk in the neutral hedges, it can be extracted by adding an index variable to the basic equation (13) regression of the first version. The second version of the test thus uses equation (14) from Chapter IV.

The results of both versions are qualitatively the same: no significant evidence of a difference in the mean hedge return in the first version and no significant evidence of a difference in the intercept term in the second version, from Period 1 to Period 2. In both versions, these were the results after the Cochrane-Orcutt procedure of adjusting for autocorrelation was applied. In addition, these results were confirmed for both versions in repetitions of the primary tests (which used all hedges) using subgroups of hedges. One subgroup of hedges consisted of only hedges of high-beta stocks; the other subgroup consisted of only hedges of low-beta stocks.

The findings of the 2 versions of the primary (all hedges) tests leads to the conclusion that the hypothesized effect of the option trading on $E(r_z)$ was not significantly evident. Moreover, the findings of the tests using the subgroups indicate that no significant shift in the slope of the risk-return relation took place. Thus, while the effects of the slope shift could have potentially obscured the observance of a change in $\bar{r}_z^e$, no significant evidence that this was the case was apparent.

The basic data from which these conclusions are extracted are contained in raw form in Table 4-3 and in Tables 5-3 and 5-4 below. Table 4-3 contains the number of available hedges for each month. Table 5-3 contains the means and standard deviations of the neutral hedges for each month. Table 5-4 contains the time series of 1 month treasury bill yields and the market index returns (CRSP index). The market index returns are not used until the second version of the test.

The numbers of available hedges in each month are used with the standard deviations in Table 5-3 to adjust for heteroskedasticity in the manner outlined in the previous chapter.

Prior to the adjustment for heteroskedasticity the treasury bill yields are subtracted from the corresponding mean hedge returns (and from the market index in the second

TABLE 5-3
Results of the Neutral Hedge Portfolio Strategy

	Month	Monthly Returns	Standard Deviations
12/73	1	0.057521	0.073142
	2	0.063433	0.054664
	3	0.049994	0.037826
	4	0.007847	0.089133
	5	0.025394	0.044350
Period 1	6	-0.008135	0.074630
	7	0.016124	0.035417
	8	-0.018948	0.074920
	9	-0.027755	0.036972
	10	-0.050112	0.069157
	11	-0.126200	0.134035
11/74	12	-0.010510	0.040210
12/74	13	0.050131	0.046967
	14	0.105327	0.106712
	15	-0.000040	0.063638
Period 2	16	0.026778	0.044363
	17	0.012212	0.044471
	18	-0.013665	0.037298
	19	0.053542	0.037964
	20	0.006455	0.030248
8/75	21	0.041263	0.030260
	Mean	.0124114	.057446
	Standard Deviation	.048246	.027200

TABLE 5-4
Time Series of Market Factor Returns and Treasury-Bill Yields

Month	Market Factor	T-Bill Yield
1	0.01400	0.00637
2	-0.00081	0.00620
3	0.00395	0.00639
4	-0.02440	0.00608
5	-0.04182	0.00737
6	-0.03664	0.00740
7	-0.02073	0.00666
8	-0.07218	0.00651
9	-0.08611	0.00647
10	-0.10892	0.00797
11	0.16429	0.00490
12	-0.04062	0.00787
13	-0.02236	0.00633
14	0.14048	0.00532
15	0.06406	0.00444
16	0.02691	0.00365
17	0.04846	0.00436
18	0.05528	0.00434
19	0.05020	0.00414
20	-0.06436	0.00467
21	-0.01747	0.00504

version) to obtain the excess return series: excess mean hedge portfolio returns (and excess market index returns). The resulting excess mean hedge return after the adjustment for heteroskedasticity is $\overline{WX}_t$ in the model of equations (13) and (14) of Chapter IV. The resulting excess market index return, for use in the second version of the tests, is $\overline{WM}_t$, again after the adjustment for heteroskedasticity.

First Version: Tests Without the Index

In the first version of the tests, equation (13) is employed for the mean excess hedge returns. The basic regression coefficients and t-values for the primary tests (using all hedges) are reported below in Table 5-5. Some of the regression summary statistics are given in the next paragraph ; included in that paragraph are some statistics relating to the Cochrane-Orcutt procedure.

The regression coefficient of determination, R^2, is .2381. The Cochrane-Orcutt procedure converged at 1 iteration. The final value of the first order serial correlation coefficient is .26631. This serial correlation coefficient is not statistically significant, since its t-value is 1.235593. The Durbin-Watson statistic <u>without</u> the Cochrane-Orcutt procedure is 1.4093; using the Cochrane-Orcutt procedure, the resulting Durbin-Watson statistic is 2.1106.

As is evident from the t-value in Table 5-5 of the coefficient of I_2, this coefficient is not significant for

TABLE 5-5
Regression Coefficients and t-Values
First Version: No Market Index
Primary Test: All Hedges
Results of Generalized Least
Squares Analysis Using Cochrane-Orcutt Procedure

Variable	Coefficient	t-Value
I_1	(U_1) -.00207218	-.168801
I_2	(U_2-U_1) .0244162	1.53876

where

U_1 is the mean hedge return for Period 1;

$U_2 - U_1$ is the mean difference between the Period 2 and Period 1 hedge returns.

the all-hedges regression of the first version test. The hypothesis of a significant change in $\bar{r}_z^e$ would be rejected, assuming no shift in the risk-return slope is causing effects that counteract the predicted movement.

The first version regressions are repeated for the high-beta stock group of hedges and again for the low-beta stock group of hedges. The results of these analyses are shown below in Table 5-6. The regression summary statistics are provided in the paragraphs following.

For the regression using the hedges of only the high-beta stocks, the R^2 is .1354. The Cochrane-Orcutt procedure converges at 1 iteration to a value for the first-order serial correlation coefficient of .120500. This correlation coefficient is not significant, since its t-value is .5428949. The resulting Durbin-Watson statistic is 2.0281. If the Cochrane-Orcutt procedure is not used, the Durbin-Watson statistic is 1.7205.

For the regression using only the hedges of the low-beta stocks, the R^2 is .0946. The Cochrane-Orcutt procedure converges at 2 iterations to a final first-order serial correlation coefficient of .080264 (t-value = .360117). The final Durbin-Watson statistic is 2.0142; the Durbin-Watson statistic without the Cochrane-Orcutt procedure is 1.7466.

The results in Table 5-6 indicate a lack of significant mean hedge return change from Period 1 to Period 2, regardless of whether the hedges are constructed using high- or low-beta stocks. It does not appear that there is evidence

TABLE 5-6
Regression Coefficients and t-Values
First Version: No Market Index
Results of Generalized Least Squares
Analysis Using Cochrane-Orcutt Procedure

Variable	Coefficient	t-Value
	Test Using High-Beta Stock Hedges	
I_1	(U_1) .00858696	.544999
I_2	(U_2 - U_1) .0157586	.794869
	Test Using Low-Beta Stock Hedges	
I_1	(U_1) -.00038156	-.0480936
I_2	(U_2 - U_1) .0126959	1.18276

of the effects of a shifting risk-return slope obstructing the view of changes in (equilibrium expected) excess zero systematic risk returns. Next are presented the results of the second version of the tests: the full interactive market-index model, equation (14) from Chapter IV.

Second Version: Tests with Index

The resulting regression coefficients and t-values of the interactive time series analysis, with market index, are shown below in Table 5-7. The results include all adjustments for heteroskedasticity and possible autocorrelation. After the adjustments for heteroskedasticity, the mean excess hedge returns are regressed on the 4 variables indicated by equation (14) from Chapter IV. The 4 variables are (a) an intercept, (b) a dummy variable, (c) the excess market index returns, and (d) the interactive dummy excess market index returns (referred to as MI_t in the previous chapter). Regression summary statistics are given in the next paragraph.

The R^2 for the regression is .2954 and the Durbin-Watson statistic is 1.9967. The Durbin-Watson statistic without the Cochrane-Orcutt procedure is 1.8173. The Cochrane-Orcutt procedure converges at 5 iterations; the final value of the first order serial correlation coefficient is .162752 (t-value is .737685).

None of the coefficients in Table 5-7 is significant, as is evidenced by the magnitude of the t-values. Despite

TABLE 5-7
Regression Coefficients and t-Values
Second Version: Using Market Index
Primary Test: All Hedges
Results of Generalized Least
Squares Analysis Using Cochrane-Orcutt Procedure

Variable	Coefficient		t-Value
I_1	(α_1)	.0128303	.742934
I_2	$(\alpha_2 - \alpha_1)$	.00880922	.444928
M	(β_1)	.322649	1.19954
MI	$(\beta_2 - \beta_1)$	.268486	-.811277

where

α_1 and β_1 are the intercept and slope of a regression on just M for Period 1, and

α_2 and β_2 are the intercept and slope of a regression on just M for Period 2.

its lack of significance, the coefficient of I_2 ($\alpha_2 - \alpha_1 =$ .0080922) in Table 5-7 has the predicted sign. The lack of significance leads to the rejection of the hypothesis of a significant change in the excess zero-systematic risk security returns. Next it will be examined whether any potential shifting-slope effects could have obscured a change in the hedge returns.

The next Table, 5-8, shows the regression coefficients and associated t-values for the second version (equation (14)) test using the high-beta stock hedges and the low-beta stock hedges. Again, it is evident that none of the coefficients are significant. The regression summary statistics are given in the next paragraph.

For the regression using the hedges of only the high beta stocks, the R^2 is .2649; the Durbin-Watson statistic is 1.9871; the Cochrane-Orcutt procedure converges at 2 iterations, with a final first-order serial correlation coefficient of -.106526 (t-value = .479124). For the regression using the hedge of only the low beta stocks, the R^2 is .1770; the Durbin-Watson statistic is 1.9532; the Cochrane-Orcutt procedure converges at 1 iteration, with a final value of the first-order serial correlation coefficient of -.093617 (t-value = -.420513).

Since none of the coefficients in Table 5-8 are significant, it appears that the potential shifting slope problem did not significantly materialize. That is, it seems safe to conclude that there was no significant obscuring of

TABLE 5-8
Regression Coefficients and t-Values
Second Version: Using Market Index
Results of Generalized Least Squares
Regression Analysis Using Cochrane-Orcutt Procedure

Variable		Coefficient	t-Value
	Test Using High-Beta Stock Hedges		
I_1	(α_1)	.0452298	1.91676
I_2	$(\alpha_2-\alpha_1)$	-.0216797	-.839015
M	(β_1)	.805411	1.94055
MI	$(\beta_2-\beta_1)$	-.812371	-1.71188
	Test Using Low-Beta Stock Hedges		
I_1	(α_1)	.0125017	1.15618
I_2	$(\alpha_2-\alpha_1)$	.00251953	.0192932
M	(β_1)	.297273	1.37960
MI	$(\beta_2-\beta_1)$	-.365492	-1.35620

changes in the hedge returns from Period 1 to Period 2 caused by the effects of a shift in the risk-return relation slope. Combining this with the I_2 coefficient in the primary regression (all hedges) of the second version leads to the conclusion that new options did not lead to a significant change in the observed hedge returns "intercept" from Period 1 to Period 2.

Some implications of the findings in this chapter for finance research are pointed out in the next chapter.

TABLE 5-9
Naive Diversification Results

	Date	Mean	30 Portfolios' Average Standard Deviation
		Portfolio Size: 5	
1.	December, 1973	0.053712	0.033437
2.	January, 1974	0.068048	0.024443
3.	February, 1974	0.050311	0.016059
4.	March, 1974	0.007598	0.040133
5.	April, 1974	0.032236	0.019183
6.	May, 1974	-0.012299	0.038504
7.	June, 1974	0.017556	0.015303
8.	July, 1974	-0.021386	0.033482
9.	August, 1974	-0.028023	0.016610
10.	September, 1974	-0.055955	0.029154
11.	October, 1974	-0.157972	0.057171
12.	November, 1974	-0.011432	0.016743
13.	December, 1974	0.051052	0.019891
14.	January, 1975	0.092992	0.047246
15.	February, 1975	0.003206	0.024398
16.	March, 1975	0.028936	0.021198
17.	April, 1975	0.005324	0.022136
18.	May, 1975	-0.012122	0.012421
19.	June, 1975	0.052101	0.018433
20.	July, 1975	0.007779	0.010142
21.	August, 1975	0.039476	0.013346
		Portfolio Size: 10	
1.	December, 1973	0.059354	0.018377
2.	January, 1974	0.071874	0.020743
3.	February, 1974	0.050044	0.008152
4.	March, 1974	0.007038	0.029007
5.	April, 1974	0.031276	0.010934
6.	May, 1974	-0.011293	0.023650
7.	June, 1974	0.016594	0.009626
8.	July, 1974	-0.026386	0.026297
9.	August, 1974	-0.027708	0.013886
10.	September, 1974	-0.057434	0.022511
11.	October, 1974	-0.149070	0.041122
12.	November, 1974	-0.009016	0.012699
13.	December, 1974	0.049522	0.017351
14.	January, 1975	0.076608	0.036854
15.	February, 1975	-0.004810	0.019798
16.	March, 1975	0.025878	0.013627
17.	April, 1975	0.002412	0.019935
18.	May, 1975	-0.011313	0.009103
19.	June, 1975	0.053854	0.011574
20.	July, 1975	0.006967	0.006539
21.	August, 1975	0.040941	0.009142

TABLE 5-9 (continued)

	Date	Mean	30 Portfolios' Average Standard Deviation
		Portfolio Size: 15	
1.	December, 1973	0.054415	0.018788
2.	January, 1974	0.070243	0.015187
3.	February, 1974	0.048424	0.007174
4.	March, 1974	0.005781	0.021929
5.	April, 1974	0.031002	0.008459
6.	May, 1974	-0.013937	0.022787
7.	June, 1974	0.015609	0.007411
8.	July, 1974	-0.023233	0.026581
9.	August, 1974	-0.026508	0.011748
10.	September, 1974	-0.055352	0.018411
11.	October, 1974	-0.154665	0.032799
12.	November, 1974	0.009442	0.010008
13.	December, 1974	0.050407	0.012684
14.	January, 1975	0.078216	0.031106
15.	February, 1975	-0.008133	0.016355
16.	March, 1975	0.027520	0.012502
17.	April, 1975	0.002886	0.015448
18.	May, 1975	-0.013077	0.009036
19.	June, 1975	0.052624	0.009570
20.	July, 1975	0.006579	0.006127
21.	August, 1975	0.040902	0.008110
		Portfolio Size: 20	
1.	December, 1973	0.059667	0.014742
2.	January, 1974	0.071634	0.014207
3.	February, 1974	0.049604	0.005022
4.	March, 1974	0.003257	0.018093
5.	April, 1974	0.031636	0.006570
6.	May, 1974	-0.009525	0.017845
7.	June, 1974	0.016463	0.007758
8.	July, 1974	-0.026744	0.021157
9.	August, 1974	-0.026742	0.008732
10.	September, 1974	-0.056334	0.017037
11.	October, 1974	-0.154119	0.029795
12.	November, 1974	-0.009403	0.008871
13.	December, 1974	0.050004	0.009392
14.	January, 1975	0.082522	0.021537
15.	February, 1975	-0.008163	0.014194
16.	March, 1975	0.028429	0.010556
17.	April, 1975	0.001036	0.013996
18.	May, 1975	-0.012328	0.009882
19.	June, 1975	0.050331	0.008047
20.	July, 1975	0.005908	0.004748
21.	August, 1975	0.040401	0.008100

TABLE 5-9 (continued)

	Date	Mean	30 Portfolios' Average Standard Deviation
		Portfolio Size: 25	
1.	December, 1973	0.056541	0.013170
2.	January, 1974	0.071205	0.008998
3.	February, 1974	0.049692	0.005461
4.	March, 1974	0.007927	0.018725
5.	April, 1974	0.031697	0.005701
6.	May, 1974	-0.007330	0.015202
7.	June, 1974	0.016229	0.007630
8.	July, 1974	-0.029652	0.019949
9.	August, 1974	-0.028471	0.007590
10.	September, 1974	-0.056761	0.013041
11.	October, 1974	-0.154098	0.028826
12.	November, 1974	-0.009601	0.006632
13.	December, 1974	0.051024	0.009290
14.	January, 1975	0.080732	0.020885
15.	February, 1975	-0.009608	0.010592
16.	March, 1975	0.027839	0.008832
17.	April, 1975	0.000350	0.013134
18.	May, 1975	-0.012534	0.008177
19.	June, 1975	0.051591	0.007880
20.	July, 1975	0.007029	0.005029
21.	August, 1975	0.040741	0.006327
		Portfolio Size: 30	
1.	December, 1973	0.055708	0.012875
2.	January, 1974	0.070910	0.011526
3.	February, 1974	0.049706	0.005629
4.	March, 1974	0.010189	0.016897
5.	April, 1974	0.031753	0.005941
6.	May, 1974	-0.007901	0.015338
7.	June, 1974	0.015690	0.006454
8.	July, 1974	-0.029090	0.015999
9.	August, 1974	-0.026861	0.006884
10.	September, 1974	-0.057076	0.015021
11.	October, 1974	-0.149755	0.023175
12.	November, 1974	-0.010159	0.005166
13.	December, 1974	0.051350	0.010440
14.	January, 1975	0.084122	0.024605
15.	February, 1975	-0.009689	0.013274
16.	March, 1975	0.027697	0.009243
17.	April, 1975	0.000615	0.009972
18.	May, 1975	-0.013265	0.007606
19.	June, 1975	0.051276	0.005110
20.	July, 1975	0.006113	0.004204
21.	August, 1975	0.040102	0.006435

TABLE 5-9 (continued)

	Date	Mean	30 Portfolios' Average Standard Deviation
		Portfolio Size: 35	
1.	December, 1973	0.055732	0.010430
2.	January, 1974	0.070428	0.010418
3.	February, 1974	0.050079	0.004837
4.	March, 1974	0.010722	0.015613
5.	April, 1974	0.031887	0.004983
6.	May, 1974	-0.006257	0.013631
7.	June, 1974	0.016671	0.005957
8.	July, 1974	-0.030639	0.017670
9.	August, 1974	-0.026633	0.006587
10.	September, 1974	-0.056227	0.015255
11.	October, 1974	-0.147105	0.022959
12.	November, 1974	-0.010423	0.006167
13.	December, 1974	0.050902	0.008814
14.	January, 1975	0.082415	0.019713
15.	February, 1975	-0.008000	0.012862
16.	March, 1975	0.027800	0.008635
17.	April, 1975	0.000973	0.008059
18.	May, 1975	-0.012071	0.007771
19.	June, 1975	0.052825	0.005827
20.	July, 1975	0.005711	0.003441
21.	August, 1975	0.040009	0.006069
		Portfolio Size: 40	
1.	December, 1973	0.056142	0.009478
2.	January, 1974	0.069852	0.008457
3.	February, 1974	0.049634	0.004366
4.	March, 1974	0.010359	0.014636
5.	April, 1974	0.032390	0.005412
6.	May, 1974	-0.005794	0.012444
7.	June, 1974	0.015597	0.005387
8.	July, 1974	-0.030596	0.014743
9.	August, 1974	-0.027505	0.005118
10.	September, 1974	-0.055080	0.011671
11.	October, 1974	-0.150575	0.022038
12.	November, 1974	-0.009326	0.004927
13.	December, 1974	0.050331	0.007276
14.	January, 1975	0.080549	0.015966
15.	February, 1975	-0.007188	0.011675
16.	March, 1975	0.028847	0.007259
17.	April, 1975	0.001434	0.008601
18.	May, 1975	-0.012599	0.005051
19.	June, 1975	0.053437	0.005582
20.	July, 1975	0.006377	0.004220
21.	August, 1975	0.040146	0.004896

TABLE 5-9 (continued)

	Date	Mean	30 Portfolios' Average Standard Deviation
		Portfolio Size: 45	
1.	December, 1973	0.056027	0.008626
2.	January, 1974	0.069602	0.009699
3.	February, 1974	0.049215	0.004848
4.	March, 1974	0.009231	0.012944
5.	April, 1974	0.032721	0.004736
6.	May, 1974	-0.008237	0.012783
7.	June, 1974	0.015416	0.005005
8.	July, 1974	-0.030186	0.013453
9.	August, 1974	-0.027374	0.005217
10.	September, 1974	-0.055519	0.011913
11.	October, 1974	-0.149412	0.017316
12.	November, 1974	-0.010149	0.005016
13.	December, 1974	0.050211	0.006593
14.	January, 1975	0.080511	0.017514
15.	February, 1975	-0.007193	0.011023
16.	March, 1975	0.027633	0.006088
17.	April, 1975	0.002257	0.007102
18.	May, 1975	-0.012831	0.005459
19.	June, 1975	0.052794	0.005658
20.	July, 1975	0.006283	0.003695
21.	August, 1975	0.039693	0.004360
		Portfolio Size: 50	
1.	December, 1973	0.057719	0.008909
2.	January, 1974	0.047710	0.005590
3.	February, 1974	0.047423	0.005953
4.	March, 1974	0.009917	0.013309
5.	April, 1974	0.031672	0.006524
6.	May, 1974	-0.005042	0.008337
7.	June, 1974	0.015333	0.004155
8.	July, 1974	-0.023236	0.013833
9.	August, 1974	-0.027642	0.004186
10.	September, 1974	-0.056221	0.010446
11.	October, 1974	-0.150384	0.020718
12.	November, 1974	-0.010107	0.005627
13.	December, 1974	0.049891	0.006198
14.	January, 1975	0.080611	0.017087
15.	February, 1975	-0.004881	0.009334
16.	March, 1975	0.030606	0.007216
17.	April, 1975	0.001290	0.009306
18.	May, 1975	-0.011075	0.005397
19.	June, 1975	0.050310	0.005742
20.	July, 1975	0.006268	0.003434
21.	August, 1975	0.039922	0.007388

Notes

[1]See Appendix A for graphs of the monthly portfolio size-variance relation.

[2]From Table 5-1, for example, it can be observed that the average portfolio standard deviation for portfolio size 5 is .02521. The average portfolio standard deviation for portfolio size 10 is .01814, and so forth.

[3]This figure is not the exact average of all the hedges, since the number of hedges in each month varied. However, the figure should be a reasonable approximation of the standard deviation of a neutral hedge.

[4]See notes 7 and 9 in Chapter III.

[5]Boyle and Emanuel's figure of 1/4 was a rough estimate and not rigorous by their own admission, and so our analysis is equally nonrigorous.

[6]The Fama-MacBeth zero beta portfolios are now understood to be only zero beta relative to the market index used in their analysis. Thus, the comparison of our standard deviations with Fama and MacBeth's is not conclusive that a naively diversified hedges portfolio is the minimum variance zero beta portfolio. The comparison may be interesting to the reader anyway.

CHAPTER VI
REVIEW AND IMPLICATIONS OF THE RESEARCH FOR OPTIONS AND THE CAPM

It was established in the dissertation's theory that there is reason to believe that the more options that are traded, the lower is the equilibrium rate of return on zero systematic risk securities. The empirical portion of this study failed to verify the hypothesis with statistical significance. Some highlights of the research are discussed in this chapter.

A Test of the CAPM

The major contribution of the research appears to be the testing of an implication of the CAPM. The direction of the empirical change in neutral hedge returns was positive, supportive of the theory. The reason(s) for the lack of statistically significant results could have been (a) the possible invalidity of the CAPM itself; (b) the possible invalidity of the theory that the role of options is to circumvent margin ceilings; or (c) the possible fact that the predicted effect of option expansion was simply not detected by our methods. Point (c) could be the case, for example, if the theorized effect of option expansion is just too gradual. Another possible explanation along the lines of point (c) could be that our own testing methods were too "blunt" (not enough data points in the time series or not enough precision in measuring the exact times at which the new options began trading). The magnitude and direction of the shift in the neutral hedge returns provides encouragement, however, that more refined techniques may eventually verify the hypothesis. The task of improving the empirical analysis is left to future research.

A unique aspect to the study has been the empirical exploration of the CAPM without the research being subject to the well-known critique of Richard Roll (1977a);[1] the CAPM implications were tested here by examining a portfolio of neutral hedges, which theoretically have zero systematic risk, and which may be constructed <u>without</u> having to identify the composition of the market portfolio.

Systematic Risk of Neutral Hedges

Another unique aspect of the study has been the manner of investigating for systematic risk in the neutral hedges, although the method was admittedly casual. Recall that the naive diversification results indicated that the standard deviation of a portfolio of neutral hedges can be reduced to about 1/4 of the standard deviation of a single hedge, a finding in line with the estimate made by Boyle and Emanuel. This finding, if it is true, would indicate an absence of systematic risk in the hedges. However, the limitation to this finding, aside from the lack of rigor of the Boyle-Emanual "1/4" figure, was the lack of a large number of available hedge positions from which to randomly pick portfolios of hedges that would not "overlap" to any great extent. Nevertheless, the findings appear promising and could be examined more thoroughly using data from the present time.[2] If it is true that the hedges had no significant systematic risk, then this finding appears to be interesting, since previously reported measures of neutral hedge systematic risk are of dubious value due to the criticisms of Roll. Like the exploration of the CAPM, the measurement of systematic risk in the neutral hedges here has avoided the problems that Roll associated with time series regression.

The results of the dissertation suggest several opportunities for further research. First, the risk and return properties of neutral hedges could be examined again using current data, which are far more plentiful than those used in this study. The rates of return on theoretically zero systematic risk positions are still of interest to capital market researchers.

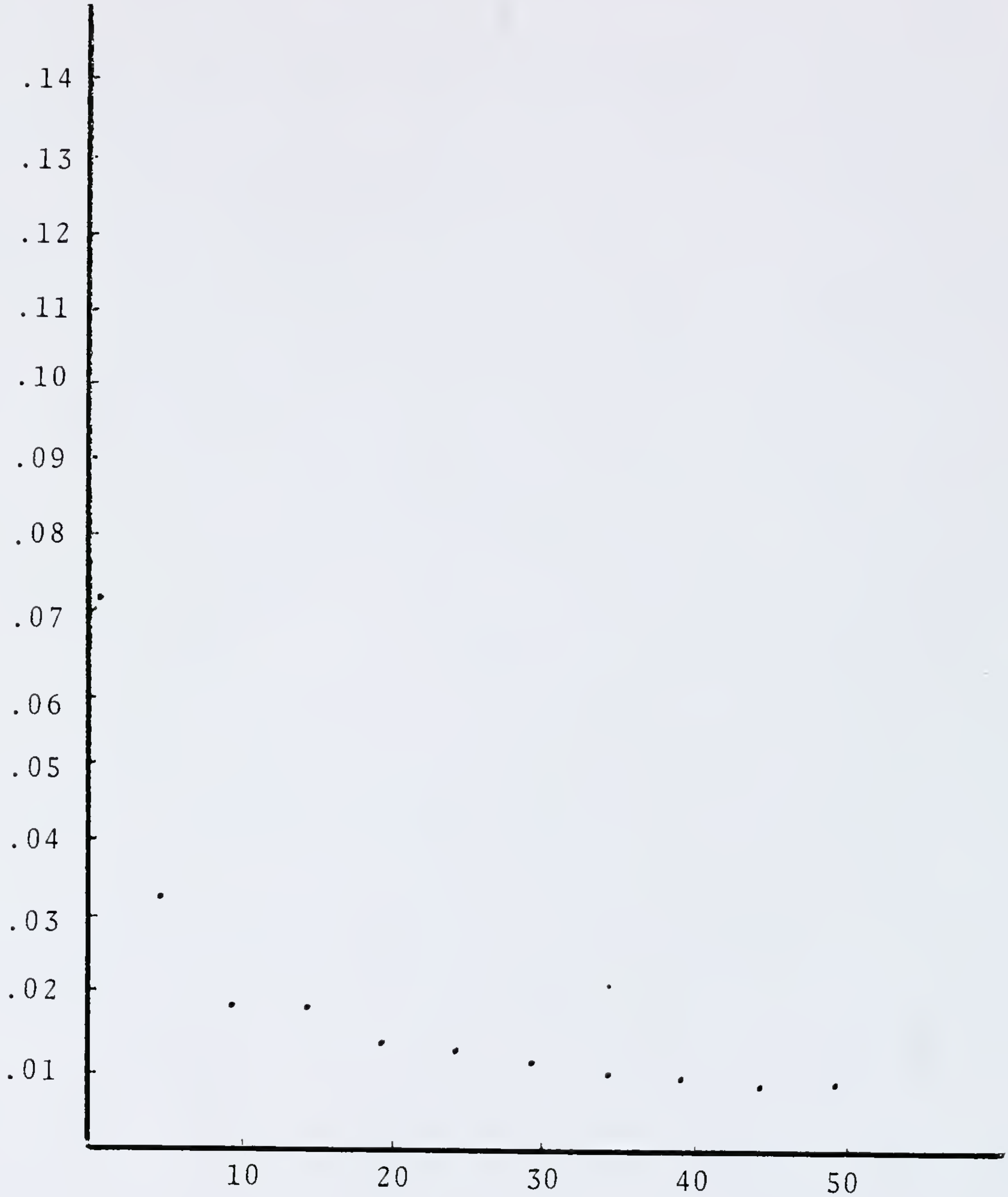
A second and related research pursuit would be an attempt to explain the results of the second version hypothesis tests. The betas of these hedges in Period 1 were higher than expected, especially for the high beta stock hedges. Since the lack of statistical significance could have resulted from not having enough time series observations, it appears that the assumption of zero systematic risk in monthly neutral hedges should be analyzed further. Also, why was this systematic risk virtually eliminated in Period 2?

Notes

[1]See Chapter III.

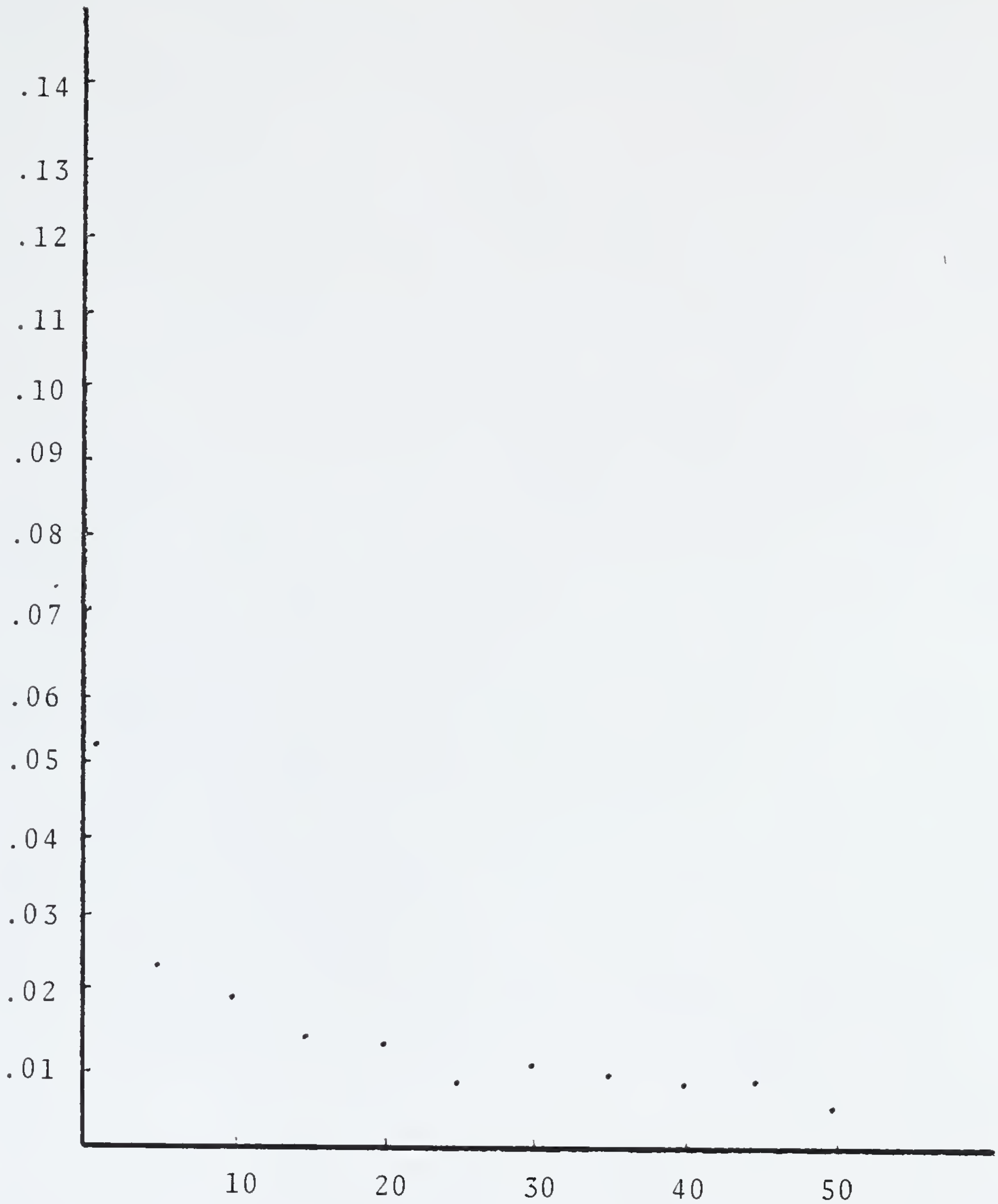
[2]There are about 250 different companies underlying stock options, about 9 times the number that were listed at the time of this study's data.

APPENDIX
MONTHLY PORTFOLIO SIZE-VARIANCE RELATION



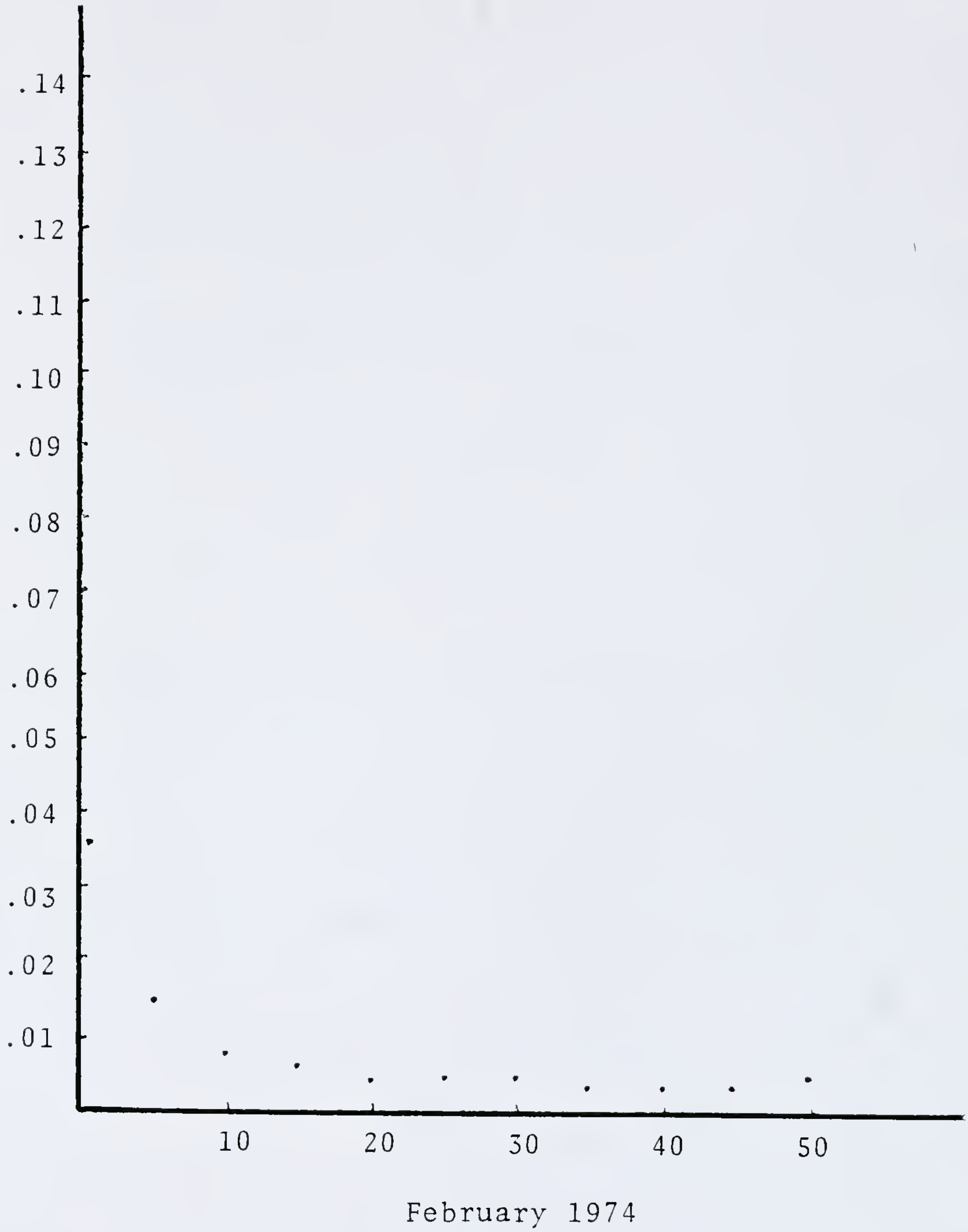
December 1973

Figure A-1



January 1974

Figure A-1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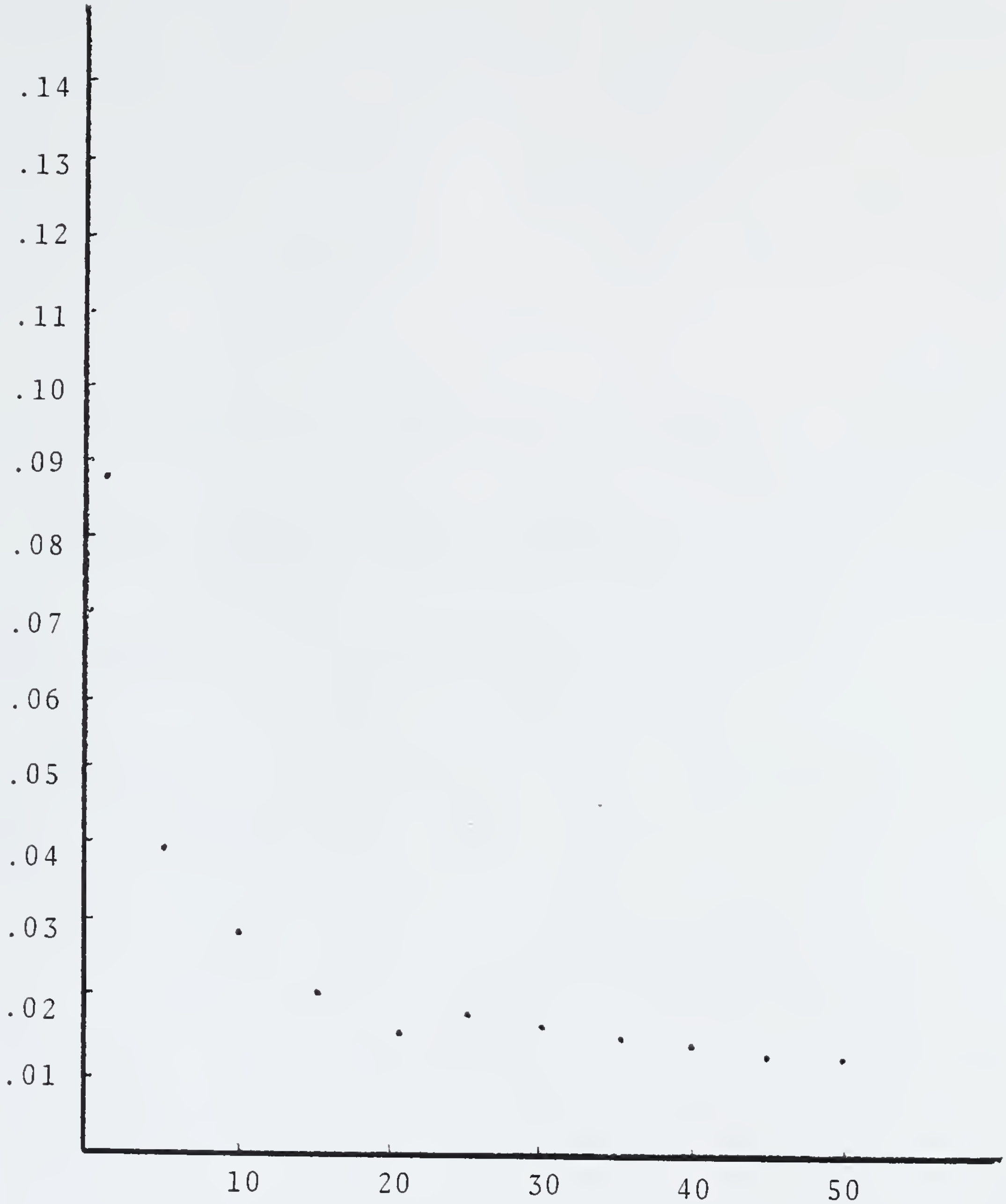
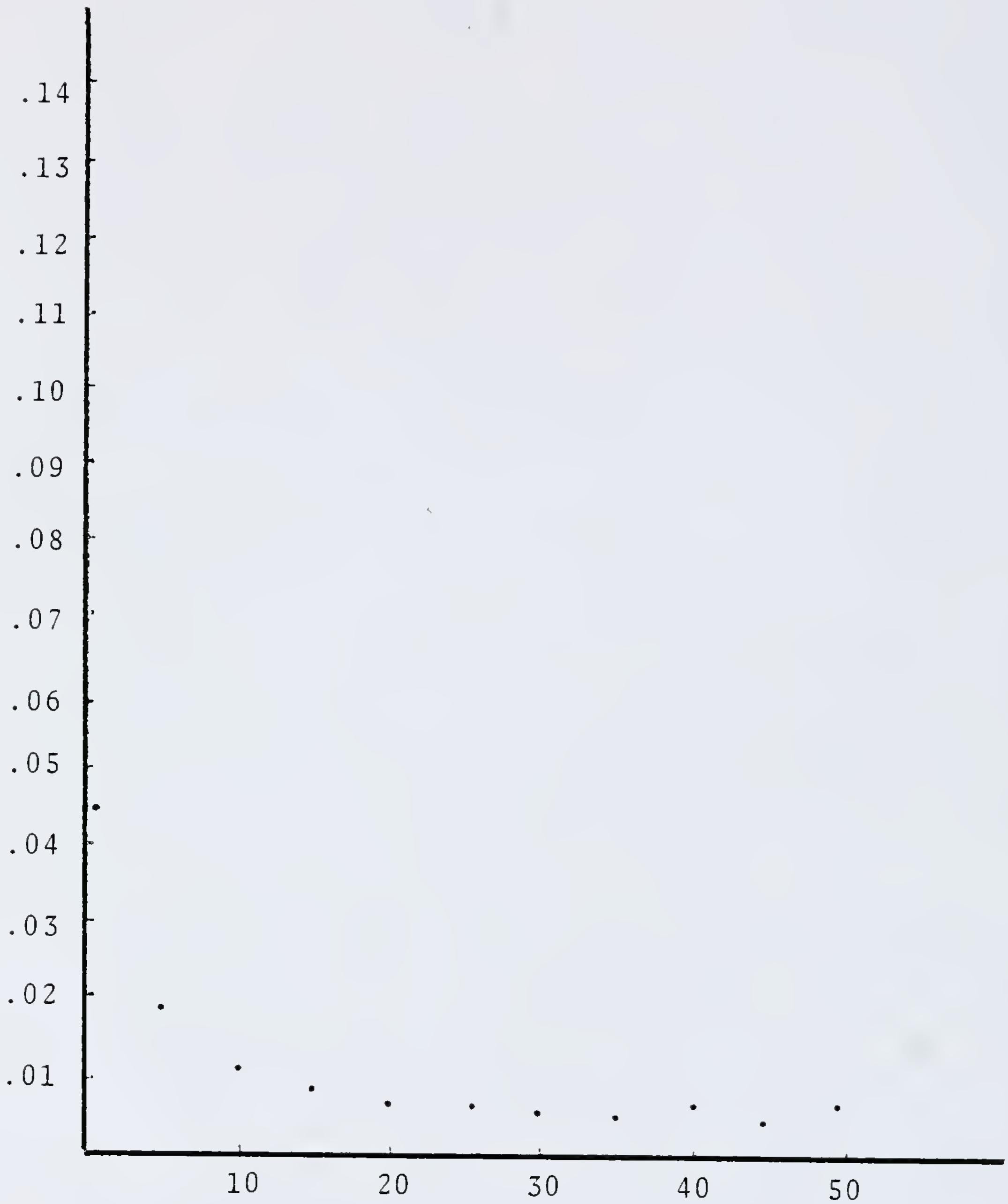
February 1974

Figure A-1



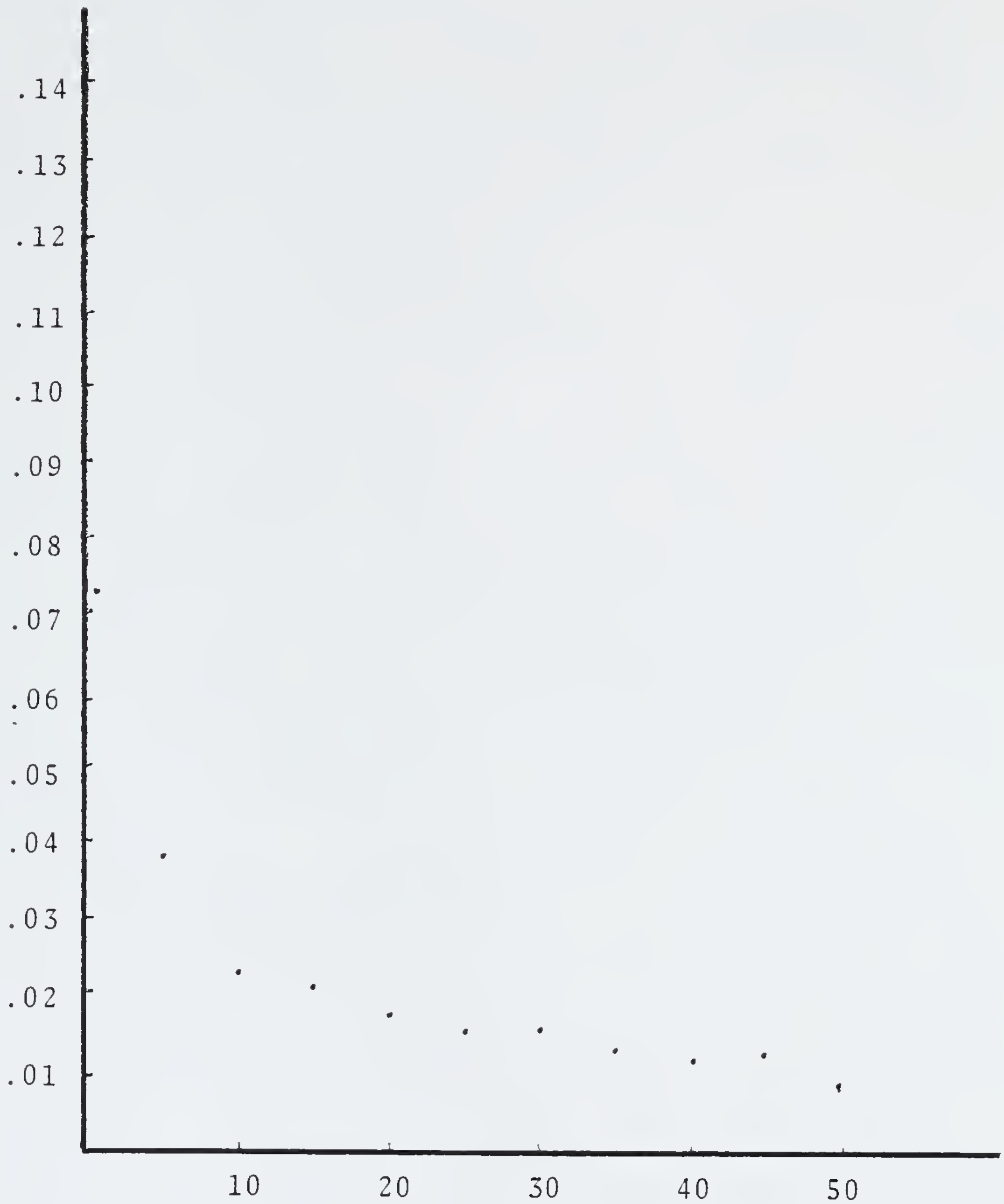
March 1974

Figure A-1



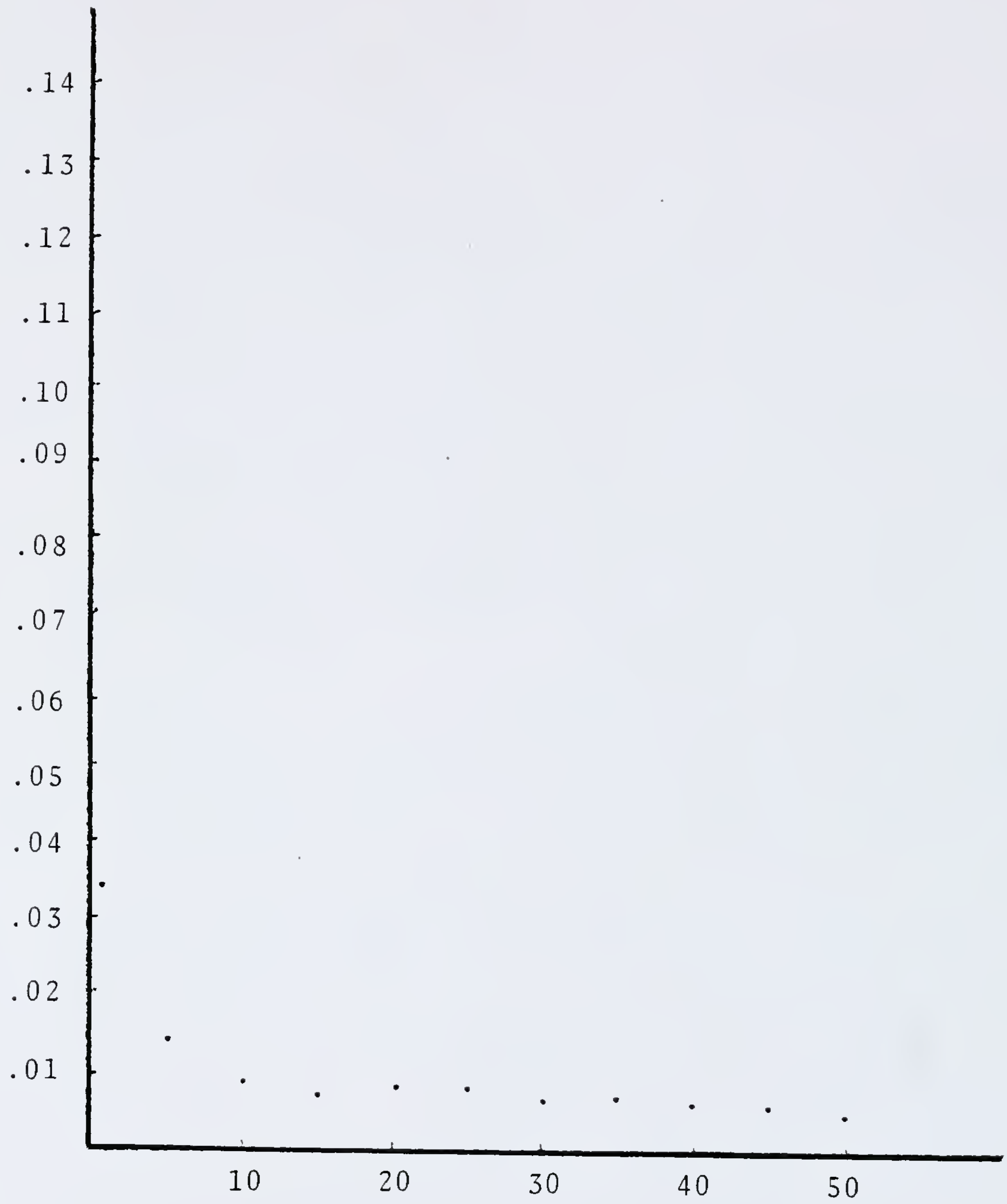
April 1974

Figure A-1



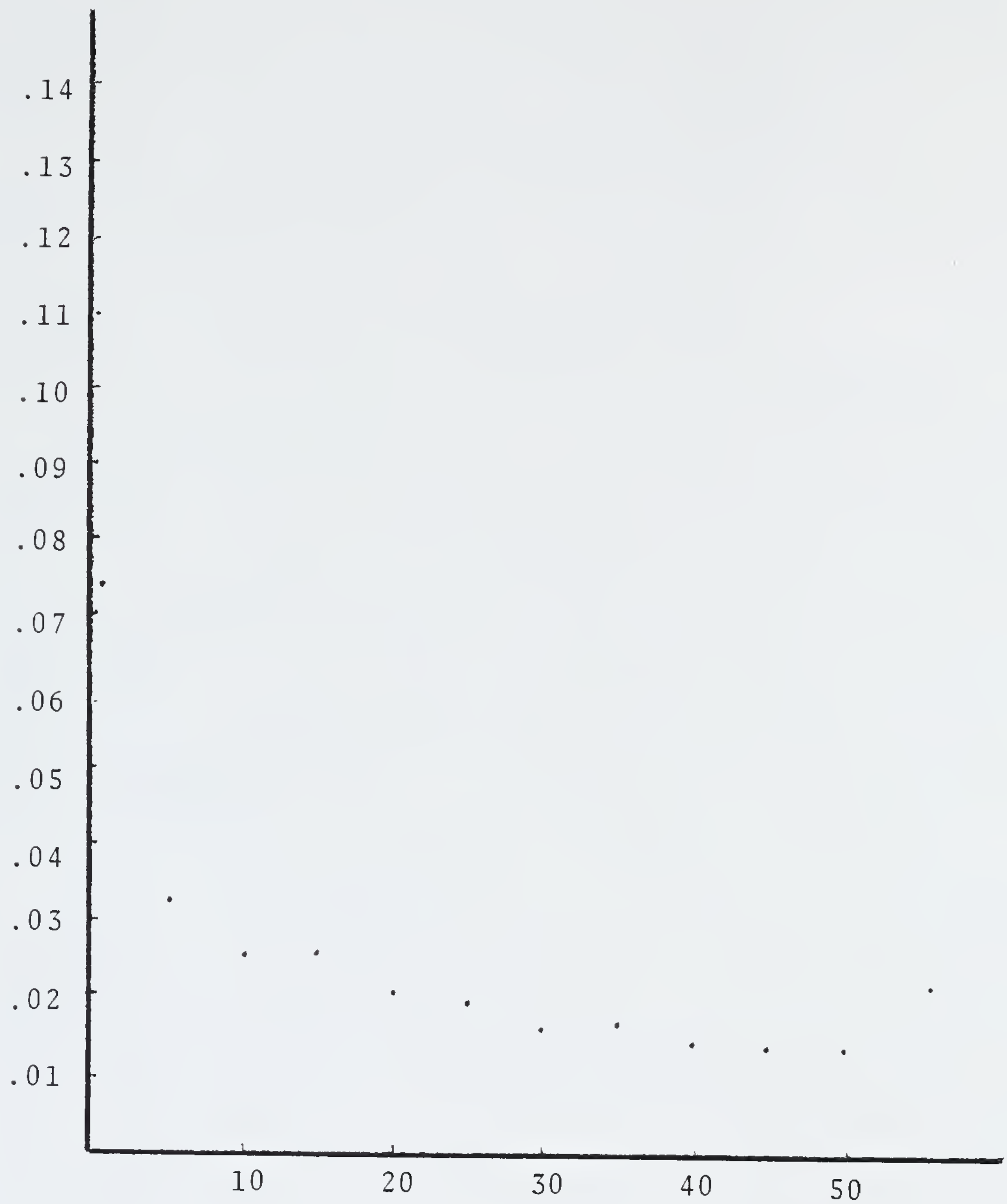
May 1974

Figure A-1



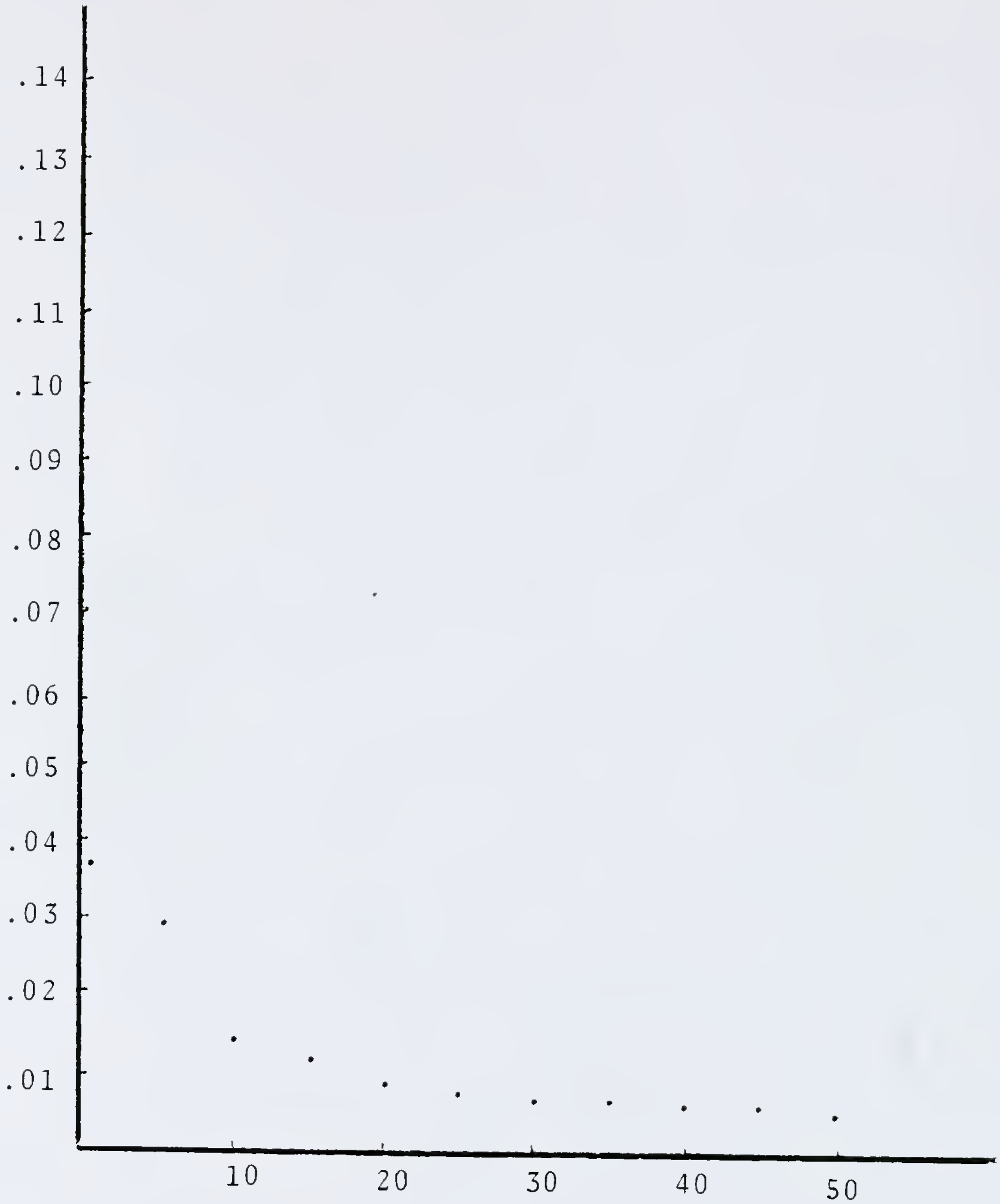
June 1974

Figure A-1



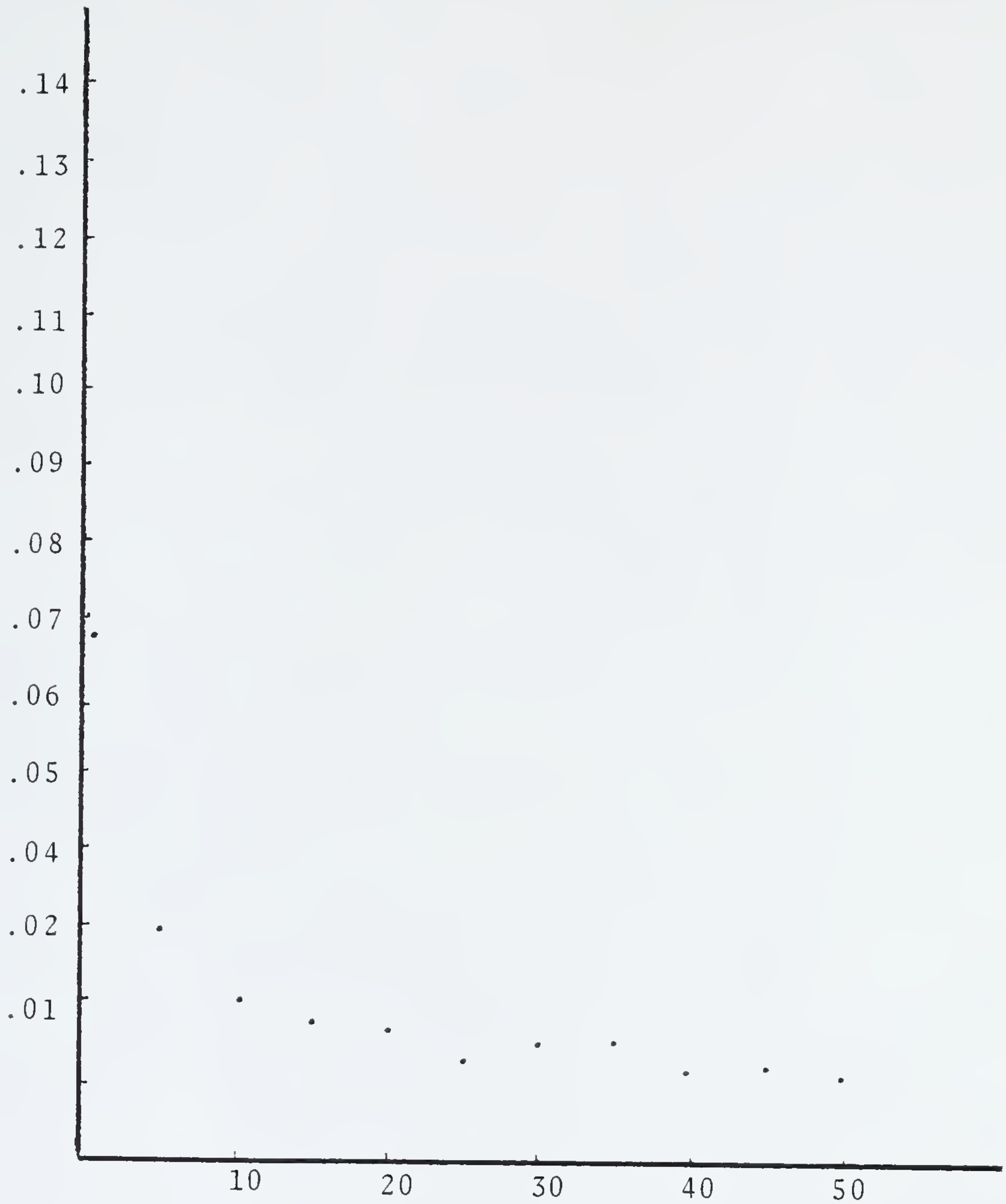
July 1974

Figure A-1



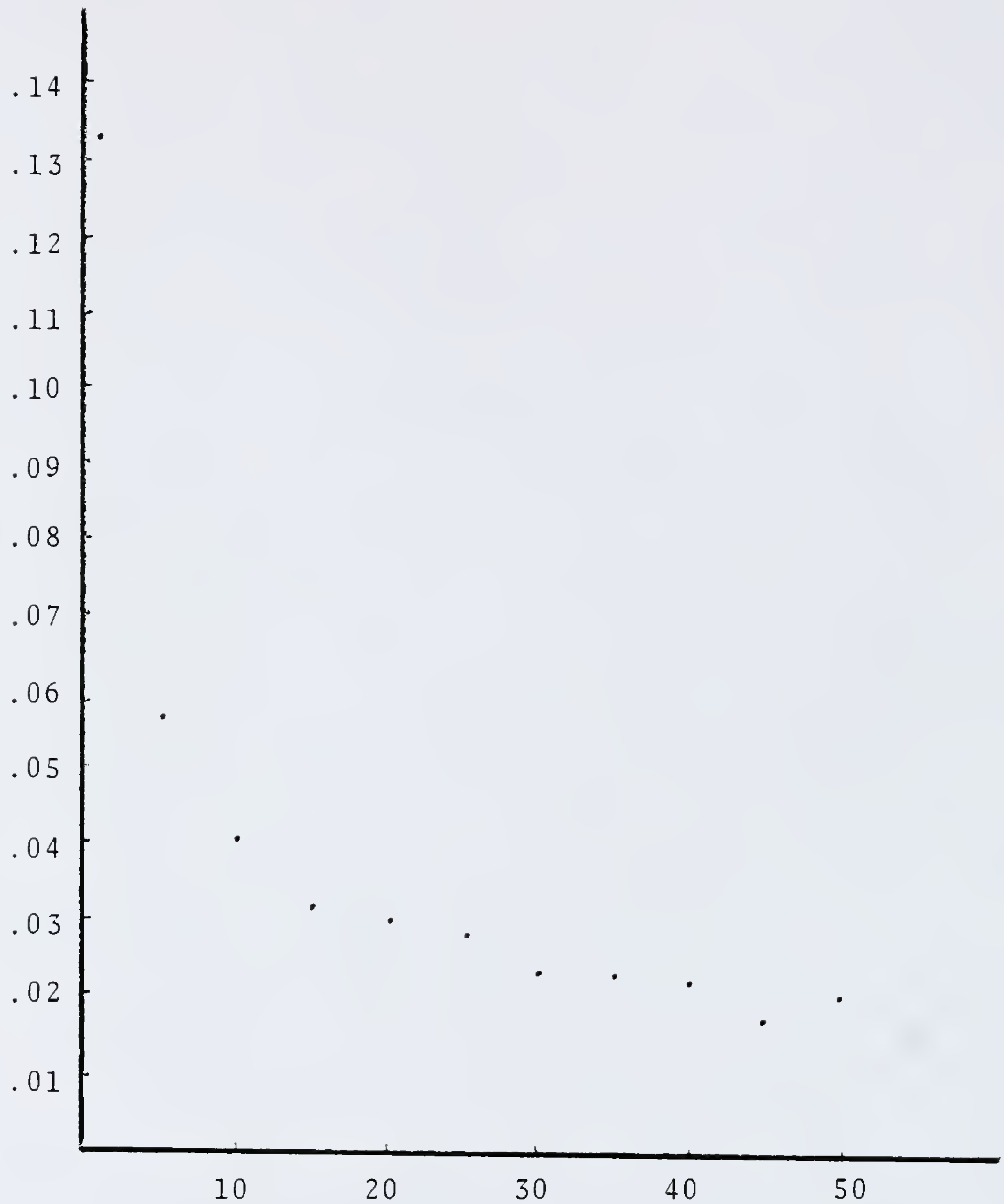
August 1974

Figure A-1



September 1974

Figure A-1

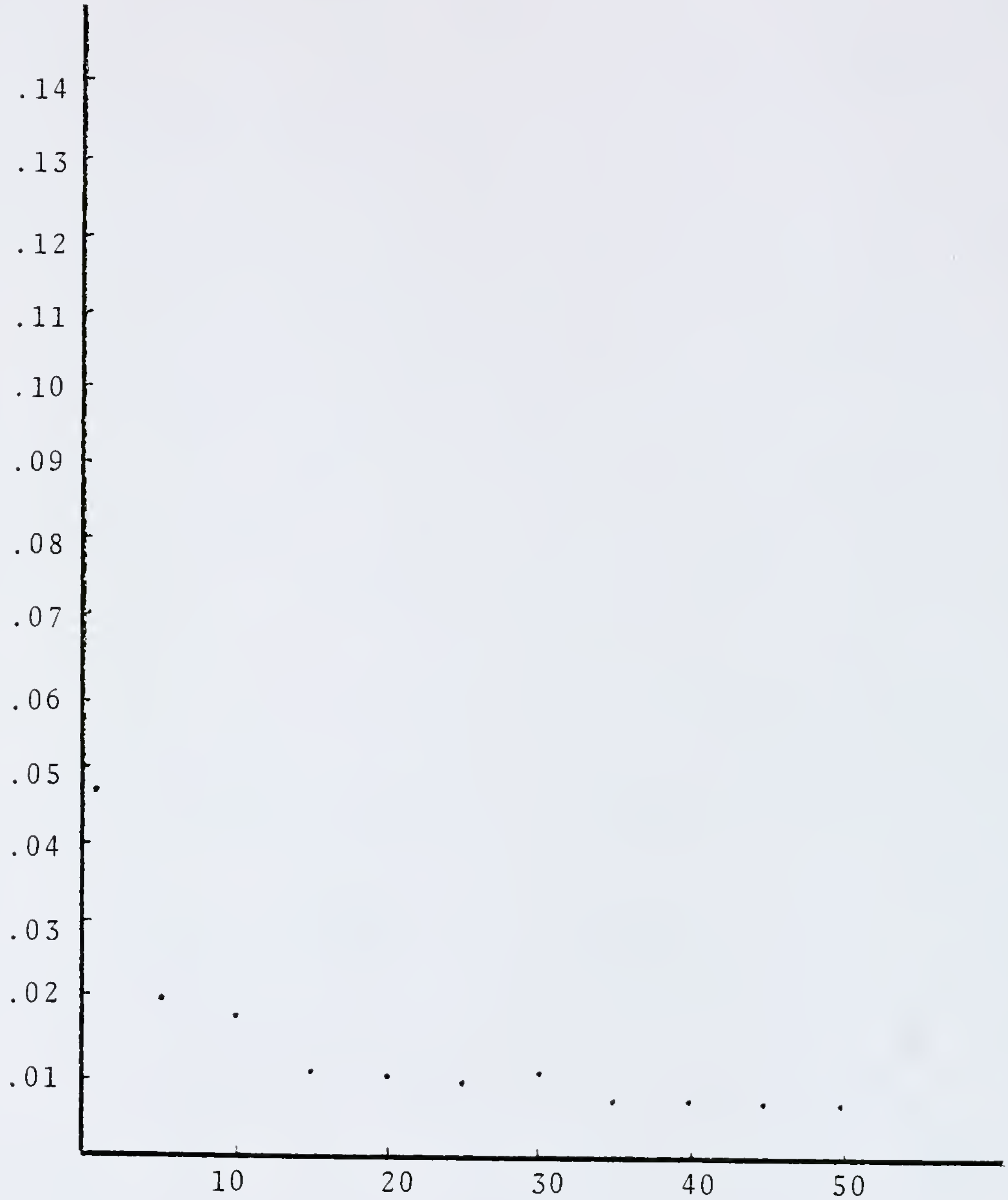


October 1974

Figure A-1

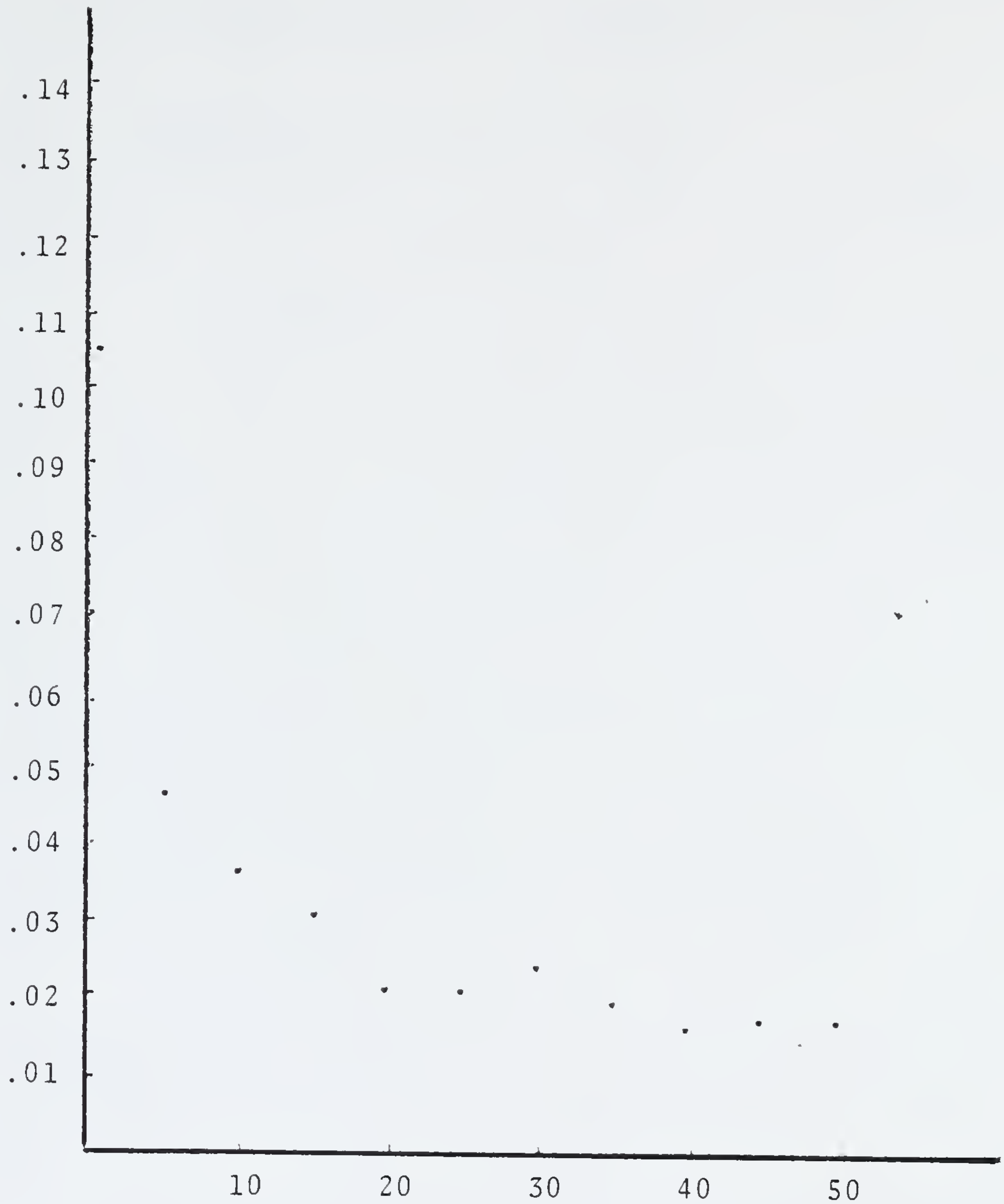
November 1974

Figure A-1



December 1974

Figure A-1

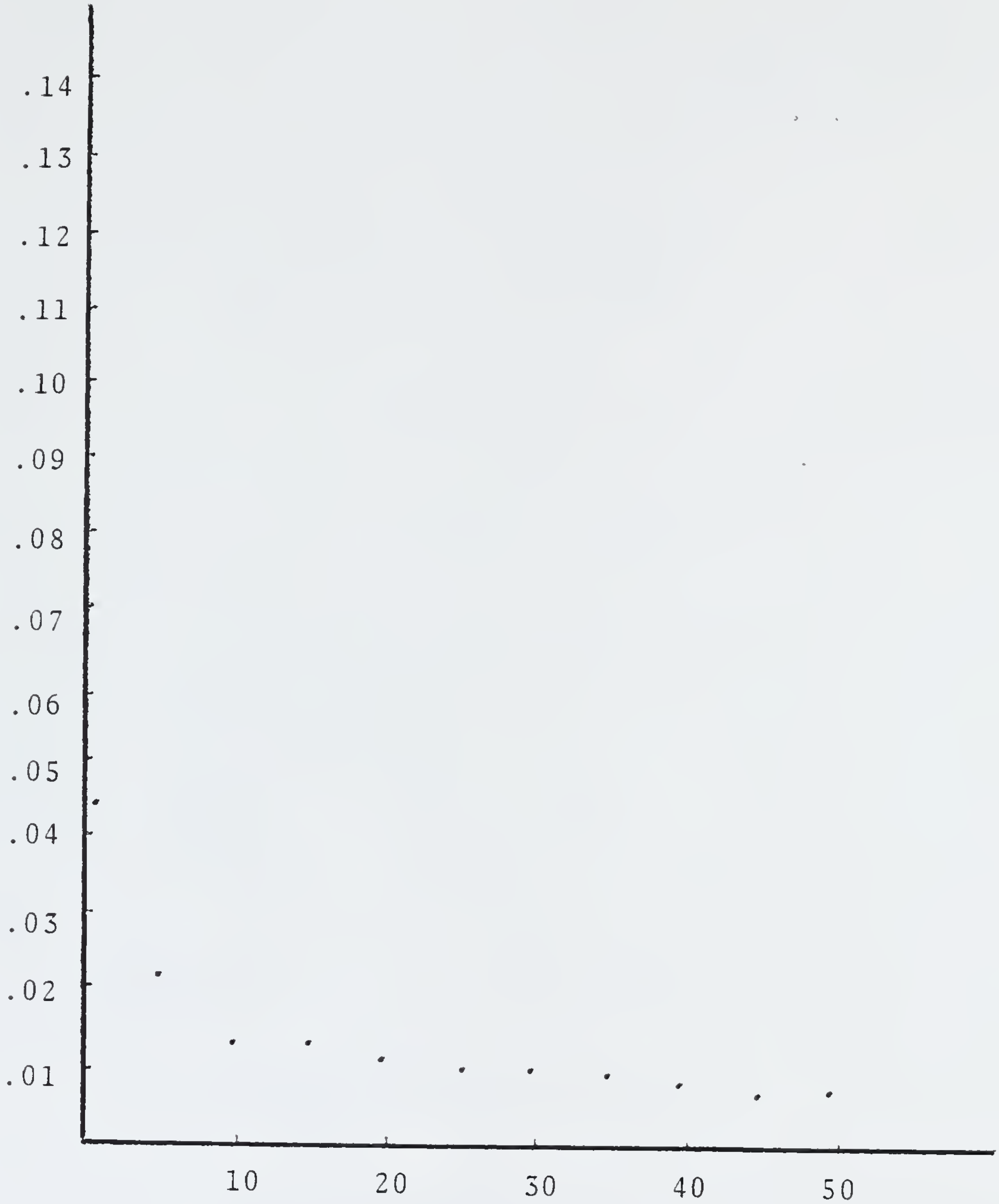


January 1975

Figure A-1

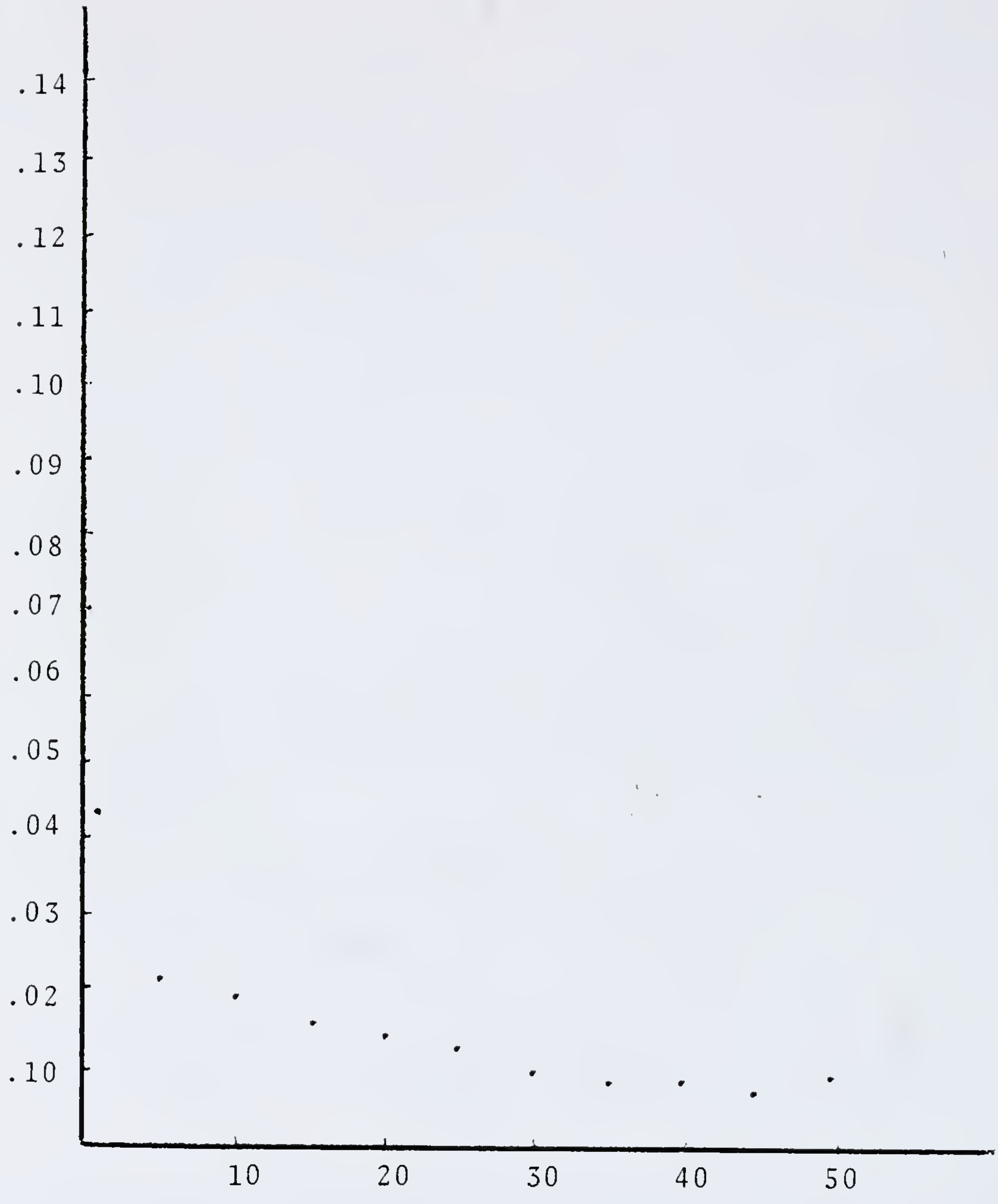
February 1975

Figure A-1



March 1975

Figure A-1

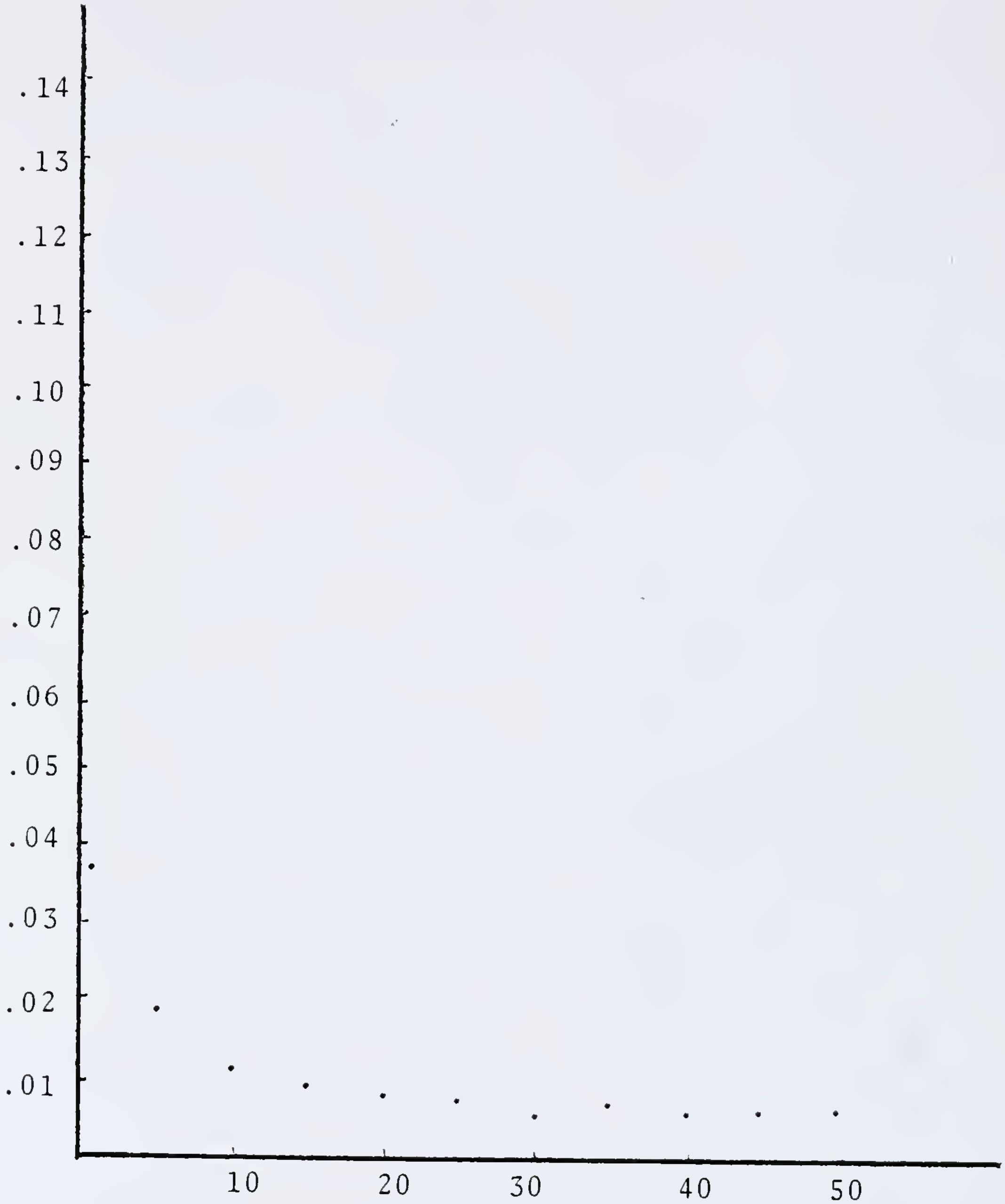


April 1975

Figure A-1

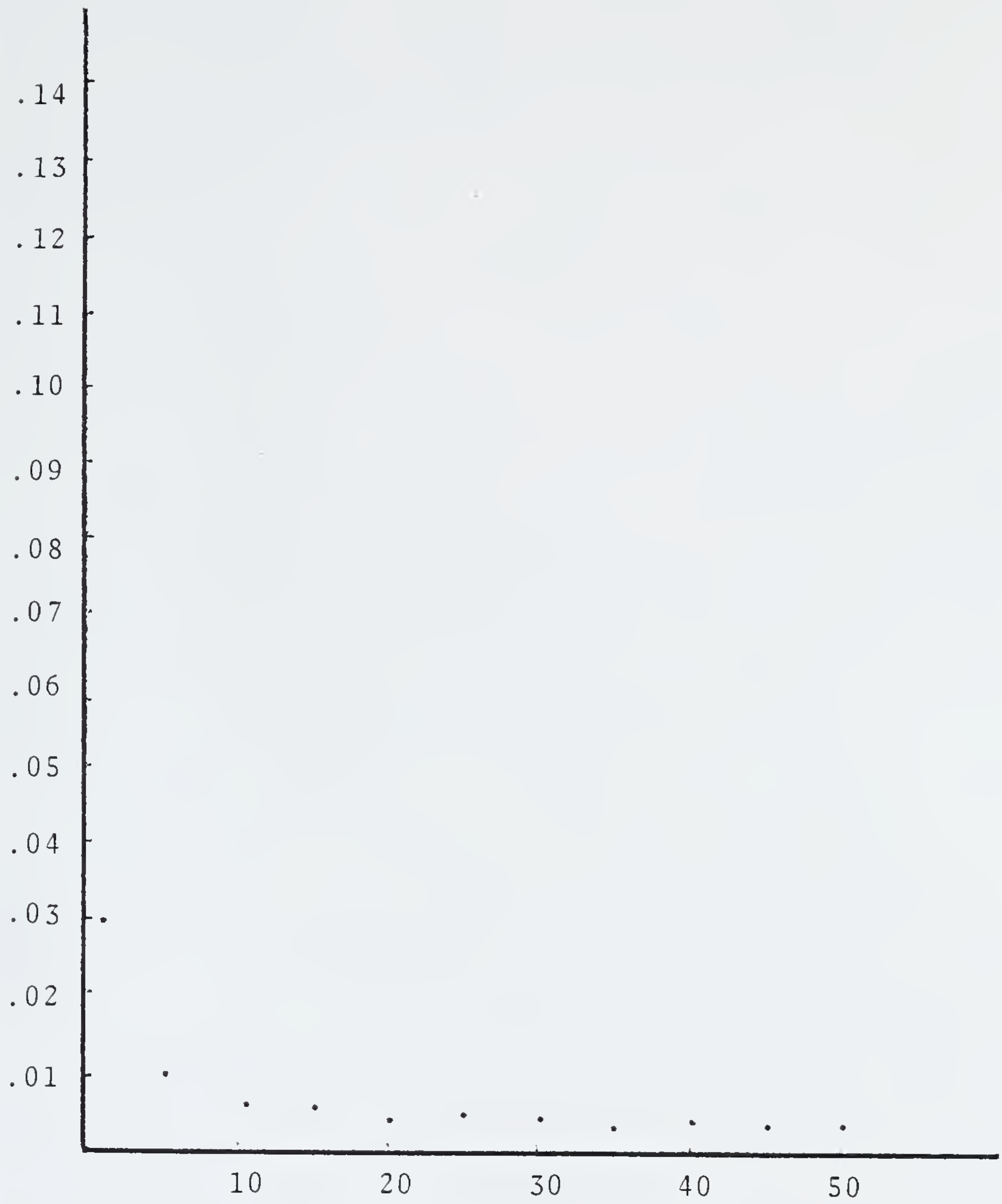
May 1975

Figure A-1



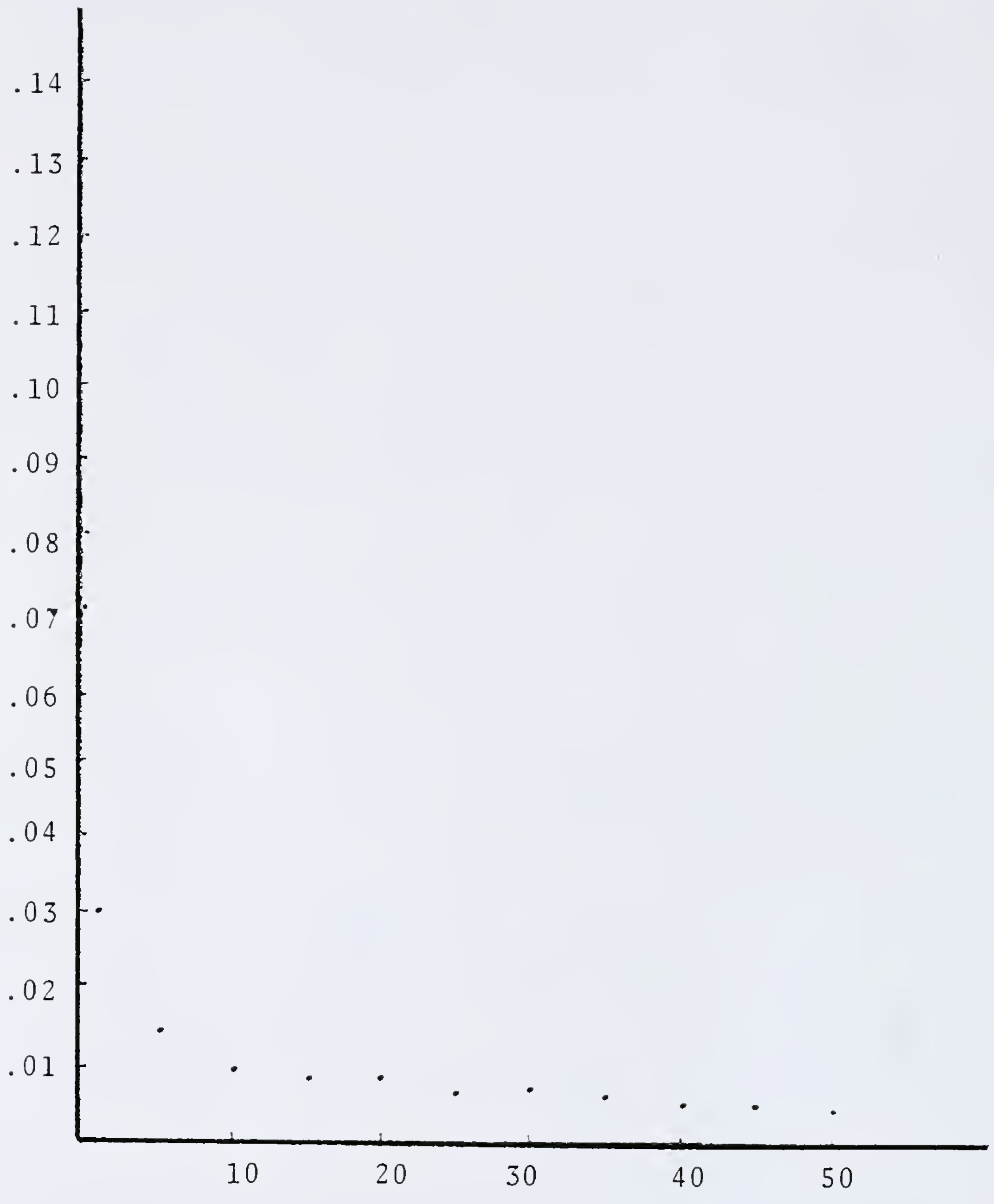
June 1975

Figure A-1



July 1975

Figure A-1



August 1975

Figure A-1

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Thomas J. O'Brien was born in Houston, Texas, on November 19, 1947. He attended St. John's School in Houston until he graduated in 1965. Mr. O'Brien attended Davidson College and graduated with an AB in economics in 1969. In 1972 he received an MBA from the University of Pennsylvania's Wharton School. In the interim of his MBA experience, Mr. O'Brien performed assignments at two southeastern commercial banks and completed Active Duty for Training on a 2nd Lieutenant in the U.S. Army.

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I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

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