A PATH OF DISCOVERY

A Life in Science, Engineering, and Agriculture

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Agricultural and Biological Engineering

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University of Florida
B.S., M.S., PhD (1965) Agricultural Engineering
North Carolina State University
Professional Societies:
American Chemical Society
American Institute of Chemical Engineers
American Society of Civil Engineers
Sigma Xi

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B.A. (1999) Anthropology
University of Florida
M.S. (2001) Library Science
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Professional Societies:
American Society for Engineering Education

Preface

This memoir is written as a collection of essays to illustrate a path of discovery in science, engineering, and agriculture. Note the emphasis on the term *a path* rather than *the path*, since there does not appear to be a single path to discovery. The key elements in this search appear to be *curiosity* and *creativity* – a curiosity about how nature works and a creative urge to modify the physical world to enhance human existence. These urges are evident in the human infant, and adults respond by supplying various needs and nurturing its development with a variety of devices and games.

The path described in this memoir contains two branches: (1) work on chemical transport follows a somewhat *reductionist* method of breaking a complex system into smaller parts which are in turn described by classical procedures of physics, chemistry, and mathematics; and (2) work on plant systems follows a somewhat *emergent* method more common in the biological sciences. The first could be called the bottom/up approach (assemble the pieces), while the second could be called the top/down approach (dissect the whole). The more recent work attempts to connect the two approaches together between plant tops and plant roots.

My goal in this memoir has been to provide more of an overview and less mathematical detail than in previous memoirs. I found this to be somewhat of a challenge, since I had been used to describing things in terms of mathematical language. Based on my reading of more popular books about mathematics, physics, chemistry, and biology I knew that it should be possible to write my subject in more popular terms as well. However, I found myself stumped with the process. So I decided to seek help from someone unfamiliar with the details of my work, but who appreciated the value of science and engineering to society.

I express my sincere gratitude to Amy Buhler, librarian at the Marston Science Library at the University of Florida, for her patience and persistence in working with me on this project. The final product reflects her dedication and determination to make this subject readable to anyone interested in learning about how science contributes to our knowledge of how nature works. My thanks also to Colleen Howard, graduate assistant in Agricultural and Biological Engineering at the University of Florida, for assistance with the figures in the document.

Readers interested in more details should consult the references in the essays.

Allen R. Overman
5 November 2009

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Chronology of Events in the Life of Allen R. Overman

(Numbers in parentheses are from list of collected works)

1937 Born March 11 to Elmer and Ruth Sullivan Overman in Wayne County, NC.

1941 My dog Dixie dies; first big loss.
Japanese bombed Pearl Harbor on December 7; dark Sunday.

1943 Began public school at Nahunta School in Wayne County, NC.

1945 US dropped atomic bombs on Japan.
WW II ended.

1947 Joined 4–H Club and entered first livestock show at Kinston, NC.
Read biography of Marie Curie.

1950 Best friend Bruce Bryant died of heart disease at Princeton, NC; second big loss.
US test of hydrogen bomb.

1951 Entered high school at Nahunta. Enroll in Vo-Ag course; bad experience with teacher; considered changing schools, but did not because of 4–H program.
Enrolled in tractor maintenance project in 4–H; met J. C. Ferguson, extension Agr. Engr.

1952 Grandfather Murray Allen Sullivan died at Kenly, NC; third big loss.
Met first girlfriend.

1954 Enrolled in chemistry and geometry; best courses in high school.
NC 4–H Swine Production winner, attended National Congress in Chicago.
Elected president of Nahunta 4–H and vice president of Wayne County 4–H.
Elected senior class president and 'Mr Chieftain' at Nahunta School.
Met William H. Danforth, founder of Purina Corp., at Wilson, NC for dedication of Purina Mill (attendance 19,000). Received Danforth Fellowship for leadership camp.

1955 Entered county livestock show; told by adult leader that I would find in life "it is not what you know, but who you know" that counts; show was stacked and winner of show predetermined.
Entered district livestock show; faired better; didn't give up due to support of parents.
Graduated from Nahunta High School. Voted most likely to succeed. Happy ending!
Attended American Youth Foundation Camp on Danforth Fellowship; received autographed copy of Danforth's *I Dare You*.
Submitted Achievement Report (160p.) for NC 4–H competition (#1 in collected works).
NC 4–H Achievement winner, attended National Congress in Chicago.

Albert Einstein died.
Entered NC State College in Agricultural Engineering.

1956 Attended Mount Olive College for one year to evaluate my direction. Elected vice president of student government and business manager for yearbook. Made many new friends. Formed gospel quartet for summer.

1957 Returned to NC State to continue in engineering.
Russians launched Sputnik. New era in American education in science and engineering.

1958 Deanye Lee and I became friends.

1959 Summer intern in Agricultural Engineering at NC State University.
Completed senior research project – valuable learning experience.
Received draft deferment to finish college.

1960 BS degree in Agricultural Engineering from NC State University.
Joined ASAE as student member. Optimistic about this profession.
Began MS program in Agricultural Engineering at NC State University.

1962 Attended International Trade Fair in Zagreb, Yugoslavia. Elected to *Sigma Xi* research society for science and engineering.

1963 MS degree in Agricultural Engineering from NC State University, minor in Math. Thesis on irrigation sprinkler to deliver a square pattern. 62p (#2 in collected works).

1964 Married Deanye Grace Lee on June 3. Complete coursework for PhD.

1965 PhD degree in Agricultural Engineering from NC State University, minor in Math. Dissertation on chemical transport by convective diffusion. 64p (#3 in collected works).
Began post-doc and faculty assignment in Agronomy at University of Illinois; great experience; audited courses in math, chemistry, soil physics, and soil fertility.

1966 Max (son) was born in August at Champaign-Urbana, IL.
Worked out solution to complex mathematical problem.

1967 Published first journal article on a multi-temperature bath in *Anal. Biochem. J.* (#4).

1968 Karen (daughter) was born in November at Champaign-Urbana, IL.
Published article on hydraulic conductivity measurements in *Soil Sci. Soc. Amer. Proc.* (#5). Published four articles on convective diffusion in *J. Phys. Chem.* (#6,7,8,9).
Declined offer of position at Colorado State University with USDA-ARS.

1969 Appointed Assistant Professor, Agricultural Engineering, University of Florida. New phase of career.
Began research on land application of dairy waste for crop production.

Taught advanced soil and water engineering course.
Supervised first student intern.

1970 Biological application of convective diffusion theory in *Plant Phys.* (#11).
Article on soil hydraulic conductivity with pressure scanning system in *Soil and Crop Sci. Soc. Fla. Proc.* (#12).
First article on land disposal of dairy waste (#13).
Began research on water reclamation/reuse with City of Tallahassee, FL. Combination of science, engineering, and agriculture.

1972 Exchange faculty summer assignment with Wilbur Mack at Florida A & M University. This facilitated summer research with City of Tallahassee.
First article in *Trans. ASAE* on hydraulic conductivity measurements (#17).
Organized First IFAS Workshop on Municipal Sewage Effluent.
Received USEPA grant ($90K) for research at Tallahassee.
Conducted research on rapid infiltration basins at Tallahassee.
Taught course on research methods in agricultural engineering.
Joined ASCE, nominated by E. T. Smerdon.

1973 Organized Second IFAS Workshop on Municipal Sewage Effluent.

1974 Promoted to Associate Professor of Agricultural Engineering.
Joe Douglas (Tallahassee treatment plant operator) died.

1975 First article in *J. Env. Engr. Div. ASCE* on crop irrigation with effluent (#27).

1976 Model of phosphorus transport in *J. Water Poll. Control Fed.* (#32).

1977 Three articles on phosphorus kinetics in *Water Res.* (#33,34,35).

1978 Conducted workshop on land treatment for USEPA Region IV in Atlanta.
Organized IFAS Workshop on Landspreading Effluent and Sludge in Florida.
Joined AIChE, nominated by Robert Walker.

1979 USEPA final report on wastewater irrigation at Tallahassee, FL.
Promoted to Professor of Agricultural Engineering.
Talk on groundwater recharge at ASCE meeting in Atlanta (#39).
Velocity dependence of phosphorus transport in *J. Water Poll. Control Fed.* (#44).

1981 Compiled data on crop response to applied nutrients for USEPA.
Jesse Lee (father-in-law) died at Goldsboro, NC.

1982 Soil and groundwater changes under land treatment of wastewater in *Trans. ASAE* (#50).

1983 Fourth article in *Water Res.* on phosphorus kinetics (#51).

1984 First article on modeling of crop growth in *J. Env. Engr. Div. ASCE* (#52).
USEPA final report on wastewater treatment by overland flow.

1985 First article on overland flow treatment of wastewater in *Trans. ASAE* (#55).

1986 Begin six volume series on crop modeling with Beth Angley (#59).
Feasibility of water reuse at Dunnellon, FL with Beth Angley (#60).

1987 Ruth Overman (mother) died at Gainesville, FL.

1988 Empirical model of crop growth in *Trans. ASAE* (#68, 69).
Process analysis of overland flow treatment in *Trans. ASAE* (#70).
Elmer Overman (father) died at Gainesville, FL.

1989 Phenomenological model of crop growth in *Agr. Sys.* (#73).
Operational characteristics of wastewater irrigation in *Appl. Engr. in Agr.* from Tallahassee studies (#80).

1990 First articles on plant nutrient concentration in *Fert. Res.* (#83, 84).
Logistic model in *Commun. Soil Sci. Plant Anal.* (#85).
Effects of water, harvest interval, and N on yields in *Agron. J.* (#90).
Appointed VC Continuing Education FS/ASAE.

1991 Multiple logistic model for NPK response of grass yields in *Agron. J.* (#91).
Conducted workshop on water reclamation/reuse for Florida Section ASAE.
Research bulletins on crop modeling (#93–95).
Irrigation of citrus with reclaimed water for Florida DEP (#96).
Conducted workshop on water reclamation/reuse for FS/ASAE. Collected $3500 in registration fees.
Elected VC Continuing Education for FS/ASAE.

1992 Yield response to applied nitrogen and clover in *Agron. J.* (#104).
Approach of yield curves to steady state in *Commun. Soil Sci. Plant Anal.* (#105).
Conducted session on Agricultural Engineering Services in Florida. Speakers from private and public sectors. Received criticism for program from faculty colleagues.
Elected VC Publicity for FS/ASAE.

1993 Design and analysis of center pivot irrigation with Sherry Allick (#110).
Worked with VC Programs FS/ASAE (Lennart Lindahl) in planning program.
Defeated for VC Programs FS/ASAE to abort my activities in Section.

1994 Extended logistic model of forage response to N in *Agron. J.* (#111).
Yield and nitrogen removal by corn in *Agron. J.* (#112).
Served on UF Task Force to recommend upgrade of water reclamation plant. Agreed to AWT plant with water reuse for landscape irrigation, including golf course.

1995 NPK model for plant nutrient uptake by forage grasses in *Trans. ASAE* (#116).
Rational basis of logistic model in *J. Plant Nutr.* (#117). Pointed out symmetry.
Model of coupling among applied, soil, root, and tops in *Commun. Soil Sci. Plant Anal.* (#118).
Results on water reclamation/reuse at Tallahassee in *Trans. ASAE* (#120).
Dry matter and nutrient accumulation with forage grass in *J. Plant Nutr.* (#124).
Forage response to harvest interval and reclaimed water in *J. Plant Nutr.* (#129).

1996 Personal memoir *OWL –Opportunity, Work, and Luck* (#131). Sent to 12 friends. My view probably too naïve; connections with proper people more important.
Three documents on population trends in Florida, US, world (#133,134,135).
Design and evaluation of center pivot at Gustafon's dairy with Scott Thourot (#136).
ASA symposium of "Use and Misuse of Crop Models" criticized my work as dead end; negative reviews from ASA and ASAE led me to drop out of ASA and end submission of manuscripts to ASAE; intolerance for independent ideas.
Elected VC Programs FS/ASAE (unopposed).

1997 VC Programs of Florida Section ASAE; history of Florida Section ASAE (#138).
Tried to decline succession to Chair FS/ASAE. Agreed to continue.

1998 Chair Florida Section ASAE. Joint meeting with ASAE. Uneventful year.
Expanded growth model for grasses in *Commun. Soil Sci. Plant Anal.* (#139).
First article on response of a vegetable to N in *HortScience* with John Willcutts (#141).
History of UF Agricultural & Biological Engineering Dept for 75th anniversary (#142). Chaired committee and edited report. Some not happy with result.
Tom Smith (Tallahassee sanitary engineer, utility director) died at Tallahassee, FL.
J. L. Morgan (Tallahassee contract farmer) died at Tallahassee, FL.
Grace Lee (mother-in-law) died at Pikeville, NC.

1999 Chair nominating committee Florida Section ASAE. Criticized for not having a UF faculty member on executive committee. None volunteered or nominated; full slate nominated by members.
Article on Langmuir-Hinshelwood model of soil P in *Commun. Soil Sci. Plant Anal.* (#144). Should have been submitted to *J. Physical Chemistry.*
Book chapter on models of crop growth and yield for Springer-Verlag (#146).
Allen Smajstrla (department colleague) died in Gainesville, FL; end of an era.

2000 Keynote talk on past in science, engineering, and agriculture at Florida Section ASAE (#149) at invitation of VC Programs (Brian McMahon).
Advised Grace Danao on UF honors project (outstanding work).
Received Distinguished Achievement Award, Florida Section ASAE.
Received Distinguished Service Award, Mount Olive College (NC).

2001 Mathematical theorem on growth model in *Commun. Soil Sci. Plant Anal.* (#151).
Model of coupling among applied, soil, and plant nutrients for vegetables in *Commun. Soil Sci. Plant Anal.* (#152).
Perspectives on future for engineers in *Resource* (#154).
Offered to chair 50th anniversary planning committee for FS/ASAE; offer declined. End of my active involvement in FS/ASAE.

2002 Corn response to irrigation and applied N in *Commun. Soil Sci. Plant Anal.* (#157).
Book *Mathematical Models of Crop Growth and Yield* by Marcel Dekker (#158); coauthored with Richard V. Scholtz.

2003 Defense of logistic model of crop yield in *Commun. Soil Sci. Plant Anal.* (#161).
Confirmation of expanded growth model in *Commun. Soil Sci. Plant Anal.* (#163).
Model of tops and roots in corn in *Commun. Soil Sci. Plant Anal.* (#164).
Model of tops and roots in bermudagrass in *Commun. Soil Sci. Plant Anal.* (#167,168).
Model analysis of overland treatment of swine waste in *Commun. Soil Sci. Plant Anal.* (#169,170).
NPK model for corn silage in *Commun. Soil Sci. Plant Anal.* (#175).
Model analysis of grass response to applied N and erosion in *Trans. ASAE* (#176).
Long-term performance of Tallahassee Southeast Farm (#177).
Water reclamation and reuse at Tallahassee–A personal perspective (#178).
Nominate Rush Choate for Golden Jubilee Award; last activity in FS/ASAE.

2004 Model of partitioning between leaf and stem in grass in *J. Plant Nutr.* (#180,181).
NPK model for bahiagrass in *J. Plant Nutr.* (#182,183).
Partitioning in alfalfa to *Commun. Soil Sci. Plant Anal.* (#185).
Model of plant population for corn to *Commun. Soil Sci. Plant Anal.* (#186).
Model of residual soil nitrogen on crop response. *Commun. Soil Sci. Plant Anal.* (#187).
Biomass partitioning in elephantgrass to *Commun. Soil Sci. Plant Anal.* (#188).
Residual and applied N to *Commun. Soil Sci. Plant Anal.* (#189; withdrawn).
Conflict with editors of *Trans. ASAE* over manuscript; insulted by president of ASAE; dropped out of ASAE after 43 years; victim of power struggle.
Awarded life membership in ASCE.

2005 Growth model for cotton to *Commun. Soil Sci. Plant Anal.* (#190; withdrawn).
Exponential model of corn population (#191; withdrawn).
Corn growth and applied nitrogen (#194; withdrawn).
Model of a broad-leaf crop (#195; withdrawn).
Biomass and plant nutrient accumulation in potato (#204; withdrawn).
Model of yield and plant P to applied P and extractable soil P (#205; withdrawn)
Mathematical characteristics of growth model (#206; withdrawn).
Simulation of biomass and plant N by perennial peanut (#208; withdrawn).
Corn response to population and row spacing (#209; withdrawn).

Orchardgrass response to applied N and clover (#215; withdrawn).
Corn response to population and solar radiation (#216; withdrawn).
Coupling among applied, soil, and plant nutrients for tall fescue (#219; withdrawn).
Model of yield response of spinach to applied N and S by spinach (#222; withdrawn).
Name change from ASAE to ASABE; end of another era. It became obvious that my productivity was too fast for journals; considered bad social policy.
Pursue scientific memoirs to provide comprehensive treatment of analytical models; documents to be deposited in UF library in hard copy and electronic format.

2006 Memoir on chemical transport with applications to soils and plants (#189).
Memoir on crop growth with focus on accumulation of biomass and mineral elements (#190).
Memoir on crop growth and nutrient uptake (#191).

2007 Memoir on model response to fertilizer and broiler litter (#194).
Memoir on model analysis of switchgrass response to applied nitrogen and calendar time (#195).
Memoir on mathematical models of crop growth and yield, miscellaneous applications (#196).

2008 Memoir on mathematical models of crop growth and yield, effect of geographical location (#197).
Memoir on mathematical models of crop growth and yield, statistical analysis of a complex system (#198).
Invited to join American Chemical Society.

2009 Memoirs criticized as not being peer reviewed. I am pleased with results (1320 pages).
Memoir on mathematical models of crop growth and yield, comparison of polynomial and logistic models (#199).
Work discredited by department colleague (anonymous). Don’t understand basis.
Memoir on ‘a path of discovery’ to be #10.
Agree to advise student for M.E. degree. Proceed cautiously because of department politics.

Graduate Students with Allen R. Overman

1973 Norman Lee-Zen Wang

1974 Quoc-Cuong Nguy

1976 Ro-Lan L. Chu

1976 Hsiao-Ching Ku

1976 Abdul A. Zakaria

1977 Brian R. McMahon

1980 Nariman E. Elfino

1984 Thomas Schanze

1985 David W. Wolfe

1986 David K. Ammerman

1987 Jeffrey F. Payne

1988 Richard H. Thompson

1989 Marcus N. Allhands

1991 Thomas C. Perry, Jr

1992 William R. Reck

1994 John F. Willcutts

1995 Denise M. Wilson

1997 C. Scott Thourot

1999 Richard V. Scholtz III

2002 Richard V. Scholtz III

2003 Kelly H. Brock

National Science Foundation (NSF) Scholars

1995 Denise M. Wilson

2002 Richard V. Scholtz III

2003 Kelly H. Brock

Student Interns with Allen R. Overman

1970 David G. Cobb

1976 Brian R. McMahon

1982 Dino M. Ricciardo

1983 Robert W. Burleson
Thomas Schanze

1984 Patrick V. Kirby
Jeffrey F. Payne

1986 Gary P. Bethune

1987 Richard H. Thompson

1988 Daniel Downey
Thomas C. Perry, Jr

1989 William R. Reck

1993 Sherry A. Allick*
Robert A. Horton

1996 C. Scott Thourot
Richard V. Scholtz III

2000 Kelly Brock
Grace Danao

2002 Sharon Brender

2009 Colleen Howard

* Work of Sherry Allick was submitted to City of Tallahassee as project report (written and oral format). Her 85 page report constituted the equivalent of a senior thesis.

COLLECTED WORKS

Allen R. Overman, PhD

Agricultural & Biological Engineering Department
University of Florida
Gainesville, FL 32611-0570

1. Overman, A. R. 1955. Achievement Record for NC 4–H competition. 160p.

2. Overman, A. R. 1963. Analysis of a Quadratic Irrigation Sprinkler. MS Thesis. Agricultural Engineering, NC State University. 62p.

3. Overman, A. R. 1965. Net Molecular Transfer in a Capillary with Diffusion and Laminar Flow. PhD Dissertation. Agricultural Engineering, NC State University. 64p.

4. Overman, A. R. and R. J. Miller. 1967. A multitemperature bath. *Anal. Biochem*. 19:195-197.

5. Overman, A. R., J. H. Peverly, and R. J. Miller. 1968. Hydraulic conductivity measurements with a pressure transducer. *Soil Sci. Soc. Amer. Proc*. 32:884-885.

6. Overman, A. R. and R. J. Miller. 1968. Convective diffusion in capillaries. *J. Phys. Chem.* 72:155-158.

7. Overman, A. R. 1968. Transient convective diffusion in capillaries. *J. Phys. Chem*. 72:3286-3288.

8. Overman, A. R. 1968. Convective diffusion across a porous diaphragm. *J. Phys. Chem*. 72:4168-4171.

9. Overman, A. R. 1968. Convective diffusion in clay systems. *Soil Sci. Soc. Amer. Proc*. 32:616-618.

10. Miller, R. J., A. R. Overman, and J. M. King. 1970. Non-linear water flow through sintered glass membranes. *Soil Sci*. 110:140-145.

11. Overman, A. R., G. H. Lorimer, and R. J. Miller. 1970. Diffusion and osmotic transfer in corn mitochondria. *Plant Physiol*. 45:126-132.

12. Cobb, D. G., A. R. Overman, and J. S. Rogers. 1970. Laboratory investigations of a pressure scanning system for soil water analysis. *Soil and Crop Sci. Soc. Fla. Proc*. 30:284-288.

13. Overman, A. R., C. C. Hortenstine, and J. M. Wing. 1970. Land disposal of dairy farm waste. Proc. Cornell Univ. Conf. on Waste Management. Rochester, NY. pp 123-126.

14. Hortenstine, C. C., J. M. Wing, and A. R. Overman. 1971. Wastewater renovation through soil and plants. *Env. Letters* 1:245-254.

15. Overman, A. R., C. C. Hortenstine, and J. M. Wing. 1971. Growth and response of plants under sprinkler irrigation with dairy waste. Proc. International Symp. on Livestock Wastes. Amer. Soc. Agr. Engr pp 331-337.

16. Overman, A. R., L. C. Hammond, and H. M. West. 1971. Comparison of measured and calculated unsaturated hydraulic conductivity for Lakeland fine sand. *Soil and Crop Sci. Soc. Fla. Proc*. 31:221-222.

17. Overman, A. R. and H. M. West. 1972. Measurement of unsaturated hydraulic conductivity by the constant outflow method. *Trans. Amer. Soc. Agr. Engr*. 15:1110-1111.

18. Harrison, D. S. and A. R. Overman. 1972. Handbook of irrigation tables and useful formulas. Agr. Engr. Report 72-12. Univ. of Florida. 17 p.

19. Overman, A. R., L. B. Baldwin, and L. C. Hammond (eds). 1972. Proc. IFAS Wastewater Workshop on Municipal Sewage Effluent. 271 p.

20. Overman, A. R. 1972. Research experience in Florida. Proc. IFAS Wastewater Workshop on Municipal Sewage Effluent. pp. 91-120.

21. Wang, L. and A. R. Overman. 1973. Field measurement of hydraulic conductivity of Lakeland fine sand. *Soil and Crop Sci. Soc. Fla. Proc*. 33:154-157.

22. Nguy, A. Q. and A. R. Overman. 1973. Comparison of electrode and optical methods for determination of ammonia in sewage effluent. *Soil and Crop Sci. Soc. Fla. Proc*. 33:168-169.

23. Overman, A. R. and H. L. Breland. 1973. Comparison of dry and wet oxidation for chemical analysis of digested sludge. *Soil and Crop Sci. Soc. Fla. Proc*. 33:174-176.

24. Overman, A. R., L. B. Baldwin, L. C. Hammond, and D. W. Jones. (eds). 1973. Proc. 1IFAS Workshop on Landspreading Municipal Effluent and Sludge. 319 p.

25. Overman, A. R. and T. P. Smith. 1973. Effluent irrigation at Tallahassee. Proc. IFAS Workshop on Landspreading Municipal Effluent and Sludge. pp 39-59.

26. Overman, A. R. and A. Zakariah. 1974. Steady-state water transport in sandy soil. *Soil and Crop Sci. Soc. Fla. Proc*. 34:4-6.

27. Overman, A. R. 1975. Effluent irrigation of pearl millet. *J. Env. Engr. Div. ASCE* 101:193-199.

28. Overman, A. R. and A. Nguy. 1975. Growth response and nutrient uptake by forage crops under effluent irrigation. *Commun. Soil Sci. and Plant Anal*. 6:81-93.

29. Overman, A. R. 1975. Effluent irrigation as a physicochemical hydrodynamic problem. *Florida Scientist* 38:215-222.

30. Overman, A. R. 1975. Ion transport through sand by convective diffusion. *Plant and Soil* 43:663-670.

31. Overman, A. R. and H. C. Ku. 1976. Effluent irrigation of rye and ryegrass. *J. Env. Engr. Div. ASCE* 102:475-483.

32. Overman, A. R., R. L. Chu, and W. G. Leseman. 1976. Phosphorus transport in a packed bed reactor. *J. Water Poll. Control Fed*. 48:880-888.

33. Overman, A. R. and R. L. Chu. 1977. A kinetic model of steady state phosphorus fixation in a batch reactor. I. Effect of soil/solution ratio. *Water Res*. 11:771-776.

34. Overman, A. R. and R. L. Chu. 1977. A kinetic model of steady state phosphorus fixation in a batch reactor. II. Effect of pH. *Water Res*. 11:777-778.

35. Overman, A. R. and R. L. Chu. 1977. A kinetic model of steady state phosphorus fixation in a batch reactor. III. Effect of solution reaction. *Water Res*. 11:779-781.

36. Overman, A. R., R. L. Chu, and Y. Le. 1978. Kinetic coefficients for phosphorus transport in packed-bed reactor. *J. Water Poll. Control Fed*. 50:1905-1910.

37. Overman, A. R. and L. E. Evans. 1978. Effluent irrigation of sorghum x sudangrass and kenaf. *J. Env. Engr. Div. ASCE* 104:1061-1066.

38. Overman, A. R. 1978. Land treatment seminar. Region IV, U. S. Environmental Protection Agency. Atlanta, GA. 69 p. Proc. IFAS Workshop on Landspreading Municipal Effluent and Sludge. 319 p.

39. Overman, A. R. 1979. Ground water recharge with municipal waste water. ASCE Meeting Paper No. 3741. 11 p.

40. Overman, A. R. 1979. Effluent irrigation of Coastal bermudagrass. *J. Env. Engr. Div. ASCE* 105:55-60.

41. Overman, A. R. 1979. Effluent irrigation at different frequencies. *J. Env. Engr. Div. ASCE* 105(3):535-545.

42. Overman, A. R. 1979. Wastewater irrigation at Tallahassee, Florida. U. S. Environmental Protection Agency. Ada, OK. EPA-600/2-79-151. 320 p.

43. Overman, A. R., B. R. McMahon, R. L. Chu, and F. C. Wang. 1980. Cation transport in a packed bed reactor of soil. *J. Env. Engr. Div. ASCE* 106:267-277.

44. Overman, A. R., B. R. McMahon, and R. L. Chu. 1980. Velocity dependence of phosphorus transport in a packed-bed reactor. *J. Water Poll. Control Fed*. 52:2471-2476.

45. Overman, A. R. 1980. Effect of parameters on cation transport in soil. ASAE Summer Meeting Paper No. 80-2025. 12 p.

46. Overman, A. R., B. R. McMahon, R. L. Chu, and F. C. Wang. 1980. Modeling of cation transport in soil-effect of velocity and ionic strength. Proc. Hydrologic modeling symposium. ASAE Publication 4-80. Amer. Soc. Agr. Engr. St. Joseph, MI. pp. 257-263.

47. Wang, F. C. and A. R. Overman. 1981. Impacts of surface drainage on ground water hydraulics. *Water Res. Bull*. 17:971-77.

48. Overman, A. R. 1981. Irrigation of corn with municipal effluent. *Trans. Amer. Soc. Agr. Engr*. 24(1):74-76.

49. Overman, A. R. 1981. Crop yields and nutrient uptake for predicting wastewater loadings in the Southeast. U. S. Environmental Protection Agency. Washington, DC. Project W-4205-NASX. 573 p.

50. Overman, A. R. and W. G. Leseman. 1982. Soil and groundwater changes under land treatment of wastewater. *Trans. Amer. Soc. Agr. Engr*. 25(2):381-387.

51. Overman, A. R., R. L. Chu, and B. R. McMahon. 1983. A kinetic model of steady state phosphorus fixation in a batch reactor. IV. Details of surface kinetics. *Water Res*. 17:15-19.

52. Overman, A. R. 1984. Estimating crop growth rate with land treatment. *J. Env. Engr. Div. ASCE* 110(5):1009-1012.

53. Overman, A. R. and T. Schanze. 1984. Overland flow treatment of municipal wastewater in Florida. U. S. Environmental Protection Agency. Ada, OK. EPA-600/S2-84-163. 182 p.

54. Overman, A. R. 1984. Land treatment of wastewater – an integral part of groundwater protection. Reuse and the Protection of Florida's Water: The Dilemma – The Challenge. The CH2M Hill Seminar Series. pp. 59-64.

55. Overman, A. R. and T. Schanze. 1985. Runoff water quality from wastewater irrigation. *Trans. Amer. Soc. Agr. Engr*. 28(5):1535-1538.

56. Overman, A. R. and D. W. Wolfe. 1986. Overland flow treatment of wastewater at Florida State Prison. *J. Water Poll. Control Fed*. 58:903-910.

57.Overman, A. R. 1986. Land treatment of sewage effluent in Florida. Florida Coop. Ext. Serv., Agr. Engr. Fact Sheet AE-27. 4 p.

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Essay on Science, Engineering, and Agriculture

My education can be divided into four phases: (1) public school, (2) college, (3) post-doc, and (4) professional. A brief description of each phase follows.

The public school phase all occurred in a single facility of twelve grades (1–12) at Nahunta School in Wayne County, North Carolina. Total enrollment was approximately 300 students. I joined the 4–H Club at age nine and was very active over the next 10 years, becoming a state winner twice (in livestock production and achievement), which allowed me to attend the National 4–H Congress in Chicago. At age ten I participated in my first livestock show. Senior year I served as local club president and vice president of the Wayne County Council. I was also county winner in public speaking. In October, 1954 I was awarded a Danforth Fellowship to attend the American Youth Foundation Leadership Training camp in Muskegon, Michigan. A part of the award was a copy of the little red book *I Dare You!* (Danforth, 1953), autographed by William H. Danforth himself. He challenged me to get ten other young people to read and sign the book, which I dutifully did over several years. Meeting him again at the camp was a unique experience since he died at age 85 on 25 December 1955. The experience of working with extension 4–H agents Bill Lancaster, Clyde Peedin, and Bill Lamb was a real highlight of those early years. By the way, 4–H (symbolized by the four leaf clover) stands for head, heart, hands, and health. Around the age of twelve I read my first biography of a scientist (Madame Curie) and found it most inspiring. This was followed later by the classic book *Silas Marner* (Eliot, 1996), from which I learned the value of compassion over gold. My best courses in high school were chemistry and geometry, both conducted by excellent teachers. These courses illustrated order in the physical world.

My next phase came in 1955 when I entered college. In high school I had met a faculty member (J.C. Ferguson) of the Agricultural Engineering Department at North Carolina State College. As a result I chose this field for a college major and career. It seemed like a good bridge between science and agriculture. A scholarship also helped with finances. One highlight of the program was working as a summer intern with a faculty member and a PhD student on a mechanization project. During this time I read the book *Origins of Modern Science* (Butterfield, 1957), which I found most enlightening about science. It was in 1957 that the Russians launched the *Sputnik* orbital satellite. This triggered a surge in focus and funding on science and engineering in the USA. It was definitely the place to be! Following a BS degree I was offered an assistantship for graduate study. Course focus was on math, physics, and chemistry. I chose an engineering project for the MS thesis. This led to further interest in science and more math, physics, and chemistry. I chose to work with a soil physical chemist (Raymond J. Miller) for the PhD thesis. My appointment in Agricultural Engineering included teaching in the water area, both lectures and labs, which was good preparation for the future.

Next I followed a path which was common in the sciences but virtually unheard of in my field – a post-doc appointment. I turned down a faculty position to continue work with Professor Miller at the University of Illinois in Agronomy. This proved to be an excellent choice. It allowed me to continue our research interests on chemical transport in porous media and to audit advanced courses in math, chemistry, soil physics, and soil fertility. These were to prove most useful in my professional career. Several journal articles came out of this work including three in *J. Physical Chemistry*. I engaged in interesting conversations with faculty about soil physics and soil chemistry. It was also during this period that I attended a seminar by the eminent physical chemist Henry Eyring. It proved to be one of the most inspiring lectures that I have ever attended, and I was to learn much more about his work. I acquired several books about science and scientists during this period.

I finally entered my professional career in Agricultural Engineering at the University of Florida. All of the work which I had conducted provided a solid foundation for teaching, research, and application. My first teaching assignment was Advanced Soil & Water Engineering, which focused on the physics and engineering of water flow in porous media. While on the books, this course had not been taught as a regular graduate course. This was followed by a course in Research Methods, which I later converted to Applied Mathematics in Agricultural Engineering. It covered both ordinary and partial differential equations and included both analytical and numerical methods. Various applications were covered, including regression analysis (linear and nonlinear) and statistics (error analysis). Over the years I also covered special topics (chemical transport, water reclamation/reuse, and advanced topics). For ten years I assumed responsibility for the undergraduate course on soil & water engineering (lecture and lab).

From the beginning I was assigned undergraduate interns and graduate students. My first research projects focused on land application of dairy waste for crop production and water quality of a large Florida lake. The dairy waste project lasted for three years and was a cooperative effort with a soil chemist and a dairy scientist. Several papers were published on chemical processes in the soil and on crop response to applied nutrients. Background in soil chemistry and soil fertility proved very useful. In my second year at UF I initiated a cooperative project with Tom Smith, utility engineer with the City of Tallahassee, on water reclamation/reuse. Support of a grant from the U.S. Environmental Protection Agency allowed us to study various irrigation systems (fixed risers, big guns, and center pivot), loading rates, crop types (hay, pasture, grain), soil characteristics (physical and chemical), and groundwater quality. The studies included field measurements, laboratory procedures, and mathematical models. This work continued for 35 years, and included many committee assignments (engineering analysis and design, farmer selection, and farm management). Research included development of models of chemical processes (particularly phosphorus kinetics and transport) and of crop growth and yield in response to various management factors (applied nutrients, water availability, crop type, soil characteristics, and environmental conditions). Results were presented through conferences, workshops, and journal articles. The work with Tom Smith (engineer), Bill Leseman (chemist), and Walter Vidak and J. L. Morgan (farmers) enhanced my interest in engineering, science, and agriculture. I provided input to the landscape architect at Walt Disney World for their reuse system. Other projects were conducted on overland flow treatment and rapid infiltration basins (Orlando Conserv II and Palm Coast). Input was provided to a number of design engineers in Florida.

Throughout the years I have maintained an interest in the work and lives of people in science. I can particularly recommend the memoirs of Richard Feynman (1985a, 1985b, 1986), a biography of Rachel Carson (Lear, 1997), and the collection of essays by Abraham Pais (Pais, 2000) about physicists and mathematicians from Niels Bohr to Eugene Wigner. Perhaps the writer who has done the most to popularize science was Isaac Asimov (cf. Asimov, 1992).

It is now convenient to divide the essay into sections on science, engineering, and agriculture.

Science

For purposes of this discussion science will consist of the subjects of mathematics, physics, chemistry, and biology.

Mathematics This subject can be divided several ways. It includes arithmetic (counting), algebra (relations), geometry (shapes), calculus (differential and integral), differential equations (ordinary and partial), group theory, topology, etc. It can also be described as *the study of patterns* (Devlin, 1994; Rucker, 1987). While it is considered that calculus was invented by Newton and Liebniz, the first standard texts in the modern sense were written by Euler (Varadarajan, 2006).

Physics One definition might be *the study of matter and energy.* It certainly involves the application of mathematics. An excellent overview of physics has been given by one of the masters of the subject (Einstein and Infeld, 1966), first published in 1938. Other summaries have been published as well (Gamow, 1961; Segré, 1980). In the early 1600s Galileo published a popular treatise on inertia and gravity (Galilei and Crew. 1914). A brief history of physics can be found in a recent book (Newton, 2007). A more technical treatment has also been published (Longair, 1984). A popular treatment of the mysteries of quantum mechanics is worth reading (Gribbin, 1995). A number of books have been published about the lives and work of individual physicists, such as James Clerk Maxwell (Everitt, 1975), Ludwig Boltzmann (Lindley, 2000), Albert Einstein (Pais, 1982), and Richard Feyman (Mehra, 1994; Sykes, 1994).

Chemistry A simple definition might be *the study of atoms and the compounds they form.* Histories of chemistry can be found in two books (Jaffe, 1976; Partington, 1989). The pioneer application of physics to chemistry has been demonstrated in the work of Pauling (Pauling and Wilson, 1985) and Eyring (Glasstone et al., 1941). Many books have been written about the lives and work of individual chemists, such as Marie Curie (Quinn, 1995) and Henry Eyring (Eyring, 2007). And the list would not be complete without the lectures of Michael Faraday at the Royal Institution during 1860-1861 for children on the chemistry of the candle (Faraday, 1904).

Biology This subject is focused on *the study of living things*, such as plants, animals, and microorganisms. A definitive treatment has been published (Mayr, 1997). A personal perspective has been given by one of the pioneers in the subject (Luria, 1973). Concerns about the impact of chemicals on biological organisms were very well addressed by Rachel Carson (Carson, 1962). Linkage between physics and biology was addressed in Erwin Schrödinger's classic book of 1944 (Schrödinger, 1992). Application of physics and chemistry to the field of biology has been addressed in several books (Johnson et al., 1974; Morton, 2007; Watson, 1980).

Engineering

Engineering involves (1) analysis, (2) design, (3) construction, (4) operation, and (5) evaluation of systems for the *production of goods and services for society*. Perhaps no one has done more to explain engineering to the public than Henry Petroski through his classic book (Petroski, 1992) and corresponding BBC documentary. The rewards of engineering have been described by the civil engineering contractor Samuel Florman (Florman, 1976).

Agriculture

Agriculture involves the *production of food and fiber for society*. A cultural perspective on agriculture has been provided in the classic book and documentary of Jacob Bronowski *The Ascent of Man* (Bronowski, 1973). An account of the political struggles of development of agriculture in the American west has also been published (Worster, 1985).

Inspiring Quotes

“Science is not holy scripture, nor do its practitioners consider themselves priests protecting a glittering grail, forever unchanging and pure. What drives scientists on is the thirst to *understand* more than to *use* nature, to build rather than to exploit a comprehensible universe.” (Newton, 1993, p. 234)

“Humanity will be much better off when the reward structure is altered so that selection pressures on careers favor the sorting out of information as well as its acquisition.” (Gell-Mann, 1994, p. 343)

“For a successful technology, reality must take precedence over public relations, for nature cannot be fooled.” (Richard Feynman as quoted in Sykes, 1994, p. 218)

“Trying to be useful and failing at it is the major source of human discontent, driving some of us crazy.” (Lewis Thomas as quoted in Judson, 1987, p. xix)

“I think it was Liebniz who described philosophy as the discipline in which you kick up a lot of dust and then complain you can’t see. That’s an attitude which many scientists share.” (Regis, 1987, p. 224)

“A model takes on the quality of theory when it abstracts from the raw data the facts that its inventor perceives to be fundamental and controlling, and puts these into relation with each other in ways that were not understood before – thereby generating predictions of surprising new facts.” (Judson, 1987, p. 229)

“The virtue of modeling, if it is good modeling, is that it nourishes physical intuition and provides a useful basis for organizing, interpreting, and communicating numerical results.” (Treiman, 1999, p. 162)

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CREATIVITY versus CONFORMITY

Allen R. Overman
(10/30/03 – 6/30/09)

This is a collection of thoughts about the battle between creativity and conformity. I have been caught in this conflict and I am choosing the former strategy. There is a price to be paid, but I believe that progress in science and service to society is better served by this choice. The choice is to either bow to authority and conform or stand on principle. There are arguments to support both sides.

I start with a quote from Mark Twain concerning the conflict over the age of the earth, which was raging during the late 19th century with Charles Darwin and Thomas Huxley on one side and Lord Kelvin and Peter Guthrie Tait on the other side:

> *It takes a long time to prepare a world for man, such a thing is not done in a day. Some of the great scientists, carefully ciphering the evidences furnished by geology, have arrived at the conviction that our world is prodigiously old, and they may be right, but Lord Kelvin is not of their opinion. He takes the cautious, conservative view, in order to be on the safe side, and feels sure it is not so old as they think. As Lord Kelvin is the highest authority in science now living, I think we must yield to him and accept his view. He does not concede that the world is more than a hundred million years old. He believes it is that old, but not older.*
> (Burchfield, 1975, p. ix)

Frederick Soddy, a collaborator of Ernest Rutherford on studies of radioactivity, was quite critical of Tait and Kelvin for "restraining the legitimate aspirations of science" by their dogmatism (Burchfield, 1975, p. 169). Kelvin had not taken account of the vast amount of heat released within the earth from transmutation of elements (Lightman, 2000, p. 54, 105). Energy release from this conversion was made explicit through Einstein's famous equation, $E = mc^2$. Further confirmation on the age of the universe has come from the work of Edwin Hubble on the expansion of the universe, which leads to estimates of 10 to 20 billion years old. So much for the rule of authority in resolving the conflict between Kelvin and Huxley.

Perhaps the most famous case in science of conflict with authority is that of Galileo (Gamow, 1988, p. 49). He was forced to kneel before the cardinals of the Church and recant the teaching of the heliocentric theory of Copernicus, which conflicted with the earth centered model of Ptolemy embraced by the Church as official doctrine. Eventually the model of Copernicus prevailed as a result of work done by Galileo and Johannes Kepler.

There are other examples of conflict with authority. The proposal of the ionization theory by Svante Arrhenius met with stiff opposition from several established professors (Jaffee, 1976, p. 164). Fortunately, his ideas were published thanks to the help of Wilhelm Ostwald through his new journal *Zeitschrift für Physikalische Chemie*. Another example is that of the bitter rejection of Boltzmann's ideas about atoms by Ernst Mach (Lindley, 2001). Mach cast a long shadow over physics during the late 19th century. Boltzmann's ideas eventually caught on and led to the establishment of statistical mechanics. A further example is the vigorous attack of Peter Guthrie Tait on the formalism of vector analysis developed by Josiah Willard Gibbs (Kramer, 1982, p. 349). In 1893 Gibbs offered a plea for tolerance in a letter to *Nature*, and by 1900 he had won the battle in favor of vector analysis. Another example relates to the speed of light through space.

Jean-Dominique Cassini had observed that one of the moons of Jupiter seemed to get off schedule, sometimes ahead of schedule and sometimes behind. Ole Roemer was hired to help resolve the question. Cassini, by now a world authority on Jupiter, had concluded that light travels instantaneously through space. Roemer concluded that it travels with a finite speed and accurately predicted a new observation, providing an estimate of the speed which agrees closely with the accepted value today of 670,000,000 mph. Rather than concede the point, Cassini discredited Roemer's work. As a result, Roemer quit science in disgust. Fifty years later Roemer's idea was confirmed and he now is credited with the first estimate of the speed of light (Bodanis, 2000, p. 43). Another famous case derives from quantum theory. In 1925 Uhlenbeck and Goudsmit concluded that the electron has intrinsic spin which is quantized. When they heard of Pauli's objection to the idea they had serious doubts about publication. However, Ehrenfest had already submitted the manuscript to a journal. Later Pauli's objection was shown to be unjustified. So Uhlenbeck and Goudsmit became famous for their discovery (Segrè, 1980, p. 141).

Much has been written about numerical simulation and its implications. A paper entitled "Verification, Validation, and Confirmation of Numerical Models in the Earth Sciences" appeared in the February 1994 issue of *Science* journal. The authors warned that verification and validation of numerical models of natural systems is impossible (Horgan, 1996, pp. 201-203). The issue of verification and validation is also discussed by Ford (1999, pp. 284-288). Long ago Quetelet warned that "Collecting data in order to support a cause already committed to, or a course of action already decided upon, is fraught with danger" (Devlin, 1998, p. 293). Dangers of excessive conformity to established ways of thinking has been pointed out (Bodanis, 2000, p. 178). The uncertainty of models has been noted by various scientists. To quote Richard Feynman: "That is the same with all our laws – they are not exact. There is always an edge of mystery, always a place where we have some fiddling around to do yet. This may or may not be a property of Nature, but it certainly is common to all the laws as we know them today" (Gleick, 1992, p. 365). The subject has also been addressed by James Watson: "No *good* model ever accounted for all the facts, since some data was bound to be misleading if not plain wrong" (Mackay, 1991, p. 256).

I prefer to think of validation as noted by Segrè (1980, p. 99): "We will mention it later, but here too, Einstein's contribution was very important, especially in his role of validating new and surprising ideas." In other words, validation should focus on ideas and concepts (relationships), not on numbers. Einstein also engaged in battles over general relativity. David Hilbert, a world figure in mathematics, was trying to figure out a proper formulation of the theory. He asked Einstein for a copy of an unpublished manuscript. After examining its contents, he adopted a procedure which Einstein had worked out, and proceeded to publish the correct version. So he appeared in print first, which led to a bitter dispute over priority. This was finally resolved in Einstein's favor in 1997 (Aczel, 1999, p. 114). After studying Einstein's general relativity, George Lemaître concluded that the universe must be dynamic, either contracting or expanding. Einstein rejected this idea, even suggesting to Lemaître that his mathematics was not very good. However, when Einstein met with Edwin Hubble to view Hubble's evidence of an expanding universe, Einstein conceded that Lemaître had been correct. This led to what Fred Hoyle later labeled the "big bang" theory of the universe (Filkin, 1997). Evidence again that even the greatest minds are not infallible.

In my view it is a mistake for editors of *Transactions ASAE* to impose the authority of self-appointed "experts" on authors through anonymous review and then to demand conformity as a condition of publication. Such experts often have a vested interest in what is published, particularly when it conflicts with their own brand of modeling. While I consider myself an expert in mathematical modeling of crop growth and yields, the editors have chosen to ignore the content and references within the manuscripts and to reject publication on the narrow issue of validation.

For an extensive account of creativity in science I strongly recommend the book *Springs of Scientific Creativity* (Aris et al., 1988). The value of tolerance of alternative ideas and interpretations in science is underscored by the four versions of quantum mechanics. The debate was heated and intense at times (Gribbin, 1995, pp. 145-146, 219). Einstein and Bohr argued the interpretation of quantum mechanics until Einstein died in 1955. Through it all they always held each other in high regard but without compromising intellectual integrity. In the end Gribbin concludes that it is best to buy the lot and learn from each version. Heinz Pagels (1982) echoes this same perspective.

I now cite a few more references in support of tolerance of a variety of ideas in science. Roger Newton has this to say: “It is, above all, because no theory fits Nature precisely that there is room for a variety of theories to account for a large body of facts” (Newton, 1997, p. 211). This book also explores the search for truth in science in great detail, and is highly recommended for the interested reader. Further insight is given by Roger Jones: “The wave function of an electron does not represent the physical electron at all. Instead, it is a source of information about an electron – a kind of mathematical almanac (Jones, 1992, pp. 158-159). My final example is the eminent theoretical physicist John Wheeler: he divides his career into three phases – phase 1 “Everything is Particles”, phase 2 “Everything is Fields”, and phase 3 “Everything is Information” (Wheeler, 1998, pp. 63-64). Perhaps what we are modeling in these efforts is information, not a physical system directly. Another quote which I like: “As the work of Newton and Einstein exemplifies, scientific breakthroughs are sometimes born of a single scientist’s staggering genius, pure and simple. But that’s rare. Much more frequently, great breakthroughs represent the collective effort of many scientists, each building on the insights of others to accomplish what no individual could have achieved in isolation. One scientist might contribute an idea that sets a colleague thinking, which leads to an observation that reveals an unexpected relationship that inspires an important advance, which starts anew the cycle of discovery. Broad knowledge, technical facility, flexibility of thought, openness to unanticipated connections, immersion in the free flow of ideas worldwide, hard work, and significant luck are all critical parts of scientific discovery.” (Greene, 2004, p. 339). Isaac Asimov views scientific law as generalization rather than as eternal truth, and points out that science is a cumulative process over time and the result of many contributors (Asimov, 1991).

A contrast between philosophical views is that between Albert Einstein and Niels Bohr (Ruhla, 1993, p. 2). Einstein belonged to the school of *realism*, which holds that a mathematical model is the image of something which really exists. Bohr was more of a *positivist*, for whom a mathematical model represents relations between observable quantities. Both schools have contributed to the progress of science. Einstein frequently started from a simple premise: ‘A theory is the more impressive the greater the simplicity of its premises, the more different kinds of things it relates, and the more extended its area of applicability.’ (Bowley and Sanchez, 1999, p. 1). Einstein underwent a profound change of heart in his attitude toward science as the result of the inspiration and insight that led to his general theory of relativity. His youthful advocacy of the empiricist position (the laws of science can logically be deduced simply from the observation of natural phenomena) eventually gave way to his more mature faith in intuition as the source of scientific theories. As he states it, “Physical concepts are free creations of the human mind, and are not, however it may seem, uniquely determined by the external world.”’ (Jones, 1992, p. 217). The special theory of relativity was developed from an appreciation for the theories of Galileo and Maxwell, not from rejecting them outright (Rovelli, 2004, p. 416-417).

In my attempts at descriptions of physical systems I have used five key concepts which have served me well:

1. Patterns – usually a graph of data
2. Relationships – equations to describe the patterns (mathematical models)
3. Connections – coupling of different components of the system
4. Consistency – among data sets and within a model
5. Mathematical beauty – such as symmetry in equations

These have been drawn from reading of science and mathematics, and form the core of my creative efforts in science and engineering. Bowing to the authority of “experts” in my experience is not the road to creativity. To quote Einstein: “Desire for approval and recognition is a healthy motive; but the desire to be acknowledged as better, stronger, and more intelligent than a fellow being or fellow scholar easily leads to an excessively egoistic psychological adjustment, which may become injurious for the individual and for the community.” (Lightman, 2000, p. 118). Two examples of mathematical beauty are particularly noteworthy. The first has been labeled the most famous equation in all of mathematics and was discovered by the mathematician Euler in the 1700s, viz. $e^{i\pi} + 1 = 0$, where e = $2.7182\cdots$, $i = \sqrt{-1}$, $\pi = 3.1415\cdots$. Note that it combines the five fundamental constants e, i, π, 1, and 0 into a single relationship. The second example has been labeled the most famous equation in all of physics and was discovered by Einstein in 1905, viz. $E = mc^2$, where E is the energy contained in an element of matter, m, and c is the speed of light in a vacuum. This established the equivalence of matter and energy.

Kaku lists three modes of thinking by great physicists (Kaku and Thompson, 1995, p. 85): Hideki Yakawa’s mode – deeply rooted in experimental data; Paul Dirac’s mode – wild speculative leap in mathematical logic; Yoichiro Nambu’s mode – combination of the other two.

Some additional thoughts have been shared by Smolin (2001, p. 216, 219, 223) about the struggle of ideas and personalities in the field of science.

I end this essay with a particular example of the conflict between creativity and conformity. It deals with the property of mathematical symmetry in the logistic model of crop response to applied nutrients (Overman, 1995). This mathematical property has proven very important in classical, quantum, and particle physics (Barrow, 1994, p. 161; Pagels, 1985, p. 188; Rothman, 1972, p. 99; Rothman, 1985, p. 222; and Weyl, 1952). Emmy Noether had established a famous theorem (Mehra, 1994, p. 132) which showed that symmetry in a dynamical system implies that something is conserved (energy, momentum, charge, spin, etc.). If this principle applied to the logistic model, then what was conserved? I searched in vain for the answer. It happened that on 29 April 1994 the eminent theoretical physicist Murray Gell-Mann was in Gainesville, FL for an award and was to appear at a local bookstore to autograph his autobiography (Gell-Mann, 1994). In his autograph to my copy of his book he wrote: “To Allen, A theory is symmetrical under an operation if the equations are same before and after the operation is performed. So there.” Murray Gell-Man. With this boost in confidence, I succeeded in identifying the conserved quantity. I promptly submitted an article to a journal for review. A reviewer summarily crossed out that section as meaningless. As a result I withdrew the article and published it elsewhere (Overman, 1995). In this case it was correct to stand on principle and refuse to delete the material. Symmetry *was* significant for my case.

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Sun(S) – Earth(E) – Environment(E) – Plant(P) – Integrated System(IS)

Preface

This memoir is written for the reader who is curious about the natural world, and who has an interest in the science which describes how nature works. It naturally includes some mathematics since this is the language of science. The reader interested in the mathematical details is directed to the references at the end of the document. We take as the goal of science the advance of knowledge and understanding of how nature works. How it *does* work, not how it *must* work by decree of some authority.

There has been development of ideas in science over the centuries from Aristotle to Ptolemy to Galileo and Kepler to Newton to Euler to Hamilton to Maxwell to Einstein and Planck to Schrödinger and Dirac to Feynman and Gell-Mann etc. In agriculture controlled field experiments have been conducted over 150 years (beginning about 1850) on crop response to various inputs, leading to a large database of information covering a wide range of soil types, crop species, and environmental conditions. It is this combination of concepts and data upon which the work covered in this document has been developed.

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Introduction

We should begin by explaining the acronym chosen for this essay. *SEEPIS* stands for

Sun(S) – Earth(E) – Environment(E) – Plant(P) – Integrated System(IS)

Sun provides the source of solar energy which drives the process of photosynthesis in plants. Earth provides the medium for plant roots and a reservoir for water and mineral elements essential to plant growth. Environment regulates the atmosphere which controls temperature and carbon dioxide concentration. Plant is the center of accumulation of biomass by photosynthesis and uptake of mineral elements from the soil. Integrated system refers to integration over space and over time by which the fundamental physical, chemical, and biological processes come together to form a system. It also refers to the approach used here by which mathematical models are used to describe system response to various inputs.

My journey begins with courses in high school in chemistry and mathematics. Chemistry illustrated order in the physical world by way of the periodic table of the elements. This was based on the *atomic hypothesis* that matter is composed of fundamental building blocks called atoms, and that these can combine to form various compounds. Mathematics illustrated order in the abstract world of algebra and geometry, by way of relationships and shapes. In college these subjects came together in a course on *analytical geometry* (Rider, 1949), which used Descartes' method of plotting graphs to show coupling between two variables x and y. This appears to have been a starting point for Isaac Newton's development of the calculus (Berlinski, 2000). A course in physical chemistry introduced the *kinetic theory of matter*, which assumes that these atoms are in a state of motion (Paul, 1962). The kinetic theory of gases was established in the 19th century by Rudolph Clausius and extended by James Clerk Maxwell and Ludwig Boltzmann. Since the number of atoms were too numerous to track individually, Maxwell introduced an element of probability to describe the distribution of molecular speeds in a gas. This work provided an explanation for Boyle's law of ideal gases, and of rate processes such as viscosity, chemical diffusion, and thermal diffusion.

Attention is first focused on research which began in 1962 as PhD studies at North Carolina State University and has continued until the present.

Part I. Chemical Transport through Porous Media

A. Convective Diffusion

This section treats the net flow of a chemical species (the solute) dissolved in water (the solvent) under the combined processes of convection (flow of the solvent) and diffusion (concentration gradient of the solute), commonly referred to as *convective diffusion*. At low flow rates it is assumed that the two processes can be simply added in a linear manner. At higher flow rates this simple addition may not be valid due to coupling between the processes.

The simplest geometry is that of a straight uniform circular capillary (Figure 1). In this case flow of the solvent is assumed to follow Poiseuille's law, which leads to a parabolic velocity distribution across the radius of the capillary. Chemical diffusion of the solute is assumed to follow Fick's law. More complex geometry occurs in the case of porous membranes and a soil column. It is then necessary to make some approximations about the mathematical description of the two processes.

1. Capillary Tube

The case was studied in detail by G. I. Taylor (1953) using a dye as solute. I chose to focus on a similar problem at very low flow rates for my PhD research (Overman, 1965). In this case it appears reasonable to replace the parabolic velocity distribution by a simple average flow velocity if the diameter of the capillary is very small compared to its length and the flow velocity is very slow. Net flow of solute can then be described by simple addition of convection and chemical diffusion along the length of the capillary. This leads to a differential equation which is second order in space and first order in time. Boundary conditions at the two ends of the capillary and an initial distribution of solute within the capillary are required to solve the equation.

The research in 1965 proved to be rather challenging. A precision bore glass capillary of 1 mm diameter and 10 cm length was chosen for the experiments (Figure 2). A glass bulb of approximately 8 cm^3 was attached to the capillary. A vertical capillary was attached to the other side of the bulb for support and to allow flow through the horizontal capillary. To achieve displacement of the contents of the horizontal capillary in 10 days required a steady flow rate of approximately 0.01 cm^3/day. Search of the literature on micro syringes and syringe pumps failed to lead to any solution along those lines. The answer turned out to be to mount the flow cell in a 40 L reservoir and to drain the reservoir at the required rate. However, observations showed that the meniscus in the vertical capillary did not fall at a steady rate. Cleaning of the unit with a dilute solution of HF acid solved this problem. The reservoir was mounted in a thermostat controlled at 30.0 ºC to within $\pm$ 0.005 ºC. For the solute deuterium oxide (D_2O) was chosen. A stock solution of 4.00% D_2O was prepared. This provided boundary conditions for the horizontal capillary of $C(x = 0) = 4.00\%$ and $C(x = 10 \text{ cm}) = 0.00\%$. The initial concentration in the capillary was $C(t = 0) = 4.00\%$. Concentrations of D_2O with time were determined with a density gradient column.

a. Steady State

The transport equation for convective diffusion was solved for the boundary and initial conditions stated above. Focus was first on the steady state case, since it is mathematically simpler than the transient case. This led to an exponential distribution of concentration with distance in the capillary (Overman and Miller, 1972). It was noted that the mathematical solution exhibited symmetry around a flow velocity of positive and negative values. This was my first direct encounter with the principle of symmetry in mathematical physics. Since we could only measure average concentration in the flow capillary, it was necessary to integrate the mathematical solution over the length of the capillary. This led to the well-known Langevin function from mathematical physics. The symmetry principle appeared again in the solution. Measurements at a flow velocity of 0.920 cm/day led to average concentration of 3.08% compared to 3.06% predicted by the theory.

b. Transient

The transient solution of the transport equation for convective diffusion led to an infinite series of sine harmonics with position in the capillary, which converged to the steady state distribution with time (Overman, 1972a). Average concentration followed an infinite series of exponential terms, which converged to the Langevin function with time. Measurements of average concentrations at times of 1, 2, 4, 6, 10, 14, and 16 days showed agreement with theory within 2%. The simple transport model for low flow velocities appeared to be verified for capillaries.

2. Porous Membrane

The next step was to test the simplified transport model for a porous membrane with a tortuous flow path. It is assumed that the membrane is uniform and very thin, so that the volume of retention of solute is very small compared to that of the reservoir adjacent to the membrane. For the case of the glass membrane it is assumed that the volume of the reservoir is constant, whereas that of the biological membrane volume can change with time due to swelling and contraction.

a. Glass Membrane

In this case the membrane consisted of a Büchner funnel with a fritted glass filter 5 micron nominal pore size, 20 mm diameter, and 2 mm thickness (Figure 3). The solute was KCl. Concentrations in the upper chamber were determined from measurements of electrical conductivity. Stirring in the end chambers was achieved with magnetic bars. The cell was placed in a thermostat controlled to within ± 0.02ºC at 25 ºC. Results are described by Overman (1972b).

Distribution of solute concentration across the membrane was assumed to be the same as for the capillary. Solution of the transport equation for convective diffusion leads to an exponential change in concentration with time in the upper chamber, with a time constant which depends on a dimensionless transport parameter (Peclet number). A diffusion experiment was conducted first to establish the time constant for diffusion. Exponential change in concentration in the upper chamber was confirmed. Flow experiments were conducted at transport parameters of –1.38, –2.71, and –7.41. Results confirmed the simple convective diffusion model for the glass membrane.

b. Biological Membrane

The theory of convective diffusion has also been applied to a biological membrane. Overman et al. (1970) have described transport of water and ions in corn mitochondria. Mitochondria are a component of plant and animal living cells. Swelling and contraction occurs in response to changes in ionic concentrations outside the membrane, which induces transfer of ions and water across the membrane. Ionic diffusion is described by Fick's law of diffusion and water moves across the membrane to maintain osmotic balance in accord with the van't Hoff equation. Swelling and contraction was measured by optical transmittance. Chemical diffusion was shown to be the rate limiting step, while water flow occurred to maintain osmotic balance.

3. Sand Column

The theory was next applied to ion transport through a column of inert sand (Overman, 1975). Ottowa sand was packed into a plastic cylinder of 1.0 cm sample length and 11.04 cm^2 area to a uniform bulk density of 1.81 g soil/cm^3 and porosity of 0.314 (Figure 4). An upper chamber of the cell consisted of a volume of 13.0 cm^3, where solute concentration was measured with time by electrical conductivity. Concentration was maintained constant in the lower chamber. Stirring in the end chambers was achieved with magnetic bars (fleas) rotating at 30 rpm. The cell was mounted in a thermostat controlled to $\pm$ 0.02 ºC at 30.0 ºC. Water flow through the column was controlled with a syringe pump and a leak-free syringe. Solute consisted of KCl.

The mathematical complexity of this system was due to the variable boundary condition in the upper chamber. Solution of the transport equation was achieved with the method of Laplace transforms. The steady state component of the solution proved to be rather straight forward, consisting of a simple exponential distribution involving the transport parameter (the Peclet number). In contrast, the transient solution contained an infinite series of sine harmonics.

A simple diffusion experiment (at zero flow rate) was first performed in order to estimate the system diffusion coefficient, which is needed to estimate the Peclet numbers for the flow experiments. Experiments were then conducted at pore velocities of 0.50, 0.98, 1.92, 2.68, and 3.76 cm/day. Measurements of electrical conductivity were continued until steady state was achieved. Analysis of the data confirmed the theory using the effective diffusion coefficient of D = 0.477 cm^2/day for all flow rates. It follows that linear addition of convection and diffusion was valid for this system.

B. Convective Diffusion and Ion Exchange

Convective diffusion theory was expanded to include ion exchange between solution and surface phases of the system. This is particularly important for chemical transport in soil, including cations and anions. Cations of interest include NH_4^{+1}, K^{+1}, Ca^{+2}, and Mg^{+2}, while anions include $H_2PO_4^{-1}$ and OH^{-1}. Kinetic models of ion exchange are combined with transport models to describe overall rate processes of transport through soil.

1. Cation Transport through Soil

The first step is to write a kinetic model of cation exchange between solution and soil particles. To simplify the analysis the focus is on exchange between the two monovalent cations NH_4^{+1} and K^{+1}. Since these two exhibit equal ionic mobility, this will avoid the necessity of diffusion potential in the model. The kinetic model of Heister and Vermeulen (1952) is adopted. Reversible adsorption and desorption is assumed and is described by overall second order kinetics. For the two cations of interest it is reasonable to assume that the coefficients of adsorption and desorption are equal. Measurements in a batch reactor show that ion exchange is extremely rapid. The cation exchange term is inserted into the equation for convective diffusion to provide an overall transport model.

Experiments were conducted to test the validity of the transport model (Overman et al., 1980). Lakeland fine sand was packed into a packed-bed reactor of 10 cm length and 4.8 cm diameter to a uniform bulk density of 1.66 g/cm^3 and porosity of 0.376. Steady flows of feed solution were provided by a peristaltic pump with speed control. Discrete samples of outflow were collected with a fraction collector. Samples were analyzed for NH_4^{+1}, K^{+1}, and Cl^{-1}. Experiments were conducted by switching between NH_4Cl and KCl of the same molar concentration and at pH = 6.5. Due to the nonlinear nature of the transport equation, it was necessary to use finite difference procedures for mathematical solution. This imposed a maximum size of time steps in order to maintain numerical stability.

The effects of flow velocity and ionic strength (feed concentration) on system response were evaluated. In all cases the theory provided excellent simulation of the concentration vs. time. Experiments were first conducted at pore velocities of 0.147, 0.297, and 0.588 cm/min. and feed concentration of 0.01 moles/liter. Results showed that the diffusion coefficient (D) exhibited quadratic dependence on pore velocity (v), consistent with findings of other investigators. In contrast, the exchange coefficient (k) showed linear dependence on flow velocity. This showed clear coupling of the rate coefficients with flow velocity for flow rates employed in this study. It follows that the diffusion coefficient is enhanced by flow, and is generally referred to as the *dispersion coefficient*. Similarly, the exchange coefficient is referred to as a *global coefficient* rather than a true kinetic coefficient (Smith, 1970). Since the batch experiment showed that kinetics of exchange at the particle surface was very fast, it was concluded that ion exchange in the packed bed reactor was limited by diffusion of ions from the bulk solution to the particle surface. Experiments were next conducted at feed concentrations of 0.0453, 0.0098, 0.0050, and 0.0013 moles/liter and a pore velocity of 0.13 cm/min. Results showed that the exchange coefficient was strongly dependent on ionic strength of the solution. This effect is believed due to compression of the electric double layer around soil colloids with increasing ionic strength. The speed of concentration response with elapsed time was greatly enhanced at higher feed concentrations.

Additional studies were conducted with a three cation system (K^{+1}, NH_4^{+1}, Na^{+1}) to evaluate response at constant ionic strength and various feed concentrations (Elfino, 1980). This brought out the complexity of the three cation system.

2. Phosphorus Transport through Soil

   a. Chemical Kinetics in a Batch Reactor

It is well known that phosphorus applied to soil tends to become unavailable for plant uptake with time, which means that a slow reaction takes place with some component in the soil. Batch studies have shown that rapid adsorption of solution phosphorus occurs onto soil particles and that this adsorption is reversible. Such systems can be described by *Langmuir-Hinshelwood kinetics* (Laidler, 1965; Moore and Pearson, 1981). This describes reversible adsorption followed by an irreversible reaction of the adsorbed species. In fact this process is similar to the Briggs-Haldane model of enzyme kinetics (Briggs and Haldane, 1925). Since adsorption of phosphorus onto soil colloids is believed to follow overall second order kinetics between molecules in solution and sites for adsorption, the resulting differential equation is nonlinear and not amenable to analytical solution. The model contains four parameters, viz. an adsorption coefficient (k_a), a desorption coefficient (k_d), a reaction coefficient (k_r), and the total quantity of sites for adsorption per unit mass of soil (S_0). It is no small task to calibrate this model for a particular set of data. Experimental design involves adding a certain mass of soil to fixed volume of solution, M g soil/liter solution. The solution contains an initial concentration of phosphorus, P_0 mg P/liter.

The soil suspension is stirred at a sufficient rate to maintain soil particles in suspension. It is also important to control pH since the process is pH dependent.

Overman and Chu (1977a, b) conducted a series of batch experiments with Lakeland fine sand to clarify the kinetics of the process (Figure 5). It was noted that upon addition of soil to the phosphorus solution the pH controller immediately began to inject acid (H^{+1} ions) into the reactor to maintain a fixed pH. It was concluded that OH^{-1} ions were being released from the soil particles. Since phosphoric acid disassociates ($H_3PO_4 \rightarrow H_2PO_4^{-1} + HPO_4^{-2} + PO_4^{-3}$) into various ions in water, an immediate question arises as to the relevant species in this case. After much thought and discussion, it was decided to design an *open batch reactor* in which phosphorus could be injected at a steady rate to drive the overall reaction to steady state. This simplified the mathematics considerably, and reduced the system of differential equations to a set of algebraic equations. Steady state concentration of solution phosphorus then related to rate of injection of phosphorus (r) by a simple hyperbolic equation similar to the Michaelis-Menten equation of enzyme kinetics. Analysis showed that the adsorption sites served as a catalyst for the reaction, and that the order of the rate coefficients followed $k_a > k_d >> k_r$. This explained the rapid adsorption followed by a slow reaction. By running the experiment at different pH values it was shown that the relevant species was the monovalent anion $H_2PO_4^{-1}$, and that adsorption involved anion exchange between $H_2PO_4^{-1}$ and OH^{-1} (Kelly and Midgley, 1943). Soil phosphorus chemistry could now be treated as a case of heterogeneous catalysis. In a later publication Overman and Scholtz (1999) used the model to describe phosphorus kinetics for a *closed batch reactor*. A detailed procedure for numerical simulation was outlined for this case.

A small point should be noted about dissociation of water into its ionic components $H_2O \leftrightarrow H^{+1} + OH^{-1}$. The more accurate representation of the hydrogen ion H^{+1} is given by the hydrated form H_3O^{+1}, generally labeled the 'hydronium' ion (Kavanau, 1964).

b. Transport in a Packed Bed Reactor

The kinetic model was incorporated into a transport equation to describe phosphorus chemistry in a packed bed reactor of soil. Since the concentration of sites for adsorption (S_0) is large compared to the concentration of phosphate ions in solution (P_0) the kinetic model can be reduced to first order adsorption, which reduces the transport equation to a linear differential equation. Laplace transforms can be used to obtain an analytical solution for the system.

Experiments were conducted with Lakeland fine sand packed in a reactor of 10 cm length and 4.7 cm diameter to a uniform bulk density of 1.73 g/cm^3 and porosity of 0.343 (Figure 6). Flow experiments with pore velocities of 0.118, 0.256, 0.539, and 0.900 cm/min. and feed concentration of approximately 10 mg P/liter were carried out until steady state concentrations were achieved (Overman et al., 1976). Steady flow rates were obtained with a peristaltic pump. Sampling ports were installed in the column at 2, 4, 6, and 8 cm positions.

Kinetics of adsorption (k_a) and desorption (k_d) were assumed to be fast compared to the reaction (k_r) so that adsorbed and solution phases were assumed to proceed in virtual equilibrium. The surface reaction was assumed to be proportional to adsorbed concentration and to follow first order kinetics. The steady state solution of the transport equation predicted an exponential distribution of phosphorus concentration with position for each flow rate. Results conformed to this prediction very closely. However, the overall rate coefficient (k) did not turn out to be constant but appeared to follow hyperbolic dependence on flow velocity (v). The model described transient data rather well. The diffusion coefficient (D) did exhibit quadratic dependence on flow velocity, as was shown for cation transport. It was shown that the dimensionless quantity $4kD/v^2 << 1$ applied to this system, which simplified the mathematics considerably. Another surprise was that the exchange coefficient ($R = k_a/k_d$) showed dependence on flow velocity. Since k was related to k_r and R by the relationship $k = k_rR$, this raised the possibility that k_r, k_a, and k_d were all dependent on flow velocity. These results suggested that kinetics of the system were more complicated than had been assumed.

Now the kinetic model was rewritten without the assumption of virtual equilibrium between surface and solution phases (Overman et al., 1978). In this case the diffusion term in the transport equation was neglected to facilitate solution by Laplace transforms. The revised model again predicted exponential distribution of phosphorus concentration at steady state. It further showed that all three rate coefficients (k_a, k_d, k_r) exhibited dependence on flow velocity. In fact the dependence appeared to follow hyperbolic equations. This led to further modification of the kinetic model to better explain experimental results (Overman et al., 1980; 1983). It was hypothesized that the adsorbed phase had to undergo reversible reconfiguration before reaction could occur, which introduced forward (k_f) and backward (k_b) rate coefficients. In addition to the reaction coefficient (k_r) it was assumed that the reconfigured species could break down directly to the solution species with rate coefficient (k_s). Analysis of the packed bed data showed that four coefficients (k_a, k_d, k_f, and k_b) could be treated as constant, while two coefficients (k_r and k_s) followed hyperbolic dependence on flow velocity (v). This model explained experimental results rather well. It was clear that $k_s \to 0$ as $v \to 0$ must hold to avoid the conclusion of perpetual motion of the chemical sequence. In fact it was concluded that the flow of liquid past the particle surface induced shear force and detachment of the reconfigured species from the surface.

Field measurements were made in Tallahassee, FL on Lakeland fine sand at the water reclamation and reuse project with treated wastewater to confirm the model. Following steady irrigation with reclaimed water for three days, measurements of soil solution phosphorus concentrations in the upper 120 cm of soil showed exponential distribution (Overman et al., 1976). Field results agreed closely with those of the packed bed reactor for a similar pore water velocity. Measurements by a student intern Sherry Allick at the Southeast Farm after 12 years of irrigation showed that extractable soil phosphorus followed an exponential distribution in the upper 150 cm of the soil (Allhands et al., 1995).

While the model of phosphorus transport described experimental results satisfactorily, it was clear that the complexity of the model had exceeded understanding of the chemistry of the system. It was decided to end this line of research. Frustration with soil phosphorus chemistry had been well expressed by a University of Illinois colleague (Kurtz, 1981).

C. Water Reclamation and Reuse by Land Application

During the 1970s the U.S. Environmental Protection Agency (USEPA) developed programs to encourage beneficial recycling and reuse of reclaimed wastewater by land application. The technology was divided into three categories: (1) slow rate irrigation, (2) overland flow, and (3) rapid infiltration basins. Slow rate irrigation applied to soils suitable for conventional irrigation and crop production. Overland flow applied to soils with low permeability in which water tended to run off the site, and also included a crop for ground cover. Rapid infiltration applied to soils with high permeability and geology suitable for the method. All three methods have been studied and implemented in Florida.

1. Slow Rate Irrigation

The City of Tallahassee, FL pioneered application of this method of land treatment of reclaimed water in 1966 under the leadership of Thomas P. Smith. The two of us began cooperative studies in 1970. Field and laboratory studies were designed to evaluate yield and nutrient uptake by crops, changes in soil characteristics, impact on groundwater, and types of irrigation systems. The studies began on a 16-acre experimental site, expanded to a 1,000-acre farm in 1980, and eventually grew to over 2,000 acres under contract management. The system has included grain (corn and soybean), and bermudagrass and winter rye for hay and pasture.

Many of the ideas developed above have been used to describe chemical changes in soil characteristics over time at the Tallahassee Southeast Farm water reuse facility (Payne et al., 1989; Overman et al., 2003). Application of reclaimed wastewater began at the Southeast Farm in 1980. The soil is Kershaw fine sand, an excessively drained soil. Water is applied at a rate of approximately 5 cm/wk through center pivot irrigation units. Crops grown at the farm include grain, hay, and pasture. Soil samples have been collected periodically in the upper 8 m. Measurements have included pH; exchangeable potassium, sodium, calcium, magnesium, and acidity; extractable phosphorus; and organic matter. Groundwater has been monitored for various elements including chloride, NH_4-N, and NO_3-N.

Gradual changes in soil and groundwater properties over several years became apparent. Soil organic matter never increased above 0.5% and soil pH stabilized near 7. Calcium became the dominant exchangeable soil cation due to its high presence in the reclaimed water (the drinking water for Tallahassee is derived from a limestone aquifer). Phosphorus was fixed in the upper 2 m of the soil profile due to reaction with iron and aluminum oxides in the soil. Nitrate stabilized at about 5 mg N/liter. There was no noticeable decrease in soil permeability over the years. The upper soil horizon did show an increase of humus with time.

2. Overland Flow

Several studies have been conducted in Florida and around the United States on this method. Results have been summarized by Overman et al. (1988). This technology requires that a field be precision graded to a uniform slope, which is then seeded to a cover crop to stabilize the soil and provide a medium for chemical and biological processes. Convective diffusion theory proved useful for description of the processes involved. It was shown that the mass transfer coefficients were dependent on flow velocity. Field scale systems were designed, constructed, and evaluated at Florida State Prison near Starke, FL and at Northeast Florida State Hospital at Macclenny, FL. In both cases turf grasses were used for vegetative cover. A system for uniform water application was perfected for the latter system, which had proven a real challenge for this technology.

The theory has also been applied to studies with swine waste (Overman, 2003a,b).

3. Rapid Infiltration Basins

Short-term studies with this method have been conducted in Florida (Overman, 1979). Convective diffusion theory proved useful in describing fixation of phosphorus in the soil and for the nitrogen series $\text{organic N} \rightarrow \text{ammonium N} \rightarrow \text{nitrate N}$. This technology has been implemented at several sites in Florida, most notably at Conserv II in Orlando, FL and at Palm

Coast, FL. Alternate loading and drying followed by disking is required to maintain the infiltration capacity of the soil. The latter system utilized innovative design to minimize cut and fill of the soil and to enhance aesthetics of the system. The design also utilized low pressure application which minimized energy consumption.

Part II. Accumulation of Biomass and Mineral Elements by Crops

A. Response to Mineral Elements

This section focuses on mathematical models which describe response of crops to applied nutrients (N, P, and K). Control variables include applied nitrogen, phosphorus, or potassium. Response variables include biomass yield and plant nutrient uptake. Models have proven useful to design and management decisions of agricultural systems.

1. Mitscherlich Model

It is generally observed that crops show positive response to applied nitrogen, phosphorus, and potassium. This response follows diminishing returns, i.e. increasing levels of applied nutrients give diminishing increments of response, approaching an upper limit. Crops also show some response even without applied nutrient, reflecting the presence of some of the nutrient in the soil. This pattern of response led to proposal of a simple exponential model early in the 20th century (Mitscherlich, 1909; Melsted and Peck, 1977). This empirical model relates biomass yield (Y) to applied nutrient (N) through an exponential function (Figure 7). Model parameters include yield for zero applied nutrient (Y_0), maximum yield at high N (Y_m), and a response coefficient (c). The model was chosen to agree with the pattern in crop response. It can actually be derived by assuming that incremental yield response (dY/dN) is proportional to the yield deficit ($Y_m - Y$).

The model was used to describe response of forage crops to applications of dairy waste on Scranton fine sand at Gainesville, FL (Overman et al., 1975). Application rates for the cool-season annual (oats) were 0.63, 1.25, and 2.50 cm/week and for the warm-season annual (sorghum-sudangrass) were 0, 2.50, and 5.00 cm/week. Yield parameters for oats were Y_0 = 5.12 Mg/ha and Y_m = 12.25 Mg/ha, while for sorghum-sudangrass these were Y_0 = 37.5 Mg/ha and Y_m = 90.7 Mg/ha. Since there were only three treatment levels, the model described the data rather well (zero degrees of freedom). However, it was not clear how to couple nutrient uptake by the forage grasses to biomass yield. Furthermore, some data exhibit sigmoid response of yield to applied nutrient (Russell, 1937), in disagreement with the Mitscherlich model.

2. Logistic Model

a. Response Equations

There is clearly a need for a more suitable model of crop response to applied nutrients. The logistic model was introduced (Overman et al., 1990a) as an alternative (Figure 8). It was first used to describe biomass yield response to applied nitrogen. Model parameters include maximum yield at high N (A, Mg/ha), an intercept parameter at N = 0 (b), and a response coefficient to applied nutrient (c, ha/kg). Application was expanded to describe effects of water availability and harvest interval for perennials as well as to applied nitrogen (Overman et al., 1990b). Effects of water availability and harvest interval were shown to be accounted for in the linear parameter (A). Data from the literature were later used to establish a quantitative relationship between yield and crop water use (Overman and Scholtz, 2002a).

The next step was to couple plant nutrient uptake with biomass yield. Overman and Wilkinson (1990a, b) used simple exponential functions to describe nutrient concentrations with applied nutrient levels. While these models provided high correlation coefficients, the search for a more cohesive approach to modeling continued. This resulted in the extended logistic model (Overman et al., 1994a), which used a second logistic equation to describe response of plant nutrient uptake (N_u) to applied nutrient. Plant nutrient concentration (N_c) was then defined as the ratio of two logistic equations ($N_c = N_u/Y$). The model was quickly applied to data for the annual corn (Overman et al., 1994b). It was later confirmed (Overman and Scholtz, 2002b) that the response coefficient (c) was the same for yield and plant nutrient uptake.

This led to what might be called the 'paradox of the two c's.' How does nature know to operate under this constraint? What are the mathematical consequences of this equivalence? A consequence of the extended logistic model was that it could be stated as three postulates: (1) yield response to applied nutrient follows a logistic equation, (2) plant nutrient response to applied nutrient follows a second logistic equation, and (3) the response coefficients of the two logistic equations are equal.

b. Phase Relation

It can be shown that a mathematical consequence of the three postulates is coupling between biomass yield (Y) and plant nutrient uptake (N_u) which follows a hyperbolic relation containing a parameter for potential maximum yield (Y_m) and response constant (K_n). This was shown to be true for a number of studies from the literature (Overman and Scholtz, 2002b), including applied nitrogen, phosphorus, and potassium (Figure 9). However, the paradox of the two c's still remained. It then occurred to restate the logistic model as two postulates: (1) plant nutrient uptake response to applied nutrient follows a logistic equation and (2) biomass yield is coupled to plant nutrient uptake by a hyperbolic equation. Combination of postulates (1) and (2) leads to a second logistic equation between biomass yield and applied nutrient with the same response coefficient. The paradox is now eliminated and the system is reduced to two postulates! It was also shown that the logistic model reduces to a simple exponential (similar to the Malthus model of population dynamics) at very low yields and to the Mitscherlich model at higher yields (Overman, 1995).

3. Multiple Logistic Model

a. Biomass Yield

The simple logistic model of crop response to applied nutrients has been shown to apply for nitrogen, phosphorus, and potassium individually. So how should the model be written for multiple levels of two or three nutrients? Such experiments are commonly referred to as *factorial*. Several mathematical possibilities were explored. After examining field data on such a study it was concluded that the model should be written in product form of individual logistic functions. In functional form this means that $Y = f(N,P,K) = f(N)\cdot f(P)\cdot f(K)$. For fixed levels of two of the nutrients the product form reduces to a simple logistic function as required. For the case of multiple levels of N, P, and K this leads to a model of seven parameters (*b* and *c* for each element and an overall *A*). Analysis of data for a complete NxPxK = 4x4x4 factorial experiment with coastal bermudagrass at Watkinsville, GA led to very satisfactory results (Overman et al., 1991). Parameters were estimated by *nonlinear regression* using a Newton-Raphson procedure.

b. Mineral Uptake

The next logical step is to extend the multiple logistic model to cover nutrient uptake for multiple levels of N, P, and K. This should be done in a way that reduces to the simple extended logistic model where two of the three elements are held at fixed levels. Again we consider the product form of the model. Examination of field data from a complete factorial experiment at Watkinsville, GA with N, P, and K shows that to first approximation concentration of a particular element depends only on the level of that element and is independent of the levels of the other two. This places a constraint on the form of the uptake function. For uptake of nitrogen (N_u) this leads to the form $N_u(N,P,K) = g(N)\cdot f(P)\cdot f(K)$. It follows that concentration of plant N (N_c) is described by $N_c(N,P,K) = N_u/Y = g(N)/f(N)$ as required. Results for uptake of phosphorus can be written in a similar manner as $P_u(N,P,K) = f(N)\cdot g(P)\cdot f(K)$ and $P_c(N,P,K) = P_u/Y = g(P)/f(P)$. Finally, uptake of potassium can be written as $K_u(N,P,K) = f(N)\cdot f(P)\cdot g(K)$ and $K_c(N,P,K) = K_u/Y = g(K)/f(K)$. Since parameters c_n, c_p, and c_k for applied N, P, and K, respectively, must be common for uptake and yield, it follows that the only change between uptake and yield is in parameters b_n, b_p, and b_k. This format introduces four additional parameters (b_n, b_p, b_k, and an overall A_{npk}). Examination of data from the same complete factorial experiment for coastal bermudagrass led to good results (Overman and Wilkinson, 1995).

B. Crop Growth

Work with engineering projects dealing with water reclamation and reuse by land application for crop production in the 1970s and 1980s led to the need to estimate accumulation of biomass and plant nutrients over time. This led over the next 30 years to a number of mathematical models, beginning with a simple empirical model for perennial grasses and culminating in more sophisticated models for perennials and annuals. Much of this work is described in the book *Mathematical Models of Crop Growth and Yield* by A.R. Overman and R.V. Scholtz (2002b).

1. Empirical Model

a. Accumulation of Biomass with Harvests

The first step is to examine accumulation of biomass over the season for a warm-season perennial grass (Overman, 1984). Data from the literature for coastal bermudagrass was selected for this purpose (Mays et al., 1980). Experiments were conducted over a 4 year period (1964-1967). Growth occurred over a 24 week period. Treatments included harvest intervals of 4 and 6 weeks on plots both with and without irrigation. Cumulative biomass yield (Y) over the growth period followed a sigmoid pattern for all treatments (Figure 10). This suggested use of the simple probability model with three parameters (A for total seasonal yield, μ for the time to the mean of the distribution, and σ for the time spread of the distribution). It was shown that a plot of normalized yield (Y/Y_t) vs. time (t) on probability paper produced a straight line which was common for both harvest intervals and irrigation treatments. This meant that individual harvests followed a Gaussian distribution over the season and that the effects of harvest interval (Δt) and irrigation could be accounted for in the linear parameter (A).

b. Accumulation of Mineral Elements with Harvests

The next step was to examine coupling between accumulation of mineral elements and biomass with time (Overman et al., 1988) using data from the literature for coastal bermudagrass grown at Watkinsville, GA (Adams et al., 1967). Treatments included applied nitrogen (N = 224, 672, and 1120 kg N/ha), nitrogen to potassium ratio (N/K = 1, 2, and 6), harvest interval (Δt = 2, 4, and 6 wk), and water availability (irrigation vs. no irrigation). Normalized yield (Y/Y_t) vs. time for N = 672 kg N/ha, Δt = 4 wk , N/K = 2, and with irrigation followed a straight line on probability paper with virtually the same μ and σ for biomass yield and cumulative plant nitrogen. For these same treatments it was also shown that a plot of cumulative plant nitrogen uptake (N_u) vs. cumulative yield followed a straight line (Figure 11). The slope of the line gave an estimate of average plant nitrogen concentration (N_c), which meant that plant nitrogen concentration was the same for each harvest. Of course plant nitrogen concentration was related to harvest interval, with the value declining with age between harvests. Response of biomass yield with applied nitrogen appeared to follow a logistic equation.

Data from a study with the warm-season bahiagrass at Gainesville, FL (Leukel et al., 1934) were used to confirm the empirical model. Accumulation of biomass and plant nitrogen were measured with calendar time (average harvest interval of 2.60 wk) for applied nitrogen of N = 295 kg N/ha and with irrigation. Response variables included biomass yield (Y) and plant nitrogen uptake (N_u). A plot of normalized yield (Y/Y_t) vs. time (t) on probability paper produced a straight line, consistent with the model. A plot of cumulative plant nitrogen vs. cumulative biomass produced a straight line, with average plant nitrogen concentration of $N_c = N_u/Y$ = 17.0 g/kg.

c. Phase Relation for Perennials

As noted above plant nitrogen concentration ($N_c = N_u/Y$) appeared to be constant for a fixed harvest interval. This simplified the analysis considerably. It meant that cumulative yield could be modeled with the simple probability model and that cumulative plant nitrogen could be estimated from $N_u = Y \cdot N_c$.

d. Dependence of Seasonal Biomass on Harvest Interval

It appeared from analysis of data from Watkinsville, GA that seasonal total yield followed a linear relationship with harvest interval ($Y_t = \alpha + \beta \, \Delta t$) up to 6 week interval. This question clearly required further analysis to reach a definitive conclusion.

2. Phenomenological Model

   a. Energy Driving Function

The empirical model provided excellent description of biomass accumulation of warm-season perennial grasses with calendar time for fixed harvest intervals up to 6 week. However, it failed to provide any insight into growth processes. The probability model suggested the presence of a Gaussian component in the system since the model involves the so-called error function, which is the integral of a Gaussian function. What could be the basis of this Gaussian distribution? Examination of data for the northern hemisphere showed that solar radiation in fact followed such a distribution over the year, at least to first approximation. This suggested that the *energy driving function* for photosynthesis could be approximated as a Gaussian distribution with calendar time (Figure 12).

   b. Intrinsic Growth Function

A growth model must also contain a function which reflects the basic growth process of the plant, viz. an *intrinsic growth function*. Again data from the literature were used as a guide (Overman and Wilkinson, 1989). Partitioning of biomass between light-gathering (leaf) and structural (stem) components appeared to follow linear dependence on harvest interval (Figure 13). It appeared logical to assume linear dependence between the rate of biomass accumulation and elapsed time since the previous harvest. Integration of the resulting differential equation led to the phenomenological growth model (Overman et al., 1989). Further analysis of data confirmed this model (Overman et al., 1990c).

   c. Dependence of Seasonal Biomass on Harvest Interval

Since the phenomenological growth model predicts linear dependence of seasonal total biomass (Y_t) on harvest interval (Δt) it should be possible to compare this prediction with field data. Fortunately Prine and Burton (1956) conducted just such a study with coastal bermudagrass at Tifton, GA during 1953 and 1954 for applied nitrogen of N = 672 kg N/ha and harvest intervals of $\Delta t = 1, 2, 3, 4, 6, \text{and } 8 \text{ wk}$. Correlation of Y_t with Δt proved linear for $0 < \Delta t \le 6 \text{wk}$ (Figure 14). Values at $\Delta t = 8 \text{wk}$ fell below the straight line prediction. This model appeared to be a step in the right direction.

There appeared to be a deficiency in the phenomenological model. It over predicted yield for harvest intervals greater than about 6 weeks, and it proved inadequate for annuals (such as corn). Now the search was on for a more suitable growth model, which required modification of the intrinsic growth function.

3. Expanded Growth Model

The next step was to examine data from a more extensive study with coastal bermudagrass by Burton et al. (1963) at Tifton, GA. In this study harvest intervals included Δt = 3, 4, 5, 6, 8, 12, and 24 wk. Response variables included seasonal total biomass yield (Y_t), plant nitrogen uptake (N_{ut}), and plant nitrogen concentration (N_c). It was concluded that Y_t and N_{ut} vs. Δt followed linear-exponential relationships with high correlation coefficients. The model was therefore expanded to include an additional factor, called an *aging function*.

a. Energy Driving Function

Since the harvest data still appeared to follow a Gaussian distribution, the energy driving function was retained as Gaussian consistent with previous results.

b. Partition Function

This result suggested the possibility of a linear-exponential intrinsic growth function. The linear function could be defined as the partition function between light-gathering and structural components of the plant, but what could the exponential term correspond to?

c. Aging Function

Since it was clear that the exponential function must be reset after each harvest for the perennial grass, it was decided to define this term as an *aging function* (Figure 15). It was expected that the aging function would apply to annuals as well as perennials.

d. Dependence of Seasonal Biomass on Harvest Interval

The expanded growth model led to a first order ordinary linear differential equation containing a Gaussian term (energy driving function), a linear term (partition function), and an exponential term (aging function). The model could be written in functional form as $dY/dt = f(E)\cdot f(P)\cdot f(A)$. Integration of the differential equation then led to an equation relating the increment of biomass accumulation (ΔY_i) for growth interval *i* in terms of calendar time (*t*). Cumulative biomass (*Y*) was then given by the sum of the increments ($Y = \sum_i \Delta Y_i$). Seasonal total biomass (Y_t) was estimated as the sum of all increments over the season. Analysis of data showed linear-exponential dependence of Y_t on harvest interval Δt (Figure 16). The question is whether or not the expanded growth model was consistent with this result. Clearly, what was needed was a mathematical theorem to connect the results together. After considerable effort the theorem was proven in 1999 (Overman, 2001). In addition, the theorem showed that the biomass over the season followed a Gaussian distribution. This provided a rational basis for the empirical model. Further confirmation of the expanded growth model was provided (Overman and Brock, 2003) by using data for the first cutting of each harvest interval for the data of Burton et al. (1963).

e. Phase Relation for Annuals

Attention was now focused on accumulation of biomass and mineral elements by an annual crop (Overman and Scholtz, 2004). Data were taken from a study at the Southeast Farm at Tallahassee, FL with corn grown on Kershaw fine sand. Measurements were made of biomass and mineral elements (N, P, K, Ca, and Mg) on a weekly basis between 15 April and 8 July 1990. Nutrient applications through reclaimed municipal water and fertilizer were: *N-P-K-Ca-Mg* = 143-13-262-470-150 kg/ha. The expanded growth model described accumulation of biomass with calendar time rather well. It was shown that accumulation of mineral elements could be coupled with biomass through hyperbolic phase relations (Figure 17). It was concluded from this analysis that mineral elements accumulated in virtual equilibrium with biomass, and that accumulation of biomass by photosynthesis was the rate limiting process in plant growth. Plant nutrient concentration is then estimated as the ratio of plant nutrient uptake to biomass.

Part III. Miscellaneous Applications of Models

A. Partitioning of Biomass

The partition function of the expanded growth model has included two components, viz. a light-gathering component (leaves) and a structural component (stems or stalks). Integration of the differential equation leads to a linear relationship between accumulated biomass (Y) and a growth quantifier (Q) of the form ($Y = A \cdot Q$) where A is a yield factor (Figures 18 and 19). Now the biomass yield can be partitioned into the two components ($Y = Y_L + Y_S$). If the model is correct, then the growth quantifier can be partitioned into two components ($Q = Q_L + Q_S$). It follows that the two components should be defined by linear relationships ($Y_L = A \cdot Q_L$ and $Y_s = A \cdot Q_S$).

1. Elephantgrass

Overman and Woodard (2006) analyzed data from a study with elephantgrass at Gainesville, FL (Woodard et al., 1993). Plots were mowed on 28 March 1989 to promote uniformity. Biomass samples were collected every 4 weeks between 16 May and 28 November 1989. Plants were partitioned into stem and leaf fractions. A linear relationship was established between biomass yield and growth quantifier (Y = 2.37 + 11.3Q). Linear relationships were also established for the two components (Y_L = 2.37 + 11.2Q_L and Y_S = 0.055 + 11.3Q_S). The intercept (2.37) represents the biomass of the leaf fraction at the initiation of simulation. Note that the slope (11.3) is essentially the same for the two plant components, as required by the model.

2. Potato

Another application of the expanded growth model can be found with a vegetable crop. Carpenter (1963) measured accumulation of biomass and plant nitrogen (vegetative and tubers) for potato at Old Town, ME. Data have been analyzed by Overman (2006b). Linear correlation was obtained for both vegetation and tuber biomass accumulation vs. growth quantifier ($Y_v = 0.069 + 1.91 Q_v$ and $Y_t = -0.069 + 5.42 Q_t$). Plant nitrogen uptake (N_u) followed a hyperbolic relationship with biomass (Y) for both plant components. Biomass response to applied nitrogen followed the logistic model for both components.

3. Accumulation of Biomass and Mineral Elements by Peanut

Application of the expanded growth model to growth of peanut has been presented by Overman (2006b). The analysis was based on data from a study at Lexington, NC (Nicholaides, 1968). Plots were planted on 6 May 1966 and sampled on 13 July, 13 and 24 August, 14 September, and 5 October 1966. Plant samples were analyzed for nitrogen, phosphorus, potassium, and magnesium. Linear correlation was obtained between biomass yield (Y) and the growth quantifier (Q). Hyperbolic phase equations were obtained for plant nitrogen, phosphorus, potassium, and magnesium. This supports the conclusion that biomass accumulation by photosynthesis is the rate limiting process in crop growth. Decline of concentration of mineral elements as plants age is explained by partitioning between light-gathering (higher nutrient concentration) and structural components (lower nutrient concentration).

4. Accumulation of Biomass by Soybean

The expanded growth model has been used to simulate biomass accumulation by soybean (Overman, 2006b). In one study results for two different soils has been compared, while the other focused on difference for two years due to rainfall.

Hammond et al. (1951) measured soybean growth on two soils (Webster silt loam and Clarion loam) at Ames, IA. Linear correlation was obtained between yield (Y) and growth quantifier (Q) for both soils ($\text{Webster}: \ Y = -0.14 + 2.37Q; \ \text{Clarion}: \ Y = 0.08 + 1.61Q$). Since the growth quantifier was the same for both soils, the difference occurred in the yield factor.

Henderson and Kamprath (1970) measured soybean growth for two years (1966 and 1967) at Clayton, NC. Linear correlation ($1966: Y = 0.008 + 1.60Q; \ 1967: Y = 0.082 + 2.82Q$) was obtained for both years. Since the growth quantifier was the same for both years, the difference occurred in the yield factor, and was due to difference in seasonal rainfall.

B. Efficiency of Nutrient Utilization by Plants

A very important question in crop production is efficiency of nutrient utilization by plants. The first step is to estimate plant nutrient uptake (N_u) in response to applied nutrient (N). It was shown in Section II.A that the extended logistic model has proven useful for this purpose. The uptake model includes three parameters: maximum uptake at high applied nutrient (A_n), an intercept parameter at zero applied nutrient (b_n), and a response coefficient (c_n). Plant nutrient uptake at $N = 0$ is defined as N_{u0}. It is common among agronomists to define efficiency of plant nutrient utilization as the ratio of increase in plant nutrient uptake above base level to applied nutrient ($E = (N_u - N_{u0}) / N$). The question is how to estimate peak efficiency of utilization?

Overman (2006c) has discussed the question in detail. First we note that maximum slope of the logistic response function ($(dN_u / dN)_{\max}$) can be defined in terms of model parameters as ($A_n c_n / 4$). This corresponds to applied nutrient where uptake reaches 50% of maximum ($Y = A_n / 2; N_{1/2} = b_n / c_n$). It was shown that the value of applied nutrient for peak uptake (N_p) occurs at 1.5 times this value ($N_p = 1.5 b_n / c_n$). Peak efficiency of nutrient utilization (E_p) can then be estimated from an equation involving the three model parameters (A_n, b_n, c_n). See Figure 20.

This procedure has been applied to a particular crop by Overman (2007b) using data from a study with switchgrass (Madakadze et al., 1999) at Montreal, Quebec, Canada. The control variables consisted of applied nitrogen of N = 0, 75, and 150 kg N/ha and harvest intervals of $\Delta t = 4, 6, \text{and } 15 \text{ wk}$. Response variables included biomass yield (Y), plant nitrogen uptake (N_u), and plant nitrogen concentration (N_c). For harvest intervals of 4 and 6 weeks peak values of applied nitrogen (N_p) and efficiency of utilization (E_p) were N_p = 147 kg N/ha and E_p = 58.5%.

C. Coupling of Plant Nutrient Uptake and Available Soil Nutrient with Applied Nutrient

It is logical to now examine connections in the sequence of nutrient steps through the system $applied \rightarrow soil \rightarrow plant\ uptake \rightarrow biomass\ yield$ for mineral elements such as nitrogen, phosphorus, and potassium. Overman (2007c) has discussed this case in detail.

1. Phosphorus

a. Rothamsted Experiment Station, UK Study

Response to phosphorus was reported for a field study by Johnston et al. (1976) at Rothamsted Experiment Station, UK. Levels of the control variable included P = 0, 82, 164, 328, and 492 kg/ha. Measurements of the response variables included extractable soil phosphorus (P_{ex}), plant phosphorus uptake (P_u), and biomass yield (Y) for the crops barley, potato (tuber), and sugar beet (sugar). A graph of extractable soil phosphorus (P_{ex}) vs. applied phosphorus (P) followed a logistic response function very closely (Figure 21). A plot of plant phosphorus (P_u) vs. extractable soil phosphorus (P_{ex}) followed a hyperbolic phase relationship very closely (Figure 22). Combination of the logistic and hyperbolic functions led to a second logistic equation between plant phosphorus uptake (P_u) and applied phosphorus (P), which was confirmed by a plot of data. The next step was to assume a hyperbolic phase relationship between biomass yield (Y) and plant phosphorus uptake (P_u). Combination of this equation with the hyperbolic relationship between plant phosphorus (P_u) vs. extractable soil phosphorus (P_{ex}) led to a third hyperbolic phase relationship between yield (Y) and extractable soil phosphorus (P_{ex}), which was also confirmed from data. Finally, a combination of the various relationships led to a third logistic equation for biomass yield (Y) vs. applied phosphorus (P).

b. Hastings, FL Study

Overman (2006a) analyzed data from a study by Rhue et al. (1981) with potato at Hastings, FL. Applied phosphorus levels included P = 0, 56, 112, and 168 kg P/ha. Response variables included extractable soil phosphorus (P_{ex}), plant phosphorus uptake (P_u), vegetative yield (Y), and plant phosphorus concentration (P_c). It was shown that P_{ex}, P_u, and Y vs. P were described by logistic equations with common response coefficient (c_p). It was further shown that P_u vs. P_{ex} and Y vs. P_u followed hyperbolic equations as predicted by the theory. It followed that P_c vs. P_u followed a linear relationship. A further consequence was that Y vs. P_{ex} followed a hyperbolic relationship, and that P_c vs. P_{ex} was described by a linear relationship.

2. Potassium

    a. New Jersey Study

Response to applied potassium was studied by Markus and Battle (1965) at New Brunswick, NJ. Levels of the control variable were K = 0, 46.5, 93, 186, 280, and 372 kg K/ha. Measurements of the response variables included extractable soil potassium (K_{ex}) and biomass yield (Y) for alfalfa. A graph of extractable soil potassium (K_{ex}) vs. applied potassium (K) followed logistic response. A plot of biomass yield (Y) vs. extractable soil potassium (K_{ex}) followed a hyperbolic phase relationship. Combination of the logistic and hyperbolic functions led to a second logistic equation between biomass yield (Y) and applied potassium (K), which was confirmed by a plot of data.

    b. Kentucky Study

Response to applied potassium was reported by Bertsch and Thomas (1985) for a study by H.C. Vaught, K.L. Wells, and K.L. Driskell (1984) in Kentucky with alfalfa. Levels of the control variable were K = 0, 112, 168, and 224 kg K/ha. Response variables included biomass yield (Y), plant potassium uptake (K_u), and extractable soil potassium (K_{ex}). Yield and plant K uptake were reported as averages over a six year period, while extractable soil K was at the end of the six year period. Extractable soil potassium (K_{ex}) vs. applied potassium (K) was described by a logistic equation. Plant potassium uptake (K_u) vs. extractable soil potassium (K_{ex}) followed a hyperbolic phase relationship. This led to a logistic relationship between K_u and K. If it is assumed that biomass yield (Y) exhibits hyperbolic dependence on plant potassium uptake (K_u), then Y vs. K_{ex} would follow a hyperbolic phase relationship and Y vs. K would follow a logistic relationship, which was confirmed from the data.

Bertsch and Thomas (1985) also reported data from a study by Thom (1984) in Kentucky with corn and soybean. Levels of applied potassium for the yield study was K = 0, 67, 134, 202, and 268 kg K/ha and for extractable soil potassium was K = 0, 55.6, 111.2, 167.6, and 222.4 kg K/ha, for both corn and soybean. Again K_{ex} vs. K and Y vs. K were described by the logistic model for both corn and soybean. It was also shown that Y vs. K_{ex} followed hyperbolic phase equations for both crops.

This analysis accomplished two things: (1) it provided a sound basis for the extended logistic model for applied phosphorus and potassium, and (2) it provided a fundamental basis for the logistic model from the soil component of the system. There is no accepted measure of available soil nitrogen. However, this conceptual picture is believed to hold for nitrogen.

D. Multiple Logistic Model

Overman (2007c) has discussed this subject in detail. The focus here is on the warm-season perennial bahiagrass and warm-season annuals corn and millet.

1. Bahiagrass

A study was conducted at Quincy, FL by Rhoads et al. (1997) with bahiagrass. The control variables were applied nitrogen, phosphorus, and potassium (N, P, K) in a complete factorial experiment with NxPxK = 3x3x3. The response variable was biomass yield (Y). Plots were harvested monthly from May through September in 1996. Plant nutrient uptake was not reported.

The analysis was segmented into response to applied nitrogen at fixed P and K, response to applied phosphorus at fixed N and K, and response to applied potassium at fixed N and P. This provided estimates of the intercept and response coefficients (b and c) of the logistic model for each element. The overall yield factor (A) was estimated from the 27 values for measured yields. A scatter plot was then constructed of estimated vs. measured yields which produced a high correlation coefficient (r = 0.978).

2. Corn

A study was conducted in Florida by Robertson et al. (1965) with five hybrids of corn. The control variables were applied nitrogen, phosphorus, and potassium (N, P, K) at four levels of N-P-K = 0-0-0, 168-50-140, 336-100-280, and 672-200-560 kg/ha. Response variables included biomass yield (Y) and plant uptake of nitrogen, phosphorus, and potassium (N_u, P_u, K_u).

Average data for the five hybrids were used in the analysis. These data were particularly difficult to analyze because the three elements were varied together in a fixed proportion. An iterative approach was used to estimate model parameters for yield (A_y, b, c) and for plant nutrient uptake (A' and b'). The model provided reasonable description of response to applied nitrogen, phosphorus, and potassium. Hyperbolic relationships of biomass yield (Y) with plant nitrogen uptake (N_u), plant phosphorus uptake (P_u), and plant potassium uptake (K_u) were confirmed. Response of plant nutrient concentrations (N_c, P_c, K_c) to applied nutrients (N, P, K) was also described by the model.

3. Millet

A study was conducted at Quincy, FL by Rhoads and Olson (1995) with millet. The control variables were applied nitrogen, phosphorus, and potassium (N, P, K) at four levels of N-P-K = 0-0-0, 123-34-120, 246-68-240, and 492-136-480 kg/ha from mushroom compost. Response variables included biomass yield (Y) and plant uptake of nitrogen, phosphorus, and potassium (N_u, P_u, K_u). Model parameters were estimated by an iterative approach as for corn. The model provided excellent description of response and phase plots.

Measurements of extractable soil phosphorus (P_{ex}) and potassium (K_{ex}) were made in this experiment. Analysis showed that P_{ex} vs. P and K_{ex} vs. K both followed logistic equations, and that the corresponding phase plots of P_u vs. P_{ex} and K_u vs. K_{ex} both followed hyperbolic equations.

E. Harvest Interval and Plant Digestibility

A significant measure of quality in forage production is digestibility of plant material by animals. Since partitioning of biomass between light-gathering (leaf) and structural (stem) fractions it would be expected that digestibility of plant material would shift with harvest interval. The expanded growth model has been used to evaluate this subject (Overman, 2006b).

1. Bermudagrass

Data for this analysis were taken from a field study by Burton et al. (1963) at Tifton, GA. Applied nitrogen was 672 kg N/ha. Harvest intervals (Δt) included 3, 4, 5, 6, 8, 12, and 24 weeks. Seasonal total biomass yield (*Y*) and *in vitro* digestible dry matter (*D*) were reported. According to the expanded growth model dependence of *Y* on Δt follows a linear-exponential relationship. It is assumed that *D* vs. Δt follows a similar relationship.

It was shown that harvest intervals for peak production of total dry matter (Δt_{py}) and digestible dry matter (Δt_{pd}) were $\Delta t_{py} = 10.4$ wk and $\Delta t_{pd} = 5.3$ wk, respectively. It was further shown that digestible dry matter fractions contained in light-gathering (f_{dL}) and structural (f_{dS}) components of the plant were $f_{dL} \cong 100\%$ and $f_{dS} \cong 37\%$, respectively. It follows that as plants age overall digestibility would decrease as observed.

2. Perennial Peanut

Data for this analysis were taken from a field study by Beltranena (1980) at Gainesville, FL. Harvest intervals included Δt = 2, 4, 6, 8, 10, and 12 wk. Seasonal total biomass yield (*Y*) and *in vitro* digestible dry matter (*D*) were reported. Peak harvest intervals for total yield and digestible dry matter were estimated to be $\Delta t_{py} = 10.7$ wk and $\Delta t_{pd} = 8.0$ wk, respectively. Digestible dry matter fractions contained in light-gathering (f_{dL}) and structural (f_{dS}) components of the plant were estimated to be $f_{dL} \cong 100\%$ and $f_{dS} \cong 50\%$, respectively. These results are consistent with those for bermudagrass.

Summary

The material in this memoir was developed on two guiding principles: (1) *simplification* and (2) *unification*. All mathematical models represent simplification of total reality. Think of a roadmap. It contains main features including roads and streets, and major landmarks such as buildings, but excludes small details such as lampposts and trees. Too much detail would render the map cluttered and useless. An example of simplification in this document is the case of crop growth. It is known that photosynthesis in which solar energy drives carbon dioxide fixation from the atmosphere and splitting of the water molecule to form green plants. And yet the full details of these processes are not fully understood (Everts, 2009; Morton, 2007; Piel, 2001, p. 279). Unification has been a central theme in science for over 400 years. This includes matter and motion (Galileo and Kepler), celestial and terrestrial mechanics (Newton), electricity and magnetism (Faraday and Maxwell), matter and energy (Einstein), and space and time (Lorentz and Einstein). In our case unification has included biomass (carbon compounds) and mineral elements (nitrogen, phosphorus, potassium, calcium, and magnesium).

The mathematical structure has followed the classical tradition of transparency in which assumptions have been stated, all equations written down, procedures of analysis specified, tables and figures of data given, and conclusions documented. This format allows readers, with sufficient discipline, to verify the results for themselves. It also allows for further refinement of the mathematical models.

The goal of science is advance of knowledge and understanding of how nature works. An implicit assumption is that there is *order* in the physical world. Associated with this order is a level of *uncertainty* in observations and measurements, which has led to statistical procedures for incorporating this uncertainty in making inferences from data. The uncertainty resides in both data (analysis of variance) and mathematical models (Fisher information), as discussed by Fisher (1922) and Frieden (1998). This belief in order was fundamental to Kepler's view of the solar system, who believed that the solar system operated like a clockwork mechanism (Newton, 2007, p. 74).

It was observed in the early 1990s that the logistic model of crop response to applied nutrient exhibited mathematical symmetry. A theorem due to Emmy Noether (Brewer and Smith, 1981) had been proven which showed that symmetry in a mathematical model corresponded to a conservation principle in the system. This property had proven important in the work of Kepler, Galileo, Newton, Maxwell, and Einstein. The search was on to identify the conservation principle in the crop system. The answer was identified in 1994 after much thought and reflection. Imagine the shock when a reviewer declared that this discovery meant nothing! Undeterred, the work was published anyway (Overman, 1995), and it has proven essential to understanding the logistic model and its applications in crop production.

Mathematical models can be constructed at various levels of complexity, from the atomic scale to field scale. Our work on crop growth and yield has been conducted at the field scale. This leads to implications and insight into the fundamental processes at work. This approach is sometimes referred to as 'top down' research. We clearly advocate tolerance for a wide range of approaches as in the best interest of science and of society.

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Glossary of Symbols and Fundamental Concepts

Chemical Symbols

N	nitrogen
P	phosphorus
K	potassium
D_2O	deuterium oxide
HF	hydrofluoric acid
KCl	potassium chloride
NH_4Cl	ammonium chloride
NO_3-N	nitrate nitrogen
Cl^{-1}	chloride anion
H^{+1}	hydrogen cation
OH^{-1}	hydroxyl anion
$H_2PO_4^{-1}$	phosphate monomer
HPO_4^{-2}	phosphate dimer
PO_4^{-3}	phosphate trimer
Ca^{+2}	calcium cation
K^{+1}	potassium cation
Mg^{+2}	magnesium cation
NH_4^{+1}	ammonium cation

Mathematical Symbols

C	chemical concentration
D	chemical diffusion coefficient, or digestible biomass yield
k	overall rate coefficient for phosphorus kinetics
k_a	adsorption coefficient for phosphorus kinetics
k_d	desorption coefficient for phosphorus kinetics
k_r	reaction coefficient for phosphorus kinetics
k_f	forward rate coefficient for phosphorus kinetics
k_b	backward rate coefficient for phosphorus kinetics
k_s	shear rate coefficient for phosphorus kinetics
v	convective flow velocity
M	mass of soil per unit volume of solution in a batch reactor
R	exchange coefficient for phosphorus kinetics
r	rate of phosphorus addition in the open batch reactor
N	applied nutrient (nitrogen, phosphorus, or potassium)
Y	biomass yield
Y_t	total biomass yield over the entire season
N_u	plant nutrient uptake (nitrogen, phosphorus, or potassium)
N_c	plant nutrient concentration (nitrogen, phosphorus, or potassium)

dY/dN	incremental yield response to applied nutrient
Y_o	intercept yield at $N = 0$ for Mitscherlich model
Y_m	maximum yield at high N for Mitscherlich model, or potential maximum yield for hyperbolic phase equation
A	maximum yield at high N for logistic model, or seasonal total yield for the empirical growth model, or yield coefficient for expanded growth model
c	nutrient response coefficient for logistic model
K_n	nutrient uptake response coefficient for hyperbolic phase equation
K_u	plant uptake of potassium
P_u	plant uptake of phosphorus
Y_t	total biomass yield for season
N_{ut}	total plant nutrient uptake for season
μ	time to mean of a Gaussian distribution
σ	time spread of a Gaussian distribution
Δt	harvest interval for perennial grass
dY/dt	incremental biomass yield response with time
ΔY_i	yield increment for i th harvest for perennial grass
Q	growth quantifier for expanded growth model
Q_L	growth quantifier for light-gathering component of plant
Q_S	growth quantifier for structural component of plant
Y_L	biomass yield for light-gathering component of plant
Y_S	biomass yield for structural component of plant
N_{u0}	plant nitrogen uptake at $N = 0$
E	efficiency of plant nutrient utilization
E_p	peak efficiency of plant nutrient utilization
N_p	nutrient application at peak efficiency of plant nutrient utilization
$N_{1/2}$	nutrient application rate to achieve 50% of maximum plant nutrient uptake
dN_u/dN	incremental increase in plant nutrient uptake with applied nutrient
A_n	maximum plant nutrient uptake at high N
b_n	intercept parameter for plant nutrient uptake
c_n	nutrient response coefficient
P_{ex}	extractable soil phosphorus
K_{ex}	extractable soil potassium
Δt_{py}	peak harvest interval for maximum biomass dry matter
Δt_{pd}	peak harvest interval for maximum digestible dry matter
f_{dL}	fraction of digestible dry matter for light-gathering component of plant for perennials
f_{dS}	fraction of digestible dry matter for structural component of plant for perennials

Fundamental Concepts

Atomic hypothesis – matter is composed of fundamental units called atoms. Atoms combine to form a range of molecules from water to complex carbon compounds.

Kinetic theory of matter – units of matter are in a state of motion. This explains rate processes such as viscosity, diffusion, heat conduction, electrical conduction, and chemical kinetics.

Chemical diffusion – movement of a chemical species from point to point due to differences in concentration of the solute dissolved in the solvent. This can occur in the direction of liquid flow (longitudinal diffusion) or between liquid and surface phases (transverse diffusion) as in ion exchange. Transport by this mechanism is assumed to follow Fick’s law.

Convective flow – transport of a chemical species (solute) due to bulk flow of the liquid (solvent) through the porous media. In a simple uniform circular capillary transport by this mechanism is assumed to follow Poiseuille’s law.

Homogeneous kinetics – chemical processes in a single phase system. An example is a chemical reaction in the liquid phase.

Heterogeneous kinetics – chemical processes in a multiphase system. An example is a chemical reaction in a liquid/solid phase system.

Langmuir Hinshelwood kinetics – a system involving reversible exchange between two phases followed by an irreversible reaction on the surface phase. An example is phosphorus chemistry in soil involving reversible ion exchange between solution and surface phases followed by an irreversible reaction on the surface phase.

Cations – ions which carry positive charge (such K^+, Na^+, NH_4^+, Ca^{+2}, Mg^{+2}).

Anions – ions which carry negative charge (such OH^{-1} and $H_2PO_4^{-1}$).

Control variable – an input variable which is controlled in the system. An example is the quantity of applied nitrogen for production of a crop in a field experiment.

Response variable – an output variable which is measured in the system. An example is the biomass yield of a crop in response to applied nitrogen in a field experiment.

Phase relation – a mathematical equation which relates two or more response variables to each other. An example is coupling between accumulation of biomass and a mineral element (such as nitrogen) by a plant in response to applied nutrient.

Mathematical model – an equation which relates a response variable to a control variable. Models range from a simple algebraic equation to complex systems of differential equations.

Model parameter – a coefficient which characterizes quantitative response of a particular system to input.

Regression analysis – a mathematical procedure for evaluating coefficients in a mathematical model. This ranges from linear regression in models based on linear algebra to nonlinear regression in models based on nonlinear algebra (such as exponential, logarithmic, and error function).

Correlation coefficient – a measure of uncertainty (goodness of fit) between response and control variables. This parameter should be used with great caution, particularly in nonlinear models.

Analysis of variance – a mathematical procedure for comparing different modes of data analysis.

Photosynthesis – formation of carbohydrates by fixation of atmospheric carbon dioxide by green plants. The process is driven by radiant energy, typically from solar energy.

Mathematics – the study of patterns in numbers, relations, and shapes. A simple division is arithmetic, algebra (including calculus), and geometry. Mathematics is the language of science.

Characteristic time – a measure of the response time of a particular process (such as convection, diffusion, or chemical reaction) in a dynamic system.

Batch reactor – a reactor for study of a chemical process in which the contents are well mixed. These may be classified as closed (no injection of reactant with time) or open (with injection of reactant at a steady rate with time).

Packed bed reactor – a reactor for study of chemical transport through a packed column of uniform bulk density of the solid material. Reactant is usually injected at a steady rate.

Annual crop – a crop which is planted each season. These can be divided into warm-season and cool-season. Examples include corn and wheat.

Perennial crop – a crop which carries over from year-to-year. These can be divided into warm-season and cool-season. Many perennials can be harvested multiple times during the season. Examples include bermudagrass, bahiagrass, and perennial ryegrass.

Harvest interval – refers to the time interval (in weeks) between harvests for a perennial crop.

Mineral elements – refers to plant nutrients which are supplied from the soil. These include nitrogen, phosphorus, potassium, calcium, and magnesium.

Forage quality – a measure of the quality of crop material for animal feed. Measures include nutrient content (such as nitrogen) and digestibility.

Efficiency of nutrient utilization – a measure of crop utilization of an applied nutrient (such as nitrogen, phosphorus, and potassium).

Goal of science – advance knowledge and understanding of how nature works.

List of Figures

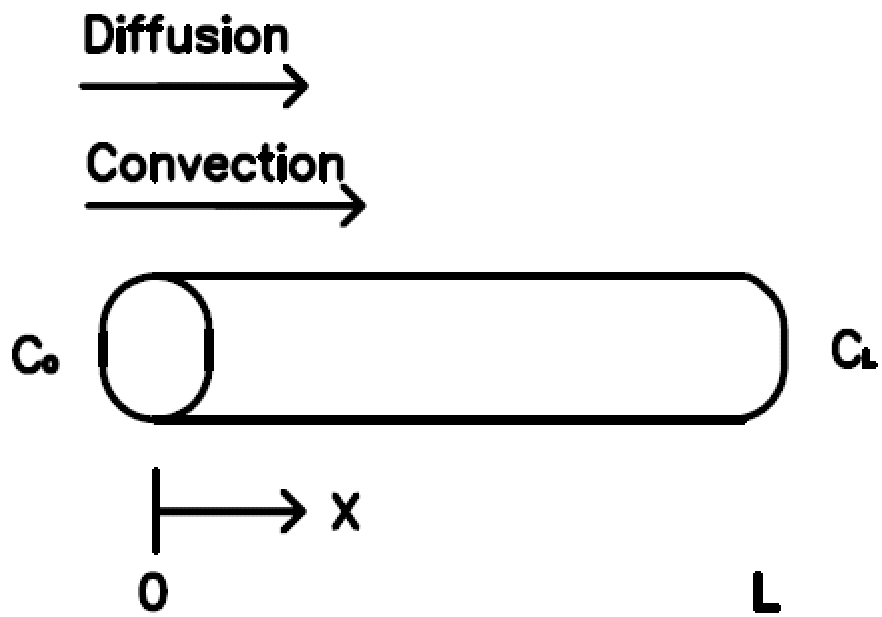


Figure 1: Convection and diffusion in a capillary tube

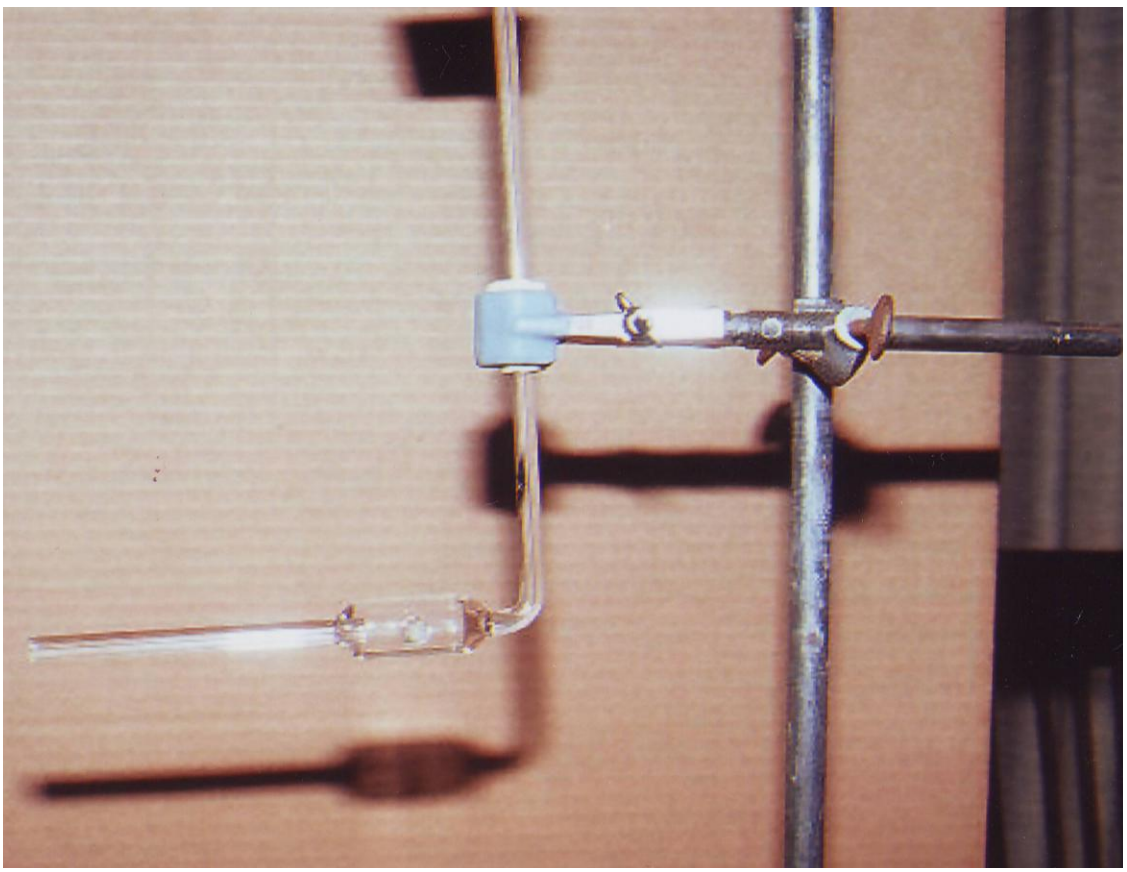


Figure 2: Flow cell for convective diffusion in a capillary tube

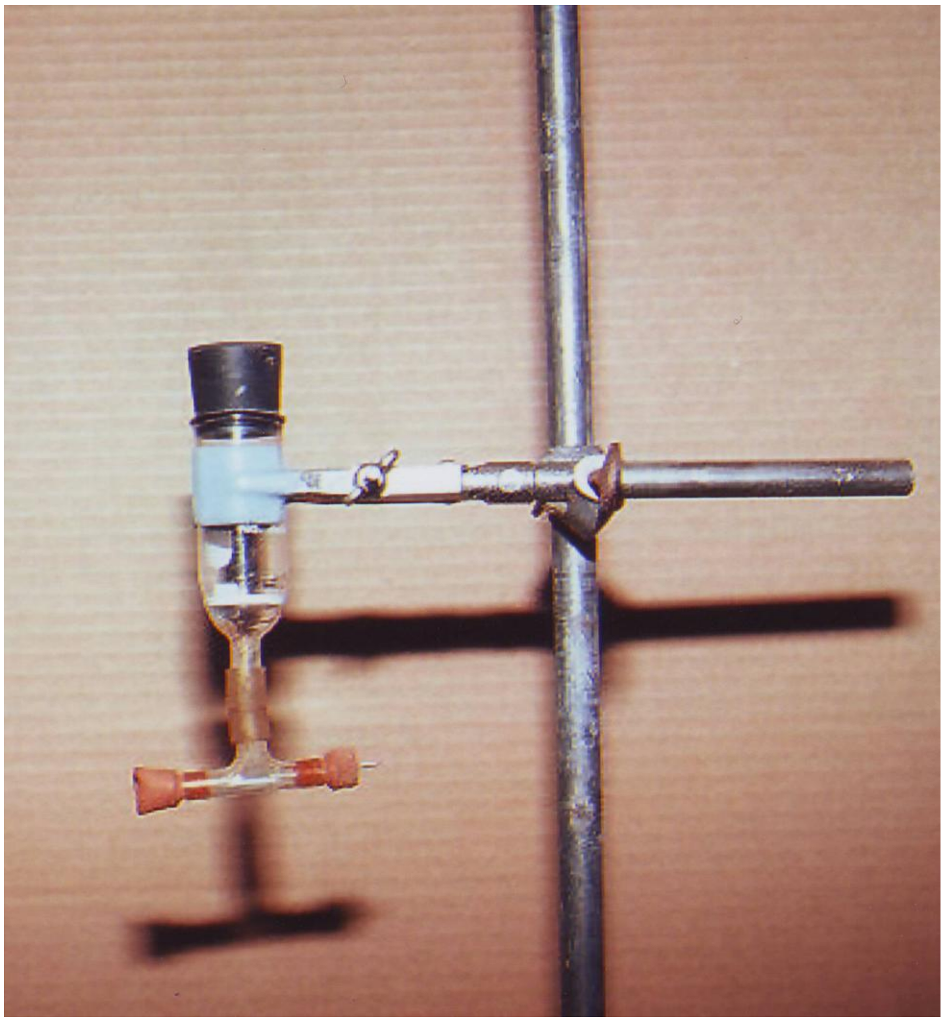
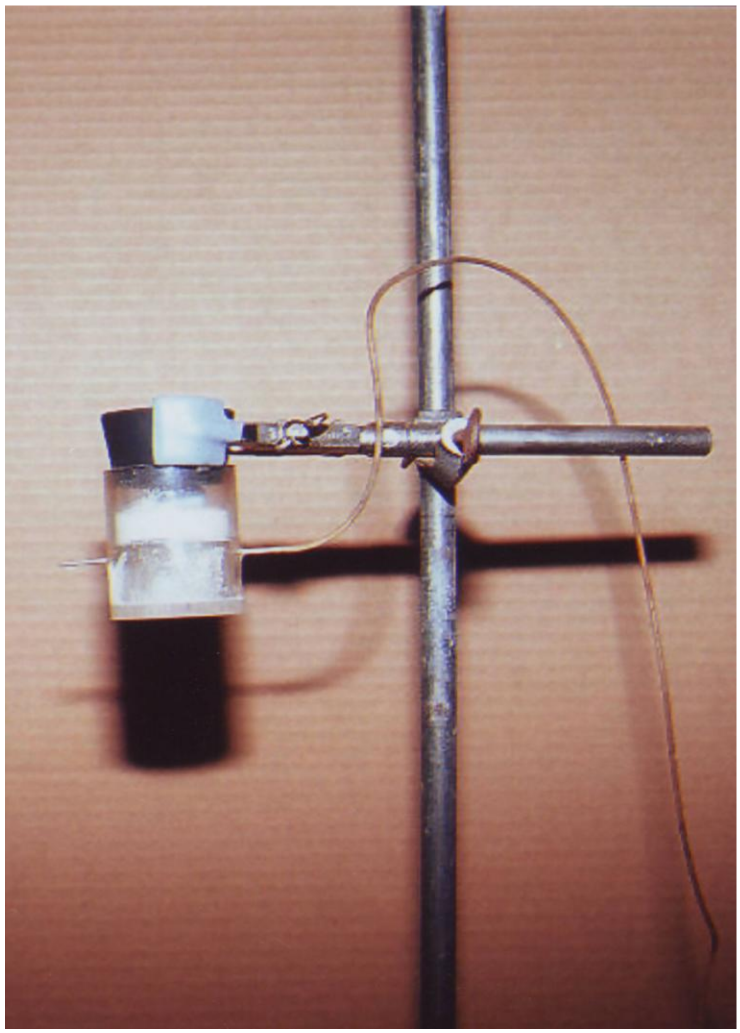


Figure 3: Flow cell for convective diffusion across a porous membrane



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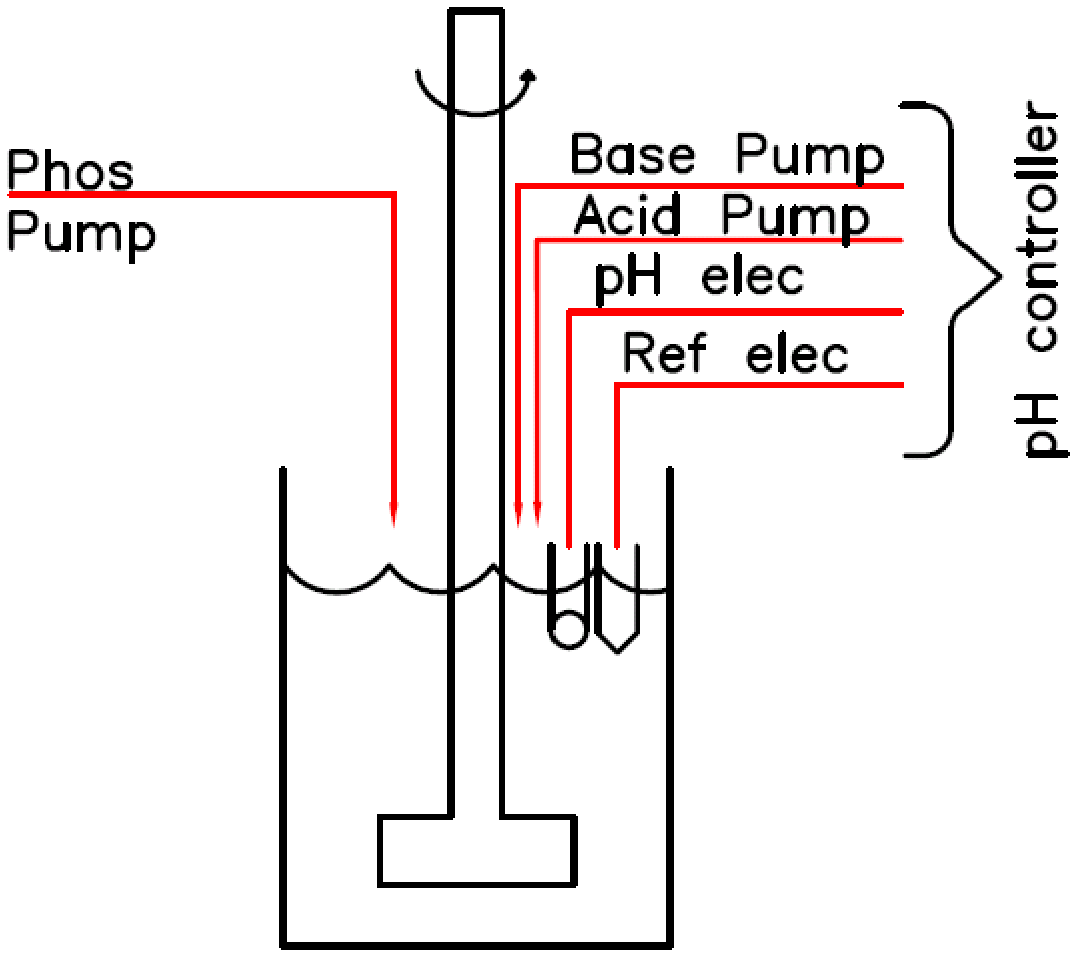

Figure 5: Phosphorus kinetics in a batch reactor

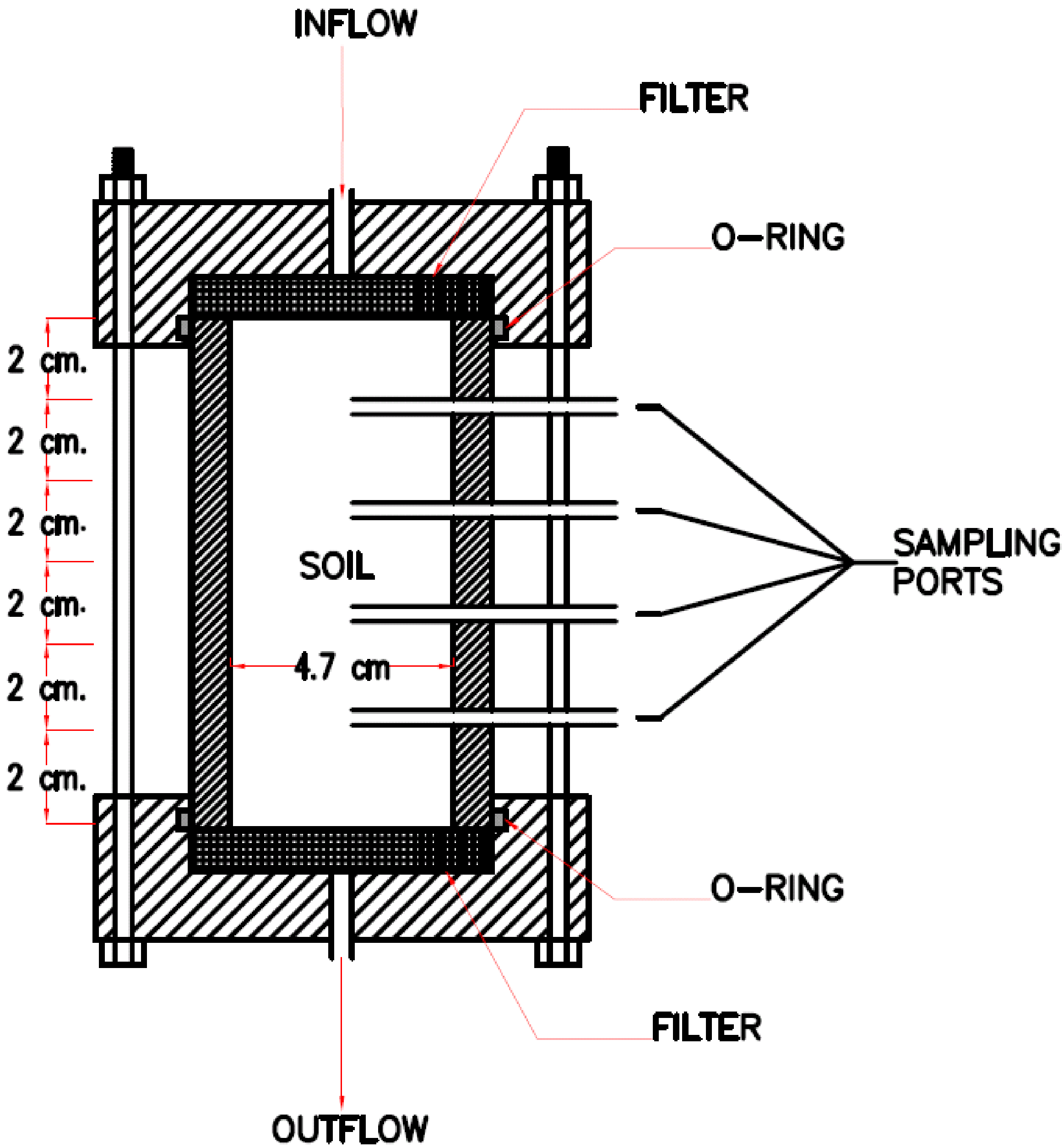

Figure 6: Phosphorus transport through a packed bed reactor

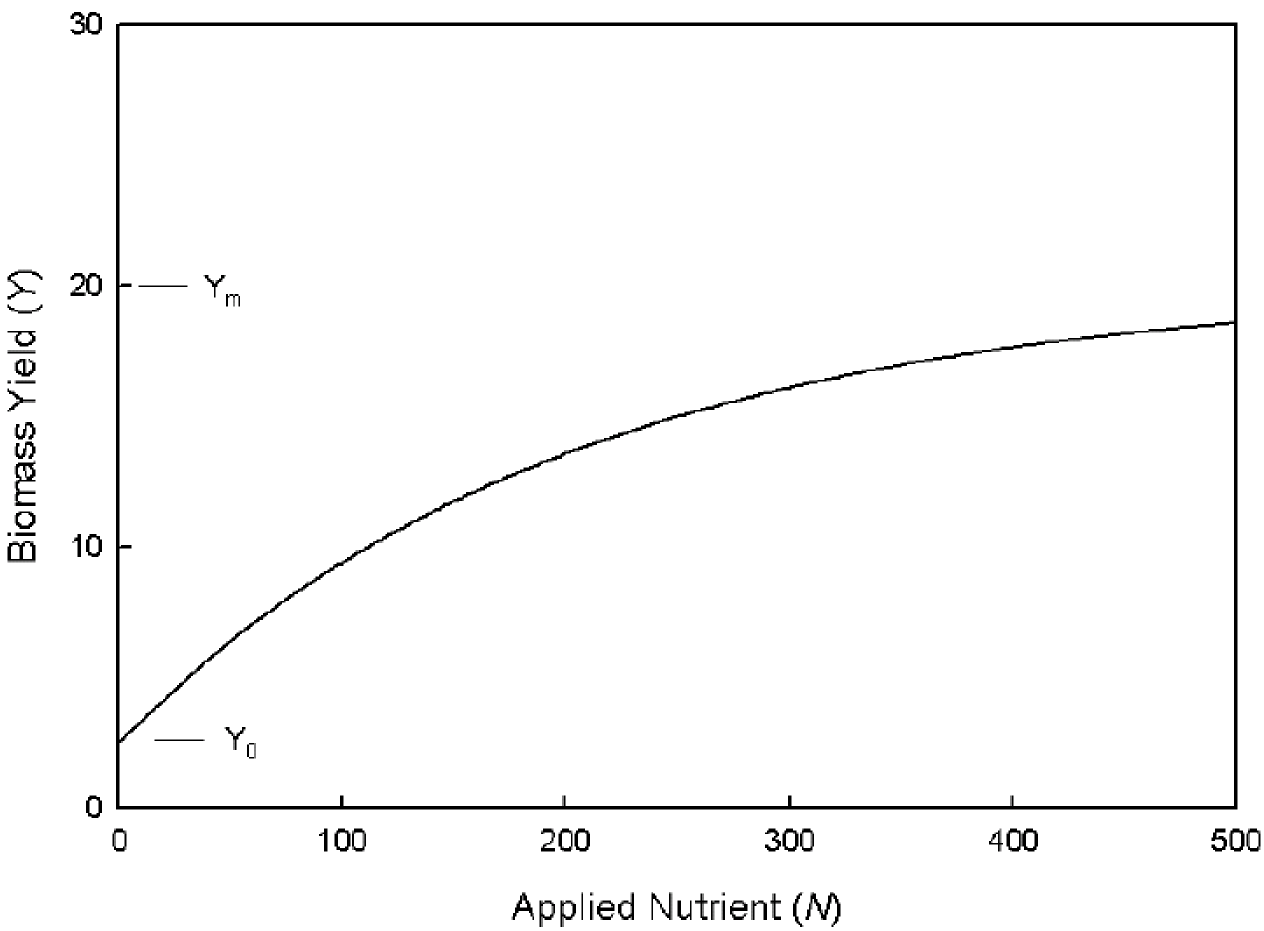

Figure 7: Typical response of Mitscherlich model to applied nutrient

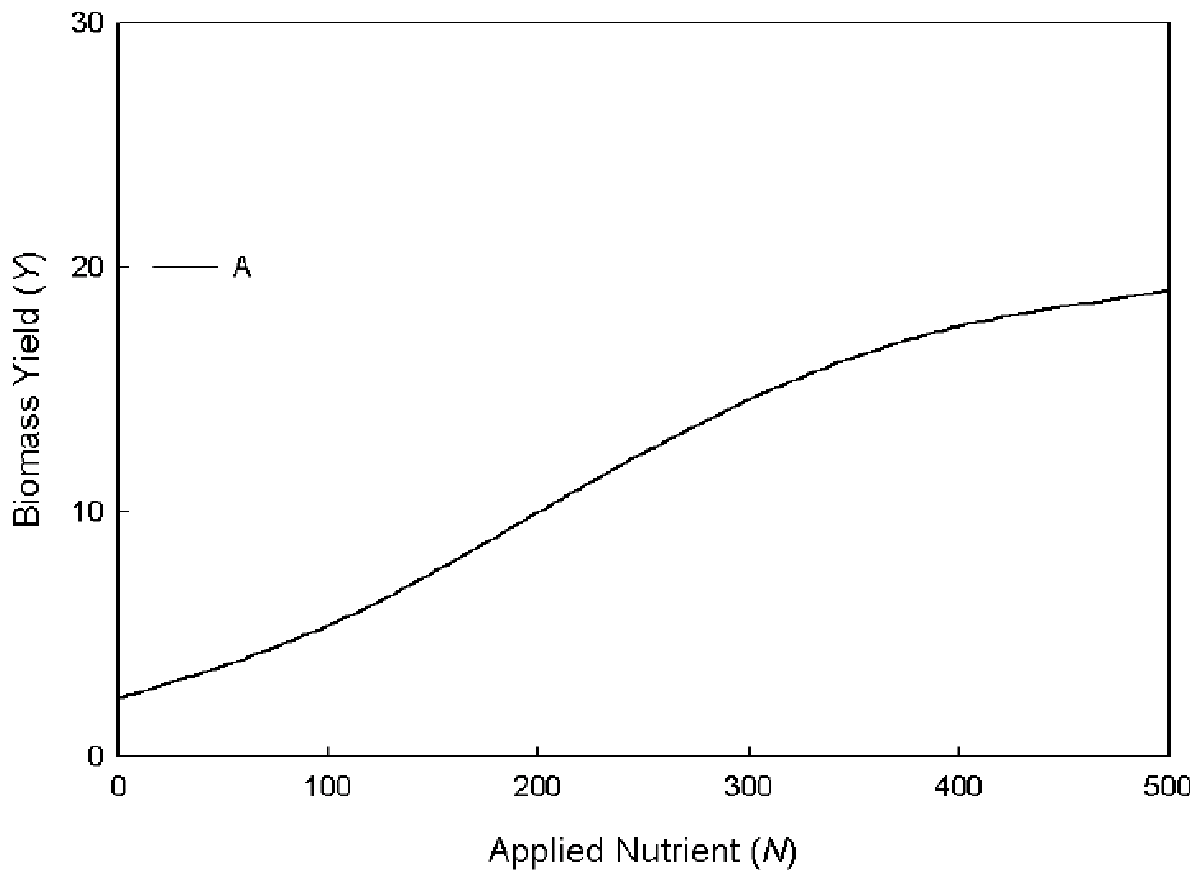

Figure 8: Typical response of logistic model to applied nutrient

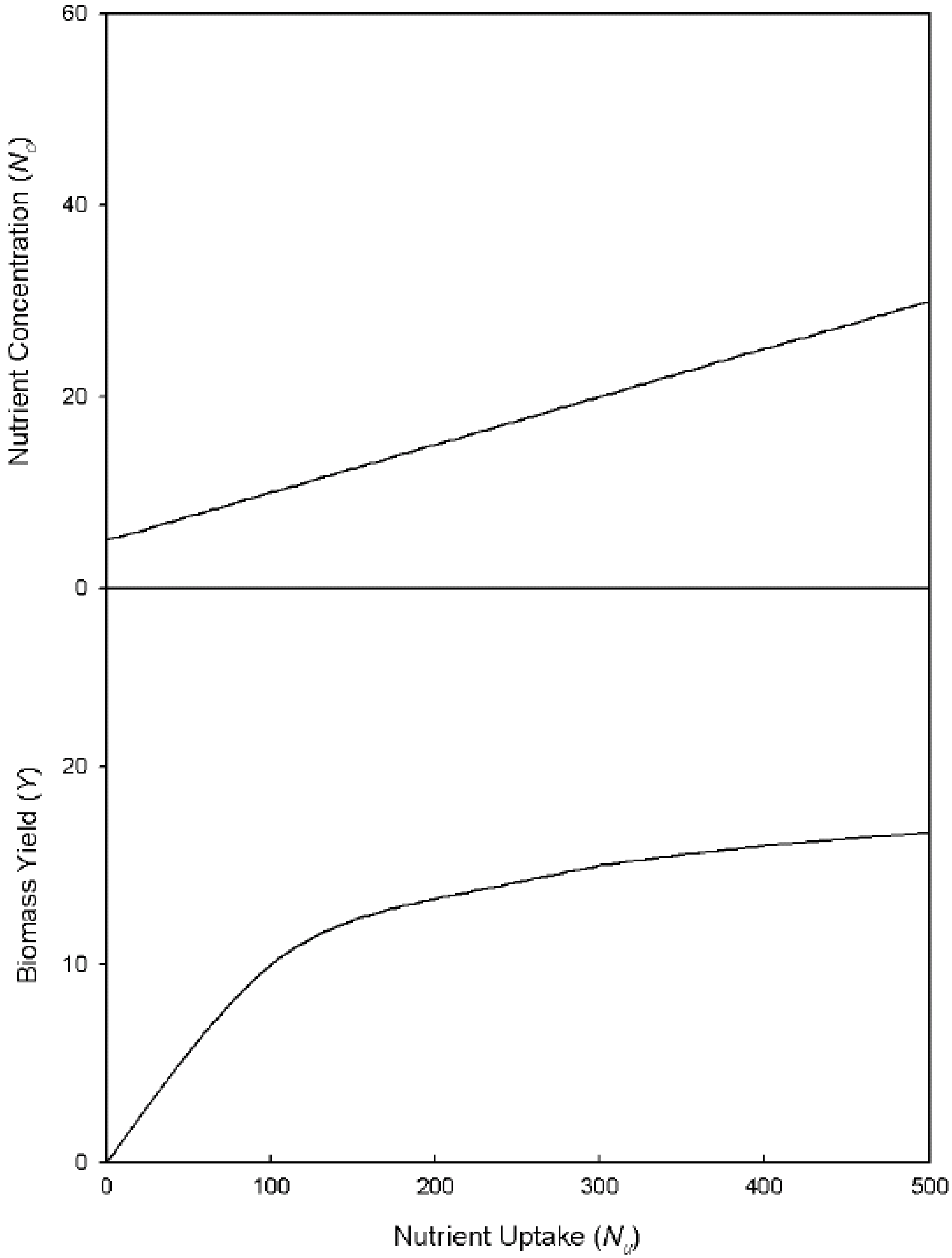

Figure 9: Phase relation for plant yield and plant nutrient concentration vs. plant nutrient uptake

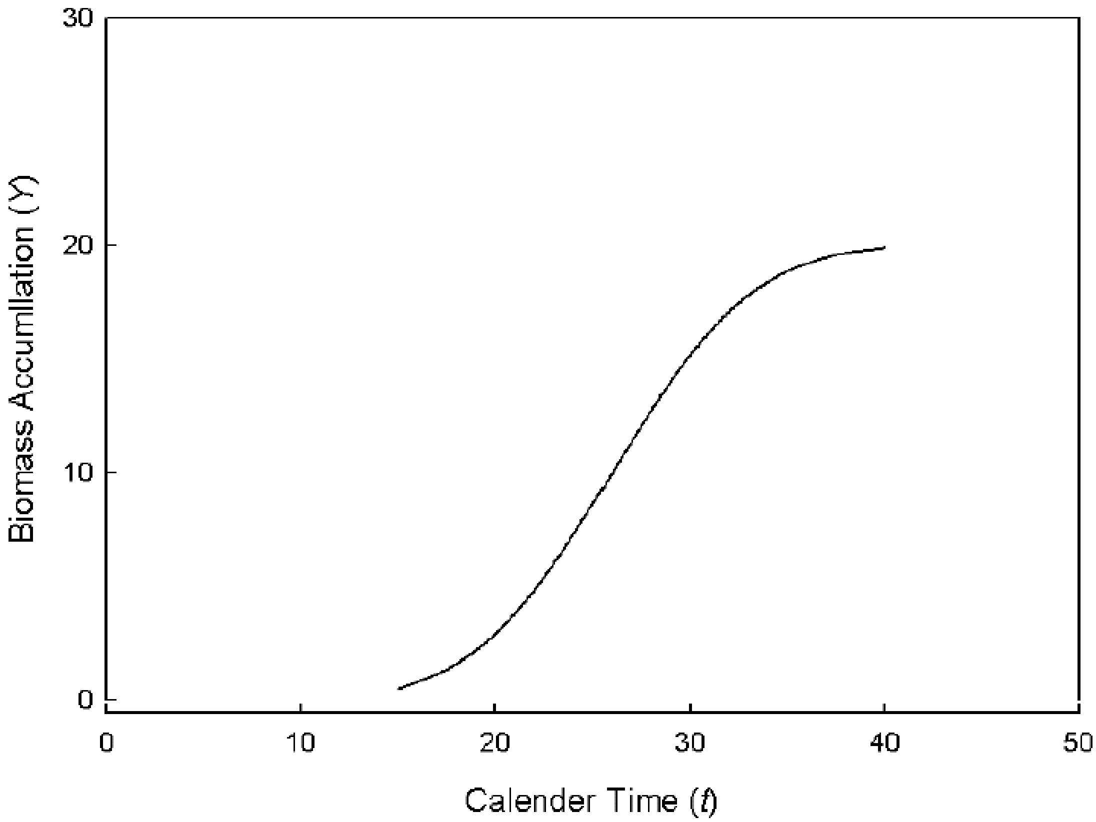

Figure 10: Accumulation of plant biomass with time for perennial grass

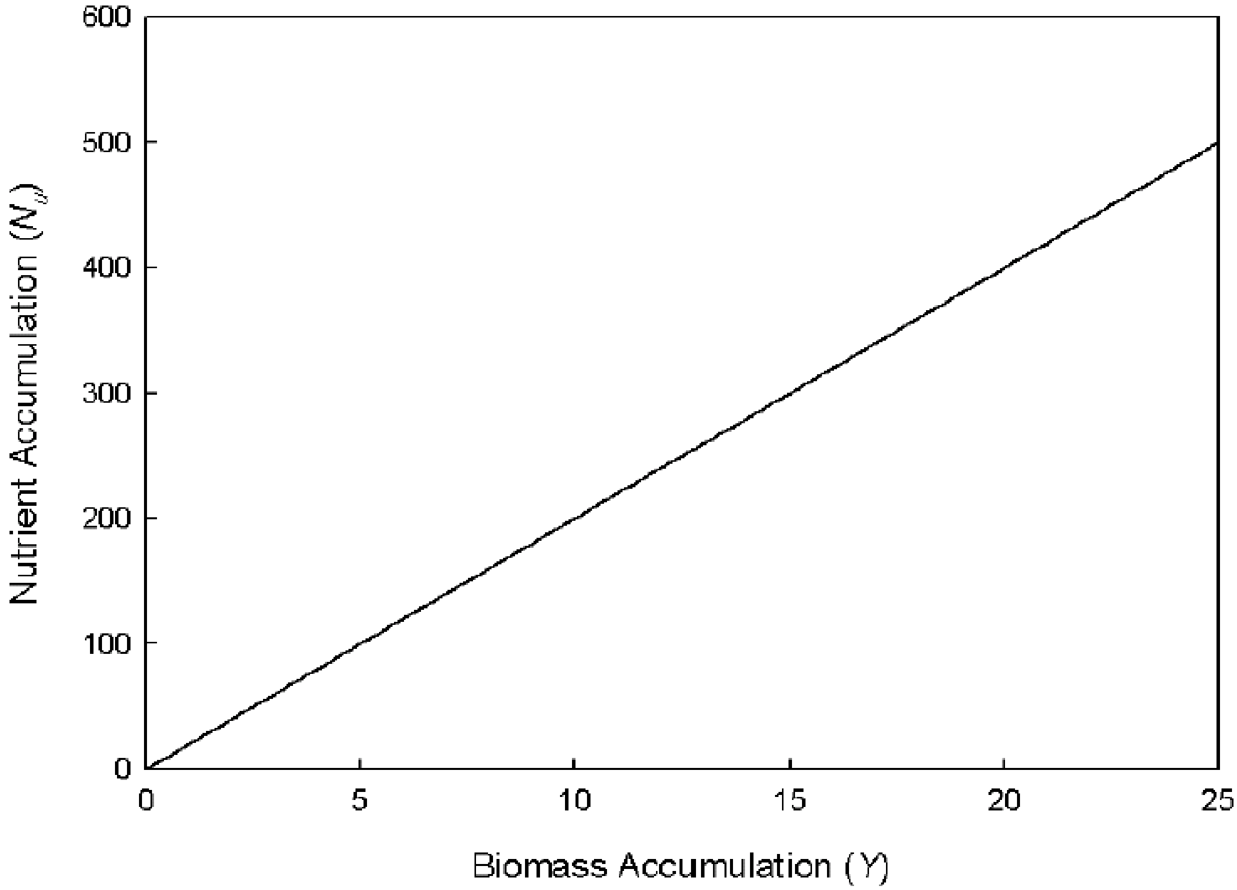

Figure 11: Coupling of plant nutrient uptake with biomass for perennial grass

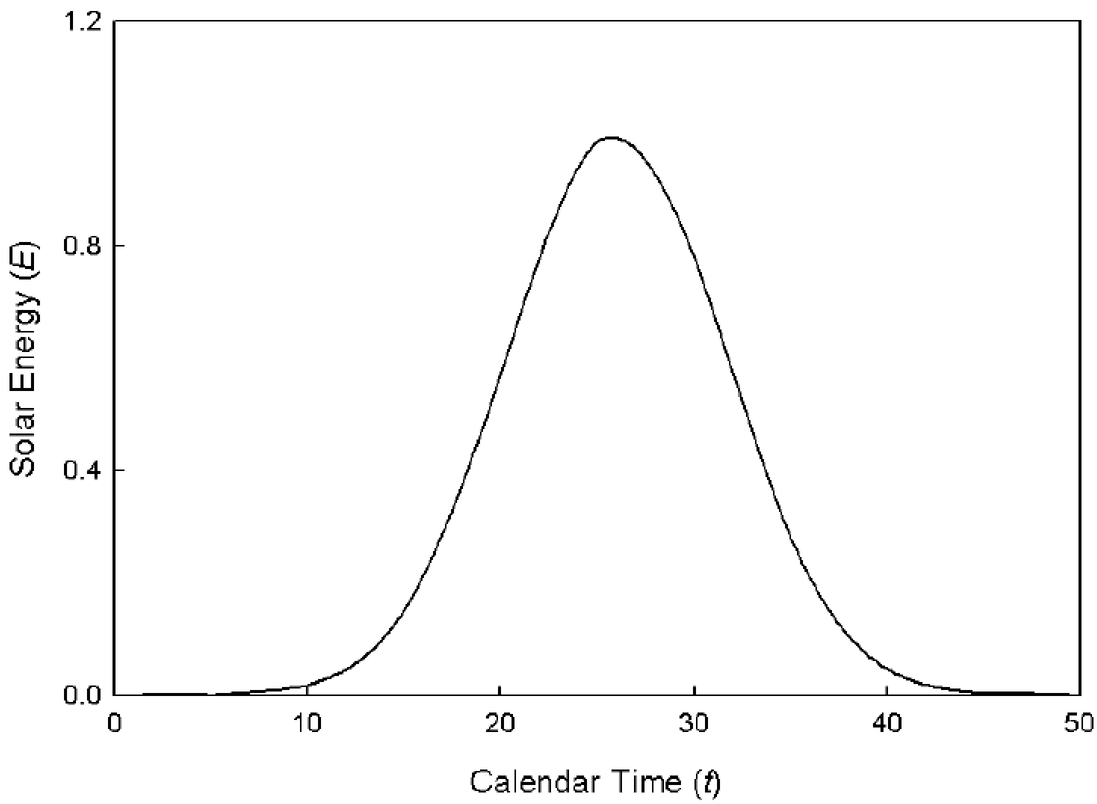

Figure 12: Distribution of solar energy with calendar time for northern hemisphere

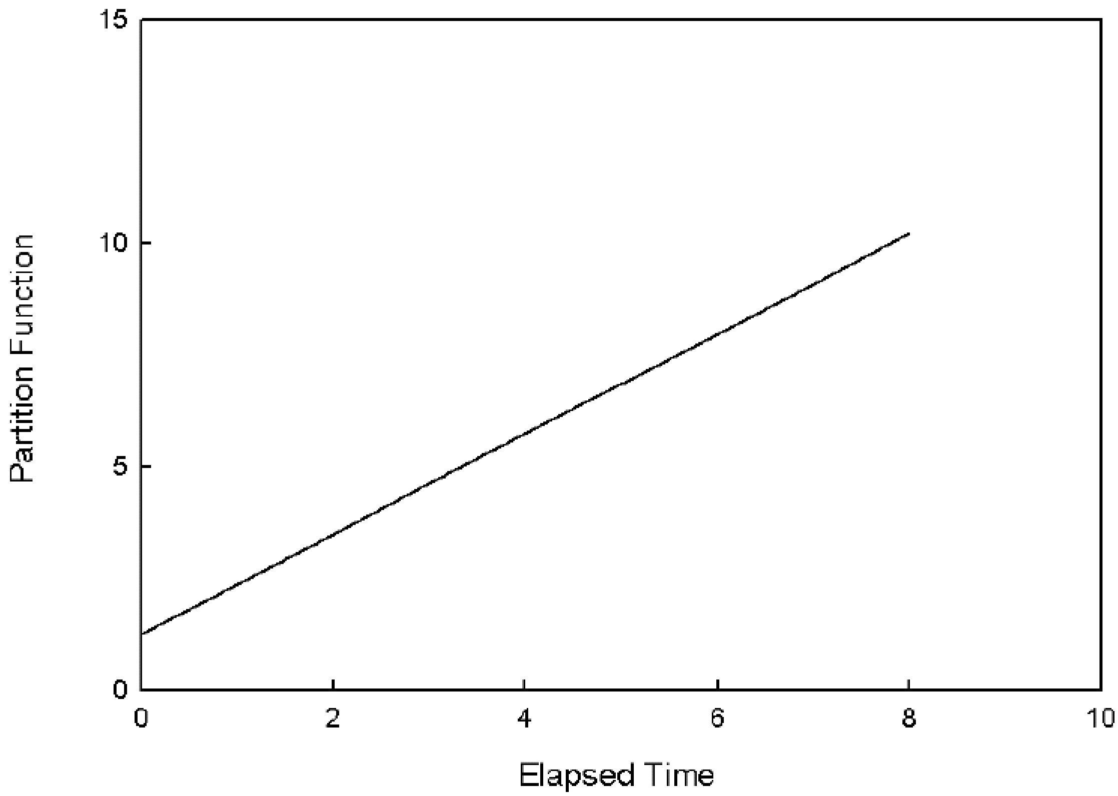

Figure 13: Dependence of partition function on elapsed time

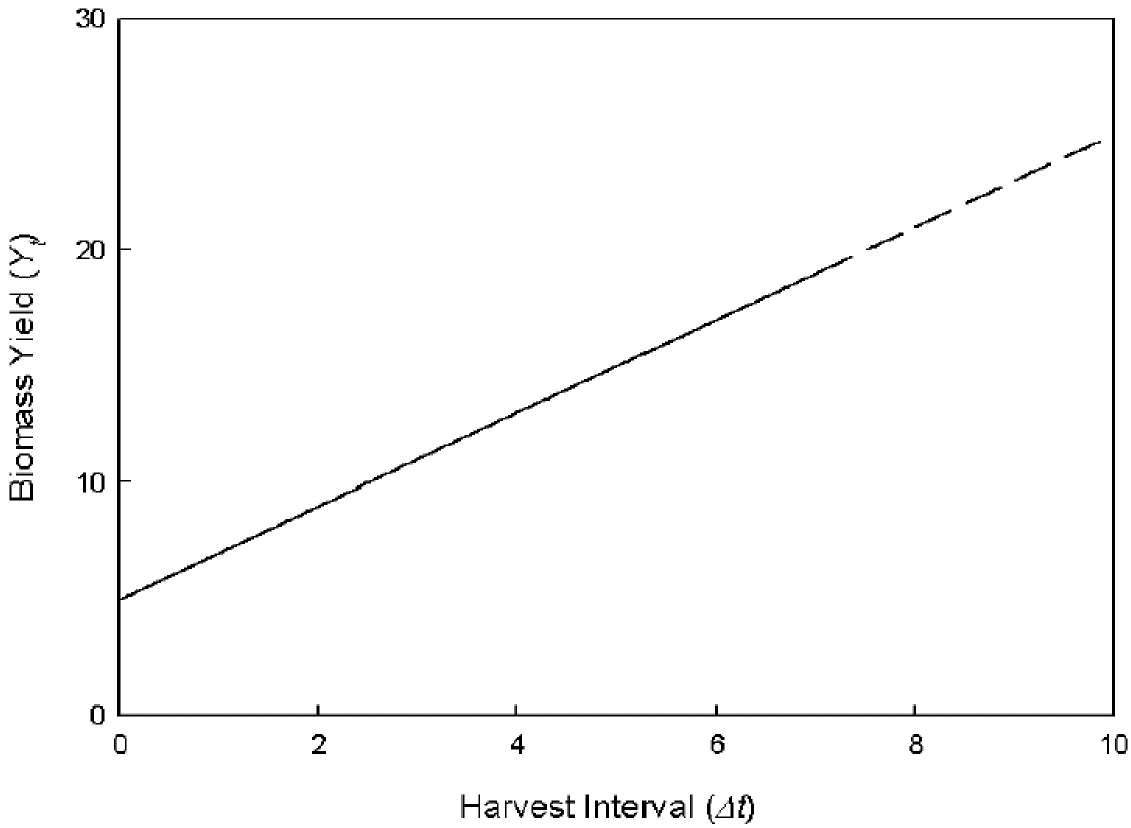

Figure 14: Dependence of biomass yield on harvest interval for phenomenological growth model

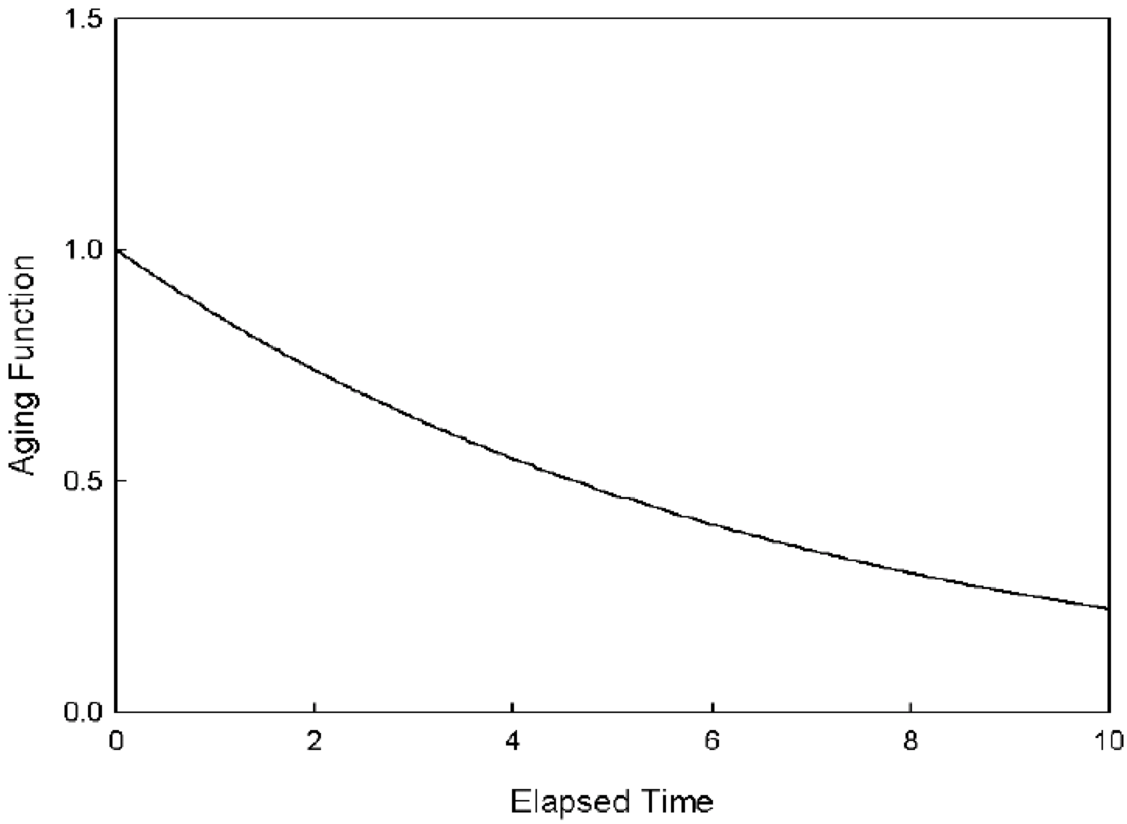

Figure 15: Dependence of aging function on elapsed time

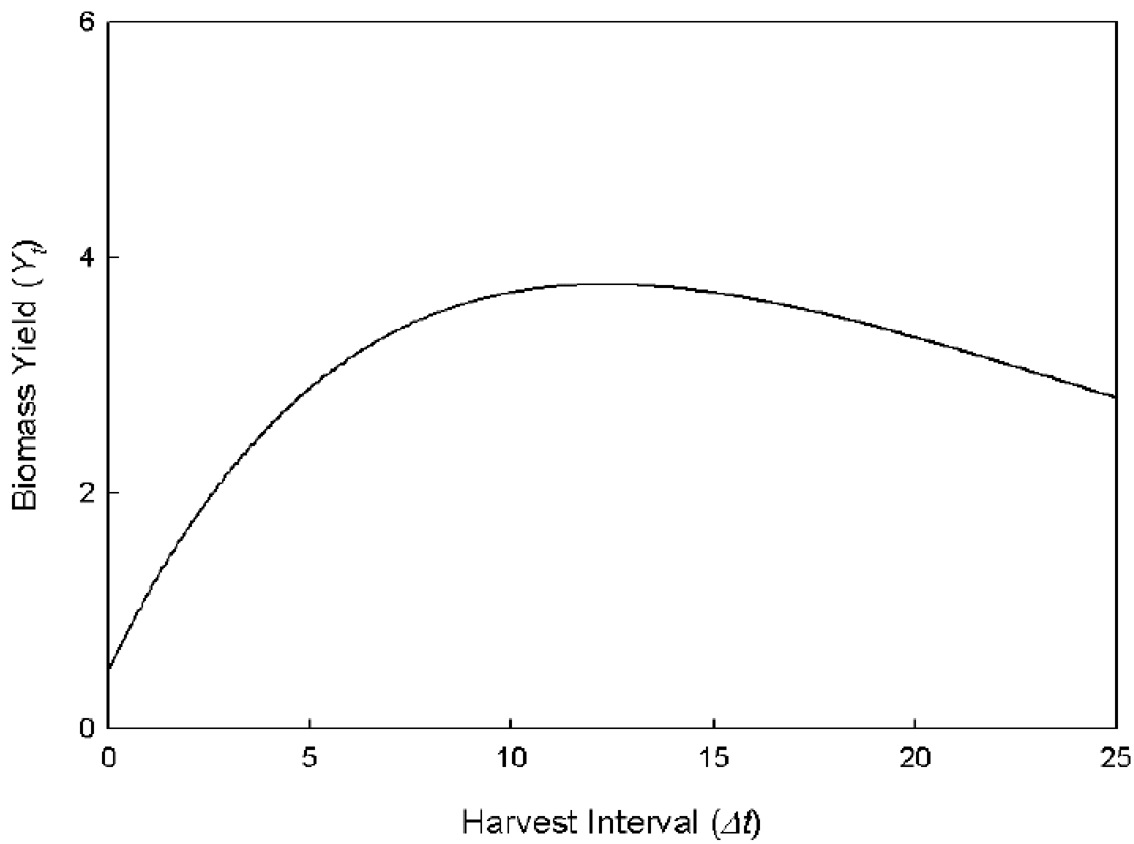

Figure 16: Dependence of biomass yield on harvest interval for expanded growth model

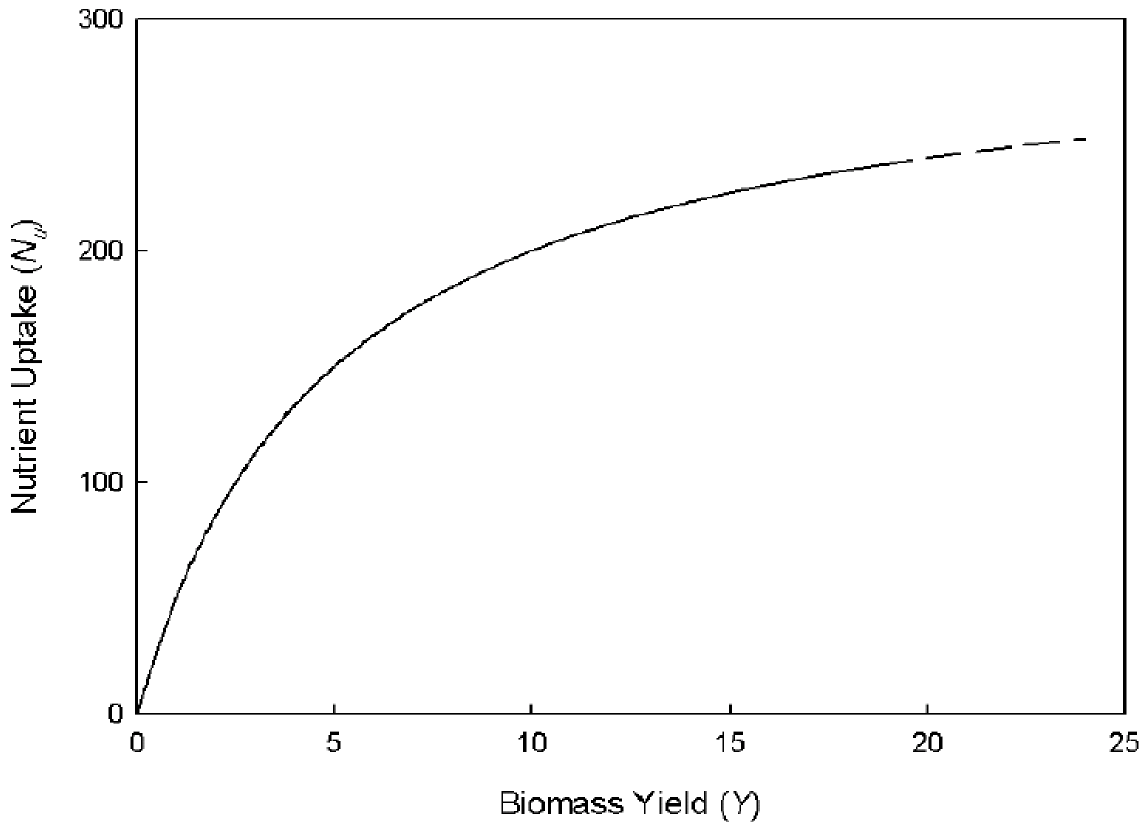

Figure 17: Phase relation for plant nutrient uptake vs. biomass yield for annual crop

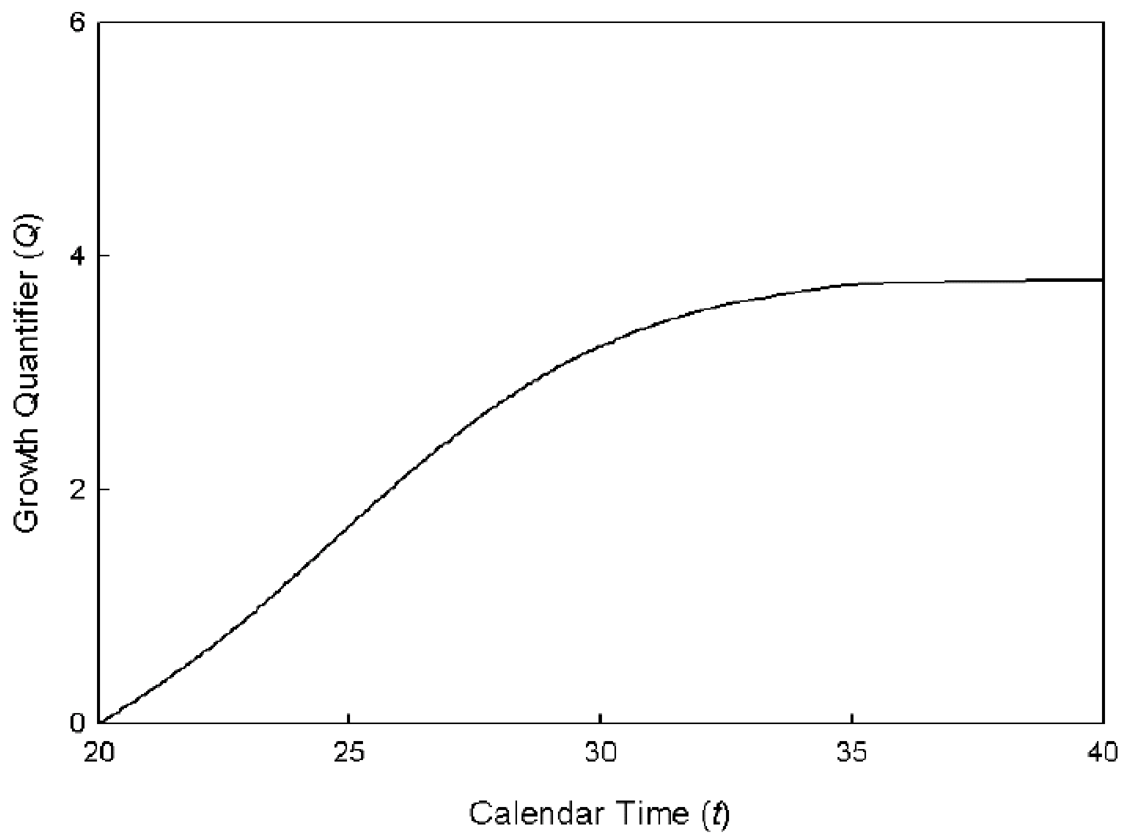

Figure 18: Dependence of growth quantifier on calendar time

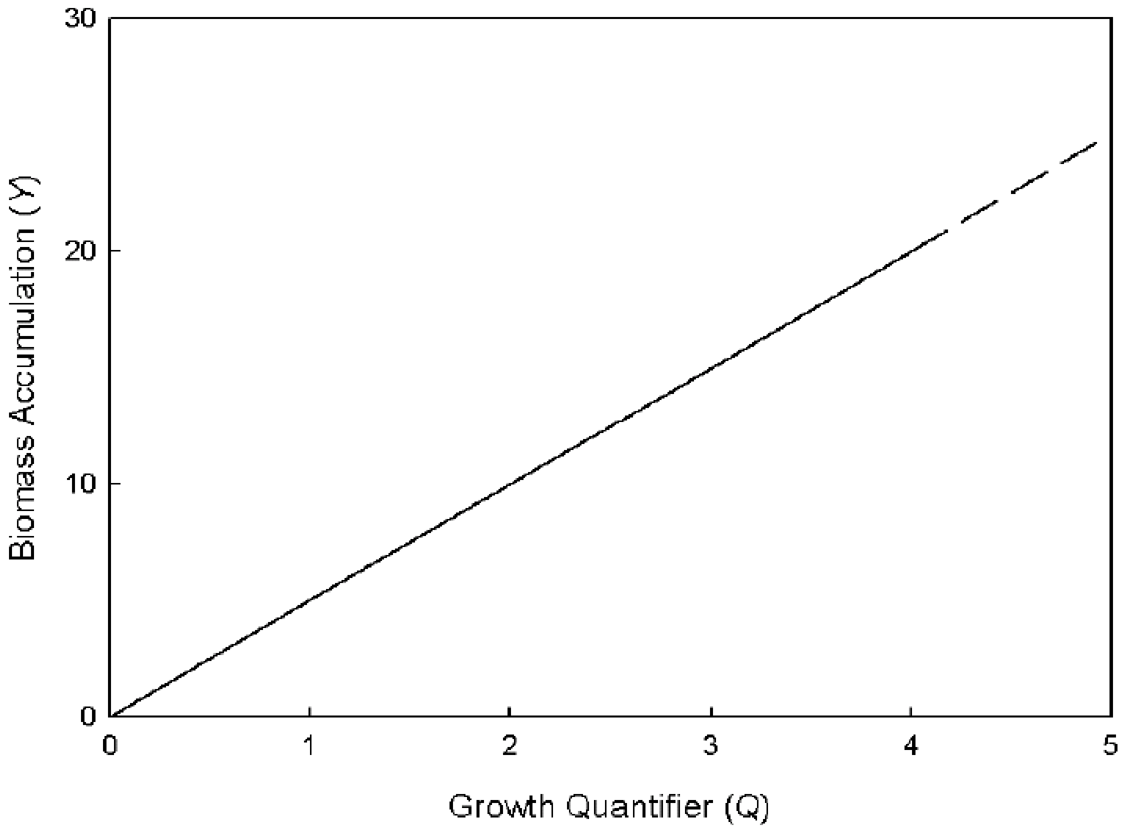

Figure 19: Dependence of biomass accumulation on the growth quantifier

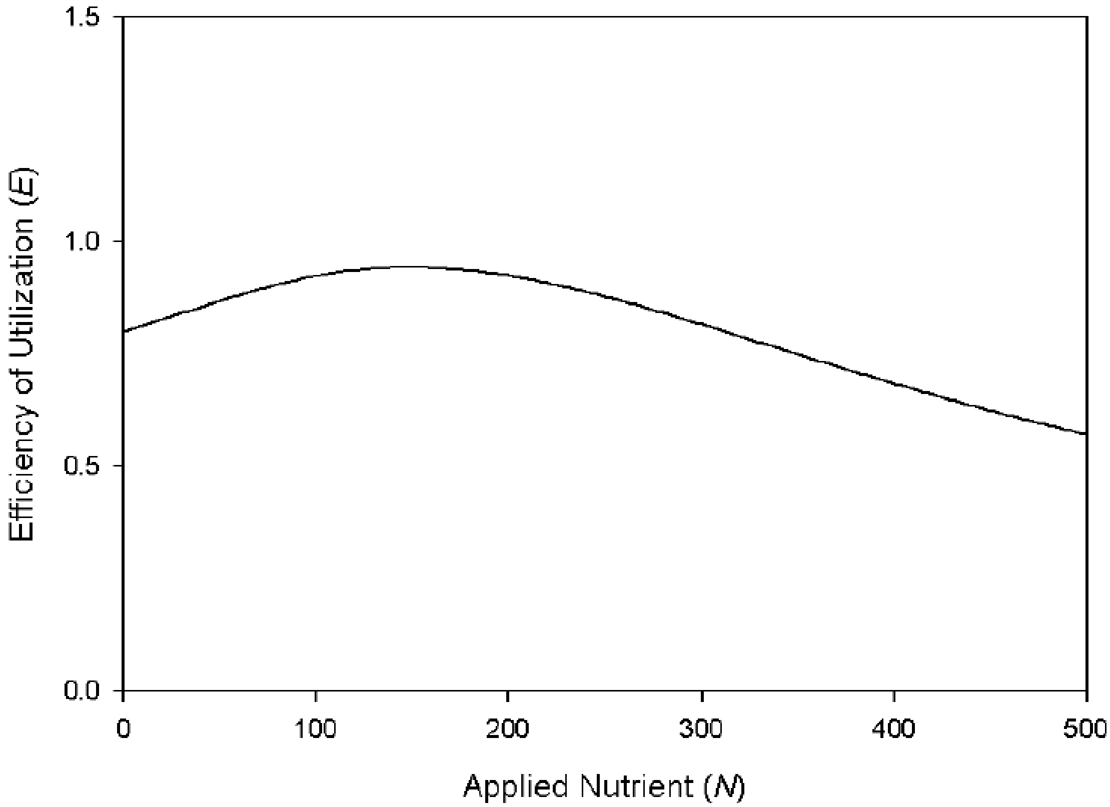

Figure 20: Efficiency of plant nutrient recovery vs. applied nutrient

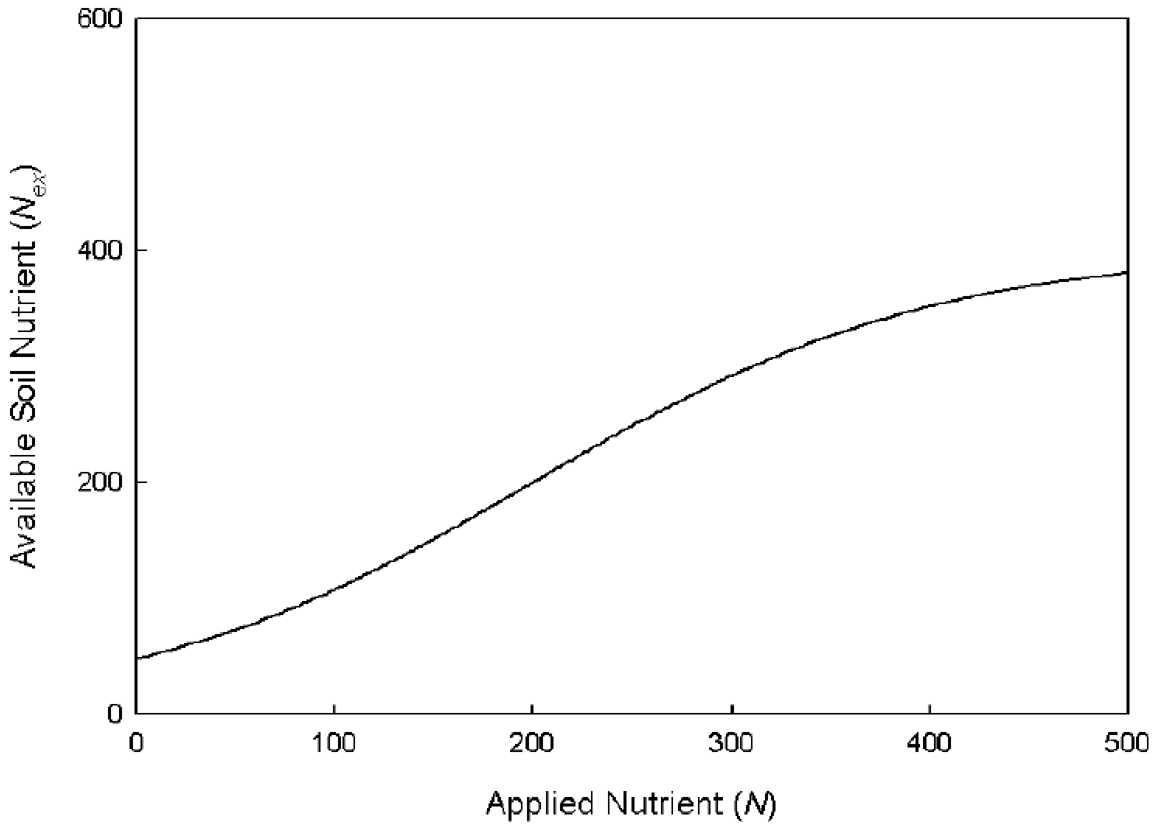

Figure 21: Available soil nutrient vs. applied nutrient

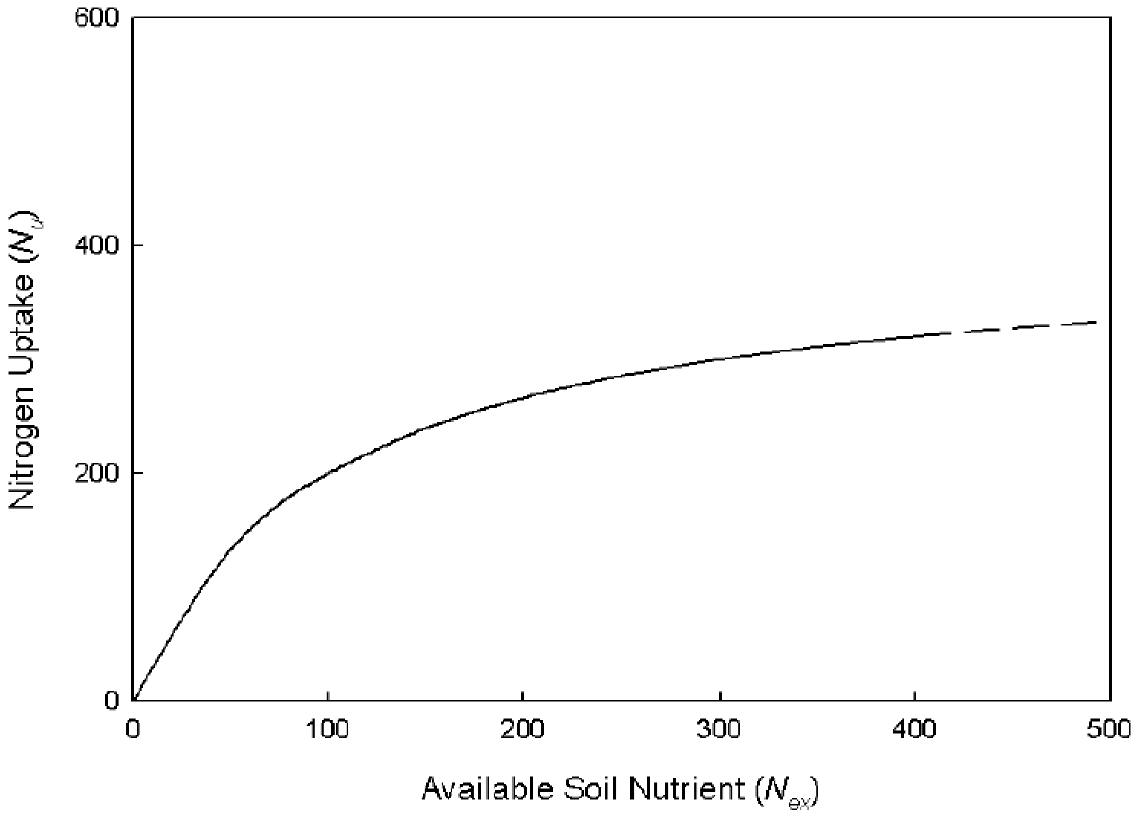

Figure 22: Phase relation for plant nutrient uptake vs. available soil nutrient