Ippen and Harleman (Ippen, 1966) considered the basic unsteady, one-dimensional (x,t) mass conservation for salt transport-diffusion, and derived the following expression for the resultant profile of salinity, s(x,t) in a non-stratified estuary:

$$\frac{s(x,t)}{s_o} = \exp\left\{-\frac{U_o}{2D_o'B}\left[N-(N-x)\exp\left(\frac{a_o}{h_o}(1-\cos\sigma t)\right)+B\right]^2\right\} \tag{9.1}$$

where $N = h_o u_o / a_o \sigma$. Here, U_o = fresh water outflow velocity, D_o' = diffusion coefficient, B = an empirical, diffusion-related coefficient, a_o = tidal amplitude, σ = tidal frequency, u_o = maximum tidal velocity at the mouth and h_o = mean water depth.

If the end of the intrusion zone is specified at s/s_o = 0.01, the maximum intrusion length, L_{max}, occurs when $\sigma t = \pi$, and the minimum, L_{min}, when σt = 0. Thus

$$L_{max} = N\left[1-\exp\left(-\frac{2a_o}{h_o}\right)\right] + B\left(3\frac{D_o'}{U_o B}-1\right)\exp\left(-\frac{2a_o}{h_o}\right) \tag{9.2}$$

$$L_{min} = B\left(3\frac{D_o'}{U_o B}-1\right) \tag{9.3}$$

In the above equations, the parameters D_o' and B must be known. In a real estuary, these parameters can be determined by measuring the average salinities at two points in the estuary at low tide.

Examples of average salinity distribution at high and low water slack as shown in Figs. 9.3a,b are based on the experiments in a model tidal channel at the Waterways Experiment Station (Harleman and Abrahams, 1966). This illustration of the difference in salinity intrusion between high water slack and low water slack (tidal range 3 cm) is essentially analogous to what would occur under an equivalent sea level rise. Note that Figs. 9.3a,b clearly show that salinity intrusion is a highly dynamic phenomenon, and that there is a 21.4 m difference in the distance of penetration between the high and low water events. It is thus evident that only a small head is required to cause a significant horizontal movement of the salt water front. Raising the sea level (increasing h_o in Eq. 9.1) would amount to pushing both curves up the estuary. The same would occur if h_o were increased by dredging. Likewise, decreasing, U_o, the outflow velocity, would cause further