4.3 PHYSICAL PRINCIPLES

4.3.1 Tidal Propagation

According to the equilibrium theory of tides, the tidal amplitude can be shown to be proportional (to leading order) to the fourth power of earth's radius, considering the moon-earth system. Since this number (6,378 km) is so large compared to any expected effect of sea level rise (i.e. increase in earth's radius), the corresponding change in the tidal range on this account would be negligible. In order to evaluate the effect of sea level rise on the tidal range, the nature of propagation of tide in very shallow waters must be considered.

The simplest description of tide in the dynamic sense is that of a shallow water wave moving along the x-direction with a speed or celerity, C_o. If a frictionless bottom is assumed, the wave equation is

$$\frac{\partial^2 \eta}{\partial t^2} = C_o^{\,2} \frac{\partial^2 \eta}{\partial x^2} \tag{4.1}$$

where $\eta(x,t)$ is the instantaneous water surface elevation. The celerity, $C_o = (gh)^{1/2}$ where g is acceleration due to gravity and h is water depth.

The effect of friction can be accounted for by including an additional term on the right hand side of Eq. 4.1. Thus, for example, this term under the assumption of linearized friction is $-gM\partial\eta/\partial t$, where M is an empirical coefficient accounting for the magnitude of bottom friction. Friction slows down the speed of propagation (celerity), decreases the current speed and reduces the tidal range compared with frictionless tide. The effect is depth-dependent, and it can be shown that in fact it varies with $h^{-1/3}$, which means that increasing the water depth would decrease frictional damping, thereby increasing the tidal range. Observations in the German Bight (southern North Sea) suggest this type of a trend, as will be noted later.

Within the estuary itself, increasing the water depth can have a drastic effect on the tidal range. The majority of present day estuaries are of holocene origin, having been formed since the last ice age and accompanying sea level rise. In some, sea level rise has caused the depths to increase while in others, sedimentation rates have been high enough for the depths to have "kept pace" with sea level rise. In a few cases, e.g. some estuaries in China (Qitang, for example), sedimentation rates have essentially exerted an