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Teaching Geometry to Students With Math Difficulties Using Graduated and Peer-Mediated Instruction in a Response-to-Intervention Model

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Geometry is a course that is increasingly required for students to graduate from high school. However, geometric concepts can pose a great challenge to high school youth with math difficulties. Although research on teaching geometry to students with mathematics difficulties is limited, teachers are challenged daily with providing support for youth who do not make adequate progress through high-quality core mathematics instruction. This article provides practical and promising practices for providing additional small group support (i.e., Tier II interventions) to youth that promote understanding of finding the area of a trapezoid. Specifically, the authors focus on the use of a graduated instructional sequence and peer-mediated instruction within an explicit instruction model. Recommendations and sample lessons are provided.

Keywords: geometry instruction, graduated instruction, mathematics difficulties, peer-mediated instruction, response to intervention, Tier II instruction

Introduction

Isn't a geometry course one of those extra courses that is not really necessary for high school students? More and more, the answer is a resounding, "No!" Changes in mathematics expectations have been fueled by the poor performance of American high school students. Results from the Program for International Assessment (Organization for Economic Co-operation and Development, 2010) indicate that 17 countries outperformed American students. Also disconcerting is that only 26% of twelfth grade students performed at or above the proficient level on the National Assessment of Educational Progress (National Center for Education Statistics, 2010).

In addition to poor performance on national and international assessments, the National Council of Teachers of Mathematics (NCTM) standards have influenced mathematics education across the United States. All but one state has revised their mathematics curriculum and content standards as a result of the NCTM standards (Woodward, 2004). For example, NCTM (2011) supports 4 years of mathematics in high school. By 2012, at least 23 states will require 3 years of high school mathematics and 11 states will require 4 years (Reys, Dingman, Nevels, & Teuscher, 2007). Geometry is among the courses that are increasingly required for a high school diploma. The most recent evidence suggests that at least 20 states have "course-based learning expectations" (although not specific course requirements) that include geometry for high school graduation (Reys et al., 2007, p. 2).

Geometry is one of the curricular areas covered by the NCTM content standards and is defined as a "branch of mathematics that deals with the measurement, properties, and relationships of points, lines, angles, surfaces, and solids; broadly: the study of properties of given elements that remain invariant under specified transformations" (NCTM, 2000). As identified in the NCTM standards (2000), the goals of geometry are for students to be able to accomplish the following:

- analyze characteristics and properties of two and three dimensional geometric shapes and develop mathematical arguments about geometric relationships
- specify locations and describe spatial relationships using coordinate geometry and other representational systems
- apply transformations and use symmetry to analyze mathematical situations
- use visualization, spatial reasoning, and geometric modeling to solve problems

In light of increasing expectations on students with regard to geometry, there are concerns that some students will have serious difficulties mastering geometric concepts. Youth classified as having a learning disability are at particular risk for failure in more advanced mathematics, such as geometry. For example, only 6% of youth with learning disabilities in 12th grade within public schools scored at or above the proficient level on the National Assessment of Educational Progress (National Center for Education Statistics, 2010).

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Youth with Math Difficulties and Geometry

Although youth with mathematics learning disabilities struggle, there exists a broader category of youth with mathematics difficulties who are also at risk for failure in mathematics. Youth with mathematics difficulties are identified as ranging from low average performance to well below average performance. More specifically, identification of mathematics difficulties is commonly defined as a standardized mathematical test score that falls below the 35th percentile (Mazzocco, 2007). Maccini, Strickland, Gagnon, and Malmgren (2008) summarized characteristics common to youth struggling in mathematics: (a) memory problems that include remembering and using multiple steps to solve a mathematics problem; (b) receptive (e.g., comprehending mathematics problems) and expressive (e.g., justifying an answer) mathematics vocabulary difficulties; and (c) cognitive deficits that may inhibit processing of mathematics concepts, procedural strategies, and rules. These characteristics may negatively affect secondary-level student performance in geometry, particularly when computing the area of geometric figures.

Geometry concepts such as finding the area of geometric shapes require the recall and recognition of the appropriate area formulas. For example, when asked to find the area of a trapezoid, youth must recall (a) geometric components of a trapezoid and (b) the formula for finding the area of a trapezoid. This instructional concept requires that youth use working and long-term memory; areas in which students with mathematics difficulties often exhibit deficits (Montague & Jitendra, 2006). Also, geometry requires the use of higher order cognitive skills (i.e., metacognitive skills). The metacognitive skills involved in computing the area of a geometric shape include prediction, planning, monitoring, and evaluation of mathematical information presented in each area problem (Carr, Alexander, & Folds-Bennett, 1994; Lucangeli & Cornoldi, 1997). More specifically, the metacognitive strategies in computing area involve (a) reading the area problem, (b) developing a plan for solving the problem, (c) monitoring understanding of the problem and completion of steps to solve for the area, (d) evaluating the accuracy of the steps and solution. Montague and Jitendra (2006) connected the development of such cognitive strategies to one's ability to understand and integrate procedural, declarative, and conceptual knowledge. To effectively compute the area of a geometric shape, youth must have a strong understanding of the declarative (i.e., factual information), procedural (i.e., steps and procedures), and conceptual (i.e., logical relationships) knowledge. Youth with mathematics difficulties often struggle with the integration and application of these three areas of knowledge, which can significantly affect achievement in secondary-level geometry classes.

Acknowledging that students with mathematics difficulties may have significant difficulties with learning mathematical concepts such as the area of geometric shapes is important in developing instructional strategies to meet the needs of a diverse group of learners. Slavin and Lake (2008) found that the most effective educational reform method to addressing the needs of a diverse group of learners involves changes to classroom instructional practices. Furthermore, youth with mathematics difficulties benefit from explicit instruction, combined with strategy instruction to assist in their ability to solve complex mathematics problems (Montague & Jitendra, 2006). One way to assist youth with mathematics difficulties within the general education environment is to provide additional small group support (Burns & Gibbons, 2008). The idea of supplementary instruction is aligned with Tier 2 within the Response to Intervention (RTI) model. Teachers may find that additional small group support, consistent with Tier 2 in RTI, may sufficiently support youth and allow them to succeed in geometry.

Meeting the Challenge

Our goal is to provide promising and practical instructional approaches to teachers that promote student learning of geometry concepts. Specifically, we focus on explicit instruction through small group, strategy-focused interventions (e.g., peer-mediated learning, use of a graduated instructional sequence), consistent with Tier 2 in the RTI model, which can be used for teaching the concept of *area*. First, we provide background on RTI and Tier II interventions, then we address the key components necessary for effectively using graduated instruction and peer-mediated learning for geometry instruction. Next, we supply ideas for combining the two approaches for geometry instruction and provide a concrete example for teachers on teaching students to find the area of trapezoids.

RTI and Tier II Interventions

The essential design of a response-to-Intervention (RTI) model for mathematics consists of three (or four) tiers of instructional support, with levels of intensity and individualization increasing at each tier. (Bryant & Bryant, 2008; Bryant, Bryant, Gersten, Scammacca, & Chavez, 2008) Tier 1, the preventative phase, ensures that all students receive high-quality core mathematics instruction. Student achievement is assessed through universal screening conducted periodically throughout the school year. Core instructional changes are made when screening data reveals a group deficit in mathematics skills. Students that are unable to make adequate gains without additional instructional support are moved into Tier 2 and receive supplemental small-group mathematics instruction. The third tier of an RTI model consists of one-on-one or small-group instruction for students who are not making sufficient growth toward grade-level mathematics benchmarks with Tier 2 supports. In some models, special education services are included within Tier 3, whereas others have an additional tier specifically for those identified as needing special education supports (Gersten & Beckman, 2009). Throughout each of the tiers, student progress toward grade-level mathematics benchmarks is continuously monitored, with the frequency of monitoring increasing at each tier. The level and form of instructional support within an RTI model is heavily influenced by a student's ability to make adequate gains in skill areas (Fuchs & Fuchs, 2006; Kashima, Schleich, & Spradlin, 2009). As noted,

students in the general education classroom that are not making adequate progress toward grade-level expectations with Tier 1 level of support are provided additional intervention support through Tier 2 of the RTI service delivery model. Empirical research on the effect of Tier II has generally shown that when groups of 4–6 students are provided at least 30 minutes of intervention in addition to core instruction for 3–5 days of the week, there are significant increases in student performance in mathematics (Fuchs, Fuchs, & Hollenbeck, 2007). At the secondary level, Tier II interventions for students with mathematics difficulties have shown to significantly improve math achievement when provided at least 2 times per week, for 30 min each session (Calhoon & Fuchs, 2003). In addition, Fuchs and colleagues (2008) postulate that there are key components to effective Tier II interventions for secondary students experiencing difficulties in mathematics. Tier II instruction must provide a connection back to the core instructional curriculum by insuring that the instructional content of Tier II aligns with that of Tier I. Tier II instruction should increase the frequency and intensity of instruction received at Tier I, which is accomplished through providing students with increased exposure to mathematics material and additional opportunities to practice learned skills. In addition, Tier II instruction includes explicit and systematic instruction, guided and independent practice, and cumulative review of previously learned material (Fuchs, 2011). By incorporating these components into Tier II interventions students will have opportunities to increase conceptual knowledge of the subject area, which may further their ability to maintain and transfer learned concepts and skills (Witzel, Riccomini, & Schneider, 2008).

Although there is support for broad essential components of Tier II mathematics interventions, limited information is available that details effective interventions that can be incorporated into secondary mathematics instruction in an RTI model. Of the research that has focused on secondary-level mathematics, two separate instructional practices have been identified as effective in teaching mathematics skills. Graduated instruction and peer-mediated instruction are small group interventions that align with the NCTM Principles and Standards (NCTM, 2000) for geometry instruction. Both instructional practices have individual lines of empirical support for application within mathematics (Allsopp, 1997; Calhoon & Fuchs, 2003; Mercer & Miller, 1992; Witzel, 2005). While limited research studies have examined the effectiveness of graduated instruction and peer-mediated instruction in relation to geometry skills (Cass, Cates, Smith, & Jackson, 2003), researchers have noted the potential of combining these instructional approaches to support struggling learners during geometry instruction (Mulcahy & Gagnon, 2007).

Moreover, we acknowledge the importance of providing recommendation for teachers currently in the field based on present understanding and available literature. As such, the combination of graduated instruction and peer-mediation instruction, two compatible and validated instructional approaches within an explicit instruction model will be the basis of our suggestions and lesson example. In this article, we discuss each instructional approach and then recommendations for combining the approaches.

Graduated Instruction

Youth with mathematics difficulties could benefit from understanding, "the fundamental mathematics concepts prior to advancing to generalization of rules, facts, or algorithms" (Maccini, Gagnon, Mulcahy, & Leone, 2006, p. 210). The National Council of Teachers of Mathematics Principles and Standards for School Mathematics emphasizes the importance of conceptual understanding of mathematics as this approach enhances students' learning and their post school opportunities (NCTM, 2000). Using a graduated sequence that includes hands on manipulatives to teach difficult mathematical concepts in a concrete and progressively more abstract manner allows students to understand abstract concepts more easily (Devlin, 2000). In addition, graduated instructional sequences have been found to be an effective method for teaching students the procedural steps associated with a mathematics problem, as well as the conceptual connections (Witzel et al., 2008).

One example of this type of instruction in mathematics is known as the Concrete–Representational–Abstract (CRA) technique (Steedly, Dragoo, Arafeh, & Lake, 2008). During the CRA sequence, instruction follows a three-part sequence beginning with the concrete stage (see Table 1). This stage involves students physically manipulating materials such as geometric shapes (i.e., parallelograms or trapezoids) to display and solve geometry area problems (Miller & Mercer, 1993). Students who use concrete materials develop more comprehensive mental representations, better apply mathematics to life, often show more motivation and on-task behavior, and exhibit a deeper understanding of mathematical concepts (Clements, 1999; Moyer, 2001; Sowell, 1989). The representational or second stage, moves the instruction from manipulatives to pictorial representations, which could include drawings or stamps of geometric shapes (Maccini & Gagnon, 2000). The final teaching stage, known as abstract, is enacted when students are ready to move to symbols, numbers, and notations to represent the area problems (Harris, Miller, & Mercer, 1995). Each of these three phases build on one another and students must understand what has been taught at each step before teachers provide opportunities for learning at the next stage (NCTM, 2000).

The CRA model has been effective in teaching a variety of mathematical concepts including basic math facts, coin sums, multiplication, place value, perimeter, fractions, and

Table 1. Graduated Instruction

Phase	Instructional practice	Examples
Concrete	Hands-on materials	Geometric shapes and figures
Representational	Picture representation	Pictures of geometric shapes and figures on a worksheet
Abstract	Abstract notations	Use of area formula for a parallelogram

algebra (Butler, Miller, Crehan, Babbitt, & Pierce, 2003; Cass, Cates, Smith, & Jackson, 2003; Mercer & Miller, 1992; Miller, Harris, Strawser, Jones, & Mercer, 1998; Miller, Mercer, & Dillon, 1992; Peterson, Mercer, & O'Shea, 1988; Witzel, 2005). It is also effective with students from multiple grade levels (Maccini & Gagnon, 2000), for those with diverse levels of mathematics achievement (Mercer & Miller, 1992; Witzel, 2005), and in a variety of instructional groupings including whole group instruction (Witzel, Mercer, & Miller, 2003), small group instruction, and one-on-one instruction with a student (Miller, Mercer, & Dillon, 1992). Last, CRA has been implicated in increased levels of retention and transfer of skills (Butler et al., 2003; Cass et al., 2003).

Specific to geometry, researchers (Cass, Cates, Smith, & Jackson, 2003) reported the efficacy of the CRA technique for teaching perimeter and area problem solving to middle and high school students. In the Cass and colleagues' (2003) study, three high school students who were classified with a learning disability were selected to receive a combination of CRA instruction in conjunction with modeling, guided practice, and independent practice. In fewer than 7 days, the instruction resulted in acquisition and maintenance of the area and perimeter skills for all three students. Cass and colleagues (2003) also noted that students maintained gains.

The results of the Cass and colleagues (2003) study, as well as numerous other studies showing benefits of the CRA (as well as the RA sequence) in other areas of secondary mathematics (see Butler, Miller, Crehan, Babbitt, & Pierce, 2003; Maccini & Ruhl, 2000; Witzel et al., 2003), indicate the promise of CRA with teaching geometry. Given our current knowledge, it is recommended that CRA be used, particularly when teaching area to youth with mathematics difficulties, as a method for supporting the understanding and integration of conceptual knowledge with procedural and declarative knowledge. The Appendix contains a practical application example of CRA instruction for geometry. This example demonstrates the graduated sequence of CRA instruction in relation to finding the area of geometric shapes. The classroom teacher facilitates this CRA sequence by incorporating explicit instruction and modeling of appropriate geometric strategies to solve area problems.

Peer-Mediated Instruction

According to the NCTM standards, mathematics instruction should be active, social, and interactive (NCTM, 2000). Effective mathematics instruction at the secondary level involves (a) the use of small, interactive groups; (b) the use of directed questioning and responses; and (c) using extended practice with feedback; all of which are components of peer-mediated instruction (Swanson & Hoskyn, 1998). Specifically, peer-mediated instruction involves pairs of students working in a collaborative manner on structured, individualized activities (Kunsch, Jitendra, & Sood, 2007).

A form of peer-mediated instruction that has received substantial support is Peer Assisted Learning Strategies (PALS; Fuchs, Fuchs, Mathes, & Martinez, 2002). Originally developed in alignment with Tennessee state mathematics curriculum standards, PALS uses peer-tutoring dyads and a systematic approach to practicing instructional content. PALS is typically used 2–3 days per week as a supplement to core mathematics instruction and consists of coaching and independent practice (see Table 2). Student dyads are configured based on level of ability, with higher and lower ability students paired together. Each student has an opportunity to act as the tutor and tutee during each PALS session. Tutors engage in teaching their peer through modeling procedural steps for problems, asking structured questions to promote conceptual knowledge, and providing immediate feedback. Independent practice is used within PALS as a method of reinforcing newly learned material and practicing previously taught information.

Table 2. PALS Implementation

	PALS Implementation Procedures
Step 1	Engage in mediated verbal rehearsal
Step 2	Provide step-by-step feedback
Step 3	Engage in frequent verbal and written interaction
Step 4	Reciprocity of roles as tutor and tutee

Calhoon and Fuchs (2003) examined the applicability of peer-mediated instructional strategies such as PALS at the secondary level with students identified as having learning difficulties and behavioral concerns. This study examined the effect of the PALS program in conjunction with curriculum-based measurement (CBM) on secondary students' mathematics performance. The participants were 92 students from 10 classrooms representing Grades 9–12. These students were identified as performing significantly below grade-level expectations and received instruction in a self-contained classroom. Classrooms were picked to receive PALS/CBM or continue mathematics instruction as usual. PALS/CBM was implemented twice weekly and CBM was conducted weekly for 15 weeks. PALS/CBM students improved in computation mathematics skills significantly more than students in classrooms conducting instruction as usual without PALS/CBM.

In relation to higher order cognitive skills and problem solving, Allsop (1997) conducted a study to determine the effects of peer-mediated instruction on algebra performance for at-risk and non–at-risk students. A sample of 262 students between the ages of 12 and 15 years was used, with 38% of the sample being students at risk for failure in mathematics. Results of the study indicated that peer-mediated instruction had a significant effect on student ability to perform higher order cognitive skills associated with algebra problem solving. In addition, analysis procedures indicated that students who were at risk for failure exhibited slightly greater gains in problem solving abilities than did students who were not at risk.

Peer-mediated instruction allows for a structured method of learning by providing students with explicit details of their roles and responsibilities in the tutoring relationship (see Table 4). In addition, this method of instruction promotes mathematical discourse and academic engagement, which can benefit youth within Tier II intervention (Maccini et al., 2008). Specifically, engaging in mathematical discourse assists students with mathematics difficulties through use of

Table 3. Instructional Steps for Classroom Teacher

Step number	Step	Example
1	Review previous lesson	"Let's review how to find the area of a square and triangle."
2	Introduce the purpose	"We will be using some of the concepts we learned in working with squares and triangles to find the area of a trapezoid."
3	Model using appropriate Concrete–Representational–Abstract stage methods	Concrete: "Let's use our trapezoid shapes and rulers to find the area." Representational: "Let's look at this picture of a trapezoid." Abstract: "Find the area using this information: b1 = 5 b2 = 7 h = 12."
4	Peer tutoring	Students work in dyads to practice learned skill using each of the Concrete–Representational–Abstract steps.
5	Independent practice	Curriculum-based measurement measures are given to each student to measure progress and maintenance of previous material.

the metacogntive strategies necessary to solve mathematical problems. Peer-mediated instruction requires that students engage in monitoring their behaviors and engagement, as well as those of their tutoring partner. This self-monitoring promotes students' ability to plan and evaluate appropriate methods to solve mathematical problems that they would face in a geometry class.

Combining CRA and Peer-Mediated Instruction

As described, CRA and peer-mediated instruction are two powerful Tier II interventions to assist youth in mathematics. Additional research is needed to evaluate the combined efficacy of these approaches, particularly with regards to geometry instruction. However, in the interim it is recommended that teachers employ a combination of the approaches to maximize student benefit (see Tables 3 and 4). Simultaneous use of CRA and peer-mediated instruction has the potential to support the learning needs of students via addressing key principles within the NCTM Standards (NCTM, 2000). Specifically, the combination of approaches will (a) provide all students with a high level of support through increased chances for feedback and assistance (Principle 1: Equity); (b) provides a coherent, focused method of instruction that will allow teachers to know and understand what areas their students need more assistance with (Principles 2 & 3: Curriculum and Teaching); (c) allow students the opportunity to learn geometry in such a way that will develop their overall conceptual understanding abilities (Principle 4: Learning); and (d) provide ongoing assessment data that can be used by teachers to show student progress and inform decisions about future interventions and instruction (Principle 5: Assessment).

Table 4. Peer Tutoring Instructional Plan

Step	Description	Example
Introduce the purpose	Tutee reads	• "You will be using some of the concepts we learned in working with squares and triangles to find the area of a trapezoid"
Guided practice	Tutor prompts tutee. Additional prompts provided as needed.	• "What parts of the trapezoid will we use to calculate the area?" (answer: bases and height) • "How do we find the base?"
	Roles switched after each has one chance in a role.	• "How do we find the height?" • "What is the formula that we use to calculate the area? Write this formula. Fill in the information we have." • "What is the area?"

By using a graduated instructional sequence and high levels of guided practice through peer-mediated instruction, classroom teachers are able to provide students with math difficulties increased exposure to curriculum content and further develop their cognitive strategies for mathematical problem solving. To illustrate the practical use of CRA and peer-mediated through explicit instruction within an RTI model for geometry, we provide a detailed lesson that integrates these approaches to help students solve contextualized area word problems (see the Appendix). In this lesson, teachers provide explicit instruction and modeling for solving for the area of specific geometric shapes. This instruction follows the CRA graduated instructional sequence within each step of the CRA model, followed by opportunities for students to practice using peer-mediated instruction.

In addition to the instructional details outlined in the sample lesson, it is recommended that teachers integrate progress monitoring throughout instruction. A key component of RTI is the ongoing assessment of students' progress toward established goal (Burns, 2008; Burns & Gibbons, 2008; Riccomini & Witzel, 2010). Curriculum-based measures containing items that represent the annual geometry expectations outlined in the NCTM standards (2000) and district-level standards should be used to monitor students' progress and growth toward grade-level benchmarks (see also Foegen, 2000, 2008; Foegen, Jiban, & Deno, 2007). Detailed skill analysis of student performance on these measures can provide teachers with information related to specific areas of difficulty for each student (Riccomini & Witzel, 2010).

Monitoring of student progress should be completed during each phase of CRA and peer tutoring to guide the teacher's decision to move on to the subsequent phase of CRA (Hudson & Miller, 2006). Specific directions pertaining to the scheduling of progress monitoring assessment are detailed in the practical intervention example in the Appendix.

Final Thoughts

General and special education geometry teachers are faced with the difficult daily challenge of assisting students with mathematics difficulties who require Tier II interventions. Although limited information is available concerning the most efficacious instructional methods for supporting youth with mathematics difficulties, it is critical that teachers rely on the current best evidence in the field. Our discussion and example of using CRA and peer-mediated instruction within an explicit instruction model for students requiring Tier II interventions, provides a promising starting point for teachers. There is great potential for these instructional approaches to promote conceptual understanding, provide for interactive learning, and provide students with disabilities the greatest opportunity for success within geometry coursework.

Author Notes

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Appendix

Example for Area of a Trapezoid

Objective: Students will solve contextualized area word problems with Tier II support that includes CRA and peer-mediated instruction.

Review: Teacher engages in reviewing previously learned skills that are perquisites for current lesson. This is used as a time to assess student understanding of prior lessons and reinforce previously taught skills.

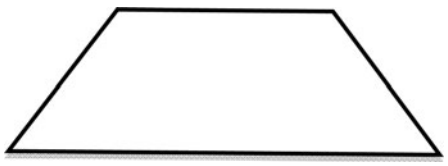
Teacher says: "Before we start today's lesson we will review the following: remember that we can represent one square unit with each of these blocks?" [Teacher shows students square unit blocks. Teacher represents various different amounts of blocks to show students that just by adding the blocks together, they get the number of square units.]

Advance Organizer and Purpose: Teacher states the objective of the lesson before modeling skills for students.

Teacher says: "Now we will use our geometric trapezoids and square-unit blocks to solve for area."

Modeling Using Concrete Phase of CRA

Sample Problem: John wants to put down new tile in his laundry room because his puppy scratched holes in the flooring. He needs to figure out the area of the room so that he can order the right amount of tile. The floor space in the room is shown below. What is the area of the floor space?



Teacher says: "The unit of measurement for the area of the floor will be squared units. How can we find the area of this trapezoid using the square unit blocks? Let's try using the square unit blocks to fill in the trapezoid shape. Now let's count the number of blocks it takes to cover the whole trapezoid. This is the area." [Teacher models for students how to use geometric shapes of trapezoids and square unit blocks to represent the problem, making sure to demonstrate how to use half unit blocks to fill the corners of the trapezoid. Teacher continually engages students during modeling process by asking for assistance/calling on individual students while completing steps of the problem.]

Peer Tutoring

Modeling: Teacher models for students how to engage in peer tutoring using the steps in Table 2 and 3 to guide this process. Following modeling, teacher prompts students to work in dyads to solve a similar problem (area of trapezoid). Students are provided with a visual aid or script to remember probing questions and corrective feedback steps.

Feedback: As the students work in dyads, teacher circulates the room providing feedback or assistance as needed. Student acting as the tutor prompts the other student (tutee) by asking questions such as "How many blocks did it take to cover the trapezoid." Refer to Table 4 for detailed student roles.

Progress Monitoring: Once each student has completed an area problem as the tutee, the teacher evaluates whether there is a need for additional practice based on student performance. Students are provided with an opportunity to engage in independent practice through completing a curriculum-based measurement (CBM) probe that targets concrete skills learned.

Phase of Instruction: Representational

Teacher provides review of previous lessons, advance organizer, and objective before beginning modeling lesson, just as in the concrete phase.

Teacher says: "Yesterday we saw that we can use the square unit blocks to represent the area of a trapezoid. Today we are going to use pictures of trapezoids and what we learned about square unit blocks to find the area."

Modeling Using Representational Phase

Students are presented with a picture of a trapezoid that has been filled in with square unit blocks. Teacher models for students how to solve area problem using a pictorial representation of the trapezoid (i.e., counting up the number of squares in the picture). Teacher then presents each student with a handout with the shape of a trapezoid on it.

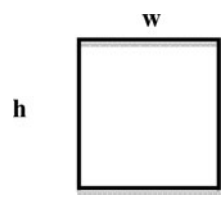
Teacher says: "Now try drawing your own square units inside the blank trapezoid at the bottom of the page. What is the area of the trapezoid? Is it the same as it was yesterday?" [Teacher provides students with a different trapezoid and prompts students to work in dyads to solve the problems using peer tutoring.]

Peer Tutoring

Feedback: As the students work in dyads, teacher circulates the room providing feedback or assistance as needed. Student acting as the tutor prompts the other student (tutee) by asking questions such as "How many square units are in the trapezoid" or "What is the area of the trapezoid." Refer to Table 4 for detailed student roles.

Progress Monitoring: Once each student has completed an area problem as the tutee, the teacher evaluates whether there is a need for additional practice based on student performance. Students are provided with an opportunity to engage in independent practice through completing a curriculum-based measure that targets concrete skills learned.

Teacher says: "So far, we have found the area of a trapezoid by counting the number of square units within the trapezoid. Let's look to see whether there is an easier way to find the area of a trapezoid. Before we begin, let's review how we used a formula to find the area of a square. The area of a square can be found by multiplying the height of the square by the width of the square ($A = h \times w$)."



"A trapezoid has some similarities to a rectangle. How can we find the height of the trapezoid?"

Teacher models how to draw a line from the top of the trapezoid to the bottom to form a right triangle on the side.

"Just as with finding the area of a rectangle, we can use a formula to find the area of a trapezoid. The formula for finding the area of a trapezoid is $A = \frac{1}{2}(B1 + B2)h$."

"B1 and B2 represent the width of the trapezoid at the top and bottom. You can see that B1 and B2 are not equal. How can we find B1 and B2 by using the picture of the trapezoid with square unit blocks drawn within it?" [Teacher models counting the unit squares that represent B1 and B2]. "Now let's find the height." [Teacher models counting the unit squares that represent the height.]

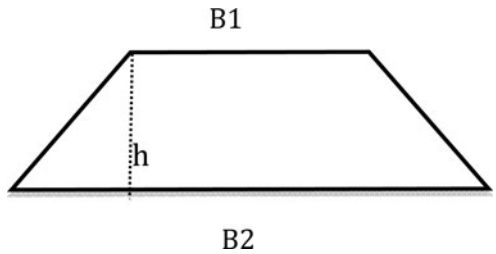
"Now that we have the measurements for h, B1, and B2, tomorrow we will use these measurements to use the formula for the area of a trapezoid."

Phase of Instruction: Abstract

The algebraic formula for the area of a trapezoid is visually presented and explained.

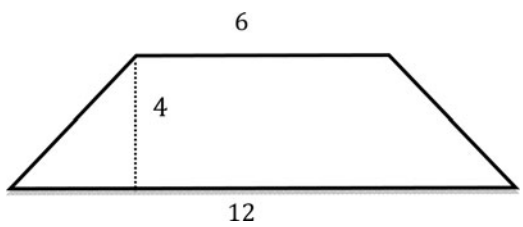
Teacher says: "So far we have been using square unit blocks to find the area of a trapezoid. Last time we saw that just like for squares, there is a formula to find the area of a trapezoid.

Let's review the components of the formula: The formula for finding the area of a trapezoid is $A = \frac{1}{2}(b1+b2)h$."



"B1 and B2 are the two base lengths of the trapezoid. H is the height of the trapezoid, which we find by drawing a line from B1 to B2 to make a right triangle."

"Yesterday we used our pictorial representation of a trapezoid to find the measurements of h, B1, and B2. Here are the measurements from yesterday:"



"Now, let's use the formula to find the area of a trapezoid." [Teacher models for students how to fill in the measurements into the formula for the area of a trapezoid]. "What is the area?" [Teacher gives students other area problems to practice with during peer tutoring.]

Peer Tutoring

Feedback: As the students work in dyads, teacher circulates the room providing feedback or assistance as needed. Student acting as the tutor prompts the other student (tutee) by asking questions such as "What parts of the trapezoid are used to find the area/what is the formula for area of a trapezoid." Refer to Table 4 for detailed student roles.

Progress Monitoring: Once each student has completed an area problem as the tutee, the teacher evaluates whether there is a need for additional practice based on student performance. Students are provided with an opportunity to engage in independent practice through completing a curriculum-based measure that targets concrete skills learned.