Cadence And Position Tracking for Switched FES-Cycling Combined with Motor Assistance for Cycle with Split-Crank

Fidel I. Esquivel Estay*

Abstract—**Functional Electrical Stimulation (FES) has proven to be an effective method to improve health and regain muscle for people with limited or reduced motor skills. FES combined with an electric motor can facilitate recovery through the implementation of a closed-loop control system. Many people with movement disorders (e.g., stroke) have asymmetries in their motor control, motivating the need for a closed-loop controller that can be implemented on a split crank cycle. In this thesis, a nonlinear sliding mode controller is designed for each side of a split crank cycle to maintain a desired cadence and a constant crank angle difference of 180 degrees, simulating standard pedaling conditions. A Lyapunov-like function is used to prove stability and tracking to the desired cadence and position for the combined FES-Motor system. Experimental results on two able-bodied individuals show the feasibility and stability of the closed-loop controller.**

I. Introduction

Functional Electrical Stimulation (FES) is the application of electrical impulses on muscle fiber to induce an involuntary muscle response or contraction. Over the last few decades, several studies have been performed analyzing the feasibility of FES exercises, such as FES induced cycling [1]. Numerous physiological and psychological benefits such as improved cardiovascular health [2], gain of muscle mass [3], and improved motor coordination [4] have been demonstrated. Previous studies performed on conventional cycles have shown how, by combining FES with an electric motor, better trajectory and cadence tracking can be obtained than by FES-cycling alone [5]. In addition, to account for people with varying degrees of impairment, [6] applied assistive, resistive and passive modes to FES induced cycling to encourage volitional pedaling. Although these studies have shown a high degree of relevance in rehabilitation treatments, the coupled dynamics of both pedals on the bike fails to isolate the individual muscle contribution of each leg. People who suffer from asymmetric neurological conditions, such as a stroke or localized muscle atrophy, might depend entirely on one limb to reach the desired torque or cadence, likely neglecting contributions of the less responsive limb. Therefore, it is desirable to study the effects of decoupled bike pedals, enabling each muscle region to perform independently of the other, maximizing muscle use and recovery.

In this thesis, a switching signal is implemented where the control signal switches between muscle groups and the electric motor as a function of effective crank angle, as in [5] and [6]. Two dynamic systems are analyzed, corresponding to each leg. For the dominant side, the objective is to track a desired cadence, whereas the non-dominant side is tasked with a position tracking objective to maintain 180 phase shift from the position of the pedal on the dominant side.

A switched-system analysis is used to design the proportional gain and sliding mode controllers for each system. Global exponential stability is obtained for both sub-systems independently. The two-leg system achieves global exponential stability, provided sufficient gain conditions are met.

*Department of Mechanical and Aerospace Engineering, University of Florida, Gainesville FL 32611-6250, USA Email: {fidelesquivel, wdixon}@ufl.edu

II. Model

The switched cycle-rider dynamics are considered as

$$\tau_{e_l}(q_l, \dot{q}_l, \ddot{q}_l, t) = \tau_{c_l}(\dot{q}_l(t), \ddot{q}_l(t), t) + \tau_{r_l}(q_l(t), \dot{q}_l(t), \ddot{q}_l(t), t), \tag{1}$$

where $q_l : \mathbb{R}_{>0} \to \mathcal{Q}$ denotes the measurable crank angle for the legs, where the subscript $l \in \mathcal{L} = \{dom, non\}$ indicates the dominant and non-dominant legs, respectively, and $\mathcal{Q} \subseteq \mathbb{R}$ denotes the set of all possible crank angles. The torques applied about the crank axis by the electric motor, the cycle, and the rider are denoted by $\tau_{e_l} : \mathcal{Q} \times \mathbb{R} \times \mathbb{R} \times \mathbb{R}_{\geq 0} \to \mathbb{R}$, $\tau_{c_l} : \mathbb{R} \times \mathbb{R} \times \mathbb{R}_{\geq 0} \to \mathbb{R}$, and $\tau_{r_l} : \mathcal{Q} \times \mathbb{R} \times \mathbb{R} \times \mathbb{R}_{\geq 0} \to \mathbb{R}$, respectively.

The torque about the crank axis by the cycle can be expressed as

$$\tau_{c_l}(\dot{q}_l(t), \ddot{q}_l(t), t) = J_{c_l}\ddot{q}_l(t) + b_c\dot{q}_l(t) + d_{c_l}(t), \tag{2}$$

where $J_{c_l} \in \mathbb{R}_{>0}$, $b_{c_l} \in \mathbb{R}_{>0}$, and $d_{c_l} : \mathbb{R}_{\geq 0} \to \mathbb{R}$ denote inertial effects, viscous damping effects, and disturbances applied by the cycle, respectively. The torque applied about the crank by the rider can be separated into passive torques, $\tau_{p_l} : \mathcal{Q} \times \mathbb{R} \times \mathbb{R} \to \mathbb{R}$, the FES induced muscle contribution, $\tau_{M_l} : \mathcal{Q} \times \mathbb{R} \times \mathbb{R}_{\geq 0} \to \mathbb{R}$, and the disturbances in the load, $d_{r_l} : \mathbb{R}_{\geq 0} \to \mathbb{R}$ as follows:

$$\begin{aligned} \tau_{r_l}(q_l(t), \dot{q}_l(t), \ddot{q}_l(t), t) \quad = \quad & \tau_{p_l}(q_l(t), \dot{q}_l(t), \ddot{q}_l(t)) \\ & -\tau_{M_l}(q_l(t), \dot{q}_l(t), t) \\ & +d_{r_l}(t). \end{aligned} \tag{3}$$

In (3), the passive torques applied by the rider are

$$\begin{aligned}\tau_{p_l}\left(q_l(t),\dot{q}_l(t),\ddot{q}_l(t)\right) &= M_{p_l}\left(q_l\left(t\right)\right)\ddot{q}_l\left(t\right) \\ &+V_l\left(q_l\left(t\right),\dot{q}_l\left(t\right)\right)\dot{q}_l\left(t\right) \\ &+G_l\left(q_l\left(t\right)\right)+P_l\left(q_l\left(t\right),\dot{q}_l\left(t\right)\right),\end{aligned} \tag{4}$$

where $M_{p_l}:\mathcal{Q}\to\mathbb{R}_{>0}$, $V_l:\mathcal{Q}\times\mathbb{R}\to\mathbb{R}$, $G_l:\mathcal{Q}\to\mathbb{R}$, and $P_l:\mathcal{Q}\times\mathbb{R}\to\mathbb{R}$, denote the inertial, centripetal-Coriolis, gravitational, and passive viscoelastic tissue forces, respectively. The torques applied by the muscles are denoted as the sum of each muscle's individual contribution by FES as

$$\tau_{M_l}\left(q_l\left(t\right),\dot{q}_l\left(t\right),t\right)=\sum_{m\in\mathcal{M}}B_{m_l}\left(q_l\left(t\right),\dot{q}_l\left(t\right)\right)u_{m_l}\left(t,q_l(t)\right), \tag{5}$$

$\forall m\in\mathcal{M}\,\forall l\in\mathcal{L}$, where $u_{m_l}:\mathcal{Q}\times\mathbb{R}_{\geq 0}\to\mathbb{R}$ is the designed muscle control current input, and the subscript $m\in\mathcal{M}=\{Q,\,G,\,H\}$ indicates the quadriceps femoris (Q), gluteal (G), and hamstring (H) muscle groups, respectively. It is assumed that the rider does not contribute volitionally to this experiment, and all volitional contributions are therefore included into the disturbance term $d_{r_l}:\mathbb{R}_{\geq 0}\to\mathbb{R}$. The uncertain muscle control effectiveness is denoted by $B_{m_l}:\mathcal{Q}\times\mathbb{R}\to\mathbb{R}_{>0},\ \forall m\in\mathcal{M}$, and can be expressed as

$$B_{m_l}=\lambda_{m_l}\left(q_l\right)\psi_{m_l}\left(q_l,\,\dot{q}_l\right)\cos\left(\beta_{m_l}\left(q_l\right)\right)T_{m_l}\left(q_l\right), \tag{6}$$

$\forall m\in\mathcal{M},\ \forall l\in\mathcal{L}$, where $\lambda_{m_l}:\mathcal{Q}\to\mathbb{R}_{>0}$ denotes the uncertain moment arm of each muscle group's force about its respective joint, $\psi_{m_l}:\mathcal{Q}\times\mathbb{R}\to\mathbb{R}_{>0}$ denotes the uncertain nonlinear function relating stimulation intensity to the force output by the muscle, and $\beta_{m_l}:\mathcal{Q}\to\mathbb{R}$ denotes the uncertain muscle fiber pennation angle. The function $T_{m_l}:\mathcal{Q}\to\mathbb{R}$ denotes the torque transfer ratio between each muscle group and the crank [7]. Definitions for the effective stimulation regions and switching laws employed during the cadence and trajectory tracking are based on [5], where the portion of the crank cycle in which a particular muscle group is stimulated is denoted by $\mathcal{Q}_{m_l}\subset\mathcal{Q}$. In this manner, $\mathcal{Q}_{m_l}$ is defined for each muscle group as

$$\mathcal{Q}_{G_l} \triangleq \left\{q_l\in\mathcal{Q}|T_{G_l}\left(q_l\right)>\varepsilon_{G_l}\right\}, \tag{7}$$
$$\mathcal{Q}_{Q_l} \triangleq \left\{q_l\in\mathcal{Q}|-T_{Q_l}\left(q_l\right)>\varepsilon_{Q_l}\right\}, \tag{8}$$
$$\mathcal{Q}_{H_l} \triangleq \left\{q_l\in\mathcal{Q}|T_{H_l}\left(q_l\right)>\varepsilon_{H_l}\right\}, \tag{9}$$

where $\varepsilon_{m_l}\in\left(0,\ \max\left(T_{m_l}\right)\right]$ is the lower threshold for each torque transfer ratio, limiting the stimulation regions so that each muscle group is engaged only when it contributes in the positive crank motion.

Based on the stimulation regions in (7)-(9), the piece-wise switching signal $\sigma_{m_l}\left(q_l\right)\in\{0,\,1\}$ is defined for each muscle group such that $\sigma_{m_l}\left(q_l\right)=1$ when $q_l\in\mathcal{Q}_{m_l}$ and $\sigma_{m_l}\left(q_l\right)=0$ when $q_l\notin\mathcal{Q}_{m_l}$, $\forall m\in\mathcal{M},\ \forall l\in\mathcal{L}$. The region of the crank cycle where FES produces efficient torques, $\mathcal{Q}_{FES_l}$, is defined as $\mathcal{Q}_{FES_l}\triangleq\bigcup_{m\in\mathcal{M}}\left\{\mathcal{Q}_{m_l}\right\},\forall m\in\mathcal{M},\ \forall l\in\mathcal{L}$ as defined in [1].

To facilitate the analysis of a combination of position-based and velocity-based switching, switching times are denoted by $\left\{t_{n,l}^{i}\right\},\ i\in\{s,\,e\},\ n\in\{0,1,2,...\},\ \forall l\in\mathcal{L}$, where each $t_{n,l}^{i}$ represents the Nth time that the system switches to activate stimulation (denoted by $i=s$) or the electric motor (denoted by $i=e$). Also in (5), $u_{m_l}:\mathbb{R}_{>0}\to\mathbb{R}$ denotes the control input and the electrical stimulation intensity applied to each muscle, defined as

$$u_{m_l}\triangleq\sigma_{m_l}k_{m_l}u_{s_l}\left(t\right),\,\forall m\in\mathcal{M}, \tag{10}$$

where the subsequently designed FES control input is denoted by $u_{s_l}\left(t\right)$ and $k_{m_l}\in\mathbb{R}_{>0}$ is a constant control gain, $\forall l\in\mathcal{L}$.

Additionally, the torque about the crank axis by the electric motor for each leg can be expressed as

$$\tau_{e_l}=B_{e_l}u_{e_l}, \tag{11}$$

where $B_{e_l}\in\mathbb{R}_{>0}$ relates the electric motor's input current to the resulting torque about the crank axis, and $u_{e_l}:\mathbb{R}_{>0}\to\mathbb{R}$ is the designed electric motor controller, $\forall l\in\mathcal{L}$.

Substituting (2)-(5) into (1) yields

$$\begin{aligned}B_{M_l}u_{s_l}+B_{e_l}u_{e_l} &= M_l\ddot{q}_l+b_{c_l}\dot{q}_l+d_{cl} \\ &+V_l\dot{q}_l+G_l+P_l+d_{r_l},\end{aligned} \tag{12}$$

where $B_{M_l}:\mathcal{Q}\times\mathbb{R}\to\mathbb{R}$ is the combined switched FES control effectiveness, defined as

$$B_{M_l}\left(q_l,\,\dot{q}_l\right)=\sum_{m\in\mathcal{M}}B_{m_l}\sigma_{m_l}k_{m_l}, \tag{13}$$

$\forall m\in\mathcal{M},\ \forall l\in\mathcal{L}$, and $M_l:\mathcal{Q}\to\mathbb{R}$ is defined as the summation $M_l\triangleq J_{c_l}+M_{p_l}$.

The subsequent development is based on the assumption that a dominant lower limb is identified and set for the controller. Therefore, a one-way system dependence is established, in the sense that the non-dominant subsystem depends on the dominant subsystem, but the reverse is not applicable. Both subsystems are autonomous and state-dependent.

The switched system in (12) has the following properties and assumptions, as listed in [8]. In addition, both legs are bounded by the same constants. In the case of severe physical differences, (i.e. severe muscle atrophy or high spasticity on one leg), the higher bound is used for both legs.

Property: 1 $c_m\leq M_l\leq c_M$, where $c_m,\,c_M\in\mathbb{R}_{>0}$ are known constants.

Property: 2 $|V_l|\leq c_V|\dot{q}_l|$, where $c_V\in\mathbb{R}_{>0}$ is a known constant.

Property: 3 $|G_l|\leq c_G$, where $c_G\in\mathbb{R}_{>0}$ is a known constant.

Property: 4 $|P_l|\leq c_{P1}+c_{P2}|\dot{q}_l|$, where $c_{P1},\,c_{P2}\in\mathbb{R}_{>0}$ are known constants.

Property: 5 $|b_c|\leq c_b|\dot{q}_l|$, where $c_b\in\mathbb{R}_{>0}$ is a known constant.

Property: 6 $|d_{r_l}+d_{c_l}|\leq c_d$, where $c_d\in\mathbb{R}_{>0}$ is a known constant.

Property: 7 By skew symmetry, $\frac{1}{2}\dot{M}_l=V_l$.

Property: 8 The unknown moment arm of each muscle group about their respective joint is non-zero, (i.e., $\lambda_l \neq 0$) [9].

Property: 9 The auxiliary term ψ_l in (6) depends on the force-length and force-velocity relationships of the muscle being stimulated and is upper and lower bounded by known positive constants, $c_\psi,\, c_\Psi \in \mathbb{R}_{>0}$, respectively, provided the muscle is not fully extended [10] or contracting concentrically at its maximum shortening velocity [8].

Property: 10 The function relating the unknown muscle fiber pennation angle to output torque is never zero, (i.e., $\cos(\beta_{m_l}) \neq 0$) [11].

Property: 11 By properties 8-10, B_{m_l} has a lower bound for all m_l, and thus, when $\sum_{m\in\mathcal{M}} \sigma_{m_l} > 0$, $c_{b_M} \leq B_{M_l}$, where $c_{b_M} \in \mathbb{R}_{>0}$.

Property: 12 $c_{b_e} \leq B_e \leq c_{B_e}$, where $c_{b_e},\, c_{B_e} \in \mathbb{R}_{>0}$.

III. Control Development

The control objective for each of the leg subsystems is defined separately and is split into the dominant subsystem, with a cadence tracking objective, and the non-dominant subsystem, with a position tracking objective to maintain a constant phase shift of 180 degrees from the dominant leg. Switching signals for the FES muscle control and and the electric motor control for both legs are defined as $\sigma_{s_l} : \mathbb{R}_{\geq 0} \to \{0,\, 1\}$ and $\sigma_{e_l} : \mathbb{R}_{\geq 0} \to \{0,\, 1\}$, respectively, $\forall l \in \mathcal{L}$, $\forall m \in \mathcal{M}$. These signals are designed as

$$\sigma_{s_l} \triangleq \begin{cases} 1 & \text{if } q_l \in \mathcal{Q}_{m_l} \\ 0 & \text{if } q_l \notin \mathcal{Q}_{m_l} \end{cases}, \tag{14}$$

$$\sigma_{e_l} \triangleq \begin{cases} 1 & \text{if } q_l \notin \mathcal{Q}_{m_l} \\ 0 & \text{if } q_l \in \mathcal{Q}_{m_l} \end{cases}. \tag{15}$$

A. Dominant System

The control objective for the dominant subsystem is to track a desired cadence which is quantified by the cadence error $e_1 : \mathbb{R}_{\geq 0} \to \mathbb{R}$, defined as

$$e_1(t) \triangleq \dot{q}_{dom_d}(t) - \dot{q}_{dom}(t), \tag{16}$$

where $\dot{q}_{dom_d},\, \dot{q}_{dom} \in \mathbb{R}_{>0}$ are the desired and measured cadences for the dominant leg. The switching signal for the FES muscle control and the electric motor control for the dominant leg are defined in (14) and (15), respectively, where $l = dom$ and $m \in \mathcal{M}$.

Taking the time derivative of (16), multiplying by M_{dom}, and substituting into yields

$$M_{dom}\dot{e}_1 = \chi_{dom} - V_{dom}e_1 - B_{e_{dom}}u_{e_{dom}} - B_{M_{dom}}u_{s_{dom}}, \tag{17}$$

where the auxiliary term $\chi_{dom} : Q \times \mathbb{R} \times \mathbb{R}_{\geq 0} \to 0$ is defined as

$$\begin{aligned} \chi_{dom} \triangleq\; & b_{c_{dom}}\dot{q}_{dom} + d_{c_{dom}} + G_{dom} + P_{dom} \\ & + d_{r_{dom}} + V_{dom}\dot{q}_{dom_d} + M_{dom}\ddot{q}_{dom_d}, \end{aligned} \tag{18}$$

and using Properties 1-6, χ_{dom} can be bounded as

$$\chi_{dom} \leq c_1 + c_2|e_1|, \tag{19}$$

where $c_1,\, c_2 \in \mathbb{R}_{>0}$ are constants, and $|\mathbf{.}|$ denotes the absolute value. Based on (16), (17), (19), and the stability analysis in Section IV, the muscle control input for the FES acting in the dominant subsystem is defined as

$$u_{s_{dom}} = \sigma_{s_{dom}}(k_1 e_1 + k_2 sgn(e_1)), \tag{20}$$

where $k_1,\, k_2 \in \mathbb{R}_{>0}$ are selectable constant control gains, and $\sigma_{s_{dom}}$ is defined in (14). Likewise, the switched control system for the electric motor is defined as

$$u_{e_{dom}} = \sigma_{e_{dom}}(k_3 e_1 + k_4 sgn(e_1)), \tag{21}$$

where $k_3,\, k_4 \in \mathbb{R}_{>0}$ are constant control gains, and $\sigma_{e_{dom}}$ is defined in (15). Substituting (20) and (21) into (17) results in the closed-loop error system for the dominant subsystem

$$\begin{aligned} M_{dom}\dot{e}_1 \;=\; & -B_{M_{dom}}\sigma_{s_{dom}}(k_1 e_1 + k_2 sgn(e_1)) \\ & -B_{e_{dom}}\sigma_{e_{dom}}(k_3 e_1 + k_4 sgn(e_1)) \\ & +\chi_{dom} - V_{dom}e_1. \end{aligned} \tag{22}$$

B. Non-dominant Subsystem

The control objective of the non-dominant subsystem is to track the desired crank angle, which is a phase shift of 180 degrees to the dominant leg. This objective is quantified by the position error $e_2 : \mathbb{R}_{\geq 0} \to \mathbb{R}$ and the auxiliary error $r : \mathbb{R}_{\geq 0} \to \mathbb{R}$, defined as

$$e_2(t) \triangleq q_{non_d}(t) - q_{non}(t), \tag{23}$$

$$r(t) \triangleq \dot{e}_2 + \alpha e_2, \tag{24}$$

where $q_{non_d},\, q_{non},\, \alpha \in \mathbb{R}_{>0}$, are the desired crank angle for the non-dominant leg, the measured crank angle for the non-dominant leg, and a known constant. The switching signal for the FES muscle control and the electric motor control for the non-dominant leg are defined in (14) and (15), respectively, where $l = non$ and $m \in \mathcal{M}$.

Taking the time derivative of (24), multiplying by M_{non}, and substituting into (12) yields

$$M_{non}\dot{r} = \chi_{non} - V_{non}r - B_{e_{non}}u_{e_{non}} - B_{M_{non}}u_{s_{non}} - e_2, \tag{25}$$

where the auxiliary term $\chi_{non} : Q \times \mathbb{R} \times \mathbb{R}_{\geq 0} \to 0$ is defined as

$$\begin{aligned} \chi_{non} \triangleq\; & M_{non}(\ddot{q}_{non_d} + \alpha\dot{e}_2) + b_{c_{non}}\dot{q}_{non} + d_{c_{non}} \\ & + P_{non} + d_{r_{non}} + V_{non}(\dot{q}_{non_d} + \alpha e_2) + G_{non}. \end{aligned} \tag{26}$$

Based on (24), and Properties 1-6, χ_{non} can be bounded as

$$\chi_{non} \leq c_3 + c_4\|z\| + c_5\|z\|^2, \tag{27}$$

where $c_3,\, c_4,\, c_5 \in \mathbb{R}_{>0}$ are known constants, and the error vector $z \in \mathbb{R}^2$ is defined as

$$z = [e_2 \ r]^T. \tag{28}$$

Based on (24), (25), (27), and the stability analysis in Section IV, the muscle control input for the FES effective region acting on the non-dominant subsystem is defined as

$$u_{s_{non}} = \sigma_{s_{non}}(k_5 r + sgn(r)[k_6 + k_7\|z\| + k_8\|z\|^2]), \tag{29}$$

where k_5, k_6, k_7 and $k_8 \in \mathbb{R}_{>0}$ are selectable control gains, and $\sigma_{s_{non}}$ was defined in (14). Likewise, the switched control system for the electric motor is defined as

$$u_{e_{non}} = \sigma_{e_{non}}(k_9 r + sgn(r)[k_{10} + k_{11}\|z\| + k_{12}\|z\|^2]), \tag{30}$$

where k_9, k_{10}, k_{11} and $k_{12} \in \mathbb{R}_{>0}$ are selectable constant control gains, and $\sigma_{e_{non}}$ was defined in (15). Substituting (29) and (30) into (25) results in the closed-loop error system for the non-dominant subsystem

$$\begin{aligned} M\dot{r} \;=\; & -B_{M_{non}}\sigma_{s_{non}}(k_5 r + sgn(r)[k_6 + k_7\|z\| \\ & +k_8\|z\|^2]) - B_{e_{non}}\sigma_{e_{non}}(k_9 r + sgn(r)[k_{10} \\ & +k_{11}\|z\| + k_{12}\|z\|^2]) + \chi_{non} - V_{non} r - e_2. \end{aligned} \tag{31}$$

IV. Stability Analysis

The Stability Analysis is divided into dominant (Section IV, A) and non-dominant (Section IV, B) subsystems. Four theorems are presented to evaluate the stability of the motor and FES controllers developed in Section III.

A. Stability of Dominant Subsystem

Let $V_{L_1} : \mathbb{R} \to \mathbb{R}$ be a positive definite, continuously differentiable, common Lyapunov-like function candidate defined as

$$V_{L_1} = \frac{1}{2} M_{dom} e_1^2. \tag{32}$$

The Lyapunov-like function is radially unbounded and satisfies the following inequalities

$$(\frac{c_m}{2})e_1^2 \leq V_{L_1} \leq (\frac{c_M}{2})e_1^2, \tag{33}$$

where c_m and c_M are defined in Property 1. After using (22), the time derivative of (32) is

$$\begin{aligned} \dot{V}_{L_1} \;=\; & \frac{1}{2}\dot{M}_{dom} e_1^2 - V_{dom} e_1^2 + \chi_{dom} e_1 \\ & -B_{M_{dom}}\sigma_{s_{dom}}(k_1 e_1 + k_2 sgn(e_1)) \\ & -B_{e_{dom}}\sigma_{e_{dom}}(k_3 e_1 + k_4 sgn(e_1)). \end{aligned} \tag{34}$$

Using Property 7, (34) becomes

$$\begin{aligned} \dot{V}_{L_1} = \;& e_1(\chi_{dom} - B_{M_{dom}}\sigma_{s_{dom}}(k_1 e_1 + k_2 sgn(e_1)) \\ & -B_{e_{dom}}\sigma_{e_{dom}}(k_3 e_1 + k_4 sgn(e_1))). \end{aligned} \tag{35}$$

Theorem 1. *For $q_{dom} \in \mathcal{Q}_{FES_{dom}}$, the cadence error system defined in (16) is exponentially stable in the sense that*

$$|e_1(t)| \leq \sqrt{\frac{c_M}{c_m}} |e_1(t^s_{n,\,dom})| \exp\left[-\frac{\lambda_{dom_1}}{2}\left(t - t^s_{n,\,dom}\right)\right], \tag{36}$$

where $\lambda_{dom_1} \in \mathbb{R}_{>0}$ is defined as

$$\lambda_{dom_1} \triangleq \frac{2}{c_M}(c_{b_M} k_1 - c_2), \tag{37}$$

where c_M was defined in Property 1, c_{b_M} was defined in Property 11, k_1 was introduced in (20), and c_2 was introduced in (18). The inequality in (36) holds for all time $t \in [t^s_{n,\,dom},\, t^e_{n+1,\,dom}]$, provided the following gain conditions are satisfied

$$k_1 > \frac{c_2}{c_{b_M}},\ k_2 > \frac{c_1}{c_{b_M}}. \tag{38}$$

Proof: When $q_{dom} \in \mathcal{Q}_{FES_{dom}}$, $\sigma_{s_{dom}} = 1$ and $\sigma_{e_{dom}} = 0$. Therefore, the expression in (35) can be written as

$$\dot{V}_{L_1} = e_1(\chi_{dom} - B_{M_{dom}}(k_1 e_1 + k_2 sgn(e_1))). \tag{39}$$

Using (19), the following upper bound on (39) can be developed

$$\dot{V}_{L_1} \leq |e_1|(c_1 - k_2 c_{b_m}) + e_1^2(c_2 - k_1 c_{b_m}), \tag{40}$$

which is negative definite, provided the sufficient gain conditions in (38) are satisfied. From (33), the inequality in (40) can be upper bounded as

$$\dot{V}_{L_1} \leq -\lambda_{dom_1} V_{L_1}, \tag{41}$$

where $\lambda_{dom_1}$is defined in (37). The differential equation in (41) can be solved as

$$V_{L_1}(t) \leq V_{L_1}(t^s_{n,\,dom})\text{exp}[-\lambda_{dom_1}(t - t^s_{n,\,dom})]. \tag{42}$$

Rearranging (42) and using (33) yields (36) for all time $t \in [t^s_{n,\,dom},\, t^e_{n+1,\,dom})$, $n \in \{0, 1, 2...\}$. ■

Theorem 2. *For $q_{dom} \notin \mathcal{Q}_{FES_{dom}}$, the cadence error system defined in (16) is exponentially stable in the sense that it can be bounded as*

$$|e_1(t)| \leq \sqrt{\frac{c_M}{c_m}} |e_1(t^e_{n,\,dom})| \exp\left[-\frac{\lambda_{dom_2}}{2}\left(t - t^e_{n,\,dom}\right)\right], \tag{43}$$

for all time $t \in [t^e_{n,\,dom},\, t^e_{n+1,\,dom}]$, where $\lambda_{dom_2} \in \mathbb{R}_{>0}$ is defined as

$$\lambda_{dom_2} \triangleq \frac{2}{c_M}(c_{b_e} k_3 - c_2), \tag{44}$$

provided the following gain conditions are satisfied

$$k_3 > \frac{c_2}{c_{b_e}},\ k_4 > \frac{c_1}{c_{b_e}}. \tag{45}$$

Proof: When $q_{dom} \notin \mathcal{Q}_{FES_{dom}}$, $\sigma_{s_{dom}} = 0$ and $\sigma_{e_{dom}} = 1$. Therefore, the expression in (35) can be expressed as

$$\dot{V}_{L_1} = e_1(\chi_{dom} - B_{e_{dom}}(k_3 e_1 + k_4 sgn(e_1))). \tag{46}$$

Upper bounding (46) using (19) and Property 12, yields

$$\dot{V}_{L_1} \leq |e_1|(c_1 - k_4 c_{b_e}) + e_1^2(c_2 - k_3 c_{b_e}), \tag{47}$$

which is negative definite provided the gain conditions in (45) are satisfied. Using (33) to upper bound (47) yields

$$\dot{V}_{L_1} \leq -\lambda_{dom_2} V_{L_1}, \tag{48}$$

where λ_{dom_2} is defined in (37). The inequality in (48) can be solved to yield

$$V_{L_1}(t) \leq V_{L_1}(t^e_{n,\,dom})\text{exp}[-\lambda_{dom_2}(t - t^e_{n,\,dom})]. \tag{49}$$

The result in (43) can be obtained from (32) and (49) for all $t \in [t_{n,\,dom},\ t_{n+1,\,dom})$ for $n \in \{0, 1, 2...\}$. ■

B. Stability of Non-dominant Subsystem

Let $V_{L_2} : \mathbb{R}^2 \to \mathbb{R}$ be a positive definite, continuously differentiable, common Lyaponov-like function candidate defined as

$$V_{L_2} = \frac{1}{2} M_{non} r^2 + \frac{1}{2} e_2^2. \tag{50}$$

The Lyapunov-like function is radially unbounded and satisfies the following inequalities

$$\lambda_i ||z||^2 \leq V_{L_2} \leq \lambda_j ||z||^2, \tag{51}$$

where $\lambda_i, \lambda_j \in \mathbb{R}_{>0}$ are known constants defined as

$$\lambda_i \leq min(\frac{1}{2}, \frac{c_m}{2}),\ \lambda_j \leq max(\frac{1}{2}, \frac{c_M}{2}), \tag{52}$$

and $||\,\mathbf{.}\,||$ is the euclidean norm of a vector. Using (31) and Property 7, the time derivative of (50) can be expressed as

$$\begin{aligned} \dot{V}_{L_2} \;=\;& r\chi_{non} - rB_{M_{non}}\sigma_{s_{non}}(k_5 r + sgn(r)[k_6 \\ &+ k_7\|z\| + k_8\|z\|^2]) - rB_{e_{non}}\sigma_{e_{non}}(k_9 r \\ &+ sgn(r)[k_{10} + k_{11}\|z\| + k_{12}\|z\|^2]) - \alpha e_2^2. \end{aligned} \tag{53}$$

Theorem 3. *For $q_{non} \notin \mathcal{Q}_{FES_{non}}$, the error system defined in (23) and (24) is exponentially stable in the sense that it is bounded as*

$$||z\,(t)\,|| \leq \sqrt{\frac{\lambda_j}{\lambda_i}} ||z(t^e_{n,\,non})|| \exp\left[-\frac{\lambda_{non_1}}{2}\left(t - t^e_{n,\,non}\right)\right], \tag{54}$$

for all time $t \in [t^e_{n.\,non}, t^s_{n+1,\,non}]$, where $\lambda_{non_1} \in \mathbb{R}_{>0}$ is defined as

$$\lambda_{non_1} \triangleq \frac{min(\alpha, c_{b_e} k_9)}{\lambda_j}, \tag{55}$$

provided the following gain conditions are satisfied:

$$k_{10} > \frac{c_3}{c_{b_e}},\ k_{11} > \frac{c_4}{c_{b_e}}\ k_{12} > \frac{c_5}{c_{b_e}}. \tag{56}$$

Proof: When $q_{non} \notin \mathcal{Q}_{FES_{non}}$, $\sigma_{s_{non}} = 0$ and $\sigma_{e_{non}} = 1$. Therefore, the expression in (53) can be reduced to

$$\begin{aligned} \dot{V}_{L_2} \;=\;& r\chi_{non} - \alpha e_2^2 \\ &+ r\left(-B_{e_{non}}\left(k_9 r + sgn(r)\left[k_{10} + k_{11}||z|| + k_{12}||z||^2\right]\right)\right). \end{aligned} \tag{57}$$

Upper bounding (57) using (27) and Property 12, yields

$$\begin{aligned} \dot{V}_{L_2} \;\leq\;& -c_{b_e} k_9 r^2 - \alpha e_2^2 - (c_{b_e} k_{10} - c_3)|r| \\ &- (c_{b_e} k_{11} - c_4)|r|\,||z|| - (c_{b_e} k_{12} - c_5)|r|\,||z||^2, \end{aligned} \tag{58}$$

which is negative definite, provided the gain conditions in (56) are satisfied. Using (55) to upper bound (58) yields

$$\dot{V}_{L_2} \leq -\lambda_{non_1} V_{L_2}, \tag{59}$$

where λ_{non_1} is defined in (55). The solution to the differential equation in (59) yields

$$V_{L_2} \leq V_{L_2}(t^e_{n,\,non})\text{exp}[-\lambda_{non_1}(t - t^e_{n,\,non}), \tag{60}$$

for all time $t \in [t^e_{n,\,non},\ t^s_{n+1,\,non}]$, $n \in \{0, 1, 2...\}$, from which (54) can be obtained. ■

Theorem 4. *For $q_{non} \in \mathcal{Q}_{FES_{non}}$, the error system defined in (23) and (24) is exponentially stable in the sense that it is bounded as*

$$|z\,(t)\,| \leq \sqrt{\frac{\lambda_j}{\lambda_i}} ||z(t^s_{n,\,non})|| \exp\left[-\frac{\lambda_{non_2}}{2}\left(t - t^s_{n,\,non}\right)\right], \tag{61}$$

for all time $t \in [t^s_{n,\,non},\ t^e_{n+1,\,non})$ for $n \in \{0, 1, 2...\}$, where $\lambda_{non_2} \in \mathbb{R}_{>0}$ is defined as

$$\lambda_{non_2} \triangleq \frac{min(\alpha, c_{b_M} k_5)}{\lambda_j}, \tag{62}$$

provided the following gain conditions are satisfied

$$k_6 > \frac{c_3}{c_{b_M}},\ k_7 > \frac{c_4}{c_{b_M}},\ k_8 > \frac{c_5}{c_{b_M}}. \tag{63}$$

Proof: When $q_{non} \in Q_{FES_{non}}$, $\sigma_{s_{non}} = 1$ and $\sigma_{e_{non}} = 0$. Therefore, the expression in (53) can be reduced to

$$\begin{aligned} \dot{V}_{L_2} \;=\;& r(-B_{M_{non}}(k_5 r + sgn(r)[k_6 + k_7||z|| \\ &+ k_8||z||^2])) + r\chi_{non} - \alpha e_2^2. \end{aligned} \tag{64}$$

Upper bounding (64) using (27) and Property 11, and rearranging, yields

$$\begin{aligned} \dot{V}_{L_2} \;\leq\;& -c_{b_M} k_5 r^2 - \alpha e_2^2 - (c_{b_M} k_6 - c_3)|r| \\ &- (c_{b_M} k_7 - c_4)|r|\,||z|| - (c_{b_M} k_8 - c_5)|r|\,||z||^2, \end{aligned} \tag{65}$$

which is negative definite provided the gain conditions in (63) are satisfied. Using (62) to upper bound (65) yields

$$\dot{V}_{L_2} \leq -\lambda_{non_2} V_{L_2}, \tag{66}$$

where λ_{non_2} is defined in (62). The inequality in (66) can be solved to yield

$$V_{L_2} \leq V_{L_2}(t^s_{n,\,non})\exp[-\lambda_{non_2}(t - t^s_{n,\,non}), \tag{67}$$

for all time $t \in [t^s_{n,\,non},\ t^e_{n+1,\,non}]$, $n \in \{0, 1, 2...\}$, from which (61) can be obtained. ■

C. Switched System Stability

From Theorems 1 and 2, the dominant leg subsystem achieves exponential cadence tracking as seen in (36) and (43). Furthermore, from Theorems 3 and 4, the non-dominant subsystem achieves exponential position tracking as indicated by (54) and (61). It is noted that both autonomous subsystems achieve exponential stability when analyzed independently. Furthermore, given that the desired trajectory of the non-dominant subsystem depends on the instantaneous position of the dominant subsystem at any moment in time, it can be asserted that provided the dominant system is stable for all time, the non-dominant system will also be stable for the given interval of time. Thus, the dynamic system outlined in (12) is stable, given that the sufficient gain conditions shown in (38), (45), (56) and (63) are satisfied.

V. Experiments

Experiments were conducted with the objective of evaluating the performance of the FES and motor controllers developed for the dominant and the non-dominant subsystems, shown in (20), (21), (29), and (30), respectively. Experiments were performed on two able-bodied 24 year old individuals, one male and one female, after written consent approved by the University of Florida Institutional Review Board was provided. The subjects were instructed to provide no volitional contribution to the pedaling effort, thus allowing the response of the system to be a direct measure of the controllers position and cadence tracking.

A. Split-Crank Motorized FES-Cycling Test Bed

To conduct the experiments, a commercially available recumbent tricycle (Terra Trike Rover X8) was modified to provide stationary cycling. The cycle crank was split into decoupled right and left sections such that there was no mechanical engagement between each side of the cycle. This test bed can be seen in Figure 1. Custom pedals were manufactured to fit orthotic boots that served to fit the rider's feet, preventing dorsiflexion and plantarflexion of the ankles while maintaining sagittal alignment of the lower legs. An optical encoder (US Digital H1) was affixed to each cycle crank to measure angular position and velocity of the dominant and non-dominant subsystems. Two 250 Watt, 24 DC motors, one for each dynamic system, were used to assist the forward pedaling motion of the rider. For each motor, an ADVANCED Motion Control (AMC) PS300W24 power supply, linked with an AMC FC15030 filter card, was used to power the motor

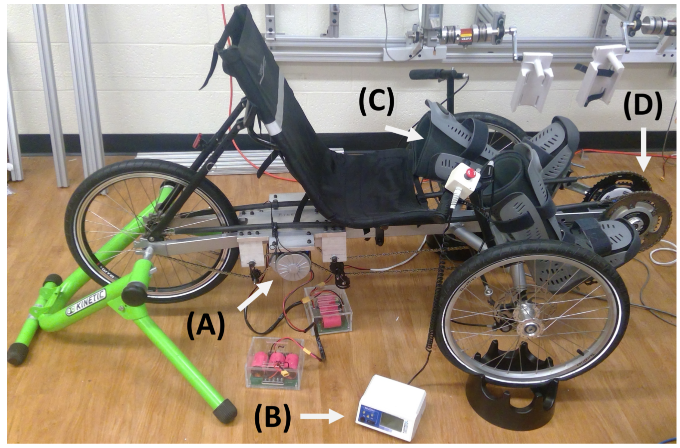

Figure 1. Motorized FES-cycling test bed with split-crank used for experiments. (A) Electric motors, (B) FES stimulator, (C) orthotic pedals, and (D) split-crank system.

and reduce electrical noise, respectively. The motors were controlled using two AMC AB25A100 motor drivers.

A current-controlled stimulator (Hasomed RehaStim) was used to deliver biphasic, symmetric, and rectangular pulses to the subject's active muscle groups via self-adhesive electrodes. The stimulation amplitude was fixed to 90mA for the quadriceps. The stimulation intensity was controlled by modulation of the pulse width according to (20) for the dominant leg, and (29) for the non-dominant leg.

A Quanser Q8-USB data acquisition device was used to collect signal data from the encoders and deliver motor current. The FES and motor controllers were implemented on a computer running real time control software (QUARC, MATLAB/Simulink, Windows 10) at a sampling rate of 500 Hz.

B. Experimental Setup

Electrodes were placed over the subjects' quadriceps femoris according to the Axelgaard electrode placement manual[1]. The subjects were then seated on the cycle and their legs secured to the orthotic boots. The seat was adjusted to ensure hyperextension of the knee would not occur at any point during the pedaling cycle. The lengths of the lower limbs as well as the distances between the crank and the hips were measured and used to calculate the effective torque transfer ratios used to determine the muscle-motor switching signals, as in [5]. The riders' legs were then approximately positioned at a 180 degree phase difference to prevent the subject from a large response caused by a large initial position error.

For the dominant subsystem, the cadence objective was set to 50 RPM. The experiment was started from rest. For the non-dominant subsystem, the desired position was defined as a 180 degree offset from the instantaneous position measured for the dominant leg. The desired crank velocity for the dominant leg

[1]http://www.palsclinicalsupport.com/videoElements/videoPage.php

$\dot{q}_{dom_d}$ and the desired crank position for the non-dominant leg q_{non_d} are defined as

$$\dot{q}_{dom_d} = \frac{5\pi}{3}\left\{1 - \exp\left[-\frac{2}{5}(t - t_0)\right]\right\}, \tag{68}$$

$$q_{non_d} = q_{dom} + \pi. \tag{69}$$

During the first 10 seconds after the simulation was initiated, no FES was applied for either leg, and the motors alone were used to bring the dominant leg to the desired cadence of 50 rpm while keeping the non-dominant leg at a constant phase difference of 180 degrees. After 10 seconds, the muscle stimulation was applied according to the switching signals and control methods described in (14), (20), and (29), respectively, where the torque transfer ratios were calculated for every subject.

C. Results

Figure 2 depicts the tracking performance for both, dominant and non-dominant subsystems for one subject during a standard experiment with a duration of 150 seconds. The tracking error is quantified by the error signals defined in Section III. Also depicted is the motor/FES control inputs over the duration of the trial.

D. Discussion

Experiments were performed to depict the system response of the decoupled dynamic systems (dominant and non-dominant legs) for cadence and position tracking on a stationary bike. The results shown in Figure 2 successfully demonstrate the implementation of the FES-muscle input controllers in (20) and (29) as well as the motor current controller in (21) and (30), to achieve exponentially stable tracking of cadence (for the dominant leg) and position (for the non-dominant leg). The results indicate exponential convergence to an ultimate bound on the tracking errors e_1 and e_2. These were found to be 2.397 RPM with a standard deviation of 3.384 RPM and 0.7179 degrees with a standard deviation of 8.546 degrees, respectively. When the controllers are active, the motor prevents the legs from going backwards, or too fast, while the FES stimulates them in the positive crank motion.

On a standard coupled cycle, mechanical engagement between the crank axles provides a counteracting force about the crank axis whenever one of the legs move, balancing forces due to gravity acting on the cycle system. On the split-crank test bed used for these experiments, when the legs are positioned in the kinematic dead zones (i.e. directly in front or behind the the crank shaft), the gravitational forces acting on each leg do not cancel each other, causing more torque deviations in this region of the crank. Therefore, while the controllers maintain the desired tracking, the added cyclic disturbance produce a forward increase in speed, that can account for the offset in the cadence error.

The position error e_2 was centered near zero degrees and had a standard deviation of 8.546 degrees. Given the nature of the experiment where both leg subsystems are mechanically independent, the phase difference between the shafts is free to pivot around 180 degrees, creating rapidly oscillating, negative to positive, position tracking error. This behavior causes the motor to provide a rapid fluctuation between positive and negative currents, which can be seen in the u_e vs time graphs in Figure 2.

The cycle-rider closed-loop systems in (22) and (31) are intended to be used by riders with asymmetrical neurological conditions. Due to this conditions, on a standard motorized FES-cycling test bed where the shafts are coupled, the working leg may dominate the cycle, effectively nullifying any input from the less effective limb. The controllers in (20), (21), (29), and (30) split the legs into two decoupled subsystems where each leg contributes only its portion of the torque needed to maintain the desired position and cadence. This implies that one leg can receive extended motor assistance without affecting the other leg's behavior or response, and the muscles from each leg will be sufficiently exercised.

VI. Conclusion

The combined FES-cycling and electric motor sliding mode controllers developed in this paper were designed to enable a rider to pedal at a desired cadence while maintaining a constant crank angle difference of 180 degrees when pedaling on a mechanically disengaged split-crank stationary cycle. A Lyapunov-like analysis was used to prove the stability of the controllers for the dominant and non-dominant subsystems, and showed exponential tracking for the error signals. Two experiments performed on two able-bodied individuals validated the control effectiveness to reach the desired cadence of 50 RPM and position offset of 180 degrees, attaining global ultimately bounded tracking.

The developed control system for a decoupled cycle crank has the potential to advance motorized FES rehabilitation procedures for people with movement disorders that result in asymmetries. The introduction of decoupled dynamics for the leg subsystems allows for individual legs to contribute proportionally to their abilities, preventing one effective leg from nullifying the input from a less effective leg or a muscle group.

In the future, potential protocols acting on the kinematic dead zones where legs accelerate downwards or decelerate upwards could be implemented to reduce the position error and improve overall tracking. The author also plans to conduct experiments on subjects with different degrees of neurological conditions to test the robustness and usefulness of the controller design.

References

[1] M. J. Bellman, T. H. Cheng, R. J. Downey, C. J. Hass, and W. E. Dixon, "Switched control of cadence during stationary cycling induced by functional electrical stimulation," *IEEE Trans. Neural Syst. Rehabil. Eng.*, vol. 24, no. 12, pp. 1373–1383, 2016.

[2] S. P. Hooker, S. F. Figoni, M. M. Rodgers, R. M. Glaser, T. Mathews, A. G. Suryaprasad, and S. C. Gupta, "Physiologic effects of electrical stimulation leg cycle exercise training in spinal cord injured persons," *Arch. Phys. Med. Rehabil.*, vol. 73, no. 5, pp. 470–476, 1992.

[3] M. Bélanger, R. B. Stein, G. D. Wheeler, T. Gordon, and B. Leduc, "Electrical stimulation: can it increase muscle strength and reverse osteopenia in spinal cord injured individuals?" *Arch. Phys. Med. Rehabil.*, vol. 81, no. 8, pp. 1090–1098, 2000.

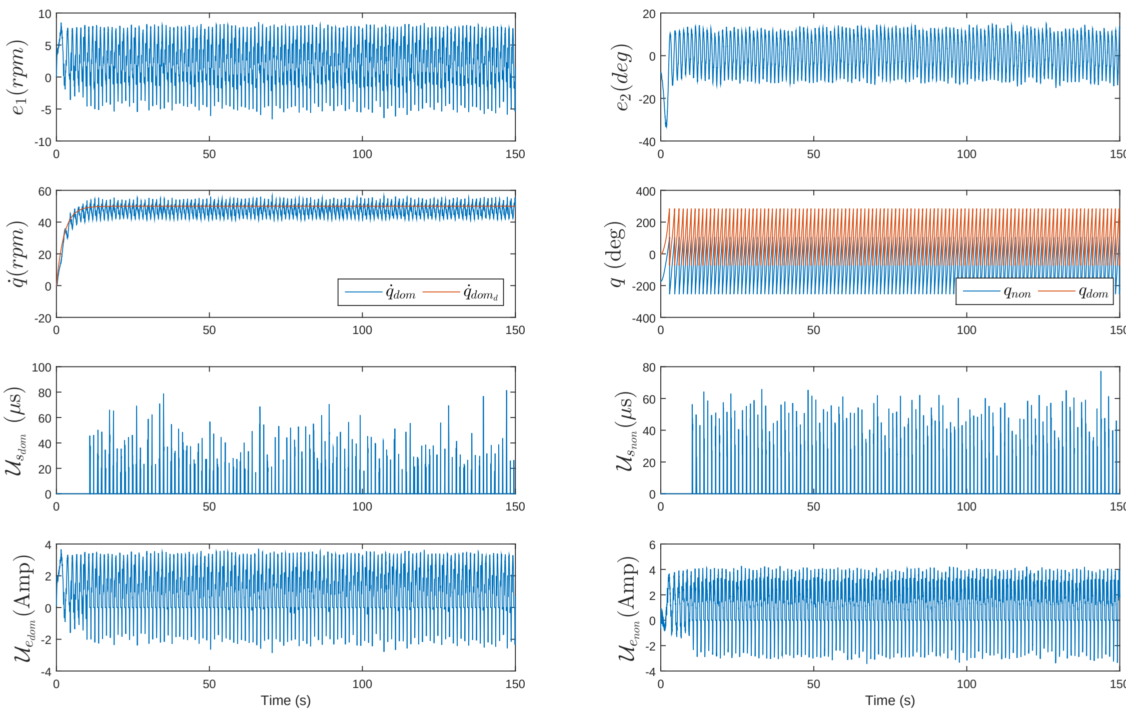

Figure 2. Tracking performance for dominant limb (left) and non-dominant limb (right). Results for the dominant limb are quantified by the cadence error e_1, trajectory tracking $\dot{q}_{dom}$, FES muscle input $u_{s_{dom}}$, and motor current input $u_{e_{dom}}$. Likewise, results for the non-dominant limb are quantified by position error e_2, position tracking q_{non}, FES muscle input $u_{s_{non}}$, and the motor current input $u_{e_{non}}$.

[4] S. Ferrante, A. Pedrocchi, G. Ferrigno, and F. Molteni, "Cycling induced by functional electrical stimulation improves the muscular strength and the motor control of individuals with post-acute stroke," *Eur. J. Phys. Rehabil. Med.*, vol. 44, no. 2, pp. 159–167, 2008.

[5] M. J. Bellman, R. J. Downey, A. Parikh, and W. E. Dixon, "Automatic control of cycling induced by functional electrical stimulation with electric motor assistance," *IEEE Trans. Autom. Science Eng.*, vol. 14, no. 2, pp. 1225–1234, April 2017.

[6] C. Rouse, C. Cousin, V. H. Duenas, and W. E. Dixon, "Cadence tracking for switched fes cycling combined with voluntary pedaling and motor resistance," in *Proc. Am. Control Conf.*, 2018.

[7] E. S. Idsø, T. Johansen, and K. J. Hunt, "Finding the metabolically optimal stimulation pattern for FES-cycling," in *Proc. Conf. of the Int. Funct. Electrical Stimulation Soc.*, Bournemouth, UK, Sep. 2004.

[8] M. J. Bellman, T.-H. Cheng, R. J. Downey, and W. E. Dixon, "Stationary cycling induced by switched functional electrical stimulation control," in *Proc. Am. Control Conf.*, 2014, pp. 4802–4809.

[9] J. L. Krevolin, M. G. Pandy, and J. C. Pearce, "Moment arm of the patellar tendon in the human knee," *J. Biomech.*, vol. 37, no. 5, pp. 785–788, 2004.

[10] D. A. Winter, *Biomechanics and Motor Control of Human Movement*. New York: Wiley, 1990.

[11] R. L. Lieber and J. Friden, "Functional and clinical significance of skeletal muscle architecture," *Muscle Nerve*, vol. 23, no. 11, pp. 1647–1666, Nov. 2000.