THE EFFECT OF A DYNAMIC GEOMETRY LEARNING ENVIRONMENT ON PRESERVICE ELEMENTARY TEACHERS' PERFORMANCE ON SIMILARITY TASKS

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CHAPTER I
INTRODUCTION

As aspects of the current mathematics reform movement become reality in the classroom, researchers question the impact of the restructuring and recontextualizing of the mathematics classroom. Interest must focus upon reform-related pedagogy with greater intellectual involvement via appropriately-chosen experiences in a context of problem solving and inquiry. In particular, the effects of implementing technology on the teaching and learning of mathematics and its impact on mathematics as reasoning must be addressed. The question is no longer whether technology belongs in the classroom, but how best to use technology to enhance learning.

Rationale

Use of technology in mathematics classrooms has increased dramatically during the past two decades and has been encouraged by leaders in the field of mathematics education (Blackwell & Henkin, 1989; English & Halford, 1995; Leitzel, 1991; National Advisory Committee on Mathematical Education [NACOME], 1975; National Council of Teachers of Mathematics [NCTM], 1980, 1989; National Institute of

Education [NIE], 1975; National Research Council [NRC], 1989; Suydam, 1976). The changing needs of our society demand that critical issues such as the role of technological advances in the learning and doing of mathematics be addressed. Computers are able to aid in visualizing abstract concepts and to create new environments which extend beyond the physical.

In addition to development of problem solving and visualization skills, technology offers enormous opportunity for curriculum reform in mathematics (Abramovich & Brown, 1996). Dynamic software (refer to Definition of Terms) such as *Geometer's Sketchpad* (Jackiw, 1995),*Cabri Géomètre II* (Laborde, 1994), and *Mathematica* (Wolfram, 1988) provide a flexibly structured mathematics laboratory: an electronic micro-world supporting the investigation and exploration of mathematical concepts at a representational level linking the concrete and the abstract. Mathematical ideas can be explored from several different perspectives in an efficient manner, resulting in deeper conceptual understanding (Kaput & Thompson, 1994).

Mathematical thinking is now often learned through problem solving activities that bury the mechanical aspects of mathematics behind interesting ideas (Vockell & Schwartz, 1992). Dynamic software allows the computer to be a powerful mathematical tool due to its ability to quickly and accurately plot, construct, measure, and perform computations. Through repetitive experiences of exploring and mathematizing, problem solving skills and one's ability to construct mathematical ideas are enhanced (Cooper, 1991). In addition, the interactive mode supports active learning--a

necessary component for effective construction of knowledge (Lewin, 1991). Furthermore, the dynamic software environment is motivational for the student because it "promotes mathematical explorations ... and helps the building of conceptual understanding" (Fraser, 1988, p. 216).

The Third International Mathematics and Science Study [TIMSS] gives mathematics educators a benchmark toward improvement of the teaching and learning of mathematics. The TIMSS data suggest that when teachers use more challenging methods to teach more complex mathematical ideas, as called for in the *Standards* trilogy: the Curriculum and Evaluation Standards for School Mathematics (NCTM, 1989), the Professional Standards for Teaching Mathematics (NCTM, 1991), and the Assessment Standards for School Mathematics (NCTM, 1995), students' performance is higher (National Center for Educational Statistics, 1998).

Statement of the Problem

There is reason to change what and how preservice elementary teachers learn mathematics. Most teachers are aware of the NCTM *Standards*, but evidence from the TIMSS video component suggests that changes in teaching practice are difficult. In the United States, lessons typically focus on acquiring mathematical skills rather than conceptual understanding, with little mathematical reasoning expected. In contrast, Japanese methodology, requiring students to think and reason, resemble the recommendations advocated in

the *Standards* (National Center for Educational Statistics, 1998).

Analyses of data from the TIMSS show, among the content areas addressed, that United States eighth and twelfth graders scored significantly below the international average in geometry and measurement. Moreover, grade twelve students' performance was relatively lowest in geometry; no country scored similar to or below the United States (National Center for Educational Statistics, 1998). This researcher posits that the lack of exposure to transformational geometry, a topic often found in other countries' curricula, may be a leading factor of poor performance. Teachers cannot teach what they themselves do not know (Fennema & Franke, 1992).

Teachers must be given the opportunity to construct and thoroughly develop their knowledge of elementary mathematics in an environment similar to the mathematics classroom they oversee (Simon & Blume, 1992). Many mathematics teacher educators advocate that teachers must be taught in ways that are consistent with how we would like them to teach (Committee on the Mathematical Education of Teachers, 1991; Fennema & Franke, 1992; NCTM, 1991).

Implementing new technologies in a reformed curriculum is not easy and the education of teachers plays a critical role. The necessity of requiring preservice teachers to learn mathematics content and methods of teaching in an environment where computer use is an integral component is clearly evident. The Mathematical Association of America stresses this point in their report <u>A call for change: Recommendations for the mathematical preparation of teachers of mathematics</u>

(Leitzel, 1991). The document states:

> [Teachers] need experience in using appropriate technology effectively in solving problems so that they can learn and adapt strategies and representations that arise while using that technology. Because they can reflect on their own learning and understanding of mathematical ideas when using technology, teachers will be better prepared to lead their own students in effective mathematical learning (p. 7).

The issue is not simply the incorporation of another gadget in the learning milieu. Rather it encompasses the careful consideration of the inclusion of technology commensurate with the current vision for mathematics education reform.

Dynamic software is generally accepted as a fertile learning environment in which students can be actively engaged in constructing and exploring mathematical ideas (Garry, 1997). This researcher postulates that the dynamic and interactive medium makes gaining an intuitive understanding more accessible to the [geometry] student (Pea, 1987). Under the psychomotor control of the user, this "dynamic" capability provides an infinite number of cases to be explored. For example, once constructed, geometric figures can be transformed by dragging a point, a vertex, or an edge. As the figure is transformed, patterns and/or relationships become visually explicit for further examination.

The theoretical foundation for using a dynamic learning environment emphasizing inquiry and exploration is based on the constructivist theory of learning as well as the role of repetitive experiences according to Cooper (1991). We have known since the days of Brownell that mathematical "practice"

often has little to do with developing habits of reasoning (Thompson, 1994). Long term problem solving is primarily a function of recognizing well known embedded principles in new situations. Cooper focuses our attention on the fact that what we want repeated is a composition of situations in ways that motivate generalization and support reflection. The constructivist theory of learning refers to the psychological level of analysis where emphasis is placed on the construction of personal knowledge and how a person acquires this knowledge (Phye, 1997). The philosophical level of analysis, although a legitimate line of inquiry, will not be addressed in this thesis.

Purpose of the Study

The purpose of the proposed study was to investigate the effect of a dynamic geometry learning environment on preservice elementary teachers' performance in the context of similarity tasks (refer to Definition of Terms). That is, this study was concerned with the mathematical evolution of students majoring in elementary education as a result of instruction with the dynamic geometry software *Geometer's Sketchpad* (Jackiw, 1995). The basic goal of the study was to detect the extent to which the dynamic environment, given its ability to efficiently display a plethora of mathematical situations, assists learning. Attention was also given to the role of prior knowledge.

Research Questions

Hence, based on the assumptions that teachers need to learn high quality mathematics and to be taught in a manner similar to the way they are expected to teach, the specific questions under investigation were:

1. After controlling for initial performance on similarity tasks, do preservice elementary teachers who work on similarity tasks in a dynamic geometry learning environment exhibit significantly different posttest performance than those who study the concept of similarity in a paper-and-pencil learning environment that employs traditional tools such as ruler and protractor?

2. After controlling for prior knowledge as measured by a standardized achievement test, do preservice elementary teachers who work on similarity tasks in a dynamic geometry learning environment exhibit significantly different posttest performance than those who study the concept of similarity in a paper-and-pencil learning environment that employs traditional tools such as ruler and protractor?

Justification of the Study

From several perspectives, the problem is important. As indicated earlier, technology is profoundly changing the teaching and learning of mathematics. Models of computer uses in education are needed to show how to incorporate this technology into daily practice (Garcia-de Galindo, 1994). To define new roles for teachers in a technologically rich classroom environment, it is necessary to explore how

computers will modify current ways of teaching and learning. Carleer and Doornekamp (1990) conclude that "to realize a valuable integration of computers in education, it will be necessary to focus the discussion no longer on computers, but on how to think and make decisions with the improvement of education as the goal"(p.5).

The acquisition of the concept of similarity is important for development of proportional reasoning and for geometrical understanding of one's immediate environment. One encounters phenomena that require familiarity with projection, scale factor, and other similarity-related concepts in everyday situations (Friedlander, 1984; Lappan, Fitzgerald, Winter, & Phillips, 1986). Similarity-related concepts are included in many parts of the mathematics curriculum. Some models for rational number concepts are based on similarity; an integral part of algebraic thinking involves ratio and proportion reasoning skills (Lappan & Even, 1988).

Similar geometric shapes would seem to provide helpful imagery for analysis of analogous situations. Dynamic software is relatively new phenomena; in particular, dynamic geometry software was not widely available a decade ago. Computerized dynamic geometry is highly visual and efficient for exploring and discovering properties of similar figures. However, realization of its potential is stifled by the lack of research using dynamic geometry software for middle school mathematics.

At this time, there is no information regarding the effect of a dynamic geometry learning environment on K-8

teachers' learning of the similarity concept. This particular study adds substance to the growing body of literature addressing the significance of dynamic geometry software in the classroom and, in particular, gives evidence whether a dynamic geometry learning environment enhances efficient construction of the concept of similarity. Moreover, this research will help substantiate the need to incorporate innovative technological tools in teacher education. One can be taught techniques and tricks, but expertise results from experience and insight.

Definitions of Terms

The following are some terms that are used throughout the manuscript; they are defined and collected here for reference:

A preserviceelementaryteacher is a student, without prior classroom teaching experience. All participants were enrolled in a required introductory mathematics methods course addressing content, methods, and materials for teaching elementary school mathematics.

The *Geometer's Sketchpad* (Jackiw, 1995), developed by Nicholas Jackiw and the Visual Geometry Project at Swarthmore College and now published by Key Curriculum Press, is an example of dynamic geometry software and was used in this study. Such software transforms the computer interface into a geometric construction micro-world capable of quickly and accurately plotting, constructing, measuring, and performing

computations. Once constructed, geometric figures can be transformed by dragging a point, vertex, or edge. As the figure is transformed, corresponding measurements and calculations change accordingly.

A dynamic geometry learning environment is defined as a learning milieu in which expository teaching is integrated with laboratory activities. The laboratory activities consist of explorations and experimentations using dynamic geometry software that allows for direct construction, manipulation, and measurement of geometric figures on the plane.

In contrast, the traditional learning environment is similar to the dynamic learning environment in that expository teaching is integrated with laboratory activities. In this case, however, laboratory activities consist of explorations using manipulatives, ruler and protractor.

A variable tension proportional divider (VTPD) is a mathematics laboratory device for drawing similar figures. The VTPD enlarges drawings and pictures in addition to geometric figures. To build a VTPD, knot together identical rubber bands. One end is held down by the thumb of one hand, acting as the projection point, and a pencil is inserted in the other end. The rubber band is stretched so that a knot traces the original figure while the pencil is moved on paper. It is of utmost importance that eyes follow the knot while the pencil draws. Although the final figures are not quite accurate, the image clearly conveys the intuitive "same shape" notion of similarity as one actively creates similar figures.

A transformation (one-to-one correspondence) on the Euclidean plane is called a similarity if there exists a positive number *k* such that the distance between the images of any two points is *k* times the distance between the points.

A dilation about the projection point (center) *P* of factor *k* is a transformation *D* in which each point *A* is mapped to its image *D(A)* in such a way that the directed distance between *P* and *D(A)* is *k* times the directed distance between *P* and *A* (Hall, 1973). In particular, the distance between the images of any two points is *k* times the distance between the points. A dilation is also referred to as a dilatation, a central similarity, or a homothety (King, 1997).

Organization of the Dissertation

In this chapter, the rationale for a study designed to examine the effect of a dynamic geometry learning environment on preservice elementary teachers strategies employed and performance on similarity tasks was described and background briefly discussed. Chapter II contains theoretical foundations and a review of related research supporting the study. Chapter III presents a description of the subjects, the composition of the instrument, the methodology and design of the study, and the results of the pilot studies. Analysis, interpretation of the data, and qualitative aspects for the present study will be given in Chapter IV. Chapter V will include a summary of the investigation, a discussion of

results, limitations of the study, implications for instruction, and suggestions for further research.

CHAPTER II
THEORETICAL FRAMEWORK AND RELEVANT RESEARCH

In the previous chapter, the study was introduced and background was given. In this chapter, the theoretical basis for the study is presented in detail. This includes a discussion of constructivist epistemology, the role of repetitive experience on learning, phases of learning, and an elucidation of the Erlanger Programm. Also included is a brief exposition on similarity and proportional reasoning in addition to relevant research in the area.

The theoretical premise that supports this study is that a dynamic geometry learning environment, supported by use of software such as the *Geometer's Sketchpad* (Jackiw, 1995), can enhance construction of knowledge and influence learning. Therefore, an examination of research focusing on software for geometric constructions follows; this section develops from static software to dynamic software as research already conducted on the use of dynamic geometry learning software is identified.

Constructivism

Using von Glasersfeld's radical theory of constructivism (1979, 1990, 1995) as a basis, Kilpatrick (1987) describes

the basic tenets of constructivism as an epistemology such that (1) all knowledge is actively constructed, not passively received, and (2) coming to know is an adaptive process that organizes one's experiential world; it is not the discovery of an independent, pre-existing world outside the mind of the knower (p. 7). Accepting von Glasersfeld's radical point of view involves rejecting ontological realism and embracing an epistemology that makes all knowing active and all knowledge subjective (Kilpatrick, 1987). This epistemological stance is in complete harmony with Piaget's theory of learning (1964), which demonstrates that human beings acquire knowledge by building it from the inside instead of internalizing it directly from the environment (von Glasersfeld, 1979; Kamii, 1990). Thus, current research on teaching from a constructivist perspective follows Piaget's biological metaphor of development and characterizes mathematical learning as a process of conceptual reorganization (Cobb, 1995).

The basic tenets of a constructivist epistemology have a direct implication upon pedagogy. Nel Noddings (1990) posits that "constructivists in mathematics education contend that cognitive Constructivism implies pedagogical Constructivism; that is, acceptance of constructivist premises about knowledge and knowers implies a way of teaching that acknowledges learners as active knowers" (p. 10). This is particularly meaningful in the case of radical constructivism; it focuses on the individual as a self-organizing system.

This is not always the case in the many variations of constructivism; some constructivists embrace Vygotsky's (1962) epistemological stance that emphasizes the cultural and social dimensions of development (Cobb, 1995; Confrey, 1995). In contrast, von Glasersfeld (1979, 1990, 1995) presents constructivism as a model of how an individual's experience forms the basis for knowing and communication. Within this model, social interaction constrains (and thus guides) the processes involved in the construction (von Glasersfeld, 1990).

Repetitive Experience

The role of experience is to generate or modify the organization of the relevant cognitive space. As conceived by Cooper (1991), the role of repetitive experience, repeated interaction with the environment, is to create, enhance, and/or reorganize cognitive space. Because the constructed cognitive space constitutes acquired knowledge, repetitive experience is of utmost importance. As in practicing typing, repetitive experience is used to mean repeating a set of similar and interrelated activities, not doing exactly the same thing over and over again.

In addition, the construction of such a cognitive space can serve as the foundation for reflective abstraction from which new knowledge results. The possibility that repetition induces reorganizations of knowledge, not just of skills, is the rationale that Cooper (1991) gives for use of the term

"repetitive experience" rather than "practice" (Thompson, 1994).

Acquisition of knowledge is experiential in that it must be actively constructed by the learner. Therefore, if cognitive development is generated by successive interactions with one's environment, then repetitive activity is necessary to provide information that facilitates problem solving. Consider the following task: Two equal-numerosity arrays in one-to-one correspondence are presented. One is then screened, and *n* objects are transferred from the visible to the screened array. The objective is to predict how many objects must be added to the visible array to make it equal to the screened array. Children learn with repeated examples that a transfer of *n* produces a difference of 2*n* (Piaget, 1974/1980; Piaget, Grize, Szeminska, & Vinh-Bang, 1977; Cooper, Campbell, & Bevins, 1983). Performance on this task highlights the role of repetitive experience because even adults frequently err until they have at least one experience with the task (Cooper, 1991). Robert Cooper's (1991) theory of repetitive experiences provides a strong foundation for reflective abstraction, and hence is a critical element for the development of insightful thought.

Phases of Learning

According to Karplus (1977), conceptual learning proceeds from an exploration stage to a concept identification stage to an application stage where new ideas

are used and extended (Simon, 1992). The application stage triggers a new level of exploration and the cycle recommences. Karplus' learning cycle, derived from Piaget's (1964) mental functioning model, provides a pedagogical framework that complements a constructivist view of learning (Fleener, 1995). Consequently, it provides a sound foundation for the middle stage of each learning session with respect to this research study.

Jurascheck (1983) suggests that learning cycle pedagogy be applied to mathematics education. Simon (1992) presents a framework utilized in two research projects for mathematics teacher learning based on what is understood about students' mathematics learning. The framework builds on Karplus' Learning Cycle and identifies a learning cycle that progresses through the following stages: exploration of a mathematical situation, discussion leading to concept identification, and application and extension of new ideas.

Also building on Piaget's work, Dienes (Dienes & Golding, 1971) advocates a learning cycle in which learners progress through a series of cyclic patterns, each comprising activities ranging from concrete to symbolic format. The earliest learning phase in each cycle begins with free play to uncover the key features of the structured materials. Following this, experiences are systematically structured to facilitate discovery of inherent relations in the material and in abstracting the concept being represented.

Dienes' learning cycle can be compared to the van Hiele's phases of learning. Although much has been communicated regarding the work of the van Hieles in terms of

a stratification of human thought (e.g. Senk, 1983), thought levels comprise only one of the three main components of the van Hiele model (Hoffer, 1983); the other components are insight and phases of learning. The van Hieles propose a sequence of five phases of learning: information, guided orientation, explication, free orientation, and integration (Clements & Battista, 1992).

Polya (1965), best known for articulating aspects of problem solving, posits that any good teaching device must be correlated somehow with the nature of the learning process and, therefore, outlines some obvious features in the form of three principles of learning: (1) the principle of active learning, (2) the principle of best motivation, and (3) the principle of consecutive phases. Specific to the third principle, Three phases are distinguished: exploration, formalization, and assimilation. Polya (1965) summarizes that "For efficient learning, an exploratory phase should precede the phase of verbalization and concept formation and, eventually, the material learned should be merged in, and contribute to, the integral mental attitude of the learner" (p. 104).

The Erlanger Programm

In 1872, Felix Klein (1893) established a structure for the analysis of geometric concepts, now referred to as the Erlanger Programm, and presented his definition of a geometry: "A geometry is the study of the properties that are

invariant when the subsets of a set *S* are mapped by the transformations of some group of transformations" (Hall, 1973, p.95). Using Klein's definition of geometry, it is possible to categorize and name various geometries according to both the invariant properties under that group and the geometries involved.

The transformations of topology are called homomorphisms; a one-to-one transformation *f* is called a homomorphism if it is continuous and reversibly continuous. Shapes can be altered by stretching, compressing, bending, and twisting, but not by tearing or joining (Mansfield, 1985). Projective geometry can be characterized as the study of properties invariant under the group of collineations; collineations are special homomorphisms which transform collinear points into collinear points and, therefore, lines into lines. Affine geometry is obtained as a subgeometry of projective geometry by restricting the group of projective transformations in such a way as to introduce parallelism and distance. In particular, affine transformations map equal distances into equal distances on the same or parallel lines, and midpoints into midpoints.

Whereas an affine transformation multiplies distances in the same direction by a constant, a similarity transformation multiplies all distances by the same positive number *k*, the ratio of the similarity transformation. Thus, the shape of a figure is preserved but not its size. The Euclidean transformation group is a subgroup of the similitude transformation group as a Euclidean transformation preserves distance (mensuration).

Similarity and Proportional Reasoning

Fuson (1978) observed that most of Piaget's (1948/1956, 1960) analyses focus on topological concepts, or on Euclidean concepts, leaving the projective, affine, and similitude transformational groups relatively neglected. Nevertheless, Piaget and Inhelder (1948/1956) describe four experiments related to the concept of similarity: (i) drawing similar triangles, (ii) sorting similar cardboard triangles, (iii) choosing and drawing similar rectangles, and (iv) drawing a similar configuration of line segments. These experiments illustrate the gradual procurement of angle and parallelism concepts. Piaget's categorization indicates that children begin to use proportions at the age of eleven; perceptual judgment of similarity is possible two years earlier (p.374). The failure to perform well in similarity tasks at an earlier age is attributed to functioning in pre-operational thought and, later, the inability to use mental constructs (i.e. proportions) involving second-order relations (Lunzer, 1968).

Freidlander's (1984) study of 675 suburban, middle-class, midwestern, predominantly white students addressed four similarity-related topics: (1) basic properties of similar shapes, (2) proportional reasoning, (3) area relationships of similar shapes, and (4) applications. Their teachers had volunteered to teach the Similarity Unit--an instructional unit developed by the Middle Grades Mathematics Project (Lappan, Fitzgerald, Winter, & Phillips, 1986) for grades six, seven and eight. Pre- and post-instructional

performance, at a significance level of 0.05, show achievement increased as a function of grade level.

In a study involving 119 high school geometry students, Chazan (1988) reported the results of an investigation into high school students' understanding of similarity. A unit was constructed for use with the *Geometric Supposers* (Schwartz & Yerushalmy, 1985). Students were observed as they learned similarity with this unit and were given pretests and posttests on fractions, ratio and proportion, and similarity. Data from the study show that even those students who show an understanding of multiplicative ratio may exhibit that understanding with certain geometric configurations and not with others. It is important to note that this study did not compare students who used the *Geometric Supposers* (Schwartz & Yerushalmy, 1985) with those who did not; instead this study focused on students' understanding of aspects of similarity.

Fleener,et al (1993) investigated the following questions: (1) Does knowing a standard algorithm for solving proportion problems interfere with the development of proportional reasoning? (2) Are stronger mathematics students more flexible or intuitive than weaker students in applying proportional reasoning strategies to solve problems? and (3) What is the relationship between a student's general level of reasoning ability (concrete, transitional, or formal) and the strategies used for solving proportional reasoning tasks? Sixteen ninth grade students engaged in proportional reasoning tasks and computational proportional problem solving in their science classes made a general positive gain in Lawson's Classroom Test of Scientific Reasoning scores,

although gains for concrete learners were mixed. Explicit teaching of the concept of ratios and student engagement in exploratory studies of the relationships between and among ratios provoked development of proportional reasoning for average students. The researchers concluded that students can benefit from experiences from which the cross-multiply-and-divide algorithm can be derived.

The understandings involved in proportional reasoning as embedded in similarity tasks are complex. According to Piaget (1975), the essential feature of proportional reasoning is the involvement of a complex quaternary relation, a relationship between two relationships, rather than a simple relationship between two directly perceivable quantities (English, 1995). Tournaire and Pulos (1985) posit one should expect this type of reasoning to develop slowly over several years. Research rational numbers and multiplicative structures suggests that many teachers do not have this understanding (Cramer & Lesh, 1988; Harel & Behr, 1995; Lacampagne, Post, Harel, & Behr, 1988).

The aforementioned studies provide a basis for the discussion of the requirements for an appropriate instructional environment to maximize the learning of the concept of similarity and, in general, the development of proportional reasoning. In view of the eclectic epistemological framework presented by this researcher, a thorough investigation of performance on similarity tasks necessitates an instructional environment where exploration and inquiry dominate the climate of the classroom.

Computer-based Geometric Construction Environment

Several studies (McCoy, 1991; Yerushalmy, 1991; Yerushalmy & Chazan, 1990; Yerushalmy & Chazan, 1993) show that, when used as intended, the *Geometric Supposers* (Schwartz & Yerushalmy, 1985) create a powerful learning environment: geometry lessons are transformed into active explorations of geometric shapes resulting in stating, testing, and proving one's own conjectures. Thus, the conception of mathematics shifts from something the learners encounter and observe to something they do and invent.

McCoy (1991) compared the geometry achievement of a tenth grade class (n=29) which used the *Geometric Supposers* (Schwartz & Yerushalmy, 1985) periodically during one school year and a similar class (n=29) which did not use the software. Analysis of Covariance was used to determine the effect of the treatment on post geometry treatment scores. To control for individual differences, the covariate was mathematics scores from the pretest, the SRA Achievement Test. The final examination provided by the publisher of the adopted textbook was used as the posttest. The items were examined and classified according to Bloom's Taxonomy (Bloom, 1956). Results revealed the experimental group had higher scores on upper hierarchical (analysis, synthesis, and evaluation) level problems and there was no statistically significant difference on lower hierarchical (knowledge and comprehension) level questions.

Forty-eight eighth graders who worked with the *Geometric Supposers* (Schwartz & Yerushalmy, 1985) outperformed a comparison group of ninety students on a test that measured knowledge of basic geometry concepts even though the comparison group was taught the same concepts and topics during the same amount of time. Yerushalmy (1991) found that the main difference between the groups was that the experimental group did not exhibit some of the frequently observed, persistent misconceptions such as having a stereotyped image of certain geometric concepts and shapes. Moreover, Yerushalmy & Chazan (1993) demonstrated that high school students working with the *Geometric Supposers* (Schwartz & Yerushalmy, 1985) seemed to have greater flexibility in interpreting figures and diagrams; in fact, the comparison group showed greater difficulty in overcoming visual obstacles such as the inability to perceive a diagram in different ways.

To explore which teaching strategies may adapt to a computer environment that facilitates discovery learning and encourages cooperative learning, Garcia-de Galindo (1994) compared the teaching strategies and evaluation methods of preservice high school geometry teachers observed in traditional classes and classes implementing the *Geometric Supposers* (Schwartz & Yerushalmy, 1985) and/or the *Geometer's Sketchpad* (Jackiw, 1995). Comparative analysis was used to categorize the teaching strategies, teaching styles, evaluation methods, classroom interactions, and computer uses of four selected teachers.

The main results were that the preferred teaching strategies and evaluation methods of the Intuitive-Feeling, Intuitive-Thinking, and Sensing-Feeling teachers may adapt to computer environments that encourage discovery and cooperative learning. The Sensing-Thinking teacher may prefer the model of teacher-centered instruction when teaching high school geometry with computers.

The four teachers reported plans to use computers in different ways. The Intuitive-Feeling teacher related plans to do calculations; the Intuitive-Thinking teacher, to generate conjectures; the Sensing-Thinking teacher, to teach the curriculum; and the Sensing-Feeling teacher, to review basic skills.

Developed for students to conjecture about Euclidean geometry, geometric construction software provides an effective means to collect a wealth of visual and numerical data to analyze. By observing the relationships revealed by the data, insightful thinking is exercised. This in turn generates need for further exploration to collect necessary data to test the conjectures and the learning cycle begins anew.

Dynamic Geometric Construction Environment

In comparison to the geometric environment described above, dynamic construction software affords the added power of dynamic manipulation of the geometric objects for efficient exploration of geometric phenomena. The construction of, for example, a quadrilateral, thus

represents the mathematical object in its fullest form due to its ability to be transformed into, in this case, another quadrilateral. Thus, the dynamic capability aids in the imagery process to see the invariance and acquire a richer global view regarding necessary and sufficient conditions for construction and analysis of the constructed objects.

Elchuck (1992) explored the effects of the dynamic capability of the geometric tool software *Geometer's Sketchpad* (Jackiw, 1995) on conjecture making ability. One hundred fifty seven grade nine students were randomly assigned to either of two treatment groups, based on capabilities of the tool software. The dynamic group had access to the full power of the *Geometer's Sketchpad* (Jackiw, 1995); the static group had access to all but the drag capabilities of this software. All participants were administered tests to determine their mathematics achievement level, spatial visualization skill, locus of control, and van Hiele level of geometric thought prior to engaging in an instructional unit. Both treatment groups underwent identical instruction in the same geometric content. The study demonstrated that mathematics achievement is a statistically significant predictor of conjecture making ability. In a post-hoc regression analysis, the type of software (dynamic versus static) was found to predict conjecture making ability when the factor school was also included in the regression model.

The purpose of Foletta's (1992) case study was to describe the nature of four high school geometry students' inquiry by observing how the students used *Geometer's*

Sketchpad (Jackiw, 1995) and by characterizing their small group interactions. Factors contributing to the students' inquiry included the role of the *Geometer's Sketchpad* (Jackiw, 1995), the design of the investigations, and the nature of peer interactions. Foletta (1992) posits that the students adapt the *Geometer's Sketchpad* (Jackiw, 1995) as an extension of the paper-and-pencil medium.

A posttest-only control-group quasi-experimental study was conducted by Lester (1996) to address the problem of improving achievement of geometric knowledge through instructional use of the software program *Geometer's Sketchpad* (Jackiw, 1995). An inductive reasoning approach was the pedagogy of instruction. Forty-seven female high school geometry students participated in the study. The *Geometer's Sketchpad* (Jackiw, 1995) was the tool used by participants in the experimental group; the control group used traditional geometry tools: ruler, pencil, protractor, and compass. The three dependent variables measured on a posttest were: geometric knowledge, construction, and geometric conjecturing. Descriptive findings for the dependent variables geometric knowledge and construction were not statistically significant at an alpha level of 0.05; descriptive findings for the dependent variable geometric conjectures were statistically significant at an alpha level of 0.05. Results from the study indicated that students learned geometry skills with greater efficiency and understood geometry concepts at higher levels as a result of creating and manipulating dynamic constructions of geometric objects on the computer screen.

The relationships among 158 high school geometry students' spatial visualization ability, mathematical ability, and problem solving strategies with and without the availability of the *Geometer's Sketchpad* (Jackiw, 1995) was explored by Robinson (1994). Following instruction using the *Geometer's Sketchpad* (Jackiw, 1995), participants were randomly assigned to one of two groups: with or without software access. The availability of the computer was not a significant factor for performance on problems related to mathematical locus. Teaching specific skills resulted in similar strategies used by participants with and without access to the *Geometer's Sketchpad* (Jackiw, 1995). Robinson (1994) posited that strategies learned with the technology transfer to paper and pencil situations.

Dixon (1995) investigated the effects of a dynamic instructional environment, English proficiency, and visualization level on the construction of concepts of rigid motion transformations. Two hundred forty-one middle school students were trained on use of the *Geometer's Sketchpad* (Jackiw, 1995). The control group was taught using the traditional textbook approach while the treatment group worked in a Macintosh computer laboratory. After controlling for initial differences, Dixon (1995) concluded that students experiencing the dynamic instructional environment significantly outperformed students experiencing a traditional instructional environment on content measures of rigid motion transformations, as well as on certain measures of visualization at an alpha level of 0.01.

The purpose of Choi's (1996) study was to investigate secondary school students' development of geometric thought during instruction based on a van Hiele model and using dynamic computer software as a tool. In particular, the students' learning process was traced in relation to van Hiele levels of geometric thought with geometric topics using an interactive computer environment. The clinical interview procedure was used. It was evidenced that the instruction based on the van Hiele's five instructional phases was well integrated with the use of the dynamic nature of the software since all students showed extensive development of geometric thought. Also, the use of the dynamic capabilities of the geometric construction software, *Geometer's Sketchpad* (Jackiw, 1995), was found to provide an advantage to students because it facilitated the movement from symbol to signal and then to implicatory character.

Research has also been conducted to investigate how dynamic geometry learning environments qualitatively impact psychological aspects of geometry learning. Yousef (1997) studied the effect of using *Geometer's Sketchpad* (Jackiw, 1994) on high school students' attitude toward geometry. The sample consisted of two groups with two classes in each group. All classes involved in the study participated in exploration activities. The exploration activities for the students in the experimental group involved the use of the *Geometer's Sketchpad* (Jackiw, 1995) and for the control group involved paper-and-pencil work only. Measurement of attitude toward geometry was conducted before and after the implementation of the experiment for both groups. Results

indicated that the scores of the pretest and the posttest of the students in the experimental group were significantly different and there was a significant difference between the gain in the scores of the control and experimental group. Qualitative data were collected from two sources: observation and interviews. The results supported the quantitative data.

Melzcarek (1996) focused on the effects of problem-solving activities using dynamic geometry computer software on readiness for self-directed learning. Attitude towards the learning of mathematics, the use of computers, and their mediating effects were also explored. Six high school classes received the experimental treatment and visited the mathematics computer laboratory once a week for a period of six weeks to work on problem-solving activities that were designed to be used with the *Geometer's Sketchpad* (Jackiw, 1995). One class served as the control group. The results of this study indicated a positive relationship between the use of dynamic geometry computer software and readiness for self-directed learning through the mediating effects of attitude towards dynamic geometry computer software.

A task-based clinical interview procedure was used by Manouchehri (1994) to study the cognitive actions of two preservice elementary teachers and their interactions with the *Geometer's Sketchpad* (Jackiw, 1995) as they worked on computer-based geometry explorations and problem solving activities. A final interview session was conducted to solicit the participants perspectives on their experiences with the computer-based activities. In the process of active engagement in exploring with the interactive computer

software, the participants moved from a naive intuitive mode to an analytical mode of thinking. Some difficulties resulted from subjects' lack of knowledge about how to explore, their lack of reflection on their operations and findings, and their inadequate content knowledge base. The collective impact of these difficulties led the subjects to drawing false conclusions and overgeneralizations. Both subjects appreciated being in control of their own learning. They articulated, however, that such experience was confusing and frustrating at times.

Summary

The review of the literature presented in this chapter commenced with a theoretical framework grounded in constructivist epistemology. According to von Glasersfeld (1995), new knowledge is constructed and reconstructed from prior knowledge within an environment of active participation as one strives to organize one's experiential world. A discussion of what repetitive experience means to research is included in addition to theories of instruction focusing on conceptual development of mathematical ideas. A review of geometric construction software, including the *Geometric Supposers* (Schwartz & Yerushalmy, 1985) and/or the *Geometer's Sketchpad* (Jackiw, 1995), revealed that it has been considerably discussed, tested, and used. The incorporation of a dynamic aspect to geometric construction software warranted further discussion.

The research provided a strong foundation for investigation into geometry learning within a dynamic environment. The research also uncovered the need for further examination of the impact of a dynamic geometry learning environment on geometry instruction effectiveness. Furthermore, the lack of research with preservice elementary teachers warrants investigation. It is the synthesis of the information gained from the review that provided the foundation and justification for this study.

CHAPTER III
RESEARCH DESIGN AND METHODOLOGY

The study methodology is detailed in this chapter which commences with the research objectives. The population addressed by this study and the participants of this particular research study are described next. This is followed by a discussion of the materials and the measurement instrument to be used. Then, a summary of the pilot sessions is described. Finally, the design of the study and the procedures to be undertaken are detailed.

Research Objectives

This research study was designed to investigate the effect of the dynamic geometry learning environment *Geometer's Sketchpad* (Jackiw, 1995) on preservice K-8 teachers' performance on similarity tasks. In light of the given eclectic theoretical framework and the literature review that reports the results of research on aspects of geometry learning, task performance was identified as a core category to analyze how preservice teachers respond to learning in a dynamic environment. The hypotheses for the study were generated from the questions posed earlier and an inspection of pilot study data. In order to facilitate

quantitative statistical analyses, following each question is the list of hypotheses pertaining to that question used for analyses of data:

1. After controlling for initial performance on similarity tasks, do preservice K-8 teachers who work on similarity tasks in a dynamic geometry learning environment exhibit significantly different posttest performance than those who study the concept of similarity in a paper-and-pencil learning environment that employs traditional tools such as ruler and protractor?

   1a. Hypothesis: There is no significant effect on posttest scores due to the interaction of treatment, course section, and pretest scores, using pretest scores as a covariate.

   1b. Hypothesis: There is no significant effect on posttest scores due to the interaction of course section and pretest scores, using pretest scores as a covariate.

   1c. Hypothesis: There is no significant effect on posttest scores due to the interaction of treatment and pretest scores, using pretest scores as a covariate.

   1d. Hypothesis: There is no significant effect on posttest scores due to the interaction of treatment and course section, using pretest scores as a covariate.

   1e. Hypothesis: There is no significant effect on posttest scores for students grouped by course section, using pretest scores as a covariate.

   1f. Hypothesis: There is no significant difference on posttest scores for students grouped by treatment, using pretest scores as a covariate.

2. After controlling for prior knowledge as measured by a standardized achievement test, do preservice K-8 teachers who work on similarity tasks in a dynamic geometry learning environment exhibit significantly different posttest performance than those who study the concept of similarity in a paper-and-pencil learning environment that employs traditional tools such as ruler and protractor?

2a. Hypothesis: There is no significant effect on posttest scores due to the interaction of treatment, course section, and SAT scores, using SAT scores as a covariate.

2b. Hypothesis: There is no significant effect on posttest scores due to the interaction of course section and SAT scores, using SAT scores as a covariate.

2c. Hypothesis: There is no significant effect on posttest scores due to the interaction of treatment and SAT scores, using SAT scores as a covariate.

2d. Hypothesis: There is no significant effect on posttest scores due to the interaction of treatment and course section, using SAT scores as a covariate.

2e. Hypothesis: There is no significant effect on posttest scores for students grouped by course section, using SAT scores as a covariate.

2f. Hypothesis: There is no significant effect on posttest scores for students grouped by treatment, using SAT scores as a covariate.

Population and Sample

The research population consisted of preservice elementary education teachers attending an American university. The 52 participants of this study were College of Education students enrolled in three sections of a mathematics methods course during a spring semester. This course, addressing methods for teaching elementary school mathematics, is a requirement of the teacher preparation program in elementary education at a major land-grant university located in the Southeastern United States. The participants were randomly assigned to two groups within each

section. One group worked in the dynamic geometry learning environment; another group worked in the traditional paper and pencil manner (refer to Definition of Terms).

Instructional Materials

The experiential-activity orientation of the Middle Grades Mathematics Project [MGMP] was used with both experimental and control groups. These materials follow recommendations of the Piagetian research on geometrical development and the instructional implications of the van Hiele studies (Friedlander, 1984). Each activity utilizes an instructional model consisting of three stages:

(1) *launch* introduces new concepts, clarifies definitions, reviews old concepts, and issues some challenge;

(2) *exploration* entails gathering data, looking for patterns, making conjectures, or developing other types of problem solving strategies;

(3) *summary* demonstrates ways to organize data, discusses the used strategies, and refines these strategies into efficient problem solving techniques.

The *launch* and *summary* stages are conducted in a whole-class mode to purposefully generate taken-as-shared knowledge (Cobb, et al., 1992); while at the *exploration* phase, individuals investigate the problem situation and follow their own path of cognition in analysis of the given task.

The MGMP material presents the concept of similarity at the van Hiele Level I, properties of shapes, and Level II, relationships among properties of shapes, of geometric

development (Friedlander, 1984). Consideration of the Van Hiele levels in the teaching of geometry in the United States facilitates progress in the learning and understanding of geometrical concepts (Burger & Shaughnessy, 1986; Carpenter et al., 1981; Clements & Battista, 1992; Crowley, 1987; Fuys, Geddes, & Tischler, 1988; Jaime & Gutierrez,1995).

Pilot Studies

In May, 1997, a pilot study was conducted using undergraduate students as defined by the population sample. The students participated in four ninety-minute sessions utilizing the *Geometer's Sketchpad* (Jackiw, 1995), a dynamic geometry software package. The first session was simply an introduction to resources made available to students enrolled in the class and did not constitute part of the treatment. Table 3.1 in Appendix B summarizes the academic content of the learning sessions conducted by the researcher.

As a result of the initial pilot, adjustments were made regarding the structure of the study. The length of each session was limited to a fifty-minute session and the number of learning sessions decreased. Participants considered the sessions too lengthy and the researcher noted some students were off task after approximately one hour. In addition, the "launch" phase for each learning session was viewed by participants as too in-depth and in need of revision for use with preservice teachers. Although appropriate for middle school students, adjustments were made to reflect the curricular needs of college students.

A second pilot study was conducted during the Fall of 1997. Undergraduate students enrolled in the mandatory introductory mathematics methods course for the preservice teacher education program participated in three fifty-minute learning sessions according to random placement in one of two groups as described above. Again, adjustments were made regarding the structure of the study. Content was further parametrized to linear measurement, therefore eliminating introduction of the tangential topic of area growth. As a consequence of the change in curriculum, the research instrument was appropriately altered, with items addressing area growth omitted. Additionally, items hypothesized to address learning attributed to the dynamic component of the geometric construction software were developed and placed into the final version of the research instrument. Table 3.2 in Appendix B summarizes the academic content of the learning sessions conducted by this researcher.

Instrumentation

Performance on the concept of similarity was measured by an instrument developed by this researcher and based, with permission, on items used by the Middle Grades Mathematics Project [MGMP] (1986). The instrument, given in Appendix A, consisted of 21 multiple choice items with up to five options for each item. Items were scored by assigning a 1 for a correct response and a 0 otherwise; no correction is made for guessing. The total score is considered as a general indicator of the level of task performance.

Content validity and reliability were considered. To insure that the instrument reflected the content domain of transformational geometry regarding the concept of similarity, a panel of five experts analyzed the twenty-one items to ascertain validity. The experts, three mathematics educators and two research mathematicians, reviewed each problem with consideration to mathematical content.

The instrument was field-tested in Spring, 1998, on sixty-six preservice elementary teachers enrolled in two sections the required elementary mathematics methods course at an American university located in the southeast sector. Measuring internal consistency of the test, the Cronbach reliability coefficient calculated from the data is 0.70. The coefficient of stability, calculated for test-retest reliability, is 0.70.

Regarding the covariates used in this study, multiple partial correlations were computed to assess to the strength of association between the dependent variable and the covariate while controlling for the other independent variables treatment and section. The research instrument, administered as the pretest, served as the covariate to statistically analyze the hypotheses related to the first question. For the second question, prior mathematical knowledge, as measured by the mathematics section of the SAT, served to adjust for initial random differences in the two treatment groups. The multiple partial correlation with respect to the covariate SAT was 0.4281 for this study and, concerning the covariate pretest, 0.5925 was calculated.

Thus, statistical analyses supported confidence for the utilization of the specified variables.

Procedure and Data Collection

The procedure consisted of a pretest, three learning sessions, and a posttest. The pretest was administered eight weeks prior to the learning sessions. The sessions for the experimental group were held in a computer laboratory on campus and involved use of the dynamic geometry software *Geometer's Sketchpad* (Jackiw, 1995); sessions for the control group were held in the traditional mathematics laboratory and involved use of manipulatives, protractor, and ruler. Written reflections were collected from all participants at the end of each learning session in order to gain further insight regarding the depth and breadth of the learning that had occurred. During the posttest, administered immediately upon completion of the research study learning sessions, participants were given access to whichever tools they had used during the course of the research study.

All components of the study were administered by this researcher during scheduled meeting times. The researcher conducted all lessons to eliminate differences in results due to instructor variability, but was aware of the possibility that a threat to validity may be introduced by the researcher influencing results. This was minimized by the use of specific lesson plans including printed activities for the participants of both groups. The contents of the learning

sessions are given in Table 3.3 of Appendix B with exemplary printed activities, showing student work, presented in Appendix C.

Prior to the study the Institutional Review Board of the given University granted permission for the investigation to take place. Students were informed; both researcher as principal investigator and those who agreed to participate in the study signed the consent form. An addendum was proposed and permission granted to access participants SAT scores. In a similar manner, students were informed and the parties involved gave written approval. The letters of informed consent are given in Appendix D.

Statistical Design of the Study

The effect of environmental intervention was examined by use of a pretest-posttest control-group design with random assignment. The Borg and Gall (1989) configuration is shown as

R O X O

R O O

where X represents the experimental treatment, O represents pretest or posttest measurement of the dependent variable, and R indicates that the experimental and control groups were formed randomly. The randomization procedure occurred within each course section.

A literature review of studies focusing on aspects of geometry learning suggested use of several statistical

procedures (Dixon, 1996; Fleener, et al, 1993; Friedlander, 1984; Lester, 1996; McCoy, 1991; Melzcarek, 1996; Robinson, 1994; Yousef, 1997). To analyze the data collected during the study, descriptive statistics were obtained for all variables under consideration and analysis of covariance [ANCOVA] was conducted. The ANCOVA allows one to test for interactions; in particular, the interaction of the treatment and course sections is of particular interest to this study. Furthermore, ANCOVA does adjust for random differences in the two groups. In addition to quantitative statistical analyses, reflections collected in written form were examined to further elucidate the geometry learning experience.

Summary

This chapter commenced with the research objectives. Information regarding the population, research sample, and materials followed. Next, the instrument used in this particular study was described and the pilots were presented. This chapter ended with the design of the study which included a description of the procedures utilized in the study.

CHAPTER IV
RESULTS

This chapter contains the results of the analyses described in the previous chapter. Descriptive statistics for the variables under investigation and ANCOVAs are presented along with the results of the qualitative component detailing the geometry learning experience. The chapter begins with descriptions of the research instrument, research objectives, and the goals of the study. Next, descriptive statistics are presented and the research hypotheses that reflect the research questions are articulated. Then, the analyses of the data are given in light of the hypotheses. Finally, an exposition of written reflections is presented to elucidate the geometry learning experience.

The Research Instrument

The research instrument measured performance on similarity tasks in a multiple choice format with up to five options for each of the twenty-one items. Appendix A contains the research instrument. Items were scored by assigning a 1 for a correct response and a 0 otherwise; no correction was made for guessing. The total score was considered as a general indicator of the level of task performance.

Research Objectives

To test for significant difference in task performance between participants grouped by learning environment, questions were developed to explore the effect of a dynamic geometry learning environment on preservice elementary teachers' performance on similarity tasks. Hypotheses were formulated to identify significant interactions between various independent variables and to adjust for initial performance on similarity tasks as measured by the pretest or by prior mathematics knowledge as measured by a standardized achievement test.

The comparison of scores on the research instrument that assessed performance on similarity tasks, administered prior to and at the end of the study, served as the basis for the analysis of the first question. The second question focused upon the effect of a dynamic geometry learning environment on preservice elementary teachers' performance on similarity tasks with prior mathematics performance adjusted for by a standardized achievement test. Hypotheses pertaining to each question allowed for statistical testing. The results of the analyses are reported at an alpha level of 0.05 in all cases.

Goals of the Study

Based upon the assumptions that teachers need to learn high quality mathematics and to be taught in a manner similar to the way they are expected to teach, two questions of

interest were generated for this particular study, focusing upon the effect of the dynamic geometry learning environment on preservice elementary teachers' performance on similarity tasks as measured by the research instrument:

1. After controlling for initial performance on similarity tasks, do preservice elementary teachers who work on similarity tasks in a dynamic geometry learning environment exhibit significantly different posttest performance than those who study the concept of similarity in a paper-and-pencil learning environment that employs traditional tools such as ruler and protractor?

2. After controlling for prior knowledge as measured by a standardized achievement test, do preservice elementary teachers who work on similarity tasks in a dynamic geometry learning environment exhibit significantly different posttest performance than those who study the concept of similarity in a paper-and-pencil learning environment that employs traditional tools such as ruler and protractor?

Thus, the goals of the study included answering the research questions in light of the given assumptions.

Descriptive Statistics

For the sake of completeness, the data set for the sample population is given in Appendix E, sorted first by treatment and then by gain score. This inclusion is to afford the reader the option to compare the relative performance of the participants on the posttest. Descriptive statistics for responses to the pretest, SAT mathematics component, and posttest for the entire sample, by group, and by group and course section, are presented in Tables 1, 2, and 3, respectively. The sample size, mean, and standard deviation are listed for each response category. The posttest mean

scores, adjusted for the covariate, were computed as 14.3699 for the control group and 16.2568 for the experimental group.

Table 1
Descriptive Statistics / Combined

	Number	Mean	Standard Deviation
Pretest	52	12.2885	3.4376
SAT	48	568.3333	80.2213
Posttest	52	15.5769	3.0120

Table 2
Descriptive Statistics / by Group

	Group	Number	Mean	Standard Deviation
Pretest	control	26	12.2308	3.2287
	experimental	26	12.3462	3.6980
SAT	control	24	580.0000	73.0098
	experimental	24	556.6667	86.8115
Posttest	control	26	14.6538	2.6542
	experimental	26	16.5000	3.1145

Research Hypotheses

The following hypotheses were generated to test for significant effect in mathematics achievement regarding preservice teachers' performance on similarity tasks as measured by the research instrument. For the first research

Table 3
Descriptive Statistics / by Group and Section

	Section	Group	Number	Mean	Standard Deviation
Pretest	a	control	7	10.0000	3.7417
		experimental	5	10.8000	4.3818
	b	control	11	13.8182	2.5226
		experimental	12	13.1667	3.7376
	m	control	8	12.0000	2.6726
		experimental	9	12.1111	3.3706
SAT	a	control	6	585.0000	55.0454
		experimental	5	556.0000	99.9000
	b	control	10	597.0000	75.1369
		experimental	11	563.6363	87.4383
	m	control	8	555.0000	83.3238
		experimental	8	547.5000	89.2428
Posttest	a	control	7	13.7143	1.4960
		experimental	5	14.0000	3.5637
	b	control	11	15.0909	2.4680
		experimental	12	17.2500	2.7010
	m	control	8	14.8750	3.6425
		experimental	9	16.4444	3.3582

question, initial differences in mathematics achievement were controlled for through the use of the research instrument administered as a pretest. The latter set of hypotheses addressed the second question and controlled for initial differences in mathematics achievement via the mathematics component of the standardized achievement test SAT.

1a. Hypothesis: There is no significant effect on posttest scores due to the interaction of treatment, course section, and pretest scores, using pretest scores as the covariate.

1b. Hypothesis: There is no significant effect on posttest scores due to the interaction of course section and pretest scores, using pretest scores as the covariate.

1c. Hypothesis: There is no significant effect on posttest scores due to the interaction of treatment and pretest scores, using pretest scores as the covariate.

1d. Hypothesis: There is no significant effect on posttest scores due to the interaction of treatment and course section, using pretest scores as the covariate.

1e. Hypothesis: There is no significant effect on posttest scores for students grouped by course section, using pretest scores as the covariate.

1f. Hypothesis: There is no significant effect on posttest scores for students grouped by treatment, using pretest scores as the covariate.

2a. Hypothesis: There is no significant effect on posttest scores due to interaction effect of treatment, course section, and SAT scores, using SAT scores as the covariate.

2b. Hypothesis: There is no significant effect on posttest scores due to the interaction of course section and SAT scores, using SAT scores as the covariate.

2c. Hypothesis: There is no significant effect on posttest scores due to the interaction of treatment and SAT scores, using SAT scores as the covariate.

2d. Hypothesis: There is no significant effect on posttest scores due to the interaction of treatment and course section, using SAT scores as the covariate.

2e. Hypothesis: There is no significant effect on posttest scores for students grouped by course section, using SAT scores as the covariate.

2f. Hypothesis: There is no significant effect on posttest scores for students grouped by treatment, using SAT scores as the covariate.

Results of Data Analyses

The initial analysis of covariance, controlling for pretest performance, yielded a p-value greater than the set alpha level of p=0.05 and the null hypothesis was accepted (Refer to Table 4). That is, no significant effect on posttest scores due to the interaction of treatment (trt), course section (scn) and pretest scores (pre), after controlling for pretest performance, was found (p=0.1017). Therefore, the following null hypothesis could not be rejected:

> Using pretest scores as the covariate, there is no significant effect on posttest scores due to the interaction of treatment, course section and pretest scores.

Finding no significant three-way interaction, the three-way interaction term was removed from the model. After removal of the three-way interaction term, hypotheses 1b-1d were then tested. There were no statistically significant two-way interactions based on the results of the ANCOVAs. The p-values for hypotheses 1b-1d were 0.2939, 0.5337, and 0.7871, respectively. Therefore, the following null hypotheses could not be rejected:

> Using pretest scores as the covariate, there is no significant effect on posttest scores due to the interaction of course section and pretest scores.

> Using pretest scores as the covariate, there is no significant effect on posttest scores due to the interaction of treatment and pretest scores.

> Using pretest scores as the covariate, there is no significant effect on posttest scores due to the interaction of treatment and course section.

Table 4
Initial Analysis Of Covariance
with Pretest as Covariate

Dependent Variable: Posttest

Source	DF	SS	MS	F Value	Pr > F
trt	1	0.3639	0.3639	0.07	0.7935
pre	1	134.4845	134.4845	25.66	0.0001
scn	2	19.1402	9.5701	1.83	0.1742
pre*trt	1	0.7196	0.7196	0.14	0.7129
trt*scn	2	24.0913	12.0457	2.30	0.1135
pre*scn	2	23.4062	11.7031	2.23	0.1204
pre*trt*scn	2	25.3796	12.6898	2.42	0.1017

Upon finding no significant two-way interactions, the interaction terms trt*pre and pre*scn were removed from the model. The term trt*scn was left in the model to guarantee control of the blocking variable course section. The ANCOVA information is presented in Table 5. After removal of the interaction terms, course section showed no statistical significance (p=0.8332) but treatment was statistically significant at the alpha level of 0.05 (p=0.0275). Thus, the following null hypothesis to the initial research question could not be rejected:

> Using pretest scores as the covariate, there is no significant effect on posttest scores for students grouped by course section.

However, the following null hypothesis could be rejected:

> Using pretest scores as the covariate, there is no significant effect on posttest scores for students grouped by treatment.

The analyses relevant to the first question indicated a statistically significant difference in performance on similarity tasks between the experimental group and the

Table 5
Analysis Of Covariance
Final Model with Pretest as Covariate

Dependent Variable: Posttest

Source	DF	SS	MS	F Value	Pr > F
trt	1	29.0884	29.0884	15.19	0.0275*
pre	1	136.3718	136.3718	24.23	0.0001
scn	2	2.0534	1.0262	0.18	0.8332
trt*scn	2	6.7176	3.3588	0.60	0.5534

*statistically significant at alpha level 0.05

control group. LSMeans, with respect to use of the pretest as covariate, was calculated at 14.74 for the control group and 16.30 for the experimental group. The remaining hypotheses, addressing the second question, were then analyzed. Again, no significant difference on posttest scores due to interaction effect of treatment (trt), course section (scn) and SAT scores (sat) was found (p=0.7926) after controlling for prior mathematics performance. That is, the analysis of covariance, controlling for prior mathematics performance, yielded a p-value greater than the set alpha level of p<0.05 and the null hypothesis was accepted (Refer to Table 6). Therefore, the following null hypothesis could not be rejected:

> Using SAT scores as the covariate, there is no significant effect on posttest scores due to the interaction effect of treatment, course section and SAT scores.

The three-way interaction term was removed from the model upon finding no significant three-way interaction. After removal of the three-way interaction term, hypotheses 2b-2d were then tested. There were no statistically

Table 6
Initial Analysis Of Covariance
with SAT as Covariate

Dependent Variable: Posttest

Source	DF	SS	MS	F Value	Pr > F
trt	1	1.5549	1.5549	0.19	0.6681
sat	1	62.8058	62.8058	7.59	0.0091
scn	2	0.9755	0.4877	0.06	0.9428
sat*scn	1	0.2838	0.1419	0.02	0.9830
trt*scn	2	3.2057	1.6028	0.19	0.8247
sat*trt	2	0.2638	0.2638	0.03	0.8592
sat*trt*scn	2	3.8684	1.9342	0.23	0.7926

significant two-way interactions based on the results of the ANCOVAs. The p-values for hypotheses 2b-2d were 0.9550, 0.7317, and 0.8297, respectively. Therefore, the following null hypotheses could not be rejected:

> Using SAT scores as the covariate, there is no significant effect on posttest scores due to the interaction of course section and SAT scores.
>
> Using SAT scores as the covariate, there is no significant effect on posttest scores due to the interaction of treatment and SAT scores.
>
> Using SAT scores as the covariate, there is no significant effect on posttest scores due to the interaction effect of treatment and course section.

The interaction terms trt*sat and sat*scn were removed from the model upon finding no significant two-way interactions. The term trt*scn was left in the model to guarantee control of the blocking variable course section. After removal of the interaction terms, course section continued to show no statistical significance (p=0.2134). However, after controlling for initial differences on SAT mathematics scores, a statistically significant difference in means

existed at the alpha level of 0.05 between treatment groups (p=0.0271). The ANCOVA information is given in Table 7. LSMeans, with respect to use of the mathematics component of the SAT as covariate, was calculated at 14.37 for the control group and 16.26 for the experimental group.

Table 7
Analysis Of Covariance
Final Model with SAT as Covariate

Dependent Variable: Posttest

Source	DF	SS	MS	F Value	Pr > F
trt	1	38.8572	38.8572	5.26	0.0271*
sat	1	79.2114	79.2114	10.71	0.0022
scn	2	23.7177	11.8588	1.60	0.2134
trt*scn	2	2.9500	1.4750	0.20	0.8199

*statistically significant at alpha level 0.05

Thus, the following null hypothesis to the second research question could not be rejected:

> Using SAT scores as the covariate, there is no significant effect on posttest scores for students grouped by course section.

However, the following null hypothesis could be rejected:

> Using SAT scores as the covariate, there is no significant effect on posttest scores for students grouped by treatment.

A Reflections Component

Written reflections on the learning experience, collected from the participants at the end of each learning session, afforded the researcher valuable insights into the preservice elementary teachers' perception of what they had

learned and the value of this knowledge in their teaching. During each session, the 52 participants logged responses on activity worksheets that prompted for description of the thought processes. After each session, participants were verbally asked to write what they had learned mathematically, their reaction to the methodology, and their views as to use in the mathematics classroom.

The data gathered from responses on the activity worksheets used in this study were collected to illuminate the cognitive process occurring within each individual during active participation. Examples are presented in Appendix C. The purpose of the collecting of reflections was to gain further insight by providing opportunities for a richer story to emerge and for reflective meaning-making to take place.

Critiquing of the written reflections granted the researcher an opportunity to gain knowledge regarding the preservice elementary teachers' perspective of the experience. The participants shared their opinions and knowledge about the mathematics content, the instruction, and the technology, in addition to the learning. Affect, as well as performance, seemed to have improved due to the correspondence from the preservice teacher to the researcher.

The following excerpts, taken from the written reflections collected at the end of each session during the course of the research study, exemplify the effect of the dynamic geometry learning environment upon cognitive development. Both treatment groups participated and are represented.

Experimental group preservice elementary teacher #1:

Day 1: "Today I learned it is easier to see this idea than if I had to draw inaccurate polygons each time to compare them. I also learned how the buttons work and how to make measurements and calculations. I would like to know how to make calculations such as $a^2+b^2=c^2$ to prove what I am doing on the computer. Very cool program!"

Day 2: "I liked today's activity because the student is guided through a process but the discovery is left to them! It asks the right questions. It really provides the basis for pre-geometry study. I don't know if the student will walk away from this exercise with a grasp on proportion geometry, but at least it gets them thinking in the right mode."

Control group preservice elementary teacher #2:

Day 1: "We learned about projective geometry mostly through the use of an instrument made up of a pencil and rubber bands. The activity was unique to say the least! The most important thing that I learned from this activity was the value of practice. I was able to participate in the activity as a novice. I had no idea what I was doing and I learned everything as I went along. As a future teacher, this is a valuable lesson and will help me better understand my students' learning."

Experimental group preservice elementary teacher #3:

Day 1: "Today I did not LEARN a whole lot. I did, however, discover quite a bit!! I did not have enough time, nor enough experience to feel as if I learned. Perhaps further practice will allow me to learn next time!"

Day 2: "Today I became comfortable with the commands on the computer!! I think because this is such a new concept, I am enjoying 'playing around' with it! Again, I AM fairly comfortable w/ dilation, rotation and transformation as we are learning this in geometry class now!"

Day 3: "I said I was not learning because I already know this stuff. But I said I discovered because I could make a conjecture and immediately see the outcome. I'm not very good on computers but I see that this tool will be great for use w/ my kids. Being able to see dimensions change is awesome. Gives one the big picture--quickly and accurately."

Experimental group preservice elementary teacher #4:

Day 3: "I learned a lot because you gave us a different way to look at the concept. I could pull together old stuff and ideas and make connections. The dynamics made a difference, like the old way cartoons were made, it made things possible to see in a short amount of time."

Control group preservice elementary teacher #5:

Day 3: "I learned I could fail a test and then pass it with a bit of practice. It was neat to see that the first test actually had some background to it. When I first took it I thought it really was a bunch of made-up information. After the couple of days, I felt like I could ace the test."

The reflections portray the preservice elementary teachers' perspective regarding the learning experience. It was interesting to note that, in both groups, many

participants regarded the active hands-on approach a form of practice that was enjoyable in addition to being productive. Furthermore, connections were established between previous knowledge and new ideas so that insightful revelations (ah-ha!) occurred; this was more often evidenced during learning sessions with the experimental group. For example, teacher #3 quickly mastered the basic skills necessary to explore Euclidean constructions and was quite enthusiastic regarding his explorations. He would "conjecture and immediately see the outcome," evaluate the situation, and repeat the cycle.

Regarding affective aspects, teacher #5 was quite skeptical regarding participation in mathematical activity. As a prospective teacher, she believed the course requirement a waste of time since she intended to teach only in the primary grades. However, by the end of the study (near the end of the semester) her participation had increased dramatically; teacher #5 had constructed a firm foundation of elementary mathematics that was deemed personally relevant. In particular, she had developed an informal, intuitive understanding of the concept of mathematical similarity by way of a novel approach. Then, teacher #5 reconstructed her knowledge base regarding the concept to conclude that she had become mathematically empowered!

Of the participants in the experimental group, most became comfortable with the software during the *launch* phase when participants were given time to experiment with the tool. Teacher #1 expressed great interest in the capabilities of the *Geometer's Sketchpad* (Jackiw, 1995), asking for assistance to develop a geometric proof of the Pythagorean

Theorem. Able to transcend physical limitations of manual dexterity, she explored the capabilities of the dynamic geometry software in order to verify the theorem. The experimental group participants, for the most part, were quite impressed by the dynamic capability of the tool. The participants in the control group, exhibiting enthusiasm in a similar manner, reported plans to share manipulatives and techniques with their future students and colleagues.

Summary

Results of the statistical analyses for the study have been described in detail in this chapter in addition to presentation of a sampling of collected reflections that exemplify certain perceptions and themes held by the research participants. Discussion of the findings, including implications and recommendations for further research, are presented in the concluding chapter.

CHAPTER V
SUMMARY AND CONCLUSIONS

This final chapter discusses the results presented in the previous chapter. An overview of the study is provided first, including a description of the sample population. Next, the significance of the study is articulated, followed by the research questions. Finally, the results and a discussion of the conclusions based upon the research results is presented. Implications from the present study are given next. The chapter concludes with recommendations for possible future research.

Overview of the Study

The purpose of the study was to explore the role of the learning environment with respect to the mathematical evolution of preservice elementary teachers as a result of active inquiry using the dynamic geometric construction software *Geometer's Sketchpad* (Jackiw, 1995). That is, this study was designed to investigate the effect of a dynamic geometry learning environment on performance in the context of similarity tasks. The basic goal of the study was to detect the extent to which the dynamic environment, given its ability to efficiently display a plethora of mathematical situations, assists learning.

Complete data were available on 52 preservice elementary teachers enrolled in three sections of an undergraduate elementary mathematics methods course in the College of Education at a major land-grant university located in the Southeastern United States. The design of the study was a pretest-posttest control group design with random assignment. Detail on the design, procedures, and methodology were presented in Chapter III. The randomization procedure occurred within each course section. The procedure consisted of the pretest, three learning sessions, and the posttest. Sessions for the control group were held in the traditional mathematics laboratory and involved use of manipulatives, ruler, and protractor; the sessions for the experimental group were held in a computer laboratory on campus and involved use of the dynamic geometric construction software *Geometer's Sketchpad* (Jackiw, 1995).

To analyze the data collected during the study, descriptive statistics were obtained for all variables under consideration and analysis of covariance [ANCOVA] was conducted. The research instrument, given to all participants eight weeks prior to the instructional sessions, served as the covariate to statistically analyze the hypotheses related to the first question. For the second question, prior mathematical knowledge, as measured by a standardized achievement test, served to adjust for initial differences in the two treatment groups.

In addition to the quantitative statistical component, written reflections were collected from all participants at the end of each session. The purpose of this exercise was to

gain further insight regarding the preservice elementary teachers' perspective of the instructional environment and to ascertain what had been learned during the session.

Significance of the Study

Implementing new technologies into the mathematics curriculum is not easy and the education of teachers plays a critical role. The necessity of requiring preservice teachers to learn mathematics content and methods of teaching in an environment where computer use is an integral component is advocated by many mathematics teacher educators (Committee on the Mathematical Education of Teachers, 1991; Curcio, Perez, & Stewart, 1994; Leitzel, 1991; McNerney, 1994; National Council of Teachers of Mathematics, 1991) It is, therefore, imperative to explore how software will modify current ways of teaching and learning in order to define new roles for teachers in a technologically rich classroom environment.

Research Questions

Based on the assumptions that teachers need to learn high quality mathematics and to be taught in a manner similar to the way they are expected to teach, the specific questions under investigation were:

1. After controlling for initial performance on similarity tasks, do preservice elementary teachers who work on similarity tasks in a dynamic geometry learning environment exhibit significantly different posttest performance

than those who study the concept of similarity in a paper-and-pencil learning environment that employs traditional tools such as ruler and protractor?

2. After controlling for prior knowledge as measured by a standardized achievement test, do preservice elementary teachers who work on similarity tasks in a dynamic geometry learning environment exhibit significantly different posttest performance than those who study the concept of similarity in a paper-and-pencil learning environment that employs traditional tools such as ruler and protractor?

Results

The hypotheses that were statistically tested, with consideration of the aforementioned two questions, are listed below. Following each set of hypotheses, the results of the analysis are presented for consideration.

1a. Hypothesis: There is no significant effect on posttest scores due to the interaction of treatment, course section, and pretest scores, using pretest scores as a covariate.

1b. Hypothesis: There is no significant effect on posttest scores due to the interaction of course section and pretest scores, using pretest scores as a covariate.

1c. Hypothesis: There is no significant effect on posttest scores due to the interaction of treatment and pretest scores, using pretest scores as a covariate.

1d. Hypothesis: There is no significant effect on posttest scores due to the interaction of treatment and course section, using pretest scores as a covariate.

1e. Hypothesis: There is no significant effect on posttest scores for students grouped by course section, using pretest scores as a covariate.

1f. Hypothesis: There is no significant effect on posttest scores for students grouped by treatment, using pretest scores as a covariate.

To investigate whether the dynamic geometry learning environment affects performance on similarity tasks as measured by the research instrument, ANCOVAs were conducted with achievement on the research instrument as a covariate. Initially, ANCOVAs were applied to assure that interaction effects were not present and to develop the final model with the following terms as independent variables: pretest, treatment, course section, and the interaction of treatment with course section. The term treatment with course section was left in the model to guarantee control of the blocking variable course section.

The initial analyses indicated that there was no statistically significant effect on posttest scores due to either three-way or two-way interaction of the independent variables. Therefore, with the exclusion of the interaction term of treatment with course section, the interaction terms were removed.

An ANCOVA was conducted using the final model. Based on the analysis, there exists supportive evidence that the difference in learning environment between the two treatment groups had a significant effect upon the preservice elementary teachers' performance on similarity tasks as measured by the research instrument. Fundamentally, the participants in the experimental group outperformed the participants in the control group

even though initial variability was taken into consideration.

Let us now focus upon the hypotheses generated to address the latter question.

2a. Hypothesis: There is no significant effect on posttest scores due to the interaction of treatment, course section, and SAT scores, using SAT scores as a covariate.

2b. Hypothesis: There is no significant effect on posttest scores due to the interaction of course section and SAT scores, using SAT scores as a covariate.

2c. Hypothesis: There is no significant effect on posttest scores due to the interaction of treatment and SAT scores, using SAT scores as a covariate.

2d. Hypothesis: There is no significant effect on posttest scores due to the interaction of treatment and course section, using SAT scores as a covariate.

2e. Hypothesis: There is no significant effect on posttest scores for students grouped by course section, using SAT scores as a covariate.

2f. Hypothesis: There is no significant effect on posttest scores for students grouped by treatment, using SAT scores as a covariate.

In a similar fashion, to further investigate whether the dynamic geometry learning environment affects performance on similarity tasks as measured by the research instrument, ANCOVAs were again conducted. For this set of hypotheses, prior knowledge, as measured by SAT mathematics scores, served as the covariate. Once more, ANCOVAs were applied to assure that interaction effects were not present and to develop the final model. There was no statistically significant effect on

posttest scores due to either three-way or two-way interaction of the independent variables.

With the following independent variables as terms: SAT, treatment, course section, and the interaction of treatment with course section, a final ANCOVA was conducted. It was concluded that the participants in the experimental group outperformed the participants in the control group when consideration was given to prior knowledge as measured by SAT mathematics scores.

This study focused on the effect of a dynamic geometry learning environment on preservice elementary teachers' performance on similarity tasks as measured by the research instrument. One can be fairly confident that this effect is attributable to the dynamic learning environment for three reasons: classes were randomly assigned to the experimental and control groups, the observed differences between groups on posttest performance were statistically significant, and extraneous variables were effectively controlled.

Discussion of the Results

> Constructivism fully agrees with Kuhn's idea of paradigm shifts (Kuhn, 1970), interpreting paradigm shifts as "change in the habitual way of constructing." The more often a construct is repeated and the greater the number of larger structures in which it is involved, the more indispensable it becomes and the more "given" or "objective" it seems (von Glasersfeld, 1979, p. 121).

The researcher concluded from the data analyses that the results support the dynamic geometry learning environment as a viable enhancement to the traditional mathematics laboratory approach. This study focused on the impact of the dynamic geometric construction software *Geometer's Sketchpad* (Jackiw, 1995) upon preservice elementary teachers' performance on similarity tasks.

Human beings acquire knowledge by building it from the inside instead of internalizing it directly from the environment (Kamii, 1990). Thus, current research from a constructivist perspective characterizes mathematical learning as a process of conceptual reorganization (Cobb, 1995). If learning is generated by successive interactions with one's environment, then repetitive activity is necessary to provide information that facilitates problem solving (Cooper, 1991).

Polya (1954, 1957, 1965) advised, when faced with a problem, to look at examples and special cases, to look at analogous situations for hints and, whenever possible, to use specialization and generalization. Polya (1965) claimed that problem solving is a practical art, like playing the piano. Such arts can only be learned by imitation and practice.

This type of software allows for direct construction, manipulation, and measurement of geometric figures. Users are actively involved in the learning process; the software supports efficiency in that it allows for dynamic manipulation of the geometric figures in order to analyze more cases in a minimal amount of time spent. As in Lester's (1996) and Yerushalmy's and Chazan's (1993) investigations of

high school students, results from the study indicated participants learned with greater efficiency as a consequence of constructing geometric figures on the computer screen. Graphic support seemed to enhance participants ability to visualize mathematical relationships and connect geometric properties to facilitate inquiry.

Calculation of posttest group means presented the researcher with additional statistics worthy of consideration. It had been previously mentioned in Chapter III that the research instrument (refer to Appendix A) was adjusted in lieu of findings from the pilot studies. Items hypothesized to address learning attributed to the dynamic component of the geometric construction software were developed, reviewed by a panel of experts, and placed into the final version of the research instrument. Thus, it was deemed appropriate to investigate individual items. A most interesting phenomena was that posttest means on parallel items, items #1 and #12, hypothesized to address learning attributed to the dynamic component of the geometric construction software, were at least 0.25 points greater for the experimental group.

This study suggests that preservice elementary teachers are able to progress quite well with the use of the *Geometer's Sketchpad* (Jackiw, 1995) through the guess-investigate-conjecture-verify process inherent to the formalization of the "phases of learning" (Karplus, 1977; Simon, 1992) and inquiry that are supported by constructivist epistemology. Polya (1965) summarizes that "for efficient learning, an exploratory phase should proceed the phase of

verbalization and concept formation and, eventually, the material learned should be merged in, and contribute to, the integral mental attitude of the learner" (p. 104).

Other Observations

In addition to the rightful inspection into quantitative aspects of this study, consideration must also be given to qualitative aspects intrinsically rich in interpretive value. The activity worksheets (refer to Appendix C) not only gave structure to the sessions; participants responses throughout the guided discovery process provided insight into their construction and reconstruction of knowledge.

During the first session, participants in either group could be observed struggling with the novel approach to understanding a concept many believed they had learned in middle school. Participants in the control group slowly drew similar geometric figures using the Variable Tension Proportional Divider (Friedlander & Lappan, 1987) to develop an intuitive notion of similarity whereas the participants in the experimental group used the dynamic capability of the *Geometer's Sketchpad* (Jackiw, 1995) to investigate basic principles of similar figures. By the end of the first session, the experimental group had measured and analyzed many different polygonal figures while the control group, although they had discovered the principles of similarity, were unsure of whether non-convex polygonal figures would observe the principles. The control group did, however,

convey an enthusiasm to use the Variable Tension Proportional Divider (Friedlander & Lappan, 1987) with students as often as the experimental group voiced opinions regarding the usefulness of the *Geometer's Sketchpad* (Jackiw, 1995) as a classroom tool.

Much information can be gained through analysis of activity worksheets and clinical reflections. The qualitative component of this study offers a mere sampling of the preservice elementary teachers' cognitive processes in action.

Limitations

This study had limitations that affect the generalizability of the results. Since the study lasted over a period of several days, variability of attendance was a problem. Absences were taken into account for the testing days, but adjustment could not be made to compensate for instructional time lost.

Concern was present that the small sample size of 52 preservice elementary teachers would have an effect upon the outcome of this research study. It is a fact that statistical power increases automatically with sample size. Furthermore, the larger the sample, the more likely its mean and standard deviation is representative of the population mean and standard deviation.

Although statistical analysis supported no statistically significant effect, course section variability was possible

in the amount of orientation the students had regarding software use prior to the study. Specifically, the researcher planned a similar amount of exposure to the software for all involved in the study, but there was no documentation of utilization time.

The researcher conducted all lessons to eliminate differences in results due to instructor variability, but was aware of the possibility that a threat to validity may be introduced by the researcher influencing results. This was minimized by the use of specific presentation plans and formalized activity worksheets.

In summary, the results of this study are generalizable only to situations involving similar participants, classroom environment, and classes of variables.

Implications and Recommendations

Our society has advanced from the industrial age into the information age where numeracy and technological literacy are important aspects of effective citizenship. The mathematics classroom must evolve from a "chalk and talk" environment into an "active" conceptually-oriented habitat that models the contemporary workplace.

Classroom teaching is undergoing great change with innovative dynamic software available. A major challenge facing mathematics and mathematics education research is the development of an indepth understanding of the function of the power technology provides, not only to revise the

mathematics that we teach, but also to transform our understanding of the teaching and learning processes. To ensure proper utilization, future teachers must be trained appropriately in order to bring to the classroom the experience, confidence, and enthusiasm necessary to effectively facilitate student learning.

This study added to the body of knowledge that already exists on technology and its impact upon curriculum and instruction. However, replication is needed before any definitive conclusions can be drawn about the effect of a dynamic geometric construction environment upon learning. Further investigations can build upon this research and continue to expand the knowledge base for educational practices in a technologically rich environment.

Future research should strive for inclusion of other factors relevant to effective learning. The impact of spatial factors, such as visualization or orientation, warrant investigation. This is particularly relevant in light of the mathematical content addressed. Furthermore, affective factors should be given consideration in order to suggest direction for proper reconceptualization of the classroom environment. In particular, differences in attitudes regarding computer or software use are rich avenues that have only recently opened up for investigation. There is also a need to continue the investigation of grade differences and learning style differences with respect to the dynamic environment.

Calculation of posttest score means presented the researcher with additional statistics worthy of

consideration. It has been previously mentioned in Chapter III that the research instrument (refer to Appendix A) was adjusted in lieu of findings of the pilot studies. Items hypothesized to address learning attributed to the dynamic component of the geometric construction software were developed and placed into the final version of the research instrument. Thus, it was deemed appropriate to investigate individual items. Research should be undertaken to study assessment of learning in a dynamic geometric construction environment in addition to the development of appropriate test items. Moreover, research should give consideration to the evaluation of programs that utilize a dynamic learning environment.

In addition to the rightful inspection into quantitative aspects of this study, consideration must also be given to qualitative aspects intrinsically rich in interpretive value. The activity worksheets (refer to Appendix C) not only gave structure to the learning sessions; participants responses throughout the guided discovery process provided insight into their construction and reconstruction of knowledge.

During the first session, participants in either group could be observed struggling with the novel approach to understanding a concept many believed they had learned in middle school. Participants in the control group slowly drew similar geometric figures using the VTPD (refer to Definition of Terms) to develop an intuitive notion of similarity whereas the participants in the experimental group used the dynamic capability of the *Geometer's Sketchpad* (Jackiw, 1995) to investigate basic principles of similar figures. By the

end of the first session, the experimental group had measured and analyzed many different polygonal figures while the control group, although they had assimilated the principles of similarity, were unsure of whether non-convex polygonal figures would observe the principles. In other words, participants of the control group were confident regarding the limited number of examples they were exposed to, but were unsure when analyzing unknown cases. Further research should be undertaken to consider possible misconceptions and, also, levels of geometric thought.

Summary

This final chapter discussed the results presented in the previous chapter. An overview of the study was provided first, including a description of the sample population. Next, the significance of the study was articulated, followed by the research questions. Then, discussion of the findings based upon the research analysis was presented. Limitations followed; the chapter concludes with implications and recommendations for future research.

APPENDIX A
RESEARCH INSTRUMENT

1. Point projections are made of the given triangle first from point P and then from point Q. Each projection uses a scale factor of 3. Which of the following is true?

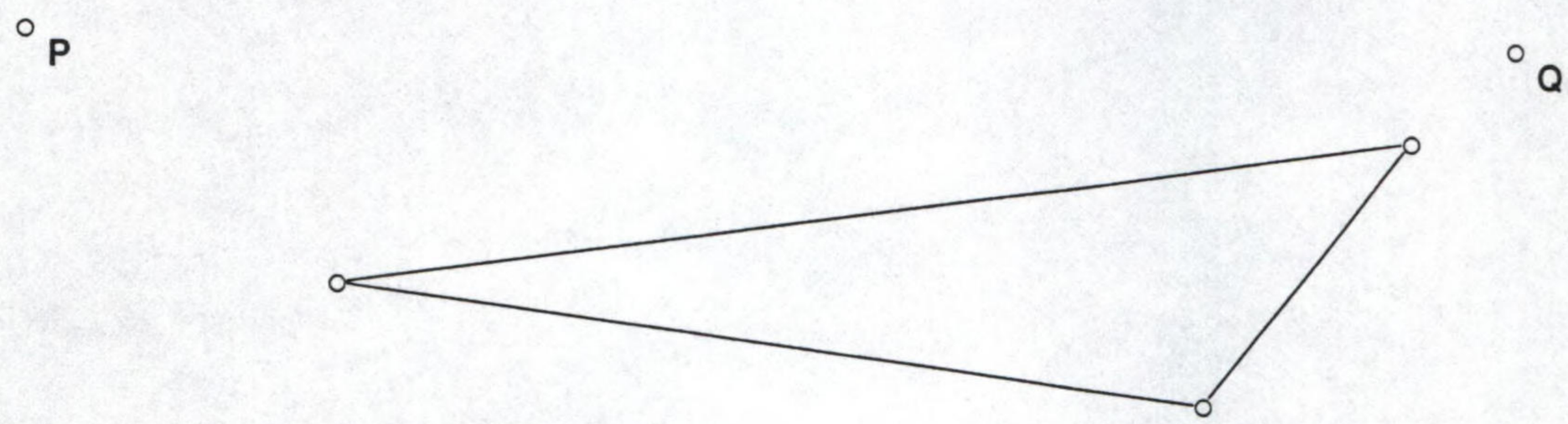
A. The triangle image from point P has angles of greater measure.

B. The triangle image from point Q has angles of greater measure.

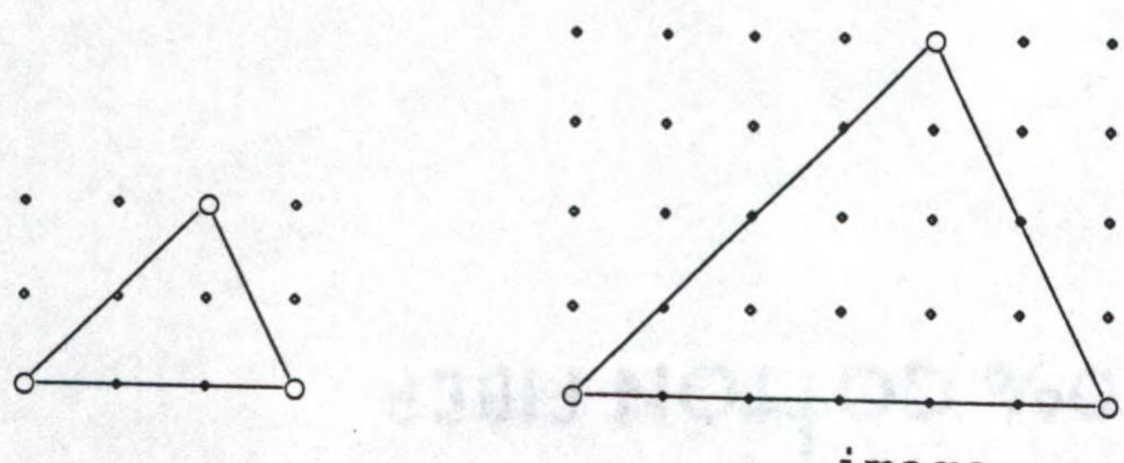
C. The triangle image from point P has a larger area.

D. The triangle images from point P and point Q have the same area.

E. The triangle image from point Q has a larger area.

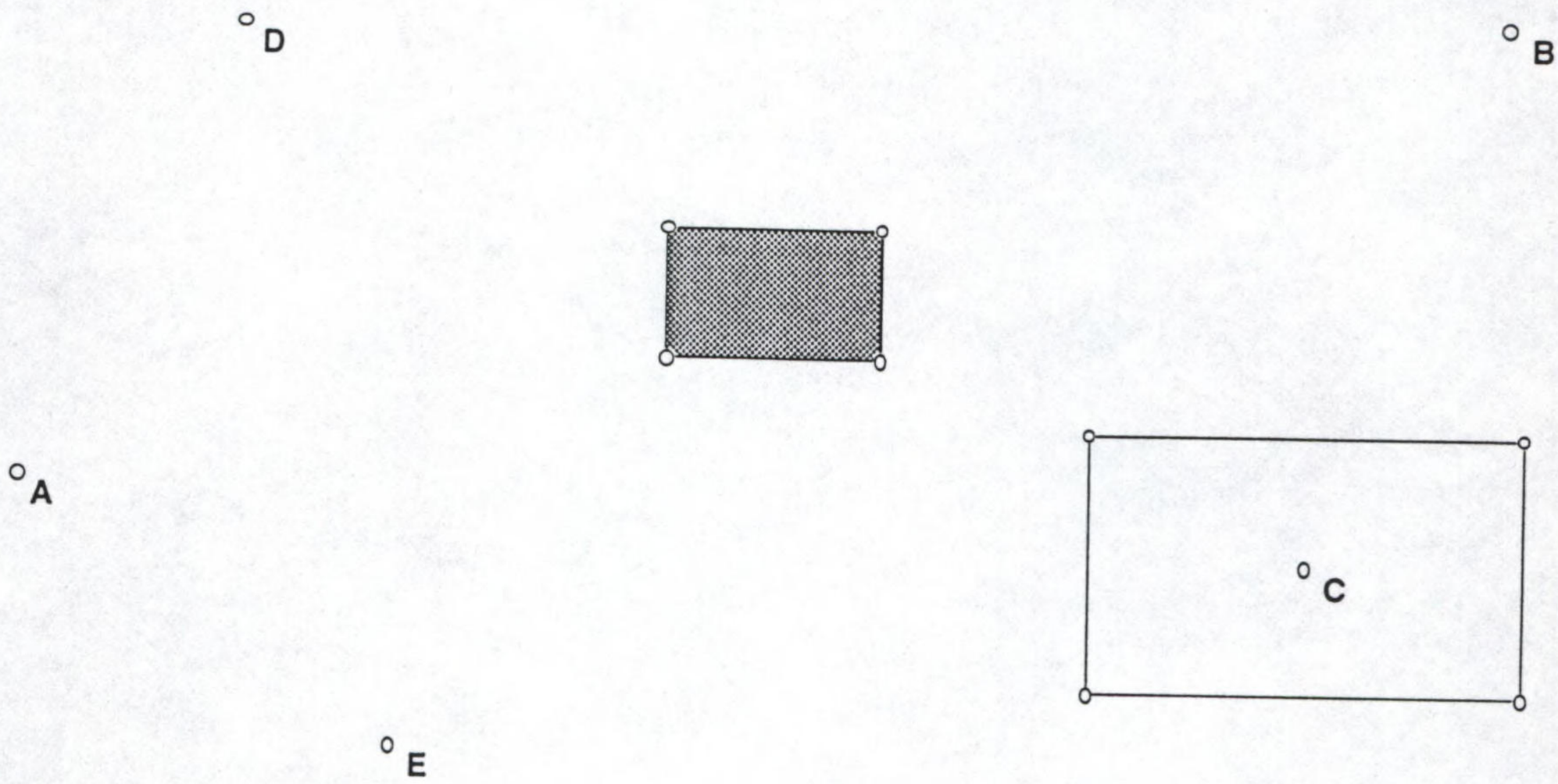


2. Which of these transformations was used for the given rectangle and its image:



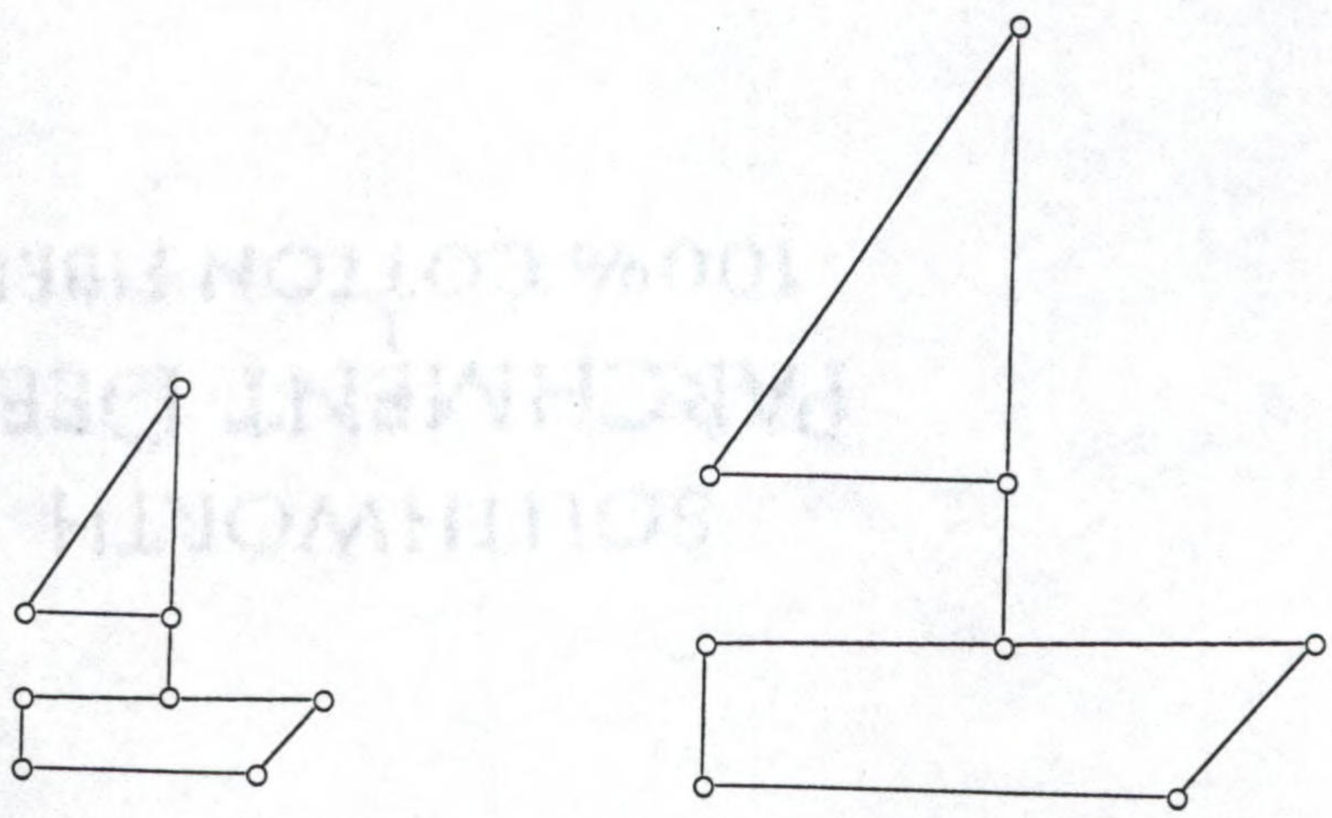
A) (x,y) -> (2x,2y)
B) (x,y) -> (x,2y)
C) (x,y) -> (2x,y)
D) (x,y) -> (2x,4y)
E) (x,y) -> (4x,2y)

3. Which projection point was used to dilate the shaded rectangle?



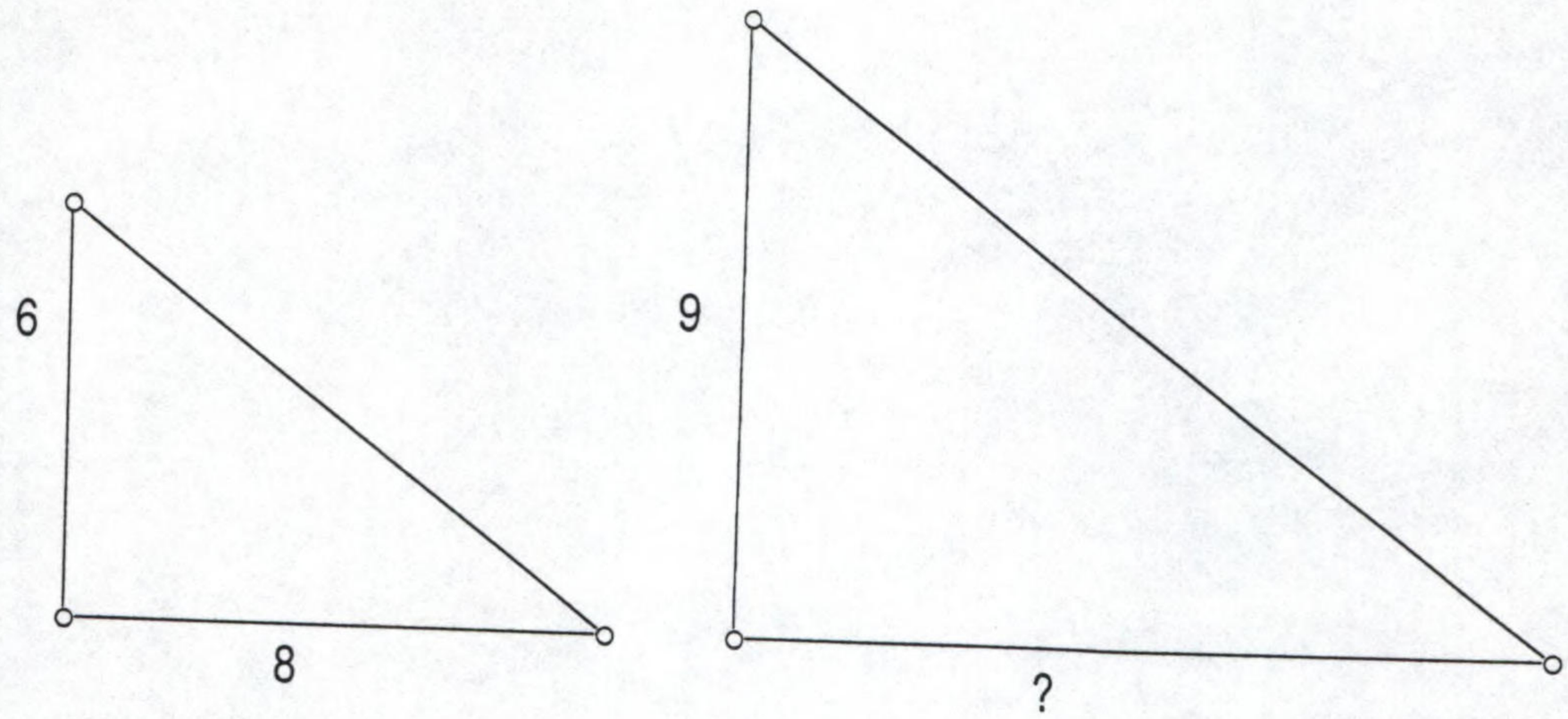
4. What scale factor has been used to enlarge the small sailboat?

A) 2 B) 3 C) 4 D) 6 E) 1/4



5. The given rectangles are similar. Find the missing length.

A) 10 B) 11 C) 12 D) 13 E) 14

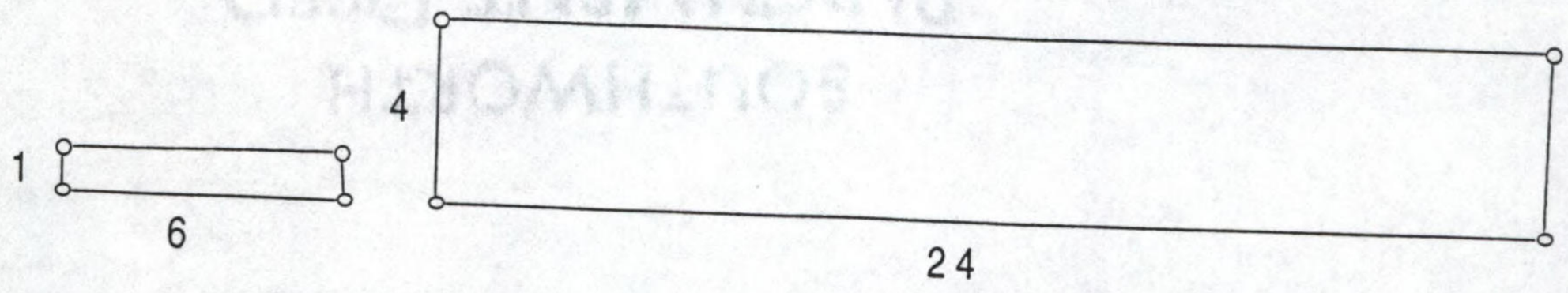


6. A man who is 6 feet tall has a shadow that is 8 feet long At the same time, a nearby tree has a shadow that is 32 feet long. How tall is the tree?

A) 30 feet
B) 21 feet
C) 24 feet
D) 42 feet
E) 48 feet

7. Given rectangles of dimensions 1x6 and 4x24, the area of the larger triangle is how many times bigger than the area of the smaller rectangle?

A) 4 times
B) 6 times
C) 8 times
D) 16 times
E) 18 times



8. If two figures are similar, which of the following might be different?

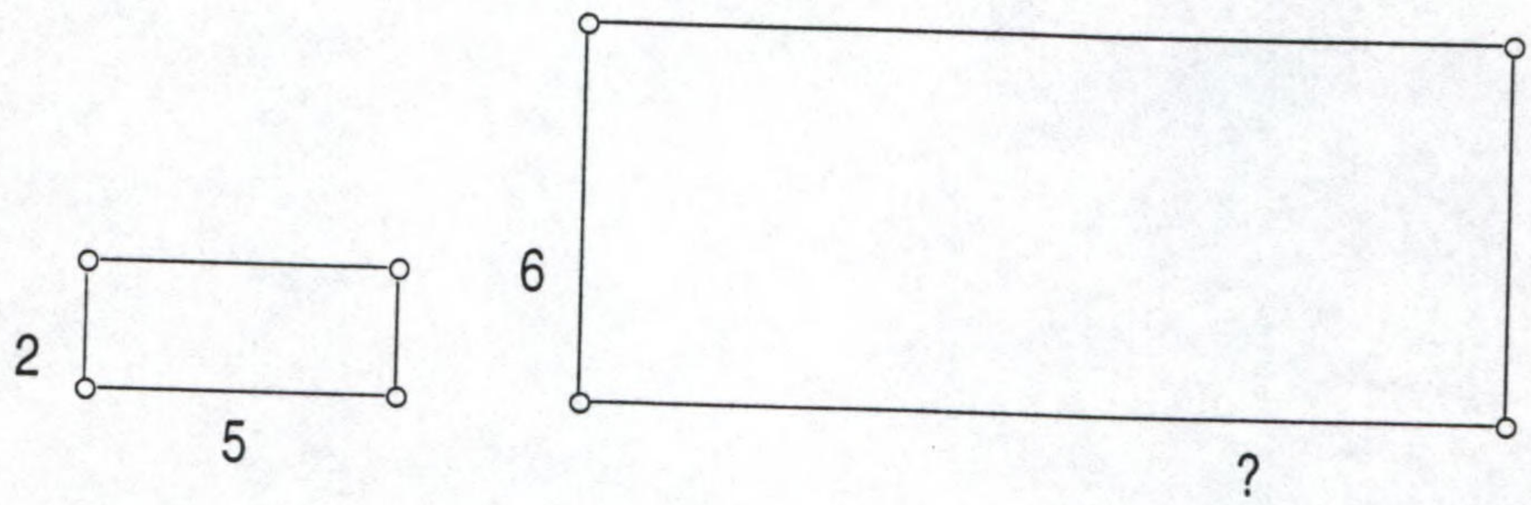
A) number of sides
B) lengths of corresponding sides
C) shape
D) size of angles
E) ratio of corresponding sides

9. A 2 meter stick has a shadow of 1/2 meter at the same time that a nearby tree has a shadow of 3 meters. How tall is the tree?

A) 6 meters
B) 12 meters
C) 1.5 meters
D) 3 meters
E) 15 meters

10. The given rectangles are similar. Find the length of the missing side.

A) 7 B) 9 C) 10 D) 11 E) 15



11. A 1x5 rectangle grows into a 4x12 rectangle. The area of the new rectangle is how many times larger than the area of the small rectangle?

A) 3 times
B) 4 times
C) 5 times
D) 15 times
E) 16 times

12. Point projections are made of the given triangle first from point P and then from point Q. Each projection uses a scale factor of 2. Which of the following is true?

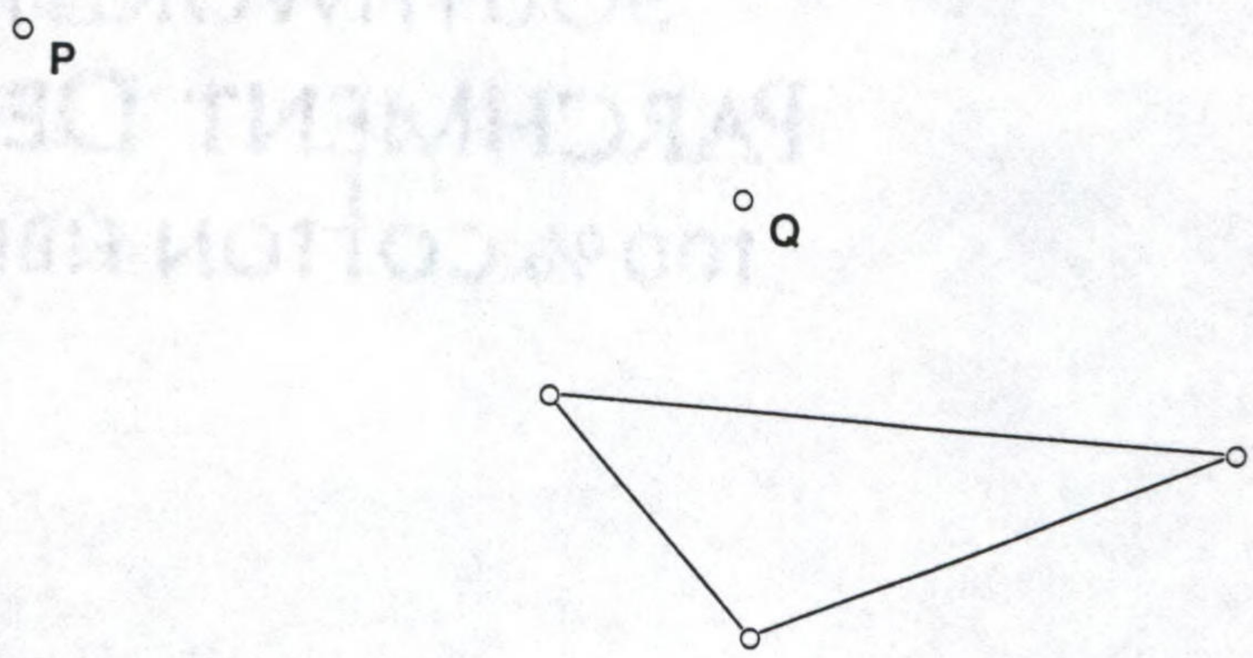
A. The triangle image from point P has angles of greater measure.

B. The triangle image from point Q has angles of greater measure.

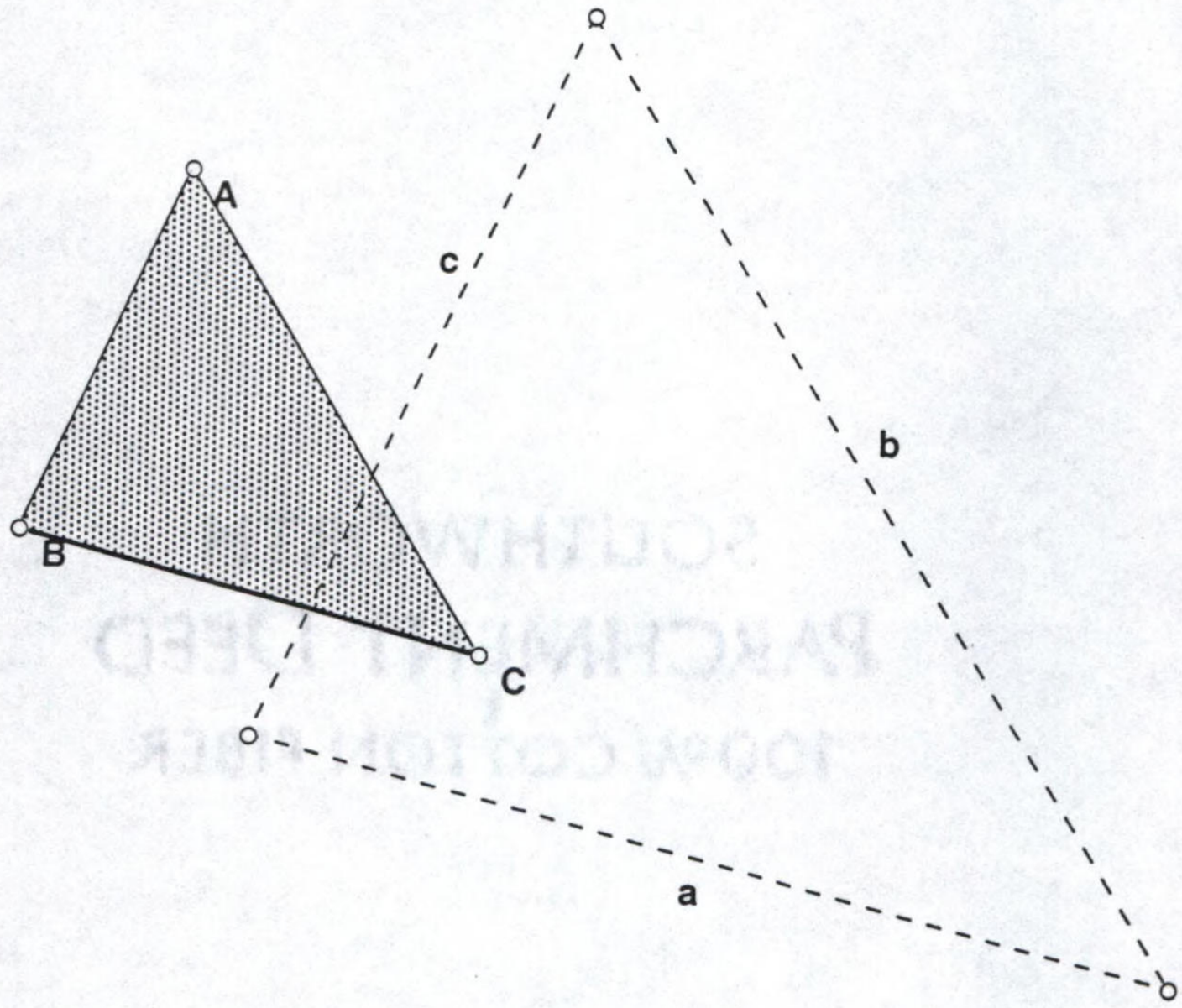
C. The triangle image from point P has a larger area.

D. The triangle images from point P and point Q have the same area.

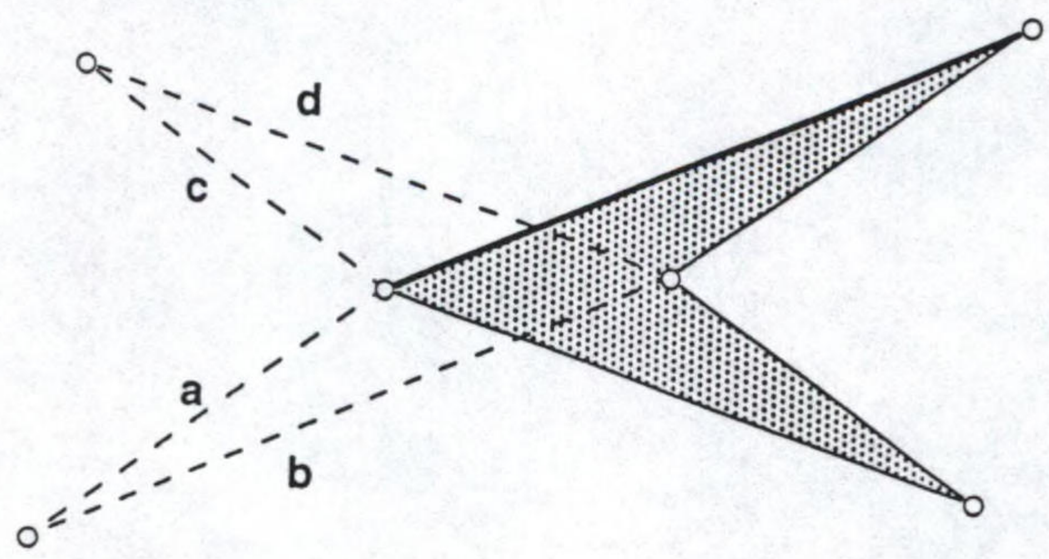
E. The triangle image from point Q has a larger area.



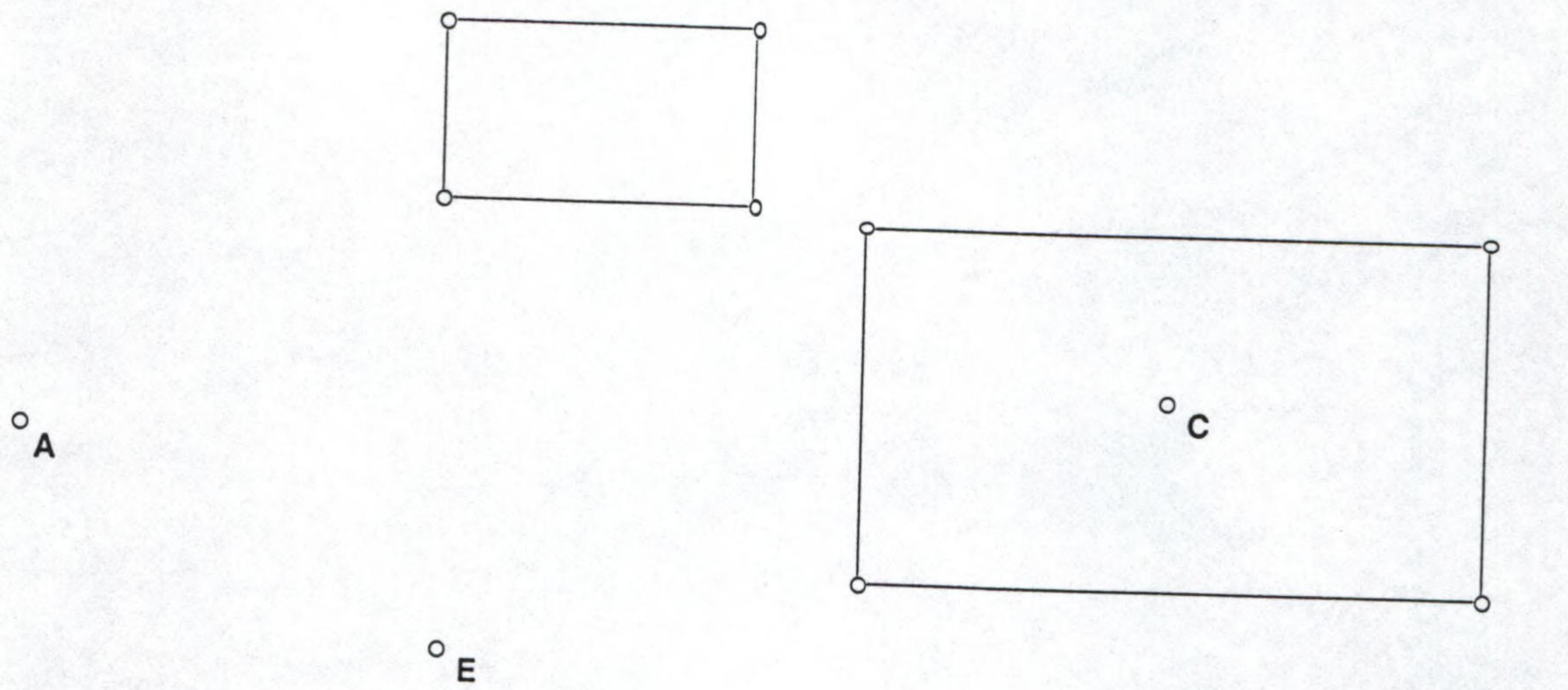
13. Given the bolded segment of the shaded triangle, the corresponding segment on the projected image is:
A) a
B) b
C) c



14. Given the bolded segment of the shaded polygon, the corresponding segment on the projected image is:
A) a
B) b
C) c
D) d

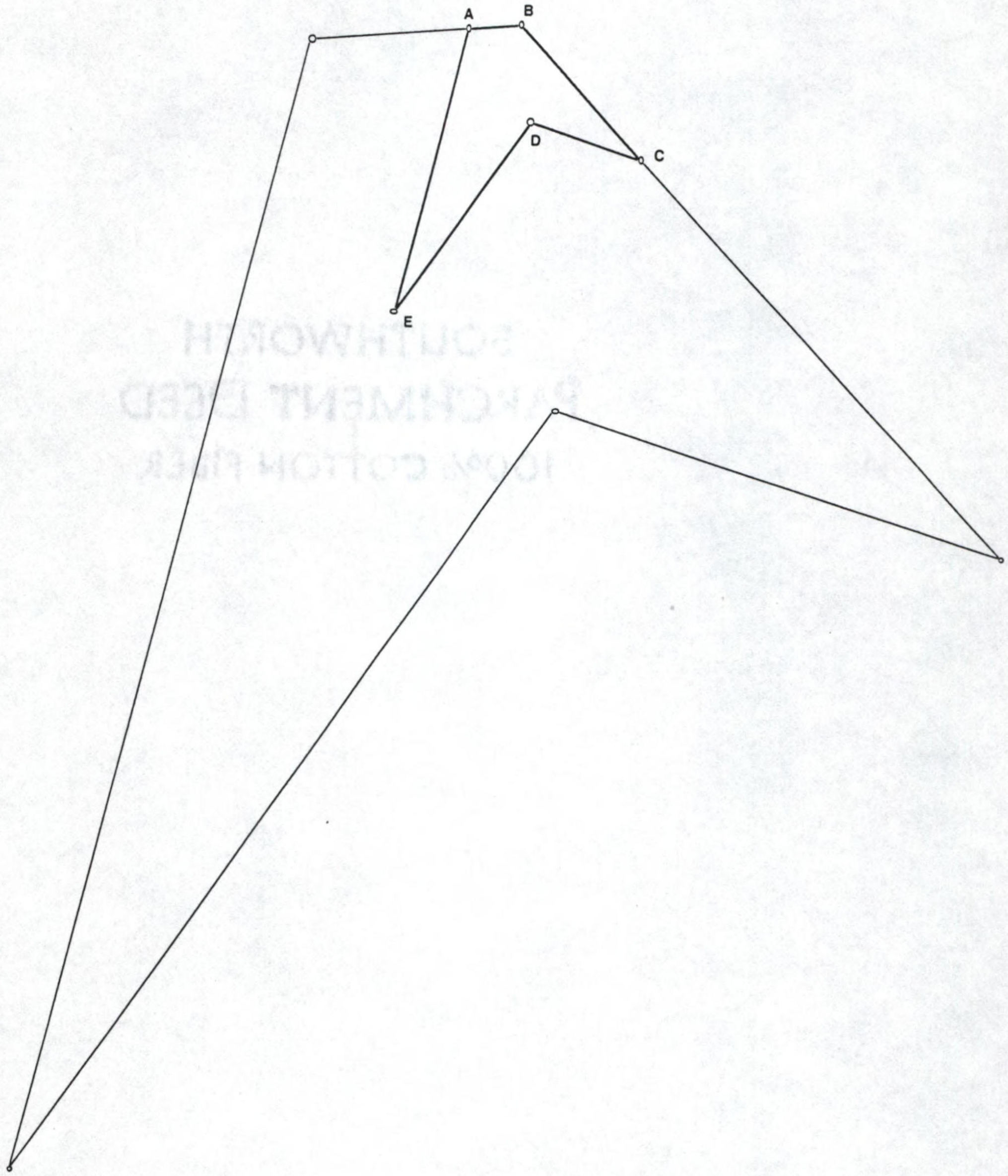


15. Which projection point was used to dilate the shaded rectangle?



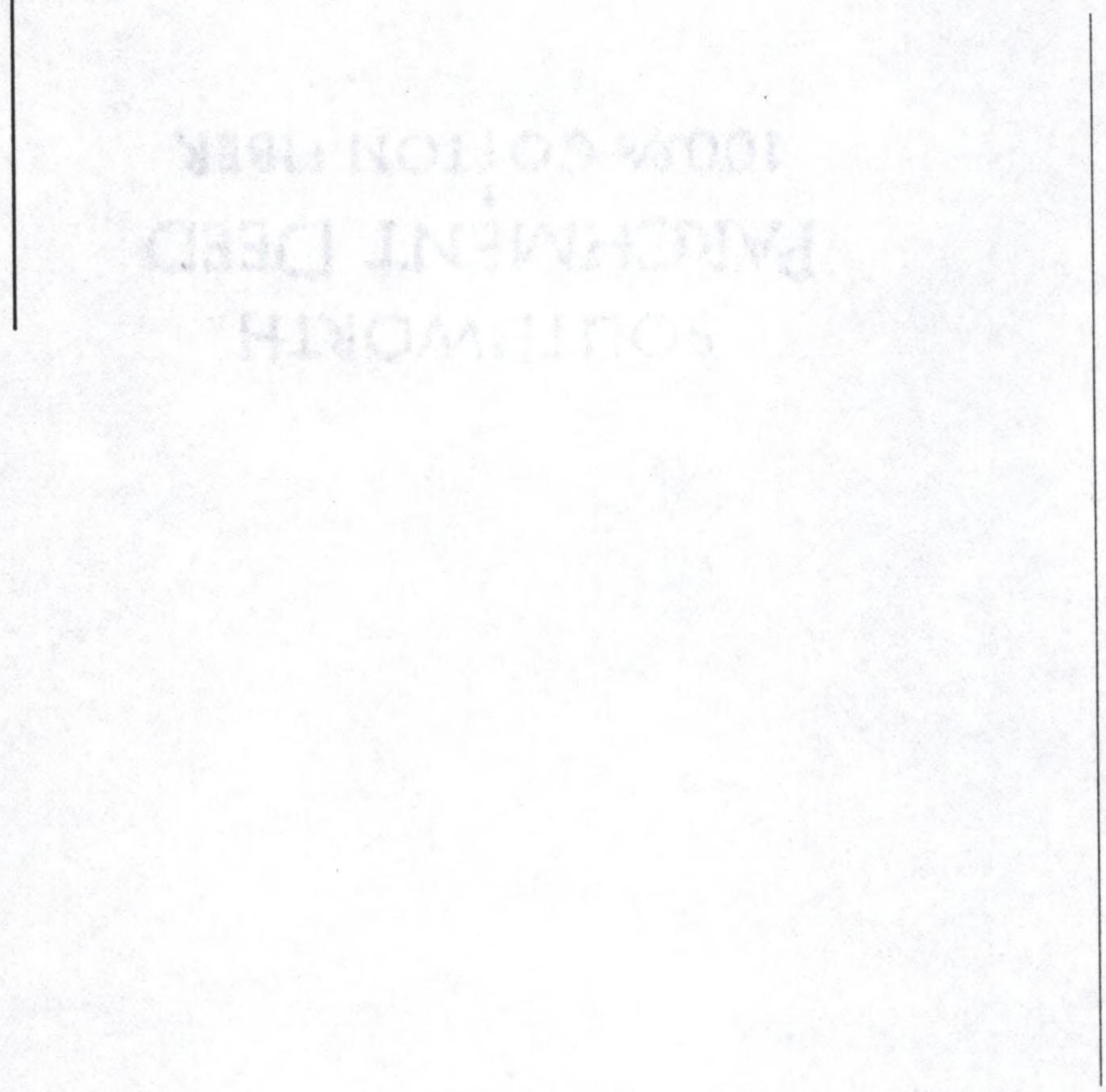
16. The point of perspective is:

A) point A
B) point B
C) point C
D) point D
E) point E



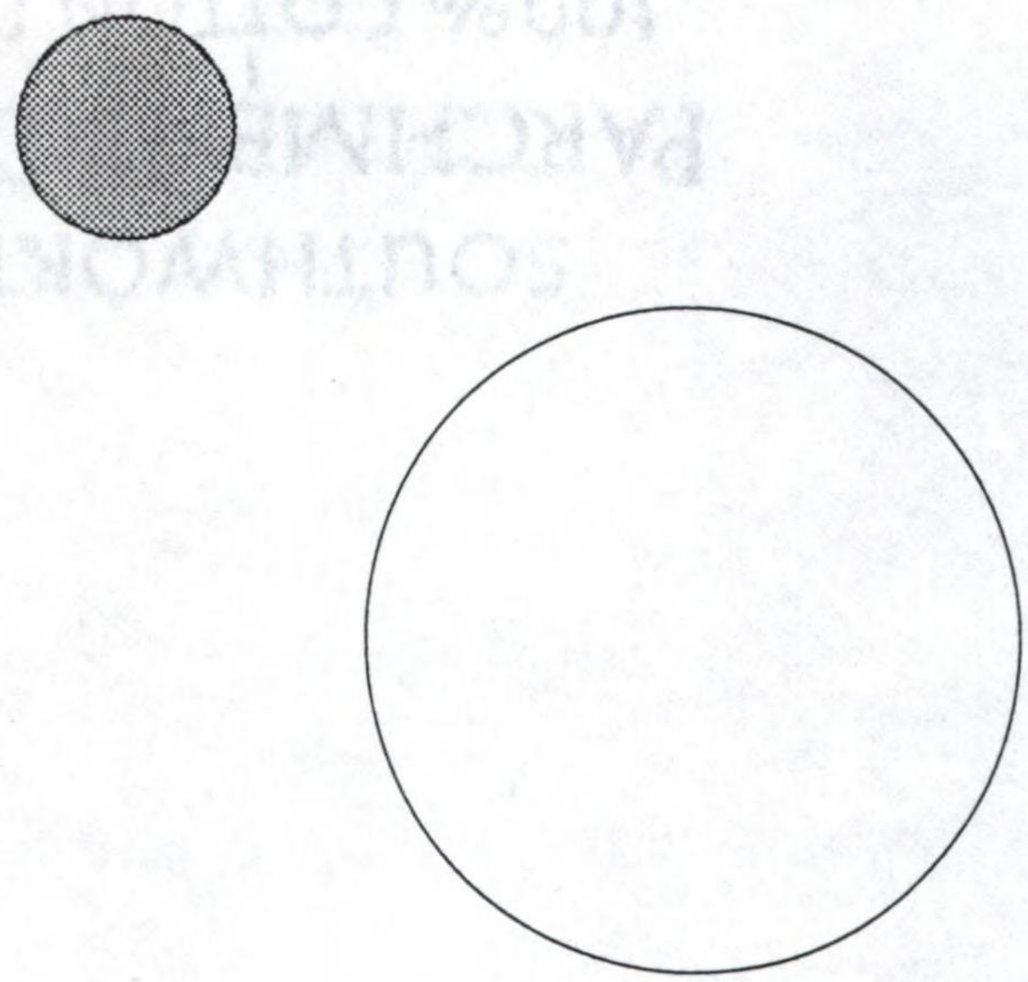
17. Choose the scale factor of the similar figures. The bolded line segment is the pre-image.The scale factor is:

A) 1
B) 2
C) 3
D) 4
E) 5

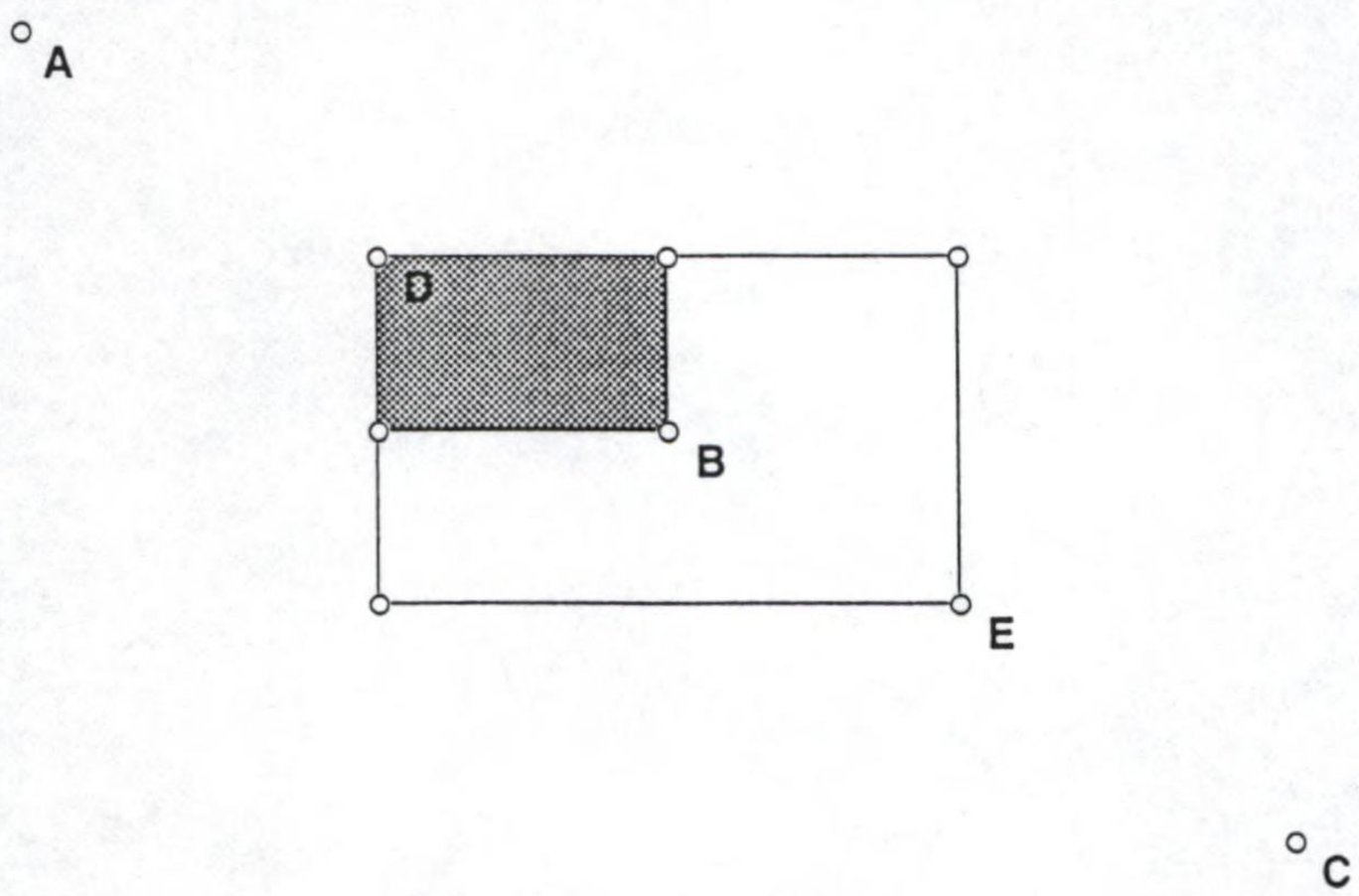


18. Choose the scale factor of the similar figures. The shaded circle is the pre-image. The scale factor is:

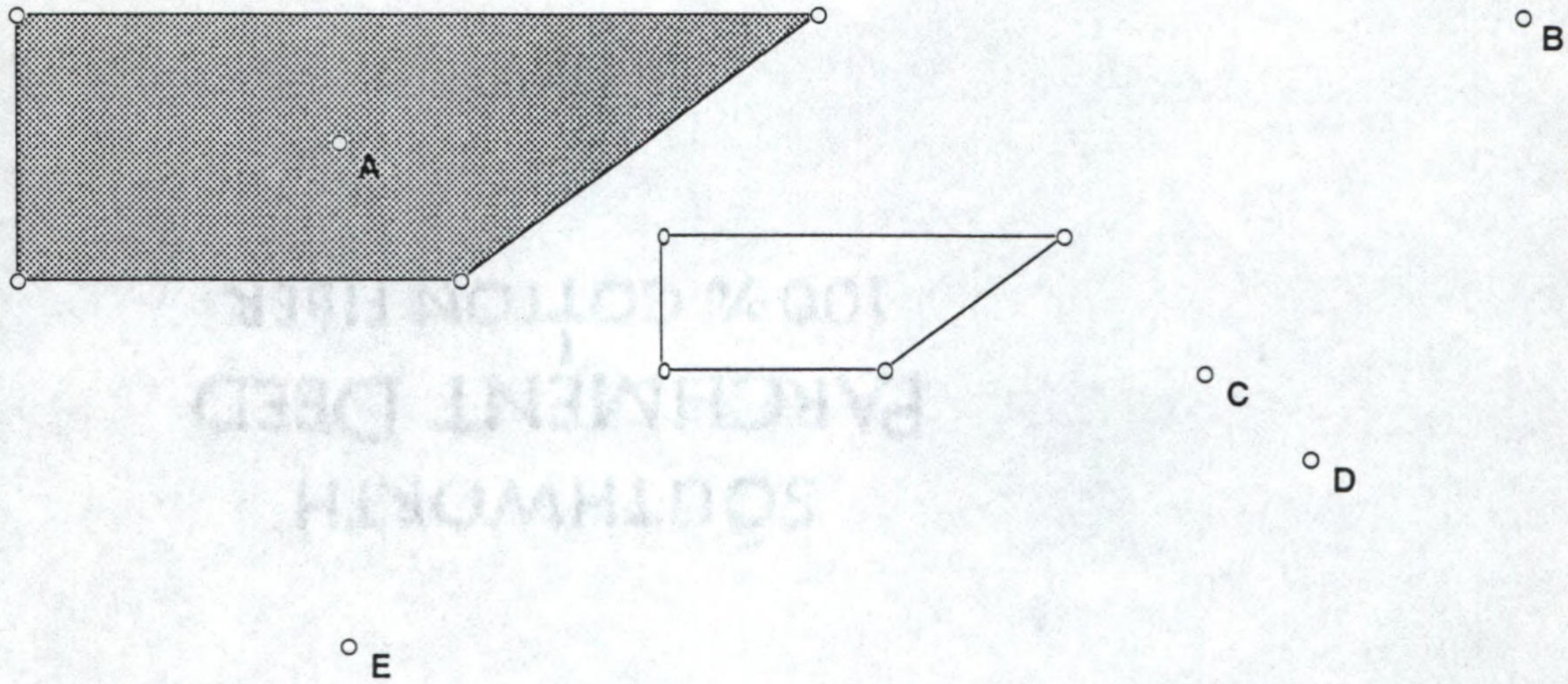
A) 1
B) 2
C) 3
D) 4
E) 5



19. Which projection point was used to dilate the shaded rectangle?

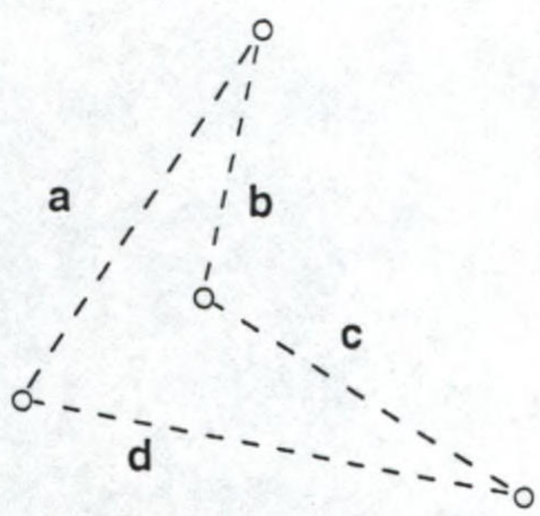


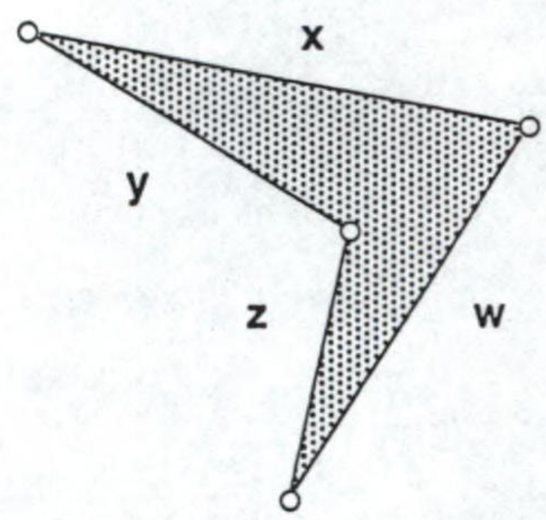
20. Which projection point was used to dilate the shaded trapezoid?



21. Solve the proportionality a/? = w/x.

? = A) a
B) b
C) c
D) d
E) e





APPENDIX B
LEARNING SESSIONS

Table 3.1
First Pilot Study

Presession
Information regarding UFIRB form.
Pretest.
Computer lab orientation.

Session 1	
Launch	Terminology review.
	Use of construction tool and measurement tools.
Exploration	Properties of similar polygons.
Summary	Class discussion of conjectures generated during individual exploration time (Corresponding angles congruent. Ratio of 2 sides of a polygon is equal to the ratio of the corresponding 2 sides of any similar polygon and they are equivalent fractions)

Session 2

Launch Review of properties of similar polygons.

Introduction to intuitive Projective Geometry.

SCALE FACTOR with DILATE tool.

Exploration Properties with respect to (1) point of perspective and (2)corresponding sides.

Summary Class discussion of conjectures generated during individual exploration time.

(The ratio of the distance from the perspective point to a vertex of a similar polygon over the distance from the perspective point to a vertex of the given polygon is equal to the scale factor.

The ratio of the measure of a side of a similar polygon over the measure of the corresponding side of the given polygon is equal to the scale factor.

Properties of similar polygons are invariant as the position of the point of perspective varies.)

Session 3

Launch	Connection of concept of similarity to ratio and proportional reasoning.
Exploration	Given pre-constructed sketches, indirect measurement tasks.
Summary	Class discussion of approaches generated during individual exploration time.

Session 4

Launch	Review of properties with respect to (1) point of perspective and (2)corresponding sides. Use of POLYGON INTERIOR tool.
Exploration	Relationship of areas of similar polygons.
Summary	Class discussion of conjectures generated during individual exploration time. (The ratio of the area of a similar polygon over the area of the given polygon is equal to the square of the scale factor.)

Table 3.2
Second Pilot Study

	Session 1
Launch	Terminology review.
	Use of construction and measurement tools.
	Introduction to intuitive Projective Geometry.
	Dynamic geometry exposition with animation or the Variable Tension Proportional Divider.
Exploration	Properties of similar figures.
Summary	Class discussion of conjectures generated during individual exploration time.
	(Corresponding angles congruent.
	Ratio of 2 sides of a polygon is equal to the ratio of the corresponding 2 sides of any similar polygon and they are equivalent fractions)

Session 2

Launch Define scale factor.

Review properties of similar polygons.

Exploration Properties with respect to

(1) point of perspective and

(2)corresponding sides.

Summary Class discussion of conjectures generated during individual exploration time.

(The ratio of the distance from the perspective point to a vertex of a similar polygon over the distance from the perspective point to a vertex of the given polygon is equal to the scale factor.

The ratio of the measure of a side of a similar polygon over the measure of the corresponding side of the given polygon is equal to the scale factor.

Properties of similar polygons are invariant as the position of the point of perspective varies.)

Session 3

Launch	Discussion of how to find the area of a figure.
Exploration	Relationship of areas of similar polygons.
Summary	Class discussion of conjectures generated during individual exploration time. (The ratio of the area of the image over the area of the given polygon is equal to the square of the scale factor.)

Launch	Review of properties with respect to (1) point of perspective and (2)corresponding sides. Connection of concept of similarity to ratio and proportional reasoning.
Exploration	Given pre-constructed sketches, indirect measurement tasks.
Summary	Class discussion of approaches generated during individual exploration time. (The measures of corresponding parts of similar triangles can be set onto a proportion to find the unknown measurement.)

Table 3.3
The Study

Session 1

Launch	Terminology review. Use of construction and measurement tools. Introduction to intuitive Projective Geometry. Dynamic geometry exposition with animation or the Variable Tension Proportional Divider.
Exploration	Metric properties of similar figures. Size and position of the image with respect to the numerosity of the scale factor.
Summary	Class discussion of conjectures generated during individual exploration time.

Session 2

Launch	Define scale factor. Review properties of similar polygons.
Exploration	Numeric properties of similar shapes with respect to (1) point of perspective and (2)corresponding sides.
Summary	Class discussion of conjectures generated during individual exploration time.

Session 3

Launch	Review of properties with respect to (1) point of perspective and (2)corresponding sides. Connection of concept of similarity to ratio and proportional reasoning.
Exploration	Numeric properties of similar shapes with respect to (1) scale factor and (2)corresponding distances.
Summary	Class discussion of approaches generated during individual exploration time.

APPENDIX C
ACTIVITIES WITH RESPONSES

Sketch

step 1: In the Display menu, choose Line Weight "thick" and Shade "100%."

step 2: Construct any polygon and change the color of each side to be distinct.

step 3: Construct a point outside the polygon and mark it as center in the Transform menu.

step 4: Select your entire polygon and dilate by a scale factor of 2/1 (Place 2 into top box and 1 into the bottom box).

Investigation: Corresponding Sides

This corresponding sides become the same color.

step 5: Measure the ratio of a side on the dilated polygon with the corresponding side on the original polygon.

step 6: Repeat step 5 using a different side. What is this ratio?

.50

step 7: Drag a vertex to change the length of a side. What changes? What stays the same?

The shape of the polygon, the angles — these all change

- The ratios stay the same!

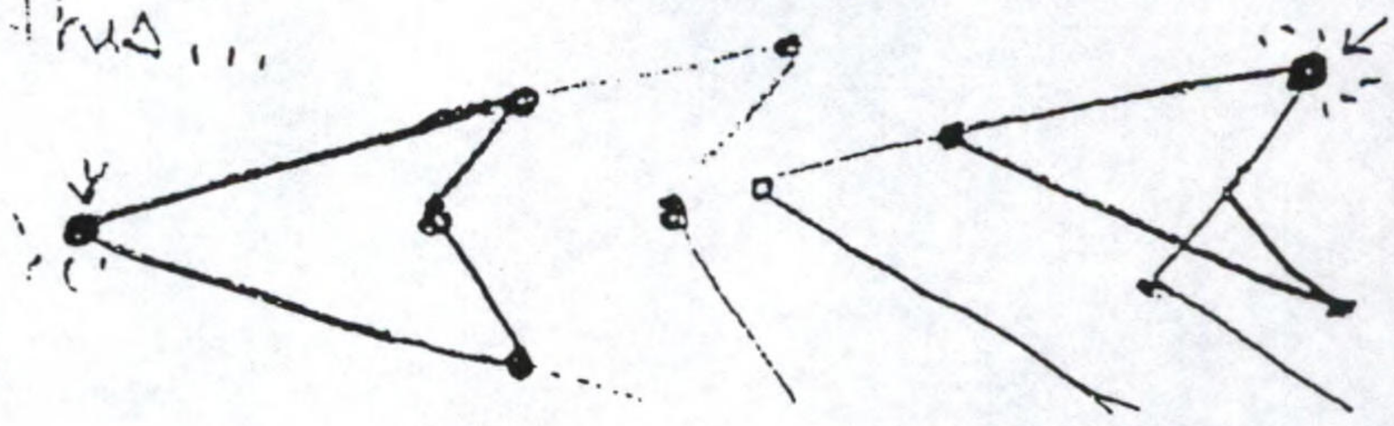
Investigation: Projection Point

step 7: Measure an angle of the dilated polygon and the corresponding angle on the original polygon. These angles are the same.

step 8: Drag the point of projection to different locations. Regarding the figures, what changes (measures, locations, ...)? What stays the same? Make sure to include placing the projection point on a vertex of the original polygon!

measures stay the same — Position changes. Size remains

When I place the point of projection on a vertex... the polygon looks something like this...

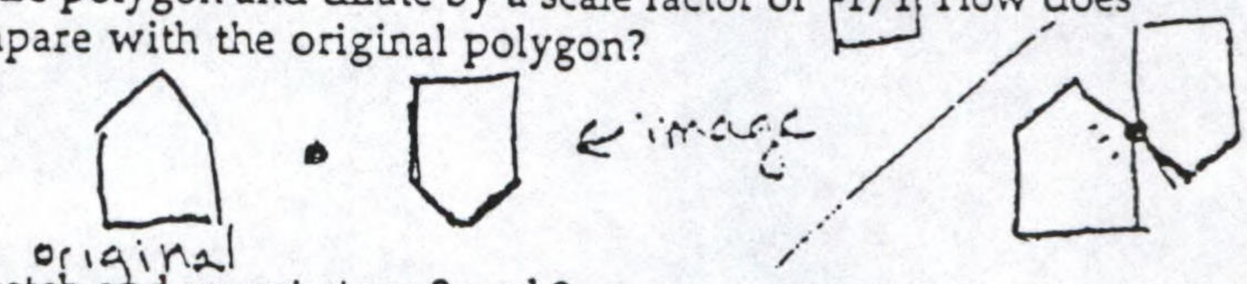


Investigation: Scale Factor

In this investigation you will discover principles of transformations with respect to the scale factor.

step 9: Open a new sketch and repeat steps 2 and 3.

step 10: Select your entire polygon and dilate by a scale factor of $\boxed{-1/1}$ How does the image compare with the original polygon? rotated 180°



step 11: Open a new sketch and repeat steps 2 and 3.

step 12: Select your entire polygon and dilate by a scale factor less than negative one. How does the image compare with the original polygon? -6

step 13: Drag the point of projection to different locations. Describe the position of the image with respect to the projection point and the original figure.

measure of the side becomes 6x the length of the original & is also rotated 180°

step 14: Open a new sketch and repeat steps 2 and 3.

step 15: Select your entire polygon and dilate by a scale factor between zero and negative one. How does the image compare with the original polygon? .50

step 16: Drag the point of projection to different locations. Describe the position of the image with respect to the projection point and the original figure.

The length of the side is ½ the original length of any one side & it is still rotated 180°

step 17: Open a new sketch and repeat steps 2 and 3.

step 18: Select your entire polygon and dilate by a scale factor between zero and one. How does the image compare with the original polygon?

Image's sides become ½ length of any side of the original. The image is not rotated & the image appears btwn the original & the point of projection.

← point of projection.

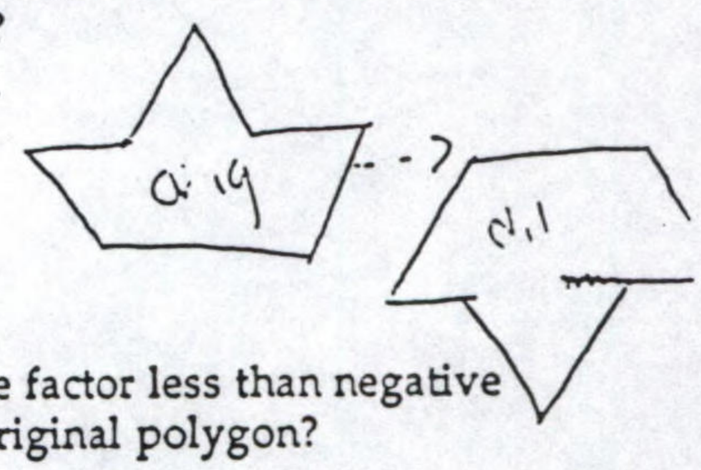
Investigation: Scale Factor

In this investigation you will discover principles of transformations with respect to the scale factor.

step 9: Open a new sketch and repeat steps 2 and 3.

step 10: Select your entire polygon and dilate by a scale factor of -1/1. How does the image compare with the original polygon?

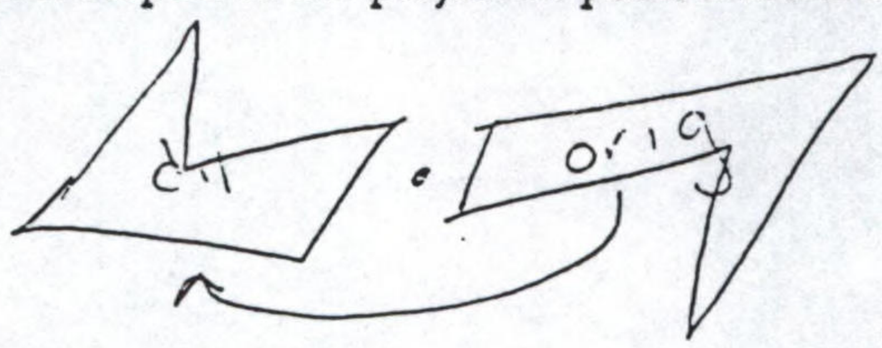
It looks like a flipped polygon



step 11: Open a new sketch and repeat steps 2 and 3.

step 12: Select your entire polygon and dilate by a scale factor less than negative one. How does the image compare with the original polygon?

step 13: Drag the point of projection to different locations. Describe the position of the image with respect to the projection point and the original figure.

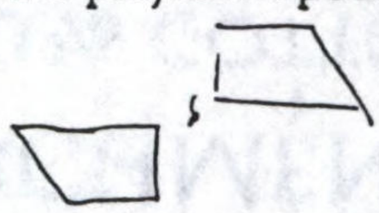


point is in line w/ two fig.

step 14: Open a new sketch and repeat steps 2 and 3.

step 15: Select your entire polygon and dilate by a scale factor between zero and negative one. How does the image compare with the original polygon?

step 16: Drag the point of projection to different locations. Describe the position of the image with respect to the projection point and the original figure.



It appears as rotated & flipped.

step 17: Open a new sketch and repeat steps 2 and 3.

step 18: Select your entire polygon and dilate by a scale factor between zero and one. How does the image compare with the original polygon?

Its smaller than the ones before

Sketch

step 1: In the Display menu, choose Line Weight "thick" and Shade "100%."

step 2: Construct any polygon and change the color of each side to be distinct.

step 3: Construct a point outside the polygon and mark it as center in the Transform menu.

step 4: Select your entire polygon and dilate by a scale factor of 2/1 (Place 2 into top box and 1 into the bottom box).

Investigation: Corresponding Sides

step 5: Measure the ratio of a side on the dilated polygon with the corresponding side on the original polygon. $\frac{c'}{mFG} = 2.00$

step 6: Repeat step 5 using a different side. What is this ratio? $\frac{A'}{mCF} = 2.00$

step 7: Drag a vertex to change the length of a side. What changes? What stays the same?

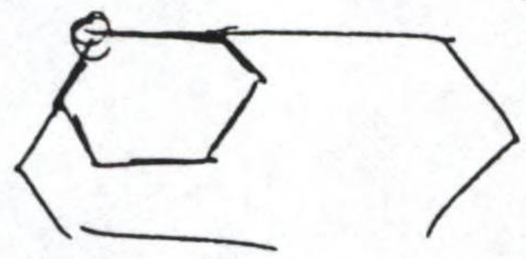
the vertex drags the two sides but the other parts of the polygon are unaffected

Investigation: Projection Point

step 7: Measure an angle of the dilated polygon and the corresponding angle on the original polygon. They are the same.

step 8: Drag the point of projection to different locations. Regarding the figures, what changes (measures, locations, ...)? What stays the same? Make sure to include placing the projection point on a vertex of the original polygon!

original stays the same dilated moves around projection point on top puts both polygons together



Investigation: Scale Factor

In this investigation you will discover principles of transformations with respect to the scale factor.

step 9: Open a new sketch and repeat steps 2 and 3.

step 10: Select your entire polygon and dilate by a scale factor of -1/1. How does the image compare with the original polygon?

ooh - a mirror image reversed about the center point (flip and rotate)

step 11: Open a new sketch and repeat steps 2 and 3.

step 12: Select your entire polygon and dilate by a scale factor ~~less than~~ b/w Ø & negative one. How does the image compare with the original polygon?

reversed mirror image but smaller not same size

step 13: Drag the point of projection to different locations. Describe the position of the image with respect to the projection point and the original figure.

the respective side gets twice as long as short side when you move it - angles stay same and still revolves around center

step 14: Open a new sketch and repeat steps 2 and 3.

step 15: Select your entire polygon and dilate by a scale factor ~~between zero and~~ less than negative one. How does the image compare with the original polygon?

bigger than orig.

step 16: Drag the point of projection to different locations. Describe the position of the image with respect to the projection point and the original figure. if I drag the proj. pt a bit to the left the new image moves 2x's as far to the left

step 17: Open a new sketch and repeat steps 2 and 3.

step 18: Select your entire polygon and dilate by a scale factor between zero and one. How does the image compare with the original polygon?

Same image just smaller.

Sketch

step 1: In the Display menu, choose Line Weight "thick" and Shade "100%."

step 2: Construct any polygon and change the color of each side to be distinct.

step 3: Construct a point outside the polygon and mark it as center in the Transform menu.

step 4: Select your entire polygon and dilate by a scale factor of 2/1 (Place 2 into top box and 1 into the bottom box).

Investigation: Corresponding Sides

step 5: Measure the ratio of a side on the dilated polygon with the corresponding side on the original polygon.

the big side is 2x's the size as the small side

step 6: Repeat step 5 using a different side. What is this ratio? same

ratio stays same even when different lengths

step 7: Drag a vertex to change the length of a side. What changes? What stays the same?

lengths change but stay at same ratio - just like similar shapes should.

Investigation: Projection Point

step 7: Measure an angle of the dilated polygon and the corresponding angle on the original polygon.

step 8: Drag the point of projection to different locations. Regarding the figures, what changes (measures, locations, ...)? What stays the same? Make sure to include placing the projection point on a vertex of the original polygon!

the measure of ~~the~~ both angles stay the same no matter length of sides or how big or small the angle

Sketch

step 1: In the Display menu, choose Line Weight "thick" and Shade "100%."

step 2: Construct any polygon.

step 3: Construct a point outside the polygon and mark it as center in the Transform menu.

step 4: Select your entire polygon and dilate by a scale factor of 2/1 (Place 2 into top box and 1 into the bottom box).

Investigation: Corresponding Distances

step 5: Select the Ray icon to your left and construct a ray starting from the projection point through a vertex of the original polygon. Does the ray intersect the image? Where?

yes - at the corresponding point on the image

step 6: Repeat *step 5* with the remaining vertices of the original polygon. Does the same phenomena happen?

yes - it happens on every vertice
So that the corresponding one on the
Image and the ray move w/ the vertice

If I use a $-1/1$ for dialation and put ⟷ lines from
the point to the vertices of orig. shape, it keeps going thru
the point to the correspond vertice of the image

Sketch

step 1: In the Display menu, choose Line Weight "thick" and Shade "100%."

step 2: Construct any polygon.

step 3: Construct a point outside the polygon and mark it as center in the Transform menu.

step 4: Select your entire polygon and dilate by a scale factor of 2/1 (Place 2 into top box and 1 into the bottom box).

Let's note that the center pt. moves the Larger shape farther outside from the small one

Investigation: Corresponding Distances

step 5: Select the Ray icon to your left and construct a ray starting from the projection point through a vertex of the original polygon. Does the ray intersect the image? Where? Yes, it intersects the similar images @ a corresponding vertex or angle.

step 6: Repeat *step 5* with the remaining vertices of the original polygon. Does the same phenomena happen? Yes, it always intersects the corresponding point.

Sketch

step 1: In the Display menu, choose Line Weight "thick" and Shade "100%."

step 2: Construct any polygon.

step 3: Construct a point outside the polygon and mark it as center in the Transform menu.

step 4: Select your entire polygon and dilate by a scale factor of 2/1 (Place 2 into top box and 1 into the bottom box).

Investigation: Corresponding Distances

step 5: Select the Ray icon to your left and construct a ray starting from the projection point through a vertex of the original polygon. Does the ray intersect the image? Where?

yes - at the corresponding point on the image

step 6: Repeat *step 5* with the remaining vertices of the original polygon. Does the same phenomena happen?

yes - it happens on every vertice so that the corresponding one on the image and the ray move w/ the vertice

if I use a -1/1 for dilation and put ⟷ lines from the point to the vertices of orig. shape, it keeps going thru the point to the correspond vertice of the image

APPENDIX D
LETTERS OF INFORMED CONSENT

LETTER OF INFORMED CONSENT

Dear _____:

I am a graduate student at the University of Florida, Department of Instruction and Curriculum; my faculty advisor is Mary Grace Kantowski. As part of my doctoral research I need to gather information on the relative effect of a dynamic geometry environment on learning. The title of my study is "The Effect of a Dynamic Geometry Environment on Preservice Teachers' Performance on Similarity Tasks."

This study involves no anticipated risks and/or direct benefits or the lack of risk/benefits to participation. This study is not designed to test individual knowledge of geometry or the individual's ability using microcomputer software. Instead, it is a study of the capabilities of technology to enhance the learning environment.

I would like to invite you to take part in my dissertation study. I will conduct all phases of the study.

If you agree to participate, you will be given two short (about 20 minutes each) multiple-choice criterion-reference quizzes addressing mathematical similarity. You do not have to answer any items you do not wish to answer. Also please note that you may terminate participation at any time for whatever reason.

This study will use a pretest-posttest control-group design. The timeline consists of a pretest, four sessions, and a posttest. You will be randomly placed into the experimental or control group. The sessions for the experimental group will be held in the computer laboratory and involve use of dynamic geometry software; sessions for the control group will be held in the traditional mathematics laboratory. The posttest will be administered upon completion of the instructional sessions.

If you have any questions concerning your participation in this study, please do not hesitate to contact me at 352-392-0761 extension 298. My home phone number is 352-466-7062; my mailing address is Route 1 Box 40-B, Micanopy, FL 32667. Furthermore, if you have any questions or concerns about your rights as a research participant, please contact the UFIRB office, P.O. Box 112250, University of Florida, Gainesville, FL 32611-2250.

Be assured that all results of this study are confidential to the extent provided by law. You will be given a code number that will appear on all results. Your identity will not be

addition, your participation or non-participation in the study will not in any way affect your grades or treatment in school. Unfortunately, I can offer no compensation for participation in this study; however, I would be happy to give you a final copy of the report. Your efforts are much appreciated.

Very Sincerely,

Helen Gerretson

I have read the procedure described on the previous page. I voluntarily agree to participate in Helen Gerretson's "The Effect of a Dynamic Geometry Environment on Preservice Teachers' Performance on Similarity Tasks" project and have received a copy of this description.

_____ _____

Participant Date

_____ _____

Investigator Date

LETTER OF INFORMED CONSENT
ADDENDUM

Dear _____:

I am a graduate student at the University of Florida, Department of Instruction and Curriculum; my faculty advisor is Mary Grace Kantowski. As part of my doctoral research I need to gather additional information on the relative effect of a dynamic geometry environment on learning. The title of my study is "The Effect of a Dynamic Geometry Environment on Preservice Teachers' Concept of Similarity."

This study involves no anticipated risks and/or direct benefits or the lack of risk/benefits to participation. This study is <u>not</u> designed to test individual knowledge of geometry or the individual's ability using microcomputer software. Instead, it is a study of the capabilities of technology to enhance the learning environment.

Thank you for your participation thus far in my dissertation study. In addition to the two short multiple-choice criterion-reference quizzes, I would appreciate permission to use your mathematics CLAST score to answer the question of whether entering mathematics knowledge is a significant predictor of performance on similarity tasks.

If you have any questions concerning your participation in this study, please do not hesitate to contact me at 352-392-0726 x 298. My home phone number is 352-466-7062; my mailing address is Route 1 Box 40, Micanopy, FL 32667. Dr. Kantowski can be reached at 392-0761 x226; her office is located in 2403 Norman Hall. Furthermore, if you have any questions or concerns about your rights as a research participant, please contact the UFIRB office, P.O. Box 112250, University of Florida, Gainesville, FL 32611-2250.

Be assured that results of this study and gathered data for this study are confidential to the extent provided by law. You were given a code number that appears on all results. Your identity will not be revealed to anyone or appear in any written work. In addition, all data will be destroyed upon completion of the study. Unfortunately, I can offer no compensation for participation in this study; however, I would be happy to give you a final copy of the report. Your efforts are much appreciated.

Very Sincerely,

Helen Gerretson

I have read the addendum described on the previous page. I voluntarily agree to participate in Helen Gerretson's "The Effect of a Dynamic Geometry Environment on Preservice Teachers' Performance on Similarity Tasks" dissertation research project and have received a copy of this addendum.

_____ _____

Participant Date

_____ _____

Investigator Date

APPENDIX E
DATA SETS

Control Group Data Set Sorted by Gain Score

OBS	ID	SECTION	TRT	SAT	TOT1	TOT2	GAIN
1	50	b	c	520	15	10	-5
2	43	b	c	580	16	14	-2
3	49	b	c	710	19	18	-1
4	21	a	c	630	14	14	0
5	63	b	c	.	15	15	0
6	1	m	c	500	12	13	1
7	6	m	c	490	15	16	1
8	8	m	c	540	8	9	1
9	54	b	c	560	13	14	1
10	55	b	c	510	12	13	1
11	2	m	c	670	13	15	2
12	25	a	c	610	12	14	2
13	29	a	c	510	9	11	2
14	56	b	c	640	13	15	2
15	5	m	c	430	8	11	3
16	22	a	c	610	13	16	3
17	28	a	c	.	11	14	3
18	53	b	c	500	15	19	4
19	57	b	c	650	13	17	4
20	4	m	c	590	12	17	5
21	10	m	c	560	14	19	5
22	19	m	c	660	14	19	5
23	31	a	c	630	8	13	5
24	46	b	c	610	10	15	5
25	48	b	c	690	11	16	5
26	33	a	c	520	3	14	11

Experimental Group Data Set Sorted by Gain Score

OBS	ID	SECTION	TRT	SAT	TOT1	TOT2	GAIN
27	16	m	e	550	11	10	-1
28	9	m	e	530	13	14	1
29	27	a	e	650	15	16	1
30	30	a	e	490	11	12	1
31	64	b	e	780	19	20	1
32	44	b	e	550	10	12	2
33	58	b	e	570	18	20	2
34	60	b	e	520	12	14	2
35	11	m	e	730	16	19	3
36	12	m	e	.	15	18	3
37	41	b	e	520	14	17	3
38	52	b	e	650	17	20	3
39	65	b	e	570	16	19	3
40	17	m	e	430	14	19	5
41	24	a	e	660	15	20	5
42	66	b	e	550	12	17	5
43	7	m	e	490	9	15	6
44	15	m	e	550	13	19	6
45	32	a	e	550	5	11	6
46	45	b	e	.	13	19	6
47	51	b	e	550	9	15	6
48	14	m	e	600	13	20	7
49	26	a	e	430	8	15	7
50	42	b	e	460	7	15	8
51	47	b	e	480	11	19	8
52	13	m	e	500	5	14	9

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BIOGRAPHICAL SKETCH

Helen Parznik Gerretson was born in Tokyo, Japan, in a year of the rabbit. As a child she lived with her parents touring Europe and finally settling near Annapolis, Maryland. Helen graduated from Archbishop Spalding High School in Arnold, Maryland. After a year of college, she married Rick Gerretson and, three years later, their twin boys were born.

While the twins, Niles and Adrian, were in middle school, Helen enrolled at the local community college, graduating with honors and a Presidential Scholarship to the University of Florida. She received her undergraduate degree with honors, majoring in mathematics, and went on to receive a Masters of Science in Teaching from the University of Florida Department of Mathematics, with minors in Statistics and Foundations of Education. The following term, she began her doctoral program.

In the fall of 1998, Helen, with husband Rick, will move to Colorado, leaving their sons to finish undergraduate work at the University of Florida. She will be an Assistant Professor in the University of Northern Colorado Department of Mathematical Sciences.

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

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This dissertation was submitted to the Graduate Faculty of the College of Education and to the Graduate School and was accepted as partial fulfillment of the requirements for the degree of Doctor of Philosophy.

August, 1998

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