THE DIFFERENTIAL PRODUCTION MODEL WITH QUASI-FIXED INPUTS: A PANEL DATA APPROACH TO U.S. BANKING

By

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A DISSERTATION PRESENTED TO THE GRADUATE SCHOOL
OF THE UNIVERSITY OF FLORIDA IN PARTIAL FULFILLMENT
OF THE REQUIREMENTS FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY

UNIVERSITY OF FLORIDA

2004

This dissertation is dedicated to my parents, Theodosios and Konstantina; my brothers, Harilaos and Ioannis; and the love of my life, Maria Chatzidaki, who made this happen.

ACKNOWLEDGMENTS

First and foremost, I would like to express my deep gratitude and sincere appreciation to my advisor, Dr. Charles B. Moss, for his outstanding guidance, encouragement, and advice during my graduate studies and the development of this dissertation. He has always been a source of motivation and inspiration. I would like especially to acknowledge Dr. Elias Dinopoulos for the endless discussions, advice, and encouragement during the research process that contributed to the quicker completion of this dissertation. Sincere appreciation is also extended to the other members of my committee Dr. James Seale, Dr. Timothy Taylor and Dr. Mark Flannery for their guidance, and constructive criticisms that led to improvements in this dissertation.

I would like to express my immeasurable gratitude to my parents, Theodosios and Konstantina Livanis; and my brothers, Harilaos and Ioannis Livanis, for their continuous love and moral support, despite the distance. I especially thank my parents, who taught me that I could achieve anything that I committed myself to fully. In the last years of my studies I was privileged to have my brother, Ioannis, studying at the same University. His humor and support made those years more enjoyable.

Finally, I would like to express my deepest love and gratitude to my partner in life, Maria Chatzidaki, for all of her love, support and sacrifice. Without her by my side, I would not have reached my goals successfully. Words cannot express how thankful I am to be sharing my life with someone so loving, patient, and thoughtful.

TABLE OF CONTENTS

LIST OF TABLES

Abstract of Dissertation Presented to the Graduate School
of the University of Florida in Partial Fulfillment of the
Requirements for the Degree of Doctor of Philosophy

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August 2004

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Major Department: Food and Resource Economics

This study assesses the empirical and policy implications of using the differential approach in opposition to dual specifications for the decisions of the multiproduct firm. In applied production analysis, the dual specifications of the firm's technology usually fail to satisfy the theoretical properties of the cost or profit function. If the validity of those properties is not examined, then empirical results should be interpreted with caution. On the other hand, the differential production model of the multiproduct firm has rarely been tested empirically, since it was first developed by Laitinen and Theil in 1978.

The novelty of this study is that it generalizes the differential production model for the multiproduct firm to account for quasi-fixed inputs in production; and to account for production technologies that are not output homogeneous, as assumed in the original model. Another objective of this study was to provide alternative parameterizations of the differential model, to account for variable coefficients over time. For this reason a supermodel was developed that contains different specifications that can be tested by

simple parameter restrictions. Further, maximum likelihood estimators were provided for the case of panel data in the differential model. The contribution of these estimators to the econometrics' literature was the consideration of nonlinear symmetry constraints for the differential model under balanced and unbalanced panel data designs.

The extended differential production model was applied to the U.S. banking industry for the period 1990–2000. To assess the empirical results of the differential model (and to provide a direct comparison with a dual specification), a translog cost function was applied to the same dataset. Results indicated that the differential model is consistent with economic theory, while the translog specification failed to satisfy the concavity property of the cost function for each year in the sample. Concerning the Allen elasticity of substitution both models found similar results. One disadvantage of the differential model was the assumption of perfect competition, which resulted in total revenue over total cost being the measure of scale economies.

CHAPTER 1
INTRODUCTION AND OBJECTIVES

1.1 Introduction

This study extends the multiproduct differential production model, developed by Laitinen and Theil (1978), to incorporate quasi-fixed inputs and applies this formulation to the U.S. banking industry. The differential approach differs from the dual specifications of cost and profit functions that have become the cornerstone of the literature in applied production analysis. Specifically, in the differential approach there is no particular specification of the firm's true technology, and thus it can describe different technologies without being exact for any particular form. The differential approach entails differentiation of the first-order conditions in a cost or profit optimization problem, to attain the input-demand and output-supply equations, respectively.

In contrast, the dual approach involves specifying a flexible functional form for the cost or profit function, to describe the firm's technology, which yields a system of equations to be estimated (e.g., a translog cost function with respective input shares). Thus, it can be considered as an approximation in the space of the variables (quantities and prices), while the differential approach is an approximation in parameter space. The disadvantage of the dual approach is that usually different functional forms lead to different results for the same dataset, as Howard and Shumway (1989) indicated, often failing to satisfy parameter restrictions. Especially, concavity restrictions tend to be nonlinear and more difficult to impose (Diewert and Wales 1987); and as a result, few empirical studies examine the concavity of their results in detail (exceptions are Featherstone and Moss 1994, Salvanes and Tjotta 1998, see also Shumway, 1995 for a recent survey of studies testing various parameter restrictions).

Numerous models have been developed for analyzing consumer demand based on the differential approach (Rotterdam, AIDS, CBS, NBR). Further, as demonstrated by Barten (1993), Lee et al. (1994) and Brown et al. (1994) a number of competing systems can be generated from alternative parameterizations of the differential system of demand that was originally introduced by Theil (1965, 1976, 1980). Thus the form of consumer demand can be selected through simple parameter restrictions.

In applied production analysis, a similar differential input-demand system was developed by Theil (1977) and Laitinen and Theil (LT, 1978). The Theil (1977) model concerns one-output transformation technologies, while the LT model extends to the multiproduct case. However, neither model (especially not the LT model) has been used much in empirical analysis because of their complexity. Exceptions include Rossi (1984), who extended the LT model to account for fixed inputs. However, he assumed that the production function was separable into variable and quasi-fixed inputs. Davis (1997) provided an application of the Theil (1977) model; while Fousekis and Pantzios (1999) generalized the Theil (1977) parameterization by including Rotterdam-type, CBS-type, and NBR-type effects. Recently, Washington and Kilmer (2000, 2002) applied the LT model in international agricultural trade. However, they assumed input-output separability and independence, which transformed the model into a single output model.

Our study extends the LT model to account for quasi-fixed inputs that are not separable from the variable inputs in the firm's technology. The model nests the Rossi

(1984) model, and a testable hypothesis is this separability. Further, in order to generalize the LT model, the output homogeneous assumption for the transformation technology is relaxed, and a comparison to the LT model is provided. Testable hypotheses were input independence, output independence, and input-output separability, as in the LT model. In the empirical section, the usual parameter restrictions of homogeneity and symmetry of the cost or profit function are imposed; and the concavity of the cost function in input prices and the convexity of the profit function in output prices were tested. Going one step farther, alternative specifications of the extended model were provided, forming the base for a test for quasi-fixity, based on a simple Hausman specification test (Schankerman and Nadiri 1986) or on direct test of the coefficients of the estimated model.

The proposed model was applied to the U.S. banking industry, giving specific attention to the concavity property of the cost function. The banking industry was selected because, most probably, the assumption of perfect competition in both input and output markets of a specific bank will hold; and because of the availability of data. The proposed model was compared with a standard transcendental logarithmic (translog) specification with quasi-fixed inputs, which is the most common specification applied to banking data. Comparison of the two models centers on whether the concavity property of the cost function is rejected. Contributions in the field of production analysis often check whether concavity is fulfilled by the estimated parameters of the cost function. Since the seminal papers of Lau (1978) and Diewert and Wales (1987), concavity is often directly imposed either locally or globally on the parameters. More recently, Ryan and Wales (1998, 2000) and Mochini (1999) discussed further techniques to impose concavity. Symmetry and homogeneity properties of the cost function can be regarded as technical properties, since they are a result of the continuity property and the definition of the cost function, respectively. On the other hand, concavity is the first property with true economic context, since it is a result of the optimization process. For instance, Koebel (2002) showed that a priori imposition of concavity may lead to estimation biases, when aggregation across goods is considered. Further, a radical failure in concavity may in fact be attributable to an inappropriate specification of the functional form.

Finally, traditional measures of efficiency (such as economies of scale) were provided for the differential model and other measures of substitutability or complementarity in the input and output sectors of the banks, such as Allen elasticities of substitution. These measures were compared with those of the translog cost specification.

1.2 Objectives

Specific objectives of our study can be summarized in the following:

1. To mathematically derive the LT differential production model of multiproduct firms under the assumption of quasi-fixed inputs and use a more general production technology that is not output homogeneous as in the LT model.

2. To provide alternative parameterizations of the extended LT model, especially for the cost-based system (input-demand system of equations). This will be useful for deriving a new test for asset quasi-fixity.

3. To provide alternative econometric procedures using Maximum Likelihood estimators for balanced and unbalanced panel data, for estimating the extended LT model.

4. To apply the extended LT multiproduct model to the U.S. banking industry and to econometrically estimate the system of derived-demand and output-supply equations using the econometric methods developed in this study.

5. To compare the results of the extended LT model with those of a flexible functional form specification, such as the translog. Specific attention was given to the concavity property of the cost function in input prices. The two models were also compared in terms of Allen elasticities of substitution and degree of economies of scale.

1.3 Overview

Chapter 2 provides the mathematical derivation of the basic model used in this analysis. It borrows heavily from the derivation techniques as presented by Laitinen (1980), but differs in terms of the added generalizations of a non-homogeneous, in the output vector, production technology; and of the existence of quasi-fixed inputs. Also, the extended model was compared with the original LT model, showing that the assumption of output-homogeneous production technology affects only the input-demand system and does not need to be imposed.

The basic parameterization of the extended model, closely following Laitinen (1980), is provided in Chapter 3. The novelty in this is the parameterization of the coefficients of the quasi-fixed inputs in the input-demand system, and the development of a "supermodel" for the cost-based system of equations. Specifically, the coefficients of the quasi-fixed inputs are a function of the respective shadow price of the quasi-fixed input. To parameterize those coefficients, the procedure of Morrison-Paul and MacDonald (2000) was used, whereby shadow prices are decomposed to their ex-ante market rental prices plus a deviation term. The "supermodel" for the cost-based system, developed in this chapter, accommodates for a new test for asset quasi-fixity and different assumptions on the estimated coefficients through simple parameter tests.

Chapter 4 concerns the econometrics of the differential approach. Section 4.1 presents the econometric issues related to the differential model, and the two step Maximum Likelihood procedure, provided by Laitinen (1980). Since this procedure does not conform to the data used in the empirical analysis, Maximum Likelihood estimators were developed for time-specific, fixed-effects, and individual-specific, random-effects panel data based on previous studies by Magnus (1982) and Biorn (2004). These procedures are useful for systems of equations with balanced or unbalanced panel data designs with nonlinear restrictions on the parameters.

Chapter 5 covers the empirical part of the present study. The time-specific, fixed-effects econometric method, presented in Chapter 4 is adapted for estimating the extended LT model and the translog specification for the banking industry. Then the results of both models are compared in terms of rejection (or not) of concavity, and elasticity measures.

Finally, Chapter 6 provides a summary, conclusions of the present study, and presents unresolved issues for future research.

CHAPTER 2
METHODOLOGY

2.1 Introduction

The Laitinen-Theil (LT, 1978) model extends previous studies by Hicks (1946) and Sakai (1974), to explicitly account for input-output separability, input independence, homotheticity and non-jointness of production. It concerns long-run behavior of risk-neutral multiproduct firms under competitive circumstances. Moreover, it is generally applicable, since it does not require specific assumptions, such as input-output separability or constant elasticities of scale or substitution. Before the LT model, Pfouts (1961, 1964, and 1973) had extended the Hicks' model to account for fixed inputs, but it was a special case since he assumed input-output separability and output independence. In the empirical literature, the LT model has hardly been applied. To my knowledge, only Rossi (1984) extended the LT model to account for fixed inputs (but he assumed separability between variable and fixed inputs) and applied the model in Italian farms. Washington and Kilmer (2000, 2002) were two other studies that used the LT model in international trade of agricultural products. However, by assuming input and output independence and input-output separability, the model became a single output model (Theil 1977).

The advantage of the LT model is that it avoids the use of a functional form for the dual specification (either cost or profit functions). That is, it does not specify a functional form for the true technology of the firm. However, the parameterization of the model provided by Laitinen (1980) implies constant price effects, and implies that the change in the cost share of the i^{th} input due to the change in r^{th} product is also constant. Therefore, there is a need for parameterization allowing for variable output and price effects. Fousekis and Pantzios (1999) provided such a general model but for the single output firm.

This chapter provides the general methodology and derivation of the short-run system of input-demand and output-supply equations for a multiproduct firm, under perfect competition in both markets of the firm. The model used was developed by LT, but it was transformed to account for a more general transformation technology that does not impose any restrictions on the returns to scale of the firm; nor imposes any restriction on homogeneity, homotheticity, input-output separability, or any other separability assumptions. These assumptions could be tested through parameter restrictions of the model. Further, the LT model was extended to account for quasi-fixed inputs. Apart from Clements (1978) and Rossi (1984), who used a transformation technology separable in the fixed inputs, there is no other attempt to specify or extend and test a more general model.

2.2 The Case of Multiple Quasi-Fixed Inputs

Let the production technology of a multiproduct, multifactor (MP-MF) individual firm be represented by a transformation function:

$$T(x, y, z) = 0 \tag{2-1}$$

where $y \in \mathbb{R}_+^m$ denotes a vector of variable outputs, $x \in \mathbb{R}_+^n$ a set of variable inputs and $z \in \mathbb{R}_+^k$ a set of quasi-fixed inputs (inputs that are difficult to adjust). Strictly positive prices of outputs and inputs are denoted by $p \in \mathbb{R}^m_{++}$ and $w \in \mathbb{R}^n_{++}$, respectively. This transformation technology satisfies certain regularity conditions (Lau 1972):

- The domain of $T(x, y, z)$ is a convex set containing the origin.
- $T(x, y, z)$ is convex and closed in $\{y, x, z\}$, in the nonnegative orthant $\mathbb{R}_{++}$.
- $T(x, y, z)$ is continuous and twice differentiable in y, x and z.
- $T(x, y, z)$ is strictly increasing in y and strictly decreasing in x .

Mittelhammer et al. (1981) showed that a single-equation multiproduct, multifactor in an implicit form production function, is not as general as it was thought to be. The production function shown by Equation 2-1 restricts each output to depend on all inputs, and other outputs that appear as arguments in the implicit form. Further, they showed that it cannot represent separability in the form of two independent functional constraints, such as $T(.) = g_1(.) + g_2(.)$, on the arguments of $T(x, y, z)$. In such cases, the gradient vector of $T(x, y, z)$ is zero, which further implies that the Kuhn-Tucker conditions do not hold. Therefore, our study did not examine separability of that form; and instead left it for future research.

Assume that a MP-MF firm minimizes variable costs of producing the vector of outputs y, conditional on the vector of quasi-fixed inputs z and fixed prices w for the variable inputs. This short-run or restricted cost function can be denoted as $VC = VC(y, w; z)$, and it is assumed that it satisfies the following properties (Chambers 1988):

- $VC(y, w; z)$ is monotonically non-decreasing, homogeneous of degree one and concave in w.
- $VC(y, w; z)$ is non decreasing and convex in y.

- $VC(y, w; z)$ is non increasing and convex in z.
- $VC(y, w; z)$ is twice continuously differentiable on $(w, y; z)$.

Applying Shephard's lemma on the restricted cost function, the conditional factor demands are then obtained as $x_i = \frac{\partial VC}{\partial w_i} = VC_w(y, w; z)$. If v denotes the vector of ex-ante market rental prices of the quasi-fixed inputs, then the short-run total cost of producing the vector y is given by $SC = VC(y, w; z) + v \cdot z'$. The long-run cost function $C(w, y)$ of the multiproduct firm is then obtained by minimizing short-run total cost with respect to quasi-fixed inputs, while holding the variable inputs and the level of output at the observed cost-minimizing levels. That is,

$$C(w, y) \equiv \min_z SC \equiv \min_z \left(VC(y, w; z) + v \cdot z'\right)$$

The first-order condition of this minimization problem implies that

$$\frac{\partial SC}{\partial z} = \frac{\partial VC(y, w; z^*)}{\partial z^*} + v = 0$$

where z^* denotes the static equilibrium levels of z. This condition can be written as $-\frac{\partial VC(y, w; z^*)}{\partial z^*} = v$, which states that a necessary condition for a firm to be in long-run equilibrium is that the shadow prices of the quasi-fixed inputs be equal to the observed ex-ante market rental prices v (Samuelson 1953). Therefore, the shadow price of a quasi-fixed input is defined as the potential reduction in expenditures on other variable inputs that can be achieved by using an additional unit of the input under consideration, while maintaining the level of outputs. Further, Berndt and Fuss (1989) showed that when this condition holds, temporary and full-equilibrium demand levels for the quasi-fixed inputs are equal. The same result holds for the short-run and long-run marginal cost and demands for variable inputs of the multiproduct firm.

2.3 Cost Minimization

For the multiproduct-multifactor firm, let y_r be the r^{th} product ($r = 1,...,m$) to which corresponds a price p_r. Let x_i be the i^{th} factor of production ($i = 1,...,n$) whose price is denoted by w_i and z_k be the k^{th} quasi-fixed factor of production ($k = 1,...,l$) with an ex-ante market rental price denoted by v_k. Assume a production function in an implicit form that is not separable into the quasi-fixed inputs, as in Rossi (1984), nor is it negatively linearly homogeneous in the output vector as in LT (1978); and assume that it satisfies the properties mentioned in Section 2.2. Thus, it can be written as

$$T(x, y, z) = 0 \tag{2-2}$$

Then in the short-run, the firm's objective is to minimize variable cost (VC) subject to its transformation technology, by varying the input quantities for given output and input prices, and for given quasi-fixed input levels. Thus, the problem that the firm faces is

$$\min_x \left\{ VC(w, x) \equiv \sum_{i=1}^{n} w_i x_i : T(x, y, z) = 0 \right\} \tag{2-3}$$

The Lagrangean of the above problem can be written as $L = \sum_{i=1}^{n} w_i x_i - \lambda T\left(x, y, z\right)$ and the first-order conditions needed to attain a minimum are given by the following equations:

$$\frac{\partial L}{\partial \ln x_i} = w_i x_i - \lambda \frac{\partial T(.)}{\partial \ln x_i} = 0 \tag{2-4}$$

$$\frac{\partial L}{\partial \lambda} = -T(x, y, z) = 0 \tag{2-5}$$

In this formulation, $\lambda > 0$ is implied by the positivity of x_i and the assumption that the marginal physical product of each input is positive ($\partial T(.)/\partial \ln x_i > 0$). Further, Equations 2-4 and 2-5 are assumed to yield unique positive values for x_i and λ; and Equation 2-4 is a vector of $n \times 1$.

The second-order conditions are given by the following equations:

$$\frac{\partial^2 L}{\partial \ln x_i \partial \ln x_j} = \delta_{ij} w_i x_i - \lambda \frac{\partial^2 T}{\partial \ln x_i \partial \ln x_j} \quad (2\text{-}6)$$

where δ is a Kronecker delta. That is $\delta_{ij} = \begin{cases} 1, & i = j \\ 0, & i \neq j \end{cases}$.

$$\frac{\partial^2 L}{\partial \lambda \partial \ln x_i} = \frac{\partial T}{\partial \ln x_i} \text{ and } \frac{\partial^2 L}{\partial \lambda^2} = 0$$

The solution of the minimization problem described in Equation 2-3 gives the conditional or compensated short-run demands of the inputs as a function of all input prices, output quantities, and quasi-fixed inputs. That is, $x^{SR} = x^{SR}(w, y, z)$ and $\lambda^{SR} = \lambda^{SR}(w, y, z)$, where x^{SR} denotes the vector of inputs and λ^{SR} is the Lagrangean multiplier. To obtain a minimum cost in the short-run, it is sufficient that the matrix of the second order derivatives that has a size $n \times n$ (Equation 2-6), is symmetric and positive definite. The minimum short-run cost is then given by

$$VC(w, y, z) = \sum_i w_i x_i(w, y, z) \quad (2\text{-}7)$$

2.3.1 Returns to Scale and Elasticities of Variable Cost

Consider first the total differential of $T(x, y, z) = 0$ in natural logarithmic form:

$$\sum_{i=1}^{n} \frac{\partial T}{\partial \ln x_i} d \ln x_i + \sum_{r=1}^{m} \frac{\partial T}{\partial \ln y_r} d \ln y_r + \sum_{k=1}^{l} \frac{\partial T}{\partial \ln z_k} d \ln z_k = 0 \quad (2\text{-}8)$$

The degree of returns to scale (RTS) is defined as the proportional increase in all outputs, resulting from a proportional increase in all inputs, variable and quasi-fixed. Letting this be the case, and defining $x^o = [x, z]$, then $d\ln x_i = d\ln x_j = d\ln z_k = d\ln z_l$ and $d\ln y_r = d\ln y_s$ can each be put before its summation sign. Then we have

$$d\ln x_i \sum_{i=1}^{n} \frac{\partial T}{\partial \ln x_i} + d\ln z_k \sum_{k=1}^{l} \frac{\partial T}{\partial \ln z_k} + d\ln y_r \sum_{r=1}^{m} \frac{\partial T}{\partial \ln y_r} = 0$$

This can also be written as

$$d\ln x_j^o \left(\sum_{i=1}^{n} \frac{\partial T}{\partial \ln x_i} + \sum_{k=1}^{l} \frac{\partial T}{\partial \ln z_k} \right) + d\ln y_r \sum_{r=1}^{m} \frac{\partial T}{\partial \ln y_r} = 0$$

Therefore,

$$RTS = \frac{d\ln y_r}{d\ln x_j^o} = \frac{-\left(\sum_{i=1}^{n} \frac{\partial T}{\partial \ln x_i} + \sum_{k=1}^{l} \frac{\partial T}{\partial \ln z_k} \right)}{\sum_{r=1}^{m} \frac{\partial T}{\partial \ln y_r}} \tag{2-9}$$

Notice that this relationship for the returns to scale is the same as the relationship derived by Caves et al. (1981).

The marginal cost of the r^{th} output can be found by taking the derivative of the optimum variable cost function (Equation 2-7) with respect to output y_r:

$$\frac{\partial VC}{\partial y_r} = \sum_i w_i \frac{\partial x_i}{\partial y_r} = \frac{VC}{y_r} \sum_i f_i \frac{\partial \ln x_i}{\partial \ln y_r} \tag{2-10}$$

where $f_i = \frac{w_i x_i}{VC}$ is the variable cost share of input i and the last expression has been derived from the second by multiplying the second term by $(y_r VC / y_r VC)$ and noting that $\partial \ln x_i = \frac{1}{x_i} \partial x_i$. Also, notice that the above equation can be written as

$$\frac{\partial \ln VC}{\partial \ln y_r} = \sum_i f_i \frac{\partial \ln x_i}{\partial \ln y_r} \tag{2-11}$$

Next, differentiating the optimum transformation technology $T(x, y, z) = 0$ with respect to $\ln y_r$, holding input prices, other outputs and quasi-fixed inputs constant, we get

$$\sum_{i=1}^{n} \frac{\partial T}{\partial \ln x_i} \frac{\partial \ln x_i}{\partial \ln y_r} + \frac{\partial T}{\partial \ln y_r} = 0 \tag{2-12}$$

However, by using the first-order condition $\frac{\partial T}{\partial \ln x_i} = \frac{w_i x_i}{\lambda}$ and by multiplying the first term by $\frac{y_r VC}{y_r VC}$, Equation 2-12 becomes

$$\frac{y_r \cdot VC}{\lambda \cdot y_r} \sum_{i=1}^{n} \frac{w_i x_i}{VC} \frac{\partial \ln x_i}{\partial \ln y_r} + \frac{\partial T}{\partial \ln y_r} = 0, \text{ where } \frac{w_i x_i}{VC} = f_i$$

Using now Equation 2-10 the above expression can be written as

$$\frac{y_r}{\lambda} \frac{\partial VC}{\partial y_r} + \frac{\partial T}{\partial \ln y_r} = 0 \tag{2-13}$$

If we sum Equation 2-13 over r then we get

$$\frac{1}{\lambda} \sum_{r=1}^{m} y_r \frac{\partial VC}{\partial y_r} + \sum_{r=1}^{m} \frac{\partial T}{\partial \ln y_r} = 0 \text{ or } \lambda = -\frac{\sum_r \frac{\partial VC}{\partial \ln y_r}}{\sum_r \frac{\partial T}{\partial \ln y_r}} \tag{2-14}$$

Letting $\gamma_1 = \frac{\lambda}{VC}$, then from Equation 2-14 we have that

$$\gamma_1 = \frac{\lambda}{VC} = -\frac{\sum_r \frac{\partial \ln VC}{\partial \ln y_r}}{\sum_r \frac{\partial T}{\partial \ln y_r}} \tag{2-15}$$

The elasticity of variable cost with respect to proportionate output changes, holding quasi-fixed inputs constant, is obtained by substituting in Equation 2-15 the expression for the lagrangean multiplier (λ) from the first-order condition (Equation 2-4). That is, we substitute $\lambda = VC / \sum_i \frac{\partial T}{\partial \ln x_i}$ in Equation 2-15 to obtain

$$\varepsilon_{vc,y} = \sum_r \frac{\partial \ln VC}{\partial \ln y_r} = -\frac{\sum_r \frac{\partial T}{\partial \ln y_r}}{\sum_i \frac{\partial T}{\partial \ln x_i}} \tag{2-16}$$

To find the elasticity of variable cost with respect to proportionate quasi-fixed input changes, we follow similar analysis as above, holding output constant. Therefore taking the derivative of the optimum variable cost function with respect to a quasi-fixed input we obtain

$$\frac{\partial VC}{\partial z_k} = \sum_i w_i \frac{\partial x_i}{\partial z_k} = \frac{VC}{z_k} \sum_i f_i \frac{\partial \ln x_i}{\partial \ln z_k} \tag{2-17}$$

Notice that $-\frac{\partial VC}{\partial z_k} = w_k$ denotes the shadow price of the quasi-fixed input. Also, from the analysis in Section 2.1, in order for the firm to be in long-run equilibrium, it has to be the case that $-\frac{\partial VC}{\partial z_k} = v_k$, where v_k is the ex-ante market rental price of the quasi-fixed input. Further, Equation 2-17 can also be transformed into the following expression

$$\frac{\partial \ln VC}{\partial \ln z_k} = \sum_i f_i \frac{\partial \ln x_i}{\partial \ln z_k} \tag{2-18}$$

Now, taking the derivative of the optimum production technology $T(x, y, z) = 0$ with respect to $\ln z_k$, holding input prices, other quasi-fixed inputs, and outputs constant, we get

$$\sum_{i=1}^{n} \frac{\partial T}{\partial \ln x_i} \frac{\partial \ln x_i}{\partial \ln z_k} + \frac{\partial T}{\partial \ln z_k} = 0 \tag{2-19}$$

Again, using the first-order condition, $\frac{\partial T}{\partial \ln x_i} = \frac{w_i x_i}{\lambda}$, multiplying the first term of the above equation by $\frac{z_k VC}{z_k VC}$, and using Equation 2-17 we obtain the following relationship

$$\frac{z_k}{\lambda} \frac{\partial VC}{\partial z_k} + \frac{\partial T}{\partial \ln z_k} = 0 \tag{2-20}$$

Summing this equation over k, we obtain the second interpretation for λ:

$$\lambda = -\frac{\sum_k \frac{\partial VC}{\partial \ln z_k}}{\sum_k \frac{\partial T}{\partial \ln z_k}} \tag{2-21}$$

However, Equation 2-21 must be equal to Equation 2-14 implying the following relationship

$$\sum_k \frac{\partial T}{\partial \ln z_k} = \frac{\sum_k \frac{\partial VC}{\partial \ln z_k}}{\sum_r \frac{\partial VC}{\partial \ln y_r}} \sum_r \frac{\partial T}{\partial \ln y_r} \tag{2-22}$$

Solving for the elasticity of cost with respect to proportionate quasi-fixed input change from the above equation, we obtain

$$\varepsilon_{vc,z} = \sum_k \frac{\partial \ln VC}{\partial \ln z_k} = \frac{\sum_k \frac{\partial T}{\partial \ln z_k}}{\sum_r \frac{\partial T}{\partial \ln y_r}} \sum_r \frac{\partial \ln VC}{\partial \ln y_r} \tag{2-23}$$

which can also be written as

$$\varepsilon_{vc,z} = \varepsilon_{vc,y} \frac{\sum_k \frac{\partial T}{\partial \ln z_k}}{\sum_r \frac{\partial T}{\partial \ln y_r}} \tag{2-24}$$

or equivalently, Equation 2-23 (through the use of Equation 2-16) can be written as

$$\varepsilon_{vc,z} = \sum_k \frac{\partial \ln VC}{\partial \ln z_k} = -\frac{\sum_k \frac{\partial T}{\partial \ln z_k}}{\sum_i \frac{\partial T}{\partial \ln x_i}} \tag{2-25}$$

Finally, taking into consideration Equations 2-16 and 2-25, the degree of returns to scale (RTS) in terms of derivatives of the variable cost function (Equation 2-9) can be written as

$$RTS = \frac{-\left(\sum_{i=1}^{n} \frac{\partial T}{\partial \ln x_i} + \sum_{k=1}^{l} \frac{\partial T}{\partial \ln z_k}\right)}{\sum_{r=1}^{m} \frac{\partial T}{\partial \ln y_r}} = \frac{1 - \sum_k \frac{\partial \ln VC}{\partial \ln z_k}}{\sum_r \frac{\partial \ln VC}{\partial \ln y_r}} \tag{2-26}$$

2.3.2 Factor and Product Shares

We have already defined the variable cost share of input i as

$$f_i = \frac{w_i x_i}{VC} \tag{2-27}$$

Taking the total differential of Equation 2-27 we have

$$df_i = f_i d \ln w_i + f_i d \ln x_i - f_i d \ln VC \tag{2-28}$$

Summing Equation 2-28 over i and noting that $\sum_i f_i = 1$ and so $d\sum_i f_i = 0$, we have

$$d \ln VC = \sum_i f_i d \ln w_i + \sum_i f_i d \ln x_i \tag{2-29}$$

or in a more compact form

$$d \ln VC = d \ln W + d \ln X \tag{2-30}$$

where $d\ln W = \sum_i f_i d\ln w_i$, $d\ln X = \sum_i f_i d\ln x_i$ are the Divisia indexes of variable input prices and variable input quantities, respectively (Divisia input price index and Divisia input volume index).

Then considering Equation 2-14 for λ, define as in Laitinen and Theil (1978)

$$g_r = \frac{y_r}{\lambda}\frac{\partial VC}{\partial y_r} = -\frac{\partial VC / \partial \ln y_r}{\sum_r \partial VC / \partial \ln y_r}\sum_r \frac{\partial T}{\partial \ln y_r} \tag{2-31}$$

as the share of the r^{th} product in total variable marginal cost multiplied by $-\sum_r \frac{\partial T}{\partial \ln y_r}$.

Notice that if we had assumed negatively linear-homogeneous production function in the output vector, which implies that $\sum_r \frac{\partial T}{\partial \ln y_r} \equiv -1$, as in LT, then g_r would be just the share of the r^{th} product in total variable marginal cost. It is the case though that at the point of the firm's optimum (from Equation 2-13):

$$g_r = -\frac{\partial T}{\partial \ln y_r} \tag{2-32}$$

Noting that $\sum_r g_r = -\sum_r \frac{\partial T}{\partial \ln y_r}$, we can define the share of the r^{th} product in total variable marginal cost as

$$\frac{g_r}{\sum_s g_s} = \frac{\partial VC / \partial \ln y_r}{\sum_s \partial VC / \partial \ln y_s} \quad \text{with } r, s = 1, \ldots, m$$

These shares are necessarily positive and have unit sum over r. Further, we can define the Divisia volume index of outputs as $d\ln Y = \sum_r \frac{g_r}{\sum_s g_s} d\ln y_r$.

Similarly, considering Equation 2-21 for λ, define

$$\mu_k = \frac{z_k}{\lambda}\frac{\partial VC}{\partial z_k} = -\frac{\partial VC/\partial \ln z_k}{\sum_k \partial VC/\partial \ln z_k}\sum_k \frac{\partial T}{\partial \ln z_k} \tag{2-33}$$

as the share of the k^{th} quasi-fixed input shadow value in total shadow value of the quasi-fixed inputs, multiplied by $-\sum_k \frac{\partial T}{\partial \ln z_k}$. Further, substituting for $\sum_k \frac{\partial T}{\partial \ln z_k}$ its equivalent form from Equation 2-22 we obtain the ratio of the k^{th} quasi-fixed input shadow value in the variable marginal cost of m outputs, multiplied by $\sum_r \frac{\partial T}{\partial \ln y_r}$:

$$\mu_k = -\frac{\partial VC/\partial \ln z_k}{\sum_r \partial VC/\partial \ln y_r}\sum_r \frac{\partial T}{\partial \ln y_r} \tag{2-34}$$

Using now Equation 2-31, the above equation transforms to

$$\mu_k = \frac{\partial VC/\partial \ln z_k}{\partial VC/\partial \ln y_r} g_r, \qquad r = 1,...,m \tag{2-35}$$

Also, at the point of the firm's optimum (from Equations 2-33 and 2-20), it holds

$$\mu_k = -\frac{\partial T}{\partial \ln z_k} \tag{2-36}$$

As in the case of outputs, note that $\sum_k \mu_k = -\sum_k \frac{\partial T}{\partial \ln z_k}$. Therefore, we can define the share of the k^{th} quasi-fixed input shadow value in total shadow value of the quasi-fixed inputs as

$$\frac{\mu_k}{\sum_e \mu_e} = \frac{\partial VC/\partial \ln z_k}{\sum_e \partial VC/\partial \ln z_e} \text{ with } k,e = 1,...,l$$

which are positive and have unit sum over k. Further, as in the case of outputs, the Divisia volume index of quasi-fixed inputs is defined as $d\ln Z = \sum_k \left(\mu_k / \sum_e \mu_e \right) d\ln z_k$.

2.3.3 Marginal Shares of Variable Inputs

Like in LT model, define the share of i^{th} variable input in the marginal cost of the r^{th} product as

$$\theta_i^r = \frac{\partial(w_i x_i)/\partial y_r}{\partial VC/\partial y_r} \tag{2-37}$$

Then multiply Equation 2-37 by $\frac{g_r}{\sum_s g_s}$ and sum over r to get

$$\theta_i = \sum_r \frac{g_r}{\sum_s g_s}\theta_i^r = \sum_r \frac{\partial VC/\partial \ln y_r}{\sum_s \partial VC/\partial \ln y_s}\frac{\partial(w_i x_i)/\partial y_r}{\partial VC/\partial \ln y_r}$$

The above equation can be written as

$$\theta_i = \frac{\sum_r \partial(w_i x_i)/\partial \ln y_r}{\sum_r \partial VC/\partial \ln y_r} \tag{2-38}$$

Equation 2-38 defines the share of the i^{th} input in variable marginal cost of outputs. Finally, as Laitinen and Theil mentioned, summation of θ_i^r, or θ_i over i gives always unity, but need not be non-negative.

In a similar fashion define the share of i^{th} variable input in the shadow price of quasi-fixed input z_k as

$$\xi_i^k = \frac{\partial(w_i x_i)/\partial z_k}{\partial VC/\partial z_k} \tag{2-39}$$

Then multiply Equation 2-39 by $\frac{\mu_k}{\sum_e \mu_e}$ and sum over k to get the share of the i^{th} variable input in variable marginal cost of m outputs:

$$\xi_i = \sum_k \frac{\mu_k}{\sum_e \mu_e} \xi_i^k = \sum_k \frac{\partial VC / \partial \ln z_k}{\sum_e \partial VC / \partial \ln z_e} \frac{\partial (w_i x_i) / \partial z_k}{\partial VC / \partial z_k},$$

which can be simplified to

$$\xi_i = \frac{\sum_k \partial (w_i x_i) / \partial \ln z_k}{\sum_k \partial VC / \partial \ln z_k} \tag{2-40}$$

As in the case of the outputs summation of ξ_i^k, ξ_i over i is always unity but need not be non-negative.

2.3.4 Input Demand Equations

The first step is to write the first-order conditions as identities and then to differentiate them with respect to their arguments. That is, with respect to each output y_r, input prices w_i, and quasi-fixed input quantity z_k, in order to determine how the optimum changes in response to changes in these given variables. Therefore, the first-order conditions as identities are

$$w_i x_i(w, y, z) - \lambda(w, y, z) \frac{\partial T(x(w, y, z), y, z)}{\partial \ln x_i} \equiv 0 \tag{2-41}$$

$$T(x(w, y, z), y, z) \equiv 0 \tag{2-42}$$

Totally differentiating Equation 2-41 with respect to $\ln y_r$, $\ln w_j$, and $\ln z_k$, it gives the following relationships, respectively

$$w_i x_i \frac{\partial \ln x_i}{\partial \ln y_r} - \lambda \frac{\partial \ln \lambda}{\partial \ln y_r} \frac{\partial T}{\partial \ln x_i} - \lambda \sum_{j=1}^{n} \frac{\partial^2 T}{\partial \ln x_i \partial \ln x_j} \frac{\partial \ln x_j}{\partial \ln y_r} - \lambda \frac{\partial^2 T}{\partial \ln x_i \partial \ln y_r} \equiv 0 \tag{2-43}$$

$$\delta_{ij} w_i x_i + w_i x_i \frac{\partial \ln x_i}{\partial \ln w_j} - \lambda \frac{\partial T}{\partial \ln x_i} \frac{\partial \ln \lambda}{\partial \ln w_j} - \lambda \sum_{\tau=1}^{n} \frac{\partial^2 T}{\partial \ln x_i \partial \ln x_\tau} \frac{\partial \ln x_\tau}{\partial \ln w_j} \equiv 0 \tag{2-44}$$

$$w_i x_i \frac{\partial \ln x_i}{\partial \ln z_k} - \lambda \frac{\partial \ln \lambda}{\partial \ln z_k} \frac{\partial T}{\partial \ln x_i} - \lambda \sum_{j=1}^{n} \frac{\partial^2 T}{\partial \ln x_i \partial \ln x_j} \frac{\partial \ln x_j}{\partial \ln z_k} - \lambda \frac{\partial^2 T}{\partial \ln x_i \partial \ln z_k} \equiv 0 \quad (2\text{-}45)$$

Notice that Equation 2-43 represents n distinct equations, equal to the number of inputs. However, if we consider all the outputs we are going to have $n \times m$ distinct equations. Similar arguments can be used to show that Equations 2-44 and 2-45 represent $n \times n$ and $n \times l$ ($k = 1,...,l$) distinct equations, respectively.

Then totally differentiating Equation 2-42 with respect to $\ln y_r$, $\ln w_j$, and $\ln z_k$ we have, respectively

$$\sum_{i=1}^{n} \frac{\partial T}{\partial \ln x_i} \frac{\partial \ln x_j}{\partial \ln y_r} + \frac{\partial T}{\partial \ln y_r} \equiv 0 \quad (2\text{-}46)$$

$$\sum_{i=1}^{n} \frac{\partial T}{\partial \ln x_i} \frac{\partial \ln x_i}{\partial \ln w_j} \equiv 0 \quad (2\text{-}47)$$

$$\sum_{i=1}^{n} \frac{\partial T}{\partial \ln x_i} \frac{\partial \ln x_j}{\partial \ln z_k} + \frac{\partial T}{\partial \ln z_k} \equiv 0 \quad (2\text{-}48)$$

Since we differentiate with respect to each output, input price and quasi-fixed input level, Equations 2-46 to 2-48 are vectors of dimension $m \times 1$, $n \times 1$ and $l \times 1$, respectively.

The next steps for the derivation of the input-demand system consist of the following

- Divide Equations 2-43 to 2-45 by variable cost (VC), use the definition of the cost shares $\frac{w_i x_i}{VC} = f_i$, and use from the first-order conditions the relationship $\lambda \frac{\partial T}{\partial \ln x_i} = w_i x_i$.
- Multiply Equations 2-46 to 2-48 by $\frac{\lambda}{VC}$ and use the following relationships $g_r = -\frac{\partial T}{\partial \ln y_r}$, $\mu_k = -\frac{\partial T}{\partial \ln \mu_k}$ and $\frac{\lambda}{VC} = \gamma_1$.

These transformations give the following relationships

$$f_i \frac{\partial \ln x_i}{\partial \ln y_r} - f_i \frac{\partial \ln \lambda}{\partial \ln y_r} - \frac{\lambda}{VC}\left[\sum_{j=1}^{n} \frac{\partial^2 T}{\partial \ln x_i \partial \ln x_j} \frac{\partial \ln x_j}{\partial \ln y_r}\right] - \frac{\lambda}{VC} \frac{\partial^2 T}{\partial \ln x_i \partial \ln y_r} \equiv 0 \qquad (2\text{-}49)$$

$$\delta_{ij} f_i + f_i \frac{\partial \ln x_i}{\partial \ln w_j} - f_i \frac{\partial \ln \lambda}{\partial \ln w_j} - \frac{\lambda}{VC}\left[\sum_{\tau=1}^{n} \frac{\partial^2 T}{\partial \ln x_i \partial \ln x_\tau} \frac{\partial \ln x_\tau}{\partial \ln w_j}\right] \equiv 0 \qquad (2\text{-}50)$$

$$f_i \frac{\partial \ln x_i}{\partial \ln z_k} - f_i \frac{\partial \ln \lambda}{\partial \ln z_k} - \frac{\lambda}{VC}\left[\sum_{j=1}^{n} \frac{\partial^2 T}{\partial \ln x_i \partial \ln x_j} \frac{\partial \ln x_j}{\partial \ln z_k}\right] - \frac{\lambda}{VC} \frac{\partial^2 T}{\partial \ln x_i \partial \ln z_k} \equiv 0 \qquad (2\text{-}51)$$

$$\sum_{i=1}^{n} f_i \frac{\partial \ln x_i}{\partial \ln y_r} \equiv \gamma_1 g_r \qquad (2\text{-}52)$$

$$\sum_{i=1}^{n} f_i \frac{\partial \ln x_i}{\partial \ln w_j} \equiv 0 \qquad (2\text{-}53)$$

$$\sum_{i=1}^{n} f_i \frac{\partial \ln x_i}{\partial \ln z_k} \equiv \gamma_1 \mu_k \qquad (2\text{-}54)$$

Now the following matrices can be defined

$$F = diag(f_1, ..., f_n),\ H = \left[\frac{\partial^2 T}{\partial \ln x_i \partial \ln x_j}\right]_{n \times n},\ H_1 = \left[\frac{\partial^2 T}{\partial \ln x_i \partial \ln y_r}\right]_{n \times m},$$

and $H_3 = \left[\frac{\partial^2 T}{\partial \ln x_i \partial \ln z_k}\right]_{n \times l}$.

Therefore Equations 2-49 to 2-54 can be written for all combinations of inputs, outputs and quasi-fixed inputs, in matrix form, as

$$\left(F - \gamma_1 H\right) \frac{\partial \ln x}{\partial \ln y'} - F \cdot i_n \frac{\partial \ln \lambda}{\partial \ln y'} = \gamma_1 \cdot H_1 \qquad (2\text{-}55)$$

$$\left(F - \gamma_1 H\right) \frac{\partial \ln x}{\partial \ln w'} - F \cdot i_n \frac{\partial \ln \lambda}{\partial \ln w'} = -F \qquad (2\text{-}56)$$

$$\left(F-\gamma_1 H\right)\frac{\partial \ln x}{\partial \ln z'}-F\cdot i_n\frac{\partial \ln \lambda}{\partial \ln z'}=\gamma_1\cdot H_3 \qquad (2\text{-}57)$$

$$i_n'\cdot F\cdot\frac{\partial \ln x}{\partial \ln y'}\equiv \gamma_1\cdot g' \qquad (2\text{-}58)$$

$$i_n'\cdot F\cdot\frac{\partial \ln x}{\partial \ln w'}\equiv 0 \qquad (2\text{-}59)$$

$$i_n'\cdot F\cdot\frac{\partial \ln x}{\partial \ln z'}\equiv \gamma_1\cdot \mu',\ \ (1\times l) \qquad (2\text{-}60)$$

Now, premultiply Equations 2-55 to 2-57 by F^{-1} and combine with Equations 2-58 to 2-60, to form Barten's fundamental matrix equation

$$\begin{bmatrix} F^{-1}\left(F-\gamma_1 H\right)F^{-1} & i_n \\ i_n' & 0 \end{bmatrix}\begin{bmatrix} F\dfrac{\partial \ln x}{\partial \ln y'} & F\dfrac{\partial \ln x}{\partial \ln w'} & F\dfrac{\partial \ln x}{\partial \ln z'} \\ -\dfrac{\partial \ln \lambda}{\partial \ln y'} & -\dfrac{\partial \ln \lambda}{\partial \ln w'} & -\dfrac{\partial \ln \lambda}{\partial \ln z'} \end{bmatrix}=\begin{bmatrix} \gamma_1 F^{-1}H_1 & -I & \gamma_1 F^{-1}H_3 \\ \gamma_1 g' & 0 & \gamma_1 \mu' \end{bmatrix}$$

and solving for the matrix of the decision variables we obtain

$$\begin{bmatrix} F\dfrac{\partial \ln x}{\partial \ln y'} & F\dfrac{\partial \ln x}{\partial \ln w'} & F\dfrac{\partial \ln x}{\partial \ln z'} \\ -\dfrac{\partial \ln \lambda}{\partial \ln y'} & -\dfrac{\partial \ln \lambda}{\partial \ln w'} & -\dfrac{\partial \ln \lambda}{\partial \ln z'} \end{bmatrix}=$$

$$\begin{bmatrix} F^{-1}\left(F-\gamma_1 H\right)F^{-1} & i_n \\ i_n' & 0 \end{bmatrix}^{-1}\begin{bmatrix} \gamma_1 F^{-1}H_1 & -I & \gamma_1 F^{-1}H_3 \\ \gamma_1 g' & 0 & \gamma_1 \mu' \end{bmatrix} \qquad (2\text{-}61)$$

From Magnus and Neudecker (1988), if A is a non-singular partitioned matrix defined as $A=\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}=\begin{bmatrix} F^{-1}\left(F-\gamma_1 H\right)F^{-1} & i_n \\ i_n' & 0 \end{bmatrix}$ and the matrix $D=A_{22}-A_{21}A_{11}^{-1}A_{12}$ is also non-singular, then the inverse of matrix A is given by

$$A^{-1}=\begin{bmatrix} A_{11}^{-1}+A_{11}^{-1}A_{12}D^{-1}A_{21}A_{11}^{-1} & -A_{11}^{-1}A_{12}D^{-1} \\ -D^{-1}A_{21}A_{11}^{-1} & D^{-1} \end{bmatrix}$$

It follows then, that

- $D = -i_n' F\left(F - \gamma_1 H\right)^{-1} F \cdot i_n$ which is a scalar. Using the property of the inverse of a scalar, we get that $D^{-1} = \dfrac{1}{-i_n' F\left(F - \gamma_1 H\right)^{-1} F \cdot i_n}$.

- $A_{11}^{-1} + A_{11}^{-1} A_{12} D^{-1} A_{21} A_{11}^{-1} = F\left(F - \gamma_1 H\right)^{-1} F - \dfrac{F\left(F - \gamma_1 H\right)^{-1} F \cdot i_n i_n' F\left(F - \gamma_1 H\right)^{-1} F}{i_n' F\left(F - \gamma_1 H\right)^{-1} F i_n}$

- $-A_{11}^{-1} A_{12} D^{-1} = \dfrac{F\left(F - \gamma_1 H\right)^{-1} F \cdot i_n}{i_n' F\left(F - \gamma_1 H\right)^{-1} F \cdot i_n}$

- $-D^{-1} A_{21} A_{11}^{-1} = \dfrac{i_n' F\left(F - \gamma_1 H\right)^{-1} F}{i_n' F\left(F - \gamma_1 H\right)^{-1} F \cdot i_n}$

As in LT, define $\psi = i_n' F\left(F - \gamma_1 H\right)^{-1} F i_n$, which is a positive scalar and implies that $D^{-1} = -\dfrac{1}{\psi}$. Then define the $n \times n$ matrix $\Phi = \left[\phi_{ij}\right]$ as

$$\Phi = \frac{1}{\psi} F\left(F - \gamma_1 H\right)^{-1} F \tag{2-62}$$

This matrix is symmetric positive definite due to H being symmetric and positive definite (sufficient condition in order to obtain a cost minimum). The above definitions imply that Φ is normalized so that its elements add up to one:

$$i_n' \cdot \Phi \cdot i_n = \sum_{i=1}^{n} \sum_{j=1}^{n} \phi_{ij} \tag{2-63}$$

Then we can define the n-element vector ϕ as the row sums of Φ:

$$\phi = \Phi \cdot i_n, \qquad \phi_i = \sum_{j=1}^{n} \phi_{ij} \tag{2-64}$$

Equation 2-64 can be written equivalently as $\phi = \Phi \cdot i_n = \dfrac{F(F-\gamma_1 H)^{-1} F \cdot i_n}{i_n' F(F-\gamma_1 H)^{-1} F \cdot i_n}$, which

implies that $\phi' = -D^{-1} A_{21} A_{11}^{-1}$, $(1 \times n)$. Also, simple algebra shows that the following relationships hold:

$$i_n'\phi = 1, \quad \phi' i_n = 1, \quad i_n' \cdot \Phi = \phi' \text{ and } i_n'(\Phi - \phi\phi') = 0, \quad (\Phi - \phi\phi') i_n = 0 \tag{2-65}$$

At this point there is one important distinction between this model and the LT analysis. In the LT model $\phi = \theta$, where θ is the vector of the marginal shares θ_i defined in Equation 2-38. However, in the present model this relationship does not hold since the proof, provided by LT, is conditional on the production function having the output homogenous of degree one property.

Using these relationships (Equations 2-62 to 2-65) the inverse of matrix A can be written as

$$A^{-1} = \begin{bmatrix} \psi(\Phi - \phi\phi') & \phi \\ \phi' & -\dfrac{1}{\psi} \end{bmatrix}$$

and so Equation 2-61 transforms to

$$\begin{bmatrix} F\dfrac{\partial \ln x}{\partial \ln y'} & F\dfrac{\partial \ln x}{\partial \ln w'} & F\dfrac{\partial \ln x}{\partial \ln z'} \\ -\dfrac{\partial \ln \lambda}{\partial \ln y'} & -\dfrac{\partial \ln \lambda}{\partial \ln w'} & -\dfrac{\partial \ln \lambda}{\partial \ln z'} \end{bmatrix} = \begin{bmatrix} \psi(\Phi - \phi\phi') & \phi \\ \phi' & -\dfrac{1}{\psi} \end{bmatrix} \begin{bmatrix} \gamma_1 F^{-1} H_1 & -I & \gamma_1 F^{-1} H_3 \\ \gamma_1 g' & 0 & \gamma_1 \mu' \end{bmatrix}$$

Solving for the individual terms we obtain the following relationships

$$F\frac{\partial \ln x}{\partial \ln y'} = \gamma_1 \psi (\Phi - \phi\phi') F^{-1} H_1 + \phi \gamma_1 g' \tag{2-66}$$

$$F\frac{\partial \ln x}{\partial \ln w'} = -\psi(\Phi - \phi\phi') \tag{2-67}$$

$$F\frac{\partial \ln x}{\partial \ln z'} = \gamma_1 \psi \left(\Phi - \phi\phi' \right) F^{-1} H_3 + \phi\gamma_1 \mu' \tag{2-68}$$

$$-\frac{\partial \ln \lambda}{\partial \ln y'} = \phi' \gamma_1 F^{-1} H_1 - \frac{1}{\psi} \gamma_1 g' \tag{2-69}$$

$$-\frac{\partial \ln \lambda}{\partial \ln w'} = -\phi' \tag{2-70}$$

$$-\frac{\partial \ln \lambda}{\partial \ln z'} = \phi' \gamma_1 F^{-1} H_3 - \frac{1}{\psi} \gamma_1 \mu' \tag{2-71}$$

Since the optimum variable input-demand equations are given by $x^* = x^*(w, y, z)$ then the differential demand for variable inputs can be found by taking the total differential of this expression (logarithmic):

$$d \ln x = \frac{\partial \ln x}{\partial \ln y'} d \ln y + \frac{\partial \ln x}{\partial \ln w'} d \ln w + \frac{\partial \ln x}{\partial \ln z'} d \ln z$$

Premultiplying now this expression by F and using the solutions above, Equations 2-66 to 2-68, we obtain the system of differential input-demand equations:

$$F d \ln x = \left[\gamma_1 \psi \left(\Phi - \phi\phi' \right) F^{-1} H_1 + \phi\gamma_1 g' \right] d \ln y - \psi \left(\Phi - \phi\phi' \right) d \ln w +$$

$$+ \left[\gamma_1 \psi \left(\Phi - \phi\phi' \right) F^{-1} H_3 + \phi\gamma_1 \mu' \right] d \ln z \tag{2-72}$$

The coefficient of the output needs further transformation in order to have some economic interpretation. For this reason, let $g' = i_m' G$, where G is an $m \times m$ diagonal matrix with $\left(g_1, ..., g_m \right)$ on the diagonal. Then, it is easy to show that

$$\left[\gamma_1 \psi \left(\Phi - \phi\phi' \right) F^{-1} H_1 + \phi\gamma_1 g' \right] = \gamma_1 \left[\psi \left(\Phi - \phi\phi' \right) F^{-1} H_1 G^{-1} + \phi i_m' \right] G \tag{2-73}$$

From Equation 2-37 we have that

$$\theta_i^r = \frac{\partial (w_i x_i) / \partial y_r}{\partial VC / \partial y_r} = \frac{w_i x_i}{\partial VC / \partial y_r} \frac{\partial \ln x_i}{\partial \ln y_r} \frac{1}{y_r} = \frac{VC \cdot f_i}{\partial VC / \partial y_r} \frac{1}{y_r} \frac{\partial \ln x_i}{\partial \ln y_r},$$

which from Equation 2-31 can be rewritten as

$$\theta_i^r = \frac{VC \cdot f_i}{\lambda g_r} \frac{\partial \ln x_i}{\partial \ln y_r} = \frac{f_i}{\gamma_1 g_r} \frac{\partial \ln x_i}{\partial \ln y_r}$$

The last member of this equation is the $(i,r)^{th}$ element of $\gamma_1^{-1}\left(F \frac{\partial \ln x}{\partial \ln y'} \right) G^{-1}$, where

$\gamma_1 = \frac{\lambda}{VC}$.

Thus, from Equation 2-73, $\left[\theta_i^r\right]$ becomes

$$\left[\theta_i^r\right] = \gamma_1^{-1} F \frac{\partial \ln x}{\partial \ln y'} G^{-1} = \gamma_1^{-1}\gamma_1 \left[\psi\left(\Phi - \phi\phi'\right) F^{-1} H_1 G^{-1} + \phi i_m'\right] G G^{-1}$$

or equivalently,

$$\left[\theta_i^r\right] = \left[\psi\left(\Phi - \phi\phi'\right) F^{-1} H_1 G^{-1} + \phi i_m'\right]$$

The last expression can be rearranged to

$$\left[\theta_i^r\right] - \phi \cdot i_m' = \psi\left(\Phi - \phi\phi'\right) F^{-1} H_1 G^{-1} \tag{2-74}$$

Therefore from Equations 2-73 and 2-74 we can write the coefficient of $d \ln y$ as

$$\gamma_1 \left[\left[\theta_i^r\right] - \phi i_m' + \phi i_m'\right] G = \gamma_1 \left[\theta_i^r\right] G \tag{2-75}$$

Following similar analysis for the coefficient of the quasi-fixed input let $\mu' = i_l' M$, where M is an $l \times l$ diagonal matrix with $(\mu_1, ..., \mu_l)$ on the diagonal. Then, as before, it holds that

$$\left[\gamma_1 \psi\left(\Phi - \phi\phi'\right) F^{-1} H_3 + \phi\gamma_1\mu'\right] = \gamma_1\left[\psi\left(\Phi - \phi\phi'\right) F^{-1} H_3 M^{-1} + \phi i_l'\right] M \tag{2-76}$$

In Equation 2-39 it was shown that the marginal share of the quasi-fixed input is given by

$\xi_i^k = \frac{\partial(w_i x_i)/\partial z_k}{\partial VC/\partial z_k} = \frac{w_i x_i}{\partial VC/\partial z_k} \frac{\partial \ln x_i}{\partial \ln z_k} \frac{1}{z_k} = \frac{VC \cdot f_i}{\lambda \mu_k} \frac{\partial \ln x_i}{\partial \ln z_k}$, which further implies that

$\xi_i^k = \frac{f_i}{\gamma_1 \mu_k} \frac{\partial \ln x_i}{\partial \ln z_k}$. This is the $(i,k)^{th}$ element of $\gamma_1^{-1}\left(F \frac{\partial \ln x}{\partial \ln z'}\right)M^{-1}$. Combining then this relationship and Equation 2-76, we obtain a simplified expression for $\left[\xi_i^k\right]$ as

$$\left[\xi_i^k\right] = \gamma_1^{-1}\left(F \frac{\partial \ln x}{\partial \ln z'}\right)M^{-1} = \gamma_1^{-1}\gamma_1\left[\psi\left(\Phi - \phi\phi'\right)F^{-1}H_3M^{-1} + \phi i_l'\right]MM^{-1}.$$

This expression can be further simplified to

$$\left[\xi_i^k\right] = \left[\psi\left(\Phi - \phi\phi'\right)F^{-1}H_3M^{-1} + \phi i_l'\right]$$

Rearranging terms in this expression, we obtain

$$\left[\xi_i^k\right] - \phi \cdot i_l' = \psi\left(\Phi - \phi\phi'\right)F^{-1}H_3M^{-1} \tag{2-77}$$

Therefore the coefficient of $d\ln z$, using Equations 2-76 and 2-77, becomes

$$\gamma_1\left[\psi\left(\Phi - \phi\phi'\right)F^{-1}H_3M^{-1} + \phi i_l'\right]M = \gamma_1\left[\left[\xi_i^k\right] - \phi i_l' + \phi i_l'\right]M = \gamma_1\left[\xi_i^k\right]M \tag{2-78}$$

Finally, using Equations 2-75 and 2-78, the system of variable input-demand equations can be written as $Fd\ln x = \gamma_1\left[\theta_i^r\right]Gd\ln y + \gamma_1\left[\xi_i^k\right]Md\ln z - \psi\left(\Phi - \phi\phi'\right)d\ln w$, with the i^{th} equation given by

$$f_i d\ln x_i = \gamma_1 \sum_{r=1}^{m} \theta_i^r g_r d\ln y_r + \gamma_1 \sum_{k=1}^{l} \xi_i^k \mu_k d\ln z_k - \psi \sum_{j=1}^{n} \left(\phi_{ij} - \phi_i\phi_j\right) d\ln w_j \tag{2-79}$$

2.3.5 Comparative Statics in Demand

The variable factor demand equation (Eq. 2-79) describes the change in the firm's demand for variable inputs due to changes in input prices, output quantities and quasi-fixed input levels. If all input price changes are proportional so that $d\ln w_j$ in Equation 2-79 can be put before the summation sign then the price term vanishes. This is obvious by noting the following relationship $\sum_{j=1}^{n}\left(\phi_{ij} - \phi_i\phi_j\right) = \sum_{j=1}^{n}\left(\phi_{ij}\right) - \phi_i\sum_{j=1}^{n}\left(\phi_j\right) = \phi_i - \phi_i = 0$, since

$\sum_{j=1}^{n}(\phi_j)$=1 from Equation 2-65. Therefore if output and quasi-fixed inputs remain unchanged and all variable input prices change proportionately then the demand for variable inputs remains unchanged. This property just verifies that the variable input demands must be homogeneous of degree zero in input prices. Further, if $(\phi_{ij}-\phi_i\phi_j)$ is less than zero then the firm will increase the use of the i^{th} factor, when absolute price of the j^{th} factor increases, ceteris paribus.

Turning now to volume changes, the total variable input decision of the firm can be obtained by summing the factor demand, Equation 2-79, over i

$$\sum_{i=1}^{n} f_i d\ln x_i = \gamma_1 \sum_{i=1}^{n}\sum_{r=1}^{m} \theta_i^r g_r d\ln y_r + \gamma_1 \sum_{i=1}^{n}\sum_{k=1}^{l} \xi_i^k \mu_k d\ln z_k - \psi \sum_{i=1}^{n}\sum_{j=1}^{n} (\phi_{ij}-\phi_i\phi_j) d\ln \tilde{w}_j$$ [1]

Noting that $\sum_{i=1}^{n}\theta_i^r=1$, $\sum_{i=1}^{n}\xi_i^K=1$, and that $\psi\sum_{i=1}^{n}(\phi_{ij}-\phi_i\phi_j)=0$ from Equation 2-65, (last relationship), then the above equation can be written as

$$\sum_{i=1}^{n} f_i d\ln x_i = \gamma_1 \sum_{r=1}^{m} g_r d\ln y_r + \gamma_1 \sum_{k=1}^{l} \mu_k d\ln z_k \tag{2-80}$$

This is the total variable input decision of the multiproduct firm and is equivalent to the total differential of the production technology of the firm. At the optimum, it has been shown that $g_r = -\frac{\partial T}{\partial \ln y_r}$ and $\mu_k = -\frac{\partial T}{\partial \ln z_k}$. Using these relationships, Equation 2-80 becomes

[1] Here LT analysis uses the relative prices equation instead of the absolute price version of the model, Equation 2-79, (see Laitinen and Theil (1978), pg. 41-45). However, this does not affect our results.

$$\sum_{i=1}^{n}\frac{f_i}{\gamma_1}d\ln x_i=-\sum_{r=1}^{m}\frac{\partial T}{\partial \ln y_r}d\ln y_r-\sum_{k=1}^{l}\frac{\partial T}{\partial \ln z_k}d\ln z_k$$

Using now the definition of γ_1 then $\frac{f_i}{\gamma_1}=\frac{w_i x_i}{VC}\frac{VC}{\lambda}=\frac{\partial T}{\partial \ln x_i}$, where the last term follows from the first-order condition. Therefore, the equivalent form of the above equation is

$$\sum_{i=1}^{n}\frac{\partial T}{\partial \ln x_i}d\ln x_i+\sum_{r=1}^{m}\frac{\partial T}{\partial \ln y_r}d\ln y_r+\sum_{k=1}^{l}\frac{\partial T}{\partial \ln z_k}d\ln z_k=0$$

This is simply the logarithmic total differential of the production technology.

However, the factor demand and the total variable input decision can be written into an equivalent form, which are more useful for the parameterization and estimation. If we proceed by multiplying the first and second term of the right hand side of Equation 2-80 by $\frac{\sum_r g_r}{\sum_s g_s}=1$, $\frac{\sum_k \mu_k}{\sum_e \mu_e}=1$ respectively, then the total variable input decision is transformed to

$$\sum_{i=1}^{n}f_i d\ln x_i=\gamma_1\sum_r g_r\sum_{r=1}^{m}\frac{g_r}{\sum_s g_s}d\ln y_r+\gamma_1\sum_k \mu_k\sum_{k=1}^{l}\frac{\mu_k}{\sum_e \mu_e}d\ln z_k$$

The Divisia volume index of variable inputs, outputs and quasi-fixed inputs have been defined as $d\ln X=\sum_{i=1}^{n}f_i d\ln x_i$, $d\ln Y=\sum_{r=1}^{m}\frac{g_r}{\sum_s g_s}d\ln y_r$ and $d\ln Z=\sum_{k=1}^{l}\frac{\mu_k}{\sum_e \mu_e}d\ln z_k$, respectively. Further, by the definition of γ_1 (Equation 2-15), $\sum_r g_r$ (Equation 2-32) and $\sum_k \mu_k$ (Equation 2-34), we have the following expressions

$$\gamma_2 = \gamma_1 \sum_r g_r = -\frac{\sum_r \dfrac{\partial \ln VC}{\partial \ln y_r}}{\sum_r \dfrac{\partial T}{\partial \ln y_r}} \left(-\sum_r \frac{\partial T}{\partial \ln y_r} \right) = \sum_{r=1}^{m} \frac{\partial \ln VC}{\partial \ln y_r} = \varepsilon_{VC,y} \tag{2-81}$$

$$\gamma_3 = \gamma_1 \sum_k \mu_k = -\frac{\sum_r \dfrac{\partial \ln VC}{\partial \ln y_r}}{\sum_r \dfrac{\partial T}{\partial \ln y_r}} \left(-\frac{\sum_k \dfrac{\partial VC}{\partial \ln z_k}}{\sum_r \dfrac{\partial VC}{\partial \ln y_r}} \right) \left(\sum_r \frac{\partial T}{\partial \ln y_r} \right) = \sum_{k=1}^{l} \frac{\partial \ln VC}{\partial \ln z_k} = \varepsilon_{VC,z} \tag{2-82}$$

Therefore, we can write the total variable input decision of the firm (Equation 2-80) as

$$d \ln X = \gamma_2 d \ln Y + \gamma_3 d \ln Z \tag{2-83}$$

where γ_2, γ_3 are the elasticities of variable cost with respect to proportionate output changes and quasi-fixed input changes, respectively.

Using the same technique as above, for the factor demand equation we obtain an equivalent form of Equation 2-79:

$$f_i d \ln x_i = \gamma_2 \sum_{r=1}^{m} \theta_i^r \frac{g_r}{\sum_s g_s} d \ln y_r + \gamma_3 \sum_{k=1}^{l} \xi_i^k \frac{\mu_k}{\sum_e \mu_e} d \ln z_k - \psi \sum_{j=1}^{n} \left(\phi_{ij} - \phi_i \phi_j \right) d \ln w_j \tag{2-84}$$

This expression is going to be useful for the parameterization of the factor demand.

The variable input allocation decision of the firm (when output changes are not proportionate) can be found by multiplying Equation 2-83 by θ_i, which gives $\theta_i d \ln X - \gamma_2 \theta_i d \ln Y - \gamma_3 \theta_i d \ln Z = 0$, and putting this expression back into Equation 2-79:

$$f_i d \ln x_i = \theta_i d \ln X + \gamma_1 \sum_{r=1}^{m} \theta_i^r g_r d \ln y_r - \gamma_1 \sum_r g_r \sum_{r=1}^{m} \theta_i \frac{g_r}{\sum_s g_s} d \ln y_r +$$
$$+ \gamma_1 \sum_{k=1}^{l} \xi_i^k \mu_k d \ln z_k - \gamma_1 \sum_k \mu_k \sum_{k=1}^{l} \theta_i \frac{\mu_k}{\sum_e \mu_e} d \ln z_k - \psi \sum_{j=1}^{n} \left(\phi_{ij} - \phi_i \phi_j \right) d \ln w_j$$

This expression is simply

$$f_i d\ln x_i = \theta_i d\ln X + \gamma_1 \sum_{r=1}^{m} \left(\theta_i^r - \theta_i\right) g_r d\ln y_r + \gamma_1 \sum_{k=1}^{l} \left(\xi_i^k - \theta_i\right) \mu_k d\ln z_k$$

$$-\psi \sum_{j=1}^{n} \left(\phi_{ij} - \phi_i \phi_j\right) d\ln w_j \tag{2-85}$$

This is the input allocation decision of the firm. This decision describes the change in the demand for the i^{th} input in terms of the Divisia volume index $d\ln X$, change in output, changes in the input prices and changes in the quasi-fixed inputs.

2.4 Conditions for Profit Maximization

Assume now that the firm's objective is to maximize profits (plus quasi-fixed costs) for given input and output prices. That, is the firm wants to

$$\max_{x,y} \left(\Pi(p,w,z) \equiv \sum_r p_r y_r - \sum_i w_i x_i \right) \quad \text{such that} \quad T(x,y,z) = 0 \tag{2-86}$$

Given the assumptions on the production technology (in the beginning of this chapter) the profit function is non-negative and well defined for all positive prices and any level of the quasi-fixed factors. Further, it is continuous, linear homogeneous and convex in all prices, it is continuous, non-decreasing and concave in the quasi-fixed factors and finally it is non-decreasing (non-increasing) in output prices (input prices) for every fixed factor (McKay et al. 1983).

Assuming that we have a first-stage of cost minimization, which gives us the input demands, then in the second stage we can maximize profits as a function only of y. Therefore, the problem that the multiproduct, multifactor firm faces is transformed to

$$\max_{y} \left(\Pi(p,w,z) \equiv \sum_{r=1}^{m} p_r y_r - VC(w,y,z) \right)$$

The first-order conditions of this maximization problem are

$$\frac{\partial \Pi}{\partial y_r} = p_r - \frac{\partial VC}{\partial y_r} = 0 \Rightarrow \frac{\partial VC}{\partial y_r} = p_r \tag{2-87}$$

Using Equation 2-31 for g_r, where $\left(g_r = \frac{y_r}{\lambda} \frac{\partial VC}{\partial y_r} \right)$, then Equation 2-87 becomes

$\lambda g_r = p_r y_r$. Summing this expression over r and using the second term of Equation 2-31 we obtain the following

$$\lambda = -\frac{\sum_r p_r y_r}{\sum_r \frac{\partial T}{\partial \ln y_r}} = -\frac{R}{\sum_r \frac{\partial T}{\partial \ln y_r}} = \frac{R}{\sum_r g_r} \tag{2-88}$$

where $R = \sum_r p_r y_r$ denotes total revenue of the firm. Also, we obtain that the share of the

r^{th} product in total revenue, multiplied by $\sum_r \frac{\partial T}{\partial \ln y_r}$ is

$$g_r = -\frac{p_r y_r}{R} \sum_r \frac{\partial T}{\partial \ln y_r} \tag{2-89}$$

Since $\sum_r g_r = -\sum_r \frac{\partial T}{\partial \ln y_r}$, notice that $\frac{g_r}{\sum_s g_s} = \frac{p_r y_r}{R}$ denotes the revenue share of the r^{th} product of the multiproduct firm. Further, using Equation 2-87, Equation 2-37 can be rewritten as $\theta_i^r = \frac{\partial(w_i x_i)}{\partial(p_r y_r)}$, which is the additional expense on the i^{th} input, incurred for the production of an additional dollar's worth of the r^{th} output.

For the second-order conditions to be valid, it must hold that $\frac{\partial^2 \Pi}{\partial y \partial y'}$ is negative definite, for which it is sufficient that $\frac{\partial^2 VC}{\partial y \partial y'}$ is symmetric positive definite, because

$\frac{\partial^2 R}{\partial y \partial y'} = 0$ follows from the assumption that the price vector is given. Therefore, we will

make the assumption that $\frac{\partial^2 VC}{\partial y \partial y'}$ is symmetric positive definite.

This maximization problem will give us the unconditional output-supply equations of the form $y^* = y^*(p, w, z)$. Taking the logarithmic total differential of the output supply we have

$$d \ln y^* = \left(\frac{\partial \ln y^*}{\partial \ln p} \right) d \ln p + \left(\frac{\partial \ln y^*}{\partial \ln w} \right) d \ln w + \left(\frac{\partial \ln y^*}{\partial \ln z} \right) d \ln z \qquad (2\text{-}90)$$

2.4.1 Output Supply

The output supply of the multiproduct-multifactor firm has the form provided by Equation 2-90. However, we need analytic expressions for the coefficients of $d \ln p$, $d \ln w$ and $d \ln z$ in order to provide an estimable, with economic meaning form. Proceeding the usual way, as in the derivation of the input-demand equation, we write the first-order condition as an identity and then we totally differentiate with respect to its arguments:

$$p_r - \frac{\partial VC\left(y(p, w, z), w, z\right)}{\partial y_r} \equiv 0 \qquad (2\text{-}91)$$

Then taking the total differential of Equation 2-91 with respect to p_s, w_i and z_k we obtain the following relationships

$$p_s: \sum_{v=1}^{m} \frac{\partial^2 VC}{\partial y_r \partial y_v} \frac{\partial y_v}{\partial p_s} = \delta_{rs} \Rightarrow \qquad \sum_{v=1}^{m} \frac{\partial^2 VC}{\partial y_r \partial y_v} y_v \frac{\partial \ln y_v}{\partial \ln p_s} = \delta_{rs} p_s \qquad (2\text{-}92)$$

$$w_i: \frac{\partial^2 VC}{\partial y_r \partial w_i} + \sum_{s=1}^{m} \frac{\partial^2 VC}{\partial y_r \partial y_s} \frac{\partial y_s}{\partial w_i} \equiv 0 \Rightarrow \qquad \frac{\partial y}{\partial w'} \equiv -\left(\frac{\partial^2 VC}{\partial y \partial y'} \right)^{-1} \frac{\partial^2 VC}{\partial y \partial w'} \qquad (2\text{-}93)$$

$$z_k: \quad \frac{\partial^2 VC}{\partial y_r \partial z_k} + \sum_{s=1}^{m} \frac{\partial^2 VC}{\partial y_r \partial y_s} \frac{\partial y_s}{\partial z_k} \equiv 0 \Rightarrow \qquad \frac{\partial y}{\partial z'} \equiv -\left(\frac{\partial^2 VC}{\partial y \partial y'} \right)^{-1} \frac{\partial^2 VC}{\partial y \partial z'} \tag{2-94}$$

However, Equation 2-92 needs further modification before it gets a familiar form. Thus, solving for y_v from Equation 2-89 we get

$$y_v = -\frac{Rg_v}{p_v} \frac{1}{\sum_v \frac{\partial T}{\partial \ln y_v}} \tag{2-95}$$

Substituting this expression back into Equation 2-92 we obtain

$$\sum_{v=1}^{m} \frac{\partial^2 VC}{\partial y_r \partial y_v} \frac{Rg_v}{p_v} \left(-\frac{1}{\sum_v \frac{\partial T}{\partial \ln y_v}} \right) \frac{\partial \ln y_v}{\partial \ln p_s} = \delta_{rs} p_s \quad ,$$

which for all (r, s) pairs in matrix form becomes

$$\left(-\frac{1}{\sum_v \frac{\partial T}{\partial \ln y_v}} \right) R \frac{\partial^2 VC}{\partial y \partial y'} P^{-1} G \frac{\partial \ln y}{\partial \ln p'} = P$$

In this expression, P denotes an $m \times m$ diagonal matrix with the output prices on the diagonal, $G = diag(g_r)$ and p is the vector of output prices. However, from Equation 2-32 we have that $Tr(G) = \sum_r g_r = -\sum_r \frac{\partial T}{\partial \ln y_r}$, where Tr denotes the trace operator.

Therefore, the above equation can be written as

$$\frac{1}{Tr(G)} R \frac{\partial^2 VC}{\partial y \partial y'} P^{-1} G \frac{\partial \ln y}{\partial \ln p'} = P$$

which is simplified to the following expression

$$\frac{G}{Tr(G)}\frac{\partial \ln y}{\partial \ln p'} = P\left(R\frac{\partial^2 VC}{\partial y \partial y'}P^{-1}\right)^{-1}$$

Finally, simplifying the right-hand side of this expression, we get

$$\frac{G}{Tr(G)}\frac{\partial \ln y}{\partial \ln p'} = \frac{1}{R}P\left(\frac{\partial^2 VC}{\partial y \partial y'}\right)^{-1} P = \psi^*\Theta^* \tag{2-96}$$

where we let $\psi^* = \frac{1}{R}p'\left(\frac{\partial^2 VC}{\partial y \partial y'}\right)^{-1} p > 0$ and $\psi^*\Theta^* = \frac{1}{R}P\left(\frac{\partial^2 VC}{\partial y \partial y'}\right)^{-1} P$.

At this point we need to bring Equations 2-93 and 2-94 into the same form as in Equation 2-96. Beginning with Equation 2-93, pre-multiplying by $\frac{1}{R}P$ and post-multiplying by $W\left(=diag(w_i)\right)$, we get

$$\frac{1}{R}P\frac{\partial y}{\partial w'}W \equiv -\frac{1}{R}P\left(\frac{\partial^2 VC}{\partial y \partial y'}\right)^{-1}\frac{\partial^2 VC}{\partial y \partial w'}W \tag{2-97}$$

Solving Equation 2-95 for R (for all (r,s) pairs) and using $Tr(G) = \sum_r g_r = -\sum_r \frac{\partial T}{\partial \ln y_r}$ we obtain the following relationship for the total revenues of the multiproduct firm:

$$R = \frac{PY}{G}Tr(G) \tag{2-98}$$

Substituting Equation 2-98 back into Equation 2-97 we obtain

$$\frac{G}{P \cdot Y \cdot Tr(G)}P\frac{\partial y}{\partial w'}W \equiv -\frac{1}{R}P\left(\frac{\partial^2 VC}{\partial y \partial y'}\right)^{-1}\frac{\partial^2 VC}{\partial y \partial w'}W$$

After canceling terms in the left-hand side of the equation, this can be simplified to

$$\frac{G}{Tr(G)}\frac{1}{Y}\frac{\partial y}{\partial w'}W \equiv -\frac{1}{R}P\left(\frac{\partial^2 VC}{\partial y \partial y'}\right)^{-1}\frac{\partial^2 VC}{\partial y \partial w'}W$$

However, the left-hand side of this expression can be further simplified to get

$$\frac{G}{Tr(G)}\frac{\partial \ln y}{\partial \ln w'} \equiv -\frac{1}{R}P\left(\frac{\partial^2 VC}{\partial y \partial y'}\right)^{-1}\frac{\partial^2 VC}{\partial y \partial w'}W$$

Using then the definition of $\psi^*\Theta^*$, we obtain

$$\frac{G}{Tr(G)}\frac{\partial \ln y}{\partial \ln w'} \equiv -\psi^*\Theta^* K', \quad \text{where } K = W\left(\frac{\partial^2 VC}{\partial w \partial y'}\right)P^{-1} \tag{2-99}$$

Using similar analysis for Equation 2-94, that is pre-multiplying by $\frac{1}{R}P$ and post-multiplying by Z, we obtain $\frac{1}{R}P\frac{\partial y}{\partial z'}Z \equiv -\frac{1}{R}P\left(\frac{\partial^2 VC}{\partial y \partial y'}\right)^{-1}\frac{\partial^2 VC}{\partial y \partial z'}Z$, which from Equation 2-98 becomes

$$\frac{G}{Tr(G)}\frac{\partial \ln y}{\partial \ln z'} \equiv -\psi^*\Theta^*\Omega', \quad \text{where } \Omega = Z\left(\frac{\partial^2 VC}{\partial z \partial y'}\right)P^{-1} \tag{2-100}$$

Therefore, pre-multiplying the differential output supply by $\frac{G}{Tr(G)}$ and using the solutions from Equations 2-96, 2-99 and 2-100 we obtain

$$\frac{G}{Tr(G)}d\ln y = \psi^*\Theta^* d\ln p - \psi^*\Theta^* K' d\ln w - \psi^*\Theta^*\Omega' d\ln z \tag{2-101}$$

where $\Theta^* = \left[\theta^*_{rs}\right]$ is an $m \times m$ symmetric positive definite matrix, which is normalized so that its elements add up to one, $\sum_{r=1}^{m}\sum_{s=1}^{m}\theta^*_{rs} = 1$. However, there is no clear interpretation for the coefficients of $d\ln w$ and $d\ln z$. Starting with the input price coefficient, define K as the $n \times m$ matrix that has the marginal shares θ_i^r (Equation 2-37) as its $(i,r)^{th}$ element. Then, from Shephard's lemma in vector form we have that $\frac{\partial VC}{\partial w} = x$. If we differentiate this relationship with respect to y' we obtain $\frac{\partial^2 VC}{\partial w \partial y'} = \frac{\partial x}{\partial y'}$. However, from Equation 2-73

we have that

$$F\frac{\partial \ln x}{\partial \ln y'} = \gamma_1\left[\psi\left(\Phi - \phi\phi'\right)F^{-1}H_1G^{-1} + \phi i'_m\right]G$$

Substituting for $[\theta_i^r]$, this expression simplifies to

$$F\frac{\partial \ln x}{\partial \ln y'} = \gamma_1\left[\theta_i^r\right]G$$

Using the definitions of the terms in both sides of the equation, this expression can be also written as

$$\frac{WX}{VC}\frac{\partial x}{\partial y'}\frac{Y}{X} = \gamma_1 KG = -\frac{R}{VC}\frac{1}{\sum_r T_{y_r}}K\left(-\frac{PY}{R}\sum_r T_{y_r}\right)$$

where $G = \left(-\frac{PY}{R}\sum_r T_{y_r}\right)$ and $\gamma_1 = -\frac{R}{VC}\frac{1}{\sum_r T_{y_r}}$ are derived from Equations 2-95 and 2-88, respectively, and $\sum_r T_{y_r} = \sum_r \frac{\partial T}{\partial \ln y_r}$. After some algebra the above equation can be transformed to $W\frac{\partial x}{\partial y'} = K\,P$. This expression can be solved for $\frac{\partial x}{\partial y'}$ or K, in order to get $\frac{\partial x}{\partial y'} = \frac{\partial^2 VC}{\partial w \partial y'} = W^{-1}K\,P$ and $K = W\frac{\partial^2 VC}{\partial w \partial y'}P^{-1}$, respectively. Therefore, the matrix of marginal shares θ_i^r can be written as

$$\left[\theta_i^r\right] = K = W\frac{\partial^2 VC}{\partial w \partial y'}P^{-1} \qquad (2\text{-}102)$$

where P is an $m \times m$ matrix with the output prices or marginal costs on the diagonal, depending on which are defined.

Given Equation 2-102, we can write the s^{th} element of $K'd\ln w$ as

$$d\ln W_s^F = \sum_{i=1}^{n} \theta_i^s d\ln w_i \quad (2\text{-}103)$$[2]

This is the Frisch variable input price index (this is denoted by the superscript F). For the coefficient of the quasi-fixed inputs notice that the $(s,k)^{th}$ element of $\Theta^*\Omega' d\ln z$ can be written as

$$\sum_{s=1}^{m} \theta_{rs}^* \sum_{k=1}^{l} \frac{\partial^2 VC}{\partial(p_s y_s)\partial \ln z_k} d\ln z_k = \sum_{k=1}^{l}\sum_{s=1}^{m} \theta_{rs}^* \frac{\partial^2 VC}{\partial(p_s y_s)\partial \ln z_k} d\ln z_k \quad (2\text{-}104)$$

Then we can define

$$\eta_{rk} = \sum_{s=1}^{m} \theta_{rs}^* \frac{\partial^2 VC}{\partial(p_s y_s)\partial \ln z_k} \quad (2\text{-}105)$$

This can be interpreted as the sum of the changes in the marginal costs of the various products due to the changes in the availability of quasi-fixed inputs, where the weights are the coefficients θ_{rs}^*, which define the substitution or complementarity relationship in production (see next section).

Noting that the r^{th} component of $\dfrac{G}{Tr(G)}$ is equal to $\dfrac{g_r}{\sum_s g_s}$ and using Equations 2-103 and 2-105 we can write the r^{th} equation of the output supply, Equation 2-101, as

$$\frac{g_r}{\sum_s g_s} d\ln y_r = \psi^* \sum_{s=1}^{m} \theta_{rs}^* d\ln p_s - \psi^* \sum_{s=1}^{m} \theta_{rs}^* d\ln W_s^F - \psi^* \sum_{k=1}^{l} \eta_{rk} d\ln z_k \quad (2\text{-}106)$$

or in an equivalent form

[2] F denotes that this is a Frisch price index, given that it has a marginal share as a weight instead of a budget share in a Divisia index.

$$\frac{g_r}{\sum_s g_s} d\ln y_r = \psi^* \sum_{s=1}^{m} \theta^*_{rs} d\left(\ln \frac{p_s}{W^F_s}\right) - \psi^* \sum_{k=1}^{l} \eta_{rk} d\ln z_k \tag{2-107}$$

The variable in the left hand side of Equation 2-107 is $\frac{g_r}{\sum_s g_s} d\ln y_r$, which is the contribution of the r^{th} product to the Divisia volume index of outputs. Note also, that $\frac{g_r}{\sum_s g_s} = \frac{p_r y_r}{R}$, which is the revenue share of the r^{th} product.

2.4.2 Comparative Statics in Supply

The supply Equation 2-107 describes the change in the firm's supply of the r^{th} product as a linear combination of all output price changes, each deflated by its own Frisch input price index and all quasi-fixed input changes. For the output-supply system, the following hold:

- If all input prices are unchanged then $d\ln W^F_s = 0$. Then Equation 2-107 becomes $\frac{g_r}{\sum_s g_s} d\ln y_r = \psi^* \sum_{s=1}^{m} \theta^*_{rs} d\ln p_s - \psi^* \sum_{k=1}^{l} \eta_{rk} d\ln z_k$.
- If the prices of all variable inputs and all outputs increase proportionately then $d\ln W^F_s = d\ln p_s$ and thus Eq. 2-107 becomes $\frac{g_r}{\sum_s g_s} d\ln y_r = -\psi^* \sum_{k=1}^{l} \eta_{rk} d\ln z_k$.

To find the total output decision of the firm, define $\theta^*_r = \sum_{s=1}^{m} \theta^*_{rs}$, and note that $\sum_r \theta^*_r = 1$ is implied by the normalization $\sum_{r=1}^{m}\sum_{s=1}^{m} \theta^*_{rs} = 1$. Therefore, the weighted means of the logarithmic price changes that occur in Equation 2-106 are $d\ln P^F = \sum_{r=1}^{m} \theta^*_r d\ln p_r$,

$d\ln W^{FF} = \sum_{r=1}^{m} \theta_r^* d\ln W_r^F$. Correspondingly, let for the coefficient of the quasi-fixed input $\sum_r \eta_{rk} = \eta_k$. Next, we sum Equation 2-106 over r and use the symmetry of Θ^* to obtain

$$d\ln Y = \psi^* d\left(\ln \frac{P^F}{W^{FF}}\right) - \psi^* \sum_{k=1}^{l} \eta_\kappa d\ln z_k \qquad (2\text{-}108)$$

This is the total output decision of the firm, which shows that ψ^* is the price elasticity of total output ($\psi^* > 0$). Next, multiplying Equation 2-108 by θ_r^* and putting the result back into Equation 2-107, we obtain the output allocation decision

$$\frac{g_r}{\sum_s g_s} d\ln y_r = \theta_r^* d\ln Y + \psi^* \sum_{s=1}^{m} \theta_{rs}^* d\left(\ln \frac{p_s}{W_s^F}\right) - \psi^* \theta_r^* d\left(\ln \frac{P^F}{W^{FF}}\right)$$
$$- \psi^* \sum_{k=1}^{l} \eta_{rk} d\ln z_k + \psi^* \theta_r^* \sum_{k=1}^{l} \eta_\kappa d\ln z_k$$

or equivalently,

$$\frac{g_r}{\sum_s g_s} d\ln y_r = \theta_r^* d\ln Y + \psi^* \sum_{s=1}^{m} \theta_{rs}^* d\left(\ln \frac{p_s / W_s^F}{P^F / W^{FF}}\right) + \psi^* \sum_{k=1}^{l} \left(\theta_r^* \eta_\kappa - \eta_{rk}\right) d\ln z_k \qquad (2\text{-}109)$$

The deflator in the price term is $d\ln \frac{P^F}{W^{FF}} = d\ln P^F - d\ln W^{FF}$, which is the same for each input-deflated, output price change in Equation 2-109. If these corrected output price changes are proportionate then the second term in the right hand side of Eq. 2-109 is equal to zero. This shows that in Equation 2-109 only relative input-deflated output price changes have a substitution effect. Therefore, if $\theta_{rs}^* < 0, r \neq s$ then r^{th} and s^{th} products are specific substitutes, while if $\theta_{rs}^* > 0, r \neq s$ then r^{th} and s^{th} products are specific complements. Further, $\theta_{rs}^* < 0, r \neq s$ implies that an increase in the s^{th} relative input-deflated output price leads to a decrease in the production of the r^{th} product. Finally, the Divisia elasticity of the r^{th} output is obtained from Equation 2-109 as

$$D_e = \frac{d \ln y_r}{d \ln Y} = \frac{\theta_r^*}{g_r / \sum_s g_s}$$

If this Divisia elasticity is negative ($D_e < 0$) then the specific output is inferior, since when firm increases total output the particular output decreases.

2.5 Rational Random Behavior in the Differential Model

According to the theory of rational random behavior (Theil 1975), economic decision-makers actively acquire information about uncontrolled variables, such as prices of inputs in the case of cost minimization and prices of outputs in the case of profit maximization, or both prices. However, this information is costly, implying that the decision-makers have incomplete information. To account for this non-optimality, Theil (1975) suggested adding a random term to the decisions of the firm. He further, showed that if the marginal cost of information is small then the decision variables of the firm (input and output levels in our case) follow a multinormal distribution with a mean equal to the full information optimum and a covariance matrix proportional to the inverse of the Hessian matrix of the criterion function.

Chavas and Segerson (1987) criticized Theil's approach to rationalize the stochastic nature of choice models because it relies on a quadratic loss function for the decision-maker. That is the error term is not an integral part of the optimization problem of the decision-maker. They instead provided a method to include it in the cost function of the firm. In this study we will follow the rational random behavior theory since otherwise it would unnecessarily complicate the analysis. Notice though that the covariances of the error terms in both systems are independent of the inclusion of quasi-fixed inputs. That is, under this theory the short-run model has the same covariances as the variable LT model. The proof is almost the same as provided by Laitinen (1980, page 209) and it will not be reproduced here.

Therefore, relying on the theory of rational random behavior, an error term is added to the variable input-demand equation (Eq. 2-84) to get

$$f_i d\ln x_i = \gamma_2 \sum_{r=1}^{m} \theta_i^r g_r' d\ln y_r + \gamma_3 \sum_{k=1}^{l} \xi_i^k \mu_k' d\ln z_k + \sum_{j=1}^{n} \pi_{ij} d\ln w_j + \varepsilon_i \tag{2-110}$$

where $g_r' = \dfrac{g_r}{\sum_s g_s}$, $\mu_k' = \dfrac{\mu_k}{\sum_e \mu_e}$ and $\pi_{ij} = -\psi\left(\phi_{ij} - \phi_i \phi_j\right)$.

Then $\varepsilon_1,...,\varepsilon_n$ have an n-variate normal distribution with zero means and variances-covariances of the form

$$Cov\left(\varepsilon_i, \varepsilon_j\right) = \sigma^2 \psi\left(\phi_{ij} - \phi_i \phi_j\right), \quad i, j = 1,...,n \tag{2-111}$$

These covariances form a singular $n \times n$ matrix, that is the sum of $\varepsilon_1,...,\varepsilon_n$ has zero variance since $\sum_{i=1}^{n} \psi\left(\phi_{ij} - \phi_i \phi_j\right) = 0$, from Equation 2-65. This further, implies that the total input decision of the firm continues to take its non-stochastic form (Equation 2-80), when the theory of rational random behavior is applied to the firm.

In the case of profit maximization, the rational random behavior theory implies that a disturbance ε_r^* must be added to the system of output-supply equations of the firm

$$g_r' d\ln y_r = \sum_{s=1}^{m} \psi^* \theta_{rs}^* d\ln p_s - \sum_{s=1}^{m} \sum_{i=1}^{n} \psi^* \theta_{rs}^* \theta_i^s d\ln w_i - \sum_{k=1}^{l} \psi^* \eta_{rk} d\ln z_k + \varepsilon_r^* \tag{2-112}$$

where the above expression was derived by taking into account Equations 2-106 and 2-103. Further, $\varepsilon_1^*,...,\varepsilon_m^*$ have an m-variate normal distribution with zero means and variances-covariances of the form

$$Cov(\varepsilon_r^*,\varepsilon_s^*)=\frac{\sigma^2\psi^*}{\gamma_2}\theta_{rs}^* \text{ with } r,s=1,...,m \tag{2-113}$$

Notice that the σ^2 is the same coefficient as in Equation 2-111. The vectors $\varepsilon=(\varepsilon_1,...,\varepsilon_n)'$ and $\varepsilon^*=(\varepsilon_1^*,...,\varepsilon_m^*)'$ are independently distributed. This implies that the system consisting of the input-demand equations and that of the output-supply equations, constitute a two-stage block-recursive system (Laitinen 1980). The first stage consists of Equation 2-112, which yields the m output changes and the second consists of Equation 2-110, which yields the n input changes for given changes in output. The independence of the input and output disturbances can be interpreted as meaning that the firm gathers information about the two sets of prices independently.

In the case of output supply, however, summation of $\varepsilon_1^*,...,\varepsilon_m^*$ over r is not equal to zero. This implies that the total output decision of the firm (Equation 2-108) takes a stochastic version, when the theory of rational random behavior is applied to the firm. This is also obvious, below

$$d\ln Y=d\ln\left(\frac{P^F}{W^{FF}}\right)-\psi^*\sum_{k=1}^{l}\eta_{rk}d\ln z_k+E^* \tag{2-114}$$

where $E^*=\sum_r\varepsilon_r^*$ and from $\sum_r\sum_s\theta_{rs}^*=1$, it follows that $Var(E^*)=\dfrac{\sigma^2\psi^*}{\gamma_2}$.

2.6 Comparison to the Original LT Model

In this section a brief comparison of the original LT model with the extended model (ELT) developed in the previous sections is provided. Laitinen and Theil (1978) assumed that the production function is negatively linear homogenous in the output vector, which implies that

$$\sum_r \frac{\partial T}{\partial \ln y_r} \equiv -1 \tag{2-115}$$

This relationship is not crucial for the derivation of the input-demand and output-supply equations, but for the definition of the coefficients in those equations. Taking into account the expression (Equation 2-9) for the returns to scale it is obvious that Equation 2-115 imposes a restriction to this measure, namely that the denominator is equal to negative unity, while in the ELT model no such assumption is imposed. As mentioned before, the main difference between the LT and ELT models relies on the coefficients g_r, π_{ij} and μ_k. Specifically, in the LT model g_r is the share of the r^{th} product in total variable cost, but in the ELT model this is true for $g_r' = g_r / \sum_s g_s$ [3]. In the case where quasi-fixed inputs are introduced to the model then similar results hold for the definition of μ_k. Concerning the price coefficients π_{ij}, in the LT model these coefficients were decomposed to $\pi_{ij} = -\psi(\theta_{ij} - \theta_i\theta_j)$, where θ_i is the marginal share defined in Equation 2-38. This relationship is entailed from assumption 2-115 and that the second derivatives of Equation 2-115 with respect to output and variable inputs are equal to zero (Laitinen 1980, page 180). In contrast, this relationship does not hold for the ELT model were no

[3] This is obvious from Equation 2-31. See also discussion below this equation.

such assumption is made and $\pi_{ij} = -\psi(\phi_{ij} - \phi_i\phi_j)$. However, as was shown in Equations 2-63 to 2-65 and the discussion below these equations, the same properties hold for both decompositions, as far as it concerns summation of these coefficients across input-demand equations or over all inputs in the same input-demand equation. The systems of equations for both models are represented below

LT Model

$$\text{ID: } f_i d\ln x_i = \gamma_1 \sum_{r=1}^{m} \theta_i^r g_r d\ln y_r + \sum_{j=1}^{n} \pi_{ij} d\ln w_j + \varepsilon_i$$

$$\text{OS: } g_r d\ln y_r = \sum_{s=1}^{m} \psi^* \theta_{rs}^* d\ln p_s - \sum_{s=1}^{m}\sum_{i=1}^{n} \psi^* \theta_{rs}^* \theta_i^s d\ln w_i + \varepsilon_r^*$$

ELT Model

$$\text{ID: } f_i d\ln x_i = \gamma_2 \sum_{r=1}^{m} \theta_i^r g_r' d\ln y_r + \gamma_3 \sum_{k=1}^{l} \xi_i^k \mu_k' d\ln z_k + \sum_{j=1}^{n} \pi_{ij} d\ln w_j + \varepsilon_i$$

$$\text{OS: } g_r' d\ln y_r = \sum_{s=1}^{m} \psi^* \theta_{rs}^* d\ln p_s - \sum_{s=1}^{m}\sum_{i=1}^{n} \psi^* \theta_{rs}^* \theta_i^s d\ln w_i - \sum_{k=1}^{l} \psi^* \eta_{rk} d\ln z_k + \varepsilon_r^*$$

Notice that in the ELT model, there are more terms in both input-demand and output-supply systems of equations, corresponding to the quasi-fixed inputs (z_k). This is one of the generalizations pursued in this study. Further, as it was shown above, there is no need to make the assumption 2-115 in order to obtain the two systems. For instance, γ_1 in the LT model is equivalent to γ_2 in the ELT model where both coefficients are defined as the revenue-variable cost ratio or as the elasticity of variable cost with respect to outputs of the firm. This assumption serves into easier derivation of the equations but it imposes a restriction in the returns to scale.

CHAPTER 3
PARAMETERIZATION AND ALTERNATIVE SPECIFICATION

3.1 Input Demand Parameterization

In order to estimate the variable input-demand and output-supply systems of the multiproduct firm, there is a need to parameterize them since both depend on the infinitesimal changes in the natural logarithms of prices and quantities. Laitinen (1980) provided a parameterization for the LT model, which is extended in the section to account for quasi-fixed inputs and the non-output-homogeneous production technology. Thus, a finite change version of the differential $d\ln q$ is defined as $Dq_t = \ln q_t - \ln q_{t-1}$, where q refers to all prices and quantities relevant to the firm and q_t is the value at time t. Further, an error term is appended to each variable input-demand equation as depicted in Equation 2-84, relying on the theory of rational random behavior (Theil 1975):

$$f_i d\ln x_i = \gamma_2 \sum_{r=1}^{m} \theta_i^r g_r' d\ln y_r + \gamma_3 \sum_{k=1}^{l} \xi_i^k \mu_k' d\ln z_k + \sum_{j=1}^{n} \pi_{ij} d\ln w_j + \varepsilon_i \tag{3-1}$$

where the following relationships were defined or proved in the previous chapter:

- Revenue share of the firm, $g_r' = \dfrac{g_r}{\sum_s g_s} = \dfrac{p_r y_r}{R}$ from Equation 2-89;
- Cost share of the firm, $f_i = \dfrac{w_i x_i}{VC}$;
- Share of the k^{th} quasi-fixed input shadow value in total shadow value of the quasi-fixed inputs, $\mu_k' = \dfrac{\mu_k}{\sum_e \mu_e} = \dfrac{\partial VC / \partial \ln z_k}{\sum_e \partial VC / \partial \ln z_e}$ from Equation 2-34;

- Negative semidefinite price terms of rank $n-1$, known as Slutsky coefficients in the Rotterdam model, $\pi_{ij} = -\psi\left(\phi_{ij} - \phi_i\phi_j\right)$;

- Revenue-Variable Cost ratio or elasticity of variable cost with respect to outputs, $\gamma_2 = \gamma_1 \sum_r g_r = \sum_r \frac{\partial \ln VC}{\partial \ln y_r} = \frac{R}{VC}$ from Equations 2-81 and 2-87;

- Elasticity of variable cost with respect to the quasi-fixed inputs, defined as $\gamma_3 = \gamma_1 \sum_k \mu_k = \sum_k \frac{\partial \ln VC}{\partial \ln z_k} = \varepsilon_{VC,z}$ from Equation 2-82;

- Share of i^{th} variable input in the shadow price of quasi-fixed input z_k, defined as $\xi_i^k = \frac{\partial\left(w_i x_i\right)/\partial z_k}{\partial VC/\partial z_k}$;

- Share of i^{th} variable input in the marginal cost of the r^{th} product, defined as $\theta_i^r = \frac{\partial\left(w_i x_i\right)/\partial y_r}{\partial VC/\partial y_r}$;

- $\gamma_1 = \frac{\lambda}{VC}$; $\sum_i \theta_i^r = 1$; $\sum_i \xi_i^K = 1$; $\sum_r g_r' = 1$; $\sum_k \mu_k' = 1$; $\sum_i \pi_{ij} = 0$;.

- Covariance of the error terms, $Cov\left(\varepsilon_i, \varepsilon_j\right) = \sigma^2\psi\left(\phi_{ij} - \phi_i\phi_j\right)$.

There are two existing problems with the estimation of the demand system. First, γ_3, ξ_i^k, μ_k, $\sum_k \mu_k$ are not observable since they involve derivatives of the variable cost function with respect to the quasi-fixed inputs. They would be observable if quasi-fixed inputs were at their full equilibrium levels, since at that point $-\frac{\partial VC}{\partial z_k} = v_k$ with v_k being the ex-ante market rental price of the quasi-fixed input. This in turn, would transform the model to a long-run with no quasi-fixed factors. A solution to this problem is to leave γ_3, ξ_i^k, μ_k, $\sum_k \mu_k$ as unknowns and estimate one coefficient $b_{ik} = \gamma_3 \xi_i^k \mu_k'$. However, as is usual with demand systems, the estimation method requires dropping one equation from the system due to singularity of the disturbances, as was shown in Section 2.5. Proceeding this way, though, the coefficient of the quasi-fixed input in the dropped equation cannot be recovered, since $\sum_i b_{ik}$ is still an unknown constant. Therefore, a complete demand system estimation method must be employed. An alternative is to transform the coefficients of the quasi-fixed inputs in order to add up in a known constant. Both methods will be discussed in the next chapter, at the choice of the econometric procedure, Section 4.1.

So far there is no distinction between the cases of one and multiple quasi-fixed inputs. As it is going to be shown in the next section, the one quasi-fixed input is a special case of the multiple quasi-fixed inputs case and the estimation method does not differ. Berndt and Fuss (1989), in their measures of capacity utilization showed that in the case of multiple inputs and multiple outputs the long-run economic capacity outputs cannot be uniquely determined unless additional demand information is incorporated in the model, such as the equality of marginal revenue with the long-run marginal cost of the firm. An alternative method though, is to consider perfect competition and specify a variable profit function as in the case examined by the present study.

3.1.1 The Case of Multiple Quasi-Fixed Inputs

Summing Equation 3-1 over i and using the definitions of the Divisia indexes as presented in Chapter 2, we obtain the total input decision of the firm:

$$d\ln X = \gamma_2 d\ln Y + \gamma_3 d\ln Z \qquad (3\text{-}2)$$

In Equation 3-1 the factor and product shares $f_i = \dfrac{w_i x_i}{VC}$ and $g'_r = \dfrac{p_r y_r}{R}$ are observable and can be calculated for any period from price and quantity data. As in the Rotterdam model or Laitinen (1980), arithmetic means are employed for these shares, since they are used to weight logarithmic changes between two periods. Therefore, by using a subscript t to denote time, the factor and product shares at period t are given by $f_{it} = \frac{w_{it}x_{it}}{VC_t}$ and $g'_{rt} = \frac{p_{rt}y_{rt}}{R_t}$, while the average factor share of the i^{th} input in t and $t-1$, and the average revenue share of the r^{th} product in $t-1$ and t are given respectively by

$$\bar{f}_{it} = \frac{1}{2}\left(f_{it} + f_{i,t-1}\right); \qquad \bar{g}'_{rt} = \frac{1}{2}\left(g'_{rt} + g'_{r,t-1}\right) \tag{3-3}$$

Further, define $Dx_t = \ln x_t - \ln x_{t-1}$, $Dy_t = \ln y_t - \ln y_{t-1}$, $Dz_t = \ln z_t - \ln z_{t-1}$ as the finite-changes version of the variables in the model, which imply that the finite-change version of the Divisia indexes can be written as $DX_t = \sum_{i=1}^{n} \bar{f}_{it} Dx_{it}$, $DY_t = \sum_{r=1}^{m} \bar{g}'_{rt} Dy_{rt}$ and $DZ_t = \sum_{k=1}^{l} \bar{\mu}'_{kt} Dz_{kt}$, respectively. According to the theory of rational random behavior the total input decision (Equation 3-2) holds without disturbance. Since γ_3 is not observable, Equation 3-2 cannot be solved for γ_2, and thus employing $\gamma_2 = \frac{R}{VC}$ from Equation 2-87 we define its geometric mean as

$$\bar{\gamma}_{2t} = \sqrt{\frac{R_t \cdot R_{t-1}}{VC_t \cdot VC_{t-1}}} \tag{3-4}$$

Then, the total input decision in its finite change version can be written as

$$DX_t = \bar{\gamma}_{2t} DY_t + \bar{\gamma}_{3t} DZ_t \tag{3-5}$$

To solve the problem of identification of $\bar{\gamma}_{3t}$, one could proceed in two ways. First, Equation 3-5 could be solved for $\bar{\gamma}_{3t} = (DX_t - \bar{\gamma}_{2t} DY_t) / DZ_t$. However, the possibility of

DZ_t being zero and that it requires specification of the unobservable term $\bar{\mu}'_{kt}$, this solution becomes unattractive. Instead, an approximation for $\bar{\gamma}_{3t}$ seems to be more plausible. Remembering that at the full equilibrium level of the quasi-fixed input $-\frac{\partial VC}{\partial z_k} = v_k$, then at any point different than this optimum, it must hold that $-\frac{\partial VC}{\partial z_k} = v_k + \delta_k$, where δ_k denotes the deviation between the ex-ante market rental price v_k and the shadow price of the quasi-fixed input (Morrison-Paul and MacDonald 2000). It follows from this definition that if $\delta_k = 0$, then the quasi-fixed input is at its full equilibrium level, while if $\delta_k > or < 0$ then we have undercapacity or overcapacity utilization of the specific quasi-fixed input, respectively. Therefore, we could use the following approximation

$$\gamma_{3t} = \sum_k \frac{\partial VC}{\partial z_k} \frac{z_k}{VC}\bigg|_t = -\sum_k \frac{v_{kt} z_{kt}}{VC_t} - \delta_{kt} \frac{z_{kt}}{VC_t} = -\sum_k \frac{v_{kt} z_{kt}}{VC_t} + \varepsilon_\gamma \tag{3-6}$$

Then taking the geometric mean of γ_{3t} and accounting for the error of the approximation ε_γ, we have that

$$\bar{\gamma}_{3t} = \sqrt{\frac{\left(\sum_k v_{kt} z_{kt}\right)\left(\sum_k v_{k,t-1} z_{k,t-1}\right)}{VC_t VC_{t-1}}} + \varepsilon_\gamma^* \tag{3-7}$$

Further, to solve the problem of identification of μ'_{kt}, we follow the same technique as in γ_{3t} and define its approximation as

$$\mu'_{kt} = \frac{\partial VC / \partial z_{kt}}{\sum_e \left(\partial VC / \partial z_{et}\right) z_{et}} = \frac{-v_{kt} z_{kt}}{-\sum_e v_{et} z_{et}} + \varepsilon_\mu \qquad k, e = 1, \ldots, l \tag{3-8}$$

while we use its arithmetic mean in our parameterization using the same argument as in the case of f_{it} and g'_{rt}:

$$\bar{\mu}'_{kt} = \frac{1}{2}(\mu'_{kt} + \mu'_{k,t-1}) + \varepsilon^*_{\mu} \qquad (3\text{-}9)$$

A problem with the finite change version $DX_t = \bar{\gamma}_{2t} DY_t + \bar{\gamma}_{3t} DZ_t$ is that it will usually be violated by the definitions of DX_t, DY_t, DZ_t $\bar{\gamma}_{2t}$ and $\bar{\gamma}_{3t}$ in the previous page. One possible explanation, as noted by Laitinen and Theil (1978), is technical change since Equation 3-2 is the total differential of the production function and Equation 3-5 entails changes from period $t-1$ to t. This could be a generalization of Hicks neutral technical change. However, in this model there is one more explanation, which is the approximation of μ'_{kt} and γ_{3t} by the use of market rental price for the quasi-fixed input since shadow price is unknown. To account for these possibilities and the errors induced by the approximation of μ'_{kt} and γ_{3t} ($\varepsilon^*_{\mu}, \varepsilon^*_{\gamma}$ respectively), we need to add a residual in the finite-change version of the total variable input decision:

$$DX_t = \bar{\gamma}_{2t} DY_t + \bar{\gamma}_{3t} DZ_t + E_t \qquad (3\text{-}10)$$

where $\bar{\gamma}_{3t} = \sqrt{\dfrac{\left(\sum_k v_{kt} z_{kt}\right)\left(\sum_k v_{k,t-1} z_{k,t-1}\right)}{VC_t VC_{t-1}}}$; $\bar{\mu}'_{kt} = \dfrac{1}{2}(\mu'_{kt} + \mu'_{k,t-1})$ and E_t contains $\varepsilon^*_{\mu}, \varepsilon^*_{\gamma}$.

From this equation the residual E_t can be calculated as

$$E_t = DX_t - \bar{\gamma}_{2t} DY_t - \bar{\gamma}_{3t} DZ_t \qquad (3\text{-}11)$$

The input changes are then corrected by computing

$$\tilde{x}_{it} = \bar{f}_{it}(Dx_{it} - E_t) \qquad (3\text{-}12)$$

This correction amounts to enforcing the finite-change version of the total input decision, since summation of the correct input over i yields

$$\sum_i \tilde{x}_{it} = DX_t - E_t = \overline{\gamma}_{2t} DY_t + \overline{\gamma}_{3t} DZ_t \tag{3-13}$$

Taking into account Equation 3-12 for the residual correction and the parameterizations of the quantities and prices, the finite-change version of the variable input-demand (Equation 3-1) can be written as

$$\tilde{x}_{it} = \sum_{r=1}^{m} \theta_i^r \tilde{y}_{rt} + \sum_{k=1}^{l} \xi_i^k \tilde{z}_{kt} + \sum_{j=1}^{n} \pi_{ij} Dw_{jt} + \varepsilon_{it} \tag{3-14}$$

In this formulation we have defined the terms $\tilde{y}_{rt} = \overline{\gamma}_{2t} \overline{g}'_{rt} Dy_{rt}$, $\tilde{z}_{kt} = \overline{\gamma}_{3t} \overline{\mu}'_{kt} Dz_{kt}$ and $\pi_{ij} = -\psi\left(\phi_{ij} - \phi_i \phi_j\right)$ as before. Further, it is assumed that θ_i^r, ξ_i^k, π_{ij} and σ^2 are constant over time so that $Cov\left(\varepsilon_i, \varepsilon_j\right) = \sigma^2 \psi\left(\phi_{ij} - \phi_i \phi_j\right)$ implies that the contemporaneous covariance matrix of demand disturbances (covariance that concerns disturbances of different equations but of the same year) is the same in each period. The effect of the correction in variable input levels, as appears in Equation 3-12, is to make Equation 3-13, which can also be written as $\sum_i \tilde{x}_{it} = \sum_r \tilde{y}_{rt} + \sum_k \tilde{z}_{kt}$, hold. This, in turn, gives that summation of Equation 3-14 over all inputs i will yield $\sum_i \theta_i^r = 1$, $\sum_i \xi_i^K = 1$, $\sum_i \pi_{ij} = 0$ and $\sum_i \varepsilon_{it} = 0$. Therefore, the variable input demand (Equation 3-14) satisfies the following properties:

- Adding up: $\sum_i \theta_i^r = 1$, $\sum_i \xi_i^K = 1$ and $\sum_i \pi_{ij} = 0$, where $i, j = 1, \ldots, n$ and $k = 1, \ldots, l$.
- Homogeneity: $\sum_j \pi_{ij} = 0$.

- Symmetry: $\pi_{ij} = \pi_{ji}$.
- Negative semi-definite matrix of the price parameter (π_{ij}) of rank $n-1$, implying that the underlying cost function is concave in input prices.

3.1.2 The Case of One Quasi-Fixed Input

As it is going to be shown below this is a special case of the multiple quasi-fixed inputs case. Notice, that when the firm employs only one quasi-fixed input then by definition $\mu'_k = 1$, and so the variable input-demand equation (Eq. 3-1) becomes

$$f_i d\ln x_i = \gamma_2 \sum_{r=1}^{m} \theta_i^r g'_r d\ln y_r + \gamma_3 \xi_i^k d\ln z_k + \sum_{j=1}^{n} \pi_{ij} d\ln w_j + \varepsilon_i \tag{3-15}$$

In this case, the Divisia index of the quasi-fixed input degenerates to

$d\ln Z = \sum_k \mu'_k d\ln z_k = d\ln z_k$ and the elasticity of the variable cost with respect to quasi-fixed input becomes $\gamma_3 = \partial \ln VC / \partial \ln z_k$, since $k = 1$. Disregarding for a moment the error term and summing Equation 3-15 over all i, we obtain the total input decision

$$d\ln X = \gamma_2 d\ln Y + \gamma_3 d\ln z_k, \quad k = 1 \tag{3-16}$$

Proceeding then, as in the case of multiple quasi-fixed inputs, the following variable input-demand equation is obtained:

$$\tilde{x}_{it} = \sum_{r=1}^{m} \theta_i^r \tilde{y}_{rt} + \xi_i^k \tilde{z}_{kt} + \sum_{j=1}^{n} \pi_{ij} Dw_{jt} + \varepsilon_{it} \tag{3-17}$$

This differential variable input demand satisfies the same properties as Equation 3-14. For instance, the assumption $\sum_i \xi_i^k$ still holds. The only difference with the case of multiple quasi-fixed inputs is that the residual term (E_t) used to correct the variable input does not contain anymore error due to approximation of $\bar{\mu}'_{kt}$, since $\bar{\mu}'_{kt} = 1$. Further, if the changes in the level of the quasi-fixed input are not zero then there is no need to use the approximation for γ_3, since it could be obtained from $\bar{\gamma}_3 = (DX_t - \bar{\gamma}_2 DY_t)/DZ_t$.

3.2 Output Supply Parameterization

As in the input demand case, we rely on the theory of rational random behavior to append an error term in the supply equation of the firm (Eq. 2-106) in order to obtain

$$g_r' d\ln y_r = \sum_{s=1}^{m} \psi^* \theta_{rs}^* \left(d\ln p_s - \sum_{i=1}^{n} \theta_i^s d\ln w_i \right) - \sum_{k=1}^{l} \psi^* \eta_{rk} d\ln z_k + \varepsilon_r^* \qquad (3\text{-}18)$$

where the following definitions were provided in the previous chapter:

- Price elasticity of total output, ψ^* with $\psi^* > 0$.
- Substitution or complementarity relationship in production denoted by θ_{rs}^*.
- The sum of the changes in the marginal costs of the various products due to the changes in the availability of quasi-fixed inputs, weighted by the coefficients θ_{rs}^*, as $\eta_{rk} = \sum_{s=1}^{m} \theta_{rs}^* \frac{\partial^2 VC}{\partial(p_s y_s)\partial \ln z_k}$
- Normalization condition, $\sum_{r=1}^{m}\sum_{s=1}^{m} \theta_{rs}^* = 1$
- Covariance of the error terms, $Cov(\varepsilon_r^*, \varepsilon_s^*) = \frac{\sigma^2 \psi^*}{\gamma_2} \theta_{rs}^*$.

Similarly, a finite-change version of the output-supply system (multiplied by $\bar{\gamma}_{2t}$ in order to make it homoscedastic) is

$$\bar{\gamma}_{2t} g_{rt}' Dy_{rt} = \sum_{s=1}^{m} \bar{\gamma}_{2t} \psi^* \theta_{rs}^* \left(Dp_{st} - \sum_{i=1}^{n} \theta_i^s Dw_{it} \right) - \sum_{k=1}^{l} \bar{\gamma}_{2t} \psi^* \eta_{rk} Dz_{kt} + \varepsilon_{rt}^{**} \qquad (3\text{-}19)$$

If it had been assumed that the coefficients $\psi^* \theta_{rs}^*$ were constants then an autoregressive scheme (AR) would be present in the supply system, since this assumption would imply that the variance-covariance matrix of the disturbances depends on γ_{2t}, which varies over time. Multiplying though, each equation in the system by $\bar{\gamma}_{2t}$ the disturbances become homoscedastic and now it is assumed that the coefficients $\alpha_{rs} = \bar{\gamma}_{2t}\psi^{*}\theta^{*}_{rs}$ and $\beta_{rk} = \bar{\gamma}_{2t}\psi^{*}\eta_{rk}$ are constant. The covariance of the disturbances is then given by $Cov(\varepsilon^{*}_{rt}, \varepsilon^{*}_{st}) = \sigma^{2}\bar{\gamma}_{2t}\psi * \theta^{*}_{rs} = \sigma^{2}\alpha_{rs}$, which is constant. The supply system can be written then in a more compact form, as

$$\tilde{y}_{rt} = \sum_{s=1}^{m} \alpha_{rs} Dp_{st} - \sum_{i=1}^{n}\sum_{s=1}^{m} \alpha_{rs}\theta^{s}_{i} Dw_{it} - \sum_{k=1}^{l} \beta_{rk} Dz_{kt} + \varepsilon^{**}_{rt} \tag{3-20}$$

The properties of the output-supply system are:

- Output supply is homogeneous of degree zero in both input and output prices.
- The coefficient matrix of the output prices, $[a_{rs}]$, must be negative definite of rank m, implying that the profit function is convex in output prices.
- Symmetry condition: $[a_{rs}] = [a_{sr}]$.
- Nonlinear symmetry condition: If linear symmetry conditions are imposed in both systems then the nonlinear coefficients of the input prices are not free parameters.

3.3 Alternative Specification for the Cost-Based System

The variable input-demand system as represented by Equation 3-1 assumes constant price effects, output and quasi-fixed effects. However, there is no reason to ex-ante impose such restrictions on the system. Fousekis and Pantzios (1999) provided a generalization of Theil's (1977) parameterization for the one product firm, based on different parameterizations for the Rotterdam model. In this section their results are extended to the multiproduct, multifactor firm.

To allow for variable output effects, θ^{r}_{i}, let us define

$$f_{i} = a_{i} + m^{r}_{i} \ln X \tag{3-21}$$

where f_i is the cost share and $\ln X$ is the variable inputs Divisia index. Note that, since $\sum_i f_i = 1$, it must hold that $\sum_i a_i = 1$ and that $\sum_i m_i^r = 0$. Multiplying then Equation 3-21 by variable cost (VC) and differentiating with respect to y_r, we get

$$\frac{\partial(w_i x_i)}{\partial y_r} = a_i \frac{\partial VC}{\partial y_r} + m_i^r \ln X \frac{\partial VC}{\partial y_r} + m_i^r \frac{VC}{y_r}\frac{\partial \ln X}{\partial \ln y_r}$$

Noting from Equation 2-10 that $\frac{VC}{y_r}\frac{\partial \ln X}{\partial \ln y_r} = \frac{\partial VC}{\partial y_r}$, then the above equation is transformed to

$$\frac{\partial(w_i x_i)}{\partial y_r} / \frac{\partial VC}{\partial y_r} = a_i + m_i^r \ln X + m_i^r$$

Making use now of Equation 3-21 and the definition of θ_i^r (see below Equation 3-1) we have that

$$\theta_i^r = \frac{\partial(w_i x_i)}{\partial y_r} / \frac{\partial VC}{\partial y_r} = f_i + m_i^r \tag{3-22}$$

Therefore, the i^{th} input demand with variable output effects becomes

$$f_i d \ln x_i = \gamma_2 \sum_{r=1}^{m} (f_i + m_i^r) g_r' d \ln y_r + \gamma_3 \sum_{k=1}^{l} \xi_i^k \mu_k' d \ln z_k + \sum_{j=1}^{n} \pi_{ij} d \ln w_j + \varepsilon_i \tag{3-23}$$

To allow for variable effects in all coefficients, let us define now

$$f_i = a_i + m_i^r \ln X + \sum_{j=1}^{n} s_{ij} \ln w_j \tag{3-24}$$

Since $\sum_i f_i = 1$, it must hold that $\sum_i a_i = 1$, $\sum_i m_i^r = 0$ and also that $\sum_i s_{ij} = 0$, $\sum_j s_{ij} = 0$, $s_{ij} = s_{ji}$, where $i, j = 1, \ldots, n$.

Totally differentiating Equation 3-24 we have

$$df_i = m_i^r d\ln X + \sum_j s_{ij} d\ln w_j \tag{3-25}$$

From Equation 2-28 it holds that the total differential of the variable cost ratio is equal to $df_i = f_i d\ln w_i + f_i d\ln x_i - f_i d\ln VC$. Also summing this expression over all inputs i, it holds that $d\ln VC = \sum_i f_i d\ln w_i + d\ln X$. Combining these two expressions we obtain

$$df_i = f_i d\ln w_i + f_i d\ln x_i - f_i \sum_i f_i d\ln w_i - f_i d\ln X \tag{3-26}$$

Equating now Expressions 3-25 and 3-26; and after some algebra we get

$$f_i d\ln x_i = (m_i^r + f_i) d\ln X + \sum_{j=1}^{n} \left(s_{ij} - f_i \left(\delta_{ij} - f_j \right) d\ln w_j \right) \tag{3-27}$$

where δ_{ij} is the Kronecker delta. To verify that the input price terms in Equation 3-27 satisfy the adding-up property we sum Equation 3-27 over all inputs i, to obtain that $\sum_{i=1}^{n}\sum_{j=1}^{n} \left(s_{ij} - f_i \left(\delta_{ij} - f_j \right) d\ln w_j \right) = 0$, which verifies that the adding-up property holds for the input price terms.

Equation 3-27 is a system of input demands that must be equal with the input-demand system presented in Equation 3-1. Forcing this equality we have

$$(m_i^r + f_i) d\ln X + \sum_{j=1}^{n} \left(s_{ij} - f_i \left(\delta_{ij} - f_j \right) d\ln w_j \right) = \gamma_2 \sum_{r=1}^{m} \theta_i^r g_r' d\ln y_r + \gamma_3 \sum_{k=1}^{l} \xi_i^k \mu_k' d\ln z_k$$
$$+ \sum_{j=1}^{n} \pi_{ij} d\ln w_j + \varepsilon_i$$

Summing this expression on both sides over i and using the previous results, we verify the total input decision of the multiproduct firm:

$$d\ln X = \gamma_2 d\ln Y + \gamma_3 d\ln Z \tag{3-28}$$

Substituting now Equation 3-28 back into Equation 3-27 we obtain

$$f_i d\ln x_i = \gamma_2 \sum_{r=1}^{m} (m_i^r + f_i) g_r' d\ln y_r + \gamma_3 \sum_{k=1}^{l} (m_i^r + f_i) \mu_k' d\ln z_k + \sum_{j=1}^{n} \left(s_{ij} - f_i \left(\delta_{ij} - f_j \right) d\ln w_j \right)$$

By rearranging terms, we get an allocation-type differential system of input demands:

$$f_i d\ln x_i = \gamma_2 \sum_{r=1}^{m} (m_i^r + f_i) g_r' d\ln y_r + \gamma_3 \left(m_i^r d\ln Z + \sum_{k=1}^{l} f_i \mu_k' d\ln z_k \right) +$$

$$+ \sum_{j=1}^{n} \left(s_{ij} - f_i \left(\delta_{ij} - f_j \right) d\ln w_j \right) \tag{3-29}$$

Letting now $m_i^r = \theta_i^r - f_i$, we get

$$f_i d\ln x_i = \gamma_2 \sum_{r=1}^{m} \theta_i^r g_r' d\ln y_r + \gamma_3 \theta_i^r d\ln Z + \sum_{j=1}^{n} \left(s_{ij} - f_i \left(\delta_{ij} - f_j \right) d\ln w_j \right) \tag{3-30}$$

Then we could combine Equations 3-23 and 3-29 into one general equation, since the left-hand side variables are the same but the right-hand side variables differ. This implies that the models are not nested. Therefore,

$$f_i d\ln x_i = \gamma_2 \sum_{r=1}^{m} (m_i^r + e_1 f_i) g_r' d\ln y_r + \gamma_3 \sum_{k=1}^{l} (m_i^r + e_1 f_i) \mu_k' d\ln z_k +$$

$$+ \sum_{j=1}^{n} \left(\pi_{ij} - e_2 f_i \left(\delta_{ij} - f_j \right) d\ln w_j \right)$$

where e_1, e_2 are two additional parameters to be estimated and the additional restriction $\sum_i (m_i^r + u_i^k) = 1 - e_1$ is imposed in the estimation.

Using a likelihood ratio test one could test which of the following restrictions are valid and so, which differential input-demand system fits the data better:

1. If $e_1 = e_2 = 0$ then we get our original differential system.
2. If $e_1 = e_2 = 1$ then we have all coefficients variable, Equation 3-29.

3. If $e_1 = 1$, $e_2 = 0$ then we have only variable output effects, Equation 3-23.

4. If $e_1 = 0$, $e_2 = 1$ then we have only input price effects being variable, Equation 3-30.

Note that the presence of $d \ln Z$ in Equation 3-29 may create problems of multicollinearity, so an instrumental variable approach is suggested for the estimation of the system. Also Equation 3-31 seems more plausible than Equation 3-29 since it alleviates the problem of multicollinearity.

3.4 Capacity Utilization and Quasi-Fixity

The most appealing alternative parameterization of the differential model is given by Equation 3-30, since it allows us to test for quasi-fixity and capacity utilization. Decomposing the Divisia index of the quasi-fixed factor in Equation 3-30, we get the following equation

$$f_i d \ln x_i = \gamma_2 \sum_{r=1}^{m} \theta_i^r g_r' d \ln y_r + \gamma_3 \theta_i^r \sum_{k=1}^{l} \mu_k' d \ln z_k + \sum_{j=1}^{n} \left(s_{ij} - f_i \left(\delta_{ij} - f_j \right) d \ln w_j \right) \quad (3\text{-}32)$$

Using the definitions of γ_3, θ_i^r and μ_k' it is easy to show that

$$\gamma_3 \theta_i^r \sum_{k=1}^{l} \mu_k' d \ln z_k = \theta_i^r \left(\sum_k \frac{\partial \ln VC}{\partial \ln z_k} \right) \sum_{k=1}^{l} \frac{\frac{\partial VC}{\partial \ln z_k}}{\left(\sum_k^{l} \frac{\partial VC}{\partial \ln z_k} \right)} d \ln z_k =$$

$$= \theta_i^r \sum_{k=1}^{l} \frac{\partial \ln VC}{\partial \ln z_k} d \ln z_k = \theta_i^r \sum_{k=1}^{l} \varepsilon_{VC,z_k} d \ln z_k$$

Substituting now this term back into Equation 3-32, it transforms the input-demand system into

$$f_i d \ln x_i = \gamma_2 \sum_{r=1}^{m} \theta_i^r g_r' d \ln y_r + \theta_i^r \sum_{k=1}^{l} \varepsilon_{VC,z_k} d \ln z_k + \sum_{j=1}^{n} \left(s_{ij} - f_i \left(\delta_{ij} - f_j \right) d \ln w_j \right) \quad (3\text{-}33)$$

Then the total input decision of the firm, $d\ln X = \gamma_2 d\ln Y + \sum_i \sum_{k=1}^{l} \varepsilon_{VC,z_k} d\ln z_k$, is obtained by summing Equation 3-33 over i and using the previous result that $\sum_i \theta_i^r = 1$.

The most important result is that the summation of Equation 3-33 over i for a specific quasi-fixed input gives us an estimate of the elasticity of variable cost with respect to the level of that quasi-fixed factor. This estimate, $\sum_i \varepsilon_{VC,z_k} = (\partial VC / \partial z_k)(z_k / VC)$, provides a tool to test for quasi-fixity of input k. Specifically, a testable hypothesis for quasi-fixity is $H_0 : (\partial VC / \partial z_k) + v_k = 0$, where v_k is the ex-ante market rental price of the quasi-fixed input. If H_0 holds then it implies that the quasi-fixed input is at its full equilibrium level and should not be included in the right-hand side of the demand equation.

Given that we have an elasticity estimate we need to transform the null hypothesis into H_0: $H_0 : \left(\frac{\partial VC}{\partial z_k} \frac{z_k}{VC} \right) * \frac{VC}{z_k} + v_k = 0$, where the first term in the parenthesis is the estimate from the input demand estimation and is being multiplied by (VC / z_k) at each data point at the sample. If the null holds at some data point then the quasi-fixed input k is at its full equilibrium level and the model is misspecified, while deviations from H_0 show that the input is quasi-fixed. A one way t-test could be developed to find the sign of capacity utilization. Note that we can test at each observation on the sample, like the Kulatilaka, (1985) t-test, providing the whole path of changes between full static equilibrium and short-run equilibrium for the input z_k. If one uses the average of the observations in the sample to construct (VC / z_k) then H_0 provides a joint test for quasi-fixity for all observations.

Schankerman and Nadiri, (1986) provided a test for quasi-fixity through a Hausman test for specification error in a system of simultaneous equations, where their system consisted of a restricted cost function, short-run demand for variable inputs and long-run demand for fixed factors. Given that in this study we do not have a functional form for the cost function we cannot apply their test for the differential model. However, a specification test between the conditional demands for variable inputs, Equation 3-1, and the long-run demands for the quasi-fixed inputs can be obtained. This would be a simultaneous-equations error specification test.

However, the estimation of Equation 3-33 requires complete system of estimation methods, since the disturbances add up to zero, implying that their variance matrix is singular. If, we would proceed by deleting one equation from the system then the coefficients of the quasi-fixed inputs in the deleted equation could not be recovered since they do not add up to a known constant. Since the focus of the present study is on the comparison of the differential model with a translog specification we will not test for quasi-fixity. However, we provide directions for the estimation methods for such systems in the following chapter.

CHAPTER 4
ESTIMATION METHODS

4.1 Choice of Estimation Method

In Chapter 3, a model for the decisions of a multiproduct firm over a period of time was presented. While this formulation seems to be restrictive for real applications, it should be noted that it can be transformed to reflect different situations. For instance, the one firm could represent one sector of the whole economy, such as agriculture. Further, if one was considering the input-demand system, then it could be transformed to reflect situations in International Trade or Marketing. Specifically, in International Trade variables in the left hand side of the equation could denote the international trade of flows of imports of a specific country from different import sources, which necessarily add up to total imports. In marketing analysis they could represent the market shares of all brands of a specific product, which add up to unity.

The purpose of this chapter is to present and develop different methods of estimation for the differential model. Specifically, in this section we present the econometric procedure for the joint estimation of the input-demand and output-supply system of a multiproduct firm, as provided by Laitinen (1980). It will form the basis for the econometric procedures in the next sections, which concern multiple multiproduct firms; that is, panel data structures. In those sections maximum likelihood estimation methods for time-specific, fixed-effects and firm-specific, random-effects panel data are developed. The novelty in those sections is the consideration of systems of equations, which are nonlinear in the parameters and have nonlinear cross-equations restrictions. For convenience, we reproduce the systems of equations

$$\tilde{x}_{it} = \sum_{r=1}^{m} \theta_i^r \tilde{y}_{rt} + \sum_{k=1}^{l} \xi_i^k \tilde{z}_{kt} + \sum_{j=1}^{n} \pi_{ij} Dw_{jt} + \varepsilon_{it} \tag{4-1}$$

$$\tilde{y}_{rt} = \sum_{s=1}^{m} \alpha_{rs} Dp_{st} - \sum_{i=1}^{n} \sum_{s=1}^{m} \alpha_{rs} \theta_i^s Dw_{it} - \sum_{k=1}^{l} \beta_{rk} Dz_{kt} + \varepsilon_{rt}^{**} \tag{4-2}$$

where $i = 1,...,n$ and $s, r = 1,...,m$ denote number of equations and we have assumed constant coefficients, θ_i^r, ξ_i^k, π_{ij}, α_{rs}, β_{rk} and $\alpha_{rs}\theta_i^s$ over time. Also, as was shown in Chapter 3, both systems of equations have homoscedastic covariance matrices, which are denoted as $Cov(\varepsilon_{rt}^*, \varepsilon_{st}^*) = \sigma^2 \bar{\gamma}_{2t} \psi^* \theta_{rs}^* = \sigma^2 \alpha_{rs}$ and $Cov(\varepsilon_i, \varepsilon_j) = \sigma^2 \psi(\phi_{ij} - \phi_i \phi_j)$. Further, the following changes in notation have been made: $\tilde{x}_{it} = \bar{f}_{it}(Dx_{it} - E_t)$, $\tilde{y}_{rt} = \bar{\gamma}_{2t} \bar{g}'_{rt} Dy_{rt}$, $\tilde{z}_{kt} = \bar{\gamma}_{3t} \bar{\mu}'_{kt} Dz_{kt}$, $\pi_{ij} = -\psi(\phi_{ij} - \phi_i \phi_j)$ $\alpha_{rs} = \bar{\gamma}_{2t} \psi^* \theta_{rs}^*$, $\beta_{rk} = \bar{\gamma}_{2t} \psi^* \eta_{rk}$, $c_{ris} = \alpha_{rs} \theta_i^s$.

Note that, since we have assumed that the matrix $[\alpha_{rs}]$ is constant over time, then ψ^*, which is the price elasticity of the firm's total supply, is proportional to the cost-revenue ratio. This can be seen from equations $\gamma_2 = R/VC$ and $\sum_r \sum_s \alpha_{rs} = \gamma_{2t} \psi^*$. An alternative parameterization can be formulated with constant ψ^* (Theil 1980). This could happen if we divide both sides of Equations 4-1 and 4-2 by γ_{2t} and treat π_{ij} / γ_{2t} and $\alpha_{rs} / \gamma_{2t}$ as constants. The disturbances $\varepsilon_{it} / \gamma_{2t}$ and $\varepsilon_{rs}^{**} / \gamma_{2t}$ are still homoscedastic.

Another problem with the parameterization of Equations 4-1 and 4-2 is that we have assumed constant technology for the firm, but this can be resolved by adding a constant term in both systems. Laitinen (1980, page 118) suggested that these terms would represent systematic changes in the firm's technology (Hicks neutral technical change).

Before we proceed into the estimation method for the joint system of Equations 4-1 and 4-2, we need to impose the adding-up restrictions, symmetry, and homogeneity properties of the two systems. We choose to impose those restrictions in order to reduce the number of coefficients to be estimated. To satisfy the adding-up property in the input-demand system, which creates the problem of a singular variance matrix of disturbances in the system of Equations 4-1, we drop one equation from this system. Following this method to deal with singular disturbances necessitates the use of a maximum likelihood (ML) estimator, which gives estimates invariant to the dropped equation (Barten, 1969).

Recently, there have been developed methods for estimating a complete system of equations with singular covariance matrix of disturbances (Equation 4-1) that do not require dropping one equation; and so do not rely on the invariance property of the ML estimator. Dhrymes (1994) considered the case of autoregressive errors in singular systems of equations. His estimation method relies on the use of a generalized inverse (Moore-Penrose) for the variance of the disturbances and on a formulation of an Aitken Minimand. Shrivastava and Rosen (2002) provided a ML estimator for a complete system of equations with unknown singular covariance matrix of disturbances. Complete system of equations estimation with singular covariance of disturbances in seemingly unrelated regression methods (SUR) and three stage least squares (3SLS) framework was provided by Kontoghiorges (2000) and Kontoghiorges and Dinesis (1997), respectively.

The initial approach was to estimate the joint system of input-demand and output-supply equations (Equations 4-1 and 4-2, respectively) by employing one of the previously mentioned methods for the input-demand system and then to provide a joint method of estimation for both systems. However, the nonlinear cross-equations restrictions on the parameters and most importantly the need for a panel data method led to the use of the more standard method, of simply dropping one equation. The transformation for the quasi-fixed inputs $-\partial VC / \partial z_k = v_k + \delta_k$ used in the parameterization of the input-demand system (Equation 4-1) serves that purpose, since the summation of z_k over i adds-up to a constant and thus the last equation can be dropped.

The homogeneity property of the input-demand system (Equation 4-1) in input prices and output-supply system (Equation 4-2) in both input and output prices is imposed by subtracting the input price that corresponds to the dropped equation from all prices in both systems. Symmetry is an important property that needs to be imposed or tested. Given the adding-up conditions, symmetry can not be tested without homogeneity already imposed. In the joint system of equations we have symmetry conditions for the price terms in the input-demand system and for the price terms in the supply system. Symmetry in the price terms of the two systems of equations (homogeneity restricted) can be imposed by including on the coefficient vector only the unique elements and rearrange the exogenous variables matrix to correspond to those elements. For instance, consider the case of one firm utilizing three variable inputs, two quasi-fixed inputs and four outputs. Then, the homogeneity and symmetry imposed input-demand and output-supply system will have the following form

$$\begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \end{bmatrix} = \begin{bmatrix} y_1 & y_2 & (w_1 - w_3) & (w_2 - w_3) & 0 & 0 & 0 \\ 0 & 0 & 0 & (w_1 - w_3) & y_1 & y_2 & (w_2 - w_3) \end{bmatrix} \left[\theta_{11}\, \theta_{12}\, \pi_{11}\, \pi_{12}\, \theta_{21}\, \theta_{22}\, \pi_{22} \right]'$$

$$\begin{bmatrix} \tilde{y}_1 \\ \tilde{y}_2 \end{bmatrix} = \begin{bmatrix} p_1 & p_2 & (w_1 - w_3) & (w_2 - w_3) & 0 & 0 & 0 \\ 0 & p_1 & 0 & 0 & p_2 & (w_1 - w_3) & (w_2 - w_3) \end{bmatrix} \begin{bmatrix} \alpha_{11} \, \alpha_{12} \, c_{11} \, c_{12} \, \alpha_{22} \, c_{21} \, c_{22} \end{bmatrix}'$$

In this formulation, we have omitted two outputs and the quasi-fixed inputs to save space, and we have dropped one input-demand equation due to the singularity of the disturbances. Notice that the input price parameters in the supply system are allowed to vary freely when the model is unrestricted or homogeneity restricted. However, under the homogeneity and symmetry restricted model, as above, these parameters are fixed (not free), since it is required $c_{ris} = \alpha_{rs}\theta_i^s$. Therefore, imposing symmetry in the joint system transforms the input price terms in the output supply to nonlinear, creating an additional complexity in the estimation procedure. Then, for our example that turns out to be the estimated model in the next chapter, the vector of coefficients has fifty free parameters in the homogeneity restricted model, including an intercept for each equation, while in the homogeneity and symmetry restricted model consists of thirty five free parameters.

Having showed how to impose adding-up, linear symmetry and homogeneity restrictions, the input-demand and output-supply systems can be written in a stacked-equation form. To account for the singular covariance matrix of disturbances in the input-demand system, the last equation was deleted. Therefore, Equations 4-1 and 4-2 are written in matrix form as

$$x_t = \Theta y_t + Kz_t + Dw_t + \varepsilon_t = Nv_t + \varepsilon_t, \qquad i = 1, \ldots, n-1 \tag{4-3}$$

$$y_t = Ap_t + C\underline{w}_t + F\underline{z}_t + \varepsilon_t^{**} = Mq_t + \varepsilon_t^{**}, \qquad r = 1, \ldots, m \tag{4-4}$$

which is subject to the following restrictions

Homogeneity

$$Ai_m + Ci_n = 0, \text{ in output supply} \tag{4-5}$$

$Di_n = 0$, in input demand (4-6)

Linear Symmetry Conditions

$A = A'$, in output supply (4-7)

$D = D'$, in input demand (4-8)

Nonlinear Symmetry

$C = -A \cdot K'$, in input demand and output supply (4-9)

The adding-up property of the input-demand system has been imposed by deleting the last equation. Homogeneity in both systems has been imposed by subtracting the input price that corresponds to the dropped input-demand equation, i.e. for $i = 3$, from all prices in both systems. The linear symmetry conditions have been imposed as shown before, but the nonlinear symmetry condition is left for the estimation procedure. Accordingly, the following conventions in the notation have been made

$$A = [\alpha_{rs}]_{m \times m},\ C = [\alpha_{rs}\theta_i^s]_{m \times n},\ F = [\beta_{rk}]_{m \times k},\ \Theta = [\theta_i^r]_{n-1 \times m},\ D = [\pi_{ij}]_{n-1 \times n},$$

$$K = [\xi_i^k]_{n-1 \times k},\ x_t = \left(\tilde{x}_{1t}, \ldots, \tilde{x}_{n-1t}\right)',\ z_t = \left(\tilde{z}_{1t}, \ldots, \tilde{z}_t\right)',\ y_t = \left(\tilde{y}_{1t}, \ldots, \tilde{y}_{mt}\right)',$$

$$\underline{z}_t = \left(Dz_{1t}, \ldots, Dz_{kt}\right)',\ \varepsilon_t = \left(\varepsilon_{1t}, \ldots, \varepsilon_{n-1t}\right)', \text{ and } \varepsilon_t^{**} = \left(\varepsilon_{1t}^{**}, \ldots, \varepsilon_{mt}^{**}\right)'.$$

The price vectors, $w_t = \left(D\breve{w}_{1t}, \ldots, D\breve{w}_{n-1t}\right)'$, $\underline{w}_t = \left(D\breve{w}_{1t}, \ldots, D\breve{w}_{n-1t}\right)'$, $p_t = (D\breve{p}_{1t}, \ldots, D\breve{p}_{mt})'$, denote the modified prices, where Dw_{nt} has been subtracted from every price. Finally, $N = [\Theta\ K\ D]$ and $M = [A\ C\ F]$ are partitioned matrices, and $v_t' = (y_t', z_t', w_t')$, $q_t = (p_t', \underline{w}_t', \underline{z}_t')$.

The joint system, as presented in Equations 4-3 and 4-4, without the nonlinear symmetry restrictions, is a triangular system. Further, relying on the theory of rational random behavior the disturbances in the demand system (Equation 4-3) are stochastically independent of those of the supply system (Equation 4-4), making the joint system block recursive. This has two implications. First, it implies that the decisions of the firm take place in two separated phases. First the output-supply decision is taken and then given this decision the input demands are determined. Accordingly, we can view $\tilde{y}_{rt}$ as a predetermined variable. However, we can observe that the marginal shares of the inputs, θ_i^r occur not only in the demand system (Equation 4-3) but also in the supply system (Equation 4-4). Therefore, in spite of independence of the disturbances of the two systems, a joint method of estimation of Equations 4-3 and 4-4 is more appropriate in order to impose these restrictions on the marginal shares. Further, in the supply system 4-4 the parameters are nonlinear if we impose symmetry and homogeneity.

Let us denote the variance across equations in the input-demand system and output-supply system, as

$$E(\varepsilon_t^{**}\varepsilon_t^{**\prime}) = \Omega^*_{m\times m} \text{ and } E(\varepsilon_t\varepsilon_t^{\prime}) = \Omega_{n-1\times n-1} \tag{4-10}$$

By relying on the rational random behavior theory, the above systems form a block recursive system and under normality we have that the joint system error covariance structure is

$$E(\varepsilon_t^{**}\varepsilon_t^{\prime}) = \Sigma_{(m+n-1)\times(m+n-1)} = \begin{bmatrix} \Omega^* & 0_{m\times n-1} \\ 0_{n-1\times m} & \Omega \end{bmatrix} \tag{4-11}$$

However, we choose not to force the off-diagonal elements of the covariance matrix to be zero. Bronsard and Salvas-Bronsard, (1984) suggested to test for the exogeneity of y_t in the input-demand system, by estimating the joint system one time with Equation 4-11 imposed and one without, and then form a likelihood ratio test for the covariance restricted versus the unrestricted model. Assuming that the disturbances are independent in different periods, Laitinen (1980, page 120), writes the log likelihood function of the joint system as

$$L = -\frac{T(m+n-1)}{2}\ln 2\pi - \frac{T}{2}\ln|\Sigma| - \frac{1}{2}\sum_{t=1}^{T}\begin{bmatrix} y_t - Mq_t \\ x_t - Nv_t \end{bmatrix}' \Sigma^{-1} \begin{bmatrix} y_t - Mq_t \\ x_t - Nv_t \end{bmatrix} \tag{4-12}$$

From Magnus and Neudecker, (1988) we have that

$$\frac{\partial \ln|\Sigma^{-1}|}{\partial \Sigma^{-1}} = \Sigma' \text{ and } \frac{\partial a'\Sigma^{-1}a}{\partial \Sigma^{-1}} = aa'$$

Then for given M, N, the first-order condition with respect to Σ^{-1} is given by

$$\frac{\partial L}{\partial \Sigma^{-1}} = -\frac{T}{2}\Sigma' + \frac{1}{2}\sum_{t=1}^{T}\begin{bmatrix} y_t - Mq_t \\ x_t - Nv_t \end{bmatrix}\begin{bmatrix} y_t - Mq_t \\ x_t - Nv_t \end{bmatrix}' = 0$$

which gives the following expression for the covariance matrix,

$$\hat{\Sigma} = \frac{1}{T}\sum_{t=1}^{T}\begin{bmatrix} y_t - Mq_t \\ x_t - Nv_t \end{bmatrix}\begin{bmatrix} y_t - Mq_t \\ x_t - Nv_t \end{bmatrix}' \tag{4-13}$$

If one wanted to assume that Σ follows the assumption of Equation 4-11 then the off-diagonal elements in Equation 4-13 would be zero. In this case one could use a two-step estimator, where in the first step $\hat{\Omega}^*$ and $\hat{\Omega}$ are estimated from each system separately (impose homogeneity in each system at this step) and then use those as an initial estimator of $\hat{\Sigma}$, where now impose the linear and nonlinear cross equations restrictions.

To apply the nonlinear symmetry constraints, it is convenient to regard the elements of M and N as functions of a vector μ that contains only the free parameters in the joint system. Further, we choose to substitute the nonlinear symmetry restrictions,

$C = -A \cdot K'$ at the objective (likelihood function). Another, equivalent way would be to include it as a constraint and maximize the constrained log-likelihood function. Magnus (1982) proposes the latter method but he also suggests substituting a large value, like 1000, to the lagrangean multiplier.

Then for given Σ the first-order conditions with respect to the i^{th} element of vector μ, are given[4] by

$$\frac{\partial L}{\partial \mu_i} = \sum_{t=1}^{T} \begin{bmatrix} y_t - Mq_t \\ x_t - Nv_t \end{bmatrix} \Sigma^{-1} \begin{bmatrix} \dfrac{\partial M}{\partial \mu_i} q_t \\ \dfrac{\partial N}{\partial \mu_i} v_t \end{bmatrix} = 0 \tag{4-14}$$

where $\begin{bmatrix} M \\ N \end{bmatrix} = \begin{bmatrix} A & -AK' & F \\ \Theta & K & D \end{bmatrix}$.

In Appendix A.1, the analytical derivatives of $\partial M / \partial \mu_i$ and $\partial N / \partial \mu_i$ are provided for a system that consists of three variable inputs, two quasi-fixed inputs and three outputs. Also, notice that we have made the substitution $C = -A \cdot K'$ in the supply system.

Finally, Laitinen (1980, page 124) shows that the information matrix for the parameters has the following form

$$-E\left(\frac{\partial^2 L}{\partial \mu_i \partial \mu_j}\right) = \sum_{t=1}^{T} \begin{bmatrix} \dfrac{\partial M}{\partial \mu_j} q_t \\ \dfrac{\partial N}{\partial \mu_j} v_t \end{bmatrix} \Sigma^{-1} \begin{bmatrix} \dfrac{\partial M}{\partial \mu_i} q_t \\ \dfrac{\partial N}{\partial \mu_i} v_t \end{bmatrix} = 0 \tag{4-15}$$

The inverse of the information matrix will yield an asymptotic estimate of the covariance matrix for the parameters that maximize L. Then for a given vector μ, we

[4] Laitinen (1980) has already derived these conditions and we reproduced them here.

define ω as the vector with i^{th} element $\partial L / \partial \mu_i$, given in Equation 4-14, and E the square and symmetric matrix, whose $(i, j)^{th}$ element is $-E\left(\frac{\partial^2 L}{\partial \mu_i \partial \mu_j}\right)$, given by Equation 4-15.

The iterative procedure that Laitinen (1980) suggests, consists of the following steps:

- Compute $\hat{\Sigma}$ using Equation 4-13 with M, N evaluated at the given vector μ.
- Use Equations 4-14 and 4-15 to evaluate ω and E.
- Let $\Delta\mu = E^{-1}\omega$. Then if $\Delta\mu < 0.000001$ use the given vector μ as the vector that maximizes L and E^{-1} as its asymptotic covariance matrix. If the previous condition does not hold then update the vector μ by using $\mu_{new} = \mu_{old} + E^{-1}\omega$.
- Get $\hat{\mu}, \hat{\Sigma}$.

Then from Equation 4-12, the concentrated log-likelihood function at the optimum becomes

$$L_{\max} = -\frac{T(m+n-1)}{2}\ln 2\pi e - \frac{T}{2}\ln\left|\hat{\Sigma}\right| \tag{4-16}$$

While Laitinen (1980) does not suggest an initial estimator for the vector μ, a consistent initial estimator for the joint system under the assumption of independent disturbances of the two systems and without linear or nonlinear symmetry restrictions imposed, could be obtained by separate iterative SUR in each system. If disturbances are not independent then a consistent estimator would be an iterative SUR in the joint system.

4.2 Fixed Effects and Pooled Model

The previous econometric procedure considers only one firm over multiple years, but our dataset consists of a large number of firms observed over a small period of time.

Therefore, we need to consider panel data techniques for the estimation of the differential model. In this section we analyze fixed-effects models and pooling across years or cross-sectional units. Recently, Baltagi et al. (2000) showed that for a dynamic specification of the demand for cigarettes, pooling was superior to heterogeneous estimators.

Before we proceed to the estimation methods, it should be noted that if one does not want to impose the disturbances in the input-demand system to be independent of those in the output-supply system, then there is no need to follow the structure of equations as initially presented in Equations 4-3 and 4-4, and was followed until Equation 4-16. Instead, we could consider that the two systems of $(n-1)$ plus m equations as one system with G equations and an equation index g, $g=1,\ldots,G$. Further, we make the following conventions regarding notation in this and subsequent sections. We suppose that $i=1,\ldots,N$ refers to the number of firms in our sample; $t=1,\ldots,T$ is the number of years that each firm is observed (balanced case); g_1 is the index for the equations of the input demands with $g_1=1,\ldots,G_1$ and g_2 is the equation index for the output-supply system with $g_2=1,\ldots,G_2$ and $G=G_1+G_2$. Then the firm i at time t we have

$$\tilde{x}_{ig_1t} = \sum_{g_2=1}^{G_2} \theta_{g_1}^{g_2} \tilde{y}_{ig_2t} + \sum_{k=1}^{l} \xi_{g_1}^{k} \tilde{z}_{ikt} + \sum_{\tilde{g}_1=1}^{G_1} \pi_{g_1\tilde{g}_1} Dw_{i\tilde{g}_1t} + \varepsilon_{ig_1t} \tag{4-17}$$

$$\tilde{y}_{ig_2t} = \sum_{\tilde{g}_2=1}^{G_2} \alpha_{g_2\tilde{g}_2} Dp_{i\tilde{g}_2t} - \sum_{g_1=1}^{G_1} \sum_{\tilde{g}_2=1}^{G_2} \alpha_{g_2\tilde{g}_2} \theta_{g_1}^{\tilde{g}_2} Dw_{ig_1t} - \sum_{k=1}^{l} h_{g_2k} Dz_{ig_2t} + \varepsilon_{ig_2t}^{**} \tag{4-18}$$

where $h_{g_2k} = \beta_{rk}$, $\alpha_{g_2\tilde{g}_2}\theta_{g_1}^{\tilde{g}_2} = c_{g_1\tilde{g}_2g_2}$, $g_1 = \tilde{g}_1 = 1,\ldots,G_1$ and $g_2 = \tilde{g}_2 = 1,\ldots,G_2$.

The notation is further simplified by assuming

$$y_{git} = x_{git}\beta_{git} + u_{git} \tag{4-19}$$

where $i = 1,...,N$, $t = 1,...,T$, $g = 1,...,G$ and $G = G_1 + G_2$; y_{git} is the dependent variable in the g^{th} equation. For instance, the vector of exogenous variables is denoted as $[y_{1it},..., y_{Git}]' = [\tilde{x}_{1it},...,\tilde{x}_{G_1it}, \tilde{y}_{1it},...,\tilde{y}_{G_2it}]'$; x_{git} is the matrix of exogenous variables for the g^{th} equation and β_g is the coefficient vector of the equation (if symmetry has been imposed then β_g has no duplicate terms).

Then stacking all G equations for each observation (i,t), we obtain

$$\begin{bmatrix} y_{1it} \\ y_{2it} \\ \vdots \\ y_{Git} \end{bmatrix} = \begin{bmatrix} x_{1it} & 0 & \cdots & 0 \\ 0 & \ddots & & \vdots \\ \vdots & & \ddots & \vdots \\ 0 & \cdots & \cdots & x_{Git} \end{bmatrix} \begin{bmatrix} \beta_1 \\ \vdots \\ \vdots \\ \beta_G \end{bmatrix} + \begin{bmatrix} u_{1it} \\ \vdots \\ \vdots \\ u_{Git} \end{bmatrix} \tag{4-20}$$

This can be written in a compact form as

$$Y_{it} = X_{it}\beta + U_{it} \tag{4-21}$$

where Y_{it} is a $G \times 1$ vector, X_{it} is a $G \times K$ matrix of exogenous variables and β is a $K \times 1$ vector with $K = \sum_{g=1}^{G} K_g$ and K_g is the number of regressors in the g^{th} equation including a constant; U_{it} is a $G \times 1$ vector of the error terms. Since we want to impose symmetry and some coefficients appear in at least two equations, then we redefine β as the complete coefficient vector, which is nonlinear and does not contain any duplicates, apart from the nonlinear terms (see example in Appendix A.2). Further, we redefine $X_{it} = [x'_{1it},..., x'_{Git}]'$, where the k^{th} element of x_{git} is redefined to contain the observations on the variable in the g^{th} equation which corresponds to the k^{th} coefficient in β. If the latter does not occur in the g^{th} equation, then the k^{th} element of x_{git} is set to zero.

A pooled model would then consist of regressing Equation 4-21 for all i and t. However, it is implicitly assumed that all firms have the same intercepts and slopes over the entire period, which is a very restrictive assumption. One way to account for heterogeneity across individuals or through time is to use variable intercept models. So following Baltagi (2001, page 31) let us decompose the disturbance term in Equation 4-21 as a two-way error component model:

$$U_{it} = a_i^* + a_t + v_{it} \qquad i = 1,...,N \text{ , } t = 1,...,T \tag{4-22}$$

where a_i^* denotes all the unobserved, omitted variables from Equation 4-21, which are specific to each firm and are time invariant; a_t denotes all unobserved, omitted variables from Equation 4-21 that are period individual-invariant variables. That is, variables that are the same for all cross-sectional units at a given point in time but that vary through time. Finally, v_{it} is white noise.

It is this ability to control for all time-invariant variables or firm-invariant variables whose omission could bias the estimates in a typical cross-section or time-series study that reveals the advantages of a panel. The way we treat a_i^* and a_t, it then differentiates between fixed-effects and random-effects models. Specifically, if we treat a_i^* as fixed parameters to be estimated as coefficients of firm-specific dummies in the sample, then we follow a fixed-effects approach. Instead, if we assume that a_i^* are random variables that are drawn from a distribution we have a random-effects model. The same arguments are true for a_t. Random-effects models are considered in Section 4.3.

So suppose that we formulate a fixed-effects model for N multiproduct firms, where the effects of omitted, unobserved, firm-specific variables are treated as fixed constants over time. Then, Equation 4-21 becomes

$$Y_{it} = a_i^* + X_{it}\beta + U_{it} \qquad i = 1,...,N\,,\ t = 1,...,T \tag{4-23}$$

In this formulation a_i^* represents the firm-specific effects. For instance, in banking it could account for all differences such as location, management skills or persistent X-inefficiency, that permanently affect the demand for inputs and supply of outputs of a particular bank relative to some other bank that face similar conditions. However, we could reject the use of a firm-specific, fixed-effects model in this study for two reasons. Our sample consists of $T \to$ fixed and $N \to$ large and a fixed-effects approach would result in a huge loss of degrees of freedom ($df = NT - N - K + 1$). Secondly, our model is already first differenced, which sweeps out the individual effects. For instance, our Y_{it} is equal to $(\ln Y_{it} - \ln Y_{it-1})$ and from Equation 4-22 it is obvious that a_i^* are differenced out. However, one could argue that fixed effects exist between $Y_{it} = \ln Y_{it} - \ln Y_{it-1}$ and $Y_{it-1} = \ln Y_{it-1} - \ln Y_{it-2}$. For that purpose we consider a random-effects model in the next section.

A more appropriate fixed-effects model would be to consider time-specific effects a_t as fixed parameters and estimate them as coefficients of time dummies (Dum_t) for each year in the sample. That is, to consider the model

$$Y_{it} = i_G \otimes \left(\sum_{t=1}^{T} a_t Dum_t \right) + X_{it}\beta + U_{it} = \underline{X}_{it}\underline{\beta} + U_{it} \qquad i = 1,...,N\,,\ t = 1,...,T \tag{4-24}$$

where i_G is a vector of ones, and we impose $\sum_{t=1}^{T} a_t = 0$ to avoid the dummy variable trap, since β contains an overall constant in each equation. Further, we make the following assumptions

Assumption 4.1: The error terms of Equation 4-24 are independent and identically distributed as

$$U_{it} \sim IIN_G\left(0_{G\times 1}, \Sigma_u\right) \tag{4-25}$$

Assumption 4.2:

$$\underline{X}_{it} \text{ and } U_{it} \text{ are uncorrelated} \tag{4-26}$$

Notice, that we made the assumption of normality since we are going to use a maximum likelihood method. If a generalized least squares method was to follow, then one should replace Assumption 1 with the following,

$$E(U_{it}) = 0_{G\times 1} \text{ and } E(U_{it}U'_{js}) = \begin{cases} \Sigma_u & if\ i = j, t = s \\ 0 & if\ i \neq j, t = s \\ 0 & if\ i \neq j, t \neq s \end{cases} \tag{4-27}$$

Also, Σ_u is defined as $\Sigma_u = \begin{bmatrix} \sigma_{11}^u & \cdots & \sigma_{1G}^u \\ \vdots & \ddots & \vdots \\ \sigma_{G1}^u & \cdots & \sigma_{GG}^u \end{bmatrix}_{G\times G}$, which is the correlation across equations for an individual at time t and is positive definite. We assume no correlation between individuals for the same year and no contemporaneous correlation across years, since we imposed in the parameterization of the model (Chapter 3) the disturbances to be homoscedastic. Notice, that the formulation in Equation 4-24 implies that only intercepts vary over time. It further implies that there are common shocks in the demand for inputs and supply of outputs for all firms in a specific year. This could be clearer by stacking the observations by year first, so

$$Y_i = \underline{X}_i \underline{\beta} + U_i \tag{4-28}$$

where Y_i, $\underline{X}_i$ and U_i are now the stacked $(GT \times 1)$ vector, $(GT \times K)$ matrix, with K including the time dummies and $(GT \times 1)$ vector of $Y's$, $X's$ and $U's$ respectively, corresponding to the T observations of individual i. That is,

$$Y_i = \begin{bmatrix} Y_{i1} \\ \vdots \\ Y_{iT} \end{bmatrix}; \quad \underline{X}_i = \begin{bmatrix} \underline{X}_{i1} \\ \vdots \\ \underline{X}_{iT} \end{bmatrix}; \quad U_i = \begin{bmatrix} U_{i1} \\ \vdots \\ U_{iT} \end{bmatrix} \tag{4-29}$$

And let $V_i = [U_{i1}, \ldots, U_{iT}]_{G \times T}$ with $U_i = vecV_i$, $i = 1, \ldots, N$ (4-30)

Then making use of Equation 4-25, the $(GT \times 1)$ vectors U_i are distributed as $IIN(0, \Omega)$, with variance matrix

$$E(U_i, U_i') = I_T \otimes \Sigma_u = \Omega \tag{4-31}$$

$$\Omega = \begin{bmatrix} \Sigma_u & 0 & \cdots & 0 \\ 0 & \ddots & & \vdots \\ \vdots & & \ddots & \vdots \\ 0 & \cdots & \cdots & \Sigma_u \end{bmatrix}_{GT \times GT} = I_T \otimes \Sigma_u$$

The model as presented in Equation 4-28 is simply a seemingly unrelated regression (SUR), first considered by Zellner (1962) but nonlinear in the parameters. We may then formulate the following proposition.

Proposition 4.1: The log-likelihood associated with the linear model (Equation 4-28), but nonlinear in the parameters, under the Assumptions 4.1 and 4.2, is given by

$$\mathcal{L} = \sum_{i=1}^{N} L_i \text{, with } L_i = \frac{1}{2} GT \ln 2\pi - \frac{T}{2} \ln |\Sigma_u| - \frac{1}{2} \sum_{t=1}^{T} U_{it}' \Sigma_u^{-1} U_{it}$$

The proof of this proposition is simple and is based on Magnus (1982). The probability density of Y_i takes the form (see Appendix of Magnus, 1982)

$$f\left(Y_i \mid \underline{\beta}, \Sigma_u\right) = 2\pi^{-GT/2} \left|\Omega\right|^{-1/2} e^{-1/2(U_i'\Omega^{-1}U_i)} \tag{4-32}$$

The log-likelihood function for firm i is then,

$$L_i = \frac{1}{2} GT \ln 2\pi - \frac{1}{2} \ln\left|\Omega\right| - \frac{1}{2}(U_i'\Omega^{-1}U_i) \tag{4-33}$$

Notice however that $\Omega = I_T \otimes \Sigma_u$ and so its inverse is equal to $\Omega^{-1} = I_T \otimes \Sigma_u^{-1}$ and the determinant is equal to $\left|\Omega\right| = T\left|\Sigma_u\right|$ (Theil 1971). Then the log-likelihood function can be written as

$$L_i = \frac{1}{2} GT \ln 2\pi - \frac{T}{2} \ln\left|\Sigma_u\right| - \frac{1}{2}\sum_{t=1}^{T} U_{it}'\Sigma_u^{-1}U_{it} \tag{4-34}$$

Now we can formulate the next proposition for the gradient vector and information matrix of the model considered in Proposition 4.1.

Proposition 4.2: Consider the linear model in Equation 4-28, but nonlinear in the parameters, under the Assumptions 4.1 and 4.2. Then the gradient vector, and the information matrix for $\mathcal{L} = \sum_{i=1}^{N} L_i$ are given by

$$\frac{\partial \mathcal{L}}{\partial \Sigma_u^{-1}} = -\frac{NT}{2}\Sigma_u + \frac{1}{2}\sum_{i=1}^{N}\sum_{t=1}^{T} U_{it}U_{it}', \quad \frac{\partial \mathcal{L}}{\partial \underline{\beta}_h} = -\sum_{i=1}^{N}\sum_{t=1}^{T}\left(\underline{X}_{it}\frac{\partial \underline{\beta}}{\partial \underline{\beta}_h}\right)' \Sigma_u^{-1}\left(Y_{it} - \underline{X}_{it}\underline{\beta}\right) \tag{4-35}$$

$$I = \begin{bmatrix} \left[\left(\sum_{i=1}^{N}\sum_{t=1}^{T}\left(\underline{X}_{it}\frac{\partial \underline{\beta}}{\partial \underline{\beta}_m}\right)' \Sigma_u^{-1}\underline{X}_{it}\frac{\partial \underline{\beta}}{\partial \underline{\beta}_h}\right)\right]_{K\times K} & 0 \\ 0 & \frac{NT}{2}\Sigma_u^2 \end{bmatrix} \tag{4-36}$$

To prove this proposition notice that for a given vector $\underline{\beta}$, we can differentiate Equation 4-34 with respect to the covariance matrix to get

$$\frac{\partial L_i}{\partial \Sigma_u^{-1}} = -\frac{T}{2}\Sigma_u + \frac{1}{2}\sum_{t=1}^{T} U_{it}U_{it}' = 0$$

Summing this equation over all individuals it gives Equation 4-35. Further, we can solve for Σ_u in the above first-order condition, to get

$$\hat{\Sigma}_u = \frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T} U_{it}U_{it}' = \frac{1}{NT}\sum_{i=1}^{N} V_i V_i' \tag{4-37}$$

Before we differentiate with respect to the nonlinear vector $\underline{\beta}$, note that $U_{it} = Y_{it} - \underline{X}_{it}\underline{\beta}$ and $\underline{\beta}$ contains no duplicates. Then, for given Σ_u we differentiate Equation 4-34 with respect to the h^{th} element of $\underline{\beta}$, $\forall h,\ h = 1,\ldots,K$ to get

$$\frac{\partial L_i}{\partial \underline{\beta}_h} = -\frac{1}{2}\sum_{t=1}^{T}\left[-Y_{it}'\Sigma_u^{-1}\underline{X}_{it}\frac{\partial \underline{\beta}}{\partial \underline{\beta}_h} - \left(\frac{\partial \underline{\beta}}{\partial \underline{\beta}_h}\right)' \underline{X}_{it}'\Sigma_u^{-1}Y_{it} + \left(\frac{\partial \underline{\beta}}{\partial \underline{\beta}_h}\right)' \underline{X}_{it}'\Sigma_u^{-1}\underline{X}_{it}\underline{\beta} + \underline{\beta}\underline{X}_{it}'\Sigma_u^{-1}\underline{X}_{it}\frac{\partial \underline{\beta}}{\partial \underline{\beta}_h}\right] = 0$$

which simplifies to

$$\frac{\partial L_i}{\partial \underline{\beta}_h} = \sum_{t=1}^{T}\left[\left(\underline{X}_{it}\frac{\partial \underline{\beta}}{\partial \underline{\beta}_h}\right)' \Sigma_u^{-1}\left(Y_{it} - \underline{X}_{it}\underline{\beta}\right)\right] = 0 \tag{4-38}$$

Summing this gradient vector over all individuals it gives Equation 4-35. In Section A.2 of the Appendix we provide the analytical gradient vector for the example of this chapter. Notice that if $\underline{\beta}$ was linear in the parameters then Equation 4-38 would give us the GLS estimator,

$$\hat{\underline{\beta}}_{GLS} = \left(\sum_{i=1}^{N}\sum_{t=1}^{T}\underline{X}_{it}'\Sigma_u^{-1}\underline{X}_{it}\right)^{-1}\left(\sum_{i=1}^{N}\sum_{t=1}^{T}\underline{X}_{it}'\Sigma_u^{-1}Y_{it}\right) \tag{4-39}$$

In order now to find the Hessian we need to take the second-order derivatives. We begin with the covariance matrix

$$\frac{\partial^2 L_i}{\partial\Sigma_u^{-1}\partial\Sigma_u^{-1}} = -\frac{T}{2}\Sigma_u^2 \text{ and so } \frac{\partial^2 \mathcal{L}}{\partial\Sigma_u^{-1}\partial\Sigma_u^{-1}} = -\frac{NT}{2}\Sigma_u^2 \tag{4-40}$$

Taking the second-order derivative of the coefficient vector, after some algebra we obtain the following expression

$$\frac{\partial\mathcal{L}}{\partial\underline{\beta}_h\partial\underline{\beta}_m'} = \left(\sum_{i=1}^{N}\sum_{t=1}^{T}\left(X_{it}\frac{\partial^2\beta}{\partial\underline{\beta}_h\partial\underline{\beta}_m'}\right)'\Sigma_u^{-1}\left(Y_{it}-\underline{X}_{it}\underline{\beta}\right)\right)-\left(\sum_{i=1}^{N}\sum_{t=1}^{T}\left(\underline{X}_{it}\frac{\partial\underline{\beta}}{\partial\underline{\beta}_m}\right)'\Sigma_u^{-1}\underline{X}_{it}\frac{\partial\underline{\beta}}{\partial\underline{\beta}_h}\right) \tag{4-41}$$

Since $E(Y_{it}-\underline{X}_{it}\underline{\beta})=0$, the expectation of the above expression gives the information matrix of the parameters

$$I_\beta = \left(\sum_{i=1}^{N}\sum_{t=1}^{T}\left(\underline{X}_{it}\frac{\partial\underline{\beta}}{\partial\underline{\beta}_m}\right)'\Sigma_u^{-1}\underline{X}_{it}\frac{\partial\underline{\beta}}{\partial\underline{\beta}_h}\right) \tag{4-42}$$

Given that the information matrix, which is defined as $I=-E\left(D^2\mathcal{L}\left(\underline{\beta},\Sigma_u\right)\right)$, is block diagonal (Heymans and Magnus, 1979), and combining Equation 4-42 with Equation 4-40 we get the expression in Equation 4-36.

Further, the inverse of the information matrix gives the asymptotic covariance of the estimates and the disturbances. Notice, that the asymptotic covariance matrix for $\underline{\beta}$ can be obtained independently from that of Ω, since the information matrix is block diagonal. The iterative procedure to find the estimates that maximize the likelihood function is similar to the one in the previous section and is based on the multivariate Gauss-Newton method (Harvey, 1993). Thus, define ω as the vector with h^{th} element $\partial\mathcal{L}/\partial\underline{\beta}_h$ and I_β as the square matrix in the upper left corner of the information matrix.

Then, the multivariate Gauss-Newton iterative procedure consists of the following steps:

- Get initial consistent estimates of the vector $\underline{\beta}$, using the GLS estimator presented in Equation 4-39 by disregarding the linear and nonlinear symmetry conditions. Impose though homogeneity and obtain the relevant estimates.
- Compute $\hat{\Sigma}_u$ using Equation 4-37 at the given vector $\underline{\beta}$.
- Use Equations 4-35 and 4-42 to evaluate ω and I_β.
- Let $\Delta\underline{\beta} = I_\beta^{-1}\omega$. Then if $\Delta\underline{\beta} < 0.000001$ use the given vector $\underline{\beta}$ as the vector that maximizes L and I_β^{-1} as its asymptotic covariance matrix. If the previous condition does not hold then update the vector $\underline{\beta}$ by using $\underline{\beta}_{new} = \underline{\beta}_{old} + I_\beta^{-1}\omega$ and go back to step 2. Continue until convergence.
- Get $\hat{\underline{\beta}}$, $\hat{\Sigma}_u$.

To avoid potential confusion between the Scoring method and the multivariate Gauss-Newton method as presented above, notice that in the case examined above those methods coincide. Specifically, if ψ denotes the vector that includes the parameters and the variance to be estimated in the model, $\hat{\psi}$ is the initial estimate of this vector and ψ^* is the revised estimate, then for step 4 in the procedure above the method of scoring consists of calculating $\psi^* = \hat{\psi} + I^{-1}(\hat{\psi})D\ln(\hat{\psi})$. The Gauss-Newton method starts by minimizing the sum of the squared error terms and in the multivariate case for systems of equations, it turns out that the updating procedure is $\psi^* = \hat{\psi} + \left[\sum_t Z_t\Sigma^{-1}Z_t'\right]^{-1}\sum_t Z_t\Sigma^{-1}\varepsilon_t$, where Z_t is equal to $-\partial\varepsilon_t'/\partial\psi$ (Harvey 1993, page 139). Given that the log-likelihood function is concentrated with respect to $\hat{\Sigma}_u$ at step 2 in the procedure, then it obvious that the inverse term in the last equation is $\left[\sum_t Z_t \Sigma^{-1} Z_t'\right]^{-1} = I_\beta$, while the last term of this

equation is equal to $\sum_t Z_t \Sigma^{-1} \varepsilon_t = \omega$.

From Proposition 4.1 we have that the log-likelihood function is

$$\mathcal{L} = \frac{1}{2} GNT \ln 2\pi - \frac{NT}{2} \ln |\Sigma_u| - \frac{1}{2} \sum_{i=1}^{N} \sum_{t=1}^{T} U_{it}' \Sigma_u^{-1} U_{it}$$

Substituting in this expression the estimates $\underline{\hat{\beta}}$ and $\hat{\Sigma}_u$, the concentrated log-likelihood function is obtained. Since $-\frac{1}{2} \sum_{i=1}^{N} \sum_{t=1}^{T} U_{it}' \Sigma_u^{-1} U_{it} = -\frac{1}{2} Tr\left(\Sigma_u^{-1}\left(NT\Sigma_u^{-1}\right)\right) = -\frac{1}{2} GNT$ then the concentrated likelihood function at the optimum becomes

$$\mathcal{L}_{\max} = \frac{1}{2} GNT(\ln 2\pi - 1) - \frac{NT}{2} \ln |\Sigma_u| \tag{4-43}$$

The previous method of estimation accommodates unbalanced panel-data designs, since it is simply a pooling of the observations across years, through the use of time-specific, dummy variables. If the data were balanced then for time-specific or firm-specific, random-effects panel data, the Magnus (1982) method could be used for the estimation of the model. Further, in the case of unbalanced panel data and random effects, but with linear symmetry conditions the maximum likelihood estimator is a straightforward extension of the one provided by Magnus (1982). Wilde et al. 1999 provide an application of this procedure.

4.3 Random Effects

In this section we consider that the individual-specific effects are random variables that follow the normal distribution. Our proposed estimation method under symmetry and nonlinear restrictions on the parameters is a special case of the Magnus (1982) maximum likelihood estimation for a balanced panel. Biorn (2004) has provided a stepwise maximum likelihood method for systems of equations with unbalanced panel data.

For convenience, we rewrite Equation 4-21 for the joint input-demand and output-supply system as

$$Y_{it} = X_{it}\beta + \varepsilon_{it}\text{ [5]} \tag{4-44}$$

$$\varepsilon_{it} = a_i + u_{it} \tag{4-45}$$

In this formulation we assume that a_i are firm-specific, random-effects, and ε_{it} are random errors. Further the coefficient vector β has no duplicates and includes an overall intercept. The matrix of exogenous variables is assumed to have the form $X_{it} = \left[x'_{1it},\dots,x'_{Git}\right]'$. Concerning the distributional form of the random variables we assume

$$a_i \sim IIN_G\left(0_{G\times1},\Sigma_a\right)\text{, that is } E(a_i)=0_{G\times1},\ E\left(a_i a_j{}'\right)=\begin{cases}\Sigma_a & if\ i=j\\ 0 & if\ i\neq j\end{cases} \tag{4-46}$$

$$u_{it} \sim IIN_G\left(0_{G\times1},\Sigma_u\right)\text{, that is } E(u_{it})=0_{G\times1} \text{ and } E(u_{it}u'_{js})=\begin{cases}\Sigma_u & if\ i=j,t=s\\ 0 & if\ i\neq j,t=s\\ 0 & if\ i\neq j,t\neq s\end{cases} \tag{4-47}$$

$$X_{it},\ a_i \text{ and } \varepsilon_{it} \text{ are uncorrelated} \tag{4-48}$$

Then we have $\Sigma_u = \begin{bmatrix}\sigma_{11}^u & \cdots & \sigma_{1G}^u\\ \vdots & \ddots & \vdots\\ \sigma_{G1}^u & \cdots & \sigma_{GG}^u\end{bmatrix}_{G\times G}$ and $\Sigma_a = \begin{bmatrix}\sigma_{11}^a & \cdots & \sigma_{1G}^a\\ \vdots & \ddots & \vdots\\ \sigma_{G1}^a & \cdots & \sigma_{GG}^a\end{bmatrix}_{G\times G}$

From Equations 4-46-4-48 it is easily shown that

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[5] The error terms in this section are not related to any disturbances in the previous sections.

$$E(\varepsilon_{it}) = 0_{G\times 1} \text{ and } E(\varepsilon_{it}\varepsilon'_{js}) = \begin{cases} \Sigma_u + \Sigma_a & \text{if } i = j, t = s \\ \Sigma_a & \text{if } i = j, t \neq s \\ 0 & \text{if } i \neq j, t \neq s \end{cases} \tag{4-49}$$

As before, we stack the observations by time to get

$$Y_i = X_i\beta + i_T \otimes a_i + u_{it} = X_{it}\beta + \varepsilon_i \tag{4-50}$$

where Y_i is a $(GT \times 1)$ vector, X_i is a $(GT \times K)$ matrix and ε_i is a $(GT \times 1)$ vector, corresponding to the T observations of firm i. Also i_T is a vector of ones. It follows that $\varepsilon_{it} \sim IIN_G(0_{G\times 1}, \Omega)$ with

$$\Omega = I_T \otimes \Sigma_u + J_T \otimes \Sigma_a \tag{4-51}$$

since $E(\varepsilon_i\varepsilon'_j) = \begin{bmatrix} \Sigma_u + \Sigma_a & \Sigma_a & \cdots & \Sigma_a \\ \Sigma_a & \ddots & & \vdots \\ \vdots & & \ddots & \vdots \\ \Sigma_a & \cdots & \cdots & \Sigma_u + \Sigma_a \end{bmatrix} = \Omega$ and I_T is a T dimensional identity matrix and $J_T = i_T i_T'$ is a $T \times T$ matrix with all elements equal to one. Then according to Biorn (2004), we could rewrite Ω as

$$\Omega = B_T \otimes \Sigma_u + A_T(\Sigma_u + T\Sigma_a) \tag{4-52}$$

where $A_T = \frac{1}{T}J_T$ and $B_T = I_T - \frac{1}{T}J_T$ are symmetric and idempotent matrices.

Following Magnus (1982) the log-likelihood for the i^{th} individual is given by

$$L_i = -\frac{GT}{2}\ln 2\pi - \frac{1}{2}\ln|\Omega| - \frac{1}{2}(Y_i - X_i\beta)'\,\Omega^{-1}(Y_i - X_i\beta) \tag{4-53}$$

Defining $\Sigma_1 = \Sigma_u + T\Sigma_a$ we have that

$$|\Omega| = |A_T \otimes \Sigma_1 + B_T \otimes \Sigma_u| \tag{4-54}$$

Using the property that A_T and B_T are symmetric and idempotent matrices, Magnus (1982) shows in his lemma 2.1 that

$$|\Omega| = |\Sigma_1||\Sigma_u|^{T-1} \tag{4-55}$$

Also note that

$$\varepsilon_i'(I_T \otimes \Sigma_u^{-1})\varepsilon_i = \sum_{t=1}^{T} \varepsilon_{it}'\Sigma_u^{-1}\varepsilon_{it} \text{ and } \varepsilon_i'(A_T \otimes (\Sigma_1^{-1} - \Sigma_u^{-1}))\varepsilon_i = (1/T)\sum_{t,s=1}^{T} \varepsilon_{it}'(\Sigma_1^{-1} - \Sigma_u^{-1})\varepsilon_{is}$$

Using then the above expressions and Equation 4-55 we can rewrite the log likelihood as

$$L_i = -\frac{GT}{2}\ln 2\pi - \frac{1}{2}\ln|\Sigma_1| - \frac{1}{2}(T-1)\ln|\Sigma_u| - \frac{1}{2}\sum_{t=1}^{T}\varepsilon_{it}'\Sigma_u^{-1}\varepsilon_{it} - \frac{1}{2T}\sum_{t,s=1}^{T}\varepsilon_{it}'(\Sigma_1^{-1} - \Sigma_u^{-1})\varepsilon_{is} \tag{4-56}$$

For given covariance matrices, Σ_u and Σ_a, we take the first and second order conditions of Equation 4-56 with respect to the h^{th} element of the nonlinear vector of parameters. Using the same techniques as in the previous section, the gradient vector and information matrix of the coefficient vector are given, respectively, by

$$\frac{\partial \mathcal{L}}{\partial \beta_h} = \sum_{i=1}^{N}\sum_{t=1}^{T}\left(X_{it}\frac{\partial \beta}{\partial \beta_h}\right)'\Sigma_u^{-1}\varepsilon_{it} + \frac{1}{T}\sum_{i=1}^{N}\sum_{t,s=1}^{T}\left(X_{it}\frac{\partial \beta}{\partial \beta_h}\right)'\left(\Sigma_1^{-1} - \Sigma_u^{-1}\right)\varepsilon_{is} \tag{4-57}$$

$$I_\beta = \sum_{i=1}^{N}\sum_{t=1}^{T}\left(X_{it}\frac{\partial \beta}{\partial \beta_h}\right)'\Sigma_u^{-1}\left(X_{it}\frac{\partial \beta}{\partial \beta_h}\right) + \frac{1}{T}\sum_{i=1}^{N}\sum_{t,s=1}^{T}\left(X_{it}\frac{\partial \beta}{\partial \beta_h}\right)'\left(\Sigma_1^{-1} - \Sigma_u^{-1}\right)\left(X_{is}\frac{\partial \beta}{\partial \beta_h}\right) \tag{4-58}$$

Asymptotic covariance matrix of $\hat{\beta}$ is obtained by taking the inverse of the information matrix above. For given coefficient vector β, we do not need to derive the first- and second-order conditions with respect to the covariance of the error terms, since the nonlinearity is in the parameters of the model. Therefore, we adapt the results from Magnus (1982), since in that paper there was a time-specific error component and not a firm-specific, to get

$$\Sigma_u = \frac{1}{(T-1)N}\sum_{i=1}^{N} V_i(I_T - A_T)V_i' \text{ , } \Sigma_a = \frac{1}{T(T-1)N}\sum_{i=1}^{N} V_i(TA_T - I_T)V_i' \qquad (4\text{-}59)$$

An iterative procedure could be employed as in the previous section for the estimates that maximize the log-likelihood function. Further, to prevent the solution to converge towards a local maximum, Magnus (1982) suggests ensuring that Σ_u and Σ_a are positive semidefinite.

CHAPTER 5
APPLICATION TO U.S. BANKING INDUSTRY

5.1 Introduction

One of the objectives of this study is to utilize the differential production model as means of estimation of the input demand, output supply and efficiency measures of US banks. To examine the robustness of the differential model results and to highlight the differences in the description of the technology that are induced by fitting the differential model, a comparison is provided against a commonly used in the literature parametric specification (translog). The discussion of the results focuses on three aspects of technology: concavity, returns to scale and input substitution as measured by the Allen-Uzawa elasticities of substitution.

The differential model is based on the total differentiation of the first-order derivatives of any arbitrary cost or profit function given a technological constraint. As it was shown in Chapter 2, this provides an input-demand and output-supply system of equations for the multiproduct-multifactor firm. The restrictive assumption of the differential assumption as presented in Chapter 2 is the one of perfect competition that may not hold in the "empirical world" and thus limiting its applications. A dual approach, instead, involves specifying a flexible functional form that achieves a second-order approximation of any arbitrary twice differentiable cost function at a given point (Diewert 1971). The translog, which was developed by Cristensen, Jorgenson and Lau (1973), can be interpreted as a Taylor series expansion and is the most popular of the

Diewert flexible forms. However, White (1980) has shown that while second-order approximations allow us to attain any arbitrary function at a given point, there is no implication that the true function is consistent at this point. Moreover, different functional forms lead to different results for the same dataset, as Howard and Shumway (1989) indicated; and often fail to satisfy parameter restrictions.

In the empirical banking literature some of the major concerns are related to the functional form specification and to the validity of the efficiency measures obtained from such specifications. For instance, Berger and Humphrey (1997) have shown that a local approximation, such as the translog, usually provide poor approximations for banking data that are not near the mean scale and product mix. The geographic restrictions on branching that have contributed to the proliferation of banks in the United States and the large amount of mergers happened when a state allowed for branching, stimulated the interest on correct efficiency measures such as economies of scale. However, early findings on economies of scale were contradictory and naturally led to the use of non-parametric measures of efficiency.

In the next section a brief review of the performance and structure of the U.S. banking industry is provided, while in Section 5.3 previous findings on the "puzzle" of economies of scale, functional form specification and the controversy on what constitutes a bank's inputs and outputs, are presented. The data used for the analysis are described in Section 5.4, while the empirical model is presented in Section 5.5. Empirical results and comparison of the differential model and translog specification in terms of satisfying concavity, Allen elasticities of substitution and economies of scale are provided in Section 5.6.

5.2 The US Banking Industry in the 90s

The banking industry constitutes a major part of the U.S. economy and it can be described as a competitive industry. In recent years, the number of commercial banks in the U.S. has begun to fall dramatically. It has decreased from 14,095 in 1984 to around 8,337 in 2000 (Table 5-1) and most of the banks exiting have been small (less than $100 million in assets). Moreover, the large banks' share of assets has increased to almost one third, while the small banks' share has decreased to less than 5% (Dick 2002).

Bank failures played an important, but not predominant, role in the decline in the number of commercial banks during 1985–1992, and bank failures have played an almost negligible role in the continuing decline seen since 1992 (Berger and Mester 1997). The primary reason for the decline in the number of commercial banks since 1985 has been bank consolidation. Until the passage of the Riegle-Neal Interstate Banking and Branching Efficiency Act (1994), U.S. commercial banks were prohibited from branching across states. This Act permitted nationwide branching as of June 1997, while some states had already allowed for intrastate and interstate branching (as early as 1978). Recently, the Gramm-Leach-Bliley Act in 1999 allowed U.S. commercial banks to participate in securities activities, such as investment banking (underwriting of corporate securities) and brokerage activities involving corporate securities.

Table 5-1 illustrates the number of banks for the period 1990–2000 along profitability measures, such as return on equity and return on assets for the "average" bank in each year. Profitability in the banking sector, as measured by the mean return on equity rose by 1.2% from 5.44% in 1990 to 11% in 2000. An alternative measure of profitability, mean return on gross total assets, rose from 0.61% in 1990 to 1% in 2000. It is obvious though that both of these measures have been stable from 1993 to 2000, even if gross total assets are increasing steadily.

Table 5-1. Financial indicators for the U.S. banking industry, 1990–2000

Year	Number of Banks	ROE	ROA	GTA
1990	12,306	5.44	0.61	350,175
1991	11,917	6.82	0.68	349,659
1992	11,456	10.90	0.98	357,869
1993	11,064	11.97	1.11	380,463
1994	10,660	11.55	1.07	420,658
1995	10,004	11.78	1.12	465,465
1996	9,593	11.86	1.12	487,833
1997	9,167	11.83	1.13	544,932
1998	8,804	11.19	1.09	618,883
1999	8,606	11.21	1.02	652,281
2000	8,337	11.09	1.01	727,936

ROE refers to return on equity, ROA to return on assets and are percentages, while GTA is the gross total assets of the bank. All these measures were calculated from the Call Reports and are means for all banks in a given year. All financial data are in real 2000 values.

5.3 Brief Literature Review

Existing efficiency studies in the banking literature can be considered as a mixture of measuring cost efficiency or profit efficiency, employment of parametric or nonparametric methods, and utilization of frontier analysis or more traditional techniques (economies of scale and scope). Regarding comparisons between frontier methods and traditional methods, Berger, Hunter, and Timme (1993) suggested that, although frontier estimation methods are theoretically correct, studies that have compared results of frontier methods to those of more traditional estimation methods have found only small differences for scale measures. However, they showed that noticeable differences exist in scope measures between frontier and more traditional estimation methods. They also concluded that the translog specification for examining scope economies causes problems due to the multiplicative nature of the outputs.

Berger and Humphrey (1997) reviewed 130 studies, which examined efficiency by using frontier methods and they concluded that nonparametric methods yield slightly lower mean efficient estimates and seem to have greater dispersion than the results of the parametric methods. Clark (1988) reviewed thirteen studies that measured economies of scope and scale for commercial banks, credit unions and savings and loans associations. He concluded that overall economies of scale exist at low levels of input, but no consistent evidence of economies of scope was found. Further, some evidence of cost complementarities also exists. However, Hunter, Timme and Yang (1990), tested for economies of scale and scope in large banks by using a minflex Laurent functional form but they found no cost complementarities. For robustness of their results they included deposits as inputs and as outputs. Noulas et al. (1990) examined returns to scale and input substitution for large U.S banks. They rejected the hypothesis of short-run or long-run returns to scale and suggested that economies or diseconomies of scale are not large enough to support the creation of only a few large banks, based on cost economies.

Berger, Hancock and Humphrey (1993) estimated scope measures using a profit function approach. They argued that a profit function method should be used to examine "optimal scope economies" (see also Berger 1995) and they emphasized the importance of the imposition of curvature properties. McAllister and McManus (1993) found that banks face increasing returns to scale to about $500 million of assets, and Berger and Mester (1997), who compare banks within size ranges, concluded that in all ranges the mean bank operates at a less than efficient scale. Featherstone and Moss (1994) measured economies of scale and scope in agricultural banking and they found that economies of scale and scope do not exist for small size banks, which include the agricultural banks.

More recently, Hunter and Timme (1995) suggested that cost inefficiencies dominate the impact of scale and scope diseconomies.

Turning now to the problem of selection of flexible functional form, Ellinger and Neff (1993) concluded that the most commonly used functional forms to measure banks' costs are the translog, generalized translog and minflex Laurent. Lawrence (1989) tested the robustness of competing flexible functional forms by using a generalized functional form, i.e. a Box-Cox transformation of all the variables in a cost function. He rejected the Box-Cox transformation of the generalized translog, but indicated that translog specifications provide an adequate fit to bank cost data. Nonetheless, a disadvantage of the log-quadratic output structure of the translog is its inability to model cost behavior when any output is zero. As a result the estimated translog cost function cannot be used to estimate economies of scope or product specific economies of scale (Pulley and Braustein 1992). This led Pulley and Braustein (1992), to propose a composite cost function for multiproduct firms, in order to estimate economies of scale in banking.

McAllister and McManus (1993) compared the results of fitting a Fourier functional form and a translog to bank data. They found that banks exhaust scale economies at a much larger output level under a Fourier specification than suggested by global estimation of a translog cost function. They concluded that the translog cost function specification gives a poor approximation when applied to all bank sizes, suggesting that nonparametric estimation procedures should be examined. Mitchell and Onvural (1996) arrived at the same conclusions about the translog specification when it is fitted across bank sizes.

Berger and Mester (1997) attribute differences in estimates of scale economies to a fundamental shift in bank costs over time that is probably associated with regulatory or technological changes. They found that for the period 1990–1995 banks operated at smaller than efficient scale irrespective of whether a global translog function or a Fourier functional form was used to estimate the bank's cost function. It should be noted though, that the Fourier flexible functional form is not free of troubles. For instance, it is left to the researcher whether to augment the underlying translog function with trigonometric terms or orthogonal polynomials, and the number of such terms to include for estimation.

The results of the aforementioned studies indicate that the average cost curve for banks may be U-shaped and that economies of scale exist only for small banks. The findings of scope economies are inconclusive. It appears that this criticism of the standard methodology is partially based on the inability of previous U.S. studies to find strong evidence of economies of scale for the largest banks. Recent evidence though suggests that sizable economies of scale exist that increase with bank size (Hughes and Mester 1998, Berger and Mester 1997).

Another problem that concerns the banking literature is that estimates of bank cost characteristics depend on changes in the definition of inputs and outputs. Specifically, concerns exist on whether deposits are inputs or outputs and which factors of production are quasi-fixed. Flannery (1982) argued that factors such as transaction and information costs, which could result into a significant proportion of the inputs into the bank's production function, are quasi-fixed in the short-run. In particular, retail bank deposits should be considered as quasi-fixed factors of production because both banks and their costumers incur "setup costs" or "transaction specific" investments costs. However, there is no consensus in the definition of inputs and outputs. For example, Hunter and Timme (1995) used core deposits and physical capital as quasi-fixed inputs to examine bank scale economies, but compared this case with a case where some of the core deposits were treated as outputs and with a case where all inputs were variable. They found that for large size banks the ray scale economy estimates were not considerably different under the three specifications, but the mean efficiency indices were significantly different. Further, Berger and Mester (2003) used physical capital and financial equity capital as fixed inputs but treated core deposits as a variable input in order to examine bank performance. Hughes et al. (2001) provided an answer to whether deposits are inputs or outputs by simply testing the sign of the partial derivative of the operating cost function with respect to the quantity of deposits. Their results strongly suggest that deposits are inputs.

5.4 Data Description

There are numerous studies that specify different inputs and outputs of a multiproduct bank and then compare which specification better fits the data. Particularly, there are three alternative approaches in defining inputs and outputs. These are (1) the intermediation or asset approach, (2) the user cost and (3) the value-added approach (Berger and Humphrey 1992). According to the intermediation approach banks act as financial intermediaries between borrowers and lenders, (Sealey and Lindley 1977). In this framework purchased funds and core deposits are considered as inputs, while bank outputs are loans and other assets. Physical inputs, as labor and premises, are specified as inputs that generate costs. Total costs include operating and interest expenses of the bank. The user cost approach classifies the output or input of a bank according to its net contribution to bank revenue, while the value-added approach determines outputs based on those having the largest value added. Thus, under the value-added approach deposits are specified as outputs since they have a large impact on value added.

In this study we view the banking firm as an intermediary (Diamond 1984), operating in competitive markets and using a multiple input-output technology. This assumption naturally leads to the selection of the intermediation approach, which is also more compatible with profit maximization (Berger and Mester 2003). This is because deposits can be considered as reducing profits because the bank borrows these funds and has an interest expense (except for checking accounts) while loans and other assets generate positive cash flows and profits.

The data are taken from Reports of Condition and Income (Call Reports) for the period 1990–2000, which contain balance sheet and income statement data for all U.S. commercial banks. Since the data contain quarterly observations for each bank, it is desirable to aggregate them into annually observations. Specifically, all the income statement data are in year to date format and so we used the last quarter of the year, while the balance sheet represents snapshots of the bank at the end of each quarter and so when selecting a variable for the balance sheet we used the average of the four quarters in each year for this variable. Observations for banks which involved in a merger were deleted for that quarter in which the merger occurred.

The specification of inputs and outputs follows closely the one of Berger and Mester (2003), with some differences though, since we have not included off-balance sheet items but have calculated a market rental price of equity capital. Table 5-2 gives the definitions of the variables that are going to be used in the empirical specification, their sample means and standard deviations for the years 1990, 1995 and 2000. Specifically, we assume that the bank transforms five inputs, three variable and two quasi-fixed, into four outputs. The variable inputs are purchased funds, core deposits and labor, while quasi-fixed inputs are the physical capital of the bank and the equity capital. The outputs of the bank are commercial loans, business loans, real estate loans and securities. The prices of the variable inputs were found by dividing the interest expense on that input over the stock of the input. For outputs, the prices are defined in a similar way and are the interest income from the specific loan category or security over the stock of that output. Berger and Mester (2003) suggest that this way of calculation is problematic, since these prices may be endogenous. They suggest, instead, for each price of a specific bank to substitute the average price that all other banks received in the same market. This method, however, may solve the problem of endogeneity but it creates another problem, that of measurement error, since local market area prices are only an approximation to the effective costs and income that are reflected in the banks' balance sheet and income statement.

For the differential model it was necessary to specify market rental prices for the two quasi-fixed inputs. For the market rental price of the physical capital was used a state average of the ratio of occupancy expense over the stock of physical capital. Before taking the state average, outliers were deleted both in the upper and lower bound of the distribution of physical capital. This measure reflects also local conditions for the market rental price of physical capital but it may introduce also a measurement error for bank holding companies (BHC) or banks that operate in different states. The market rental price of equity was obtained by an average return on equity in the U.S. state that each bank belongs. The observations with negative or very high return on equity were deleted before taking those averages. Then for each bank the state average was substituted. These approximations, however, are taken into account into the differential model since all the inputs of the bank are corrected for this approximation as was shown in the parameterization of the model. Instead, McAllister and McManus (1993) arbitrarily picked a required return on equity, which they assumed was identical across all banks. Clark (1996) used the Capital Asset Pricing Model to determine a market-based required return on equity. Hughes et al. (2001) argue that a plausible range of market return for banks' equity is between 0.14 and 0.18 and they evaluate cost minimization (optimality) at this range of prices.

Observations with less that 1% of capital to equity ratio were deleted since they look suspicious as Berger and Mester (2003) noted. Also, observations with zero values for physical capital were deleted. The only environmental variable that was available from the Call Reports and was used in this study was a one-digit code indicating the chartering authority of the bank. Specifically, banking-type entities, thrifts, credit unions and Edge corporations have federal charters, while agreement corporations have state charters. We used banks with state charters as the base case. Regulatory variables, such as unit branching state, were not made available and so they were not included in the study. Finally, the use of the differential model necessitates first differencing of the individual series and so we deleted the observations of the banks that did not exist in the previous or after year of analysis. This resulted to a loss of 15,000 observations, leaving us with 96,584 observations for the period 1990–2000. Instead, in the translog where there is no differencing the total number of observations was 111,908.

Table 5-2. Definition of variables and descriptive statistics (mean and standard deviation)

Symbol	Definition	1990	1995	2000
Cost and Revenue				
VC	Variable cost	27,683 (324,861)	25,185 (258,501)	40,620 (545,901)
R	Revenues	33,757 (384,240)	37,822 (396,330)	58,878 (806,564)
Variable input quantities				
x_L	Number of employees	125 (955)	146 (1,145)	192 (2,360)
x_{PF}	Quantity of purchased funds (time deposits over \$100,000, foreign deposits, federal funds purchased, demand notes issued to US Treasury, trading liabilities, other borrowed money, mortgage indebtedness and obligations under capitalized leases, and subordinated notes and debentures)	118,737 (1,949,831)	167,161 (3,025,916)	312,929 (5,781,246)
x_{CD}	Quantity of core deposits (time deposits under \$100,000, domestic transaction accounts and savings deposits)	198,770 (1,181,062)	250,320 (1,525,891)	340,506 (3,859,352)
Variable input prices				
w_L	Bank's price of labor (Salaries and employee benefits/Number of employees)	34.7558 (8.7998)	37.9986 (9.7581)	43.3118 (12.3015)
w_{PF}	Bank's price of purchased funds (expense of purchased funds / x_{PF})	0.0755 (0.0165)	0.0539 (0.0113)	0.0567 (0.008)
w_{CD}	Bank's price of core deposits (expense of core deposits / x_{CD})	0.0627 (0.0085)	0.041 (0.0068)	0.0422 (0.0079)
Variable output quantities				
y_{CL}	Loans to individuals in domestic offices	36,391 (284,169)	47,520 (385,290)	57,315 (653,672)
y_{BL}	All loans other than consumer loans, real estate loans	92,036 (1,350,062)	107,721 (1,770,320)	186,389 (3,585,618)
y_{RE}	Real estate loans in bank's domestic offices	88,701 (656,245)	120,489 (817,765)	201,930 (2,363,300)

Table 5-2. (Continued)

Symbol	Definition	1990	1995	2000
y_s	Securities: All other non financial assets	129,364 (1,203,502)	183,781 (2,130,830)	273,778 (4,237,691)
Variable output prices				
p_{CL}	Bank's price of consumer loans (interest income on consumer loans less provisions for loan and lease losses and allocated transfer risk allocated to consumer loans / y_{CL})	0.1032 (0.0339)	0.0923 (0.0288)	0.0914 (0.0304)
p_{BL}	Bank's price of business loans (interest income on business loans less provisions for loan and lease losses and allocated transfer risk allocated to business loans / y_{BL})	0.1291 (0.0351)	0.1123 (0.0291)	0.106 (0.0291)
p_{RE}	Bank's price of real estate loans (interest income on real estate loans less provisions for loan and lease losses and allocated transfer risk allocated to real estate loans)	0.0958 (0.023)	0.0825 (0.0192)	0.0803 (0.0195)
p_s	Bank's price of securities = interest income on securities / y_s	0.0786 (0.0141)	0.0612 (0.0133)	0.0628 (0.0147)
Quasi-fixed input quantities				
z_1	Premises and fixed assets	5,086 (49,190)	6,577 (65,107)	8,879 (109,805)
z_2	Quantity of financial equity capital	22,274 (156,851)	36,767 (302,486)	60,178 (767,529)
Quasi-fixed input market rental prices				
w_{z_1}	Proxy for the price of z_1 (State average of occupancy expense / z_1)	0.3221 (0.0452)	0.3114 (0.0402)	0.295 (0.0478)
w_{z_2}	Proxy for the price of z_2 (Modified State average Return on Equity)	0.1196 (0.0168)	0.1303 (0.0158)	0.1298 (0.0165)

All financial variables are measures in 1000s of constant 2000 dollars by using the implicit GDP price deflator. All prices are interest rates

5.5 Empirical Model

Following the parameterization of Chapter 3, the empirical differential model for three variable inputs, two quasi-fixed and four outputs consists of the following system of input-demand and output-supply equations

$$\tilde{x}_{it} = a_i + \sum_{r=1}^{4} \theta_i^r \tilde{y}_{rt} + \sum_{k=1}^{2} \xi_i^k \tilde{z}_{kt} + \sum_{j=1}^{2} \pi_{ij} Dw_{jt} + \varepsilon_{it}, \qquad i, j = 1,2 \tag{5-1}$$

$$\tilde{y}_{rt} = a_i^* + \sum_{s=1}^{4} \alpha_{rs} Dp_{st} - \sum_{i=1}^{2}\sum_{s=1}^{4} \alpha_{rs} \theta_i^s Dw_{it} - \sum_{k=1}^{2} \beta_{rk} Dz_{kt} + \varepsilon_{rt}^{**}, \qquad r, s = 1,\ldots,4 \tag{5-2}$$

where we have superseded the firm subscript for notational convenience and $i, j = 1,\ldots,2$ denote inputs of a multiproduct firm and $r, s = 1,\ldots,4$ denote its outputs. The output variables are defined as $\tilde{y}_{rt} = \overline{\gamma}_{2t} \overline{g}'_{rt} Dy_{rt}$, while the quasi-fixed inputs as $\tilde{z}_{kt} = \overline{\gamma}_{3t} \overline{\mu}'_{kt} Dz_{kt}$. For any variable we define the finite, first difference as $Dq_t = \ln q_t - \ln q_{t-1}$. Prices of variable inputs and outputs are given by Dw_{it} and Dp_{st}, respectively. Then,

$\overline{f}_{it} = \frac{1}{2}\left(f_{it} + f_{i,t-1}\right)$ and $\overline{g}'_{rt} = \frac{1}{2}\left(g'_{rt} + g'_{r,t-1}\right)$ are the arithmetic means of the variable cost ratio and the revenue-cost ratio, respectively. In this formulation we have also corrected the variable inputs $\tilde{x}_{it}$ for the approximation in the quasi-fixed inputs and for the difference approximation, since we use $\tilde{x}_{it} = \overline{f}_{it}(Dx_{it} - E_t)$, where the correction $E_t = DX_t - \overline{\gamma}_{2t} DY_t - \overline{\gamma}_{3t} DZ_t$ comes from the total input decision of the firm. Further,

$DX_t = \sum_{i=1}^{n} \overline{f}_{it} Dx_{it}$, $DY_t = \sum_{r=1}^{m} \overline{g}'_{rt} Dy_{rt}$ and $DZ_t = \sum_{k=1}^{l} \overline{\mu}'_{kt} Dz_{kt}$ denote the Divisia indexes of variable inputs, outputs and quasi-fixed inputs, respectively. For the quasi-fixed inputs we have defined $\overline{\mu}'_{kt} = \frac{1}{2}(\mu'_{kt} + \mu'_{k,t-1})$ as the arithmetic mean of the market rental price ratio $\left(v_{kt}z_{kt} / \sum_k v_{kt}z_{kt}\right)$ where v_{kt} is the market rental price of quasi-fixed input k at time t. Similarly, we have defined $\bar{\gamma}_{3t}$ as the geometric mean of the market rental value of quasi-fixed inputs over variable cost ratio $\left(\sum_k v_{kt}z_{kt}\right)/VC_t$. The geometric mean of the revenue variable cost ratio is given by $\bar{\gamma}_{2t} = \sqrt{\frac{R_t \cdot R_{t-1}}{VC_t \cdot VC_{t-1}}}$. Further, we assume that the coefficients θ_i^r, ξ_i^k, π_{ij}, α_{rs} and β_{rk} are constant, while we have shown that the disturbances in both systems are homoscedastic.

The coefficients of the differential model do have economic interpretation unlike other parametric models. Specifically, π_{ij} are negative semidefinite price terms of rank $n-1$, known as Slutsky coefficients in the Rotterdam model; ξ_i^k is the share of i^{th} variable input in the shadow price of quasi-fixed input; θ_i^r is the share of i^{th} variable input in the marginal cost of the r^{th} product; α_{rs} are the output price terms, which must be positive definite; a_i and a_i^* are technology shifters in the input-demand and output-supply system, respectively; and β_{rk} are the summation of the changes in the marginal costs of the various products due to the changes in the availability of quasi-fixed inputs, weighted by the coefficients θ_{rs}^* and total output price elasticity ψ^*, where $\beta_{rk} = \bar{\gamma}_{2t}\psi^*\eta_{rk}$. Laitinen (1980) showed how one could recover the θ_{rs}^* and ψ^* terms.

The adding up restrictions $\sum_i \theta_i^r = 1$, $\sum_i \xi_i^K = 1$, $\sum_i \pi_{ij} = 0$ and $\sum_i a_i = 0$, where $i, j = 1,...,n$ and $k = 1,...,l$, homogeneity in the input-demand system $\sum_j \pi_{ij} = 0$ and homogeneity in the output-supply system have been imposed by subtracting the price of labor from all input prices, Dw_{it}, and output prices, Dp_{st}. Symmetry in the input-demand system, $\pi_{ij} = \pi_{ji}$, in the output supply $[a_{rs}] = [a_{sr}]$ and the nonlinear symmetry conditions for the coefficients of the input prices in the output-supply system were imposed in the estimation procedure, as was shown in Chapter 4.

Considering the above specification of the input-demand and output-supply system, we could derive the elasticity of i^{th} input with respect to the r^{th} output (ε_{xy}), own and cross price elasticities (ε_{xw}), fixed factor elasticities (ε_{xz}), cost elasticity with respect to fixed factor (ε_{VC,z_k}), cost elasticity with respect to r^{th} output (ε_{VC,y_r}), and technology shifts in the input-demand and output-supply curves, respectively, as

$$\varepsilon_{xy} = \frac{d\ln x_i}{d\ln y_r} = \frac{\bar{\gamma}_2 \bar{g}_r}{\bar{f}_i}\hat{\theta}_i^r \tag{5-3}$$

$$\varepsilon_{xw} = \frac{d\ln x_i}{d\ln w_j} = \frac{\hat{\pi}_{ij}}{\bar{f}_i} \tag{5-4}$$

$$\varepsilon_{xz} = \frac{d\ln x_i}{d\ln z_k} = \frac{\bar{\gamma}_3 \bar{\mu}_k}{\bar{f}_i}\hat{\xi}_i^k \tag{5-5}$$

$$\varepsilon_{VC,z_k} = \sum_i \bar{f}_i \frac{d\ln x_i}{d\ln z_k} = \sum_i \bar{\gamma}_3 \bar{\mu}_k \hat{\xi}_i^k \tag{5-6}$$

$$\varepsilon_{VC,y_r} = \sum_i \bar{f}_i \frac{d\ln x_i}{d\ln y_r} = \sum_i \bar{\gamma}_2 \bar{g}_r \hat{\theta}_i^r \tag{5-7}$$

$$\varepsilon_a = \frac{\hat{a}_i}{\bar{f}_i} \text{ and } \varepsilon_{a^*} = \frac{\hat{a}_i^*}{\bar{g}_r} \tag{5-8}$$

Similarly, one can find the elasticities from the output-supply system. Since a comparison with the translog specification is the main focus of this study estimates for all these elasticities are not going to be provided. Estimates are going to be provided for the returns to scale (RTS) and for Allen elasticity of substitution derived by Laitinen (1980, page 42) for the differential model, as

$$\varepsilon_{xw}^{A} = \frac{\hat{\pi}_{ij}}{\bar{f}_i \bar{f}_j}, \quad i \neq j \tag{5-9}$$

Similarly Allen elasticities of transformation for the output-supply system can be written, as

$$\varepsilon_{yp}^{*A} = \frac{\hat{\alpha}_{rs}}{\bar{g}_r \bar{g}_s}, \quad r \neq s \tag{5-10}$$

The system of Equations 5-1 and 5-2 is estimated by using the maximum likelihood procedure, outlined in Chapter 4, Section 4.2 under the assumption of normal distributed disturbances for a panel of banks for the period 1990–2000. Consequently, we append time dummy variables in each equation to capture time specific effects that are not captured by the overall constant term, representing common shocks in the technology of all firms in a given year. As mentioned in Chapter 4, the differential model is a first-differences model, implying that any firm-specific, time-invariant fixed-effects have been eliminated. However, we have appended one firm-specific variable, which is the charter type of the bank.

While we have assumed that first differences make each series stationary, there is no a priori reason for a one period lag in each time series. One could test for an ARIMA (0,d,0) model and then difference each time series accordingly. Given that the sample used is highly unbalanced and tests should be performed for each bank in the sample for the whole period, we assume that the differential follows an ARIMA (0,1,0) and so the first-differences model is appropriate. Therefore the differential model has already accounted for heterogeneity in size, age, management, employee's education, technology and location under the assumption that these effects can be represented by a bank-specific fixed intercept term and this term is not correlated with the intercept terms.

Estimation of the model as presented in Equations 5-1 and 5-2 for all banks and for the period 1990–2000, makes the implicit assumption that all banks have the same slope coefficients and intercept for each year in the sample and the same slope coefficients across years. This seems to be a quite odd assumption, given earlier results in the literature suggesting that economies of scale differ across bank sizes and product mix. Berger and Mester (2003) estimated a Fourier flexible functional form for each year in the sample in order to allow for all the coefficients to vary over time and captured heterogeneity across firms through the use of firm-specific dummies (bank belongs to unit banking states, limited branching states, primary regulator of the bank and other). However, these dummy variables account only for changes in the intercept of the model for a given cross section and so implicitly they made the assumption of same slope coefficients for all firms in a given year. Differences on asset size and product mix may be also manifested in the slope coefficients. The slope coefficients in the differential model are actually marginal shares, e.g. θ_i^r is the share of i^{th} variable input in the marginal cost of the r^{th} product of a specific firm. Making the assumption of common slopes across firms it implies that these firms should have a homogeneous technology in average. To account for slope heterogeneity, a random coefficients model could be implemented (Hsiao 1986, page 128), but this task is left for future research.

To capture size effects in the intercept of the differential model two more dummy variables relating to asset size have been included. Specifically, we defined small banks as those with asset size below 300 million US dollars, medium scale banks as those with asset size from 300 million to 2 billion and we excluded the large banks with asset size above 2 billion as the base case. In order to capture scale effects these dummies were not defined in real 2000 asset values but in their annual values. The discussion of the results focus on three different years in the sample: (1) 1991, which is in the midpoint of the "credit crunch" period (1989–1992), (2) 1998, which is right after the enactment of the Riegle-Neal Interstate Banking and Branching Efficiency Act (1994) that permitted nationwide branching in 1997 and (3) in 2000, which is the year after the Gramm-Leach-Bliley Act of 1999 that permitted commercial banks in participating in securities activities such as investment banking. Another reason for centering the discussion on these years was the observation from Table 5-1 of stability in the mean return of equity and assets from 1993 and after. Finally, note that these years in the differential model are actually changes from the previous years (i.e 1991 is the change from 1990 to 1991).

The translog cost specification with the input shares equations, where homogeneity has been imposed by dividing the input prices and the variable cost by the price of labor, has the following form

$$\ln\left(\frac{VC}{w_L}\right) = \alpha + \sum_{i=1}^{2}\beta_i \ln(w_i / w_L) + \frac{1}{2}\sum_{i=1}^{2}\sum_{j=1}^{2} b_{ij}\ln(w_i / w_L)\ln(w_j / w_L) + \sum_{r=1}^{4} g_r \ln y_r$$

$$+\frac{1}{2}\sum_{r=1}^{4}\sum_{s=1}^{4} g_{rs}\ln y_r \ln y_s + \sum_{k=1}^{2} f_k \ln z_k + \frac{1}{2}\sum_{k=1}^{2}\sum_{l=1}^{2} f_{kl}\ln z_k \ln z_l + \sum_{i=1}^{2}\sum_{r=1}^{4} th_{ir}\ln(w_i / w_L)\ln y_r$$

$$+\sum_{i=1}^{2}\sum_{k=1}^{2} ks_{ik}\ln\left(\frac{w_i}{w_L}\right)\ln z_k + \sum_{r=1}^{4}\sum_{k=1}^{2} p_{rk}\ln y_r \ln z_k + \sum_{t=1}^{11} d_t D_t + \sum_{i=1}^{2}\sum_{t=1}^{11} d_{it}\ln w_i D_t + \sum_{r=1}^{4}\sum_{t=1}^{11} c_{rt}\ln y_r D_t$$

$$+\sum_{k=1}^{2}\sum_{t=1}^{11} e_{kt}\ln z_k D_t + \sum_{s=1}^{3} a_s A_s + \sum_{i=1}^{2}\sum_{s=1}^{3} a_{is}\ln w_i A_s + \sum_{r=1}^{4}\sum_{s=1}^{3} h_{rs}\ln y_r A_s + \sum_{k=1}^{2}\sum_{s=1}^{3} m_{ks}\ln z_k A_s \quad (5\text{-}11)$$

and by Shephard's lemma the input shares equations are given by

$$S_i = b_i + \sum_{j=1}^{2} b_{ij} \ln\left(\frac{w_j}{w_L}\right) + \sum_{r=1}^{4} th_{ir} \ln y_r + \sum_{k=1}^{2} ks_{ik} \ln z_k + \sum_{t=1}^{11} d_{it} T_t + \sum_{s=1}^{3} a_{is} A_s \tag{5-12}$$

The usual symmetry conditions have been imposed on the estimation procedure, while we have dropped on share equation due to singularity of the disturbances. In this specification we have included time dummies T_t for each year in the sample, which capture technological changes between the periods. The 1998 year dummy was dropped as a base period. In 1997 the Riegle-Neal Act became effective and therefore a comparison with the other years could reveal any possible effects of the Act in the technology or economies of scale of banks.

To test for technological changes between the base year and any other year in the sample, it is only required a Wald test restricting all the coefficients for the dummy that needs to be tested, to zero. Further, A_s is a set of three dummies that capture asset size of banks and we have dropped the large banks as a base. In this specification we allow not only for variable intercept terms but also for variable slope coefficients in the input prices, the output levels and in the quasi-fixed inputs. Economies of scale (SCE), also called Ray scale economies, measure the elasticity of cost with respect to proportional changes in the scale of outputs holding the product mix unchanged (Baumol, Panzar and Willig, 1988). That is,

$$SCE = \sum_{r=1}^{4} g_r + \sum_{r=1}^{4}\sum_{s=1}^{4} g_{rs} \ln y_s + \sum_{i=1}^{2}\sum_{r=1}^{4} th_{ir} \ln \frac{w_i}{w_L} + \sum_{r=1}^{4}\sum_{k=1}^{2} p_{rk} \ln z_k +$$

$$+ \sum_{r=1}^{4}\sum_{t=1}^{11} c_{rt} D_t + \sum_{r=1}^{4}\sum_{s=1}^{3} h_{rs} \ln y_r A_s \tag{5-13}$$

The Allen-Uzawa elasticities of substitution for the translog can be calculated (Binswanger 1974), as follows

$$\varepsilon_{ij}^{A} = \frac{b_{ij}}{S_i + S_j} + 1 \quad for \quad i \neq j \text{ and } \varepsilon_{jj} = \frac{b_{ij} + S_j(S_j - 1)}{S_j^2} \tag{5-14}$$

5.6 Empirical Results

The differential system of equations, Equations 5-1 and 5-2, and the translog cost function (Equation 5-11) along with the input share equations (Equation 5-12) were estimated by the nonlinear Full Information Maximum Likelihood (FIML) method developed in Chapter 4, Section 4.2. The multivariate Gauss-Newton procedure for time-specific fixed-effects panel data was implemented in SAS by using the proc model and fit statements. The symmetry and homogeneity restrictions were imposed on the estimation procedure as it was shown in Chapter 4, while the concavity property of the cost function was tested for both models.

In Table 5-3, the parameter estimates for the differential model are presented. All the coefficients are statistically significant at the 1% level of confidence and the own input price coefficients π_{ii} are negative, satisfying the monotonicity assumption of the underlying cost function. Insignificant terms were mostly the coefficients of the time dummy variables. In particular, insignificant were the following terms: the 1991–92 and 1996–97 period in the core deposits (CD) input-demand equation; the 1990–91 and 1992–93 in the consumer loans (CL) output-supply equation; the 1993–94, 1996–97 and 1998–99 in the business loans (BL) equation and 1991–92 in the securities (SR) equation. These results indicate that only systematic changes in the technology exist between these periods and 1997–98 period and those are captured by the common intercept term. However, in the CL equation the intercept is insignificant indicating no systematic changes in the technology for this output in a Hicks neutral sense. The CL and BL had the lowest fit in the data as measured by the equation adjusted R-square, while the input-demand equations had the largest fit with the CD equation explaining 88% of the cases in the sample.

One of the advantages of the differential system is that the coefficients of the model have economic interpretation. For instance, in the output-supply system, the coefficients of the output prices are the product of the price elasticity of total output ψ^* and the terms θ^*_{rs}, which reveal the substitution or complementarity relation in production. In Table 5-3, this product appears as α_{rs} and since ψ^* is always positive (proved in Chapter 2), the sign of those coefficients shows whether the outputs are specific substitutes or complements. For instance, consumer loans and business loans are specific substitutes (since $\alpha_{12} < 0$), which implies that an increase in the relative input-deflated price of business loans will result to a decrease in the production of consumer loans.

The parameter estimates and standard errors for the translog cost function and the input share equations are presented in Table 5-4. It appears that only nine parameters are insignificant at levels of confidence greater than 10%, while all other estimates are significant at the 1% level of confidence. This result was expected given the large sample, which consists of 111,908 observations for the period 1990–2000. The insignificant parameters are the cross terms of outputs and of quasi-fixed inputs with time dummies in the translog cost function. This implies that the slope coefficient of the specific output or quasi-fixed input and for the specific year does not change with respect to the base year. Also, the estimate of medium size banks (a_2) is insignificant, implying that the intercept of the cost function does not vary with respect to medium size banks.

Table 5-3. Parameter estimates and standard errors for the differential model, 1990–2000

Equation	Parameter estimate		Standard error	Equation	Parameter estimate		Standard error
CD	θ_1^1	0.4242	0.00245	LB	θ_3^1	0.2345	0.00175
CD	θ_1^2	0.5779	0.00198	LB	θ_3^2	0.2547	0.00141
CD	θ_1^3	0.6415	0.00168	LB	θ_3^3	0.2124	0.00119
CD	θ_1^4	0.6907	0.00181	LB	θ_3^4	0.1434	0.00129
CD	ξ_1^1	0.4348	0.00568	LB	ξ_3^1	0.5770	0.00399
CD	ξ_1^2	0.1952	0.00531	LB	ξ_3^2	0.5854	0.00375
CD	π_{11}	-0.0773	0.00093	LB	π_{13}	0.0823	0.00068
CD	π_{12}	-0.0049	0.00061	LB	π_{23}	0.0188	0.00066
CD	c_1	0.0128	0.00069	LB	π_{33}	-0.1011	0.00068
CD	a_0	-0.0196	0.00113	LB	a_{ob}	-0.0045	0.00079
CD	d_1	0.0140	0.00101	LB	r_b	0.0010*	0.00024
CD	d_7	0.0108*	0.00112	LB	d_{1b}	0.0041	0.00070
CD	p_1	-0.0009*	0.00071	LB	d_{2b}	0.0010	0.00078
CD	e_1	-0.0053	0.00071	LB	c_b	0.0096	0.00048
CD	f_1	0.0014	0.00070	LB	p_b	0.0317	0.00051
CD	g_1	0.0175	0.00072	LB	e_b	0.0246	0.00050
CD	h_1	0.0030	0.00072	LB	f_b	0.0042	0.00049
CD	k_1	0.0004*	0.00073	LB	g_b	-0.0207	0.00051
CD	n_1	-0.0136	0.00074	LB	h_b	-0.0005	0.00050
CD	r_1	-0.0009	0.00034	LB	k_b	-0.0013	0.00050
CD	s_1	-0.0069	0.00075	LB	n_b	0.0074	0.00051
				LB	s_b	-0.0094	0.00052
PF	θ_2^1	0.3412	0.00241	PF	d_8	-0.0119	0.00107
PF	θ_2^2	0.1673	0.00194	PF	p_2	-0.0305	0.00067
PF	θ_2^3	0.1460	0.00164	PF	e_2	-0.0189	0.00067
PF	θ_2^4	0.1658	0.00177	PF	f_2	-0.0052	0.00067
PF	ξ_2^1	-0.0118	0.00549	PF	g_2	0.0030	0.00069
PF	ξ_2^2	0.2193	0.00516	PF	h_2	-0.0023	0.00068
PF	π_{22}	-0.0118	0.00062	PF	k_2	0.0012	0.00069
PF	c_2	-0.0224	0.00066	PF	n_2	0.0064	0.00071
PF	a_1	0.0242	0.00108	PF	r_2	-0.0000	0.00033
PF	d_2	-0.0181	0.00097	PF	s_2	0.0165	0.00071

Table 5-3. (Continued)

Equation	Parameter	estimate	Standard error	Equation	Parameter	estimate	Standard error
CL	c_3	-0.0013*	0.00086	BL	c_4	-0.0212	0.00108
CL	α_{11}	-0.0038	0.00045	BL	α_{12}	-0.0014	0.00039
CL	$c_{11}{}^{a}$	0.0003*	0.00038	BL	α_{22}	-0.0123	0.00066
CL	$c_{12}{}^{a}$	-0.0008	0.00015	BL	$c_{21}{}^{a}$	-0.0053	0.00049
CL	β_{11}	0.0147	0.00071	BL	$c_{22}{}^{a}$	-0.0019	0.00016
CL	β_{12}	0.0947	0.00142	BL	β_{21}	0.0372	0.00089
CL	a_2	0.0002*	0.00143	BL	β_{22}	0.1442	0.00177
CL	d_3	-0.0036	0.00127	BL	a_3	0.0164	0.00178
CL	d_9	-0.0058	0.00141	BL	d_4	-0.0001*	0.00159
CL	p_3	-0.0056	0.00089	BL	d_{10}	-0.0086	0.00175
CL	e_3	-0.0013*	0.00088	BL	p_4	-0.0326	0.00111
CL	f_3	0.0100	0.00088	BL	e_4	-0.0251	0.00110
CL	g_3	0.0074	0.00091	BL	f_4	-0.0005*	0.00110
CL	h_3	0.0075	0.00090	BL	g_4	-0.0021	0.00113
CL	k_3	0.0039	0.00091	BL	h_4	-0.0033	0.00112
CL	n_3	0.0054	0.00093	BL	k_4	-0.0004*	0.00114
CL	r_3	0.0000*	0.00043	BL	n_4	-0.0006*	0.00116
CL	s_3	0.0067	0.00094	BL	r_4	-0.0024	0.00054
				BL	s_4	0.0057	0.00117
RE	c_5	-0.0110	0.00126				
RE	α_{13}	-0.0008	0.00046	RE	p_5	-0.0179	0.00130
RE	α_{23}	-0.0007*	0.00056	RE	e_5	-0.0205	0.00128
RE	α_{33}	-0.0129	0.00093	RE	f_5	-0.0034	0.00128
RE	$c_{31}{}^{a}$	-0.0029	0.00060	RE	g_5	-0.0075	0.00132
RE	$c_{32}{}^{a}$	-0.0008	0.00018	RE	h_5	-0.0023	0.00131
RE	β_{31}	0.0583	0.00103	RE	k_5	0.0068	0.00133
RE	β_{32}	0.2583	0.00206	RE	n_5	0.0189	0.00136
RE	a_4	0.0155	0.00208	RE	r_5	-0.0028	0.00063
RE	d_5	0.0155	0.00186	RE	s_5	0.0288	0.00137
RE	d_{11}	0.0171	0.00205				

Table 5-3. (Continued)

Equation	Parameter estimate		Standard error	Equation	Parameter estimate		Standard error
SR	α_{14}	0.0048	0.00051	SR	d_{12}	-0.0083	0.00190
SR	α_{24}	0.0042	0.00063	SR	p_6	-0.0009*	0.00121
SR	α_{34}	0.0089	0.00078	SR	e_6	-0.0200	0.00120
SR	c_6	0.0047	0.00117	SR	f_6	-0.0297	0.00119
SR	α_{44}	-0.0212	0.00120	SR	g_6	-0.0306	0.00123
SR	c_{41} [a]	-0.0044	0.00063	SR	h_6	-0.0077	0.00122
SR	c_{42} [a]	0.0001	0.00019	SR	k_6	-0.0201	0.00123
SR	β_{41}	0.0252	0.00095	SR	n_6	0.0012	0.00126
SR	β_{42}	0.2672	0.00190	SR	r_6	0.0013	0.00058
SR	a_5	0.0256	0.00193	SR	s_6	-0.0215	0.00127
SR	d_6	-0.0197	0.00172				

* Insignificant even at the 10% level of confidence. All other estimates are significant at the 1% level of confidence. [a] Denotes $c_{is} = \sum_s \alpha_{rs}\theta_i^s$, $i = 1,2$. The period 1998–1997 was dropped as the base. PF=Purchased Funds, CD=Core Deposits, LB=Labor Equation, CL=Consumer Loans with subscript 1, BL=Business Loans with subscript 2, RE=Real Estate Loans with subscript 3, SR=Securities with subscript 4.

For CD, PF, CL, BL, RE, SR at that order and respectively across equations, the coefficients are as follows: Intercept terms = a_0-a_5 ; For instance a_4 is the intercept term for the RE equation. Dummies: r_1-r_6 = charter-type; d_1-d_6 = Asset size below 300 million; d_7-d_{12} = Asset size above 300 million but below 2 billion; c_1-c_6 = 1990–1991; p_1-p_6 = 1991–1992; e_1-e_6 = 1992–1993; f_1-f_6 = 1993–1994; g_1-g_6 = 1994–1995; h_1-h_6 = 1995–1996; k_1-k_6 = 1996–1997; n_1-n_6 = 1998–1999; s_1-s_6 = 1999–2000.

Table 5-4. Parameter estimates and standard errors for the translog, 1990–2000

Symbol	Parameter estimate	Standard error	Symbol	Parameter estimate	Standard error
a_0	3.7163	0.07361	p_{31}	0.0053	0.00017
b_1	1.3322	0.00426	p_{32}	0.0338	0.00031
b_2	0.0436	0.00440	p_{41}	-0.0068	0.00044
b_{11}	0.2129	0.00060	p_{42}	-0.0798	0.00099
b_{22}	0.0574	0.00059	a_1	-0.7527	0.04660
b_{12}	-0.0628	0.00047	a_2	0.0463*	0.03620
g_1	0.2002	0.00324	d_1	-0.0348	0.01110
g_2	0.1365	0.00087	d_2	-0.1612	0.01110
g_3	0.1520	0.00348	d_3	-0.2322	0.01130
g_4	0.0180	0.00874	d_4	-0.1971	0.01110
g_{11}	0.0486	0.00010	d_5	-0.0584	0.01130
g_{22}	0.0052	0.00000	d_6	-0.0754	0.01130
g_{33}	0.1113	0.00015	d_7	-0.0685	0.01080
g_{44}	0.2584	0.00104	d_{10}	0.0265	0.01050
g_{12}	-0.0026	0.00002	d_0	-0.0419	0.01090
g_{13}	-0.0288	0.00010	a_{11}	0.1092	0.00150
g_{14}	-0.0640	0.00023	a_{12}	0.0839	0.00121
g_{23}	-0.0075	0.00002	a_{21}	-0.0957	0.00141
g_{24}	-0.0004	0.00007	a_{22}	-0.0754	0.00104
g_{34}	-0.0927	0.00027	h_{11}	0.0029	0.00111
f_1	-0.1098	0.00537	h_{12}	0.0104	0.00076
f_2	0.4282	0.01042	h_{21}	-0.0048	0.00039
f_{11}	0.0061	0.00027	h_{22}	-0.0039	0.00033
f_{22}	0.0051	0.00128	h_{31}	0.0829	0.00120
f_{12}	-0.0121	0.00049	h_{32}	0.0236	0.00077
th_{11}	-0.0048	0.00014	h_{41}	0.1468	0.00332
th_{12}	-0.0017	0.00004	h_{42}	0.0542	0.00244
th_{13}	0.0267	0.00018	m_{11}	0.0243	0.00212
th_{14}	0.0355	0.00036	m_{12}	0.0048	0.00155
th_{21}	-0.0017	0.00013	m_{21}	-0.2070	0.00434
th_{22}	0.0007	0.00004	m_{22}	-0.1034	0.00322
th_{23}	-0.0097	0.00016	d_{11}	0.0062	0.00098
th_{24}	-0.0051	0.00033	d_{12}	0.0186	0.00098
ks_{11}	-0.0300	0.00021	d_{13}	0.0283	0.00103
ks_{12}	-0.0220	0.00042	d_{14}	0.0218	0.00105
ks_{21}	0.0007	0.00019	d_{15}	0.0168	0.00103
ks_{22}	0.0326	0.00040	d_{16}	0.0154	0.00104
p_{11}	0.0052	0.00014	d_{17}	0.0100	0.00105
p_{12}	0.0409	0.00029	d_{18}	0.0025	0.00103
p_{21}	0.0005	0.00004	d_{110}	-0.0064	0.00104
p_{22}	0.0014	0.00009	d_{111}	-0.0192	0.00104

Table 5-4. (Continued)

Symbol	Parameter estimate	Standard error	Symbol	Parameter estimate	Standard error
d_{21}	0.0040	0.00094	c_{37}	-0.0069	0.00097
d_{22}	-0.0129	0.00097	c_{38}	0.0070	0.00084
d_{23}	-0.0378	0.00105	c_{310}	0.0074	0.00088
d_{24}	-0.0487	0.00109	c_{311}	0.0123	0.00108
d_{25}	-0.0420	0.00104	c_{41}	0.0137	0.00221
d_{26}	-0.0240	0.00100	c_{42}	0.0061	0.00230
d_{27}	-0.0165	0.00102	c_{43}	0.0149	0.00242
d_{28}	-0.0066	0.00099	c_{44}	0.0054	0.00246
d_{210}	0.0083	0.00098	c_{45}	-0.0024*	0.00227
d_{211}	0.0322	0.00094	c_{46}	-0.0034	0.00249
c_{12}	0.0055	0.00076	c_{47}	0.0061	0.00242
c_{13}	0.0019	0.00087	c_{48}	0.0132	0.00237
c_{14}	0.0016	0.00079	c_{410}	0.0113	0.00243
c_{15}	0.0047	0.00071	c_{411}	0.0166	0.00240
c_{16}	-0.0019	0.00079	e_{11}	-0.0016*	0.00144
c_{17}	0.0025	0.00079	e_{12}	-0.0038	0.00143
c_{18}	0.0008*	0.00069	e_{13}	-0.0076	0.00150
c_{110}	0.0061	0.00071	e_{14}	-0.0034	0.00149
c_{111}	0.0050	0.00077	e_{15}	-0.0062	0.00139
c_{21}	0.0026	0.00021	e_{16}	-0.0024	0.00151
c_{22}	0.0007	0.00020	e_{17}	-0.0013*	0.00151
c_{23}	0.0003*	0.00023	e_{18}	-0.0065	0.00137
c_{24}	-0.0009	0.00022	e_{110}	-0.0029	0.00136
c_{25}	-0.0011	0.00022	e_{111}	-0.0039	0.00144
c_{26}	-0.0008	0.00023	e_{21}	-0.0022*	0.00223
c_{27}	-0.0009	0.00024	e_{22}	0.0049	0.00230
c_{28}	-0.0005	0.00023	e_{23}	-0.0065	0.00238
c_{210}	-0.0002*	0.00023	e_{24}	0.0032	0.00242
c_{211}	0.0013	0.00044	e_{25}	0.0147	0.00226
c_{31}	-0.0050	0.00089	e_{26}	0.0183	0.00260
c_{32}	-0.0033	0.00085	e_{27}	0.0050	0.00250
c_{33}	0.0058	0.00084	e_{28}	-0.0127	0.00238
c_{34}	0.0007*	0.00082	e_{210}	-0.0208	0.00249
c_{35}	-0.0042	0.00084	e_{211}	-0.0306	0.00271
c_{36}	-0.0068	0.00088	d_9	-0.0294	0.01070

* Insignificant even at the 10% level of confidence. All other estimates are significant at the 1% level of confidence. The year 1998 was dropped as the base.

The striking result is that concavity is rejected for the mean size bank in each year of our sample under the translog specification. Table 5-5 presents a comparison between the translog specification and the differential in terms of satisfying or rejecting concavity.

Table 5-5. Concavity test

Year	Translog	Differential
1990	Reject	—
1991	Reject	N.s.d.
1992	Reject	N.s.d.
1993	Reject	N.s.d.
1994	Reject	N.s.d.
1995	Reject	N.s.d.
1996	Reject	N.s.d.
1997	Reject	N.s.d.
1998[a]	—	—
1999	Reject	Reject
2000	Reject	N.s.d.

[a] Base year, excluded from the estimation. In the differential, year denotes the change between current and previous year.

The concavity test was an examination of the eigenvalues of the Hessian of the cost function with respect to input prices, evaluated for the mean size bank and for the average product mix in the industry, for the translog specification. In the differential model the test for concavity requires the price matrix in the input demands $[\pi_{ij}]$ to be negative semidefinite. Since these coefficients are constant through time, accepting concavity for the pooled model may not imply that concavity is accepted for each year in our sample. Further, rejection of concavity implies possible misspecification of the model. For that purpose, separate regressions for each year in our sample were performed (cross-section) and provided further evidence that concavity is rejected for each year in the translog specification, while in the differential, concavity is rejected only for the period 1998–1999. Further, we performed the same test for the price matrix $[\alpha_{rs}]$, which should be positive definite in the output-supply system. Our results indicate that convexity in the output prices is rejected for every year in the sample, not a promising result for the differential, since it already satisfies concavity. However, this may be attributed to a number of reasons, as we have not included in the analysis, off-balance sheet items that appear to be significant outputs of the U.S. commercial banks in the past decade. Quasi-fixity of some outputs including the off-balance sheet items of the banks is another reason that may have driven this rejection. It should be noted though, that the fit of the consumer loans and business loans output-supply equations was poor. For instance, consumer loans explained only 7% of the cases in our sample, while the business loans explained only 12%, as represented by the adjusted R-square of each equation. These results imply possible misspecification in the output-supply system where we come back to what constitutes inputs and what outputs in banking.

In terms of Allen-Uzawa elasticities of substitution, significant differences are found between the two models. Evaluating Equation 5-14 for the translog at the sample mean for each year in our sample and Equation 5-9 for the differential again at the sample mean of each year we obtain the results of Table 5-6. It appears that the translog specification overestimates the elasticities between purchased funds and labor and between core deposits and purchased funds (or the differential underestimates). In both models labor is an Allen substitute for purchased funds but in the translog it appears to have the greater substitutability among the inputs, while this substitutability is reduced through the period analyzed. Instead, in the differential greater substitutability among the inputs appear to be between both pairs of labor-core deposits and labor-purchased funds.

Table 5-6. Allen-Uzawa elasticities of substitution

Year	Translog			Differential		
	CD - PF	CD - LB	PF - LB	CD - PF	CD - LB	PF - LB
1990	0.2322	-0.0614	1.1750	—	—	—
1991	0.1017	0.0181	1.1901	-0.0639	0.5650	0.4653
1992	-0.2116	0.1685	1.1922	-0.0802	0.4950	0.4849
1993	-0.4483	0.2363	1.1838	-0.1018	0.4370	0.4814
1994	-0.2955	0.2250	1.1562	-0.1038	0.4236	0.4317
1995	0.0147	0.1464	1.1398	-0.0853	0.4470	0.3742
1996	0.0579	0.1353	1.1330	-0.0733	0.4735	0.3477
1997	0.1128	0.1201	1.1228	-0.0697	0.4803	0.3271
1998[a]	—	—	—	—	—	—
1999	0.1730	0.1061	1.1033	-0.0629	0.4937	0.2782
2000	0.2903	0.0345	1.0927	-0.0574	0.5130	0.2489

[a] Base year, excluded from the analysis. In the differential model the elasticities are the percentage change between the current and previous year.

In the case of core deposits and labor there is an increasing degree of substitutability over the years. The striking result is in the case of purchased funds and core deposits, where both models find them to be Allen complements for the period 1992 to 1994, which is the period after the credit crunch and before the Riegle-Neal Act in 1994. However, the differential is consistent through the whole period and reports complementarity between those two inputs. Recalling that the Allen measure of substitution in the differential is the ratio of the cross-price elasticity over the product of the average cost ratio of each input, and that this term is statistically significant, it implies that the differential provides a purer measure. However, the assumption of constant price effects through time, that is these own and cross price elasticities are constant through time might have driven this implausible result. Therefore, one could use the variable price effects developed in Chapter 3 in order to verify the robustness of these results in the differential model.

The final point of comparison between the two models is in terms of economies of scale. Before we start with the discussion of the results we need to provide an equation for the economies of scale in the differential model, given that there is no explicit functional form of the cost function as in the translog. Economies of scale (SCE) measures the elasticity of cost with respect to a proportionate increase in all outputs and is given by

$$SCE = \sum_{r=1}^{m} \frac{\partial \ln TC}{\partial \ln y_r} \tag{5-15}$$

Increasing returns to scale (costs increase proportionately less than output increases), decreasing or constant returns to scale exist if Equation 5-15 is less than, greater than or equal to unity, respectively (Baumol, Panzar and Willig 1988, page 50). Notice that in the case of profit maximization the first-order conditions imply equality between the price of each output and their corresponding marginal costs. This result was used by Laitinen and Theil (1978) for the long-run model in order to parameterize the coefficient which is equivalent to our γ_2 coefficient in the input-demand and output-supply system and in their model was also equal to the returns to scale. By assuming profit maximization in the short-run, this parameterization transfers through in the short-run model. Substituting, then the first-order condition in the scale economies measure SCE, it simply becomes the revenue total cost ratio of the multiproduct firm. To make this result obvious note that Equation 5-15 can be written, as

$$SCE = \sum_{r=1}^{m} \frac{\partial \ln TC}{\partial \ln y_r} = \sum_{r=1}^{m} \frac{\partial TC}{\partial y_r} \frac{y_r}{TC}$$

Given the assumption of quasi-fixed inputs the total marginal cost can be written as

$$\frac{\partial TC}{\partial y_r} = \frac{\partial VC}{\partial y_r} = \frac{VC}{y_r}\sum_i f_i \frac{\partial \ln x_i}{\partial \ln y_r}$$

By substituting this relationship in the SCE equation and using the results from equations (2-66) and (2-75) we get that

$$SCE = \sum_{r=1}^{m} \frac{\partial \ln TC}{\partial \ln y_r} = \frac{VC}{TC}\sum_{r=1}^{m}\sum_i f_i \frac{\partial \ln x_i}{\partial \ln y_r} = \frac{VC}{TC}\sum_{r=1}^{m}\sum_i \gamma_2 \theta_i^r g_r$$

From the adding-up properties of the input-demand system the term inside the summation is equal to γ_2, which from the profit maximization case is equal to the revenue-variable cost ratio. Making substitution of these results into the scale economies measure, it yields that in the differential model under the assumption of profit maximization, is equal to the revenue total cost ratio. This further implies that constant returns to scale exist when there is a zero profit condition. Therefore, in the differential model there is no explicit measure for the economies of scale, if profit maximization is assumed. The system of demand equations could be estimated separately and regard the γ_2 term as a parameter to be estimated. However, this implies that a complete system of demand equations should be implemented and in the case of panel data it is cumbersome.

Table 5-7 presents the results for the economies of scale, derived from the translog model (Equation 5-13) for the mean size bank in the sample and also those calculated from the revenue-total cost ratio (for the differential model). While most of the previous studies that use the translog specification (except Stiroh 2000, Hughes and Mester 1998, Berger and Mester 1997) could not find economies of scale for the banking industry, in this study using a translog cost function specification with variable slopes for the outputs, quasi-fixed inputs and input prices, we found significant economies of scale evaluated at the empirical mean of our sample.

Table 5-7. Economies of scale for the mean size U.S. bank, 1990–2000

Year	Translog Economies of scale	Total revenue-Total cost ratio
1990	0.898922	1.035225
1991	0.892359	1.045724
1992	0.899825	1.066944
1993	0.877201	1.072805
1994	0.858988	1.090447
1995	0.848583	1.073222
1996	0.861026	1.059456
1997	0.877344	1.052327
1998[a]	—	1.049784
1999	0.876549	1.044226
2000	0.881262	1.037623

[a] No estimates for the translog, since 1998 was dropped as the base year.

This result can explain the wave of mergers that happened in the industry. Comparing the overall economies of scale results of our study with those of Stiroh (2000), who examined economies of scale for Bank Holding Companies and by asset size, we found similarities within a range of 0.01-0.14, with our results showing stronger economies of scale. However, concavity has been rejected in the translog specification which may be caused from heterogeneity in the data. Specifically, Berger and Mester (1997) argued that there is a scale bias by including banks of different sizes in a single regression and one needs to correct for possible heteroscedasticity. Moreover, the economies of scale as measured in the translog are actually the returns to the quasi-fixed factors, which may explain the finding of unusually high scale economies. Finally, we observe that in our sample the total revenue-total cost ratio is almost equal to one over the whole period, implying constant returns to scale or slight diseconomies of scale for the U.S. commercial banks, if the measure of economies of scale from the differential model is adapted.

CHAPTER 6
SUMMARY AND CONCLUSIONS

The Laitinen-Theil (1978) or differential model concerns long-run behavior of a multiproduct firm under the assumptions of perfect competition and output homogeneous production technology. The purpose of this study was to provide an extension of the Laitinen-Theil (1978) model for the multiproduct-multifactor firm by examining firm behavior in the short-run. Specifically, the extended model accounted for quasi-fixed inputs in the transformation technology of the multiproduct firm, while the assumption of output homogeneous technology was relaxed. It turns out that the assumption of output homogeneous technology does not play a crucial role in the derivation of the input demands apart from the returns to scale measure, and in the price terms of the input-demand system.

The mathematical derivation of the system of input-demand and output-supply equations is presented in Chapter 2, where for the convenience of the reader a detailed representation of the derivations followed. To capture stochastic behavior of the multiproduct firm we appended disturbances in the system of input-demand and output-supply equations, relying on the theory of rational random behavior, developed by Theil (1975). It appears that quasi-fixity does not play a role in obtaining the covariance matrices of the two systems of equations, since both in our study and in Laitinen-Theil (1978), are the same. Further, as in Laitinen and Theil (1978), the covariance matrix of the disturbances in the input-demand system was singular.

Chapter 3 deals with the parameterization of the joint system of input-demand and output-supply equations, when quasi-fixity is present. In the original model of Laitinen and Theil the parameterization of the input-demand system was adding up to the total input decision of the firm with all coefficients being observed. However, in the parameterization of the short-run model, developed in Chapter 2, the coefficients of the quasi-fixed inputs in the total input decision of the firm were unobserved, since they entailed their shadow prices.

To overcome this "problem", a technique devised by Morrison-Paul and MacDonald (2000), was used. In sort, it can be assumed that the shadow price of the quasi-fixed input is equal to its market rental price plus a deviation term. The input-demand system then can be corrected for this introduction of the error term the same way that Laitinen and Theil corrected for the finite differences approximation in their model. However, this method is not required for the estimation of the model. This method was only used, in order for the input-demand system to add-up into known constants, and thus deletion of one equation (since disturbances are singular) would impose no problem in the estimation.

Recently, various econometric techniques have been developed for the estimation of the full system of input-demand equations that one could follow. However, if a joint estimation of the input-demand and output-supply system of equations is desired, then it is more parsimonious to follow the above described method, since when symmetry is imposed, the nonlinear restrictions on the parameters of the model should also be considered.

As in the Rotterdam model there is a similar misspecification in the differential model of production, since it is assumed that all coefficients of the input-demand and output-supply systems are constant through time. To alleviate this restriction, alternative parameterizations of the differential model were developed, accounting for variable price and output effects in the input-demand system. It turns out that all alternative parameterizations could be tested through parameter restrictions in a more general model developed for that purpose. Further, one of the alternative parameterizations was used for the development of a test for quasi-fixed inputs in the multiproduct firm. This test could be implemented at each sample point to provide us with the behavior of the quasi-fixed input over time.

In Chapter 4 the econometrics of the differential model were presented. Several econometric methods were provided for the estimation of the differential model. In Section 4.1 the Laitinen (1980) maximum likelihood procedure was presented, which concerned the estimation of the differential model for one multiproduct firm over time. However, the need to account for panel data structures led to the development of two new procedures. In Section 4.2 the cases of time-specific and firm-specific effects panel data were considered. It was argued that firm-specific fixed effects may be already accounted for in the differential model if first differences of each time series are observed (i.e. ARIMA (0,1,0)). This would imply that these firm-specific fixed effects enter multiplicatively in the unknown cost function of the firm, which further implies variable slopes in the cost function of the firm over time, in a Hicks neutral sense. Therefore, a maximum likelihood method was provided only for the case of time-specific, fixed effects panel data; given also that the available dataset had large cross-sectional units and few time periods. This method is a sub-case of the maximum likelihood method for systems of equations and panel data, developed by Magnus (1982). However, it was adapted to account for time-specific, fixed effects panel data, since it originally concerned time-specific, random-effects panel data; and also to account for the nonlinear cross-equations symmetry restrictions. In the last part of Chapter 4, a firm-specific, random effects maximum likelihood method was presented, which also accounts for the nonlinear symmetry restrictions. It is left for future research a random coefficients approach and the case of random effects with unbalanced panel data for the differential model.

In Chapter 5, the developed model was applied to the U.S. banking industry for the period 1990–2000, where the discussion of the results focused in the technology that is implied by the differential model. To provide a direct comparison with dual functional forms, a translog cost specification was also implemented and tested for the same dataset. The two models were compared in terms of, rejecting or satisfying the concavity property of the cost function, Allen elasticities of substitution, and an efficiency measure, such as economies of scale.

The most striking result of Chapter 5 is that even if the translog fails to satisfy concavity for each year in our sample, the differential model satisfies concavity for the whole period. Considering that in the differential model the test for concavity is a simple test on the price coefficients in the input-demand system of equations that are assumed to be constant over time, the differential model was regressed for each year in the sample. Results indicated that the differential model satisfied the concavity property for all years, apart from 1999. However, the differential system failed to satisfy convexity in the output supply system, which could simply imply that for the specific dataset used there is a misspecification on the supply side or that significant variables were excluded from the analysis, as indicated in Chapter 5. For instance, possible quasi-fixed outputs that have not been included or included as variable could have driven this result.

In terms of Allen elasticities of substitution the differential model provides consistent estimates through time, while there was an inconsistency between the models in one pair of inputs. That is, the translog finds that purchased funds and core deposits are Allen substitutes, apart from three years in the sample, but the differential model always finds that they are Allen complements. However, both results are inconsistent with previous findings in the literature, where these inputs are found to be Allen substitutes. For the differential model, it may imply that the price effects are not constant through time, as assumed, and one should use the variable price effects parameterization of the differential developed in Chapter 3.

One of the disadvantages of the parameterization that was used to estimate the differential model is that the measure for economies of scale becomes the total revenue-total cost ratio, restricting this way the analysis. A descriptive analysis of the sample showed that this ratio was almost equal to one or slightly higher, implying constant or decreasing returns to scale under the assumptions of the differential model. It also implies that in the U.S. banking industry a zero profit condition holds, given the data used in the analysis. To alleviate this restriction on the returns to scale one of the alternative parameterizations and a complete system estimation method could be followed. It should be noted, finally, that when using the translog model significant economies of scale were found for the mean-size U.S. commercial bank over the period 1990–2000, providing an explanation for the mergers activity in the industry. Considering that the measure of economies of scale in the short-run is simply a measure of returns to the quasi-fixed factors, the economies of scale that were found from the translog system were surprisingly high and further analysis should be done in this area. Moreover, the concavity property of the cost function was rejected under the translog specification for all of the years in the analysis, implying that the results for the economies of scale measure should not be valid. In the banking literature is a common practice (with few exceptions) not to test for the validity of the concavity property but one should empirically test for all the theoretical properties of the cost or profit function. Failure to do so may lead to incorrect policy-making decisions.

APPENDIX
ANALYTICAL GRADIENT VECTOR

A.1 Gradient Vector for Section 4.1

Consider one firm that uses three variable inputs, two quasi-fixed inputs in the production process and produces four outputs. After dropping one input-demand equation and imposing homogeneity and symmetry, the model presented by equations 4.3 and 4.4 has the following coefficient matrices

$$A=\begin{bmatrix}\alpha_{11} & \alpha_{12} & \alpha_{13} & \alpha_{14}\\ \alpha_{12} & \alpha_{22} & \alpha_{23} & \alpha_{24}\\ \alpha_{13} & \alpha_{23} & \alpha_{33} & \alpha_{34}\\ \alpha_{14} & \alpha_{24} & \alpha_{34} & \alpha_{44}\end{bmatrix},\quad F=\begin{bmatrix}\beta_{11} & \beta_{12}\\ \beta_{21} & \beta_{22}\\ \beta_{31} & \beta_{32}\\ \beta_{41} & \beta_{42}\end{bmatrix},\quad D=\begin{bmatrix}\pi_{11} & \pi_{12}\\ \pi_{12} & \pi_{22}\end{bmatrix},$$

$$\Theta=\begin{bmatrix}\theta_{11} & \theta_{12} & \theta_{13} & \theta_{14}\\ \theta_{21} & \theta_{22} & \theta_{23} & \theta_{24}\end{bmatrix}$$

$$C=\begin{bmatrix}-(\alpha_{11}\theta_{11}+\alpha_{12}\theta_{12}+\alpha_{13}\theta_{13}+\alpha_{14}\theta_{14}) & -(\alpha_{11}\theta_{21}+\alpha_{12}\theta_{22}+\alpha_{13}\theta_{23}+\alpha_{14}\theta_{24})\\ -(\alpha_{12}\theta_{11}+\alpha_{22}\theta_{12}+\alpha_{23}\theta_{13}+\alpha_{24}\theta_{14}) & -(\alpha_{12}\theta_{21}+\alpha_{22}\theta_{22}+\alpha_{23}\theta_{23}+\alpha_{24}\theta_{24})\\ -(\alpha_{13}\theta_{11}+\alpha_{23}\theta_{12}+\alpha_{33}\theta_{13}+\alpha_{34}\theta_{14}) & -(\alpha_{13}\theta_{21}+\alpha_{23}\theta_{22}+\alpha_{33}\theta_{23}+\alpha_{34}\theta_{24})\\ -(\alpha_{14}\theta_{11}+\alpha_{24}\theta_{12}+\alpha_{34}\theta_{13}+\alpha_{44}\theta_{14}) & -(\alpha_{14}\theta_{21}+\alpha_{24}\theta_{22}+\alpha_{34}\theta_{23}+\alpha_{44}\theta_{24})\end{bmatrix}$$

The other matrices are similar. The unique element vector consists of

$$\mu=\begin{Bmatrix}\theta_{11},\theta_{12},\theta_{13},\theta_{14},\xi_{11},\xi_{12},\pi_{11},\pi_{12},\theta_{21},\theta_{22},\theta_{23},\theta_{24},\xi_{21},\xi_{22},\pi_{22},\\ \alpha_{11},\alpha_{12},\alpha_{13},\alpha_{14},\beta_{11},\beta_{12},\alpha_{22},\alpha_{23},\alpha_{24},\beta_{21},\beta_{22},\alpha_{33},\alpha_{34},\beta_{31},\\ \beta_{32},\alpha_{44},\beta_{41},\beta_{42}\end{Bmatrix}$$

Then recall that $\dfrac{\partial M}{\partial \mu_i}=\dfrac{\partial A}{\partial \mu_i}p_t+\dfrac{\partial C}{\partial \mu_i}w_t+\dfrac{\partial F}{\partial \mu_i}z_t$ and differentiate with respect to the elements of the unique vector. For instance,

$$\frac{\partial M}{\partial \theta_{11}} v_t = \begin{bmatrix} -\alpha_{11} & 0 \\ -\alpha_{12} & 0 \\ -\alpha_{13} & 0 \\ -\alpha_{14} & 0 \end{bmatrix} w_t \text{, since } \theta_{11} \text{ appears in matrix } C \text{ in the output supply.}$$

$$\frac{\partial M}{\partial a_{11}} v_t = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} p_t + \begin{bmatrix} -\theta_{11} & -\theta_{21} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} w_t$$

The partial derivatives of M with respect to elements that belong uniquely to the input demand are all zero. All the other can be found similarly.

A.2 Gradient Vector for Section 4.2

The unique coefficient vector now consists of

$$\beta' = \left\{ \begin{matrix} \theta_{11}, \theta_{12}, \theta_{13}, \theta_{14}, \xi_{11}, \xi_{12}, \pi_{11}, \pi_{12}, \theta_{21}, \theta_{22}, \theta_{23}, \theta_{24}, \xi_{21}, \xi_{22}, \pi_{22}, \\ \alpha_{11}, \alpha_{12}, \alpha_{13}, \alpha_{14}, c_{11}, c_{12}, \beta_{11}, \beta_{12}, \alpha_{22}, \alpha_{23}, \alpha_{24}, c_{21}, c_{22}, \beta_{21}, \beta_{22}, \\ \alpha_{33}, \alpha_{34}, c_{31}, c_{32}, \beta_{31}, \beta_{32}, \alpha_{44}, c_{41}, c_{42}, \beta_{41}, \beta_{42} \end{matrix} \right\}$$

where now we have more elements corresponding to the nonlinear terms. That is,

$$\begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \\ c_{31} & c_{32} \\ c_{41} & c_{42} \end{bmatrix} = \begin{bmatrix} -(\alpha_{11}\theta_{11} + \alpha_{12}\theta_{12} + \alpha_{13}\theta_{13} + \alpha_{14}\theta_{14}) & -(\alpha_{11}\theta_{21} + \alpha_{12}\theta_{22} + \alpha_{13}\theta_{23} + \alpha_{14}\theta_{24}) \\ -(\alpha_{12}\theta_{11} + \alpha_{22}\theta_{12} + \alpha_{23}\theta_{13} + \alpha_{24}\theta_{14}) & -(\alpha_{12}\theta_{21} + \alpha_{22}\theta_{22} + \alpha_{23}\theta_{23} + \alpha_{24}\theta_{24}) \\ -(\alpha_{13}\theta_{11} + \alpha_{23}\theta_{12} + \alpha_{33}\theta_{13} + \alpha_{34}\theta_{14}) & -(\alpha_{13}\theta_{21} + \alpha_{23}\theta_{22} + \alpha_{33}\theta_{23} + \alpha_{34}\theta_{24}) \\ -(\alpha_{14}\theta_{11} + \alpha_{24}\theta_{12} + \alpha_{34}\theta_{13} + \alpha_{44}\theta_{14}) & -(\alpha_{14}\theta_{21} + \alpha_{24}\theta_{22} + \alpha_{34}\theta_{23} + \alpha_{44}\theta_{24}) \end{bmatrix}$$

The gradient vector then has to take into consideration the above decomposition of the c_{ij} terms. Thus, $\frac{\partial \beta}{\partial \alpha_{11}} = \left\{ \begin{matrix} 0, \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 0, \\ 1, 0, 0, 0, -\theta_{11}, -\theta_{21}, 0, \ldots\ldots\ldots\ldots\ldots 0, \\ 0, \ldots\ldots\ldots\ldots\ldots, 0 \end{matrix} \right\}$

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BIOGRAPHICAL SKETCH

Grigorios Livanis was born on September 4, 1974, in Athens, Hellas. After graduating from high school, he worked for 1 year in a private company in Athens. His quest for knowledge made him take the national examinations for entrance into a Hellenic university. In 1993, he was admitted to the Department of Energy Technology at the Technological Institute of Athens. However, he was not satisfied with their program of study, and he decided to retake the national examinations. Higher scores admitted him in fall 1994 to the Department of Agricultural Economics at the Agricultural University of Athens. His devotion to academic excellence and merit was demonstrated by ranking first in his department for the entire duration of his studies. For this reason, he received annually recognition awards and scholarships from the State Scholarships Foundation of the Hellenic Republic.

After receiving his B.Sc./M.Sc. combined degree with high honors in the summer of 1999, he was offered assistantships to pursue graduate studies in agricultural economics at the University of Wisconsin, Michigan State University, University of Missouri, and the University of Florida (UF). His choice to join the graduate school at UF was highly influenced by the fact that it was the only one where both, he and his fiancée, Maria Chatzidaki, were offered a scholarship among their combined university choices.

In the fall of 1999, he enrolled in the Department of Food and Resource Economics at the University of Florida. He received his M.Sc. degree with a minor in mathematics in December 2000. He continued on toward his doctoral degree in spring 2001, in the same department, with fields of specialization in financial and production economics, and applied econometrics. In spring 2004, he received a Presidential Recognition Award for his outstanding achievements and contributions to the University of Florida. He fulfilled all the requirements and coursework, with a 3.95 overall G.P.A., and was awarded a Doctor of Philosophy degree in August 2004.

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

Charles B. Moss, Chair
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I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

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