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Economic Efficiency and Cost Allocation for Water Resource Projects with Economies of Scale

by

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ECONOMIC EFFICIENCY AND COST ALLOCATION FOR WATER RESOURCE PROJECTS WITH ECONOMIES OF SCALE

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INTRODUCTION

Many economic optimization techniques are available for evaluating various water resource project designs. Also, numerous methods have been proposed or are now used to apportion costs of a water resource project among participants and/or purposes. However, no extensive research has been directed towards evaluating the relationship between the economic optimization and cost allocation phases of planning water resource projects. The purpose of this study is to provide a basis for establishing and evaluating the relationship between economic efficiency and project cost allocation. First, a review of some recently developed theories on the formation and sustainability of natural monopolies is presented. Several important implications concerning the efficiency and fairness of project coalition formation may be derived from these economic theories. To provide a basis for quantifying the implications derived from economic theory, methods for evaluating economic efficiency and for apportioning the cost among participants of a joint water resource project are reviewed. Finally, through the simple example of a small wastewater reuse project, several of these methods are applied to demonstrate how efficiency and equity might be related in order to evaluate various joint project designs. In particular, implications for the special case of economies of scale in production for three different demand relationships are examined.

SECTION I

LITERATURE REVIEW

Some insight into the current dilemma regarding the interaction of efficiency and equity can be gained by reviewing the work of Loughlin (1977) and Rossman (1978). Loughlin examines the efficiency of three cost allocation schemes based on a set of efficiency and equity criteria. He contends that economic efficiency and cost allocation are independent of each other since costs allocated to a participant or project purpose are not the costs to consider in deciding project feasibility, optimal scale of development, or which participants to include in the project. Rossman notes that by judging the cost allocation methods in terms of economic evaluation criteria, Loughlin contradicts the argument that cost allocation is not required for economic evaluation. In fact Zajac (1978) proposes that from the viewpoint of either efficiency or equity, pricing and entry-exit (from the joint project) must be considered jointly, not in isolation. This section examines the theoretical basis for these two differing viewpoints. The theory of natural monopoly provides the basis of the argument for joint consideration of efficiency and equity, while the principles of engineering cost allocation often dictate independent financial and economic analyses. In addition to these conceptual views of the problem, methods for quantifying efficiency and equity are reviewed.

Natural Monopoly

Recent developments in the theory of natural monopoly provide some important implications for the roles of economic optimization and fairness in coalition formation of joint projects. A brief overview of some of the important concepts on natural monopolies will provide an understanding of these implications.

A competitive market prevails if the number of sellers is large enough so that no one seller is able to influence the market price by a unilateral change in output. Competitive equilibrium is assumed to occur where marginal costs equal marginal benefits, i.e. where the supply and demand curves intersect. A competitive equilibrium has the property known as Pareto optimality where net benefits are maximized so that no other feasible distribution of outputs can improve the welfare (measured in terms of net benefits) of one individual without reducing the welfare of another. However, as demonstrated in the next section, maximum net benefits do not always correspond to marginal conditions. Therefore maximum net benefits is a more generally applicable condition for economic optimality.

In contrast to the competitive market is the monopolistic market with only one active seller (or firm) in the market. Sharkey (1982a) describes conditions for the formation and stability of a natural monopoly which results when a single firm can produce at lower cost than any combination of two or more firms in a market where firms are able to compete on an equal basis. Entry of firms in a natural monopoly could only reduce social welfare (measured in terms of cost of producing a given output) by raising total costs of production. Assuming all firms in the market have the same cost function C, a single firm is more efficient than two or more firms if

$$C(q) < C(q^1) + \ldots + C(q^k) \tag{1}$$

where q = vector of outputs in a particular market = $(q_1, \ldots q_n)$

$q^1, \ldots q^k$ = output vectors that sum to q.

If inequality (1) holds for all feasible disaggregations of q, then C is subadditive at q and the market is a natural monopoly. Indeed, a natural monopoly exists if and only if the cost function is subadditive, that is if

$$C(q) < C(x) + C(q-x) \tag{2}$$

for any x such that $0 < x < q$.

Economies of scale exist where average costs fall with increasing output or where

$$C(\lambda q) < \lambda\, C(q) \tag{3}$$

for all λ such that $1 < \lambda \leq 1 + \varepsilon$ where ε is a small positive number. Economies of scale are sufficient but not necessary conditions for subadditivity of single output cost functions. However, in a multiple output market, subadditivity holds only if there are also economies of joint production. One such measure is economy of scope which exists if a single firm can produce any vector of outputs more efficiently than two or more specialty firms for a constant level of production of each output. While economy of scope is a useful intuitive concept, it is not particularly helpful in deriving actual sufficient conditions for subadditivity. Therefore, Sharkey presents other specific properties of the cost function which may be empirically verified. See Sharkey (1982a) for a discussion of these properties.

Sharkey (1982a) observes that economies of scale in the production of a private good are closely related to the concept of public goods which are consumed collectively rather than individually. A fixed cost in the production of such a good has the property of collectiveness since it is consumed and must be paid by the consumers collectively. Economists have long recognized that collective goods cannot be efficiently allocated through the market system. As a result, various forms of public control have evolved for financing and allocating collective goods. Therefore, natural monopolies (such as public utilities) are often referred to as regulated firms in the economics literature.

Given that under subadditive cost conditions, the natural monopoly is the least cost (or maximum welfare) market configuration, the question remains as to how to set regulated prices. Since the competitive equilibrium is economically efficient, one alternative might be to regulate prices to equal marginal costs for a given level of production. However, in many regulated industries where there are often large fixed costs, marginal cost is below average cost. With prices set at marginal costs, total revenues fall short of total costs. A regulated firm would be able to survive only if subsidized by revenues raised in other parts of the economy. Such subsidies might be distributed by the government. Alternatively, owners of a regulated firm might receive compensation by the firm's customers for the use of the owner's capital. The legal precept in the United States requires that regulated firms operate at a zero profit, where all costs are considered. The problem of determining regulated, economically efficient prices is addressed in many theoretical models. Zajac (1978) considers several simple models such as Ramsey prices, two-part tariffs, and self-selecting two-part (block) tariffs. As Zajac (1978) observes, economic theories on regulated firm pricing have focused mainly on the question of economic efficiency with very little emphasis on issues of justice and fairness.

Zajac (1978) further observes that even if the cost structure is subadditive and a single supplier can provide all services cheaper than some combination of separate suppliers, the possibility may exist that some group of services can be provided cheaper by a separate supplier. This then is the focus of the modern theory of natural monopoly in which models by Zajac (1972) and Faulhaber (1972, 1973) demonstrate some of the inherent contradictions involved in setting prices that are both economically efficient and sustainable (no cross-susidization). Following is a brief discussion of these concepts of efficiency and cross-subsidization and how they relate to issues of economic analysis and cost allocation of joint projects.

Although natural monopoly is by definition the most efficient form of production, there may be no price that simultaneously satisfies total market demand, provides revenues to cover total cost, and discourages entry by other firms. This situation is illustrated in Figure 1 for the case of a single output natural monopoly. To satisfy demand and revenue requirements, the price is set at $P \geq P_m$. However, to discourage entry the price must be set so that $P = P_o < P_m$. This natural monopoly is therefore unsustainable. A natural monopoly for which an entry deterring price exists is termed a sustainable natural monopoly. This concept was first defined by Baumol, Bailey, and Willig

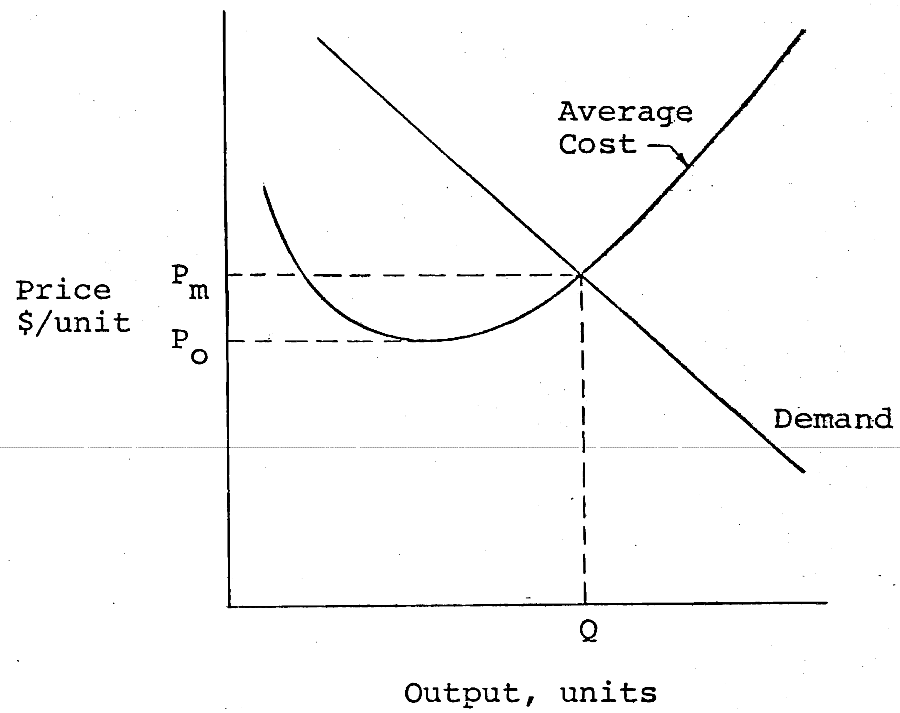


Figure 1. Example of an unsustainable natural monopoly.

(1977) and Panzar and Willig (1977). When an unsustainable multiple output natural monopoly operates under conditions where entry by rival firms is barred, prices must be such that one output is subsidizing another. The theory of cross-subsidization is proposed by Faulhaber (1975), Zajac (1972, 1978), Sharkey (1982b), and Sandberg (1975).

Sharkey (1982a) describes conditions under which both single output and multiple output natural monopolies are sustainable by two approaches; the intuitive concept of sustainability and the theory of cooperative games. Applying the former approach, Sharkey (1982a) defines conditions for sustainability based on various cost function properties including declining average costs which are extended to the concept of the supportable cost function. In the cooperative game approach, buyers in a market are viewed as players in a cooperative game. The objective is to form coalitions for which production and distribution of outputs are most favorable. A natural monopoly exists if the coalition of all buyers achieves a better outcome than any partition of buyers into subcoalitions. A necessary condition for natural monopoly is that the characteristic function be superadditive (revenue maximization) or subadditive (cost minimization). The characteristic function is given by the outcomes in terms of costs, benefits, or savings for the various subcoalition combinations. The characteristic function C for a cost game is subadditive if

$$C(S) + C(T) \geq C(S \cup T) \quad S \cap T = \Phi \tag{4}$$

where S and T are any two subsets of the grand coalition N and $S \cup T \neq \Phi$. The core of the game defines stable outcomes for which no player or subcoalition of players would benefit by leaving the grand coalition. Therefore, if the core exists, the market will be a stable natural monopoly.

Sharkey (1982a) defines two constraints. If p = $(p_i \ldots p_n)$ is the price vector, q = $(q_1, \ldots q_n)$ is the output vector, and N represents the set of all buyers, then

$$\sum_{i \varepsilon N} p_i q_i = C(N) \tag{5}$$

This constraint requires prices such that total revenues cover total costs. In addition, to assure that individual players or subcoalitions cannot be served more cheaply, the price vector must satisfy the constraint

$$\sum_{i \varepsilon S} p_i q_i \leq C(S) \tag{6}$$

for all subsets $S \subseteq N$. Sharkey then proves several theorems that guarantee the existence of the core. These conditions include properties of the cost function that are essentially the same as the sufficient conditions for subadditivity and sustainability. As will be seen in a later discussion, these same cooperative game theoretic results have been directly applied to several current cost allocation methods.

Finally, Sharkey (1982a) provides two warnings with respect to the cooperative game model. First, the model assumes that coalition formation is costless. However, real coalitions form with an additional transaction cost (Heaney, 1983). Secondly, the cooperative game model is derived from the underlying noncooperative game where the monopolistic firm and rival firms are players. As a result, important aspects of the interaction between players may be lost. Zajac (1978) provides some additional, very important warnings. First, for games involving more than just a few players, the number of possible groupings that must be evaluated may be formidable. Perhaps most importantly, the approach generally implies consumption of prespecified quantities of goods or services. This assumption might be valid where all buyer demands are completely insensitive to price variations. However, this is rarely the case. Therefore, analyses must examine variations in quantities demanded with varying prices. As price and quantity vary so do total revenues and costs, resulting in important implications for the efficiency of various stable price vectors. For example, if efficiency is defined in terms of maximizing overall benefits (derived from demand) minus costs (derived from supply), the optimal single firm or grand coalition configuration allocates output for an implied price vector.

This price vector need not be stable even though a set of stable price vectors may exist. Any of these other price vectors would not encourage efficient output consumption and so would yield a net benefit necessarily less than that implied by strict optimization. Herein lies the inherent conflict between efficiency and equity in regulated firm pricing. From the game theoretic approach, this concept may be extended to the case where costs must be apportioned among groups in a joint project.

Optimization

Evaluation of the efficiency of an endeavor implies optimization which involves the determination of a highest or lowest value over some range. Jelen and Black (1983) group optimization problems into three categories. The first, preferential optimization is subject to preference and taste only. The second, mathematical or physical optimization is not subject to these considerations. The third category is economic optimization which is a combination of preferential and mathematical optimization where preference elements are expressed quantitatively. Economists define economic efficiency in terms of Pareto optimality where resources are allocated so as to most effectively serve consumers' tastes and preferences. Efficiency in the choice of quantities of different outputs requires that for each output, marginal costs of production equal marginal benefits to each buyer. As noted, the competitive equilibrium automatically satisfies this condition. However, as noted earlier marginal conditions do not always correspond to maximum net benefits. When the marginal cost curve lies above the marginal benefit curve for lower output levels, the point where the two curves meet actually corresponds to minimum net benefits. In addition, where production capacity constraints restrict output, marginal conditions may never even be reached. Here, economic efficiency can be defined only in terms of the difference between total benefits and total costs with the optimum corresponding to maximum net benefits. This situation is examined more closely for a special case in the wastewater reuse example problem. For benefits and costs measured in terms of dollar worth, the conditions for economic efficiency are defined as in Figure 2.

Application of these economic principles to the evaluation of engineering projects requires analysis of the demand for project output and project costs. Marginal costs represent supply while marginal benefits depict the demand curve. Project evaluation, then, requires determination of costs and benefits as a function of the various outputs followed by maximization of their difference. Determination of project costs and benefits involves application of various cost engineering principles and techniques (e.g., see James and Lee, 1971). Rather, this study focuses on efficiency and equity issues for previously determined

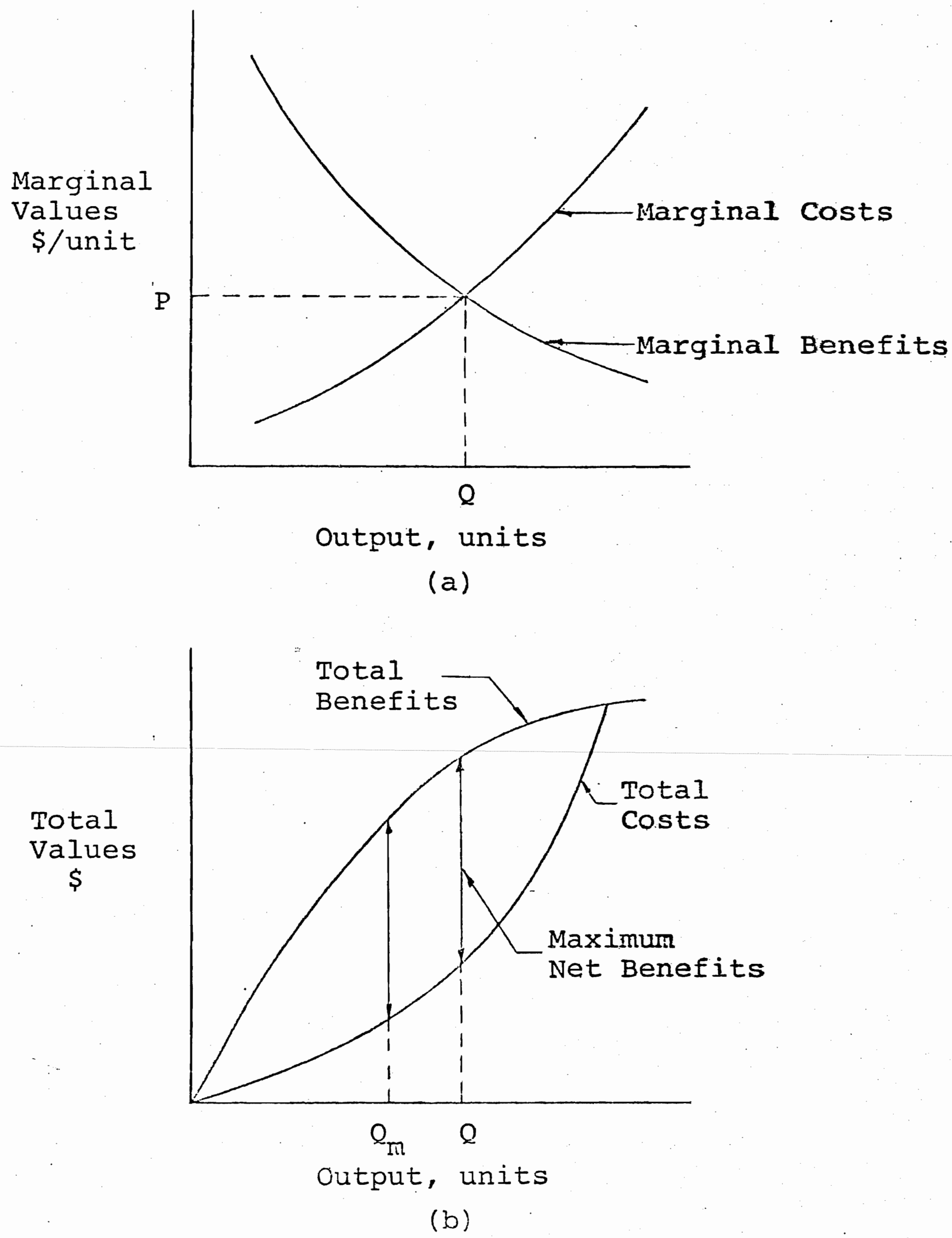


Figure 2. Conditions for economic efficiency: (a) marginal benefits equal marginal costs for output Q correspond to (b) maximum net benefits for output Q. (Q_m corresponds to optimal output conditions when production is limited to Q_m).

project costs and benefits. For a given set of cost and benefit information, various mathematical results and numerical techniques of optimization theory may be applied to project evaluation. An overview of some of the more important results and techniques for engineering applications is presented next.

Optimization theory addresses problems requiring minimization or maximization of a real-valued objective function subject to satisfying a number of equality and/or inequality constraints. The objective function and constraints are expressed in terms of the decision variables (output quantities). Linear programming (LP) defines a class of optimization problems in which the objective function and constraints are linear. The problem is solved by means of an iterative procedure. The most widely used is the simplex method which proceeds by moving from one feasible solution to another in such a way so as to improve the value of the objective function. For a more complete discussion, see any of several texts including Beightler et al. (1979), McMillan (1975), and S.P. Bradley et al. (1977).

With the general availability of several commercial software packages, linear programming provides an effective means of solving many classes of engineering problems. In particular, network models which involve the distributions of products from plants to consumer markets exhibit a special LP structure that can be exploited in developing an efficient solution algorithm. S.P. Bradley et al. (1977)

describe a general solution approach derived from specializing the rules of the simplex method. Many water resource problems are conveniently modeled as network problems and are readily solved using appropriate solution algorithms. Mandl (1981) surveys the state-of-the-art in network models and algorithms that can be applied to planning of irrigation and wastewater systems. For the case where total treatment plant and pipe costs are linear functions of capacity and all capacities and fixed demands are known, the problem is formulated so that the objective is to minimize costs subject to conservation of flow constraints. To solve this problem G.H. Bradley et al. (1977) and Maurras (1972) have developed codes that are specializations of the simplex algorithm. While both algorithms find the global optimum, the suitability of a linear objective cost function restricts application of the model. Furthermore, the codes are not generally available and can only be obtained from the authors. Integer programming models can incorporate features to form a more complex network model than the strict LP model. The technique has generally been applied to the fixed-charge model which assumes a linear cost function but with fixed project costs. However, integer programming may be used to approximate nonlinear functions by means of linear equations coupled with logical restrictions (integer variables). This piecewise linearization method is analogous to the delta-method applied in separable programming as described later in this paper. Using a special branch-and-bound algorithm by Rardin and Unger (1976), Jarvis et al. (1978) apply integer programming as a fixed-charge model for design of wastewater systems exhibiting economies of scale. Standard integer programming codes are commercially available, e.g. the integer programming code by IBM Corporation.

Although many real world problems are nonlinear, the availability of powerful linear programming methods provides a major incentive for approximating nonlinear problems in linear form. Therefore, one common approach to the general problem is to replace nonlinearities with linear approximations and solve the resulting linear program. Reklaitis et al. (1983) explore three basic strategies for employing linear approximations in solving nonlinear problems. All of these methods approximate a nonlinear function f(x) in the vicinity of a point X_o by a Taylor series expansion where higher order terms are ignored. The point X_o is called the linearization point. The three basic linearization methods differ primarily in the manner and frequency with which the linearizations are updated. In the direct successive LP approach, the LP solution defines the direction for a line search. Two successive LP algorithms, the Frank-Wolfe algorithm for linearly constrained problems, and the Griffith and Stewart method for nonlinear constraints, are presented in Reklaitis et al. (1983) and S.P. Bradley (1977). The Frank-Wolfe algorithm forms a linear approximation at the point X_o by replacing the objective function with its current value plus a linear correction term. The resulting LP solution is used to define a search direction. The search direction is given by the line segment joining the LP solution and the linearization point. Therefore, the successive LP approach may be viewed as an alternating series of LP and line-search subproblems. To maintain feasibility in the nonlinearly constrained problem, the Griffith and Stewart method bounds the step size for each intermediate LP solution. Since this algorithm proceeds rather slowly towards a solution, the successive LP approach is most appropriate for problems with only a few nonlinear terms.

The idea of the cutting plane approach is to successively improve the linear approximations to the constraint boundary in the region near the solution as the solution is approached from outside the feasible region. Gottfried and Weisman (1973) describe Kelley's algorithm in which linearized constraints (cutting-planes) are introduced one at a time to successively eliminate portions of the previous approximation of the feasible region. The resulting series of LP subproblems can be solved using specialized LP methods. However, this approach is not applicable for equality-constrained problems. In addition, convergence is ensured only for convex problems. In spite of these difficulties this strategy has been effectively used to solve some specially structured problems.

The third linearization technique is separable programming which utilizes piecewise linear approximations of separable nonlinear functions over the full range of the problem variables. The method is applicable to problems of the form

$$\text{Maximize or (Minimize)} \quad \sum_{j=1}^{n} f_j(x_j)$$

$$\text{Subject to} \quad \sum_{j=1}^{n} g_{ij}(x_j) \leq (= \text{ or } \geq) 0 \qquad (i = 1,2,\ldots m)$$

$$x_j \geq 0 \quad (j = 1,2,\ldots n)$$

Because decision variables appear separately in each function f_j and g_{ij}, the objective and constraint functions are separable. Instead of solving this nonlinear problem directly, an effective strategy is to make appropriate approximations so that linear programming can be used. Hadley (1964) and S.P. Bradley et al. (1977) examine two commonly used approximation techniques, the delta (δ)-method and the lambda (λ)-method. Although ensuring only a locally optimal solution, this approach is particularly suited to large network models since the separable programming mode of commercially available LP codes can efficiently solve very large problems. This model frequently arises in engineering applications particularly water resource planning formulations.

Currently, most mathematical programming applications to water resource problems incorporate linearization techniques and linear programming solutions. The linearization-based algorithms often cannot give acceptable estimates for the boundaries of the feasible region or the objective function. Rather than rely on inaccurate linearization to define the location of a point, linear approximations might be used only to determine a good local direction for search. Examination of values of the original objective and constraint functions can then yield the optimal point along the search direction. This strategy is analogous to unconstrained gradient search methods such as the conjugate gradient and quasi-Newton methods where linear approximation is used to determine a good search direction and actual function values guide the search along this direction. In the constrained case directions must be chosen to yield feasible points. Reklaitis et al. (1983) provide an excellent discussion of these direction-generation methods.

The group of direction-generation techniques called feasible direction methods requires solution of an LP subproblem to determine a direction that is both a descent (or ascent) direction and feasible. The objective of the LP subproblem is to solve for the direction that maximizes the increase (or decrease) in the objective function subject to feasibility constraints. This method has disadvantages of a slow rate of convergence and inability to directly accommodate nonlinear equality constraints.

Another group of methods known as generalized reduced gradient methods (GRG) simply solves a set of linear equations instead of the LP solution to determine a favorable search direction. This method uses all nonbasic variables to define the direction and so is a generalization of the convex simple method, the direct analog to the linear simplex method. Incorporation of conjugate gradient or quasi-Newton strategies accelerates convergence of this method. The method may also be extended to accommodate nonlinear constraints. A number of GRG codes are currently available. Reklaitis et al. (1983) survey the major comparative studies which examine the relative merits of the various methods and codes.

In addition to strategies for exploiting linear approximations to nonlinear problem formulations are methods that use higher order approximations, specifically quadratic approximations. Essential to the strategy for employing quadratic approximations is the method of quadratic programming. Quadratic programming involves use of a simplex-like algorithm to obtain a solution for a problem consisting of a quadratic objective function and linear constraints. Since the partial derivatives of a quadratic function are linear, a modified linear programming problem is generated by application of Kuhn-Tucker conditions (and thus forming partial derivatives). The method yields a local optimum which is guaranteed globably optimal only for strictly concave or convex functions. Hadley (1964) discusses several computational techniques for solving quadratic programming problems.

Reklaitis et al. (1983) examine several approaches using full quadratic approximation of objective and constraint functions which prove no easier to solve than the general nonlinear problem. In addition, formulation of quadratic programming subproblems (quadratic objective function and linear constraints) results in no significant improvement over the successive LP approaches. However, formulation of a subproblem objective function with the quadratic term as the second derivative of the Lagrangian function does provide the basis for an efficient algorithm for generating good search directions. The difficulty of providing second derivatives for the problem functions is resolved using quasi-Newton methods which only require differences of gradients of the Lagrangian function to approximate and update the second derivative. The result is a sequential quadratic programming (SQP) algorithm in which the solution to the quadratic programming subproblem defines the search direction. Reklaitis et al. (1983) survey some of the major studies which examine the relative merits of the various quadratic approximation methods and codes. The survey also includes studies which compare various nonlinear programming methods and codes based on criteria such as efficiency, global convergence, ability to solve various types of problems, and ease of use. In general, nonlinear programming techniques have not been applied to water resource problems. However, results of the comparative studies indicate that GRG and SQP have features that are important to engineering optimization.

Cost Allocation

Economic efficiency in many water resource projects may be achieved by taking advantage of (1) economies of scale in production and distribution facilities, (2) the assimilative capacity of the receiving environment, (3) excess capacity in existing facilities, (4) multipurpose opportunities, and/or (5) multigroup cooperation (Heaney and Dickinson, 1982). The results of economic analysis often indicate that participants should be combined into a cooperative joint venture. Like natural monopolies and public goods, the market system may not efficiently allocate the benefits (or costs) of joint water resource projects. Therefore, given an economically efficient joint project design the task of distributing the economic impacts among all of the participants remains. The objectives of cost allocation include (1) satisfaction of the financial requirement that project revenue equals project cost; and (2) satisfaction of economic requirements so as to encourage optimum use of project output. As noted previously, financial and economic requirements are satisfied simultaneously under pure competition. The market reaches equilibrium where marginal cost equals marginal benefits. At this intersection marginal cost also equals average cost so that the resulting price vector P and output vector Q satisfy both economic and financial requirements.

The absence of such a competitive market system requires some form of public control to administer prices which properly allocate resources. For example, when average costs are decreasing, a price based on efficiency does not satisfy the financial requirement. This can be seen in Figure 3 where the optimal output Q and corresponding marginal price P_m are still given by the intersection of marginal cost and marginal benefit. However, average cost exceeds marginal cost so that a price P_a is required so that revenues will just cover costs. Unfortunately, average cost pricing at P_a restricts use and forces a suboptimal situation.

In contrast to economies of scale, conditions of increasing average cost result in an optimum pricing scheme that more than satisfies the financial requirement. Figure 4 shows that for increasing average costs, marginal cost lies above average cost so that marginal pricing produces a net revenue of $Q(P_m - P_a)$. In water resource applications this presents a problem of what to do with the net revenue. According to project financial requirements, only the cost legally obligated during project construction and operation must be recovered.

With the competitive market as a model for a "first best" pricing scheme, a common pricing approach is to pretend that a market exists for project outputs. The

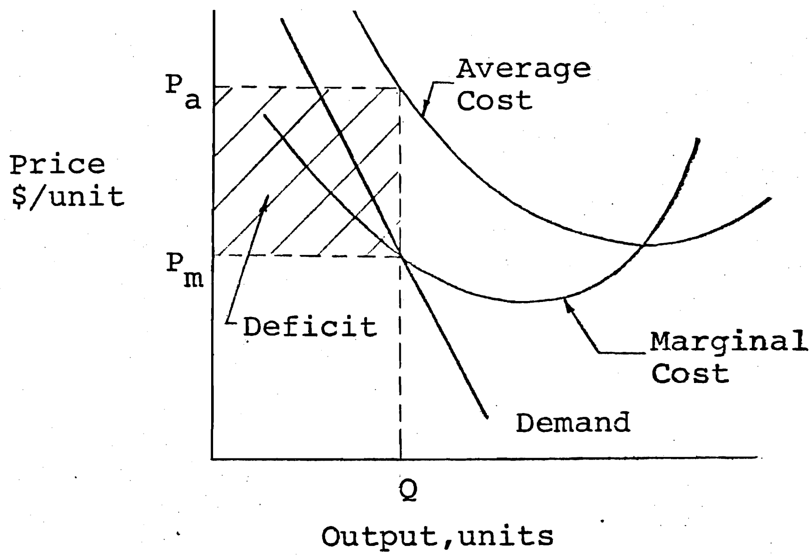


Figure 3. Selection of price under decreasing average cost conditions.

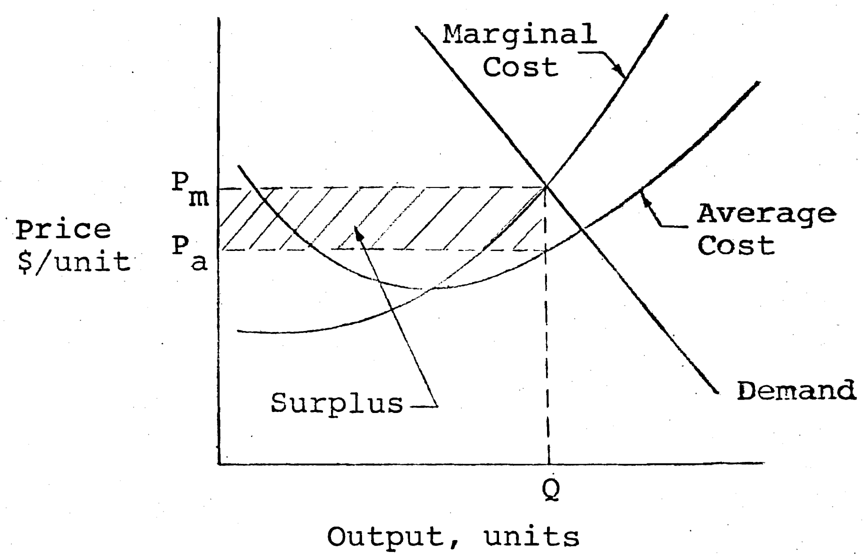


Figure 4. Selection of price under increasing average cost conditions.

objective of the resulting market analysis is to determine a "second best" allocation of resources given that prices are constrained to deviate from marginal cost. This strategy has dominated regulated firm pricing schemes where decreasing average cost conditions are predominant (Zajac, 1978). James and Lee (1971) describe three basic approaches for this pricing strategy as applied in water resources planning. One approach is to employ price discrimination to capture some of the consumer surplus. The objective in price discrimination is to charge those users receiving marginal benefits greater than P_m (refer to Figure 3) an extra fee to satisfy the financial requirement. Even though prices above marginal cost reduce full economic utilization, the net effect may be a reasonable compromise between the economic and financial requirements. A second approach is to maintain marginal pricing while raising the additional required revenue through a fixed, nonmarginal charge. Each user would pay a flat fee plus a per-unit charge equal to marginal cost. A final approach is to set price equal to marginal cost while supplementing the resulting revenue through subsidies. Such subsidies are usually distributed by the government and involve redistribution of income from those paying taxes to those receiving subsidies.

Despite the potential of marginal cost pricing, actual practice often dictates application of charges based on some measurable unit called a charging vehicle. James and Lee (1971) describe three such charging vehicles which include

(1) ability to pay, (2) benefit received, and (3) quantity of output or average cost. While schemes involving each of these vehicles can be formulated to satisfy financial requirements, a theoretical difficulty centers on economic grounds. Economists argue that unless price equals marginal cost, users are not given the proper incentive to balance the value they derive from use of incremental output against its marginal opportunity cost. Indeed, there has been continued interest in the application of marginal cost pricing techniques to water resource planning. For example, Hanke and Davis (1973) report significant potential for marginal cost pricing for municipal water services, industrial and municipal sewage treatment, navigation, and flood control. Guariso et al. (1981) present an iterative algorithm for determining optimal water supplies and demands in a regional network where marginal cost equals marginal benefits. The authors propose use of resulting marginal cost prices as reference points for evaluating prices determined by conventional methods.

Despite continued interest in marginal cost pricing, most water resource planners and governing agencies rely on numerous (some ad hoc) methods of dividing project costs among participants. Loughlin (1977) expresses the prevailing view that economic and financial analysis are independent of each other. The rationale is that joint costs which cannot be directly attributed to a participant are not marginal and so do not influence optimal design (except for total project justification). Since joint costs must be paid, financial analysis is required to allocate them among participants. Consequently, economic efficiency in water resources planning is usually defined by the maximum positive difference between total benefits and total costs. Cost allocation, then, involves apportionment of the costs of the economically optimal solution among participants. Notably absent from this concept of economic efficiency is the complete notion of Pareto optimality which requires that for an economically efficient allocation of resources, no other feasible distribution of output can benefit one participant without harming another. In practice, the use of an incomplete definition for economic efficiency results in cost allocation schemes that imply prespecified demand quantities relatively insensitive to price. This model may or may not be appropriate depending on the actual supply and demand conditions.

The objective of the cost allocation phase of water resources planning is to determine a vector of charges that is acceptable to all project participants. If charges are not acceptable to all, some may choose not to participate resulting in a suboptimal final project design. In other words, the objective is to define an equitable vector of charges for which no individual or group of participants would benefit by leaving the joint project. This exactly corresponds to the concept of sustainability of natural monopoly pricing which can be described using cooperative

N-person game theory as in Sharkey (1982a). In fact, several of the conventional or proposed procedures for allocating costs directly incorporate cooperative game theoretic concepts.

Heaney (1979a) describes the evolution of current practice beginning with the Tennessee Valley Authority (TVA) studies in the 1930's and including the Federal Inter-Agency Studies in the 1950's. Comparison of conventional method criteria and game theoretic concepts reveals important similarities including the requirement for full recovery of costs and the notion that no participant should be charged more than he would pay if he acted independently. These requirements constitute the set of imputations of a cooperative game. Heaney and Dickinson (1982) propose an extension of current practice to require that charges satisfy the full core constraints so that no group of participants is charged more than it would pay if acting alone. Following is a brief description of some commonly used conventional and proposed allocation methods.

James and Lee (1971) identify cost allocation methods by the definition of cost used, the cost directly assigned to each participant, and the cost allocation vehicle. The result is a matrix of 18 possible ways to apportion costs. Three of the most commonly used cost allocation methods include the use of facilities, alternative justifiable expenditure (AJE), and separable costs remaining benefits (SCRB) methods.

Loughlin (1977) describes the use of facilities method which allocates joint costs in proportion to some measure of the relative use of the central project facilities by each participant. Joint costs are determined by subtracting either direct costs or separable costs. Direct costs are defined to be the costs of the elements of the project which are used solely by that participant. Separable costs are the differences between the total multigroup project cost and the cost of the project without the participant. They include direct costs as well as the incremental costs of changing the size of joint cost elements. The use of facilities method is generally considered acceptable only where joint use is clearly determinable on a comparative basis (Loughlin, 1977). The method does not incorporate any of the stability criteria included in other conventional methods.

Recommendations by the Federal Inter-Agency River Basin Committee extend the notion of equity in cost allocation by requiring that participants receive a proportional share of the savings resulting from joint projects. Consequently, the Committee recommends use of the SCRB method and the AJE method when the effort or expense to obtain information for the SCRB method is prohibitive (Loughlin, 1977). Heaney (1979a) describes the AJE method for which each participant is assigned his direct cost plus a share of the remaining joint costs (total cost less all direct costs) in proportion to his alternative costs avoided. Alternative costs avoided are defined as the difference between the participant's stand alone cost and direct cost. The method recognizes that the benefit to each individual participant could be less than the go-it-alone cost. In addition, the alternative justifiable expenditure method is often much easier to calculate than the SCRB method described below. The method has been favored by the Tennessee Valley Authority as well as the U.S. Environmental Protection Agency.

In contrast to the AJE method, the SCRB method uses separable costs rather than direct costs to determine joint costs. The SCRB procedure assigns to each participant his separable costs plus a share in the remaining joint costs in proportion to the remaining benefits (as limited by alternative costs). For convex games the SCRB solution lies in the center of the core. A game is convex if

$$C(S) + C(T) \geq C(S \cup T) + C(S \cap T) \qquad S \cap T \neq \Phi \qquad (7)$$

for all S and T subsets of the grand coalition N and C defined previously as the characteristic cost function. The Federal Power Commission, the U.S. Army Corps of Engineers, and the Bureau of Reclamation have relied on the SCRB method almost exclusively. All water resource agencies are applying the SCRB method for multiobjective and multipurpose federally assisted reservoir projects (Loughlin, 1977).

Although conventional methods such as the SCRB method incorporate fairness criteria similar to some of those employed in cooperative game theory, corrections are required to ensure that the resulting vector of charges is sustainable (lie within the core if the core exists). Heaney (1979b) proposes that incorporation of individual benefits in the SCRB method should be extended to include subcoalition benefits. In addition, Heaney and Dickinson (1982) propose a generalization of the SCRB method to incorporate full core constraints. The rationale here is that for games where the separable costs which are the incremental costs for each participant (player) to join the coalition last are not the lowest incremental costs, the SCRB method prorates joint costs based on upper and lower bounds that are not in the core. Consequently, for nonconvex games the SCRB solution does not lie in the center of the core and may not lie within the core at all for extreme cases. The proposed method involves solution of a system of linear programs to delineate the core bounds followed by proration of joint costs based on these actual core bounds. For the case where no core exists, Heaney and Dickinson (1982) suggest relaxation of intermediate coalition constraints to determine a compromise solution. This generalized SCRB procedure is called the minimum cost remaining savings (MCRS) method.

Analogous to the "fair solutions" of the conventional SCRB and MCRS methods are some of the unique solution concepts used in game theory. Two of the most popular unique solution notions are the Shapley value and the nucleolus. The idea of the Shapley value is that each participant should pay the incremental cost of adding him to the coalition. With the assumption that all coalition formation sequences are equi-likely, the Shapley value assigns to each player the incremental cost he brings to coalitions expected over all coalition formation sequences. The resulting charge to the ith participant is

$$X(i) = \sum_{S \subseteq N} \alpha_i(S)[C(S) - C(S - \{i\})] \tag{8}$$

where $\alpha_i(S) = \dfrac{(S - 1)(n-S)!}{n!}$

and n is the total number of participants. If the game is convex, the Shapley value is in the center of the core. However, for nonconvex games the Shapley value may fall outside of the core. In addition, computations for projects involving more than just a few participants are quite tedious. Littlechild and Owen (1973) present a simplified Shapley value for application in a special class of multipurpose projects where the purpose with the largest cost of separate action in a coalition determines the characteristic function cost for the entire coalition. Littlechild and Thompson (1977) demonstrate the advantages of the simple Shapley value for determining aircraft landing fees. Heaney (1979a) demonstrates how the simple Shapley value might be applied to multipurpose water resource projects through an example for pollutant pricing in a wastewater treatment plant.

In addition to the computational problems, Loehman et al. (1979) find another deficiency in application of the Shapley value to real situations. Loehman et al. (1979) argue that it may be unrealistic to assume that all orders of users are equi-likely as assumed for the Shapley value. Loehman and Whinston (1976) have developed a generalized Shapley value that represents the expected incremental cost where all orders are not equi-likely. Loehman et al. (1979) apply the general Shapley value to an eight-city regional wastewater treatment system in which coalition sequences that do not occur are identified and their probabilities set to zero in Shapley value computations. In the eight-city example, "impossible" coalition formation sequences are given as those that are not economically viable. This application of the general Shapley value is not correct for the conventional definition of the characteristic cost function. The characteristic function C(S) is usually defined as the optimal solution for that coalition. At worst, no lower cost results in coalition formation in which case the coalition is said to be inessential, that is

$$C(S) + C(T) = C(S \cup T) \qquad S \cap T = \Phi \tag{9}$$

for all subsets S and T in N. By setting to zero the probabilities of inessential coalitions, weak players that acutally contribute the smallest savings to the grand coalition become stronger players and are undercharged while actual strong players are overcharged. This distorts the game and may lead to very unfair charges. If the criteria for identifying impossible coalition sequences are restricted to include only political or strictly physical considerations, application of the general Shapley value might be appropriate.

The nucleolus maximizes the minimum savings of any coalition and requires N-1 linear programs. The solution satisfies all of the core constraints, always exists, and is unique. See Heaney (1979a) or Lucas (1981) for more information on computing the nucleolus.

At this point in the discussion, two particular points warrant further consideration. First, the matter of defining an appropriate characteristic function presents some problems. Throughout much of the game theory and cost allocation literature the characteristic function is assumed given with little consideration of how it is derived. Conventional and game theory concepts and methods are of little value unless the required characteristic function values can be unambiguously defined. Sorenson (1972) defines the following four alternative definitions for the characteristic cost function.

$C_1(S)$ = value to coalition if S is given preference over N-S

$C_2(S)$ = value of coalition to S if N-S is not present

$C_3(S)$ = value of coalition in a strictly competitive game between coalition S and N-S

$C_4(S)$ = value of coalition to S if N-S is given preference

As will be demonstrated in this study, alternative definitions can be used depending on how the problem is defined.

The second consideration involves the notion of strict optimization in engineering design. Often economic requirements call for a large or complex project design that may be difficult if not impractical to implement. The preferred solution may be a relatively efficient design that is easier to implement. The argument for good suboptimal solutions is founded on several grounds. Heaney (1983) observes that as the size of a regional project increases, transaction costs might be expected to increase at the margin due to multiple political jurisdictions, growing administrative costs and shifting of environmental impacts. Although most economic analyses ignore these transaction costs, Heaney (1983) estimates that they may run from 2 to 10 percent of total costs. When transaction costs are high an intermediate but simpler solution might be preferred. Unfortunately, little work has been done examining viable intermediate economic solutions. One reason for this is that the availability of powerful optimization techniques such as linear programming, reduced gradient, and sequential quadratic programming techniques ignore suboptimal solutions that may be close enough to the optimum to be satisfactory. Wilde (1978) develops the concept of "satisfactory design" as an approach to convert relatively difficult optimization problems into the construction of easy to compute bounds and estimates of the ideal optimum. Finally, the viability of suboptimal designs is especially apparent given the uncertainty of physical and economic data. With regard to wastewater treatment systems, cost estimates are usually based on power function approximations of available data. Consequently, first order cost estimates which merely specify treatment process types such as primary sedimentation or activated sludge may be in error by nearly 60 percent. Even more detailed second order estimates involving specific component cost estimates such as filtration media, backwash pumping, surface washing facilities may err by more than thirty percent (Clark and Dorsey, 1982). Likewise, performance estimates are often highly variable. Clearly, emphasis on strict economic optimization may be inappropriate when uncertainty of data is high. Suboptimal solutions may be obtained with less effort and expense while achieving a satisfactory result. Given this wide variability in cost estimates, it is not surprising that practicing professionals do not seem to concern themselves with the average cost/ marginal cost controversy.

Summary

Several important observations may be extracted from this literature review. First, economic theory addresses only the case where the optimal design corresponds to marginal benefits equal to marginal costs. Economists argue that given this optimal condition, the proper pricing scheme in a monopolistic (economies of scale) market is one that produces the least damaging deviation from strict marginal pricing. Economic theory fails to address the problem that arises when the economic optimum does not correspond to marginal benefits equal to marginal costs. While recent theories on both natural monopoly pricing and engineering cost allocation incorporate the concept of sustainability, current cost allocation methods typically ignore marginal cost considerations. Instead marginal considerations are viewed as strictly a part of the economic evaluation phase of project planning and therefore independent of the cost allocation phase. A major point of discussion in this paper concerns the applicability of these two conflicting view-points. Finally, with regard to economic optimization, most water resource applications involve linear programming solutions to the linear cost and fixed-charge problems. Although economies of scale in project construction and operation are typical for many water resource projects, incorporation of concave cost functions has generally been avoided due to conceptual and computational problems. However, recently developed codes may provide a means of formulating more realistic models for engineering application.

Section II

METHODOLOGY

The purpose of this section is to develop a general approach for examining the relationship between the economic and financial objectives in water resources planning. Following is a suggested sequence of steps that provides a means of describing and evaluating this relationship. The steps are listed below in general terms and are followed by a discussion of possible results and conclusions for a simple wastewater reuse example. The discussion includes three different cases defined according to problem objectives and corresponding cost and benefit relationships.

Project Evaluation Steps

1. Examine overall project objectives.
2. Define the system for study.
3. Formulate the problem in terms of specific objectives and constraints for the system defined in step 2.
4. Examine how the problem objectives and constraints relate to theory and available methodology, e.g. economic theory, optimization techniques, and cost allocation methods.

5. Apply appropriate methodologies based on the analysis in step 4.
6. Examine implications for economic efficiency and equity in terms of tradeoffs and possible compromise solutions.

Discussion and Wastewater Reuse Example

Step 1. Examine Overall Project Objectives

This first step is required to formulate the problem. Often the motivation behind a proposed water resource project involves political, social, and environmental considerations as well as economic and financial objectives. Clear understanding of the major and underlying objectives is essential to the evaluation of tradeoffs and viability of possible compromise solutions. Donovan et al. (1980) discuss several of the possible considerations involved in wastewater reuse planning. One objective might be to reduce water supply costs to a group of users by taking advantage of economies of scale in a wastewater reuse project. Other objectives might include increasing available water during drought by decreasing demand on the community's existing water supplies. In this way potable water supplies can be protected for more valuable use. Other goals may require a reuse system to provide social benefits such as development of municipal recreational facilities. Still other motivating factors might involve improvement or shifting of environmental quality impacts. An underlying issue for any objective is the question of how best to apportion project costs (or benefits). The objective of the resulting financial analysis might be an equitable but simple allocation of project costs (or benefits). Inherent tradeoffs may develop between these two financial objectives (equity and simplicity) as well as among financial and economic, social, or political objectives. Finally, planning objectives must incorporate any legal or institutional constraints.

The objective for the example wastewater reuse study is to determine the system design that minimizes total water supply costs to potential wastewater users and to fairly allocate these costs among participants.

Step 2. Define the System for Study

Before the problem can be properly formulated for analysis, the system under consideration must be carefully defined. A proper definition includes identification and description of technical, economic, social, legal, and political relationships. Although this is an extremely important phase of water resources planning, it is often quite complex. See Sample (1983) for detailed treatment of this phase in an actual wastewater reuse study for the South Florida Water Management District. Detailed consideration of how the system is determined for this example might detract from the emphasis of this paper. Therefore, the final system for the example problem is simply given as shown in Figure 5. The system consists of a secondary treatment plant as the single wastewater supply source and four irrigation sites. Locations of pipes connecting the treatment plant to each site are fixed by right-of-way restrictions.

Costs for consideration in subsequent economic and financial analysis include the cost of additional treatment at the plant and transportation costs. Additional information includes the demand schedule for each user. This is derived from water use patterns or from alternative water supply costs. Alternative water supply costs may be referred to as on-site costs while treatment and transportation costs may be termed off-site costs. In actual practice, costs for consideration may also include costs for water quality monitoring, replumbing, storm water runoff control, and future capacity expansions.

Application of cost engineering principles results in estimates for treatment and transportation costs. For planning studies, cost estimates are often derived from statistical cost equations corrected with appropriate updating and localizing factors. The general form for the equations is a power relationship of the major input and output variables for construction and operation and maintenance of the various project components and unit processes. For the wastewater reuse example, a reasonable representation for treatment costs and on-site costs is given by a

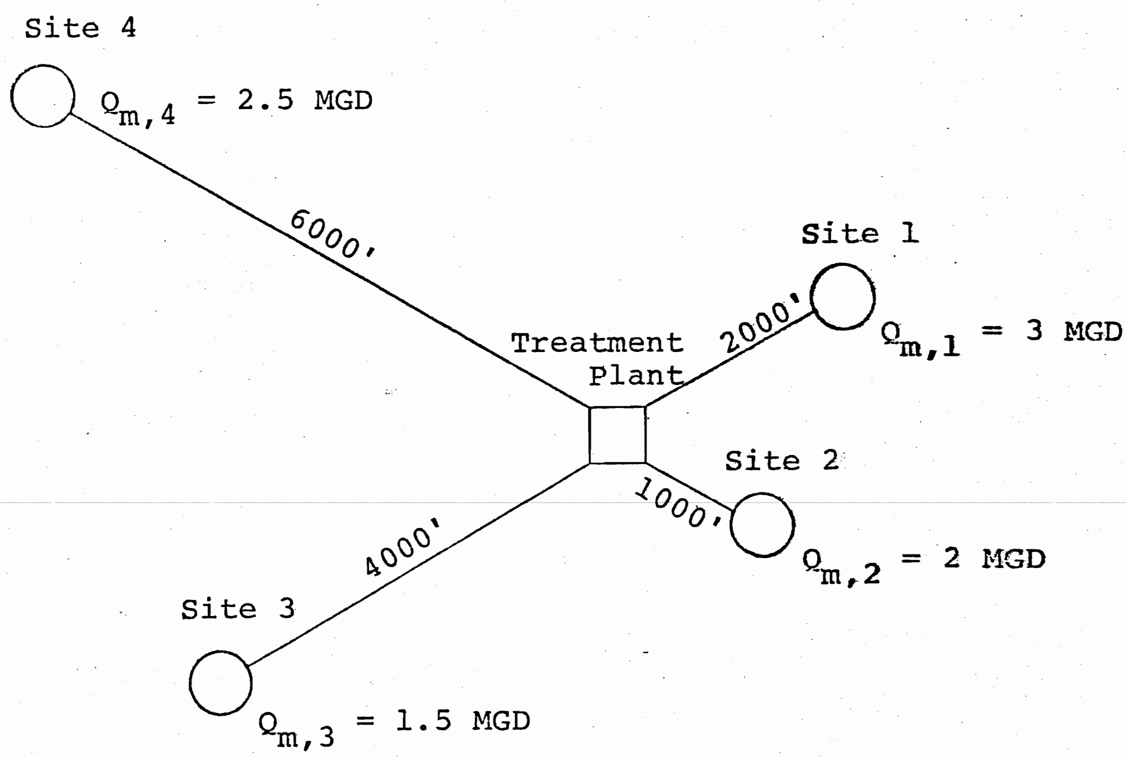


Figure 5. System for wastewater reuse example.

power function of the flow variable. In contrast, transportation costs which include pipe and pumping costs are best estimated from static head, pipe diameter, and pipe length as well as flow. For a given pipeline path, the pipe length is fixed and static head may be reasonably estimated. As a result, cost estimates for the example transportation costs may be reduced to functions of flow and pipe diameter. In addition, for a given flow a tradeoff exists between pipe costs which increase with increasing pipe diameter and pumping costs which decrease with increasing diameter. Deb (1978) presents an optimization model which selects the optimal pipe diameter to minimize total transportation costs. Sample (1983) modifies this procedure to include a more detailed analysis of transportation component costs. The result is a scheme to compute optimum pipe diameter as a function of flow so that total transportation costs may be presented as strictly dependent on treated wastewater flow. All costs are estimated by a power function of the form

$$C = a\,Q^{b}$$

where

C = annual cost, 10^3\$

Q = treated wastewater flow, MGD

a,b = constant function parameters

All estimates include construction and operation and maintenance costs and are assumed to be properly updated with local information.

The specific equations for the example problem are given in Table 1. Cost relationships for off-site costs exhibit economies of scale as indicated by the exponent parameters which are less than one for both treatment and transportation. This is consistent with conditions found in actual practice. For example, Sample (1983) found significant economies of scale in both treatment and transportation costs in the South Florida study. The example off-site relationships are consistent with these general results. On-site equations for the example problem are also given in Table 1 for three cases including economies and diseconomies of scale as well as linear on-site costs. These three cases are examined to demonstrate different results and implications for economic and financial analysis.

A final consideration for defining the system involves specification of maximum demand and plant capacity constraints. Two different plant capacity limits are considered corresponding to the case where supply does not limit system design (plant capacity, $Q_{m,t} = 10$ MGD) and, alternatively, to the case where system design is supply limited (plant capacity, $Q_{m,t} = 6$ MGD). This analysis assumes that each potential wastewater user's current quantity of total water use is equal to the maximum demand fixed by physical requirements. However, each user's demand for recycled

Table 1. Cost Functions for Wastewater Reuse Example

Cost Component	Site, i: 1	2	3	4
On-site Treatment, C_t	$60(Q_{of,1} + Q_{of,2} + Q_{of,3} + Q_{of,4})^{.7}$			
Transportation, $C_{p,i}$	$8(Q_{of,1})^{.4}$	$5(Q_{of,2})^{.3}$	$13(Q_{of,3})^{.5}$	$17(Q_{of,4})^{.6}$
On-site, $C_{on,i}$				
<u>Case 1</u> Linear Costs	$80Q_{on,1}$	$70Q_{on,2}$	$90Q_{on,3}$	$100Q_{on,4}$
<u>Case 2</u> Economies of Scale	$100(Q_{on,1})^{.9}$	$80(Q_{on,2})^{.9}$	$100(Q_{on,3})^{.8}$	$120(Q_{on,4})^{.7}$
<u>Case 3</u> Diseconomies of Scale	$25(Q_{on,1})^{1.7}$	$40(Q_{on,2})^{1.4}$	$45(Q_{on,3})^{1.2}$	$70(Q_{on,4})^{1.1}$

water is price dependent as given by on-site cost relationships. Therefore, the demand constraint is that total water supplied to each user must equal the fixed demand specified in Figure 5. However, the quantities obtained on-site and off-site are determined by the economic and financial analysis in the steps that follow.

Step 3. Formulate the Problem in Terms of Specific Objectives and Constraints

As stated in step 1 the objectives of the wastewater reuse study are twofold. The first objective involves economic efficiency for which the desired design is one that minimizes total irrigation water costs for the defined system. The second objective involves financial considerations for which an equitable apportionment of final project costs is desired. The economic objective and corresponding constraints are specified in terms of the system defined in step 2. The resulting economic optimization problem is to

$$\text{Minimize } Z = \sum_{i=1}^{4} (C_{on,i} + C_{p,i}) + C_t$$

$$\text{subject to} \qquad Q_{on,i} + Q_{of,i} = Q_{m,i} \qquad i = 1, 2, 3, 4$$

$$\sum_{i=1}^{4} Q_{of,i} \leq Q_{m,t}$$

where

$C_{on,i}$ = on-site water supply cost for site i, 10^3\$/year

$C_{p,i}$ = transportation cost for pipeline to site i, 10^3\$/year

C_t = treatment cost , 10^3\$/year

$Q_{on,i}$ = on-site water supply to site i, MGD

$Q_{of,i}$ = off-site water supply to site i, MGD

$Q_{m,i}$ = maximum water demand for site i, MGD

$Q_{m,t}$ = treatment plant capacity, MGD.

This cost minimization problem is equivalent to the net benefit maximization problem for benefits defined as alternative (on-site) costs avoided. In general, a correct specification of core constraints incorporates individual and subcoalition benefits as well as costs. For this example, the problem is defined so as to include benefits implicitly as alternative costs avoided.

With regard to financial objectives, recall that if the core exists it contains the set of outcomes for which no coalition would be better off not participating. This seems a reasonable set of minimum criteria for an equitable solution. Since the subsequent analysis shows that the core does exist for each of the three cases in this example, we can justifiably eliminate any allocation result not in the core. The core may be represented by the following set of constraints.

$$X(i) \leq C(i) \qquad i = 1, 2, 3, 4 \tag{10}$$

$$\sum_{i \varepsilon S} X(i) \leq C(S) \qquad S \subset N \tag{11}$$

$$\sum_{i \varepsilon N} X\ (i) = C(N) \tag{12}$$

where

C(S) = the characteristic cost function for coalition S

X(i) = charge to player i

N = grand coalition of players.

Step 4. Examine How the Problem Objectives and Constraints Relate to Theory and Available Methodology

This step involves examining how economic and cost allocation theory apply to the problem as defined in step 3. The water resource planning and pricing literature often defines problems where the total potential demand lies within the economies of scale range of project cost functions. This is analogous to assuming that the project design has no production capacity limit or that the capacity limit is not binding. For the wastewater reuse problem, the size of the project is limited by either total maximum demand or treatment plant capacity. In addition, most conventional cost allocation and game theory solutions do not consider demand. Charges determined by methods such as SCRB or the Shapley value can be stable in the economic sense only for inelastic user demand. This assumption may be valid when potential participants must satisfy a fixed demand with no alternatives other than joining the project coalition or constructing an independent project with the same cost function(s) as the joint project. Where there are economies of scale, benefits grow as coalition size increases, and the analysis becomes a problem of determining how the benefits (costs) are to be distributed. A similar effect results in the wastewater reuse example but is complicated by production limits and demand considerations. For the example we will examine three types of demand relationships and the implications of production limits on economic efficiency and cost allocation. As will be demonstrated later in this section, economic optimization for the wastewater reuse problem may be achieved rather easily for certain types of cost relationships. However, in other cases mathematical programming techniques may be required to solve the general nonlinear optimization problem. Three optimization methods: separable programming, generalized reduced gradient, and sequential quadratic programming are considered mainly on the basis of the availability of computer codes.

Recall that separable programming is a linearization technique which utilizes piecewise linear approximations of separable nonlinear functions. This technique is applicable to the example problem since all problem decision variables appear separately in the objective and constraint equations.

Separable programming is contained within the linear programming procedures of the Mathematical Programming System (MPS/360) available through the IBM Corporation (1971). The separable programming procedure employs the delta-method described by Hadley (1964) in which the separable nonlinear functions are approximated by means of linear equations coupled with logical restrictions. The solution is given by a simplex procedure modified to incorporate the required logical restrictions. One caution in applying this technique is that the solution may converge to a local optimum. This possibility may be reduced by solving both the dual and primal problem or solving the problem for different initial values for the decision variables to verify results.

Application of the MPS/360 code proved rather tedious for the wastewater reuse example. The user is required to construct appropriate piecewise linearizations for each separable function. The example problem objective function contains several different separable functions that must be approximated. Furthermore, each set of computer input data is very specific to the particular problem definition. Relatively small changes in the problem definition often require extensive alterations in the computer input. Therefore, this method may not be practical for planning studies involving many different nonlinear cost relationships or studies requiring flexibility in defining these relationships and problem constraints. This is the case for the wastewater reuse example where the problem is defined for various demand and constraint conditions. As a result, this technique was abandoned in favor of a more flexible code.

One such code is NPSOL by Gill et al. (1983) which uses a sequential quadratic programming algorithm in which the search direction is the solution of a quadratic programming subproblem. The algorithm requires that the user define the problem in terms of the specific objective and constraint functions and their partial derivatives. Compared to separable programming, this code requires much less preliminary work by the user and easily accommodates changes in problem definition. However, this code is very sensitive to problem conditioning and is best suited to small, dense problems containing nonlinear constraints. Attempts to apply NPSOL to the example problem resulted in problems of ill-conditioning not easily remedied.

The third code, GRGA by Abadie (1975), employs the generalized reduced gradient method. As with NPSOL the user must define the problem in terms of the specific objective and constraint functions as well as partial derivatives. Comparative studies indicate that this code is among the most efficient and least sensitive to problem condition (see Reklaitis et al., 1983). Unfortunately the only available documentation for this code is in French. Additionally, the program generates all output in French. However, application of the code to the example results in no conditioning problems. Therefore, results presented for the example problem are those determined from GRGA.

Financial analysis for the example problem involves evaluation of various cost allocation schemes including the Shapley value and MCRS method as well as marginal and average cost pricing strategies. As the financial objective is to determine a set of charges that satisfy core constraints, the characteristic cost function must be defined. In conventional cost allocation theory, the characteristic cost function is defined under an implicit assumption of inelastic demand. That is, the quantity of output demanded by each player is assumed fixed. The player, then, must satisfy his fixed demand through either independent action or cooperation in a joint project. The characteristic function is computed for fixed quantities using fixed cost relationships. However, it is often unrealistic to assume an inelastic demand. In the wastewater reuse example, the total quantity of water required for irrigation at each site is assumed fixed, but the demand for wastewater is price sensitive since each user has an alternative water supply option with its own cost relationship. The player may satisfy his demand through varying degrees of participation in a joint project as opposed to the usual all or nothing restriction. Independent action, then, may involve a quantity of project output different from the quantity assigned for joint project cooperation. Determination of the characteristic function requires specification of appropriate quantities for independent action. For the wastewater reuse example, these quantities may be taken as the off-site supply corresponding to the minimum cost combination of on-site and off-site water supply for each player. However, where potential demand exceeds production capacity, it is necessary to define how the restricted supply is to be apportioned not only for the final project but for independent action as well. Therefore, to determine the optimal go-it-alone solution for coalition S, it is necessary to apportion an output quantity to S by defining the behavior of the complementary coalition N-S.

Recall that Sorenson (1972) suggests four possible definitions for the characteristic function. We will examine each of these definitions to determine which might be the most appropriate for application to the example problem. Sorenson defines $C_1(S)$ as the value to the coalition if S is given preference over N-S. If S has preference over N-S, S gets all the wastewater it wants while entering the project after N-S. This way S would get all of the highest savings from off-site economies of scale without having to share them with other groups. However, this would result in a game that is not subadditive. Coalition S would never want to cooperate with other groups and have to share savings.

A second definition, $C_2(S)$, is the value to the coalition if N-S is not present. This would require that S has the option to build and operate its own facility or be allowed to use the central facility up to its willingness to pay (as given by alternative costs), its maximum demand, or plant capacity, whichever comes first. This may or may not lead to a subadditive game depending on actual relative costs and capacity limitations, e.g. S may gain more savings by independent action as defined by $C_2(S)$ than by joining a coalition where its level of activity may be restricted.

A third definition, $C_3(S)$k involves a strictly competitive game between S and N-S. Since joint cost economies of scale imply a basic cooperative attitude among participants, $C_3(S)$ is certainly not an appropriate definition for this example.

As a final definition, $C_4(S)$ is the value of coalition S if N-S is given preference. Giving preference to N-S means letting N-S go last so that it can realize highest marginal savings. There are at least two ways $C_4(S)$ can be computed using this definition. First, given that Q_N is the economically efficient output allocation vector for the grand coalition N, C(S) may be computed using the go-it-alone cost for its allocated portion of Q_N. This is equivalent to making S go first to receive its allocated quantity of resource. This implies a cost to N-S equal to C(N) - C(S). With economies of scale C(S) will always be more than the cost in a larger coalition for a given Q_N, thus ensuring subadditivity. However, this method is not really in keeping with the concept of independent action since it depends on results of economic analysis involving N. However, it does provide a means of ensuring that all members of the grand coalition will have a right to some of the potential savings whether or not they cooperate. In effect, this method of computing C(S) uses the results of economic analysis to establish ownership. Perhaps a better approach is the idea of letting the members of N-S go ahead and form their own coalition and least cost solution. If the plant capacity limit is reached, S will get nothing and C(S) is just S's alternative (on-site) cost. In this way not everyone is guaranteed some of the potential savings. This result may be appropriate if S is such a weak player that it contributes savings only when joining relatively large coalitions. If there were some remaining capacity, we would define C(S) as the value to S of the least cost solution for the remaining supply. The implied cost to N-S is again C(N) - C(S). Coalition S might object since N-S gets some additional savings by having S go first. On the other hand, coalition N-S could argue that to charge S less would require some degree of cooperation by S. This last definition is probably the best for this particular application since it ensures game subadditivity and is in keeping with notions of independent action.

Before discussing the various specific cases for the example problem, a few comments are in order to clarify some possible points of confusion. First, in the analysis that follows it is often easier to discuss benefits or net benefits rather than alternative costs avoided or cost savings. This interchangeability of terms is a result of equivalence in problem formulations. Recall that defining the economic objective as a cost minimization problem is equivalent to defining it as a net benefit maximization problem. Benefits are computed from on-site cost as follows:

$$B(i) = C_{on,i}(Q_{m,i}) - C_{on,i}(Q_{m,i} - Q_{of,i}) \tag{13}$$

where $B(i)$ = benefits to user i, 10^3/year

and $C_{on,i}$, $Q_{m,i}$, and $Q_{of,i}$ are as defined previously in Table 1.

An additional point involves inclusion of direct costs in the analysis. It may seem reasonable to allocate direct costs separately from joint costs. However, since each participant in the optimal project benefits from the inclusion of all the other participants, each participant should be willing to include direct transportation costs in the total cost that must be allocated. In particular, if direct costs are not included, the relatively high direct costs assigned to some users might discourage their cooperative participation, resulting in a suboptimal final project design. Inclusion of direct costs in the analysis means that, in general, a single cost function will not apply. A single cost function often applies only to joint project components. Usually, different cost relationships apply to different cost elements. This is the case for the wastewater reuse problem where estimating functions for transportation costs are different for each site.

Case 1. Single-Step Demand Function

A single-step user demand is derived from linear on-site costs which in turn imply a constant unit water price. This situation may be common in actual practice where a user's current unit cost for water is the only information easily available. The single-step demand curve for site 1 in the wastewater example is shown in Figure 6(a). The flow level $Q_{m,1}$ corresponds to the maximum demand flow for site 1. The unit cost of $80,00 per MGD represents the average (or marginal) benefit to the site and corresponds to a linear benefit curve as shown in Figure 6(b).

Economic Optimization

First we consider joint project components, for the moment ignoring direct cost components. This corresponds to consideration of central facility treatment costs while ignoring transportation costs. For this simple case, the potential user (player) with the highest average benefit gets priority, i.e. $Q_{of,i}$ = Min[$Q_{m,i}$, remaining plant

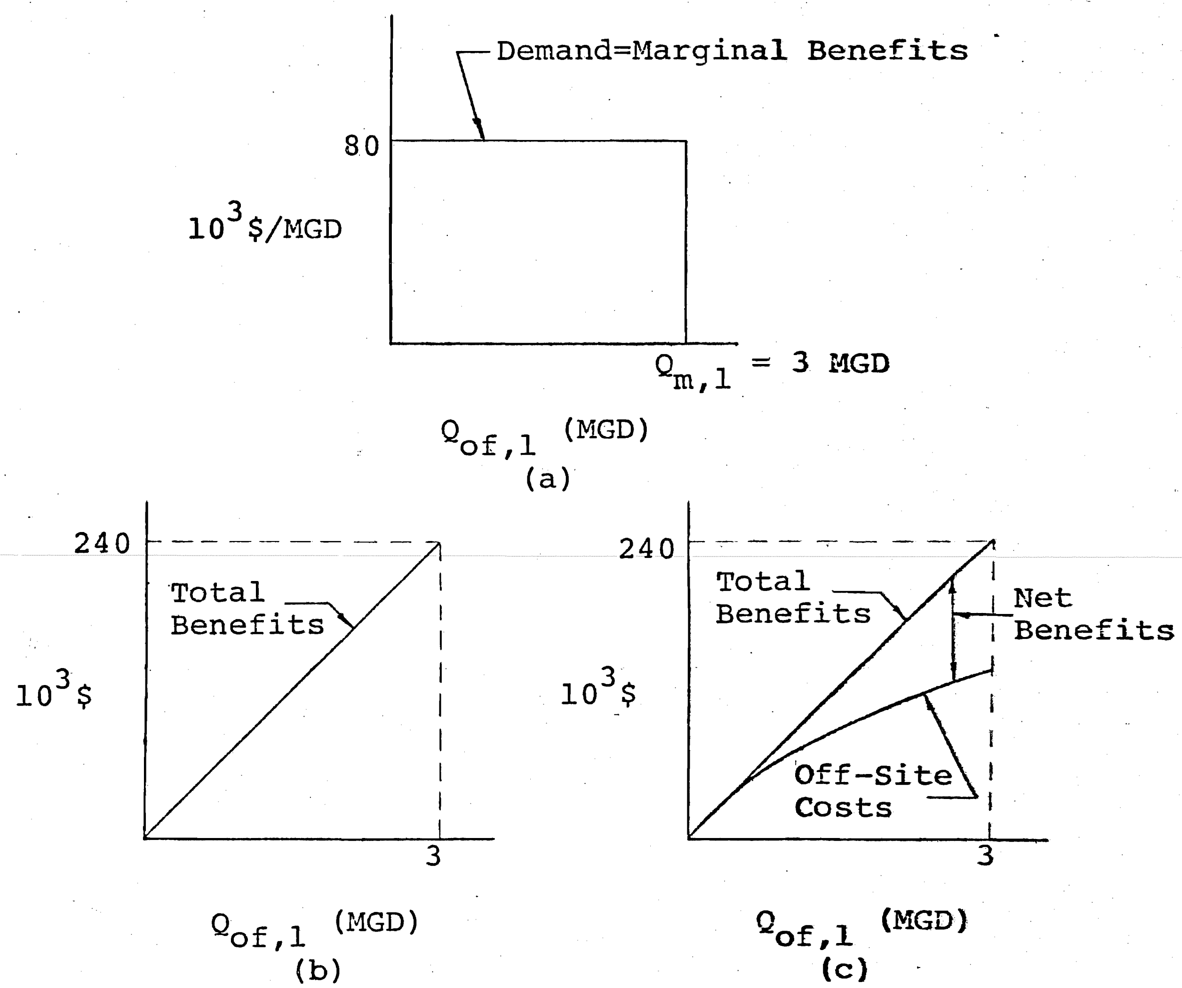


Figure 6. Case 1 benefit and cost relationships for site 1: (a) marginal benefits, (b) total benefits, and (c) net benefits.

capacity]. This is true because net benefits are continuously increasing as shown in Figure 6(c) for site 1. If some plant capacity remains, the user with the next highest average benefit receives output up to his maximum demand or plant capacity. This rank order assignment of wastewater supply continues until either potential demand or the plant capacity limit is reached. The result for this simple case is a bandwagon effect for which the order that players join the bandwagon has important implications for economic optimization. In particular, if plant capacity is less than total potential demand, some players may be partially or totally excluded from the project. The excluded players contribute lower benefits and so are given lower priority in joining the project.

Results are not quite so easily seen when direct costs are added to the analysis. Since direct cost components such as the transportation costs here usually involve cost relationships which differ from joint costs and differ from other direct costs, the player with the highest average benefit may not contribute the greatest net benefit to the project. As shown in Figure 7 player A's average benefit is less than player B's average benefit, but A also has relatively low direct costs so that his net benefits are actually greater than those for B for a given level of output Q_p. To allow for this possibility, analyses which include direct costs must consider total net benefits instead of just average benefits. The result is similar to

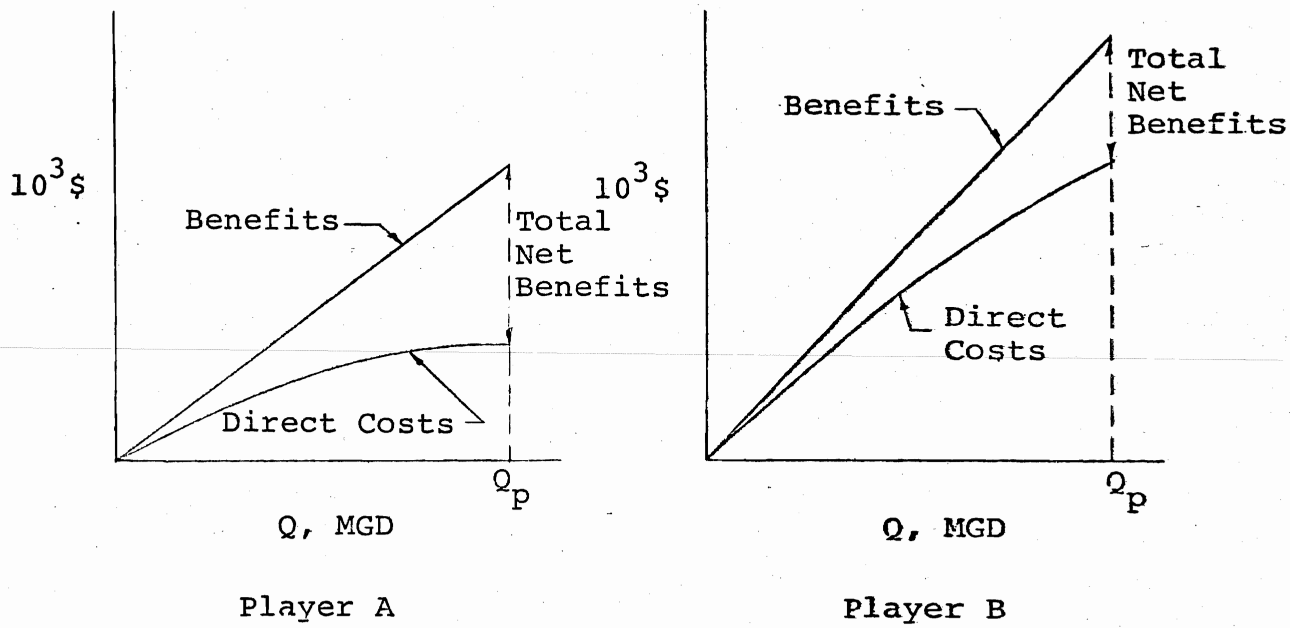


Figure 7. Priority ranking example for direct cost considerations for case 1.

the rank order effect for joint costs. To maximize total net benefits, the player with the highest total net benefits for any arbitrary output level receives highest priority. This result is valid even when each player's benefits are not strictly greater than costs since priority ranking is determined by total net benefits. An important result for case 1, then, is that the economic optimum is achieved by assigning output to each user according to a priority ranking based on comparison of total net benefits. The net benefits a user contributes to the project are limited only by maximum demand or plant capacity constraints since net benefits are continuously increasing. In step 5 we apply the general result for case 1 to the wastewater reuse example.

Cost Allocation

With the above general results for case 1, we next consider several cost allocation schemes in search of an appropriate strategy to satisfy financial objectives. The methods for consideration include marginal pricing from economic theory, the Shapley value from game theory, and MCRS (as a generalization of SCRB) and average cost pricing methods from conventional practice. First, we completely eliminate the possibility of applying marginal pricing strategies. Clearly, where net benefits are continuously increasing within the range of maximum demand and plant capacity limits, marginal costs will equal marginal benefits only at a point of minimum net benefits. Consequently, the concept of marginal pricing is not applicable for case 1.

To evaluate the remaining methods, we must first define the core for the final project design in characteristic function form as defined by $C_4(S)$. When considering joint cost functions such as central facility treatment costs, economies of scale guarantee the existence of a core and in particular a strictly convex core (Tschirhart, 1975). Note that the direct costs have no bearing on overall convexity. Cost allocation solutions for a convex game exhibit certain properties. Namely, the Shapley value will be in the center of the core and extremely close to the MCRS solution (Heaney and Dickinson, 1982). Also, for a convex game the MCRS and SCRB solutions are identical. Furthermore, since the marginal cost curve falls below a decreasing average cost curve, a vector of charges based on average cost pricing also satisfies core constraints. Computations presented in step 5 demonstrate these general results for the wastewater reuse example. In step 6 we examine some important implication of these results.

Case 2. Upward - Sloping Demand Function

An upward-sloping demand for reclaimed water is derived from decreasing on-site marginal costs. This situation might correspond to economies of scale in on-site pumping costs or a schedule of declining unit or block prices for purchased water. A sketch of the on-site cost curve and corresponding upward-sloping demand is shown in Figure 8 for site 1. Again, the flow level $Q_{m,1}$ corresponds to the maximum demand for the site. The resulting total benefit curve is also shown in Figure 8.

Economic optimization

The situation here is very similar to that in case 1. As demonstrated in Figure 8, net benefits are continuously increasing for a given cost function. Therefore, the same general results apply in which the economic optimum is achieved through priority assignment of output based on comparison of net benefits for each user. However, unlike the previous case, the output level for comparison of net benefits may not be strictly arbitrary where the net benefit curve for one player intersects the net benefit curve for another player. The output level corresponding to an intersection implies a switch in priority from one player to another. This is demonstrated in Figure 9. At flow levels less than Q_k player A dominates while at flow levels greater than Q_k, player B dominates. Therefore, priority between A and B depends on quantity of output available to the players. In general, then, priority among players in a subcoalition S may not be given by the priority ranking in the grand coalition. On the other hand, if the net benefit curves do not intersect within the range of interest, priority among players is determined by comparison of net benefit contributions for any arbitrary flow level within the range of

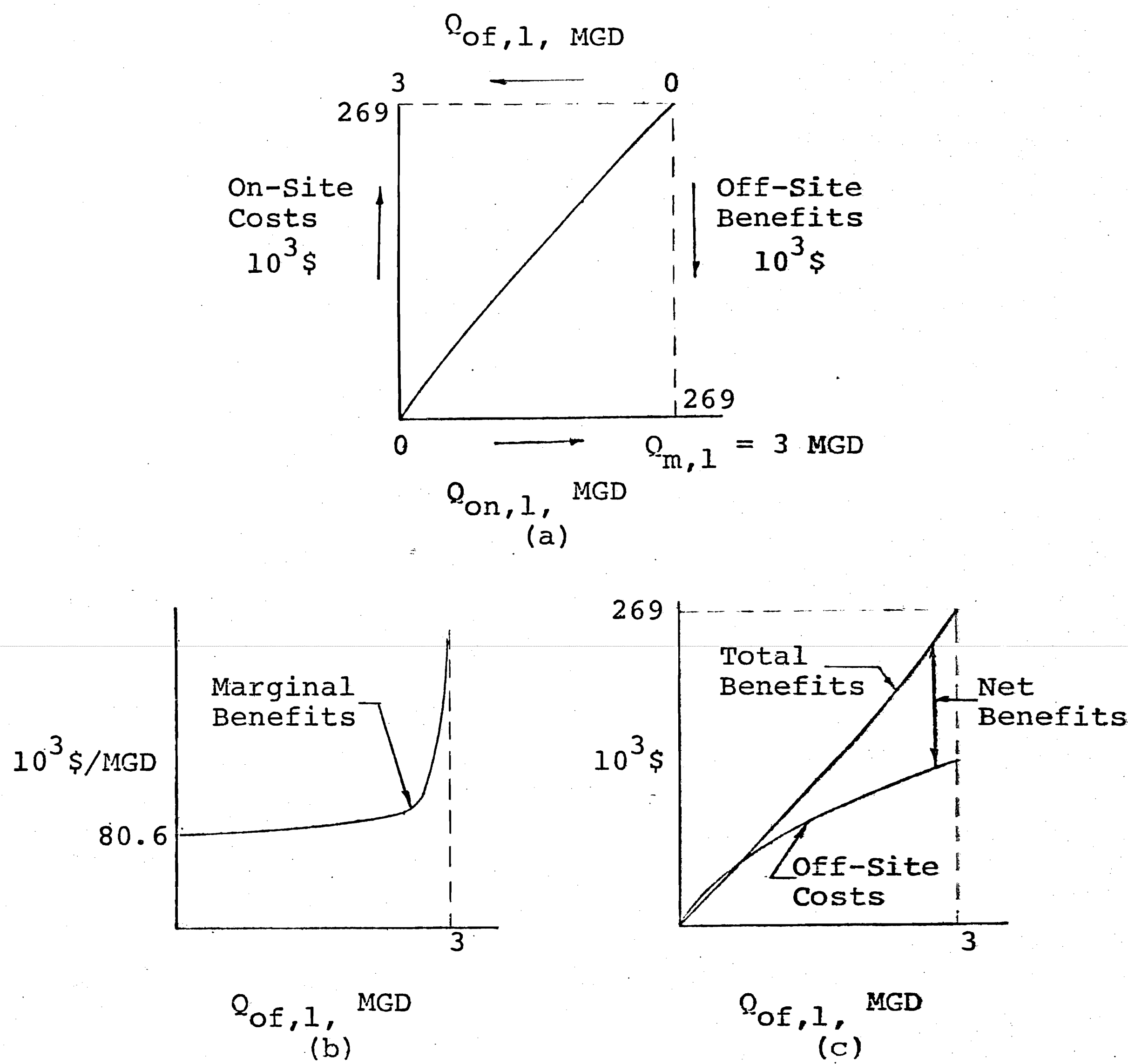


Figure 8. Case 2 benefit and cost relationships for site 1: (a) total on-site costs, (b) marginal benefits, and (c) net benefits.

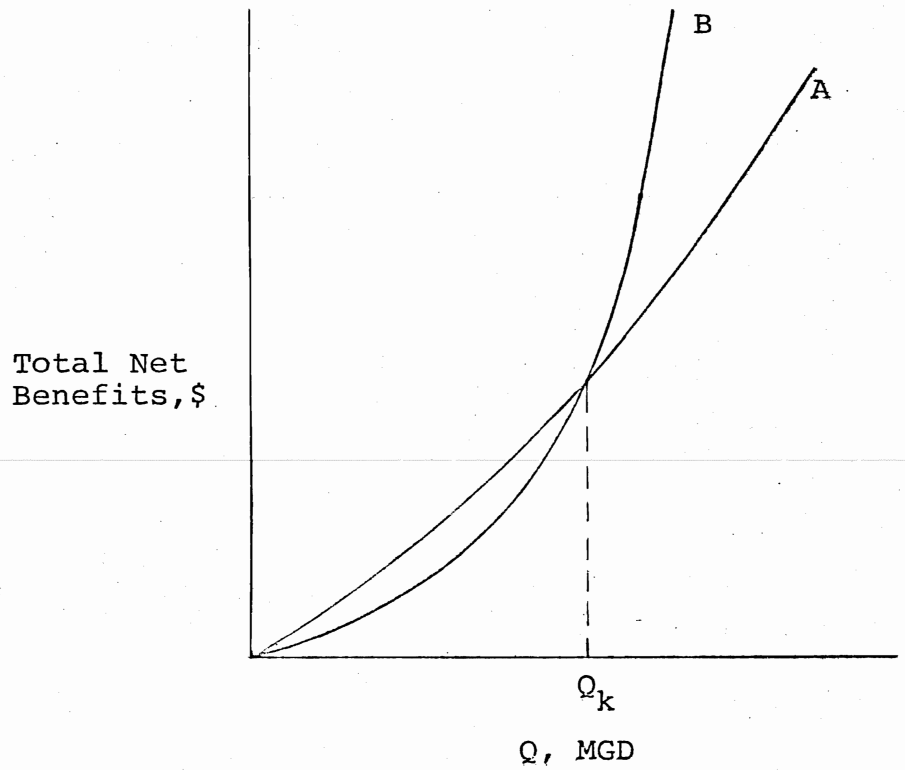


Figure 9. Example of shift in priority for case 2.

interest. The resulting rank order priority holds for all subcoalitions as well as the grand coalition.

Cost allocation

The discussion regarding financial analysis for case 1 is applicable to case 2 as well. General conclusions for financial analysis are identical for the two cases as demonstrated in steps 5 and 6.

Case 3. Downward-Sloping Demand Function

A downward-sloping wastewater demand is derived from increasing on-site marginal costs. This situation might correspond to diseconomies of scale in on-site operating costs or increasing unit or block prices for purchased water. The on-site cost curve and corresponding demand and benefit curves are sketched in Figure 10 for site 2.

Economic optimization

As for the first two cases, the least cost solution results when each user is assigned wastewater supply on the basis of his contribution to total net benefits. In contrast to cases 1 and 2, if case 3 benefits are compared to off-site costs, say treatment costs, their difference is not strictly increasing. As demonstrated in Figure 10 benefits increase for lower flow levels, reaching a maximum at Q* before beginning to decrease. The flow level Q* corresponds to the point where marginal costs equal marginal benefits. Therefore, when a user's demand is downward sloping, his

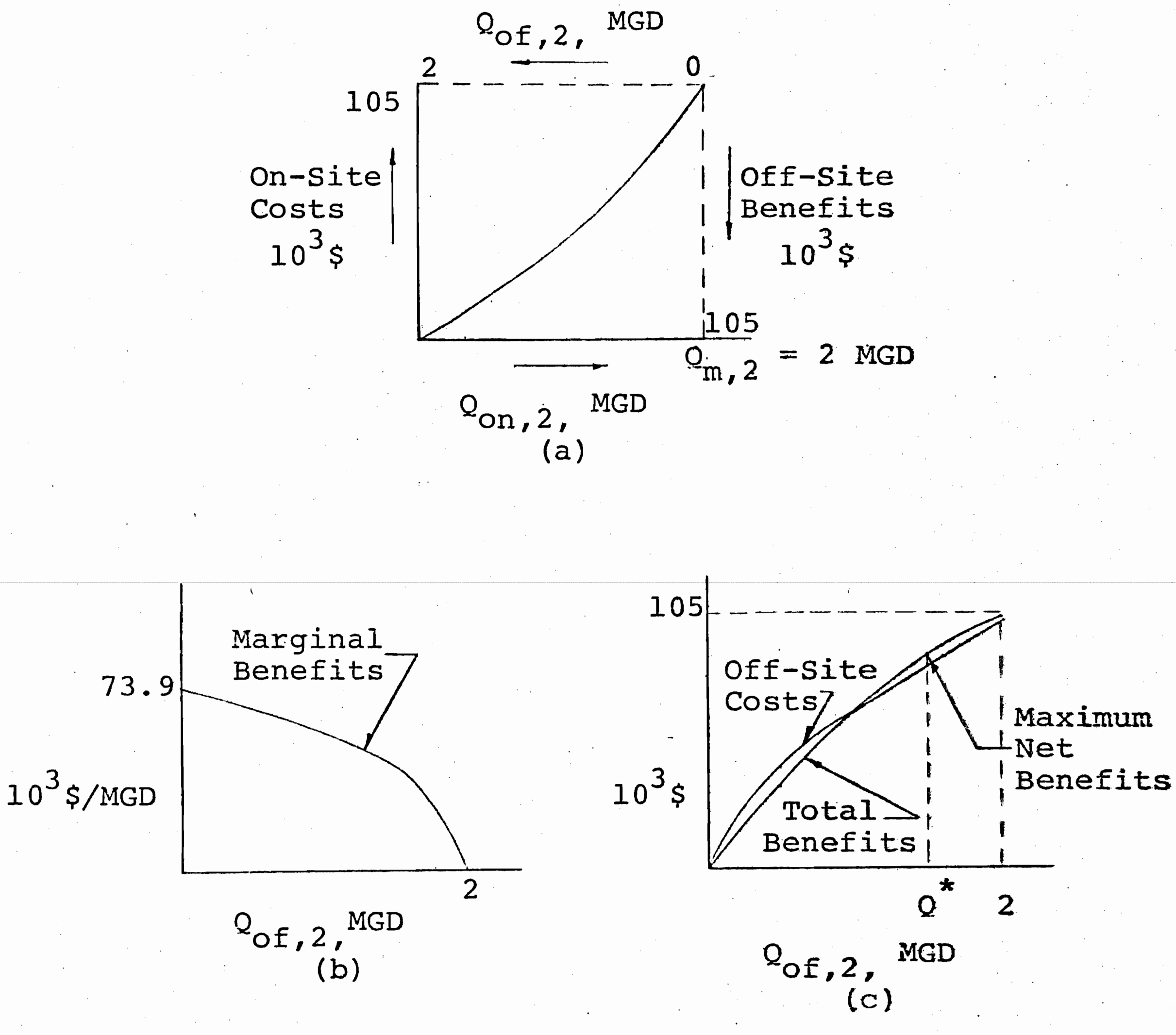


Figure 10. Case 3 benefit and cost relationships for site 2: (a) total on-site costs, (b) marginal benefits, and (c) net benefits.

contribution to the project is limited by marginal considerations as well as maximum demand and plant capacity limitations. For single-step or upward-sloping demand conditions, the only possibility for a mix of on-site and off-site supply to a participant occurs when the plant capacity limit is reached before the last ranking participant fulfills his maximum demand. For downward-sloping demand conditions, the result may be a mix of supply to any or all of the project participants.

Wastewater supply is still allocated according to priority based on net benefit contributions, but that priority is not so easily determined. As for case 2 the quantities for comparison of net benefit contributions cannot be arbitrarily chosen. In particular, the player contributing the greatest net benefits at a given flow level may not contribute the greatest net benefits at another larger flow level. This is true since a player's marginal benefits decrease with increasing quantity of acquired output. Therefore, as a high priority player's marginal benefits decrease with increasing supply, the potential contributions of other players become increasingly attractive perhaps reaching an output level where a new player gains priority. This output level corresponds to a point where one player's net benefit curve intersects another player's net benefit curve. This is analogous to the situation in case 2. Consequently, priority among players in any coalition S depends upon the quantity of resource available to S as defined by $C_4(S)$. The result is that determining priority among players requires additional information regarding relative net benefits at various output levels and quantities corresponding to marginal conditions (marginal benefits equal to marginal costs). For this example, the nonlinear nature of the problem and inclusion of direct as well as joint costs may preclude simple solution by hand calculation in favor of an appropriate mathematical programming code. Such is the case in step 5 where we apply the GRGA reduced gradient code to solve the economic optimization problem for case 3.

Cost allocation

General conclusions for cases 1 and 2 regarding the nature of the core and various cost allocation methods are not applicable to this case. First, since the optimal design may include quantities corresponding to marginal conditions, a charge system based at least in part on marginal pricing might be viable. However, recall that strict marginal pricing does not satisfy total cost requirements.

To evaluate the remaining methods, we again define the core for the final project design in characteristic function form from $C_4(S)$. Intuitively, one might expect that the resulting cost allocation game is convex as in the previous two cases. However, as we will see, economies of scale guarantee convexity only for C(S) defined for fixed quantities for an assumed inelastic demand or for quantities given by $C_4(S)$ for single-step or upward-sloping demand, i.e. cases 1 and 2. For case 3 the $C_4(S)$ definition ensures subadditivity since the quantity of resource a player receives in each subcoalition does not exceed the quantity he receives in the grand coalition. However, with diseconomies of scale in on-site costs, the largest incremental savings from avoided costs correspond to the first units of acquired output. At the same time, incremental savings due to off-site economies of scale are greatest at larger output levels. If on-site diseconomies are relatively strong, the incremental savings from avoided on-site costs at low output levels may be greater than the incremental off-site savings at higher output levels. So, even though a player receives less output and less total savings by joining smaller coalitions he may realize greater incremental savings than by joining a larger coalition. The result is that the cost allocation game may not be convex. That is, a player or group of players could have higher incremental savings (or lower incremental costs) by joining the project earlier, say second or third, instead of last. Recall that in cases 1 and 2 savings from off-site economies of scale and alternative costs avoided both increase with output so that the highest incremental savings occurs at the highest output levels, i.e. by being the last player to join the grand coalition.

Step 5. Apply Appropriate Methodologies

Case 1. Single-Step Demand Function

Economic optimization

The optimization problem for case 1 is to

$$\text{Minimize } Z = 80Q_{on,1} + 70Q_{on,2} + 90Q_{on,3} + 100Q_{on,4}$$

$$+ 8(Q_{of,1})^{.4} + 5(Q_{of,2})^{.3} + 13(Q_{of,3})^{.5}$$

$$+ 17(Q_{of,4})^{.6} + 60(Q_{of,1} + Q_{of,2} + Q_{of,3}$$

$$+ Q_{of,4})^{.7}$$

S.T.

$$Q_{on,1} + Q_{of,1} = 3 \text{ MGD}$$

$$Q_{on,2} + Q_{of,2} = 2 \text{ MGD}$$

$$Q_{on,3} + Q_{of,3} = 1.5 \text{ MGD}$$

$$Q_{on,4} + Q_{of,4} = 2.5 \text{ MGD}$$

$$Q_{of,1} + Q_{of,2} + Q_{of,3} + Q_{of,4} \leq 10 \text{ MGD}$$

6 MGD

To demonstrate the proposed general results for the case of single-step demand functions, we first examine potential contributions from each player and establish the priority ranking that minimizes total water supply costs. Referring to Table 1 and equation 13 we compute off-site costs and benefits for any arbitrary flow level. We then rank each player according to his net benefit contribution. As shown in Figure 11, site 4 receives highest priority followed in rank order by sites 3, 1 and 2.

Since the total potential demand for wastewater is only 9 MGD, the problem constraint for a treatment plant capacity limit of 10 MGD is not tight. The optimal project design includes all sites contributing a positive net benefit to the project. Results of the rank order calculations indicate that all sites are viable and should be included to achieve the optimal design. The resulting project consists of all sites each receiving a total wastewater supply equal to its maximum demand quantity. Project costs total $343,000 per year, a 55 percent savings over alternative on-site costs. Results are summarized in Table 2.

For treatment plant capacity equal to 6 MGD, the maximum supply of wastewater cannot satisfy total water supply requirements for the four sites. The wastewater supply must be allocated among the sites so as to achieve overall minimum water supply costs. As stated previously, wastewater supply is allocated according to a rank order procedure. Using the rank order established above we assign wastewater as given in Table 2. As the highest ranking player, site 4 receives a quantity of wastewater equal to its maximum water demand. With next highest priority, site 3 receives a quantity also equal to its maximum demand. At this point, there are only 2 MGD of wastewater left. As

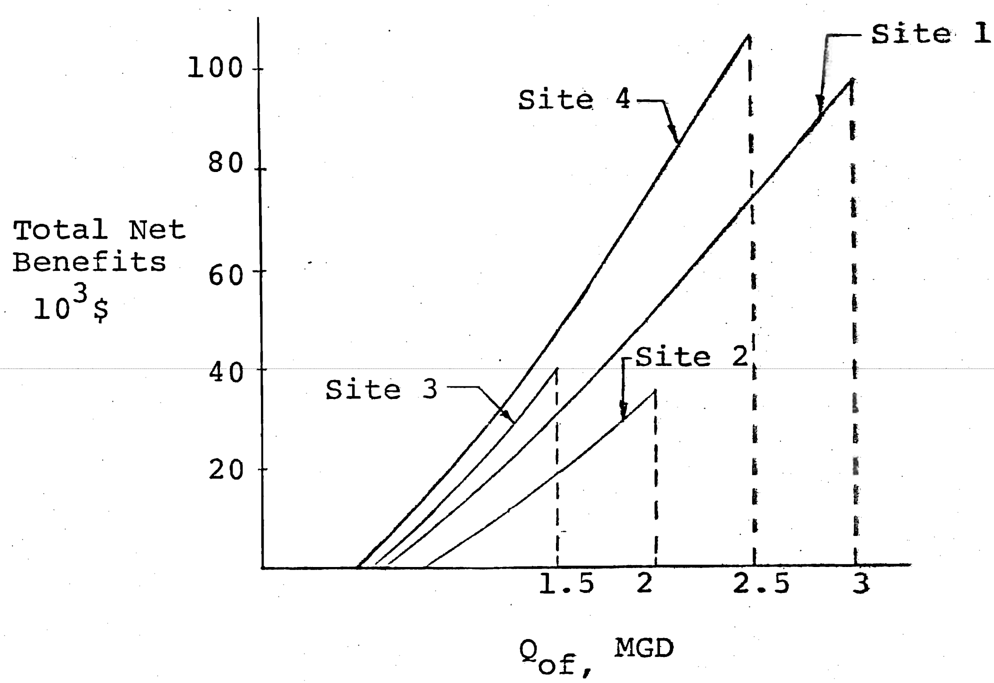


Figure 11. Net benefits for rank order determination for case 1.

Table 2. Optimal Resource Allocation for Case 1

Site	$Q_{m,t}$ MGD	$Q_{m,i}$ MGD	$Q_{of,i}$ MGD
1	10	3.0	3.0
2	10	2.0	2.5
3	10	1.5	1.5
4	10	2.5	2.5
	Total	9.0	9.0
1	6	3.0	2.0
2	6	2.0	0.0
3	6	1.5	1.5
4	6	2.5	2.5
	Total	9.0	6.0

the third ranking player, site 1 receives all of this remaining supply to satisfy two-thirds of its total irrigation requirement. The resulting project design completely excludes site 2. The annual final system cost is $346,000.

Cost allocation

To apply and evaluate the various cost allocation methods we first compute the characteristic function for both plant capacity limits. Recall that C(S) is defined by $C_4(S)$ which gives N-S priority over S. For the 10 MGD capacity limit, giving coalition N-S priority does not restrict the quantity of wastewater available to coalition S. Coalition S receives wastewater flow equal to its total maximum demand but is assigned the cost of supplying the quantity alone, without benefit of the economies of scale from including N-S. The resulting characteristic cost functions are

C(1) = 142	C(2) = 104	C(3) = 96	C(4) = 143
C(12) = 204	C(13) = 200	C(14) = 240	C(23) = 166
C(24) = 208	C(34) = 204	C(123) = 257	C(124) = 294
C(134) = 292	C(234) = 262	C(1234) = 343	

where the corresponding subadditive game is convex as verified in the Appendix.

The resulting minimum fairness criteria are given by the following core constraints.

$$
\begin{array}{llllll}
X(1) & & & & \leq & 142 \\
 & X(2) & & & \leq & 104 \\
 & & X(3) & & \leq & 96 \\
 & & & X(4) & \leq & 143 \\
X(1) & + X(2) & & & \leq & 204 \\
X(1) & & + X(3) & & \leq & 200 \\
X(1) & & & + X(4) & \leq & 240 \\
 & X(2) & + X(3) & & \leq & 166 \\
 & X(2) & & + X(4) & \leq & 208 \\
 & & X(3) & + X(4) & \leq & 204 \\
X(1) & + X(2) & + X(3) & & \leq & 257 \\
X(1) & + X(2) & & + X(4) & \leq & 294 \\
X(1) & & + X(3) & + X(4) & \leq & 292 \\
 & X(2) & + X(3) & + X(4) & \leq & 262 \\
X(1) & + X(2) & + X(3) & + X(4) & = & 343
\end{array}
$$

Since the game is convex, the first four and last five conditions stipulate the upper and lower bounds on $X(i)$. That is, a vector of charges within the core must satisfy

$$
\begin{aligned}
81 \leq X\ (1) \leq 142 \\
51 \leq X\ (2) \leq 104 \\
49 \leq X\ (3) \leq 96 \\
86 \leq X\ (4) \leq 143 \\
X(1) + X(2) + X(3) + X(4) = 343
\end{aligned}
$$

The resulting Shapley, MCRS (equivalent to SCRB for a convex game), and average cost pricing solutions are presented in Table 3. As expected the Shapley value and MCRS solutions are in the center of the core. Although the set of charges derived from average cost is positioned away from the core center in favor of the highest priority sites, 3 and 4, we see that the shift is not extreme and that each site is still awarded large savings. This particular result has important implications for the final selection of an appropriate pricing scheme.

For the 6 MGD capacity limit, giving N-S priority does restrict the quantity of wastewater available for coalition S. The available supply must be allocated among the players in S according to the established priority order. For the 6 MGD three-site system, priority among sites 1, 3 and 4 in any subcoalition is the same as that already established, i.e. site 4 has highest priority, site 3 second, and site 1 lowest. A sample calculation of the characteristic function is shown below.

C(1): let (3,4) go first, $Q_{of,3} + Q_{of,4} = 4$ MGD

$6 - 4 = 2$ MGD left

$$C(1) = 8(2)^{0.4} + 60(2)^{0.7} + 80 = 188$$

Similarly,

$$C(3) = 136 \qquad C(4) = 201$$

$$C(13) = 251 \qquad C(14) = 292 \qquad C(34) = 258$$

$$C(134) = 346$$

Table 3. Charges for Case 1

Site	$Q_{m,t}$ MGD	Shapley	MCRS	Average Charge	Cost Savings
		Total cost, 10^3\$/year			
1	10	102.92	102.27	114.33	52%
2	10	69.25	69.48	76.22	46%
3	10	64.58	65.39	57.17	58%
4	10	106.25	105.87	95.28	62%
	Total	343	343	343	
1	6	126.33	125.82	148.30	38%
3	6	83.33	85.07	74.14	45%
4	6	136.33	135.11	123.56	51%
	Total	346	346	346	

The resulting subadditive cost game is convex since

$$C(13) + C(14) \geq C(134) + C(1)$$
$$251 + 292 \geq 346 + 188 \qquad \text{OK}$$

$$C(13) + C(34) \geq C(134) + C(3)$$
$$251 + 258 \geq 346 + 136 \qquad \text{OK}$$

$$C(14) + C(34) \geq C(134) + C(4)$$
$$292 + 258 \geq 346 + 201 \qquad \text{OK}$$

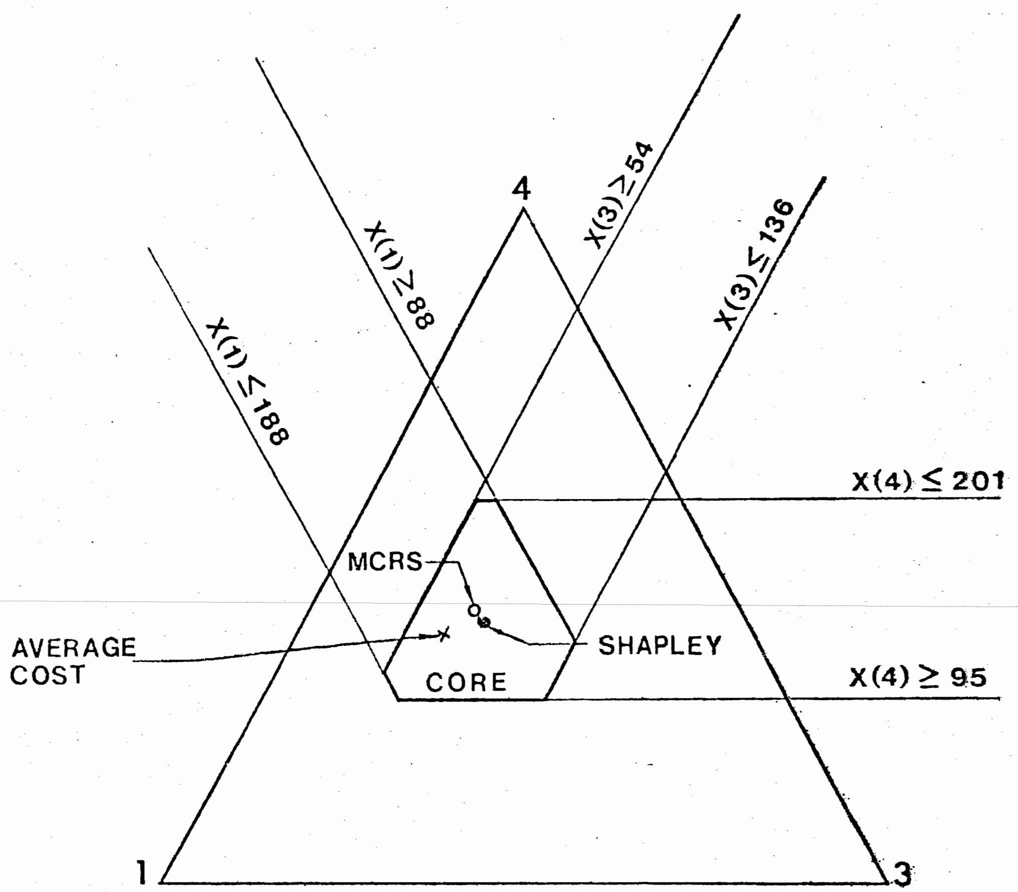
The core bounds are given by

$$\begin{aligned}
X(1) \qquad\qquad\qquad\qquad &\leq 188 \\
X(3) \qquad\qquad &\leq 136 \\
X(4) &\leq 201 \\
X(1) + X(3) \qquad\qquad &\leq 251 \\
X(1) \qquad\qquad + X(4) &\leq 292 \\
X(3) + X(4) &\leq 258 \\
X(1) + X(3) + X(4) &= 346
\end{aligned}$$

or

$$\begin{aligned}
88 \leq X(1) &\leq 188 \\
54 \leq X(3) &\leq 136 \\
95 \leq X(4) &\leq 201 \\
X(1) + X(3) + X(4) &= 346
\end{aligned}$$

The Shapley, MCRS, and average cost pricing solutions are shown in Table 3 and in Figure 12. Again, as expected,



C(134) = 346

Figure 12. Core and charges for three-person game for case 1.

the Shapley value and MCRS solution are in the center of the core. Again the average cost solution shifts away from the core center in favor of the strongest players. However, even as the lowest ranking player, site 1 still receives a significant savings over exclusive on-site water supply.

Case 2. Upward-Sloping Demand Function

Economic optimization

The optimization problem for case 2 is to

$$\begin{aligned} \text{Minimize } Z = {} & 100(Q_{on,1})^{.9} + 80(Q_{on,2})^{.9} + 100(Q_{on,3})^{.8} \\ & + 120(Q_{on,4})^{.7} + 8(Q_{of,1})^{.4} + 5(Q_{of,2})^{.3} \\ & + 13(Q_{of,3})^{.5} + 17(Q_{of,4})^{.6} \\ & + 60(Q_{of,1} + Q_{of,2} + Q_{of,3} + Q_{of,4})^{.7} \end{aligned}$$

$$\begin{aligned} \text{S.T.} \quad & Q_{on,1} + Q_{of,1} = 3 \text{ MGD} \\ & Q_{on,2} + Q_{of,2} = 2 \text{ MGD} \\ & Q_{on,3} + Q_{of,3} = 1.5 \text{ MGD} \\ & Q_{on,4} + Q_{of,4} = 2.5 \text{ MGD} \\ & Q_{of,1} + Q_{of,2} + Q_{of,3} + Q_{of,4} \le \begin{matrix} 10 \text{ MGD} \\ 6 \text{ MGD} \end{matrix} \end{aligned}$$

To determine the least cost solution for the case of upward-sloping demand functions, we apply the principles established for case 1. Note that in the range of interest (0 to 9 MGD) only two net benefit curves intersect. The net benefits for sites 1 and 3 are equal for a flow level of

1.38 MGD. The consequences of this shift in priority are demonstrated in Figure 13 where net benefit contributions are compared for flow levels in the range of interest. Priority between sites 1 and 3 depends on whether the supply available to the two sites is greater than or less than 1.38 MGD.

Again the constraint for a plant capacity of 10 MGD is not tight. For wastewater supply equal to maximum demand, each site contributes positive net benefits. Therefore, the optimal project includes all four sites, each receiving an output quantity equal to its maximum demand. This result is identical to case 1.

For a plant capacity of 6 MGD the limited supply is allocated according to the priority ranking established in Figure 13. The most efficient resource allocation is given in Table 4. Site 4 is completely excluded from the project so that the final design consists of only the three highest ranking sites. Sites 1 and 3 each receive off-site supply quantities which satisfy total water demand, while site 2 must settle for a mix of on- and off-site supply. The project represents a 48% savings that must be allocated among the three participants.

Cost allocation

For the 10 MGD capacity, the characteristic function and cost allocation solutions are identical to those for case 1 as shown in Table 5. For the 6 MGD capacity limit,

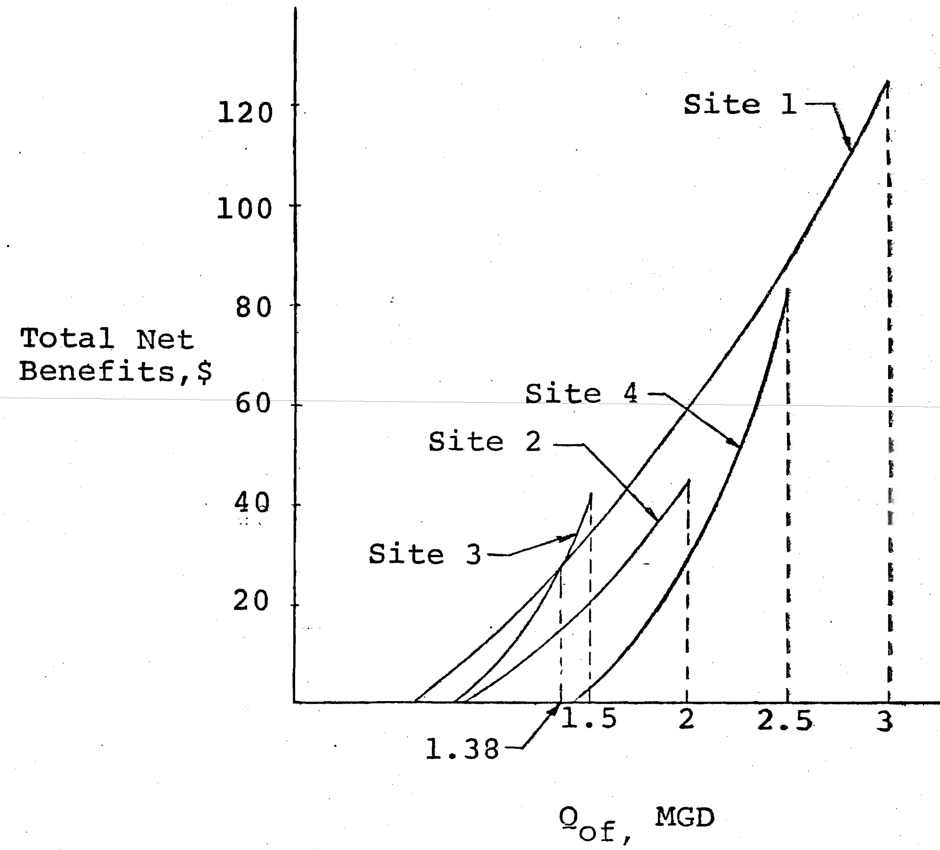


Figure 13. Net benefits for rank order determination for case 2.

Table 4. Optimal Resource Allocation for Case 2

Site	$Q_{m,t}$ MGD	$Q_{m,i}$ MGD	$Q_{of,i}$ MGD
1	10	3.0	3.0
2	10	2.0	2.0
3	10	1.5	1.5
4	10	2.5	2.5
	Total	9.0	9.0
1	6	3.0	3.0
2	6	2.0	1.5
3	6	1.5	1.5
4	6	2.5	0.0
	Total	9.0	6.0

Table 5. Charges for Case 2

		Total Cost, 10^3\$/year		Average Cost	
Site	$Q_{m,t}$ MGD	Shapley	MCRS	Charge	Savings
1	10	102.92	102.27	114.33	52%
2	10	69.25	69.48	76.22	46%
3	10	64.58	65.39	57.17	58%
4	10	106.25	105.87	95.28	62%
	Total	343	343	343	
1	6	126.33	125.66	132.40	51%
2	6	78.33	78.45	88.30	41%
3	6	82.33	82.89	66.23	52%
	Total	287	287	287	

the available supply is allocated according to the established priority ranking. The characteristic cost function for player 1 is computed by

$$C(1): \text{ let } (2,3) \text{ go first, } Q_{of,2} + Q_{of,3} = 3.5 \text{ MGD}$$

$$6 - 3.5 = 2.5 \text{ MGD left}$$

$$C(1) = 8(2.5)^{0.4} + 60(2.5)^{0.7} + 100(0.5)^{0.9} = 179$$

Similarly, $C(2) = 128 \quad C(3) = 130$

$$C(12) = 233 \quad C(13) = 239 \quad C(23) = 194$$

$$C(123) = 287$$

This game is convex since

$$C(13) + C(12) \geq C(123) + C(1)$$

$$239 + 233 \geq 287 + 179 \quad \text{OK}$$

$$C(12) + C(23) \geq C(123) + C(2)$$

$$233 + 194 \geq 287 + 128 \quad \text{OK}$$

$$C(13) + C(23) \geq C(123) + C(3)$$

$$239 + 194 \geq 287 + 130 \quad \text{OK}$$

The core bounds are given by

$$
\begin{array}{llll}
X(1) & & & \leq 179 \\
& X(2) & & \leq 128 \\
& & X(3) & \leq 130 \\
X(1) & + X(2) & & \leq 233 \\
X(1) & & + X(3) & \leq 239 \\
& X(2) & + X(3) & \leq 194 \\
X(1) & + X(2) & + X(3) & = 287
\end{array}
$$

or

$$
\begin{array}{c}
93 \leq X(1) \leq 179 \\
48 \leq X(2) \leq 128 \\
54 \leq X(3) \leq 130 \\
X(1) + X(2) + X(3) = 287
\end{array}
$$

The Shapley value, MCRS, and average cost pricing solutions are given in Table 5 and in Figure 14. They demonstrate the applicability of important case 1 general results to case 2.

Case 3. Downward-Sloping Demand Function

Economic optimization

The optimization problem for case 3 is to

$$
\begin{aligned}
\text{Minimize } Z = \; & 25(Q_{on,1})^{1.7} + 40(Q_{on,2})^{1.4} + 45(Q_{on,3})^{1.2} \\
& + 70(Q_{on,4})^{1.1} + 8(Q_{of,1})^{.4} + 5(Q_{of,2})^{.3} \\
& + 13(Q_{of,3})^{.5} + 17(Q_{of,4})^{.6} \\
& + 60(Q_{of,1} + Q_{of,2} + Q_{of,3} + Q_{of,4})^{.7}
\end{aligned}
$$

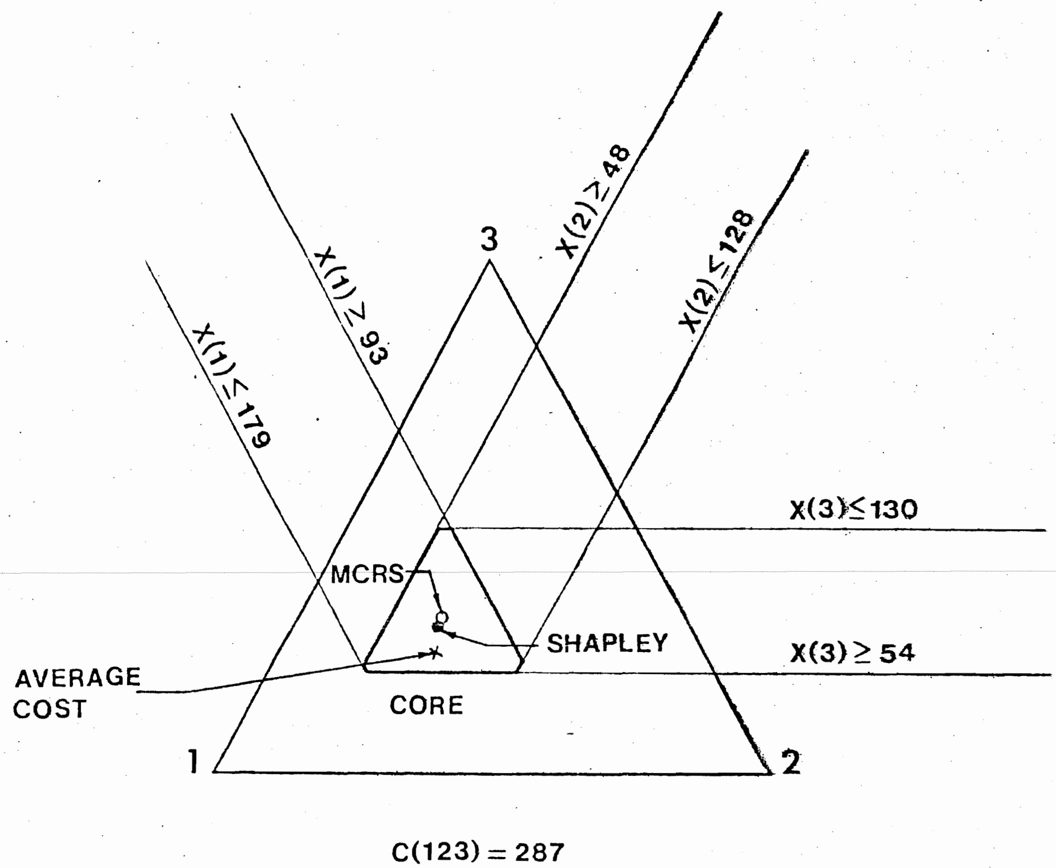


Figure 14. Core and charges for three-person game for case 2.

S.T. $$Q_{on,1} + Q_{of,1} = 3 \text{ MGD}$$

$$Q_{on,2} + Q_{of,2} = 2 \text{ MGD}$$

$$Q_{on,3} + Q_{of,3} = 1.5 \text{ MGD}$$

$$Q_{on,4} + Q_{of,4} = 2.5 \text{ MGD}$$

$$Q_{of,1} + Q_{of,2} + Q_{of,3} + Q_{of,4} \leq \begin{matrix} 10 \text{ MGD} \\ 6 \text{ MGD} \end{matrix}$$

Application of the GRGA code to determine the minimum cost solution for the case of a downward sloping demand function yields the optimal project designs given in Table 6. The role of marginal considerations in determining optimal resource allocation is evident. Although plant capacity is not a limiting factor for project size in the first design, the optimal output level of 8.41 MGD does not correspond to the 9 MGD potential demand. Only site 4 receives its maximum demand. That the remaining sites are assigned quantities less than their maximum demand results from the effect of marginal conditions in which the quantity assigned to the site corresponds to an output level where marginal costs equal marginal benefits. The results of marginal considerations and shifting priorities are also evident in the 6 MGD capacity design. Here, the capacity constraint is tight, and the final project includes only three participants.

Table 6. Optimal Resource Allocation for Case 3

Site	$Q_{m,t}$ MGD	$Q_{m,i}$ MGD	$Q_{of,i}$ MGD
1	10	3.0	2.56
2	10	2.0	1.89
3	10	1.5	1.46
4	10	2.5	2.5
	Total	9.0	8.41
1	6	3.0	1.99
2	6	2.0	1.51
3	6	1.5	0.0
4	6	2.5	2.5
	Total	6.0	6.0

Cost allocation

Since analysis of either of the above project designs can be used to effectively demonstrate all relevant results and conclusions, we arbitrarily select the three-site design for computational ease only. To determine the characteristic function for this design, we apply GRGA to establish the optimal output level for coalition N-S. Using any remaining capacity as the new maximum supply constraint, we then apply the code to determine the optimal "go-it-alone" output quantity for coalition S. Computations are summarized below.

C(1): let (2,4) go first, $Q_{of,2} + Q_{of,4} = 4.32$ MGD

$6 - 4.32 = 1.68$ MGD left

$$C(1) = 8(1.68)^{0.4} + 60(1.68)^{0.7} + 25(1.32)^{1.7} = 136$$

C(2): let (1,4) go first, $Q_{of,1} + Q_{of,4} = 4.95$ MGD

$6 - 4.95 = 1.05$ MGD left

$$C(2) = 5(1.05)^{0.3} + 60(1.05)^{0.7} + 40(0.95)^{1.4} = 104$$

C(4): let (1,2) go first, $Q_{of,1} + Q_{of,2} = 4.24$ MGD

$6 - 4.24 = 1.76$ MGD left

$$C(4) = 17(1.76)^{0.6} + 60(1.76)^{0.7} + 70(0.74)^{1.1} = 163$$

$C(12)$: let (4) go first, $Q_{of,4} = 2.5$ MGD

$$6 - 2.5 = 3.5 \text{ MGD left}$$

$$C(12) = 8(1.99)^{0.4} + 5(1.51)^{0.3} + 60(3.5)^{0.7} + 25(1.01)^{1.7} + 40(0.49)^{1.4} = 200$$

$C(14)$: let (2) go first, $Q_{of,2} = 1.64$ MGD

$$6 - 1.64 = 4.36 \text{ MGD left}$$

$$C(14) = 8(1.88)^{0.4} + 17(2.48)^{0.6} + 60(4.36)^{0.7} + 25(1.12)^{1.7} + 70(0.02)^{1.1} = 239.$$

$C(24)$: let (1) go first, $Q_{of,1} = 2.24$ MGD

$$6 - 2.24 = 3.76 \text{ MGD left}$$

$$C(24) = 5(1.29)^{0.3} + 17(2.47)^{0.6} + 60(3.76)^{0.7} + 40(0.71)^{1.4} + 70(0.03)^{1.1} = 212$$

$$C(124) = 8(1.99)^{0.4} + 5(1.51)^{0.3} + 17(2.5)^{0.6} + 60(6)^{0.7} + 25(1.01)^{1.7} + 40(0.49)^{1.4} = 296$$

We then check the appropriate conditions for game convexity.

$$C(12) + C(14) \geq C(124) + C(1)$$

$$200 + 239 \geq 296 + 136 \quad \text{OK}$$

$$C(12) + C(24) \geq C(124) + C(2)$$

$$200 + 212 \geq 296 + 104 \quad \text{OK}$$

$$C(14) + C(24) \geq C(124) + C(4)$$

$$239 + 212 \geq 296 + 163 \quad \text{NO}$$

Failure of this last condition indicates that the core is nonconvex. Sites 1 and 2 would each rather join the grand coalition second after site 4. The core constraints are

$$\begin{array}{rcl}
X(1) & & \leq 136 \\
X(2) & & \leq 104 \\
X(4) & & \leq 163 \\
X(1) + X(2) & & \leq 200 \\
X(1) + X(4) & & \leq 239 \\
X(2) + X(4) & & \leq 212 \\
X(1) + X(2) + X(4) & & = 296
\end{array}$$

Nominal bounds for the vector of charges are given by separable cost considerations, i.e. incremental costs of joining the grand coalition last. Nominal bounds correspond to actual bounds only for convex games. Actual maximum and minimum charges that satisfy all core constraints may be determined from the solution of a system of six linear programs (See Heaney and Dickinson, 1982). A comparison of nominal and actual core bounds reveals only a relatively small difference in the upper bound for X(4).

nominal bounds,

$$84 \leq X(1) \leq 136$$
$$57 \leq X(2) \leq 104$$
$$96 \leq X(4) \leq 163$$
$$X(1) + X(2) + X(4) = 296$$

actual bounds,

$$84 \leq X(1) \leq 136$$
$$57 \leq X(2) \leq 104$$
$$96 \leq X(4) \leq 155$$
$$X(1) + X(2) + X(4) = 296$$

Even though the core is nonconvex it is still relatively close to convexity. In general the proximity of the core to convexity depends on the specific cost relationships.

The Shapley value, MCRS, and average cost solutions are presented in Table 7 and in Figure 15. All three solutions satisfy core constraints but only the MCRS solution is in the center of the core. The average cost solution is close to the lower core bound for site 4, relatively far from the core center. As discussed in the final step of analysis, the implications here are not so clear as for the first two cases.

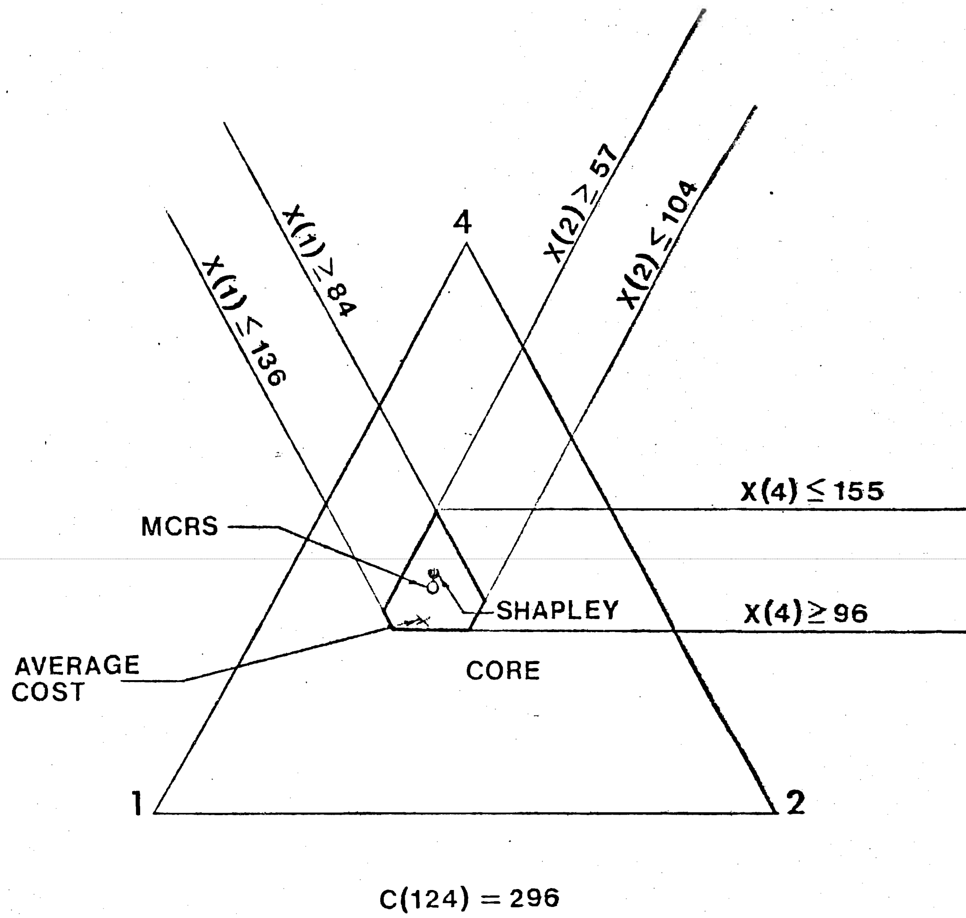


Figure 15. Core and charges for three-person game for case 3.

Table 7. Charges for Three-Person Game for Case 3

		Total Cost, 10^3 \$/year			
				Average Cost	
Site	$Q_{m,t}$ MGD	Shapley	MCRS	Charge	Savings
1	6	102.00	103.41	118.40	5%
2	6	72.50	74.58	78.93	25%
4	6	121.50	118.01	98.67	48%
	Total	296	296	296	

Step 6. Examine Implications of Economic and Financial Analysis

In this step we examine implications of results for the three cases. Recall that economic optimization for economies of scale in production with nondecreasing demand functions (cases 1 and 2) results in a strict rank order priority among potential participants which may or may not vary with output. Under these circumstances, proper economic analysis guarantees a convex set of sustainable charges. The Shapley value, MCRS, and average cost solution vectors are contained within this convex set. Although each of these pricing solutions satisfies the game theory stability criteria recognized by both economic and cost allocation theory, economists might argue that only those schemes based on marginal cost pricing will encourage economically efficient use of available resources. As demonstrated, marginal cost pricing is not applicable for single-step or upward-sloping demand relationships. Furthermore, where production capacity constraints are not tight each player receives his maximum demand quantity of output. Therefore, any sustainable charge vector encourages this economically efficient maximum participation.

Where the project design is supply limited, any sustainable charge vector encourages efficient resource use for all but the last ranking player "cut off" at the capacity output level. At a unit price corresponding to his assigned charge, this player wants to buy additional output to satisfy his maximum demand. In addition, any player excluded during the optimization phase of analysis might want to participate arguing that he should have an opportunity to share in the savings. It might be reasonable to expect that an appropriate governing agency could enforce consumption restrictions to ensure optimal resource allocation. This is in contrast to the public goods and natural monopoly pricing problem where there are many consumers as opposed to a relatively small group of potential buyers. Restrictions on how much of a public good or natural monopoly product consumers may purchase are generally impractical and often inappropriate.

Of the unique solution methods considered, only the Shapley value and MCRS solution are guaranteed to be in the center of the core for cases 1 and 2. The notion that the core center is the most equitable solution persists in game theory but is not universally accepted in water resource applications. In addition, uncertainties in technical and cost estimation data may indicate that emphasis on the center of the core is inappropriate. A suitable alternative cost allocation method might be average cost pricing which has several advantages. First, it is computationally easy, avoiding the problem of defining and computing the characteristic function. This advantage is especially apparent for projects involving more than just three or four groups. In addition to computational ease average cost pricing has the advantage of being easily understood and accepted by participants and administrators. As a case in point, Loehman et al. (1979) observe that decision makers involved in a proposed regional wastewater treatment system for the Meramec River Basin prefer uniform unit prices based on average costs to nonuniform charges even at the expense of economic efficiency.

Finally, we discuss implications for the third case where demand relationships are downward sloping. Rather than a strict rank order based on continuously increasing net benefits, priority among players depends on marginal considerations as well as plant capacity and maximum demand constraints. As a result, marginal pricing might be a viable cost allocation strategy. Any marginal pricing scheme requires some means of recovering total project costs and is further complicated by the fact that not all allocated quantities are necessarily determined from marginal conditions. Some allocated quantities may be the result of production capacity or maximum demand considerations. The problem is how to apply marginal cost pricing strategies where such strategies are applicable to only a portion of the total system price vector. With regard to the Shapley value, MCRS, and average cost solutions, we examine the nature of the resulting cost allocation game. Recall that the game is subadditive but not necessarily convex. Only the MCRS solution is guaranteed to be in the core. However, except for extreme conditions the Shapley value will probably satisfy core constraints and may even be relatively close to the core center as in the wastewater reuse example.

The location of the average cost solution is less predictable and may lie outside of the core even for relatively attractive games that are barely nonconvex. Therefore, for a nonconvex game, other approaches such as the Shapley value and particularly the MCRS method may be more appropriate than average cost pricing.

CONCLUSIONS

Results of this analysis indicate several important implications regarding the relationship between economic optimization and cost allocation for joint water resource projects. These implications may be generalized for various supply and demand relationships. This paper examines the special case of economies of scale in production for three types of demand relationships. Two groups of general results are derived. First, where the nature of the supply and demand curves are such that net benefits are continuously increasing, the resulting cost allocation game is convex. Furthermore, optimal resource allocation is achieved through a rank order assignment of output to each participant. The resource quantity allocated to each participant is limited only by demand or production capacity constraints. Consequently, economic optimization and cost allocation may indeed be independent of each other. A simple average cost pricing scheme is not only sustainable but is easily determined and implemented. However, if efficient resource utilization is not enforceable for supply limited projects, a suboptimal solution may result.

Where net benefits are not strictly increasing, the relationship between economic efficiency and cost allocation is not so clear. Joint consideration of efficiency and equity analysis might be warranted in some instances. In particular, where marginal considerations are determining factors for efficient resource allocation, marginal cost pricing strategies may be appropriate. Thus, results from economic analysis are applied to financial analysis so that the two are directly related to each other. Where efficient resource allocation can be enforced, other charging schemes including the Shapley value, MCRS, and average cost pricing solutions may be appropriate. However, since the cost allocation game may be nonconvex, only the MCRS solution is guaranteed to be sustainable. Again, where consumption restrictions are not enforced, implementation of a nonmarginal cost price vector may result in a suboptimal final system. Herein lies the tradeoff between efficiency and equity. Evaluation of this tradeoff is an important topic for future research.

APPENDIX

Convexity Conditions for Four-Person Game for

Cases 1 and 2

C(12)	+	C(13)	$\geq$	C(123)	+	C(1)	
204	+	200	$\geq$	257	+	142	OK
C(14)	+	C(12)	$\geq$	C(124)	+	C(1)	
240	+	204	$\geq$	294	+	147	OK
C(13)	+	C(14)	$\geq$	C(134)	+	C(1)	
200	+	240	$\geq$	292	+	147	OK
C(12)	+	C(23)	$\geq$	C(123)	+	C(2)	
204	+	166	$\geq$	257	+	104	OK
C(24)	+	C(12)	$\geq$	C(124)	+	C(2)	
208	+	204	$\geq$	294	+	104	OK
C(23)	+	C(24)	$\geq$	C(234)	+	C(2)	
166	+	208	$\geq$	262	+	104	OK

$$C(13) + C(23) \geq C(123) + C(3)$$
$$200 + 166 \geq 257 + 96 \quad \text{OK}$$

$$C(34) + C(13) \geq C(134) + C(3)$$
$$204 + 200 \geq 292 + 96 \quad \text{OK}$$

$$C(23) + C(34) \geq C(234) + C(3)$$
$$166 + 204 \geq 262 + 96 \quad \text{OK}$$

$$C(14) + C(24) \geq C(124) + C(4)$$
$$240 + 208 \geq 294 + 143 \quad \text{OK}$$

$$C(34) + C(14) \geq C(134) + C(4)$$
$$204 + 240 \geq 292 + 143 \quad \text{OK}$$

$$C(24) + C(34) \geq C(234) + C(4)$$
$$208 + 204 \geq 262 + 143 \quad \text{OK}$$

$$C(123) + C(14) \geq C(1234) + C(1)$$
$$257 + 240 \geq 343 + 142 \quad \text{OK}$$

$$C(124) + C(13) \geq C(1234) + C(1)$$
$$294 + 200 \geq 343 + 142 \quad \text{OK}$$

$$C(134) + C(12) \geq C(1234) + C(1)$$
$$292 + 204 \geq 343 + 142 \quad \text{OK}$$

$$C(123) + C(24) \geq C(1234) + C(2)$$
$$257 + 208 \geq 343 + 104 \quad \text{OK}$$

$$C(124) + C(23) \geq C(1234) + C(2)$$
$$294 + 166 \geq 343 + 104 \quad \text{OK}$$

$$C(234) + C(12) \geq C(1234) + C(2)$$
$$262 + 204 \geq 343 + 104 \quad \text{OK}$$

$$C(123) + C(34) \geq C(1234) + C(3)$$
$$257 + 204 \geq 343 + 96 \quad \text{OK}$$

$$C(134) + C(23) \geq C(1234) + C(3)$$
$$292 + 166 \geq 343 + 96 \quad \text{OK}$$

$$C(234) + C(13) \geq C(1234) + C(3)$$
$$262 + 200 \geq 343 + 96 \quad \text{OK}$$

$$C(124) + C(34) \geq C(1234) + C(4)$$
$$294 + 204 \geq 343 + 143 \quad \text{OK}$$

$$C(134) + C(24) \geq C(1234) + C(4)$$
$$292 + 208 \geq 343 + 143 \quad \text{OK}$$

$$C(234) + C(14) \geq C(1234) + C(4)$$
$$262 + 240 \geq 343 + 143 \quad \text{OK}$$

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