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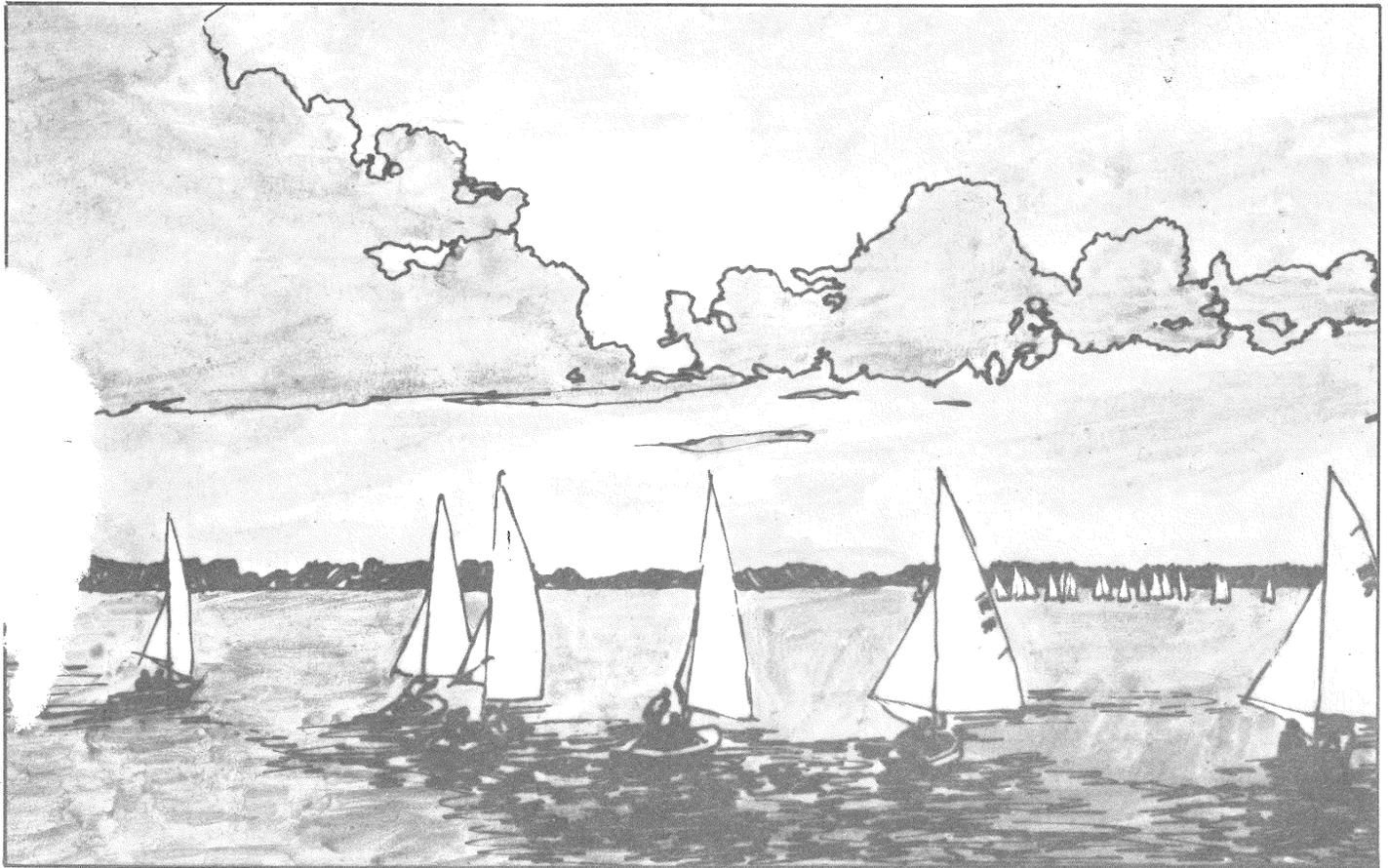
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*Analysis of Storm Water Seepage Basins
In Peninsular Florida*

By

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Dr. Sylvester Petryk initiated the study and served as its principal investigator through October, 1975. His initiative and participation in this research are greatly appreciated.

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ABSTRACT

Rapid urbanization, currently proceeding in Florida, has resulted in significant problems with regard to both flood control and pollution abatement.

The objective of the reported study was to search for the design procedures that may improve efficiency, safety and adequacy of drainage systems in peninsular Florida. Special attention was given to possibilities of recharging groundwater aquifers with excess storm water. Through such a system, partial solution to problems of inadequate potable water supply in some areas can be achieved simply as a by-product of flood control systems.

In Chapter 2 of this report a conceptual design framework is presented. In developing this framework, a variety of disciplines involved in the solution of urban drainage systems in peninsular Florida are considered.

Comparatively high levels of groundwater in many locations require consideration of two factors relating to the ability of seepage ponds to divert surface water to the aquifer. The engineer should study the ability of the pond to seep water and he should analyze the response of the groundwater table to the seeping water. With respect to these two factors, analyses and solutions adapted for engineering application are presented in Chapter 3 of this report. *STOP Here*

The continuing reduction in the availability of potable water in coastal zones of Florida has prompted several communities to design recharging systems utilizing treated storm water as well as effluents. Chapters 4 to 6 of this report concern flow conditions in the Floridan

Aquifer that should be considered in connection with this subject. Methods by which the particular phenomena associated with flow conditions in the aquifer can be evaluated are presented. These methods cover several spectral expansion approaches as well as a complete numerical approach.

CHAPTER 1
GENERAL INTRODUCTION

SCOPE OF THE STUDY

Since ancient times, people throughout the world have had to cope with periodic floods and inundations of lands and communities. Notably pragmatic solutions to flood control were developed in various locations. These solutions were dependent on human imagination and ingenuity applied through available resources to a broad variety of situations and conditions. Even for a relatively limited area such as the United States, we cannot imagine a singular methodology applicable in every situation. Effective flood control depends not only on resources and materials provided by 'Mother Nature' but, also on man's attitude toward the application of these resources. The very definition of the problem depends on the attitude of the people and their understanding.

Peninsular Florida is a unique system in many respects. In the early part of this century, flood control projects were begun in earnest over an area frequently exposed to devastating hurricanes and extensive flooding. Since World War II, tremendous changes, inherent in the rapid development of the state, brought about improved techniques and better understanding of flood control. Presently, the state of Florida faces simultaneous problems of excessive storm water and limited water supplies particularly in coastal communities. Rapid growth and the influx of new people also resulted in reduced water quality, thus adding another factor to water management and flood control. This project has been initiated with the idea of simultaneously improving flood control techniques and enriching groundwater resources in peninsular Florida.

OBJECTIVES

The investigators found it necessary to direct their efforts toward three principle objectives:

- (a) General management and development of conceptual design framework.
- (b) Development of a simplified analysis for the evaluation of seepage ponds in diverting stormwater to groundwater storage.
- (c) Development of approaches in the analysis of migration of contaminants in groundwater due to natural conditions as well as situations induced by artificial seepage.

METHODOLOGY

With respect to the listed objectives, Chapter 2 of this report concerns the development of the conceptual design framework as related to urban drainage systems in central and southern Florida.

Chapter 3 of this report concerns management of water quantities. This effort in the investigation involved determining the ability of seepage ponds to divert collected stormwater to the groundwater aquifer. Emphasis was given to development of simplified methods that can easily be applied by drainage design personnel.

Chapters 4 through 6 concern mathematical methods that can be used for the analysis of flow conditions in an aquifer similar to the Floridan Aquifer. Basic models are suggested and a variety of mathematical methods are checked from the point of view of applicability, efficiency and accuracy. These chapters form an introduction to analyses of water quality problems that have yet to be resolved in Florida.

RESEARCH PROGRAM

An initial research program was outlined by Dr. Sylvester Petryk in 1974 when he submitted the research proposal. The study, as envisioned in that proposal, would consist of the following five tasks

- 1) Review of existing literature
- 2) Modeling of precipitation, runoff, and storm water seepage basins
- 3) Economic analysis and optimization of design
- 4) Design procedure
- 5) Experimental measurements.

The research conducted during the past two years has been concerned with all of these areas. However, the program was modified with respect to the emphasis and the degree of effort devoted to each task.

A review of the existing literature indicated that our efforts should be directed more toward improvement of the complete design framework and project management rather than involvement with particular design techniques or procedures. Chapter 2 of this report covers this part of our activity, which is partially related to each of tasks 3, 4, and 5 mentioned above.

The design of seepage ponds in Florida often presents special problems because of high groundwater tables. These problems are the subject of Chapter 3 and are related to task 2 mentioned above.

Problems of water quality have become increasingly important in Florida as well as in other parts of the country. One of our primary concerns in this regard was the effect that injection of impure water might have on the quality of Florida's groundwaters. Chapters 4 to 6 of this report are concerned with this problem, which is related to task 2 mentioned above.

A series of field studies and measurements was conducted as suggested in task 5. Good results were obtained but the job was more or less routine in nature and we did not find it necessary to include them in this report. Our main objective in the field study program was to acquaint ourselves with local problems and to advise local people with respect to these subjects.

CHAPTER 2

FACTORS AFFECTING THE DESIGN OF URBAN DRAINAGE SYSTEMS IN PENINSULAR FLORIDA

INTRODUCTION

The demand for housing in Florida continues to climb as Americans seek a place in the sun. Retirement and tourism are rapidly displacing agriculture in developing and utilizing an attractive natural environment. Growth, since the end of World War II, has been phenomenal. Land development and home construction have become an integral part of the economy. These activities draw heavily on Florida's natural resources which are not limitless and, in many instances, are key elements in a sensitive environmental system. The demands of a progressive economy cannot be ignored, however, these demands can be adjusted to provide optimal use of a limited system of supply. Concerted efforts are being made to provide both understanding of a complex and dynamic system, and a reasonable balance of supply and demand.

Water, one of Florida's most abundant resources, is a critical factor in this dynamic system. Early management concentrated on drainage and flood control to such an extent that damage to the system resulted. The past twenty years has seen a shift in management emphasis as water shortages and environmental damage became more apparent. Contemporary management practices are undergoing rapid change. The idea of water as a scarce commodity has prompted basin-wide regulation of consumptive use and natural flow. Degradation of water quality has led to more stringent pollution control laws and the development of improved abatement techniques.

All of these efforts have recognized and addressed urban development as a leading cause of problems in maintaining water quality and managing flow. Urban storm runoff has been found to be a source of water pollution that is of equal or greater magnitude when compared with any other identifiable source. Home building and land development require drainage systems that accelerate runoff, consequently, creating or aggravating flood prone situations. Homes, streets, parking lots and commercial buildings retard or prevent infiltration of rainwater and necessary recharge of groundwater aquifers.

In view of the current activity in water management and pollution control, the drainage engineer must be aware of the many facets of contemporary design and he must apply sound and systematic methods in development of his design. The natural environment of Florida presents a unique set of circumstances. Laws, regulations, procedures and techniques, although rooted in historic precedent and established standards, have all been developed with some consideration for these circumstances. The drainage engineer should have a thorough knowledge, not only of applied techniques, but of social, environmental and political impacts of development. He should understand recent changes in legal and governmental philosophies. The engineer needs to know sources of information, plan requirements, and the procedural aspects of government regulation and approval. Most importantly, systematic method in design synthesis permits the engineer to organize and evaluate his data, identify and augment weak points and effectively manage the design process.

The objective of the present study is to review and suggest a conceptual design framework as related to the following topics:

1. Basic legal principles associated with drainage, groundwater, land use and water courses.
2. Highlights and purposes of legislative and regulatory efforts on the federal, state and local level.
3. Elements of the feasibility study as the first phase in the design process.
4. Considerations and techniques applied to preliminary and detailed design of drainage systems.
5. Integration of the various design factors by means of a flow diagram.

Finally, this discussion is oriented toward design factors particular to peninsular Florida.

LEGAL PRINCIPLES

Surface runoff is of legal concern for a variety of reasons. Damage done by concentrated runoff, either as a floodwave passing downstream or a backwater flooding of upstream lands, may result in injury to the affected parties. Alteration of flow directions, capacities or other drainage characteristics can and often does lead to environmental damage, e.g., lowered water tables, water pollution, damage to vegetation, erosion and accretion. Property damage from flooding of homes and businesses may be the direct result of poorly managed runoff and improper land use.

Two basic principles of law concerning disposal of surface waters are 'the civil law rule' and 'the common enemy rule'. Under the civil law rule, the upland or dominant owner has an easement on the downstream owner for passage of surface runoff in its natural manner. The common enemy rule stipulates that the servient or lower owner may take measures neces-

sary to keep these waters off of his land. Generally, Florida courts have followed the civil law rule modified by 'reasonable use'. For example, the general rule regarding drainage into a natural watercourse states that a riparian owner may discharge surface runoff without regard to either the 'civil law rule' or the 'common enemy rule'. However, this right is subject to three limitations which have been imposed by the courts in varying degrees. These limitations include:

1. Drainage must be reasonable.
2. Drainage must not come from outside the natural basin.
3. The natural capacity of the stream must not be exceeded.

The concept of reasonable use has been applied in cases involving both land use and water rights. In several instances, the courts have ruled that land use, adversely affecting surface or groundwater flows, incurs no liability on the owner as long as his use is reasonable and legitimate [2, 3]. Rulings have also been made in light of the riparian doctrine, requiring reasonably unimpaired or undiminished flow.

In conclusion, the engineer must consider possible liabilities and legal consequences in his design of drainage works. Stormwater systems designed to parallel natural discharges from a site are desirable from both an engineering and a legal viewpoint. Further, conflicts between water rights and legitimate land use have not been resolved in an entirely consistent manner. It is apparent that the body of law governing these activities and rights is not well defined and does not truly recognize the interrelation of land use and natural flow.

FEDERAL, STATE AND LOCAL REGULATIONS

Florida's rapid growth has produced heavy and often conflicting demands on existing water resources. The geology of Florida provides extensive groundwater sources that have been utilized in satisfying these demands. Wholesale acceleration and channelization of runoff and failure to provide adequate recharge to supplying aquifers combined with concentrated withdrawal from these aquifers has led, in several instances, to serious problems that cannot be resolved on a single situational basis. Total basin management has become the imperative solution. Comprehensive legislation at all levels of government has been enacted to provide a framework for management. Agencies with permitting and other regulatory powers have been created to affect solutions to apparent conflicts on a regional basis.

The Federal Water Pollution Control Act of 1948 began what has become a massive effort to improve the quality of our national water resources. Subsequent legislative efforts authorized federal assistance in research and development of methods of controlling pollution. Most recently, the F.W.P.C.A., Amendments of 1972 placed stronger emphasis on storm runoff as a source of pollution. Section 208 of this act is directed toward area-wide planning and management of both 'point source' and 'non-point source' discharges. Stormwater runoff, a non-point source, is to be evaluated on the basis of extensive data collection and monitoring in the field. Methods for reducing pollutant loading are to be developed at the community level and, subsequently integrated into a basin-wide management plan [1].

Current technology in this area has not established a total 'cause and effect' relationship between runoff and stream quality. Loading rates

are highly variable and depend on land use, storm duration and intensity, and seasonal weather patterns. This relationship or a reasonable approximation is singular in its importance with regard to effective modeling and comprehensive understanding of the problem.

A second program of particular interest to the drainage engineer stems from the National Flood Insurance Act of 1968 and the Flood Disaster Protection Act of 1973. Through this program, federally subsidized flood insurance is available to homes and businesses located in flood-prone communities. A prerequisite to qualification requires that designated communities adopt restrictive land use ordinances for areas subject to flooding on a frequency of once in one-hundred years (equivalent to a one-percent chance in any one year) [4].

The drainage engineer should recognize three significant points in these programs. First, the federal government has initiated comprehensive efforts to maintain and improve the quality of natural waters through strengthened programs at the state and local levels. Secondly, the flood insurance program is a substantial legislative effort directed toward better land use practices and flood control measures. Finally, these efforts actively involve the community and its citizens by promoting improved local ordinances and related decision making.

During 1975, Florida's environmental agencies were reorganized and consolidated into two major agencies, the Department of Environmental Regulation and the Department of Natural Resources. Additionally, the state's water management districts were redefined and were invested with broadened duties and powers. The Department of Environmental Regulation is responsible for enforcing pollution control laws and maintaining or

improving water quality. Permitting of sewage treatment plants is a duty of this agency. In granting permits for point sources such as sewage treatment plants, the D.E.R. will review and actively consider stormwater control, treatment and disposal as part of the permit request [7]. Other highlights of agency guidelines include [7]:

1. Impact of stormwater discharges on the receiving waters will be viewed considering designated use of the waters, practical considerations and cost-effectiveness.
2. Design of stormwater management systems should include a variety of techniques in reducing impact. Sod filters, vegetated buffers, sediment traps, primary considerations in design.
3. Detention basins should be considered as essential in residential, business, industrial and highway development. Interim drainage systems, serving construction sites, should be anticipated and included.

The Florida Water Resources Act of 1972 empowers the Department of Natural Resources to accomplish the conservation, protection, management and control of the waters of the state. In effect, this act makes all waters in the state subject to regulation [6]. This act further directed the environmental agencies to formulate a state-wide water use plan. Elements of the plan are being prepared at the district level and will be comprehensive in scope and objectives. Other activities of the Department of Natural Resources through the water management districts include permitting for consumptive use by all users, constructing and maintaining lands and works incident to management activities, permitting and regulation of wells, and management and storage of surface waters.

It is readily apparent that the state is assuming the role of the riparian owner in many respects. Maintaining undiminished water quality and quantity is now a valid public concern as opposed to a strictly riparian right. In addition, the proposed water use plan will include recommendations as to flood plain zoning and protection from flood hazards. Thus, the state is assuming an ever increasing responsibility over land use management and water resources which, traditionally, have been inherent in private land ownership.

Subdivision regulations and zoning ordinances form development regulations and standards applicable at the community level. Further, these regulations outline procedures in obtaining community approval for a proposed development. They provide an orderly exchange or basis of communication between the engineer and the community. Frequent discussions during the approval process, enable the engineer and community planners and officials to effectively demonstrate their needs and desires. Individual citizens may express their opinions during public hearings before the local governing body as a matter of state law.

Subdivision regulations vary considerably across the state. Community resources and extent of development tend to impose practical limits on regulatory programs and requirements. Obviously, the most sophisticated regulations available are relatively ineffective without adequate staffing and community support. Federal and state efforts are aimed at supplementing community programs and preventing abusive and costly development such as has occurred in the past.

Development standards tend to fall into two broad categories. The first is concerned with conditions or programs unique to a community.

Such factors as flood control, aquifer recharge and salt water intrusion may require regulations or procedures written specifically for these conditions. For example, Orange County has recognized the need for conserving natural recharge areas. Their regulations outline special measures for maintaining natural high-infiltration in these areas. "Typical methods include the use of filtered recharge wells, bottomless inlets, perforated pipe, grading to retard runoff, artificial seepage basins, swales in street rights-of-way, and utilization of natural percolation areas" [5].

The second category of regulations covers more detailed requirements. Minimum pipe size and material specification, street and lot grading, allowable overland flow distances, design methodology, allowable velocities in swales and ditches, drainage right-of-way requirements, and storm water details are typical items included in this category. Serviceability and ease of maintenance are important considerations in these specifications relating primarily to the 'hardware' of drainage. Routine maintenance of drainage systems and streets is a major cost item in municipal budgets. Flooded streets and overgrown ditches are common complaints from taxpayers and every effort is made to minimize such situations in writing these regulations.

In summary, legal principles governing surface flow and groundwater are derived from rights associated with private ownership. Conflicts arising from alteration of natural flow patterns or from consumptive uses have been resolved considering reasonable use of land and water. It is apparent that these principles are not adequate to resolve conflicts considering contemporary demands. As a result, regional or basin-wide management of water resources is being affected through combined legisla-

tive efforts on all levels of government. Preemption of private water rights appears to be justified in that the burden of flood damages and optimization of consumptive uses rests with the entire community. Finally, lawmakers have broad powers in preserving and protecting natural resources in the public interest. Thus, considerable legislation has been enacted to improve land use, relieve flood hazards and reduce pollution of natural waters.

PROJECT FEASIBILITY

Project initiation usually begins with a discussion including the client and key staff professionals. Subsequent efforts depend to a great extent on several factors. Among these are the intended scope of the project, potential market, capital availability, anticipated scheduling of planning, engineering, construction and sales, legal considerations and immediate technical needs. Certainly, forty acres in an established residential area will have significantly different needs than ten-thousand acres of undeveloped and agricultural lands. The experience of the client or developer will also influence project initiation. An experienced developer often will have established project economics, scope and approximate timing prior to discussion with design professionals. Project feasibility may, however, include multi-disciplinary studies and analyses of the previously mentioned factors.

One of the most fruitful approaches to evaluation and design is organization and coordination of all related efforts. Project planning draws each factor into the design process as it becomes pertinent. A systematic approach to project management helps to ensure optimization of

technical effort and production and, that the quality of the effort will be thoroughly professional.

Drainage design as an engineering process, is well suited to an organized approach. Design synthesis and coordination of effort in drainage engineering is graphically demonstrated in the flow diagram as shown in the Appendix adapted from Woodson [8]. Flow diagrams are an accepted and proven management tool. Their inherent flexibility makes them adaptable to a variety of situations and design conditions. The included flow diagram, presented in the Appendix, is divided into three distinct phases paralleling the balance of this discussion. These phases include project feasibility, preliminary design and detailed design.

Intensive data collection is the first step following project initiation. Gathering information on existing drainage patterns and both current and future land use follows several avenues. A thorough understanding of the existing drainage situation is essential not only in determining required improvements but, also in demonstrating the effectiveness of proposed improvements. Standard sources of information that will readily provide data on drainage, land use and flood potential should first be investigated. Among these sources are:

1. U. S. Geological Survey for topographic maps, flood studies, groundwater studies, geologic investigations, well surveys, stream and lake gage records and general hydrologic information.
2. Municipal or County Engineering and Planning for subdivision regulations, zoning ordinances, aerial mapping, established drainage networks and other information directly related to the site.

3. Soil Conservation Service for soil surveys, drainage techniques proven in the area, recommended best use of the site and extensive technical expertise in drainage and hydrology.

4. State agencies for regulatory information and design guidelines in stormwater management. Specific information as to local environmental considerations and current practices.
5. Federal agencies for flood insurance and flood plain information, possible federal permits, design standards for qualification under federal loan programs and many other technical services.

The reconnaissance survey of the site of a proposed development is particularly important in that information resulting from visual inspection is of primary importance and is generally available from no other source. Initial data may be summarized and plotted or noted on a site plan to facilitate observations and comparisons during the survey. Verification of land use, drainage patterns and improvements, both on the site and in the surrounding catchment, is an important objective. Other factors to be considered during 'recon' are:

1. Attractive or beneficial features on the site that will contribute to aesthetics and environmental quality.
2. Hydraulic constrictions and control points, ponds, lakes, streams, springs and natural depressions collecting runoff.

3. High water marks on trees and drainage structures, water surface elevations and groundwater or water table elevations.
4. Soil types and areal distribution. Soil deposits such as organic muck and peat that must be considered in design and construction.
5. Identification of likely drainage schemes, outfalls and receiving waters. Probable quality of current runoff and receiving waters.
6. Identification of chronic drainage problems and existing improvements both on the site and in the immediate catchment.

During or following field investigation, the engineer should outline requirements for additional topographic surveys, soils investigations, stream gaging, water quality evaluations, environmental surveys and similar efforts. Methods and techniques for field investigations are dependent on site conditions and the extent of proposed development. Detailed field notes and supplementary photographs are common to all surveys and provide documentation to support subsequent judgments, public presentations and technical discussions.

Evaluation and incorporation of collected information involves simulation of existing drainage conditions, potential for improvement, delineation of drainage systems and land use planning. 'Rough' calculations or approximations should be made in evaluating the hydraulic and hydrologic characteristics of the existing system and the proposed improvements. The drainage engineer should work very closely with those involved in land

planning. Land use, of course, will have multiple effects on any proposed drainage system. Compatibility of land use and soil characteristics is of particular importance in Florida. Planners must be aware of probable water table elevations and seasonal fluctuations. Much of Florida consists of fine sands with small percentages of silt in the surficial layers. Often, a semi-permeable layer of consolidated fines and organic material underlies surface sands. Infiltration of stormwater is severely retarded and lateral or surficial flow may be very slow, particularly on the marine terraces and in the 'piney flat woods'. The drainage engineer should evaluate cost-effective measures in reducing high water tables and flat gradients and, he should advise land planners of his findings.

Land planning should be coordinated to reflect drainage methods and alternatives best suited for the site, required rights-of-way, pond locations, flood elevations, fill sources and quantities, recharge sites and natural features to be preserved. Every effort should be made at this time to promote an aesthetically pleasing drainage network. An open ditch or floodway is patently obvious and prominent in appearance. The additional costs associated with a landscaped and meandered channel are more than justified considering heightened sales potential and consumer ideals.

The preliminary engineering report or feasibility study reflects a combined effort in engineering, planning and project economics. Form and content for such a study are variable and, generally, depend on the scope of the project and desires of the client. It cannot be over-emphasized that such a report is preliminary in nature and that subsequent efforts may contradict stated conclusions, cost projections and

quantity estimates. Misunderstandings are frequent in this respect and the engineer should use care and discretion in making this point clear. Regulations, permitting and government coordination are often disregarded at this stage of the process. The feasibility study should include a thorough discussion of all aspects of the permitting and approval process.

Preliminary engineering design provides the client with valuable information for project scheduling and funding. The feasibility report serves the developer in anticipating construction costs, cash flow, marketing and sales. Finally, the engineer and client can selectively evaluate alternatives and determine preliminary design objectives.

PRELIMINARY AND DETAILED DESIGN

Preliminary design should be approached considering overall site hydrology. Simulation techniques commonly used include the rational method, unit hydrographs, synthetic hydrographs, regression equations and more complex computer modeling techniques such as those developed by the Corps of Engineers and the Soil Conservation Service. The object of all of these methods is to systematically quantify runoff and to route these quantities through the system.

Typically, the outfall point is identified together with available gage records or some estimation of appropriate water surface elevation. The flat gradients, so common to much of Florida, necessitate a thorough investigation and a reasonable approximation of downstream water surfaces corresponding to the events being simulated. Collector systems are usually a combination of open channels and storage basins that depend on

both physical ground slopes and hydraulic head in maintaining positive flow. Central Florida's highlands may require grade stabilization structures where slopes are relatively steep. Peak velocities in unlined channels generally should not exceed two feet per second in Florida's sandy soils. Less frequent storm events may be permitted to generate somewhat higher velocities. Storage-elevation curves and stage-discharge relationships may be approximated in routing surface runoff through the system. Routing begins with generation of water quantities at the upstream end of the system and proceeds down the slope to the receiving waters. Peak discharges and water surface elevations, derived from the routing process, may then be used to calculate water surface profiles. Most systems generate peak flows over a relatively short period of time so that only peaks may be considered in calculating the profiles. Subcritical flow profiles must begin at the receiving waters and, based on the appropriate surface elevation, proceed upstream. Adjustments are made in channel geometry, storage, control structures and overland flow times until close agreement is achieved between the water surface profiles and the elevations generated by routing. This procedure begins in the preliminary design phase and is continually refined through the end of the design sequence. Many factors that are elemental to simulation will have considerable variation depending on the event being considered. Rainfall intensities, for example, will generate system responses markedly different for various events. It is advisable to simulate storms of several frequencies and durations even though regulations may specify design based on one or two standard events. Practical considerations, such as allowable design time, must be considered in managing this procedure.

Internal drainage systems direct runoff to the collectors. The rational method is almost universally used to design local drainage such as swales, culverts, inlet size and location, street slopes and lot grading. Home building and related land development require simultaneous evaluation of earthwork and drainage. Generally, surface slopes may be adjusted to minimize drainage improvements. Cutting and filling required to achieve ideal slopes may, however, result in excessive costs in earth moving. Once again, several iterations of site engineering may be required to achieve optimal conditions in detailed design.

Incorporation of detention-retention ponds in the drainage system is desirable to attenuate increased discharges resulting from development. Ponding is a recommended method in reducing pollutant loading. Common practice is to provide sufficient storage within the system for the volume of runoff resulting from a given storm event. These events or design rainfalls generally require enough storage so that runoff from high frequency events, such as afternoon thunderstorms, is totally retained or detained for an extended period. Silts and other suspended solids have a chance to settle, evaporation and infiltration effectively reduce the volume of stored runoff, biological action and dilution reduce the concentration of pollutants, filtering devices prevent discharge of surface borne contaminants such as oil and grease, and trash and debris are prevented from entering the receiving waters. Natural storage areas, including cypress ponds and mangrove swamps, may be utilized in developing detention. These areas have the advantage of acting as filters for runoff, thus improving discharge quality. Care must be used to maintain minimum water levels in these areas and to prevent

damaging pollutants from entering such a system that utilizes natural vegetation. Other considerations necessary to retention-detention ponds are:

1. Storage capacity recovery through evaporation, infiltration, structural devices regulating outflow, modified underdrains and pumping.
2. Siltation or clogging in the pond. Two-stage ponds should be provided where sediment loads will be high. Periodic cleanout of settling basins in the first stage will be required. A subsurface berm or equivalent structure should separate the settling basin from the second stage.
3. Maintenance, safety, vegetation, normal depth, fill needs, aesthetics, emergency overflow, and drawdown capability are among the additional considerations.

Systems of interconnected lakes and ponds are frequently employed in Florida to provide fill material, aesthetic appeal and recreational benefits. In addition, extensive open systems tend to lower surrounding water tables in chronically wet areas.

All drainage systems should be viewed considering the effects of the maximum probable storm. Surface drainage systems may be inundated in some areas causing streets and low lying areas to act as open channels or ponding areas. Utility services may be interrupted. High water in the outfall system may result in flood conditions for an extended period. In this regard, auxiliary floodways may be desirable, road elevations should be sufficiently high to allow limited continuous access, floor elevations near points of concentration should be higher than arbitrary

minimums and off-site structures and embankments should be evaluated as flood hazards.

In effect, preliminary design involves evaluation of a selected development plan. Design efforts should reflect sufficient detail such that very limited alternatives remain together with final details necessary to construction. The flow diagram, presented in the Appendix, outlines elements of this phase through client and agency approval. Presentation of the preliminary plan together with supporting data, becomes part of a comprehensive review by the client and staff. These plans may then be submitted to local agencies for preliminary approval and required zoning changes. Review and discussion with state and federal agencies should be conducted on an informal basis. Results of these interviews and evaluations should be summarized for incorporation into detailed design.

The third and final phase, detailed design, completes this sequence. Construction plans, specifications and detailed cost estimates are primary objectives. Revisions to the preliminary design should be solicited from staff, client, agencies and potential construction contractors. The design manager must provide a thorough check of every detail and should coordinate with other members of the staff in locating and resolving possible conflicts. Almost invariably, insufficient clearances between sanitary sewerage, storm mains and water supply will become evident and necessitate change. Experienced contractors will often point out plan discrepancies, quantity errors and unfavorable site conditions during bid preparation or contract negotiation. During this phase, the engineer should coordinate closely with permitting and approving agencies in providing supporting data for selected design features. Unnecessary and

costly changes, based on arbitrary interpretation or judgmental differences, may be avoided through foresight and preparation. All parties should be informed of required revisions and all plans should be noted accordingly.

CONCLUSIONS

Drainage design has become considerably more than estimating runoff and selecting culverts. The design professional is obliged to assume a broad viewpoint in developing his program and directing design efforts. In summary of this discussion, the following conclusions are presented:

1. The engineer should be familiar with basic legal principles related to drainage and how they have been applied in resolving conflicts and water rights litigation.
2. The engineer must be aware of rapid changes taking place with regard to water rights and resource management. Federal and state legislation in this area has shifted rights and responsibilities away from the landowner and toward the public entity.
3. Local regulations and ordinances are being re-written or supplemented in order to effect better land use practices and flood protection measures.
4. Substantial efforts are being made to develop and implement improved pollution abatement techniques. Urban runoff, as a prime source, may be improved through a variety of techniques presently available to the designer.
5. Legislative mandates have been very direct. The engineering profession should take precautions in protecting their design

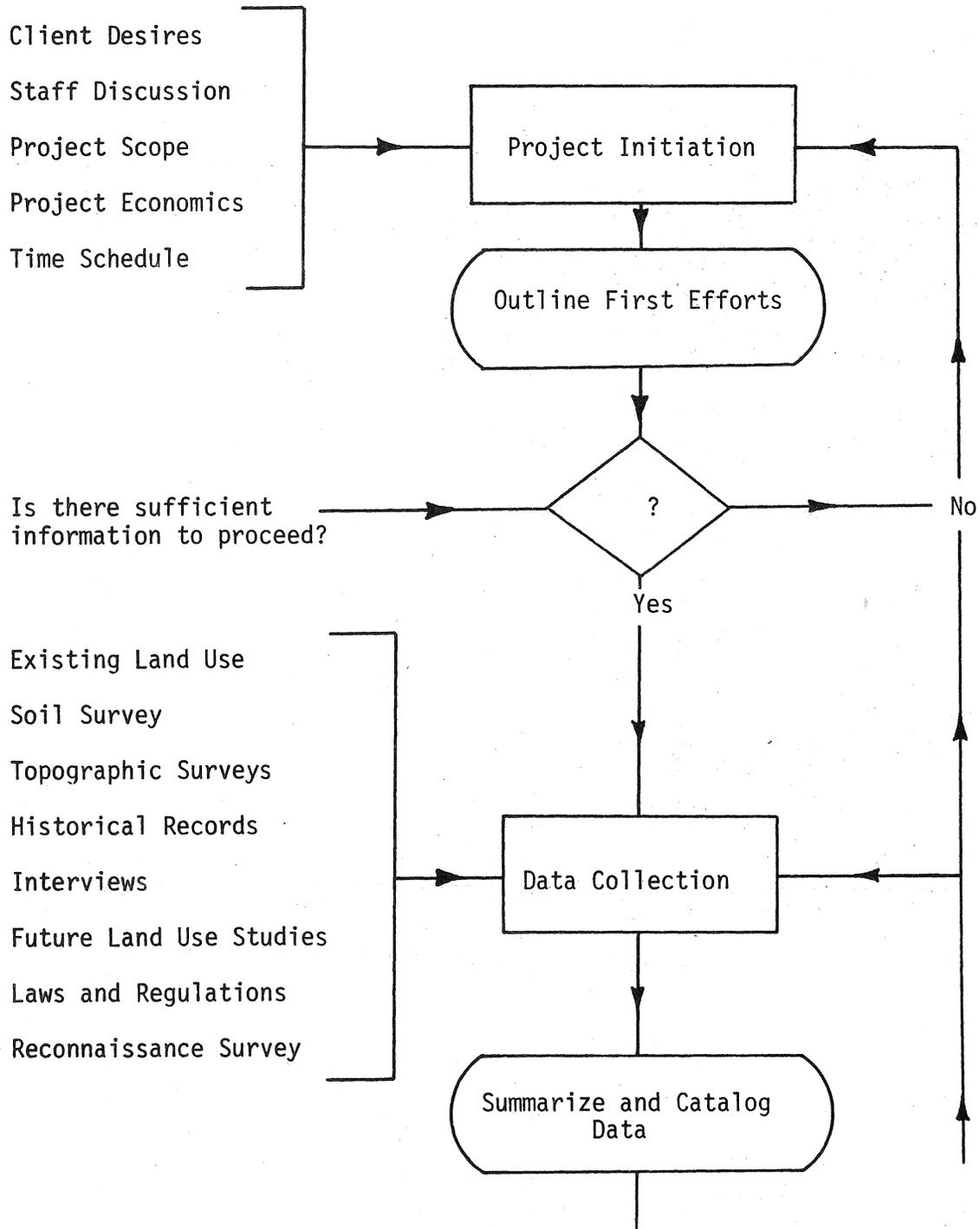
prerogatives and, at the same time, align their efforts with these mandates.

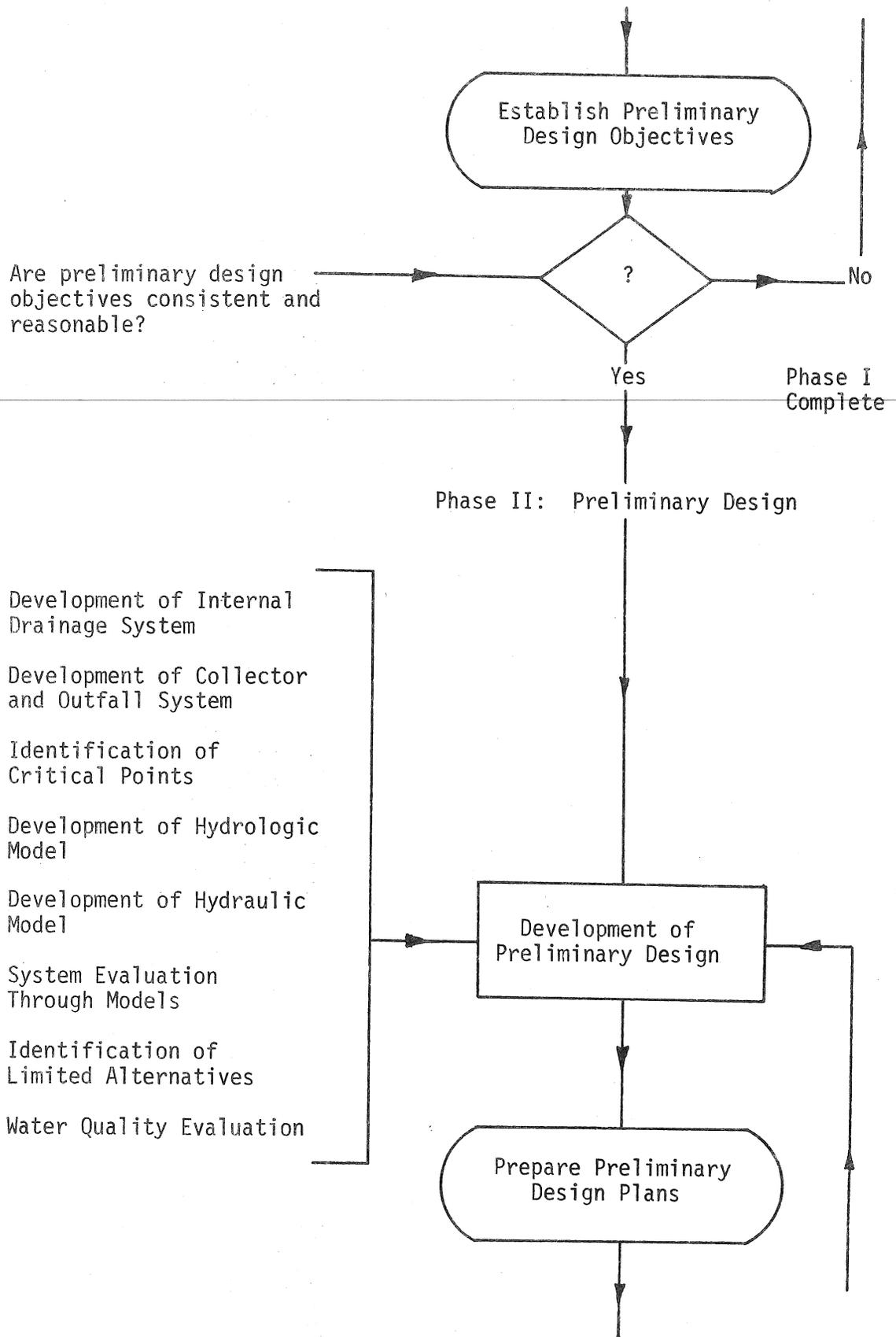
6. The feasibility study is, possibly, the most important phase in design synthesis. This study develops a set of conditions influencing the entire project.
7. Florida's environment and dependence on well managed water resources emphasizes the efforts of the drainage engineer. He must actively participate in several areas of project development.
8. Detention-retention systems have become a necessity in designing improvements. Ponding or storage improves water quality, reduces flood potential and provides increased aquifer recharge.
9. Management of the design process and attainment of technical efficiency may be aided through the use of a flow diagram presented in the Appendix. Considerable flexibility allows this technique to be adapted to most projects.

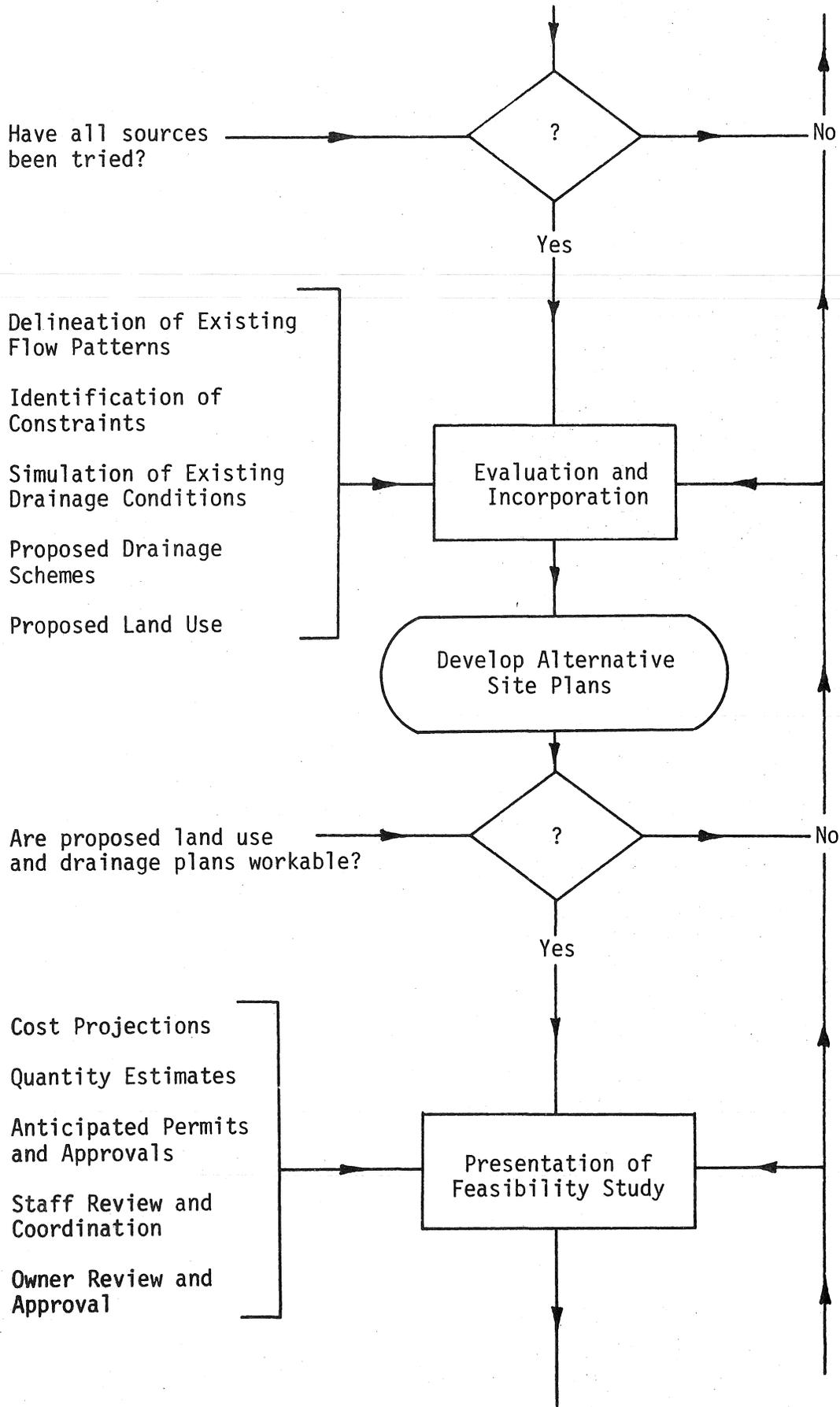
Finally, the attitude of the engineer must be receptive to change and innovation. Bending to accommodate is not the answer. Embracing and assertive leadership is a trademark of the professional.

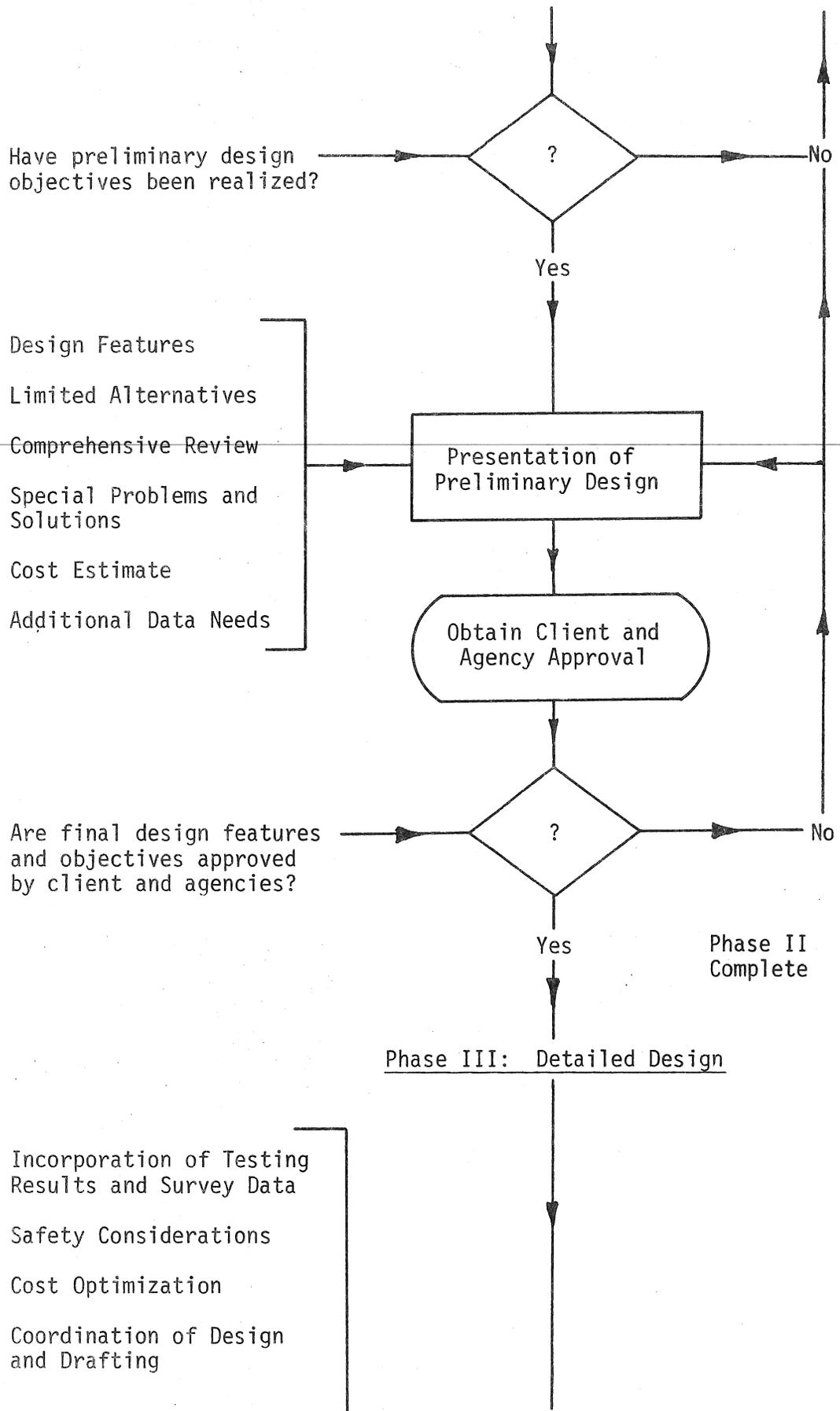
APPENDIX
FLOW DIAGRAM FOR URBAN DRAINAGE DESIGN

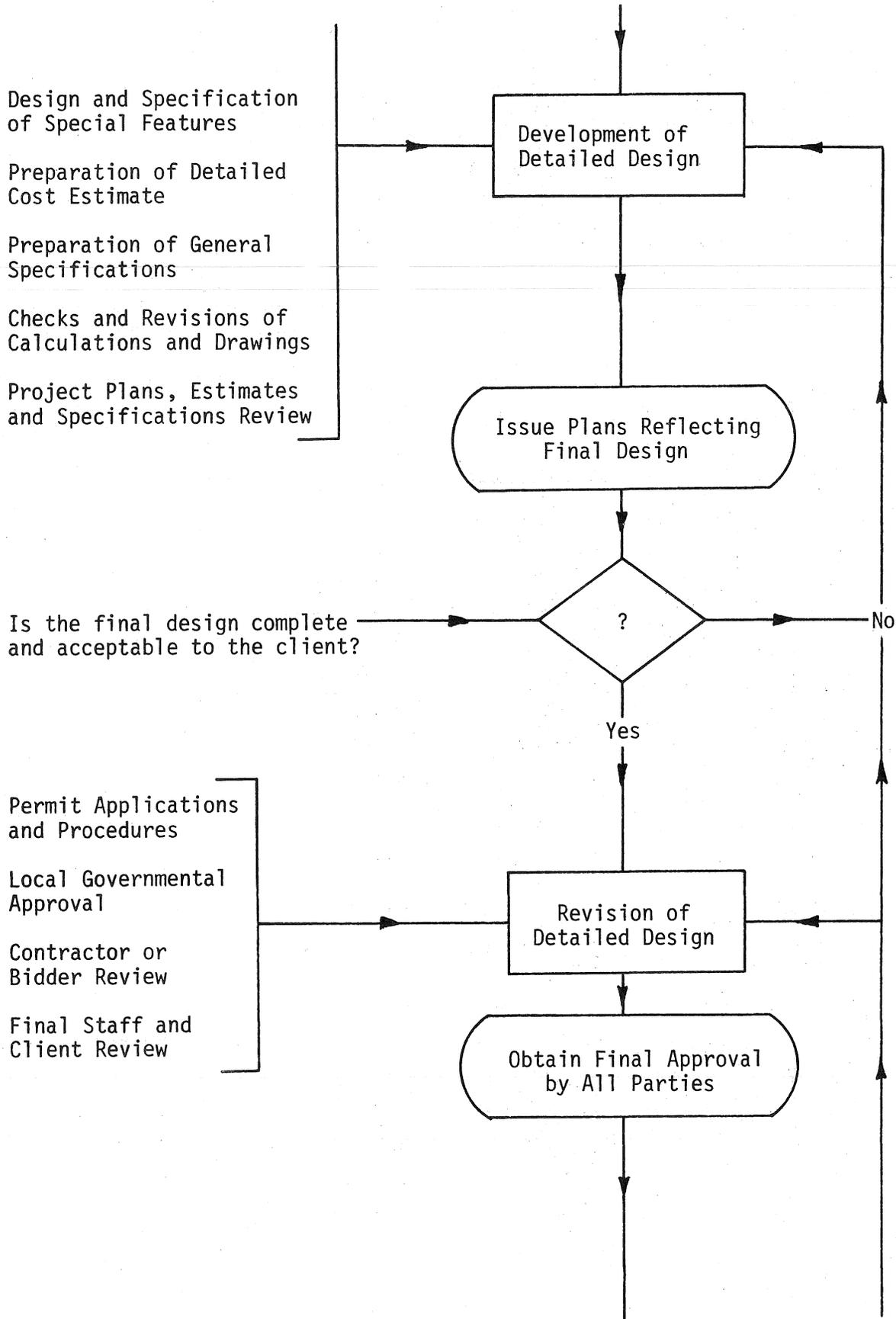
Phase I: Project Feasibility



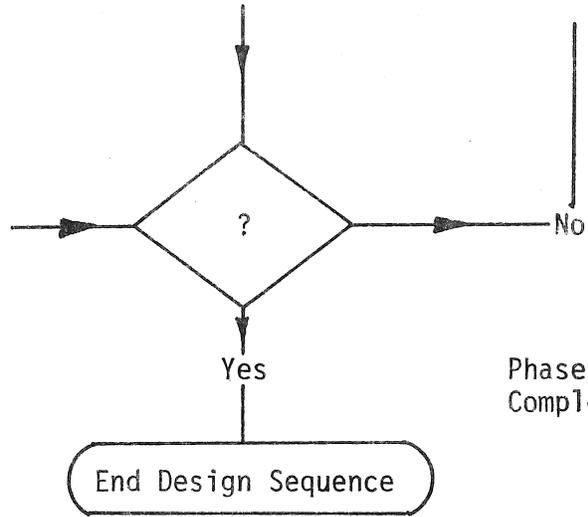








Are plans, specifications, estimates, and permits complete? Are approvals in receipt?



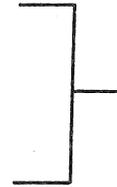
Phase III Complete

Symbols Key

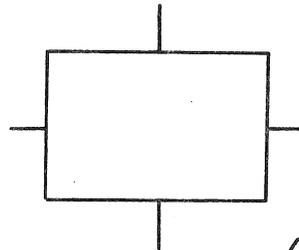
Flow Direction:



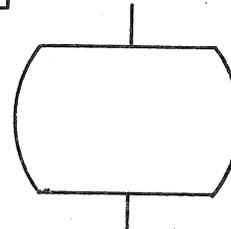
Specific Activity or Information:



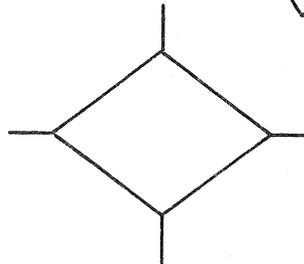
General Activity:



Executive Action:



Decision:



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CHAPTER 3

ANALYSIS OF TRANSIENT GROUNDWATER FLOW FROM SEEPAGE PONDS

INTRODUCTION

The ability of a seepage pond to divert storm runoff into the groundwater aquifer depends on two processes: (a) the rate of seepage from the pond, and (b) the reaction of the groundwater table.

The action of a seepage pond in disposing of storm water is a transient phenomenon. The inflow to the pond, specified as a runoff hydrograph, causes variations in the depth of ponded water. Outflow is considered here to be entirely by infiltration, which varies not only because of changes in pond depth but also due to variations in such soil properties as hydraulic conductivity and storativity. The growth and decay of the groundwater mound in the underlying aquifer is also time dependent.

These facts are well known among scientists but are usually not taken into consideration in the design of a seepage pond. The objectives of this study are to develop methods by which the designer will be able to estimate seepage pond effectiveness by considering both the rate of seepage from the pond and the variation in the groundwater table while taking into account the transient nature of the problem.

CALCULATION OF UNSTEADY SEEPAGE FROM A POND

The adequacy of a seepage pond is evaluated by a storage routing procedure, which is basically an account of the inflow, outflow, and change of stored volume over successive discrete time increments. The inflow is represented by the runoff hydrograph for the design storm on the area to be drained. For the purposes of the present exposition it

will be considered a given function of time. The outflow, on the other hand, is dependent on time, depth of ponding, and the properties of the soil.

Consider an initially dry seepage pond constructed in unsaturated soil. As it fills, the water begins to escape from it by vertical unsaturated flow, or infiltration. The most convenient way to describe this flow is by the formula of Green and Ampt [1911] as modified by Bouwer [1969]. It has been shown that this approach, which was long thought to be purely empirical, is soundly based on physical principles [Morel-Seytoux & Khanji, 1974] and gives very good answers [Whistler and Bouwer, 1970].

Green and Ampt based their derivation on a simplified model of infiltration which treats the soil as a bundle of vertical capillary tubes. The vertical hydraulic conductivity and moisture content of the unsaturated flow are considered constant, as is the capillary suction potential of the advancing wetting front. Applying Darcy's law to this idealized flow, the infiltration rate is described as

$$w = K_t \frac{H + L + \psi_n}{L} \quad (1)$$

where w = infiltration rate

K_t = hydraulic conductivity of the transmission zone

H = depth of ponded water

ψ_n = capillary suction potential

L = depth of penetration of the wetting front.

The rate of advance of the wetting front is

$$\frac{dL}{dt} = \frac{w}{f} \quad (2)$$

where t = time

f = the volumetric fraction of fillable pore space.

The equation of continuity applied to the pond yields

$$\frac{dv}{dt} = A \frac{dH}{dt} = I - f \frac{d}{dt}(A_f L) \quad (3)$$

where v = stored volume in the pond

I = inflow

A = area of pond surface

A_f = area involved in infiltration.

Introducing equation 3 into 1 and 2 we obtain, after differentiation, the following non-linear differential equation

$$\frac{d^2}{dt^2}(L^2/2) + (fA_f/A-1)\frac{dL}{dt} + fL/A \frac{dA_f}{dt} - I/A = 0 \quad (4)$$

which can be solved numerically.

However, in most practical cases the rate of change of H is about an order of magnitude smaller than the rate of change of L . In such cases an integration of equation 2 between t_i and t_{i+1} provides a simple discrete presentation of the problem, as follows

$$\int_{t_i}^{t_{i+1}} dt = \int_{L_i}^{L_{i+1}} \frac{w}{f} dL.$$

The integration leads to

$$t_{i+1} - t_i = f/K_t [L_{i+1} - L_i - (H_{i+\frac{1}{2}} + \psi_n) \ln \left(\frac{H_{i+\frac{1}{2}} + L_{i+1} + \psi_n}{H_{i+\frac{1}{2}} + L_i + \psi_n} \right)]. \quad (5)$$

In dimensionless form this can be expressed as

$$\frac{\Delta t K_t}{f L_i} = \frac{\Delta L}{L_i} - \frac{\Gamma_{i+\frac{1}{2}}}{L_i} \ln \left(\frac{L_{i+1}/L_i + \Gamma_{i+\frac{1}{2}}/L_i}{1 + \Gamma_{i+\frac{1}{2}}/L_i} \right). \quad (6)$$

where

$$\Gamma_{i+\frac{1}{2}} = H_{i+\frac{1}{2}} + \psi_n$$

$$L_i = \text{value of } L \text{ at time } t_i$$

$$L_{i+1} = \text{value of } L \text{ at time } t_{i+1}$$

$$\Delta t = t_{i+1} - t_i$$

$$\Delta L = L_{i+1} - L_i$$

$$H_{i+\frac{1}{2}} = \left(\frac{1}{2}\right)(H_i + H_{i+1}), \text{ the mean value of } H.$$

A chart that can be applied for engineering design purposes is shown in Figure 1. The curves are based on the solution to equation 6.

The application of the discrete approach of equation 6 was checked against experimental data reported by Weaver and Kuthy [1975]. In this experiment a seepage pond was constructed and filled with water at a controlled rate. The variation of the pond volume and area vs. depth is shown in Figure 2. The experimenters listed soil test data which led to the following values of soil parameters.

$$K_t = 1.2 \text{ ft/hr} \quad f = 0.2 \quad \psi_n = 0.5 \text{ ft.}$$

According to the reported measurements, the rate of change of H was 5 to 10 times smaller than the rate of change of L. This would indicate

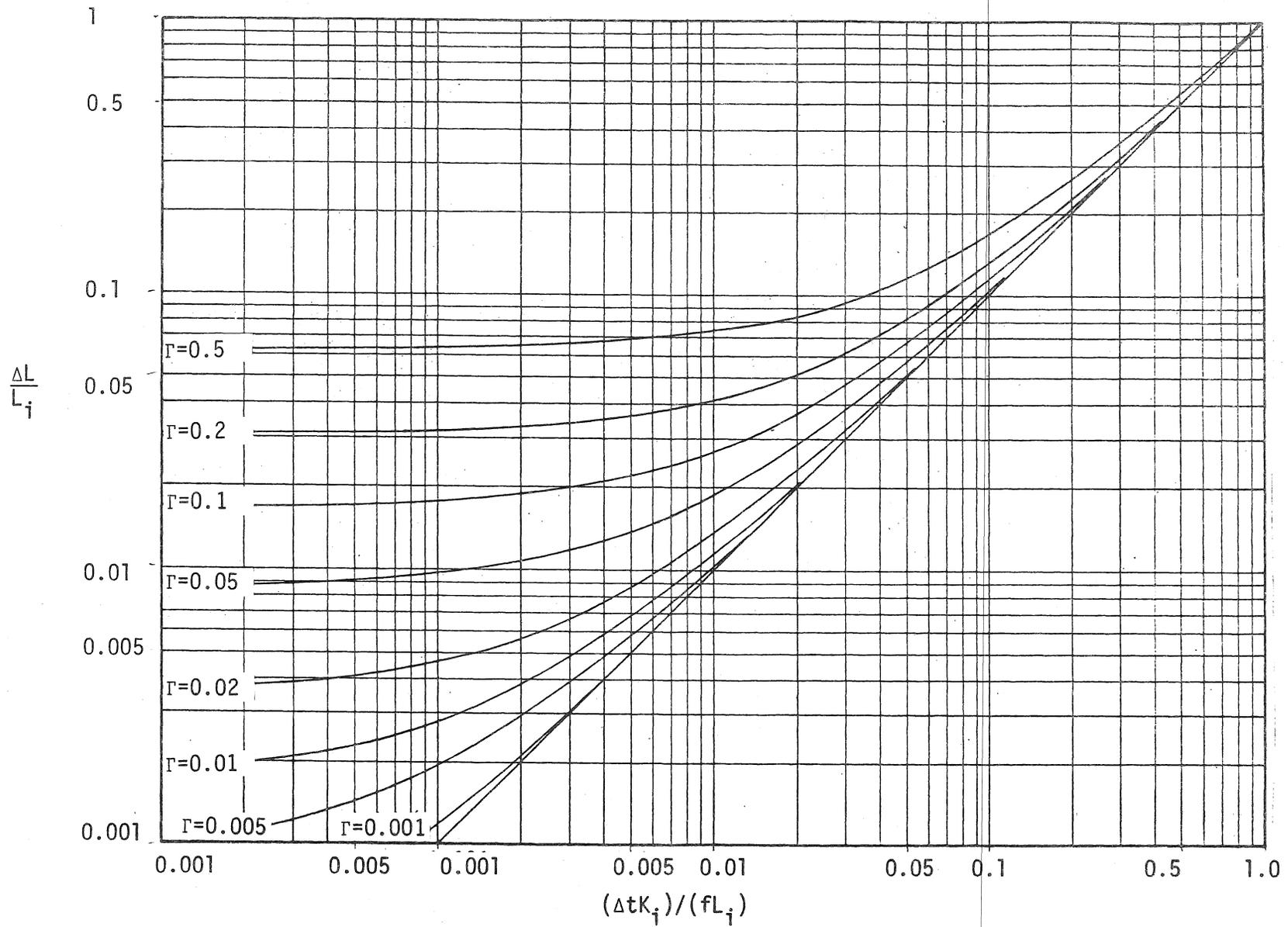


Figure 1. Solutions to Equation 6 for $L_{i+1}/L_i = 1.2$

AVERAGE AREA (thousand square feet)

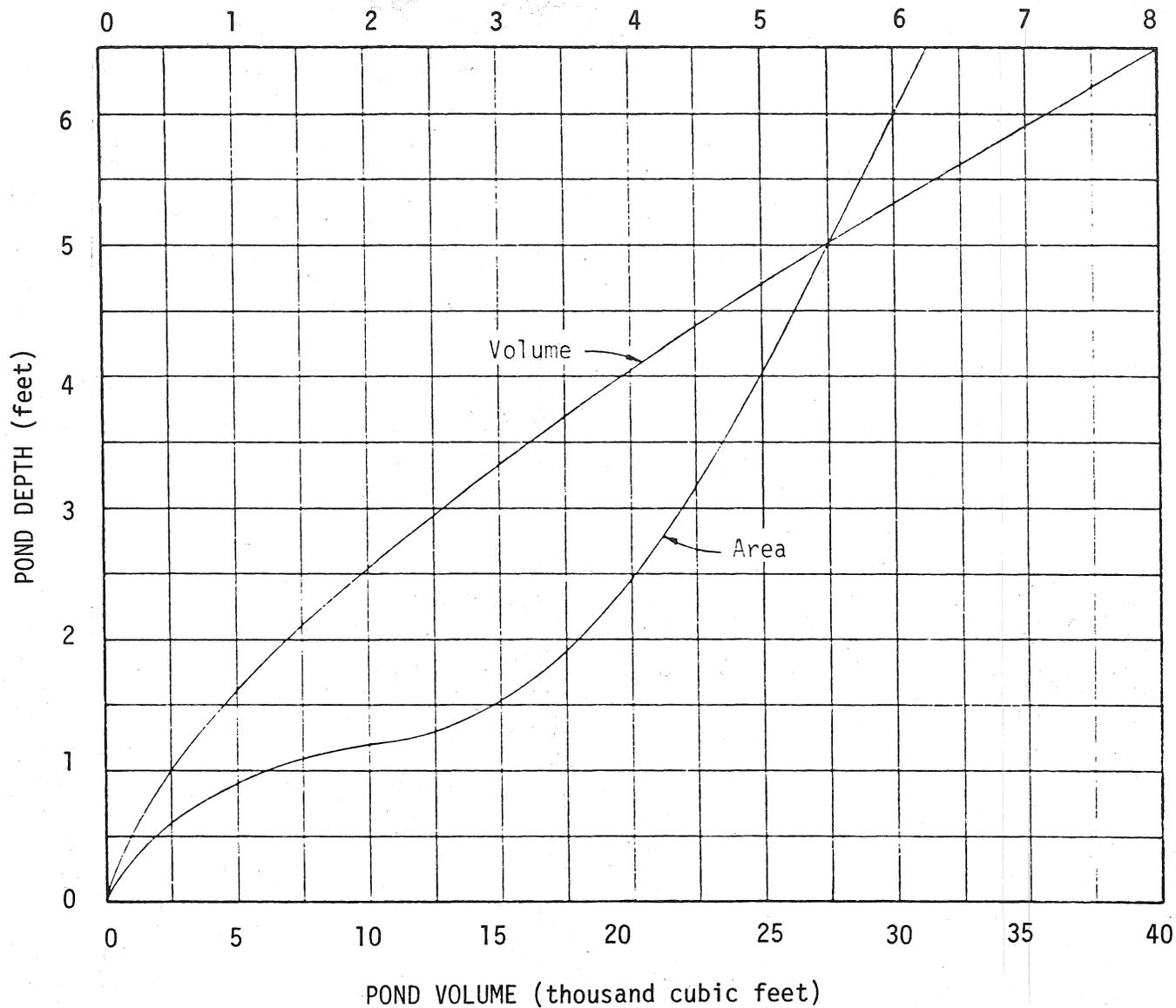


Figure 2. Depth vs. Average Area and Volume for Test Pond

that the method utilizing equation 6 should give good results. The comparison between the experimental data and the theoretical prediction is shown in Figure 3. The agreement is, indeed, quite good.

CALCULATION OF THE RESPONSE OF THE GROUNDWATER TABLE

The reaction of the water table aquifer to vertical infiltration from a seepage pond is characterized by the growth of a groundwater mound. For analysis we consider a circular pond constructed in homogeneous and isotropic soil. The aquifer is underlain by a horizontal impervious layer and initially has a horizontal free surface. When the wetting front of vertical infiltration reaches this free surface the mound begins to form. From this time the infiltration is assumed to continue at a constant rate.

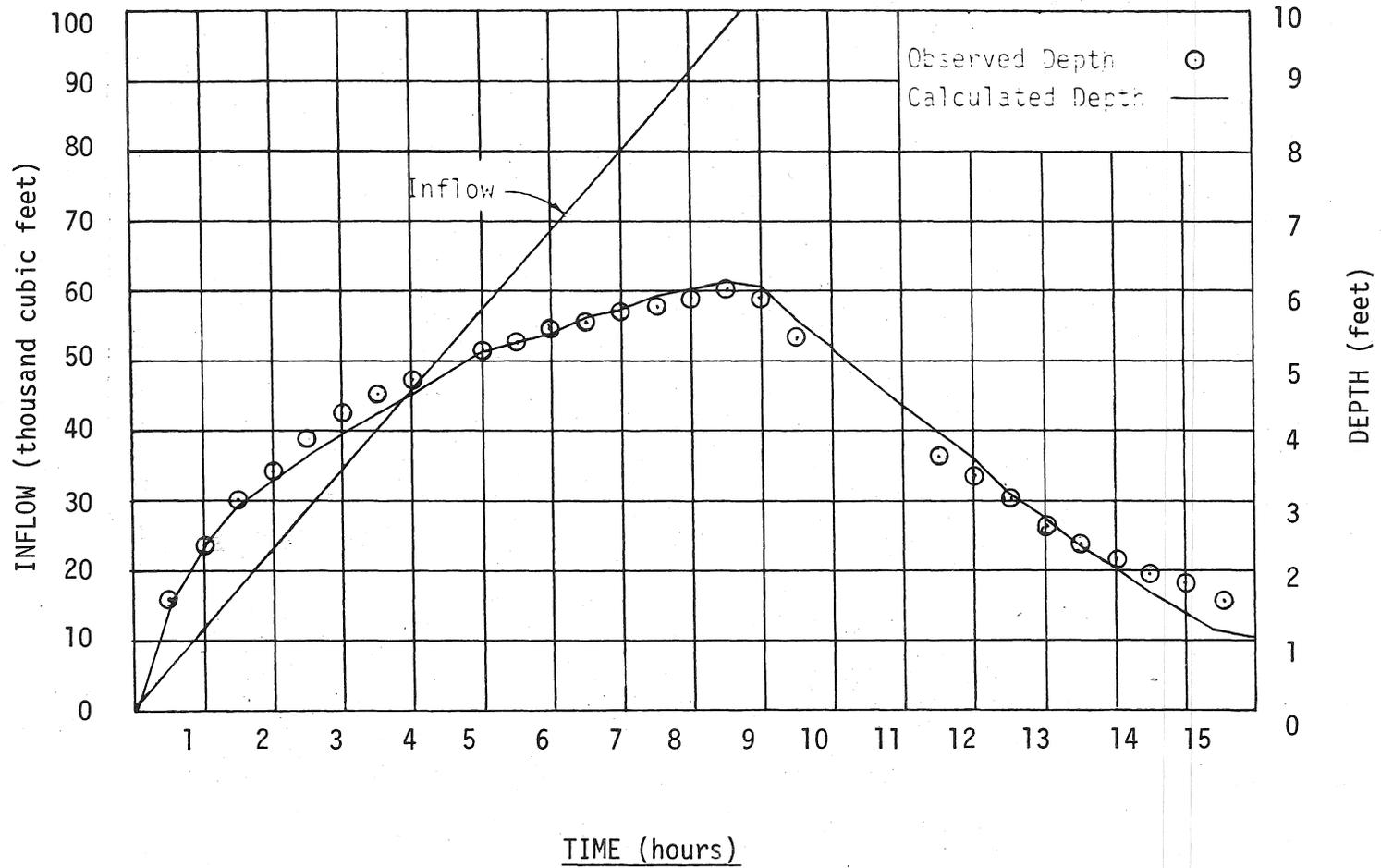


Figure 3. Comparison of Experimental and Calculated Storage Routing

In a cylindrical coordinate system centered on the seepage pond as shown in Figure 4 the axisymmetric flow in the saturated mound is described by

$$\frac{\partial h}{\partial t} = \frac{K_s}{n} \frac{\partial}{\partial r} \left(h \frac{\partial h}{\partial r} \right) + \frac{K_s h}{nr} \frac{\partial h}{\partial r} + \frac{w}{n} \quad (7)$$

where K_s = saturated hydraulic conductivity

n = specific yield

w = rate of vertical infiltration.

Both w and n are functions of radial distance, r . The value of specific yield is greatly reduced in the zone of infiltration under the pond.

Based on the experimental evidence of Bodman and Coleman [1943] it seems reasonable to assign a specific yield to the zone of infiltration with a value of 20% of the specific yield elsewhere. The rate of infiltration, w , is zero when r is greater than the pond radius, R_0 .

Noting that $h = S + a$, equation 7 can be rearranged:

$$\frac{\partial S}{\partial t} = \frac{K_s}{n} \left(S + a \right) \frac{\partial^2 S}{\partial r^2} + \frac{K_s}{n} \left(\frac{\partial S}{\partial r} \right)^2 + \frac{K_s}{rn} \left(S + a \right) \frac{\partial S}{\partial r} + \frac{w}{n} \quad (8)$$

To make the equation dimensionless, introduce the following dimensionless variables:

$$t' = tk_s a/R_0^2, \quad r' = r/R_0, \quad S' = S/a$$

Substitution of these variables along with some manipulation and omitting the primes we obtain:

$$n \frac{\partial S}{\partial t} = \frac{\partial^2}{\partial r^2} (S^2/2) + \frac{\partial^2 S}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} (S^2/2) + \frac{1}{r} \frac{\partial S}{\partial r} + WR_0^2/(K_s a^2) \quad (9)$$

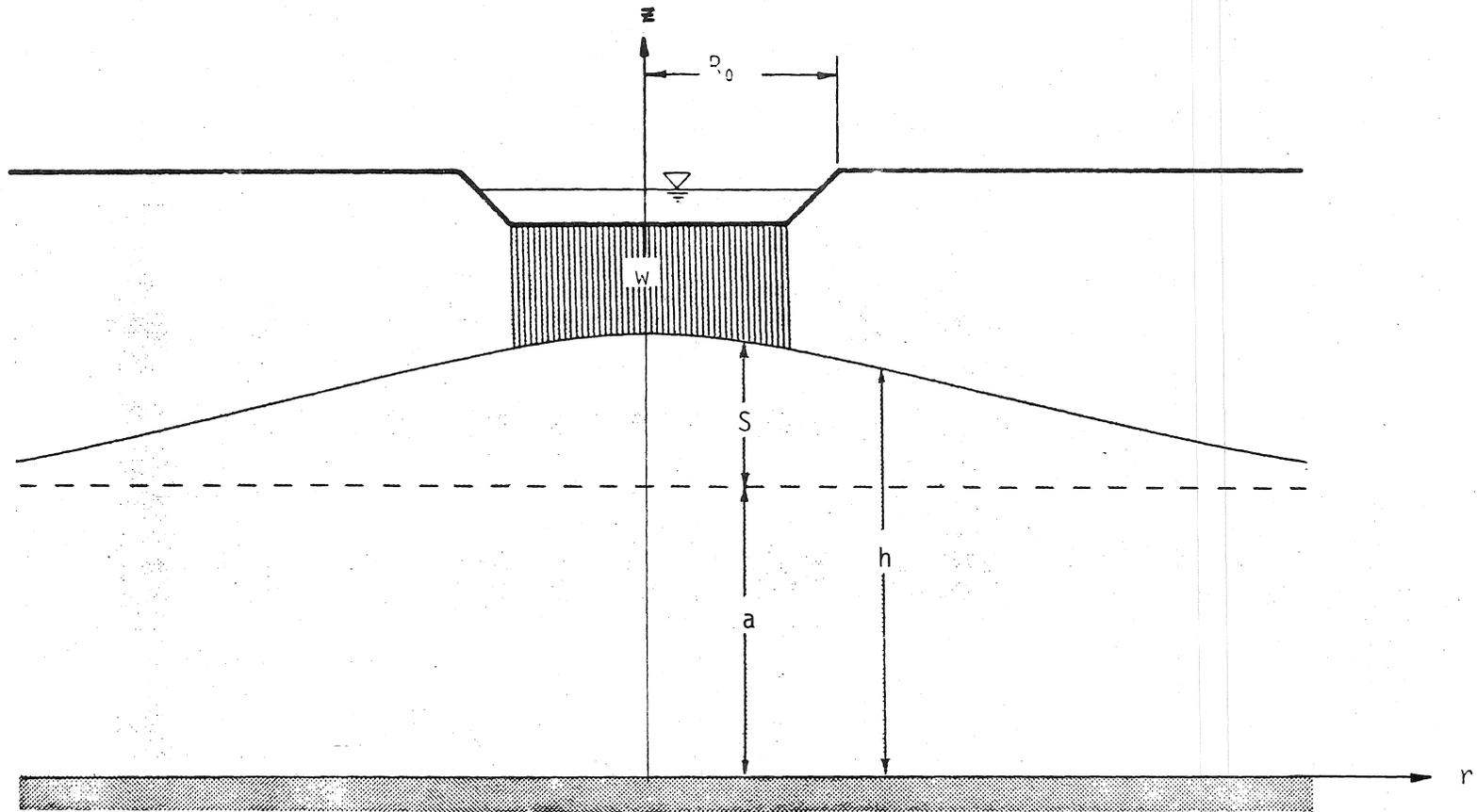


Figure 4. Groundwater Mound

In this form, the non-linear terms are distinctly separated from the linear terms. They are treated differently in the numerical analysis.

$$\frac{\partial S}{\partial r} = \frac{\partial}{\partial r} \left(\frac{S^2}{2} \right) = 0 \text{ at } r = 0 \quad (10)$$

and

$$S = 0 \text{ at } r = \infty$$

The initial condition is:

$$S = 0 \text{ at } t = 0 \quad (11)$$

To solve the problem by finite differences, we must set up an appropriate space - time grid. The time increments are designated by i 's and the space increments by j 's. The time derivative of S is approximated by a forward difference.

$$\frac{\partial S}{\partial t} \approx \left(S_{i+1,j} - S_{i,j} \right) / \Delta t \quad (12)$$

The non-linear spatial derivatives are represented by central difference operators:

$$\frac{\partial}{\partial r} (S^2/2) \approx \left(S^2_{i,j+1} - S^2_{i,j-1} \right) / (4\Delta r) \quad (13)$$

$$\frac{\partial^2}{\partial r^2} (S^2/2) \approx \left(S^2_{i,j+1} - 2S^2_{i,j} + S^2_{i,j-1} \right) / 2\Delta r^2 \quad (14)$$

The linear space derivatives are the most influential terms in the equation. If the solution were to become numerically unstable, it would

probably be because of these terms. Therefore, they are approximated by the mean of the finite difference representations on the $(i + 1)$ th and the (i) th time rows. This is the implicit method of solution developed by Crank and Nicolson [1947] which has good stability and convergence characteristics:

$$\frac{\partial S}{\partial r} \approx \frac{1}{2} \left[(S_{i+1,j+1} - S_{i+1,j-1})/(2\Delta r) + (S_{i,j+1} - S_{i,j-1})/(2\Delta r) \right] \quad (15)$$

$$\frac{\partial^2 S}{\partial r^2} \approx \frac{1}{2} \left[(S_{i+1,j+1} - 2S_{i+1,j} + S_{i+1,j-1})/\Delta r^2 + (S_{i,j+1} - 2S_{i,j} + S_{i,j-1})/\Delta r^2 \right] \quad (16)$$

When these finite difference approximations are substituted into equation 9, a tridiagonal system of equations is generated which can be solved by Gaussian elimination.

The computer program developed from these finite difference operators was tested with several combinations of grid mesh ratio and grid size. It was found to give stable results with a ratio of $\Delta t/\Delta r^2 \leq 0.11$. The mesh size chosen was $\Delta r = 0.03$. To simulate the boundary condition at infinity, the calculations were carried out to $r = 25$ at which point S was required to equal zero always.

For comparison with the linearized analytical solution of Hantush [1967], the finite difference program was run with constant specific yield, n . The results of this run are shown by the dashed lines in

Figure 5. They are practically identical to the results obtained by the method of Hantush. When n was allowed to vary as a function of r , the results were very much different, as shown by the solid lines in Figure 5.

DISCUSSION AND CONCLUSIONS

An analysis of the hydraulic operation of a seepage pond should be conducted in two phases. The first phase is concerned with the rate at which water seeps out of the pond by vertical infiltration through its bottom. The equation for vertical unsaturated flow developed by Green and Ampt [1911] can be used to describe the outflow in a storage routing process. The storage routing is simple enough to be done by hand when the soil is assumed to be homogeneous. The Green and Ampt equation can also be adapted for use in soil where the hydraulic conductivity varies monotonically with depth [Bouwer, 1969] which is a very common occurrence. This calculation is about as simple as the storage routing procedure but the combination of variable ponded depth and variable hydraulic conductivity would be cumbersome enough to make the use of a computer or a programmable calculator desirable.

The second phase of the analysis deals with the response of the water table to recharge from the seepage pond. The determination of adequate capacity of the groundwater aquifer requires use of the design charts for each specific case. The design chart itself is based on the numerical solution of the non-linear differential equation.

The importance of the finite difference solution of the groundwater mound is that it makes possible the consideration of an axisymmetric variation of the specific yield. It would also be possible to include the effects of axisymmetric variation in other soil properties.

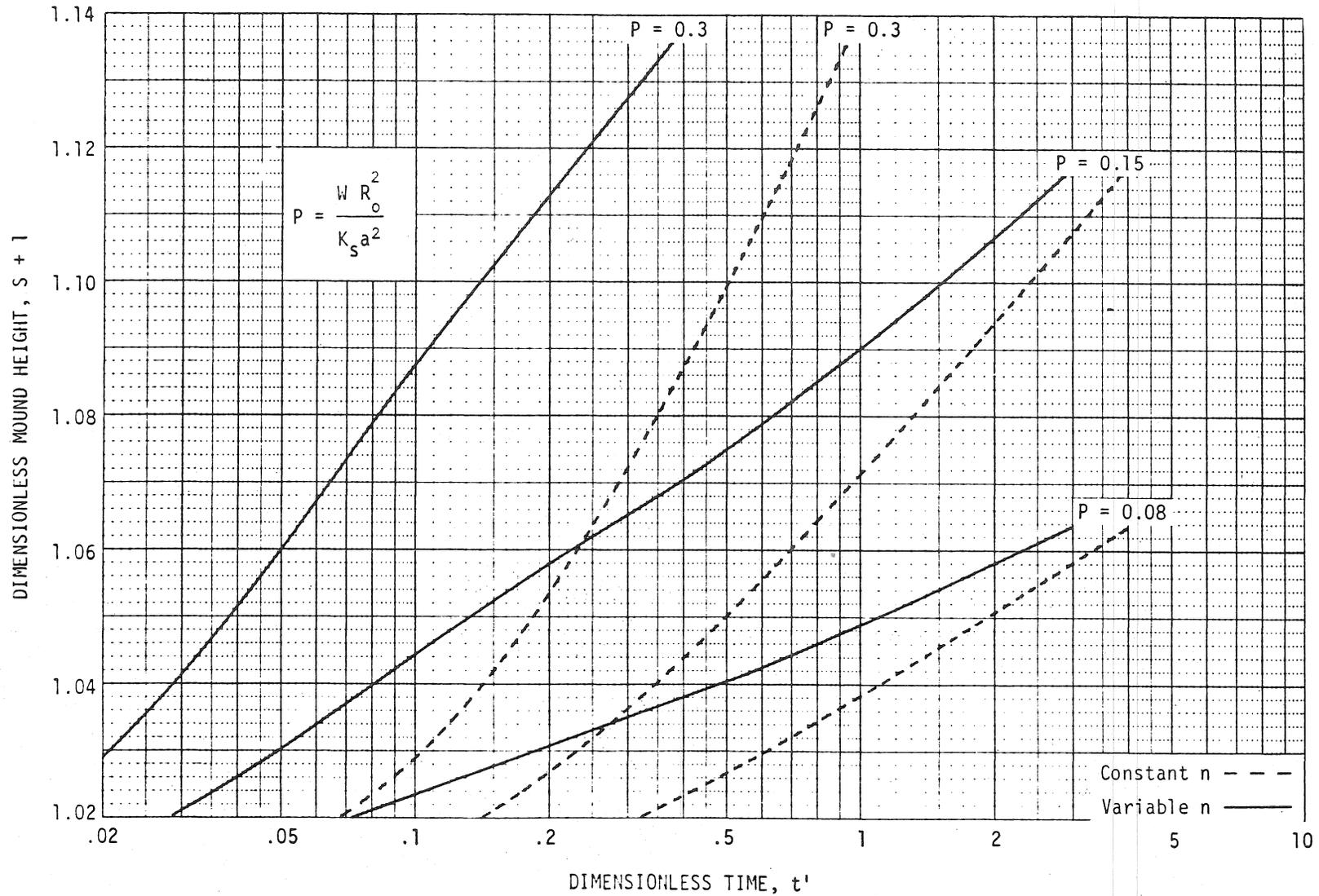


Figure 5. Mound Height at $r' = 0$, Comparison of Constant and Variable n

It should be noted that the saturated hydraulic conductivities referred to in the two phases of the analysis are, very often, not the same. Infiltration is concerned with vertical flow while the groundwater mound is mainly influenced by horizontal flow. The difference between the saturated hydraulic conductivities in the two directions usually stems from horizontal layering of the soil.

NOTATION

a	-	initial thickness of aquifer [L]
A	-	area [L ²]
A _f	-	area of infiltration [L ²]
f	-	volumetric fraction of fillable pore space
h	-	thickness of aquifer [L]
H	-	depth of ponded water [L]
I	-	inflow to pond [L ³]
K _s	-	hydraulic conductivity of saturated soil [L/T]
K _t	-	hydraulic conductivity of the transmission zone in unsaturated flow [L/T]
L	-	depth of penetration of the wetting front [L]
n	-	specific yield
P	-	dimensionless recharge intensity, $wR_0^2/K_s a^2$
r	-	radial distance [L]
r'	-	dimensionless radial distance, r/R_0
R ₀	-	radius of seepage pond [L]
S	-	rise of groundwater mound above water table [L]
S'	-	dimensionless mound height, S/a
t	-	time [T]

- t' - dimensionless time, $tK_s a/nR_0^2$
 v - stored volume in pond [L^3]
 w - infiltration rate [L/T]
 Γ - $\psi_n + H$ [L]
 ψ_n - capillary suction potential at field capacity [L]

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CHAPTER 4

THERMAL CONVECTION IN A CAVERNOUS AQUIFER

INTRODUCTION

In a previous article [Rubin, 1976], referred as I in this paper, the author analyzed instability criteria related to onset of thermohaline convection in an aquifer whose properties are similar to those of the Boulder Zone of the Floridan Aquifer. Such an aquifer is characterized by extremely large pore size and transmissivity leading to very intensive solute and heat dispersion as well as to invalidity of the laminar Darcy law even when flow velocities are extremely small.

In I it was found that for moderate Reynolds numbers the doubly diffusive convection can be approximated by the singly diffusive convection, for in such cases mechanical dispersion is larger than molecular solute and heat diffusion.

The objective of this study is to analyze transport phenomena in the cavernous aquifer subjected to singly diffusive convection.

BASIC EQUATIONS

The analysis is related to a flow field similar to the Boulder Zone (the deep regions) of the Floridan Aquifer. We assume that density gradients are induced by a single component only, referred to as temperature. In the cavernous strata, turbulent effects, as well as mechanical heat dispersion, are induced by even extremely slow fluid motions. In I the basic equations related to the calculation and approximated by the Boussinesq approach were presented as follows:

$$\frac{\partial u_i}{\partial x_i} = 0 \quad (1)$$

$$\frac{\partial p}{\partial x_i} + \rho g n_i + \frac{\mu \phi}{K} (1 + b) u_i = 0 \quad (2)$$

$$\gamma \frac{\partial T}{\partial t} + u_i \frac{\partial T}{\partial x_i} = \frac{\partial}{\partial x_i} (E_{ij} \frac{\partial T}{\partial x_j}) \quad (3)$$

$$\rho = \rho_0 [1 - \alpha(T - T_0)] \quad (4)$$

where u_i = velocity vector
 p = pressure
 ρ = fluid density
 K = permeability
 ϕ = porosity
 μ = viscosity
 T = temperature
 α = coefficient of thermal expansion

ρ_0, T_0 = density and temperature of reference

γ = a coefficient defined by

$$\gamma = \frac{\rho C \phi + \rho_s C_s (1 - \phi)}{\rho C \phi} \quad (5)$$

Through a brief literature survey presented in I it was suggested that the friction function, b , can be approximated by:

$$b = 0.014 \text{ Re} \quad (6)$$

In an isotropic medium the dispersion tensor, E_{ij} , can be expressed as a sum of the isotropic molecular diffusivity and the second order

symmetric mechanical dispersion tensor, as in

$$E_{ij} = \kappa \delta_{ij} + E_{ij}^* \quad (7)$$

where

$$E_{ij}^* = E_t^* \delta_{ij} + (E_\ell^* - E_t^*) u_i u_j / U^2 = (E_t - \kappa) \delta_{ij} + (E_\ell - E_t) u_i u_j / U^2 \quad (8)$$

Here subscripts t and ℓ refer to transversal and longitudinal components respectively.

An assumption of singly diffusive convection is justified for a multi-component system for moderate Reynolds numbers. A model suitable for the description of the mechanical dispersion tensor in such cases was suggested by Saffman [1959]. According to this model

$$\frac{E_t^*}{\nu} = \frac{s + 2}{2(s + 1)(s + 3)} \left(\frac{Re}{\phi} \right) \quad (9)$$

$$\frac{E_\ell^*}{\nu} = \frac{(s + 1)^2}{2(1 - s)(s + 2)(s + 3)} \left(\frac{Re}{\phi} \right) \quad (10)$$

where s is a power coefficient describing the dependence between the velocity and the pressure drop (s varies between unity, for laminar flow, and half, for turbulent flow).

The flow field model considered in this study while unperturbed conditions prevail, is described in Figure 1. This is a saturated porous layer of infinite horizontal extent bounded by two impermeable planes located at bottom and top of the aquifer. Temperatures on bottom and top of the porous layer are T_0 and $T_0 - \Delta T$, respectively. Through the

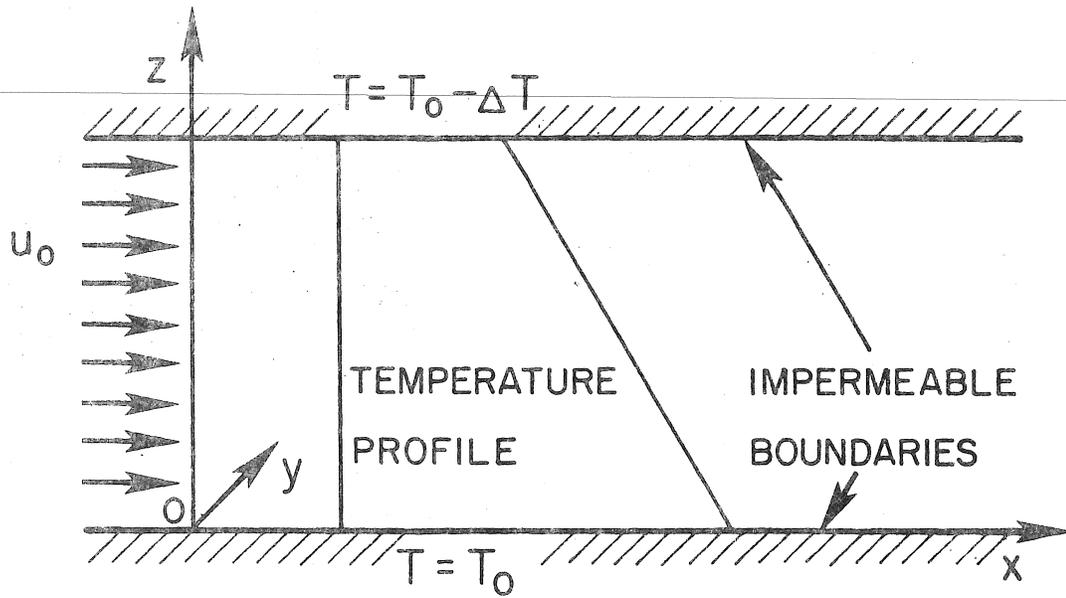


Figure 1. Schematical description of unperturbed conditions

porous layer the fluid flows uniformly in the longitudinal, x , direction. y and z are transversal and vertical coordinates respectively.

The flow field variables can be nondimensionalized as follows:

$$\begin{aligned} x'_i &= (x_i - u_0 t \lambda_i) / d & T' &= (T - T_0) / T \\ u'_i &= u_i d / E_t & U' &= Ud / E_t \\ t' &= t E_t / d^2 & E'_{ij} &= E_{ij} / E_t \\ p' &= \frac{(p - \rho_0 g z) K}{\mu \phi E_t (1 + b)} + \frac{x u_0}{E_t} \end{aligned} \quad (11)$$

where λ_i is a unit vector in the x direction. Substituting the dimensionless variables of (11) in (1)-(3) and omitting the primes we obtain

$$\frac{\partial p}{\partial x_i} - RT n_i + (1 + \beta) u_i = 0 \quad (12)$$

$$\gamma \frac{\partial T}{\partial t} + u_i \frac{\partial T}{\partial x_i} = \frac{\partial}{\partial x_i} (E_{ij} \frac{\partial T}{\partial x_j}) \quad (13)$$

Here R is the Rayleigh number defined by

$$R = \frac{\alpha g \Delta T K d}{\nu \phi E_t (1 + b)} \quad (14)$$

The power s in (9) and (10) according to I can be calculated through

$$s = \frac{\lambda n(\text{Re})}{\lambda n(1 + b) + \lambda n(\text{Re})} \quad (15)$$

However, (15) can be utilized only when $\text{Re} \geq 2.77$ if $b = 0.014 \text{ Re}$.

For unperturbed conditions (12) and (13) yield

$$u_i = 0 \quad (16)$$

$$\beta = 0 \quad (17)$$

$$T = -z \quad (18)$$

$$p = p_0 - Rz^2/2 \quad (19)$$

$$E_{xx} = E_d/E_t \quad E_{yy} = E_{zz} = 1 \quad E_{ij}(i \neq j) = 0 \quad (20)$$

LINEAR STABILITY ANALYSIS

Stability criteria of the flow field are determined through the linear stability analysis. The flow field is subjected to small disturbances in the velocity (u, v, w), temperature (θ), dispersion tensor (ϵ_{ij}), friction function (β), friction power coefficient (ξ) and pressure. Disturbances are very small and through the linear stability analysis second order terms are negligible.

For the linear stability analysis the module of the velocity vector in the perturbed flow field is approximated by

$$U = [(u_0 d/E_t + u)^2 + v^2 + w^2]^{0.5} \approx u_0 d/E_t + u \quad (21)$$

Substituting (21) in (15), (9) and (10) we obtain expressions for the principal components of the dispersion tensor. By applying (8), all components of the dispersion tensor in the perturbed flow field are obtained.

It is convenient to express the velocity components by utilizing a scalar function Ω as follows

$$u = \frac{\partial^2 \Omega}{\partial x \partial z} \quad v = \frac{\partial^2 \Omega}{\partial y \partial z} \quad w = -\nabla_1^2 \Omega \quad (22)$$

where $\nabla_1^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ (23)

Introducing flow field perturbations in (12) and (13), neglecting second order terms and eliminating the pressure perturbation we obtain

$$\nabla_1^2 (R\theta + \nabla^2 \Omega) = 0 \quad (24)$$

$$\gamma \frac{\partial \theta}{\partial t} + \nabla_1^2 \Omega = \frac{E_l}{E_t} \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \theta}{\partial z^2} + c_1 \frac{\partial}{\partial x} \nabla_1^2 \Omega - c_2 \frac{\partial^3 \Omega}{\partial x \partial z^2} \quad (25)$$

where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$

$$c_1 = \frac{E_l - E_t}{u_0 d} \quad c_2 = \frac{E_t^*}{u_0 d} \quad (26)$$

Assuming vanishing values of θ and Ω on top and bottom of the aquifer, these disturbances can be expanded by the following normal modes

$$(\theta, \Omega) = (\theta_1, \Omega_1) \sin(\pi z) \exp [i(a_x x + a_y y) + (\sigma_r + i\omega)t] \quad (27)$$

where θ_1 and Ω_1 , are constants, a_x and a_y are the wave number components.

For point of stability ($\sigma_r = 0$) substitution of (27) in (24) and (25) yields the following secular equation

$$\begin{vmatrix} a^2 + \pi^2 & R \\ a^2 - ia_x(c_1 a^2 - c_2 \pi^2) & \chi a^2 + \pi^2 + i\gamma\omega \end{vmatrix} = 0 \quad (28)$$

where

$$\chi = \frac{(E_l/E_t)(a_x/a_y)^2 + 1}{(a_x/a_y)^2 + 1} \quad (29)$$

The minimal value of R satisfying (28) is the critical Rayleigh number. Therefore, (28) yields the following criteria of point of instability

$$\begin{aligned} \chi &= 1 & a_x &= 0 & a_y &= a & \omega &= 0 \\ a_0 &= \pi & R_0 &= 4\pi^2 \end{aligned} \quad (30)$$

Hence convection cells are two dimensional rolls whose axes are parallel to the unperturbed velocity vector. Superposition of the unperturbed velocity and the convection velocity leads to a helical flow field.

Convection currents conducted in two dimensional rolls were also obtained for the ordinary Bénard problem [Malkus and Veronis, 1958; Schlüter et al., 1965] as well as for free convection in a porous layer while mechanical dispersion effects are negligible [Straus, 1974]. However, in those cases only nonlinear stability analysis associated with stability analysis of the steady convection motion yield such a result, whereas, in our study the linear stability analysis indicated that phenomenon. In our case, calculations concerning the anisotropy of the dispersion led to the conclusion that convection cells should be two dimensional rolls.

Convection sets out in planes where the effective Rayleigh number attains maximal values, namely, where the coefficients of hydrodynamic dispersion attain minimal values.

Inertial effects associated with the invalidity of the laminar Darcy law introduce the friction function, b , in the expression for R but do not affect the linear stability analysis and predictions.

FINITE AMPLITUDE DISTURBANCES AND NONLINEAR STABILITY ANALYSIS

The effect of the convection motion on transport processes through the aquifer can be predicted through the solution of the nonlinear equations of motion and heat transport related to supercritical conditions.

The convection motion is two dimensional, therefore, the velocity components can be expressed by the stream function as follows

$$\psi = \frac{\partial \Omega}{\partial y} \quad v = \frac{\partial \psi}{\partial z} \quad w = -\frac{\partial \psi}{\partial y} \quad (31)$$

Substituting the finite amplitude disturbances in (12), (13) and eliminating the pressure perturbation we obtain

$$R \frac{\partial \theta}{\partial y} + \nabla^2 \psi = B(\psi, \beta) \quad (32)$$

$$-\gamma \frac{\partial \theta}{\partial t} + \nabla^2 \theta - \frac{\partial \psi}{\partial y} = H(\psi, \theta) + D(\epsilon_{iz}) + F(\epsilon_{ij}, \theta) \quad (33)$$

where B and H are the friction and the heat advection spectra, D and F are two parts of the heat dispersion spectrum, all of which are of the form

$$B(\psi, \beta) = -\frac{\partial}{\partial x_i} \left(\beta \frac{\partial \psi}{\partial x_i} \right) \quad (34)$$

$$H(\psi, \theta) = - \frac{\partial(\psi, \theta)}{\partial(y, z)} \quad (35)$$

$$D(\epsilon_{iz}) = \frac{\partial}{\partial x_i} (\epsilon_{iz}) \quad (36)$$

$$F(\epsilon_{ij}, \theta) = - \frac{\partial}{\partial x_i} (\epsilon_{ij} \frac{\partial \theta}{\partial x_j}) \quad (37)$$

Here β and ϵ_{ij} are finite amplitude disturbances in the friction function and the dispersion tensor respectively.

As long as the convection velocity is smaller than the unperturbed velocity the absolute value of the velocity in the flow field can be approximated by

$$U = [(u_0 d/E_t)^2 + V^2]^{0.5} = (u_0 d/E_t)(1 + \lambda) \quad (38)$$

where $V^2 = v^2 + w^2$ (39)

$$\lambda = \left(\frac{E_t}{u_0 d}\right)^2 \frac{V^2}{2} \left[1 - \left(\frac{E_t}{u_0 d}\right)^2 \frac{V^2}{4} + \left(\frac{E_t}{u_0 d}\right)^4 \frac{V^2}{8} - 5 \left(\frac{E_t}{u_0 d}\right)^6 \frac{V^6}{64} + \dots \right] \quad (40)$$

Applying (40), (6) and (15) we obtain

$$\beta = [b/(1 + b)]\lambda \quad (41)$$

$$\xi = \frac{(1 + b)[\lambda \ln(1 + b)] - b[\lambda \ln(\text{Re})]}{(1 + b)[\lambda \ln(\text{Re})] \ln[(1 + b)\text{Re}]} s\lambda \quad (42)$$

Introducing (42) in (9), (10) and the dimensionless form of the dispersion tensor we obtain after minor approximations

$$\begin{aligned}
\epsilon_{ij} = & \frac{E_t^*}{E_t} \left[\lambda + \xi \left(\frac{1}{s+2} - \frac{1}{s+1} - \frac{1}{s+3} \right) \delta_{ij} \right. \\
& + \frac{E_t^2}{(u_0 d)^2} \left\{ \left(\frac{E_\ell}{E_t} - 1 \right) (1 + \lambda) + \xi \frac{E_t^*}{E_t} \left[\frac{1}{1-s^2} [1-s \right. \right. \\
& \left. \left. + \frac{E_\ell^*}{E_t^*} (3-s)] - \frac{2s+5}{(s+2)(s+3)} \left(\frac{E_\ell^*}{E_t^*} - 1 \right) \right] (1-\lambda) \right\} u_i u_j \quad (43)
\end{aligned}$$

As long as the Rayleigh number is not very high, λ is very small and can be approximated by the first term of (40), leading to the following approximations

$$\beta = \beta_1 V^2 \quad (44)$$

$$\xi = \xi_1 V^2 \quad (45)$$

$$\epsilon_{ij} = \hat{\alpha}_1 V^2 \delta_{ij} + \hat{\alpha}_2 u_i u_j \quad (46)$$

where

$$\begin{aligned}
\beta_1 &= \frac{b}{2(1+b)} \left(\frac{E_t}{u_0 d} \right)^2 \\
\xi_1 &= \frac{E_t^2 s}{2(u_0 d)^2} \frac{(1+b)[\ln(1+b)] - b[\ln(\text{Re})]}{(1+b)[\ln(\text{Re})]\ln[(1+b)\text{Re}]} \\
\hat{\alpha}_1 &= \frac{E_t^* E_t}{2(u_0 d)^2} + \zeta_1 \left(\frac{1}{s+2} - \frac{1}{s+1} - \frac{1}{s+3} \right) \frac{E_t^*}{E_t} \quad (47)
\end{aligned}$$

$$\hat{\alpha}_2 = \frac{E_t^2}{(u_0 d)^2} \left(\frac{E_\ell}{E_t} - 1 \right)$$

If (32) and (33) are subject to the following homogeneous boundary conditions

$$\psi, \theta = 0 \text{ at } z = 0, 1 \quad (48)$$

then the system (32) and (33) can be solved by means of a set of truncated eigenfunctions expressing the finite amplitude disturbances.

Double Fourier series expansions may conveniently be applied for such purposes as follows

$$\psi = \sum_{p,q=1}^{\infty} \hat{\psi}_{p,q} \sin(p\alpha y) \sin(q\pi z) \quad (49)$$

$$\theta = \sum_{\substack{p=0 \\ q=1}}^{\infty} \hat{\theta}_{p,q} \cos(p\alpha y) \sin(q\pi z) \quad (50)$$

The calculation can be simplified by using the complex variable presentation of $\sin(p\alpha y)$ and $\cos(p\alpha y)$ leading to

$$\psi = -i \sum_{\substack{p=-\infty \\ q=1}}^{\infty} \psi_{p,q} e^{ip\alpha y} \sin(q\pi z) \quad (51)$$

$$\theta = \sum_{\substack{p=-\infty \\ q=1}}^{\infty} \theta_{p,q} e^{ip\alpha y} \sin(q\pi z) \quad (52)$$

provided that

$$\psi_{p,q} = -\psi_{-p,q} = \frac{1}{2} \hat{\psi}_{p,q} \quad (53)$$

$$\theta_{p,q} = \theta_{-p,q} = \frac{1}{2} \hat{\theta}_{p,q} \quad (54)$$

The solution of the system (32) and (33) through the Fourier series expansion means the determination of the Fourier series coefficients which can be done by power series expansion. Such a method was first used by Kuo [1961] for the analysis of the ordinary Bénard convection. Palm et al. [1972] and Rubin [1975] through different modifications applied this approach for the analysis of free convection with no dispersion and turbulence in porous media. Rubin and Christensen [1975] suggested some guidelines for the utilization of such an approach when analyzing instabilities induced by salinity gradients in a saturated porous layer. The advantage of this method lies in its simplicity.

According to (30) the inception of convection motion is of the marginal instability of exchange, namely, steady convection follows point of instability. Therefore, the first term in (33) vanishes when steady convection is attained. In such conditions the coefficients $\psi_{p,q}$ and $\theta_{p,q}$ of the Fourier series expansion can be expressed through a power series expansion as follows:

$$(\psi_{p,q}, \theta_{p,q}) = \sum_{n=1}^N (\psi_{p,q}^{(n)}, \theta_{p,q}^{(n)}) \eta^n \quad (55)$$

where η is a small parameter defined by

$$\eta = [(R-R_0)/R]^{0.5} \quad (56)$$

The Rayleigh number is also expanded by a finite power series as follows:

$$R = R_0 + R_{0s} \sum_{j=1}^S \eta^{2j} \quad (57)$$

where $R_{OS} = R_0 / (1 - \eta^{2S})$ $S = N/2$ (58)

Other flow field perturbations like $v, w, \beta, \xi,$ and ϵ_{ij} as well as perturbation spectra $H, B, D,$ and F can be also expanded in double Fourier and power series. Expressions for the coefficients $H_{p,q}^{(n)}, B_{p,q}^{(n)}, D_{p,q}^{(n)}$ and $F_{p,q}^{(n)}$ are presented in the Appendix.

If the analysis is conducted for $N = 1,$ it reduces to the linear stability analysis, yielding critical values of wave number and Rayleigh number as given in (30).

There is a positive relationship between increases in Rayleigh numbers and wave numbers. However, taking the assumption that under supercritical conditions the wave number remains constant does not significantly affect predictions of transport phenomena for quite a wide range of Rayleigh numbers [Straus, 1974]. Such an assumption is not required by the method used here but considerably simplifies the analysis.

Substituting the series expansions in (32) and (33) we obtain

$$R_0 p\pi \theta_{p,q}^{(n)} + R_{OS} p\pi \sum_{i=1}^{\infty} \theta_{p,q}^{(n-2i)} + \pi^2(p^2+q^2)\psi_{p,q}^{(n)} + B_{p,q}^{(n)} = 0 \quad (59)$$

$$\pi^2(p^2+q^2)\theta_{p,q}^{(n)} + p\pi \psi_{p,q}^{(n)} + H_{p,q}^{(n)} + D_{p,q}^{(n)} + F_{p,q}^{(n)} = 0 \quad (60)$$

The functions ψ and θ are generated by superpositions of trigonometric functions. For $n = 1$ the only nonvanishing coefficients are $\psi_{1,1}^{(1)}$. Therefore, only coefficients $\psi_{p,q}^{(n)}$ and $\theta_{p,q}^{(n)}$ with even values of $|p|+q$ have nonvanishing values. Moreover the values of the subscripts $|p|$ and q are smaller or equal to n . Coefficients with other subscripts vanish. According to (60).

$$\theta_{0,q}^{(n)} = - (H_{0,q}^{(n)} + D_{0,q}^{(n)} + F_{0,q}^{(n)}) / (\pi^2 q^2) \quad (61)$$

For $p \neq 0$, (59) and (60) yield

$$\begin{aligned} \psi_{p,q}^{(n)} \left[\frac{(p^2 + q^2)^2 - 4p^2}{4p\pi(p^2 + q^2)} \right] + \frac{R_{os}}{R_o} \sum_{i=1}^{\infty} \theta_{p,q}^{(n-2i)} + B_{p,q}^{(n)} / (\pi p R_o) \\ - (H_{p,q}^{(n)} + D_{p,q}^{(n)} + F_{p,q}^{(n)}) / [\pi^2 (p^2 + q^2)] = 0 \end{aligned} \quad (62)$$

For $p=q=1$

$$\theta_{1,1}^{(n)} + \sum_{i=1}^{\infty} \theta_{1,1}^{(n-2i)} + B_{1,1}^{(n+2)} / (\pi R_{os}) - 2(H_{1,1}^{(n+2)} + D_{1,1}^{(n+2)} + F_{1,1}^{(n+2)}) / R_{os} \quad (63)$$

Through (61), (62) and (63) all the coefficients $\psi_{p,q}^{(n)}$ and $\theta_{p,q}^{(n)}$ are determined. According to (63) the coefficients $\psi_{1,1}^{(n)}$, $\psi_{2,2}^{(n+1)}$ and $\theta_{0,2}^{(n+2)}$ must be determined simultaneously. However, by simple arrangements $\theta_{1,1}^{(n)}$ can be expressed explicitly. Such a procedure avoids any trial process.

Calculations of the mean horizontal temperature and the Nusselt number followed the determination of the series coefficients. These parameters are given by

$$\bar{T} = -z + \sum_{n=1}^N \sum_{q=1}^{\infty} \theta_{0,q}^{(n)} [\sin(q\pi z)] \eta^n \quad (64)$$

$$\begin{aligned} Nu = 1 - \sum_{n=1}^N \eta^n \left\{ \pi \sum_{q=1}^{\infty} q \theta_{0,q}^{(n)} + \hat{\alpha}_1 \pi \sum_{i=1}^{n-1} \sum_{p=-\infty}^{\infty} \sum_{q,s=1}^{\infty} sq \psi_{p,q}^{(i)} \psi_{-p,s}^{(n-i)} \right. \\ \left. - \hat{\alpha}_1 \pi^3 \sum_{i=2}^{n-1} \sum_{j=i}^{i-1} \sum_{p,k=-\infty}^{\infty} \sum_{q,\ell,s=1}^{\infty} q\ell s \psi_{p,q}^{(j)} \psi_{k,\ell}^{(i-j)} \theta_{|p+k|,s}^{(n-i)} \right\} \end{aligned} \quad (65)$$

Calculation of Nu determines the convergence of the method and the termination of the series expansion. Through the calculation, presented in the next section, we took $N = 6, 8, 10, 12, 14$ and 16 according to the variation in the Nusselt number. If Nu varied by less than 2% as N was increased from N to N + 2 then the expansion was terminated.

RESULTS AND DISCUSSION

According to (44)-(47) convection effects are not determined only by the Rayleigh number but also by the Reynolds (Re) and Prandtl (Pr) numbers, as well as by the ratio between the porous layer thickness and the characteristic pore size (d/d_p). The whole analysis approach is applicable only when mechanical dispersion is at least comparable with the molecular diffusion. This relationship is determined by the magnitude of Re and Pr. It seems that if $Pr \geq 1$, which is reasonable for practical purposes of hydrology [Somerton, 1958], then dispersion effects are significant even for $Re \approx 3$. However, if the Prandtl number is smaller than unity (if the solid fraction is a good conductor) then dispersion effects become significant at higher Reynolds numbers.

The effect of Re, Pr and d/d_p on the convection phenomenon is introduced in the analysis through β_1 , $\hat{\alpha}_1$ and $\hat{\alpha}_2$ (and $\hat{\alpha}_3 = \hat{\alpha}_1 + \hat{\alpha}_2$) which determine effects of turbulence and dispersion induced by the convection motion. Figure 2 demonstrates changes in these coefficients due to variations in Re, Pr and d/d_p when the porosity is 0.4. The effect of Pr vanishes for high Reynolds numbers as in such cases mechanical dispersion affects transport processes more than the molecular diffusion. When the Reynolds number increases, according to model (9) and (10), the mechanical dispersion tensor becomes more and more isotropic. This phenomenon leads to a reduction in the value of $\hat{\alpha}_2$. Turbulent effects become more and more significant when the Reynolds number increases, leading to a positive relationship between increases in the Reynolds number and the coefficient β_1 .

According to (63) turbulent effects introduced through the friction spectrum B amplify dispersion effects. However as $\hat{\alpha}_2$ decreases when Re increases, it was found through the calculation that the net effect of an increase in Re led to a minor reduction in the influence of the spectra B, D and F.

According to Figure 2 the main parameter that determines the effect of the spectra B, D and F on the convection phenomenon is d/d_p . The coefficients $\hat{\alpha}_1$, $\hat{\alpha}_2$ and β_1 are almost inversely proportional to the square of this parameter. It should be mentioned that the absolute value of the coefficient ξ_1 is usually much smaller than $\hat{\alpha}_1$. Hence changes in s due to the convection motion are almost negligible. We may usually neglect terms depending on this coefficient in expressions (47).

As an example we present in Figures 3 and 4 descriptions of mean horizontal temperature and Nusselt numbers for an aquifer when $\phi = 0.4$, $Re = 3$ and $Pr = 1$.

Figure 3 demonstrates profiles of mean horizontal temperature for various Rayleigh numbers and values of d/d_p . As explained before, a decrease in d/d_p increases the effects of dispersion and turbulence induced by the convection motion. The convection motion increases the value of the effective dispersion coefficient. However, this effect leads to a reduction in values of temperature gradients at top and bottom of the porous layer as demonstrated in Figure 3.

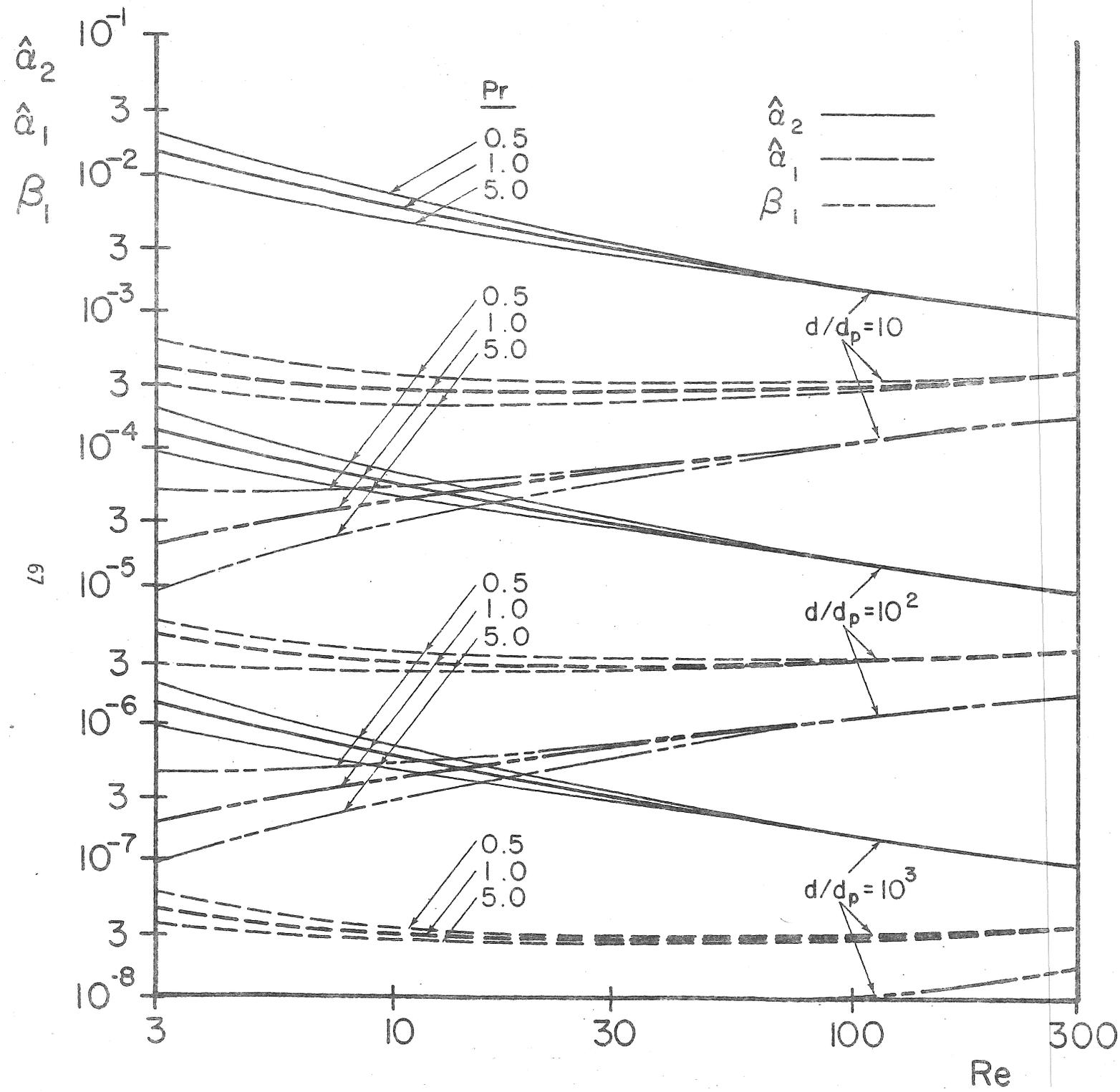


Figure 2. Description of $\beta_1, \hat{\alpha}_1$ and $\hat{\alpha}_2$ vs. Re for various values of Pr and d/d_p ($\phi=0.4$).

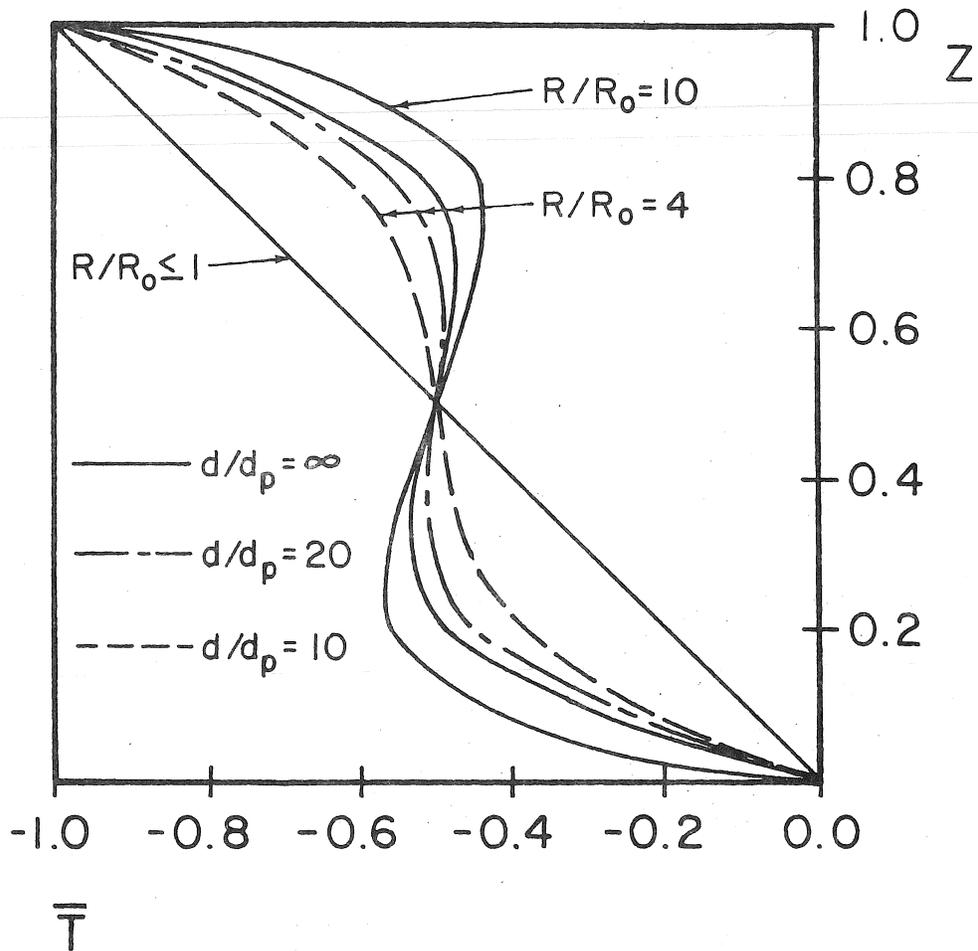


Figure 3. Mean horizontal temperature profiles for various values of R/R_0 and d/d_p ($\phi=0.4$, $Pr=1$, $Re=3$).

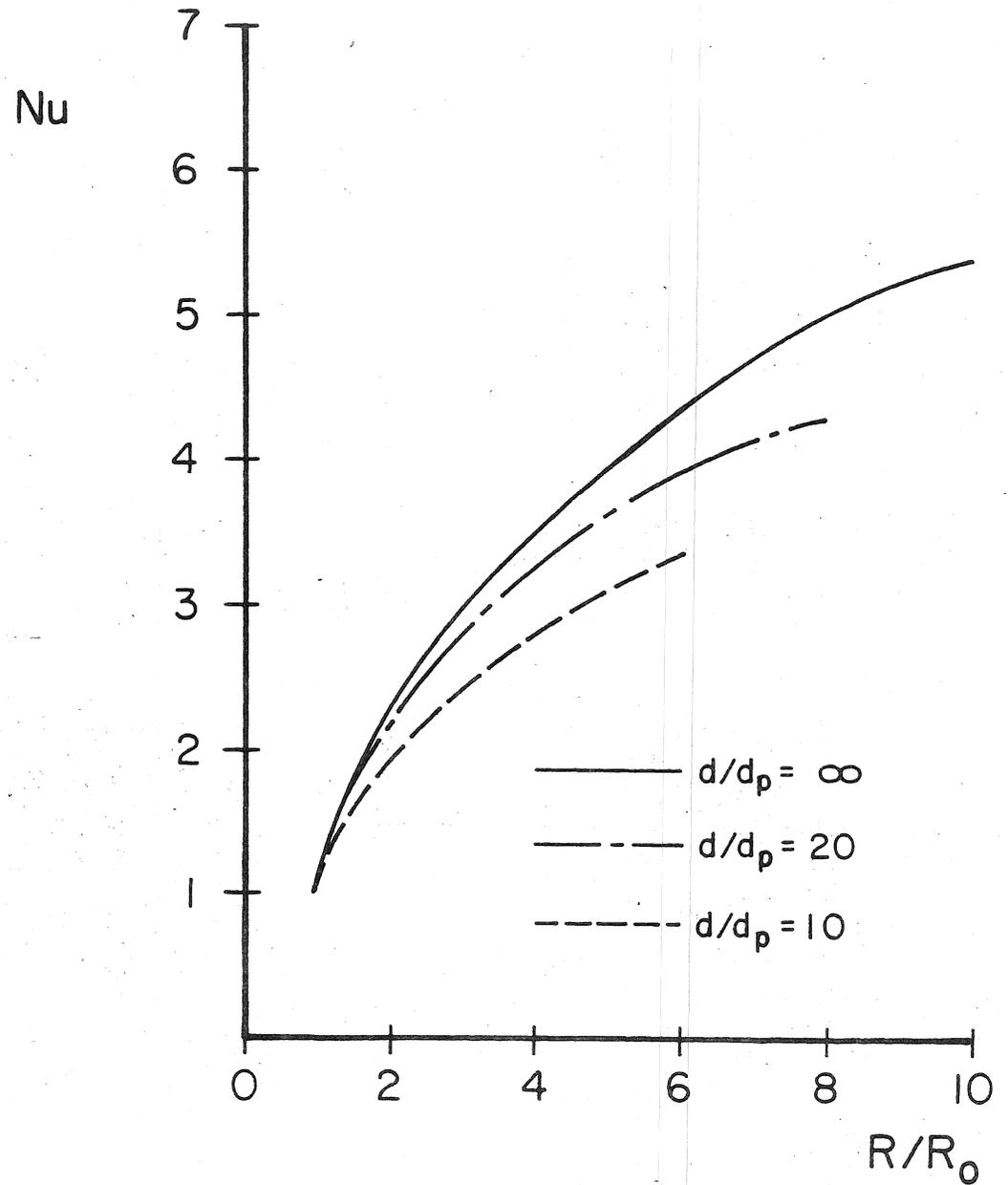
At high Rayleigh numbers a thick region in the center of the porous layer should achieve a nearly isothermal state in the mean. However, as presented in Figure 3 such conditions lead to a positive temperature gradient or reversal of temperatures. Such a phenomenon was also identified in the ordinary Bénard problem [Kuo, 1961; Veronis, 1966]. Veronis [1966] tried to explain the origin of such a strange phenomenon. However, better choice of wave numbers diminishes this effect. Figure 3 also demonstrates the creation of boundary layers on top and bottom of the aquifer leading to invalidity of the continuum approach even for moderate Rayleigh numbers if d/d_p is not very large.

Figure 4 presents variations of Nusselt number with Rayleigh number for various values of d/d_p . According to this presentation, the net effect of the mechanical dispersion induced by the convection motion leads to a reduction in the heat transport through the aquifer. Dispersion and turbulence induced by the convection motion act as a stabilizing mechanism in the flow field (ironical interpretation).

However, this mechanism is more complicated than just a stabilization. In the calculation it was found that the phenomenon was associated with increased values of the higher modes of the series expansion, leading to a reduction in the convergence of the series expansions.

Through the calculation, the maximal values of λ and V were continuously checked in order to examine the validity of approximations (44)-(47) and to follow changes in the Reynolds and the Peclet numbers. It

Figure 4. Description of Nu vs. R/R_0 for various values of d/d_p ($\phi=0.4$, $Pr=1$, $Re=3$).



was found that for the range $R/R_0 \leq 10$, λ_{\max} was much smaller than unity which justifies utilization of (44)-(47) and yields only minor changes in the Reynolds number.

CONCLUSIONS

The anisotropic character of the dispersion tensor leads to convection motions conducted in two dimensional rolls whose axes are parallel to the unperturbed velocity vector. By choosing a new definition for the Rayleigh number, turbulence and dispersion effects can be introduced in the linear stability analysis with no substantial complications in the calculations and results.

Finite amplitude analysis for homogeneous boundary conditions can be conducted by Fourier series and power series expansions.

The significance of mechanical dispersion and turbulence induced by the convection motion depends on Pr , Re and mainly on d/d_p . An increase in Re diminishes the effect of Pr and reduces effects of dispersion and turbulence associated with the convection motion. An increase in d/d_p reduces significantly effects of dispersion and turbulence due to the convection motion. For $d/d_p \geq 10^2$ these effects practically vanish. However, for smaller values of d/d_p these effects lead to a reduction in Nu .

The singly diffusive analysis presented in this article can be applied when density gradients are induced by a single component or when boundary conditions and effective dispersion coefficients are identical for all components in a multicomponent system. The latter condition is approximately satisfied when mechanical dispersion effects are larger than any molecular diffusion in the aquifer.

APPENDIX

Expressions for coefficients of heat advection, friction and heat dispersion spectra

$$H_{|p|,q}^{(n)} = \frac{\pi^2}{2} \frac{n-1}{i \equiv 1} \sum_{k=-\infty}^{\infty} \sum_{\ell=1}^{\infty} \Psi_{k,\ell}^{(i)} \{ \ell(p-k) [\theta_{|p-k|,s_v}^{(n-i)} (v=1,2;3)] + k [s_v \theta_{|p-k|,s_v}^{(n-i)} (v=2;1,3)] \} \quad (66)$$

$$D_{|p|,q}^{(n)} = \frac{\pi^3}{2} \frac{n-1}{i \equiv 1} \sum_{k=-\infty}^{\infty} \sum_{\ell=1}^{\infty} \ell \Psi_{k,\ell}^{(i)} \{ [(\hat{\alpha}_1 + \hat{\alpha}_3)k(p-k) - \hat{\alpha}_2(p-k)^2 + 2\hat{\alpha}_1 s_v^2] [\psi_{p-k,s_v}^{(n-i)} (v=1,2;3)] \} \quad (67)$$

where

$$s_1 = q - \ell \quad s_2 = q + \ell \quad s_3 = \ell - q \quad (68)$$

$$\theta_{|p-k|,s_v}^{(n-i)} (v=2;1,3) = \theta_{|p-k|,s_2}^{-\theta} |p-k|, s_1^{-\theta} |p-k|, s_3 \quad (69)$$

$$B_{p,q}^{(n)} = \beta_1 \frac{\pi}{4} \frac{n-1}{i \equiv 2} \sum_{j=1}^{i-1} \sum_{k,m=-\infty}^{\infty} \sum_{\ell,h=1}^{\infty} \Psi_{k,\ell}^{(j)} \Psi_{m,h}^{(i-j)} \{ \ell h [4k(p-k-m) + (p-k-m)^2 + 3s_v^2] [\psi_{p-k-m,s_v}^{(n-i)} (v=1,2,3,4;5,6,7)] + [3k^2 h s_v - \ell^2 m(p-k-m)] [\psi_{p-k-m,s_v}^{(n-i)} (v=3,4,5;1,2,6,7)] \} \quad (70)$$

$$F_{|p|,q}^{(n)} = \frac{\pi}{16} \frac{n-1}{i \equiv 2} \sum_{j=1}^{i-1} \sum_{k,m=-\infty}^{\infty} \sum_{\ell,h=1}^{\infty} \Psi_{k,\ell}^{(j)} \Psi_{m,h}^{(i-j)} \{ \ell h [(\hat{\alpha}_1 + \hat{\alpha}_3)k(p-k-m) + \hat{\alpha}_3(p-k-m)^2 + \hat{\alpha}_1 s_v^2] [\theta_{|p-k-m|,s_v}^{(n-i)} (v=5,6,7;1,2,3,4)] + m [2\hat{\alpha}_1 k^2 - \hat{\alpha}_2 \ell^2] (p-k-m) + \hat{\alpha}_1 k(p-k-m)^2 + \hat{\alpha}_3 k s_v^2 [\theta_{|p-k-m|,s_v}^{(n-i)} (v=3,4,5;1,2,6,7)] + h [\hat{\alpha}_2 k^2 + 2\hat{\alpha}_2 k(p-k-m) - 2\hat{\alpha}_1 \ell^2 - (\hat{\alpha}_1 + \hat{\alpha}_3)mk] [s_v \theta_{|p-k-m|,s_v}^{(n-i)} (v=1,3,5,7;2,4,6)] \} \quad (71)$$

In (70) and (71) the subscript s_v may obtain seven different values as follows:

$$\begin{aligned} s_1 &= q - \ell - h & s_2 &= q + \ell + h & s_3 &= q - \ell + h & s_4 &= q + \ell - h & s_5 &= \ell + h - q \\ s_6 &= h - \ell - q & s_7 &= \ell - h - q \end{aligned} \quad (72)$$

NOTATION

a	wave number
a_x, a_y	components of wave number
a_0	critical wave number
b	friction function
B	friction spectrum
$B_{p,q}, B_{p,q}^{(n)}$	coefficients in series expansions for B
c_1, c_2	coefficients defined in (26)
C	specific heat of fluid
C_s	specific heat of solid
d	porous layer thickness
d_p	characteristic pore size
D	part of heat dispersion spectrum
$D_{p,q}, D_{p,q}^{(n)}$	coefficients in series expansions for D
E_{ij}^*, E_{ij}	heat dispersion tensors (mechanical and hydrodynamical respectively)
E_ℓ^*, E_ℓ	longitudinal heat dispersion coefficients
E_t^*, E_t	transversal heat dispersion coefficients
F	part of heat dispersion spectrum
$F_{p,q}, F_{p,q}^{(n)}$	coefficients in series expansions for F
g	gravity acceleration
H	heat advection spectrum
$H_{p,q}, H_{p,q}^{(n)}$	coefficients in series expansions for H
K	permeability
ℓ_i	unit vector in the longitudinal direction
n_i	unit vector in the vertical direction

N	total number of terms in the series expansion
Nu	Nusselt number
p	pressure
p_0	pressure at the coordinates origin
Pr	Prandtl number (ν/κ)
R	Rayleigh number defined in (14)
R_0	critical Rayleigh number
R_{0s}	parameter defined in (58)
Re	Reynolds number ($\phi U d_p/\nu$)
s	power coefficient
S	= N/2
t	time
T	temperature
T_0	temperature at $z = 0$
u	longitudinal velocity perturbation
u_i	velocity vector
u_0	unperturbed velocity
U	module of velocity vector
v	lateral velocity perturbation
V	module of velocity perturbation
w	vertical velocity perturbation
x_i	coordinate
x,y,z	coordinate system
α	coefficient of volumetric thermal expansion
$\hat{\alpha}_1, \hat{\alpha}_2, \hat{\alpha}_3$	coefficients defined in (47), ($\alpha_3 = \alpha_1 + \alpha_2$).
β	perturbation in the friction function

β_1	coefficient defined in (47)
γ	parameter defined in (5)
δ_{ij}	Kronecker's delta
ΔT	difference in temperature between bottom and top of the porous layer
ϵ_{ij}	dispersion tensor perturbation
η	small parameter defined in (56)
θ	temperature perturbation
θ_1	constant defined in (27)
$\hat{\theta}_{p,q}^{\theta}, \hat{\theta}_{p,q}^{\theta}, \hat{\theta}_{p,q}^{\theta(n)}$	coefficients in series expansions for θ
κ	thermal diffusivity of saturated porous medium
λ	parameter defined in (40)
μ	viscosity
ν	kinematic viscosity
ξ	perturbation in s
ξ_1	coefficient defined in (47)
ρ	fluid density
ρ_0	density at $z = 0$
ρ_s	solid density
σ_r	parameter expressing amplification of small disturbances
ϕ	porosity
χ	parameter defined in (29)
ψ	stream function
$\Psi_{p,q}^{\psi}, \Psi_{p,q}^{\psi}, \Psi_{p,q}^{\psi(n)}$	coefficients in series expansions for ψ
ω	parameter expressing oscillations

Ω scalar function defined in (22)

Ω_1 constant defined in (27)

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CHAPTER 5

SEMINUMERICAL APPROACH FOR THE MATHEMATICAL MODELING OF SINGLY DISPERSIVE CONVECTION IN GROUNDWATERS

INTRODUCTION

Considerable interest is now focused in the State of Florida, as well as in other locations in the United States, for the possible utilization of deep saline aquifers for waste disposal. Vernon (16) delineates the properties of the deep zones of the Floridan aquifer which makes this stratum available for such application. Henry and Kohout (2) mention the fact that in a thick system, like the Floridan aquifer, the effect of geothermal activity should be considered, too. One of these authors has postulated in previous articles (3,4) that geothermal activity induces groundwater circulation in the Floridan aquifer. Singly diffusive convection is the convection motion induced by a single dissolved component (e.g. temperature or salinity) in a fluid layer. The hydrodynamics of this phenomenon in a saturated porous media has been studied while, in most instances, assuming that the fluid is initially at rest (7,8). However, groundwaters are generally subject to hydraulic gradients leading to slow, effectively horizontal flow of the subsurface water. This movement through a formation, similar to the Boulder Zone in the deep saline region of the Floridan aquifer (1,4), leads to intensive mechanical dispersion of heat and soluted materials in the aquifer, as well as inertial effects, as demonstrated by invalidity of the laminar Darcy law. In such a system molecular diffusion effects are usually less significant than mechanical dispersion. Convection phenomenon under such conditions may therefore, be called dispersive convection. A system is subject to singly dispersive convection if one of the following criteria is satisfied: a) density gradients are

introduced by a single component; or b) all molecular diffusivities of the dissolved components are much smaller than the mechanical dispersion, and all of them have the same boundary conditions. The objective of this article is to present a rather simple method by which flow conditions in such an aquifer can be simulated.

BASIC EQUATIONS

The basic equations applied for the analysis are the equations of continuity, motion, and dispersion, subject to the Boussinesq approximation,

$$\nabla \cdot \vec{u} = 0 \quad \dots \dots \dots (1)$$

$$\nabla p + \rho g \vec{h} + \frac{\mu \phi}{K} (1+b) \vec{u} = 0 \quad \dots \dots \dots (2)$$

$$\gamma \frac{\partial T}{\partial t} + \vec{u} \cdot \nabla T = \nabla \cdot (\vec{E} \cdot \nabla T) \quad \dots \dots \dots (3)$$

The coefficient γ appearing in Equation 3 is defined by:

$$\gamma = [\rho C_W \phi + \rho_S C_S (1-\phi)] / \rho C_W \phi \quad \dots \dots \dots (4)$$

In a brief literature survey, presented in a previous study (10), it was suggested that the friction function, b , appearing in Equation 2 can be approximated by:

$$b = 0.014 \text{ Re} \quad \dots \dots \dots (5)$$

It is assumed that the fluid density depends linearly on the dissolved component, which is the temperature, as follows:

$$\rho = \rho_0 [1 - \alpha (T - T_0)] \quad \dots \dots \dots (6)$$

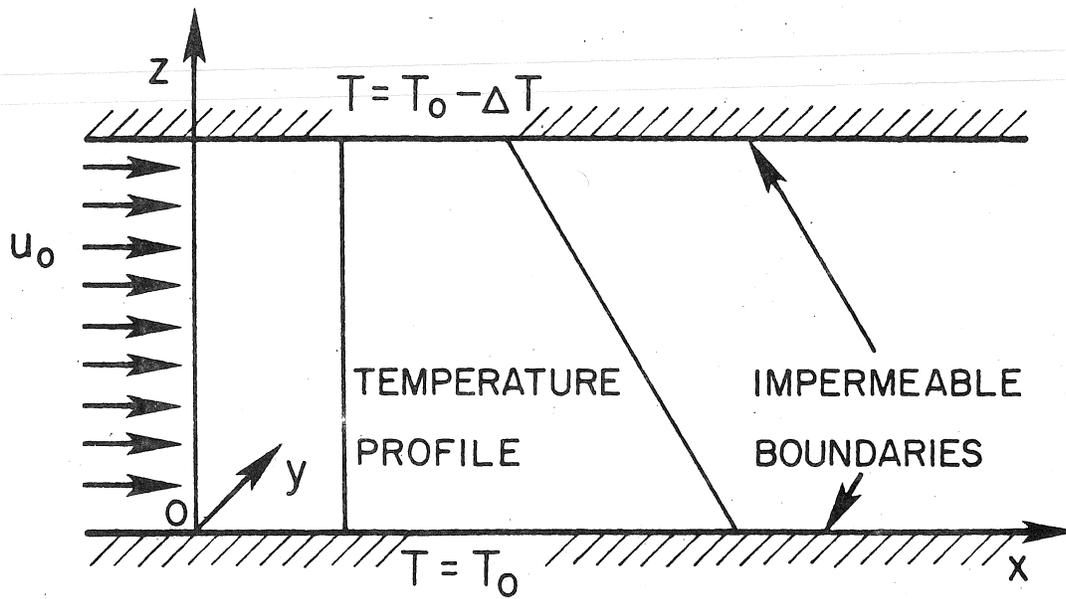


Figure 1. Schematical description of the aquifer with no convection motion.

The flow field model considered prior to the inception of the convection motion, as presented in Figure 1, is a cavernous aquifer consisting of a porous layer of infinite horizontal extent. It is bounded by two impermeable planes on which the temperature is constant. Through the porous layer the fluid flows uniformly in the longitudinal x direction. The transversal and vertical coordinates are y, and z, respectively.

A moving coordinate system is applied with the velocity u_0/γ in the x direction, and consider

$$d, \Delta T, e_t, e_t/d, d^2/e_t, \mu\phi e_t(1+b)/K \dots \dots \dots (7)$$

as characteristic length, temperature, dispersion coefficient, velocity, time, and pressure, respectively.

In such a manner the following dimensionless basic equations are obtained (in further expressions all variables are dimensionless):

$$\nabla \cdot \vec{u} = 0 \dots \dots \dots (8)$$

$$\nabla p - RT\vec{n} + (1+\beta)\vec{u} = 0 \dots \dots \dots (9)$$

$$\gamma \frac{\partial T}{\partial t} + \vec{u} \cdot \nabla T = \nabla \cdot (\vec{E} \cdot \nabla T) \dots \dots \dots (10)$$

Here the Rayleigh number, R, and the variable β , are defined as follows:

$$R = \frac{\alpha g \Delta T K d}{\nu \phi e_t (1+b)} \quad \beta = b[(U/u_0) - 1]/(1+b) \dots \dots \dots (11)$$

In an isotropic medium the dimensionless dispersion tensor can be expressed by:

$$\vec{E} = E_t \vec{I} + (E - E_t) \vec{u} \vec{u} / U^2 \dots \dots \dots (12)$$

As long as there is no convection motion Equations 8-10 yield:

$$\begin{aligned} \vec{u} = 0 \quad \beta = 0 \quad T = -z \quad p = p_0 - Rz^2/2 \\ E_{xx} = E_l \quad E_{yy} = E_{zz} = 1 \quad E_{xy} = E_{xz} = E_{yz} = 0 \dots \dots \dots (13) \end{aligned}$$

THE FLOW FIELD STABILITY

The stability of flow conditions presented by Equations 13 can be determined by analyzing the growth of small disturbances in the aquifer. These are disturbances in the velocity, \vec{u} , temperature, θ , dispersion tensor, $\bar{\bar{E}}$ and pressure, p .

Second order terms depending on these disturbances are negligible.

It is convenient to express the velocity vector through the scalar variable Ω as follows:

$$\vec{u} = \vec{\delta}\Omega \quad \text{where} \quad \vec{\delta} = \vec{n} \times \nabla \times \nabla \dots \dots \dots (14)$$

By substituting the small disturbances in the basic equations, and applying the boundary conditions

$$\theta, \nabla_1^2 \Omega = 0 \text{ at } z = 0, 1. \dots \dots \dots (15)$$

(where $\nabla_1^2 = -\vec{n} \cdot \vec{\delta}$)

we obtain:

$$\begin{aligned} \gamma \frac{\partial}{\partial t} \nabla^2 \Omega - R \nabla_1^2 \Omega &= \nabla^2 \nabla_d^2 \Omega - R[(E_d - 1)/u_0][\nabla(\nabla_1^2 \Omega)] \cdot \vec{x} \\ &+ R(\partial E_t / \partial u_0)[\nabla(\vec{\delta}\Omega \cdot \vec{x})] \cdot \vec{n} \dots \dots \dots (16) \end{aligned}$$

where

$$\nabla^2 = \nabla \cdot \nabla \quad \nabla_d^2 = (E_d - 1)\vec{x}\vec{x} : \nabla \nabla + \nabla^2 \dots \dots \dots (17)$$

By expanding Ω in the following normal mode,

$$\Omega = a_1 [\sin(\pi z)] \exp[i(a_x x + a_y y) + (\sigma_1 + i\sigma_2)t] \dots \dots \dots (18)$$

we obtain for the point of instability ($\sigma_1 = 0$, and minimum value of R)

$$a_x = 0 \quad a_y = a \quad \sigma_2 = 0 \quad a_0 = \pi \quad R_0 = 4\pi^2 \dots \dots \dots (19)$$

Hence the instability is of the marginal of exchange type. Convection cells are two dimensional rolls whose axes are parallel to the unperturbed velocity vector. Superposition of the unperturbed motion and the convection motion yields a helical flow field.

ANALYSIS OF THE STEADY STATE CONVECTION

In the previous section it was proved that convection cells are two dimensional rolls. It is convenient to express the velocity by the stream function

$$\vec{u} = -\vec{\ell} \times \nabla\psi \quad \psi = -[\vec{\ell}, \vec{n}, \nabla\Omega] \dots \dots \dots (20)$$

where the square brackets symbolize the box product.

Substituting Equation 20 in the basic equations, and eliminating the pressure perturbation, we obtain:

$$R[\vec{n}, \vec{\ell}, \nabla T] + \nabla^2\psi = B(\psi, \beta) \dots \dots \dots (21)$$

$$-\gamma \frac{\partial \theta}{\partial t} + \nabla^2\theta - [\vec{n}, \ell, \nabla\psi] = H(\psi, \theta) + D(\bar{\varepsilon} \cdot \vec{n}) + F(\bar{\varepsilon}, \theta) \dots \dots \dots (22)$$

where $\bar{\varepsilon}$ is the finite amplitude disturbance in the dispersion tensor.

Variables B, H, D and F are nonlinear terms defined as follows:

$$B(\psi, \beta) = -\nabla \cdot (\beta \nabla \psi) \dots \dots \dots (23)$$

$$H(\psi, \theta) = -[\vec{\ell}, \nabla\psi, \nabla\theta] \dots \dots \dots (24)$$

$$D(\bar{\varepsilon} \cdot \vec{n}) = \nabla \cdot (\bar{\varepsilon} \cdot \vec{n}) \dots \dots \dots (25)$$

$$F(\bar{\varepsilon}, \theta) = -\nabla \cdot (\bar{\varepsilon} \cdot \nabla\theta) \dots \dots \dots (26)$$

As long as convection velocity is smaller than the unperturbed velocity the absolute value of the flow field velocity can be approximated by:

$$U = u_0(1 + \lambda - 1/2\lambda^2 + 1/2\lambda^3 - \dots) \dots \dots \dots (27)$$

where

$$\lambda = v^2/(2u_0^2) \quad v^2 = \vec{u} \cdot \vec{u} \dots \dots \dots (28)$$

If $\lambda \ll 1$ terms depending on high orders of λ can be neglected. In such conditions, the expressions for β and the dispersion tensor can be approximated by:

$$\beta = \beta_1 v^2 \dots \dots \dots (29)$$

$$\bar{\varepsilon} = \alpha_1 v^2 \bar{I} + \alpha_2 \vec{u}\vec{u} + \alpha_3 (\vec{u}\vec{\ell} + \vec{\ell}\vec{u}) + \alpha_4 v^2 \vec{\ell}\vec{\ell} \dots \dots \dots (30)$$

where

$$\beta_1 = b/[2(1+b)u_0^2] \dots \dots \dots (31)$$

$$\alpha_1 = (1/2u_0)(\partial E_t / \partial u_0) \quad \alpha_2 = (E_\ell - 1)/u_0^2$$

$$\alpha_3 = (E_\ell - 1)/u_0 \quad \alpha_4 = (1 - E_\ell)/u_0^2 + (1/2u_0)[\partial(E_\ell - E_t)/\partial u_0] \dots \dots \dots (32)$$

The system of the differential equations 21 and 22 is subject to the following boundary conditions:

$$\psi, \theta = 0 \text{ at } z = 0, 1 \dots \dots \dots (33)$$

Such boundary conditions are simple according to the definitions presented by Orszag (9). In such cases, accurate simulation of incompressible flows can be obtained by spectral methods.

Assuming that ψ and θ are periodic in the horizontal direction, these variables can be presented by sets of truncated eigenfunctions as suggested by Veronis in similar studies (17,18).

$$\psi = \sum_{p,q=1}^{\infty} \hat{\psi}_{p,q} \sin(p\alpha y) \sin(q\pi z) \dots \dots \dots (34)$$

$$\theta = \sum_{\substack{p=0 \\ q=1}}^{\infty} \hat{\theta}_{p,q} \cos(p\alpha y) \sin(q\pi z) \dots \dots \dots (35)$$

The calculation can be simplified by using the complex variable presentation of $\sin(p\alpha y)$ and $\cos(p\alpha y)$ leading to

$$\psi = -i \sum_{\substack{p=-\infty \\ q=1}}^{\infty} \psi_{p,q} [\sin(q\pi z)] \exp(ip\alpha y) \dots \dots \dots (36)$$

$$\theta = \sum_{\substack{p=-\infty \\ q=1}}^{\infty} \theta_{p,q} [\sin(q\pi z)] \exp(ip\alpha y) \dots \dots \dots (37)$$

provided that

The convergence of the expression for the Nusselt number also determines the truncation of the Fourier series expansion (the value of N).

NUMERICAL CALCULATIONS

Experiments concerning heat transfer characteristics of porous rocks (e.g. 5, 6) showed that the phenomenon of heat dispersion due to the fluid movement in the stratum is very similar in nature to the characteristics of solute dispersion in porous media. We may adopt, then, models of mechanical dispersion, which are available in the literature, for the quantitative evaluation of the effect of the convection phenomenon on the intensity of transport processes in the aquifer.

Saffman (11) suggested the following expressions for the coefficients of dispersion.

$$\frac{e_t}{v} = \frac{1}{Pr} + \frac{s+2}{2(s+1)(s+3)} \left(\frac{Re}{\phi}\right) \dots \dots \dots (42)$$

$$\frac{e_l}{v} = \frac{1}{Pr} + \frac{(s+1)^2}{2(1-s)(s+2)(s+3)} \left(\frac{Re}{\phi}\right) \dots \dots \dots (43)$$

where s is a power coefficient describing the dependence between the flow velocity and the pressure gradient. In a previous article (10) it was suggested that this coefficient can be calculated through the following expression

$$s = \ln(Re) / \{\ln[(1+b)Re]\} \dots \dots \dots (44)$$

Equation 44 can be applied for $Re \geq 2.77$. Through the calculation, it was assumed that $Pr = 1$, which seems to be a reasonable value for limestone and dolomite aquifers (13).

The convergence of the numerical integration was very moderate. Several approaches were applied to speed convergence of the calculations. These were:

- (a) Each calculation was conducted in two steps, in the first step we took

$d/d_p = \infty$ (which leads to $B=D=F=0$), and, after obtaining results for such conditions the actual value of d/d_p was introduced into the program; (b) Results obtained with lower values of N and/or R were used as initial quantities for higher values of these parameters. As a criterion for steady state, Straus (14,15) suggested, in such calculations, to take

$$\{ |(d\theta_{p,q}/dt)/\theta_{p,q}| \}_{\max} < 10^{-4} \dots \dots \dots (45)$$

In our calculation, it was found that such a criterion is very conservative. Usually, when applying a criterion of one hundred times less conservative, variations of less than 1% were obtained in the Nusselt number. Through the convergence process, decaying oscillations were detected in the value of the Nusselt number.

Figure 2 presents variations of the Nusselt number with Rayleigh number for various values of d/d_p . These are the maximal values of Nu obtained when varying the value of the wave number (12). The wave number increased with increasing values of R/R_0 .

Variations in the wave number induced minor changes in Nu for a constant Rayleigh number.

According to Figure 2, mechanical dispersion, due to the convection motion, leads to a reduction in the intensity of transport processes through the aquifer. The calculation indicated that in small values of d/d_p the magnitude of the dispersion coefficients increases, when convection occurs, leading to a reduction in the magnitude of the temperature gradients on top and bottom of the aquifer. This reduction in the temperature gradients affects transport processes more than the increase in the value of the dispersion coefficients. It was also found that lower values of d/d_p give rise to the higher modes of the Fourier series expansions and reduce the convergence of the calculation. For very

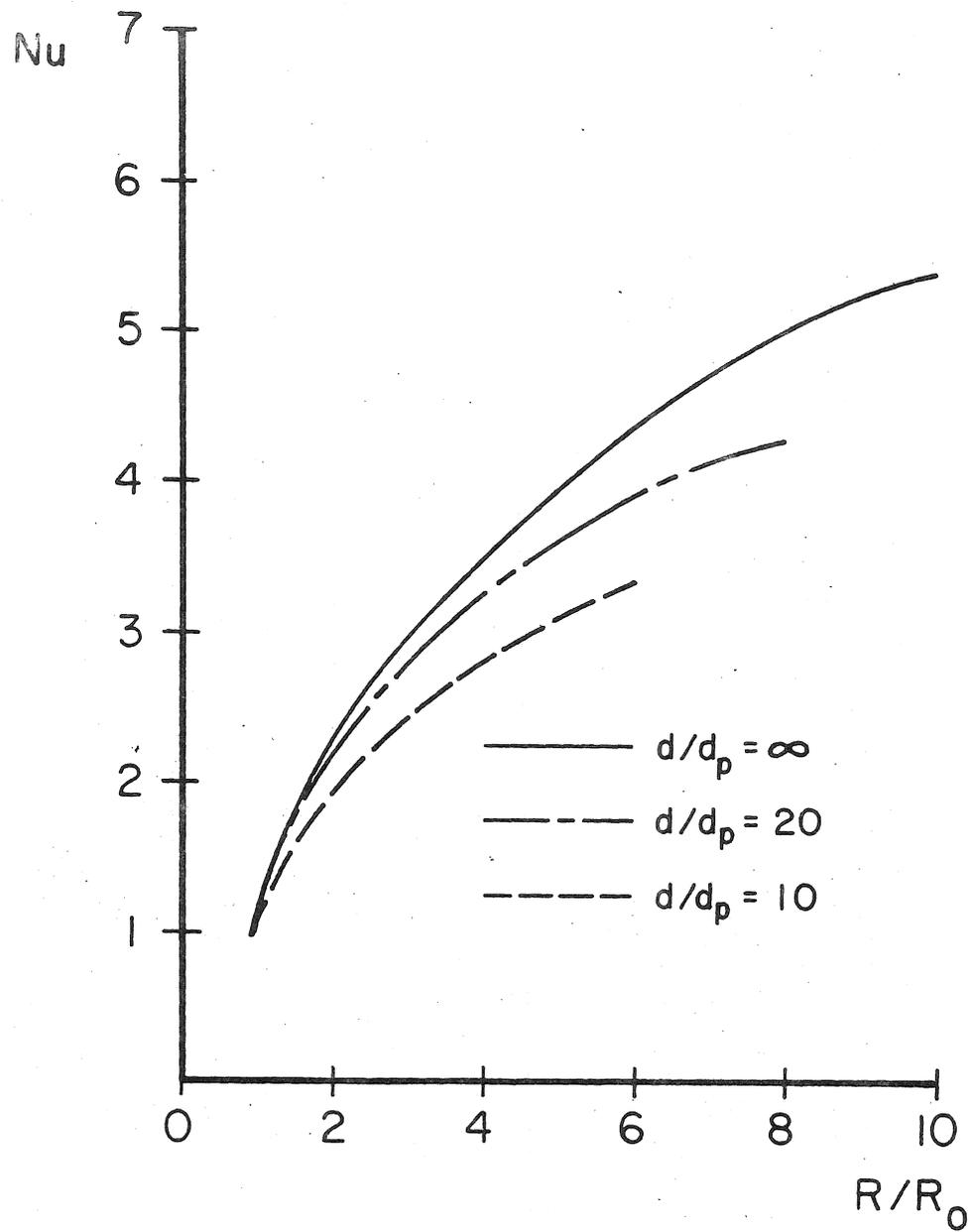


Figure 2. Description of Nusselt number vs. R/R_0 for various values of d/d_p ($\phi=0.4$; $Pr=1$; $Re=3$).

small values of d/d_p the analysis fails to follow the physical phenomenon, even at moderate Rayleigh numbers, as the thickness of the boundary layers developed on top and bottom of the aquifer is of the same order of magnitude as the characteristic pore size. In such cases the continuum approach applied through the analysis is not satisfied. Through the calculation, the maximal value of λ was continuously checked in order to examine the validity of the approximations presented in Equations 27-32. It was found that for $R/R_0 \leq 10$, the maximal value of λ was much smaller than unity.

CONCLUSIONS

Singly dispersive convection in aquifers can be analyzed by expanding the flow field disturbances in eigenfunctions.

The anisotropic character of the mechanical dispersion determines the plane in which convection motions are conducted. According to the analysis, convection cells are two dimensional rolls whose axes are parallel to the unperturbed velocity vector.

Analysis of steady state convection can be conducted by transforming the equations of motion and continuity to a set of general first order differential equations that can be integrated through available subroutines.

The effect of mechanical dispersion and turbulence, induced by the convection motion, depends mainly on the ratio between the porous layer thickness and the characteristic pore size; Prandtl and Reynolds numbers have less significant effects on the physical phenomenon.

The analysis of the singly dispersive convection presented in this study can be applied when density gradients are induced by a single component, or when boundary conditions and effective dispersion coefficients are identical for all components in a multicomponent system.

APPENDIX I - EXPRESSIONS FOR THE COEFFICIENTS OF THE SPECTRAL FUNCTIONS

$$H_{p,q} = H_{-p,q} = (\pi a/2) \sum_{k=1-N}^{N-1} \sum_{s=1}^{N-|k|} \Psi_{k,s} \{s(p-k)[\theta_{|p-k|}, w_v^{(v=1,2;3)}] + k[w_v^{\theta_{|p-k|}}, w_v^{(v=2;1,3)}]\} \dots \dots \dots (46)$$

$$D_{p,q} = D_{-p,q} = (\pi a^2/2) \sum_{k=1-N}^{N-1} \sum_{s=1}^{N-|k|} s \Psi_{k,s} \{[(\alpha_1 + \alpha_3)k(p-k) + 2\alpha_1(\pi^2/a^2)w_v^2 - \alpha_2(p-k)^2][\Psi_{p-k,w_v}^{(v=1,2;3)}]\} \dots \dots \dots (47)$$

where

$$w_1 = q-s \quad w_2 = q+s \quad w_3 = s-q$$

$$\theta_{|p-k|}, w_v^{(v=2;1,3)} = \theta_{|p-k|}, w_1^{-\theta_{|p-k|}}, w_2^{-\theta_{|p-k|}}, w_3 \dots \dots \dots (48)$$

$$B_{p,q} = -B_{-p,q} = (\pi^2 a^2/4) \sum_{k,m=1-N}^{N-1} \sum_{s=1}^{N-|k|} \sum_{h=1}^{N-|m|} \Psi_{k,s} \Psi_{m,h} \{sh[4k(p-k-m) + (p-k-m)^2 + 3(\pi^2/a^2)w_v^2][\Psi_{p-k-m,w_v}^{(v=1,2,3,4;5,6,7)}] + [3(a^2/\pi^2)k^2 h w_v - s^2 m(p-k-m)][\Psi_{p-k-m,w_v}^{(v=3,4,5;1,2,6,7)}]\} \dots \dots \dots (49)$$

$$F_{p,q} = F_{-p,q} = (\pi^2 a^2/4) \sum_{k,m=1-N}^{N-1} \sum_{s=1}^{N-|k|} \sum_{h=1}^{N-|m|} \Psi_{k,s} \Psi_{m,h} \{sh[(\alpha_1 + \alpha_3)k(p-k-m) + \alpha_3(p-k-m)^2 + \alpha_1(\pi^2/a^2)w_v^2][\theta_{|p-k-m|}, w_v^{(v=5,6,7;1,2,3,4)}] + m[(p-k-m)(2\alpha_1 k^2 a^2/\pi^2 - \alpha_2 s^2) + \alpha_1 k(a^2/\pi^2)(p-k-m)^2 + \alpha_3 k w_v^2][\theta_{|p-k-m|}, w_v^{(v=3,4,5;1,2,6,7)}] + h[\alpha_2 k^2 + 2\alpha_2 k(p-k-m) - 2\alpha_1 s^2(\pi^2/a^2) - (\alpha_1 + \alpha_3)mk][w_v^{\theta_{|p-k-m|}}, w_v^{(v=1,2,3,4;5,6)}]\} \dots \dots \dots (50)$$

where

$$w_1 = q-s-h \quad w_2 = q+s+h \quad w_3 = q-s+h \quad w_4 = q+s-h \\ w_5 = s+h-q \quad w_6 = h-s-q \quad w_7 = s-h-q \dots \dots \dots (51)$$

NOTATION

- a = wave number;
- a_0 = critical wave number;
- a_x, a_y = wave number components;
- b = friction function defined in Equation 5;
- B = friction spectrum defined in Equation 23;
- $B_{p,q}$ = coefficients in the Fourier series expanded for B ;
- C_s, C_w = specific heats of soil and water, respectively;
- d = porous layer thickness;
- d_p = characteristic pore size;
- D = part of heat dispersion spectrum defined in Equation 25;
- $D_{p,q}$ = coefficients in the Fourier series expanded for D ;
- e_l, e_t = longitudinal and lateral dispersion coefficients, respectively;
- E_l, E_t = dimensionless dispersion coefficients (dispersion coefficients divided by e_t existing prior to convection conditions);
- \bar{E} = dispersion tensor;
- F = part of heat dispersion spectrum defined in Equation 26;
- $F_{p,q}$ = coefficients in the Fourier series expanded for F ;
- g = gravity acceleration;
- H = heat advection spectrum defined in Equation 24;
- $H_{p,q}$ = coefficients in the Fourier series expanded for H ;
- I = unit matrix;
- K = permeability;
- \vec{l} = unit vector in the longitudinal direction;
- \vec{n} = unit vector in the vertical direction;
- N = truncation parameter;
- Nu = Nusselt number;
- p = pressure;

p_0 = pressure at $z = 0$;

Pr = Prandtl number ($=\nu/\kappa$);

R = Rayleigh number defined in Equation 11;

R_0 = critical Rayleigh number;

Re = Reynolds number ($=\phi U d_p/\nu$);

s = power coefficient;

t = time;

T = temperature

\vec{u} = velocity vector;

u_0 = longitudinal velocity existing prior to convection conditions;

U = module of velocity vector;

V = module of convection velocity;

x, y, z = coordinates;

α = coefficient of volumetric thermal expansion;

$\alpha_i (i=1, \dots, 4)$ = coefficients defined in Equation 32;

β = perturbation in friction function;

β_1 = coefficient defined in Equation 32;

γ = parameter defined in Equation 4;

$\vec{\delta}$ = operator defined in Equation 14;

ΔT = temperature difference between bottom and top of the aquifer;

$\vec{\varepsilon}$ = dispersion tensor perturbation;

θ = temperature perturbation;

$\hat{\theta}_{q,q}, \theta_{p,q}$ = coefficients in the Fourier series expanded for θ ;

κ = thermal diffusivity of saturated porous media;

λ = parameter defined in Equation 28;

μ = viscosity;

ν = kinematic viscosity;

ρ = fluid density;

ρ_0 = fluid density at $z = 0$;

ρ_s = solid density

σ_1, σ_2 = parameters expressing growth and oscillation of disturbances;

ϕ = porosity;

ψ = stream function;

$\hat{\psi}_{p,q}, \Psi_{p,q}$ = coefficients in the Fourier series expanded for ψ ;

Ω = scalar variable defining velocity perturbations;

Ω_1 = constant defined in Equation 18;

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CHAPTER 6
NUMERICAL SIMULATION OF SINGLY
DISPERSIVE CONVECTION IN GROUNDWATERS

INTRODUCTION

Considerable interest is now focused in the State of Florida as well as other locations in the U. S. for the possible utilization of the deep and saline aquifers for waste disposal. Vernon [1970] counted the properties of the deep zones of the Floridan Aquifer which he believed makes this stratum capable of such applications. Henry and Kohout [1972] mentioned the fact that in a thick system like the Floridan Aquifer the effect of geothermal activity should be considered too. That statement was based on previous articles [Kohout, 1965; 1967] postulating that geothermal activity induces groundwater circulation in the Floridan Aquifer.

Only sophisticated numerical approach may be used for the simulation of all possible phenomena that may occur in an aquifer like the Floridan Aquifer subject to geothermal activity. However, the applicability of such an approach should be determined according to ranges of variations of the parameters determining the physical phenomena.

The purpose of this study is to determine such ranges for the possible application of finite difference numerical simulation.

BASIC EQUATIONS

The basic equations utilized for the analysis were presented in a previous study [Rubin, 1976b]. These are the following equations of continuity, motion and dispersion subject to the Boussinesq approximation

$$\nabla \cdot \vec{u} = 0 \quad (1)$$

$$\nabla p - \rho g \vec{n} + \frac{\mu \phi}{K} (1+b) \vec{u} = 0 \quad (2)$$

$$\gamma \frac{\partial T}{\partial t} + \vec{u} \cdot \nabla T = \nabla \cdot (\vec{E} \cdot \nabla T) \quad (3)$$

where

\vec{u} , velocity vector

p , pressure

ρ , fluid density

K , permeability

ϕ , porosity

μ , viscosity

\vec{n} , unit vertical vector in the downward direction

T , temperature

\vec{E} , heat dispersion tensor

γ , coefficient defined by

$$\gamma = \frac{\rho C_w \phi + \rho_s C_s (1-\phi)}{\rho C_w \phi} \quad (4)$$

In a brief literature survey presented in a previous study [Rubin, 1976a] it was suggested that the friction function, b , would be approximated by

$$b = 0.014 Re \quad (5)$$

It is assumed that the fluid density depends linearly on the temperature as follows

$$\rho = \rho_0 [1 - \alpha(T - T_0)] \quad (6)$$

where

α , coefficient of thermal expansion

ρ_0, T_0 , density and temperature of reference.

The flow field model considered in this study prior to the inception of convection motion is described in Fig. 1. This is a saturated porous layer of infinite horizontal extent bounded by two impermeable planes on which the temperature is constant. Through the porous layer the fluid flows uniformly in the longitudinal, x , direction. The transversal and vertical coordinates are y and z , respectively.

By considering

$$d, \Delta T, e_t, e_t/d$$

as units of length, temperature, dispersion coefficient and velocity, respectively, we nondimensionalize the flow field variables as follows

$$\begin{aligned} \vec{x}' &= (\vec{x} - u_0 t \vec{\hat{x}}/\gamma)/d & T' &= (T - T_0)/\Delta T \\ \vec{u}' &= \vec{u}d/e_t & u'_0 &= u_0 d/e_t & U' &= Ud/e_t & (7) \\ t' &= te_t/d^2 & \vec{E}' &= \vec{E}/e_t \\ p' &= \frac{(p + \rho_0 gz)}{\mu \phi e_t (1+b)} + \frac{xu_0}{e_t} \end{aligned}$$

Substituting the dimensionless variables of (7) in (1)-(3) and omitting the primes we obtain

$$\nabla p + RT\vec{\hat{n}} + (1+\beta)\vec{u} = 0 \quad (8)$$

$$\gamma \frac{\partial T}{\partial t} + \vec{u} \cdot \nabla T = \nabla \cdot (\vec{E} \cdot \nabla T) \quad (9)$$

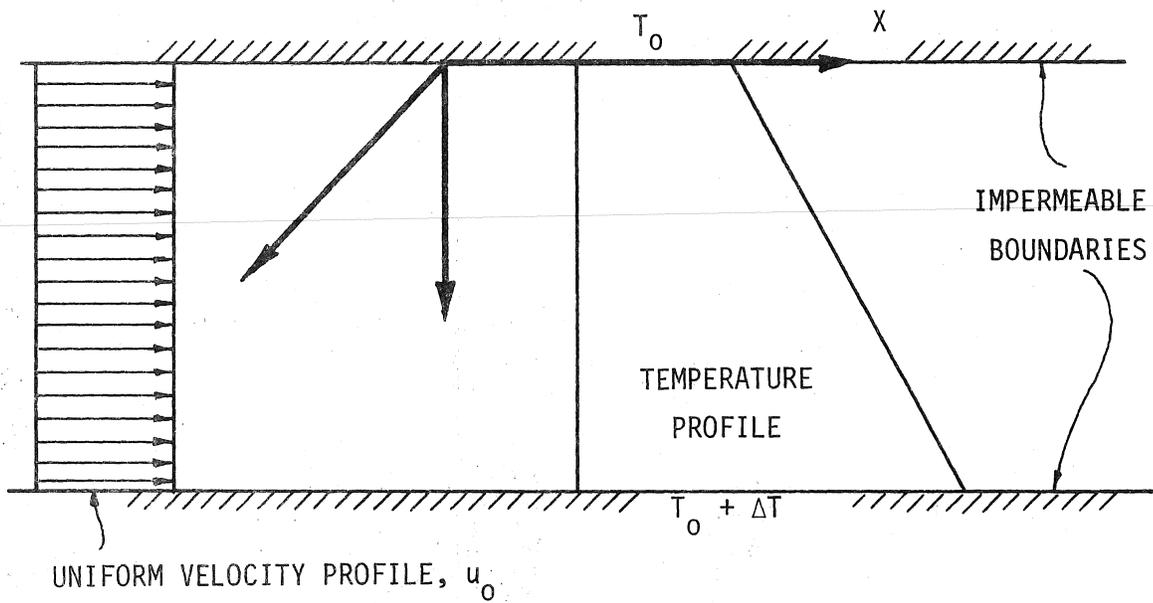


Figure 1. Schematical description of the aquifer with no convection motion.

Here the Rayleigh number R and the coefficient β are defined respectively as follows

$$R = \frac{\alpha g \Delta T k d}{\nu \phi e_t (1+b)} \quad \beta = b[(U/u_0) - 1]/(1+b) \quad (10)$$

In an isotropic medium the dispersion tensor can be expressed by

$$\bar{\bar{\epsilon}} = \bar{\bar{I}} + (E_\ell - 1) \vec{u}\vec{u}/U^2 \quad (11)$$

where

$\bar{\bar{I}}$, unit matrix

$$E_\ell = e_\ell/e_t \quad (12)$$

As long as there is no convection motion, (8) and (9) yield

$$\begin{aligned} \vec{u} = 0 \quad \beta = 0 \quad T = z \quad p = p_0 - Rz^2/2 \\ E_{xx} = E_\ell \quad E_{yy} = E_{zz} = 1 \quad E_{xy} = E_{xz} = E_{yz} = 0 \end{aligned} \quad (13)$$

THE FLOW FIELD STABILITY

Criteria of instability of the flow field can be determined easily by analyzing the growth of small perturbations in the flow field. These are disturbances in the velocity vector (\vec{u}), temperature (θ), dispersion tensor ($\bar{\bar{\epsilon}}$), and pressure. Second order terms depending on these disturbances are negligible. Such conditions yield that the variation W , in the absolute value of the velocity vector is

$$W = \vec{u} \cdot \vec{\ell} \quad (14)$$

Therefore $\bar{\bar{\epsilon}}$ can be obtained through the following expression

$$\bar{\varepsilon} = \frac{\partial E_t}{\partial u_0} W \bar{I} + (E_\ell - 1)(\vec{u}\vec{\ell} + \vec{\ell}\vec{u})/u_0 + \frac{\partial(E_\ell - E_t)}{\partial u_0} W \vec{\ell}\vec{\ell} \quad (15)$$

It is convenient to express the velocity components by utilizing a scalar function Ω as follows

$$\vec{u} = \vec{\delta}\Omega \quad \text{where} \quad \vec{\delta} = \vec{n} \times \nabla \times \nabla \quad (16)$$

Substituting the small perturbations in (1)-(3) and eliminating the pressure disturbance we obtain

$$\nabla_1^2 (R\theta - \nabla^2 \Omega) = 0 \quad (17)$$

$$\gamma \frac{\partial \theta}{\partial t} - \nabla_1^2 \Omega = \nabla_d^2 \theta - \frac{(E_\ell - 1)}{u_0} [\nabla(\nabla_1^2 \Omega)] \cdot \vec{\ell} + \frac{\partial E_t}{\partial u_0} [\nabla(\vec{\delta}\Omega \cdot \vec{\ell})] \cdot \vec{n} \quad (18)$$

where

$$\nabla_1^2 = \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial y^2} \quad \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \quad (19)$$

$$\nabla_d^2 = E_\ell \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

The disturbance θ and the vertical component of the velocity perturbation vanish on top and bottom of the aquifer. Hence (17) and (18) are subject to the following boundary conditions

$$\theta, \nabla_1^2 \Omega = 0 \quad \text{at} \quad z = 0, 1 \quad (20)$$

Applying (20) and substituting θ by Ω through (17) in (18) we obtain

$$\begin{aligned} \gamma \frac{\partial}{\partial t} \nabla^2 \Omega - R \nabla_1^2 \Omega &= \nabla^2 \nabla_d^2 \Omega - R \frac{(E_\ell - 1)}{u_0} [\nabla(\nabla_1^2 \Omega)] \cdot \vec{\ell} \\ + R \frac{\partial E_t}{\partial u_0} [\nabla(\vec{\delta}\Omega \cdot \vec{\ell})] \cdot \vec{n} &\quad (21) \end{aligned}$$

We expand Ω in the following normal mode

$$\Omega = \Omega_1 [\sin(\pi z)] \exp[i(a_x x + a_y y) + (\sigma_1 + i\sigma_2)t] \quad (22)$$

Substituting (22) in (21) for point of instability ($\sigma_1 = 0$) and separating real and imaginary parts we obtain

$$R = (a^2 + \pi^2)(\chi a^2 + \pi^2)/a^2 \quad (23)$$

$$\sigma_2 = \frac{Ra_x}{\gamma(a^2 + \pi^2)} [(E_\ell - 1)a^2/u_0 + \pi a_x \partial E_t / \partial u_0] \quad (24)$$

where

$$\chi = [E_\ell (a_x/a_y)^2 + 1] / [(a_x/a_y)^2 + 1] \quad (25)$$

The critical Rayleigh number is the minimal value of R satisfying (23). Therefore instability is demonstrated by

$$\begin{aligned} \chi &= 1 & a_x &= 0 & a_y &= a & \sigma_2 &= 0 \\ a_0 &= \pi & R_0 &= 4\pi^2 \end{aligned} \quad (26)$$

Hence the principle of exchange of stabilities holds. Convection cells are two dimensional rolls whose axes are parallel to the unperturbed velocity vector. Superposition of the unperturbed and the convection motions yields a helical flow field. The anisotropy of the hydrodynamic dispersion led to the characteristics of the point of instability presented in (26).

NUMERICAL CALCULATION OF THE STEADY STATE CONVECTION

In the previous section we proved that the convection cells are two dimensional rolls. Hence, the convection velocity components can be

expressed by the stream function as follows

$$\vec{u} = -\vec{\ell} \times \nabla \psi \quad \psi = -[\vec{\ell}, \vec{n}, \nabla \Omega] \quad (27)$$

where the square brackets symbolize the box product.

Substituting (27) in (1)-(3) and eliminating the pressure we obtain

$$R[\vec{n}, \vec{\ell}, \nabla T] - \nabla^2 \psi - \nabla \cdot (\beta \nabla \psi) = 0 \quad (28)$$

$$\gamma \frac{\partial T}{\partial t} - [\vec{\ell}, \nabla \psi, \nabla T] = \nabla \cdot (\vec{E} \cdot \nabla T) \quad (29)$$

If the model correlating the dispersivities with the velocity vector is known, then (28) and (29) can be solved while being subject to the specific boundary conditions of the aquifer.

In order to save computer time quantities, it is convenient to expand the absolute value of the velocity vector in a power series as follows

$$U = u_0 [1 + 0.5(V/u_0)^2 - \dots] \quad (30)$$

where V is the absolute value of the convection velocity ($V = \vec{u} \cdot \vec{u}$).

The variable β appearing in (28), according to (5), can be expressed through

$$\beta = \beta_1 V^2 \quad (31) \quad \text{where} \quad \beta_1 = \frac{b}{2(1+b)u_0^2} \quad (32)$$

The dispersion tensor can be expanded in power and Taylor's series as follows

$$\vec{E} = (1 + \alpha_1 V^2) \vec{I} + \alpha_2 \vec{u} \vec{u} + \alpha_3 (\vec{u} \vec{\ell} + \vec{\ell} \vec{u}) + (E_{\ell}^{-1} + \alpha_4 V^2) \vec{\ell} \vec{\ell} + \dots \quad (33)$$

where

$$\alpha_1 = \frac{1}{2u_0} \frac{\partial E_t}{\partial u_0} \quad \alpha_2 = (E_\ell - 1)/u_0^2 \quad \alpha_3 = (E_\ell - 1)/u_0 \quad (34)$$

$$\alpha_4 = \frac{1 - E_\ell}{u_0^2} + \frac{1}{2u_0} \frac{\partial (E_\ell - E_t)}{\partial u_0} \quad (35)$$

From all the terms appearing in the right hand side of (33) only the first two are important for the calculation of transport phenomena and flow conditions. It should be mentioned that approximations (30)-(35) are valid only for convection velocities slower than the flow velocity induced by the horizontal hydraulic gradient.

According to (22) the boundary conditions determining the convection motions are

$$\begin{aligned} \psi, \frac{\partial \psi}{\partial y}, T = 0 & \quad \text{at } z = 0 \\ \psi, \frac{\partial \psi}{\partial y} = 0, T = 1 & \quad \text{at } z = 1 \\ \psi, \frac{\partial \psi}{\partial z}, \frac{\partial T}{\partial y} = 0 & \quad \text{at } y = 0, L \end{aligned} \quad (36)$$

where L is the width of the half convection cell ($L = \pi/a$). This parameter is determined through the maximization of the Nusselt number.

The system (28) and (29) was solved in this study by a combination of the SOR (Successive Over Relaxation) method for the solution of (28) and the ADI (Alternating Direction Implicit) method for the solution of (29).

The FD (Finite Difference) approximation for (28) by applying central difference approximations yields the following SOR scheme for the $m+1$ iteration

$$\begin{aligned}
\psi_{i,j}^{(m+1)} = & \psi_{i,j}^{(m)} + \omega \{ \psi_{i-1,j}^{(m+1)} + \psi_{i,j-1}^{(m+1)} + \psi_{i+1,j}^{(m)} + \psi_{i,j+1}^{(m)} - 4\psi_{i,j}^{(m)} \\
& - R(h/2)(T_{i,j+1} - T_{i,j-1}) + \beta_1 [(V_{i,j}^{(m)})^2 (\psi_{i-1,j}^{(m+1)} + \psi_{i,j-1}^{(m+1)} \\
& + \psi_{i+1,j}^{(m)} + \psi_{i,j+1}^{(m)} - 4\psi_{i,j}^{(m)})] + (\beta_1/4) [(V_{i,j+1}^{(m)})^2 - (V_{i,j-1}^{(m)})^2] (\psi_{i,j+1}^{(m)} \\
& - \psi_{i,j-1}^{(m)}) + (\beta_1/4) [(V_{i+1,j}^{(m)})^2 - (V_{i-1,j}^{(m)})^2] (\psi_{i+1,j}^{(m)} - \psi_{i-1,j}^{(m)}) \}
\end{aligned}
\tag{37}$$

where ω is the relaxation coefficient. As a first approximation for its optimal value we took its value for the Poisson's equation given by [Smith, 1969]

$$\omega = \frac{2 - [4 - (\cos\pi/N + \cos\pi/M)^2]^{0.5}}{(\cos\pi/N + \cos\pi/M)^2}
\tag{38}$$

Usually β_1 is very small. Therefore, (38) is applicable, leading to a good convergence of (37).

The solution of (29) according to the ADI method was conducted in two successive schemes; the first yields the approximate temperature distribution T^* from the given temperatures T^n (at time t_n) implicitly in the y direction through the following scheme

$$\begin{aligned}
& \frac{T_{i,j}^* - T_{i,j}^n}{k/2} + v_{i,j} \frac{T_{i,j+1}^* - T_{i,j-1}^*}{2h} + w_{i,j} \frac{T_{i+1,j}^n - T_{i-1,j}^n}{2h} \\
&= \left(\frac{\partial E_{yy}}{\partial y} + \frac{\partial E_{zy}}{\partial z} \right)_{i,j} \frac{T_{i,j+1}^* - T_{i,j-1}^*}{2h} \\
&+ \left(\frac{\partial E_{yz}}{\partial y} + \frac{\partial E_{zz}}{\partial z} \right)_{i,j} \frac{T_{i+1,j}^n - T_{i-1,j}^n}{2h} \\
&+ (E_{yy})_{i,j} \frac{T_{i,j+1}^* - 2T_{i,j}^* + T_{i,j-1}^*}{h^2} + (E_{zz})_{i,j} \frac{T_{i+1,j}^n - 2T_{i,j}^n + T_{i-1,j}^n}{h^2} \\
&+ 2(E_{yz})_{i,j} \frac{T_{i+1,j+1}^n - T_{i-1,j+1}^n - T_{i+1,j-1}^n + T_{i-1,j-1}^n}{4h^2} \tag{39}
\end{aligned}$$

The next step is the determination of the solution for T^{n+1} by applying an implicit scheme in the z direction as follows

$$\begin{aligned}
& \frac{T_{i,j}^{n+1} - T_{i,j}^*}{k/2} + v_{i,j} \frac{T_{i,j+1}^* - T_{i,j-1}^*}{2h} + w_{i,j} \frac{T_{i+1,j}^{n+1} - T_{i-1,j}^{n+1}}{2h} \\
&= \left(\frac{\partial E_{yy}}{\partial y} + \frac{\partial E_{zy}}{\partial z} \right)_{i,j} \frac{T_{i,j+1}^* - T_{i,j-1}^*}{2h} \\
&+ \left(\frac{\partial E_{yz}}{\partial y} + \frac{\partial E_{zz}}{\partial z} \right)_{i,j} \frac{T_{i+1,j}^{n+1} - T_{i-1,j}^{n+1}}{2h} \\
&+ (E_{yy})_{i,j} \frac{T_{i,j+1}^* - 2T_{i,j}^* + T_{i,j-1}^*}{h^2} + (E_{zz})_{i,j} \frac{T_{i+1,j}^{n+1} - 2T_{i,j}^{n+1} + T_{i-1,j}^{n+1}}{h^2} \\
&+ 2(E_{yz})_{i,j} \frac{T_{i+1,j+1}^* - T_{i-1,j+1}^* - T_{i+1,j-1}^* + T_{i-1,j-1}^*}{4h^2} \tag{40}
\end{aligned}$$

Schemes similar to (39) and (40) were applied in previous studies concerning the ordinary Bénard convection as well as singly diffusive convection in porous medium [Aziz and Hellum, 1967; Holst and Aziz, 1972; Cabelli and De Vahl Davis, 1971].

The value of the derivatives at the convection cells boundaries were calculated according to approximations with round off error of $O(h^2)$ as follows

$$\begin{aligned}
 \left(\frac{\partial \Gamma}{\partial z}\right)_{i=1,j} &= (-3\Gamma_{i,j} + 4\Gamma_{2,j} - \Gamma_{3,j})/2h \\
 \left(\frac{\partial \Gamma}{\partial y}\right)_{i=m+1,j} &= (3\Gamma_{m+1,j} - 4\Gamma_{m,j} + \Gamma_{m-1,j})/2h \\
 \left(\frac{\partial \Gamma}{\partial y}\right)_{i,j=1} &= (-3\Gamma_{i,1} + 4\Gamma_{i,2} - \Gamma_{i,3})/2h \\
 \left(\frac{\partial \Gamma}{\partial y}\right)_{i,j=N+1} &= (3\Gamma_{i,N+1} - 4\Gamma_{i,N} + \Gamma_{i,N-1})/2h
 \end{aligned}
 \tag{41}$$

where Γ is a dummy variable.

The truncation error in (37) is of $O(h^2)$. The truncation error in (39) and (40) is of $O(h^2 + k^2)$. The velocity and dispersion tensor components appearing as coefficients in (39) and (40) were calculated for the time $t_{n+\frac{1}{2}}$. Therefore, these coefficients were obtained by linear extrapolation or interpolation before being applied in (39) and (40).

For the initial conditions we assumed that the temperature is distributed according to

$$T_{i,j} = ih + 0.2(0.5 - jh)\sin(i\pi h) \tag{42}$$

Such an assumption was recommended by Combarous [1970] in a study concerning natural convection without hydrodynamic dispersion effects. The computation process was ended when maximum changes in ψ and T in one time step was smaller than 10^{-5} . However, we also followed changes in the Nusselt number. The Nusselt number was calculated along the row presenting the centerline of the cell through the following expression

$$Nu = \frac{1}{L} \int_0^L (-wT + E_{zz} \frac{\partial T}{\partial z} + E_{zy} \frac{\partial T}{\partial y}) dy \quad (43)$$

Through the calculations the time step was adjusted to assure convergence of the numerical computation. The mesh size was determined according to the variations in the Nusselt number. If the value of Nu was changed by less than 2% by changing M to $M+2$ (where M is the number of intervals in the vertical direction) then it was assumed that the mesh size is sufficiently small. For $R/R_0 \leq 6$ the mesh size varied in the range $0.1 \geq h > 0.05$. In order to speed up the convergence, values of ψ and T , obtained for negligible effects of the convection motion on the dispersion tensor were used as initial values for calculations considering these effects.

NUMERICAL RESULTS AND DISCUSSION

For the identification of the relationship between the velocity vector and the dispersivities, we applied the model suggested by Saffman [1959]. According to this model

$$\frac{e_t^*}{v} = \frac{s + 2}{2(s+1)(s+3)} \left(\frac{Re}{\phi} \right) \quad (44)$$

$$\frac{e_{\ell}^*}{\nu} = \frac{(s+1)^2}{2(1-s)(s+2)(s+3)} \left(\frac{Re}{\phi}\right) \quad (45)$$

where s is a power coefficient describing the dependence between the flow velocity and the pressure gradient. In a previous article [Rubin, 1976a] it was suggested that s can be calculated through the following expression

$$s = \frac{\ln(Re)}{\ln[(1+b)Re]} \quad (46)$$

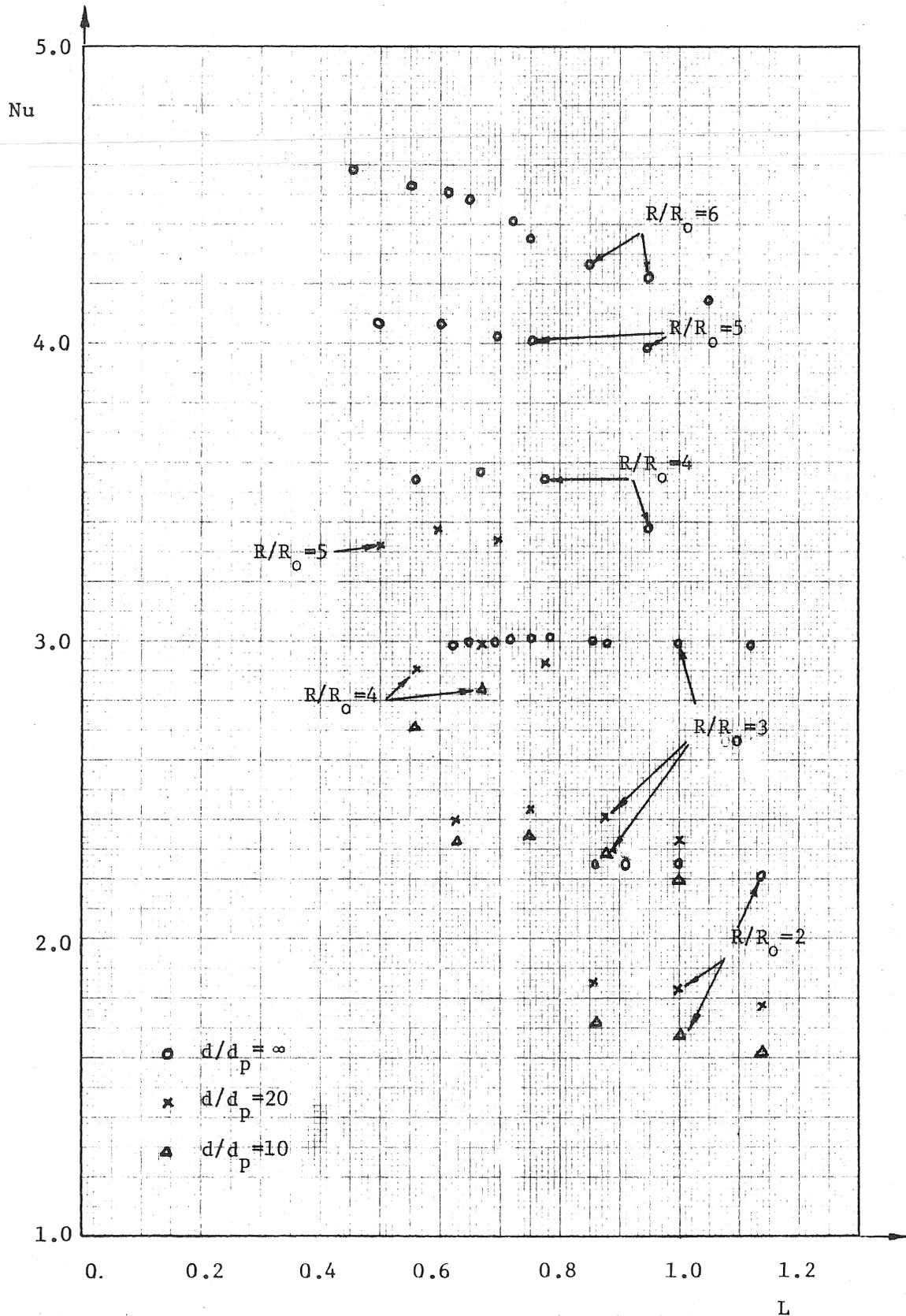
The transversal and longitudinal dispersion coefficients are given by

$$\frac{e_t}{\nu} = \frac{1}{Pr} + \frac{e_t^*}{\nu} \quad \frac{e_{\ell}}{\nu} = \frac{1}{Pr} + \frac{e_{\ell}^*}{\nu} \quad (47)$$

where Pr is the Prandtl number. Through the calculations we assumed $Pr = 1.0$ which seems to be a reasonable value for limestone and dolomite aquifers [Somerton, 1958].

In Fig. 2 we present the dependence between the Nusselt number and the convection cell width for various values of R/R_0 and d/d_p . The effect of d/d_p is significant when its value is comparatively small. It reduces the value of the Nusselt number as well as the width of the convection cell. The half width of the convection cell is the value of L associated with the maximal Nusselt number. This assumption was based on investigation concerning the ordinary Bénard problem [Schülter et al., 1965] as well as diffusive convection in porous media [Straus, 1974], and cannot be proved through simple numerical experiments. However, through the calculation it was found that for wide ranges of L , the Nusselt number changed very little as shown in Fig. 2.

Fig. 2. Nusselt numbers vs. half cell size for various values of R/R_o and d/d_p ($\phi = 0.4$, $Re = 3.0$, $Pr = 1.0$)



At high Rayleigh numbers boundary layers are created at the boundaries of the convection cell. The simulation of processes occurring in these regions requires fine mesh size. Such requirements limit the applicability of the schemes (37)-(40) related to a constant mesh size for the whole grid.

Terms depending on the effect of the convection motion on the dispersion tensor have convective nature. The magnitude of such terms mainly depends on the parameter d/d_p . Therefore, simulation of processes related to small values of d/d_p require fine mesh size although in such cases the Nusselt number is smaller than what expected in the diffusive convection.

In Fig. 3 we present profiles of mean horizontal temperatures for various values of the Rayleigh number and the parameter d/d_p . In small values of d/d_p mechanical dispersion due to the convection motion is significant. Fig. 3 demonstrates that such an effect is associated with a reduction in the values of the temperature gradients at the convection cell boundaries.

In Fig. 4 is shown the dependence between the Nusselt number and the Rayleigh number for various values of d/d_p . According to this figure mechanical dispersion induced by the convection motion leads to a reduction in the effectiveness of transport processes through the aquifer. Such effects can be significant when the ratio between the aquifer thickness and the characteristic pore size is comparatively small.

The validity of the calculations was checked by conventional methods as described in the previous section (changing time steps and intervals while following variations in the Nusselt number) as well as comparing

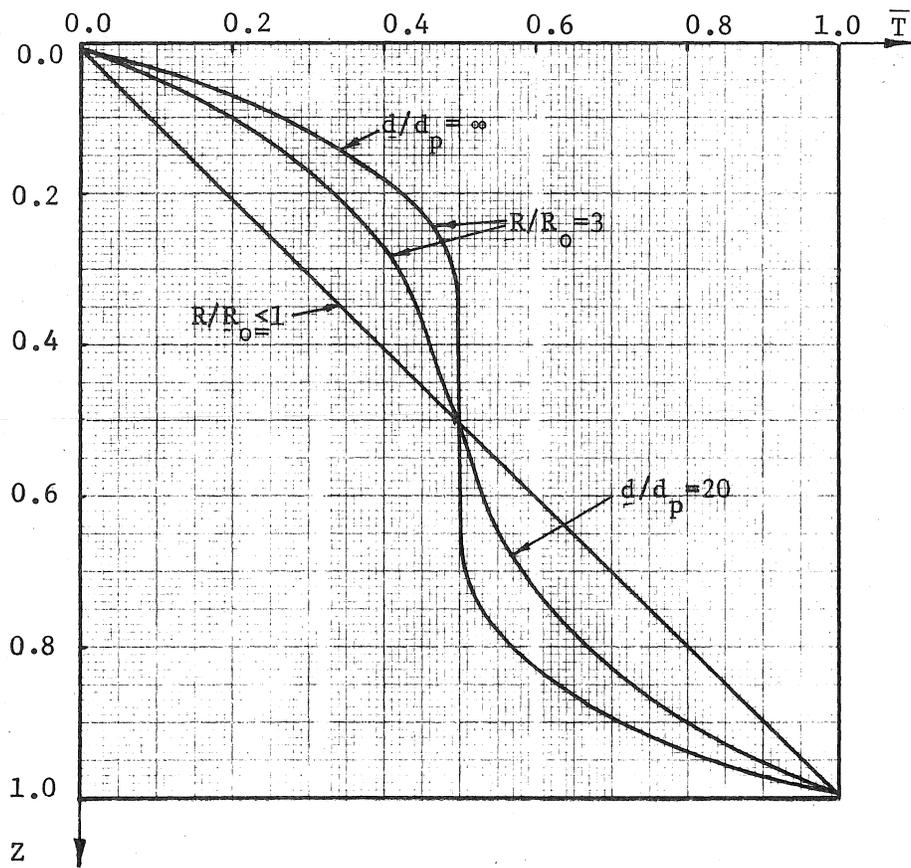


Fig. 3. Profiles of mean horizontal temperatures for various values of d/d_p when $R/R_o = 1, 3$. ($\phi = 0.4$, $Re = 3.0$, $Pr = 1.0$)

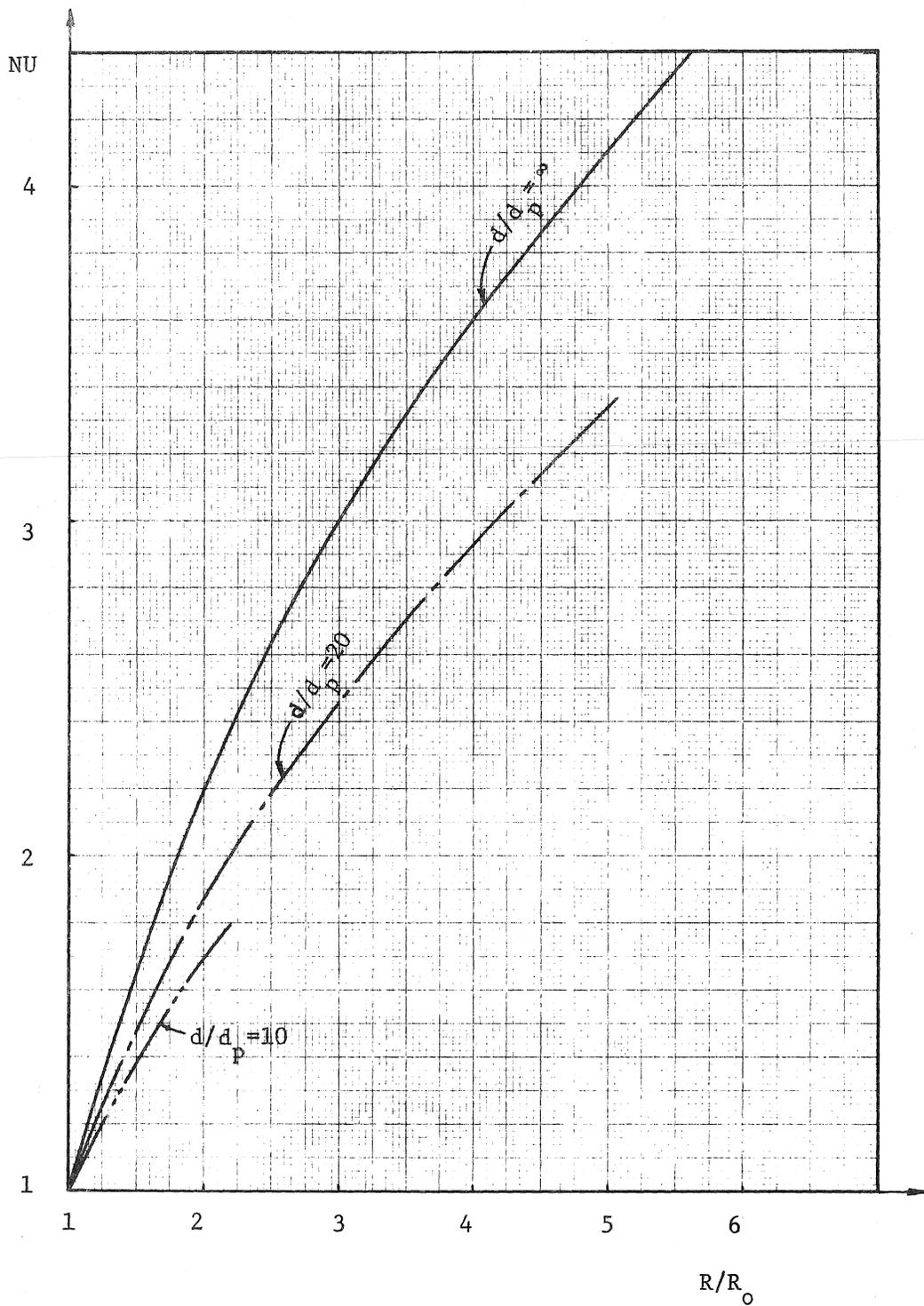


Fig. 4. The Nusselt number vs. R/R_0 for various values of d/d_p ($\phi = 0.4$, $Re = 3.0$, $Pr = 1.0$)

the results with those obtained through Fourier series expansion. It was shown by Orszag [1971] that spectral methods for numerical simulation of incompressible flows within simple boundaries lead to more accurate solution of the mathematical problem. In the case of thermal convection it is possible to utilize series expansions leading to analytical solution of the problem [Rubin, 1975; 1976b]. By comparing our numerical results with the analytical ones we could present further evaluation of the applicability of the numerical schemes applied in this study. From such a comparison it was found that if mechanical dispersion effects are negligible the combination of the SOR and ADI can be applied up to $R/R_0 \approx 6$. The applicability of this procedure reduces if mechanical dispersion due to the convection motion is significant.

CONCLUSIONS

A combination of the SOR and ADI methods can be applied for the analysis of singly dispersive convection in porous media. The procedure is practically applicable up to $R/R_0 \approx 6$. If mechanical dispersion due to the convection motion is significant the applicability of this method reduces. The requirement for coordinate intervals and time steps are determined by the formation of boundary layers at the convection cell boundaries as well as by the production of terms having the convection character.

NOTATION

a	wave number
a_0	critical wave number
a_x, a_y	wave number components
b	friction function defined in (5)
C_s	specific heat of solid
C_w	specific heat of water
d	porous layer thickness
d_p	characteristic pore size
e_ℓ, e_ℓ^*	longitudinal dispersion coefficients (hydrodynamical and mechanical, respectively)
e_t, e_t^*	transversal dispersion coefficients
E_ℓ	dimensionless longitudinal dispersion coefficient
E_t	dimensionless transversal dispersion coefficient
\bar{E}	dispersion tensor
h	coordinate interval
\bar{I}	unit matrix
k	time step
K	permeability
L	half cell width
$\vec{\ell}$	unit vector in the longitudinal, x direction
m	number of iteration
M	number of intervals producing the cell height
\vec{n}	unit vector in the vertical, z direction
N	number of intervals producing the cell width
Nu	Nusselt number

p	pressure
p_0	pressure at the coordinates origin
Pr	Prandtl number ($=\nu/\kappa$)
R	Rayleigh number
R_0	critical Rayleigh number
Re	Reynolds number
s	power coefficient
t	time
T	temperature
\bar{T}	mean horizontal temperature
T_0	temperature at $z = 0$
\vec{u}	velocity vector
u_0	velocity existing prior to the inception of the convection motion
U	absolute value of the velocity vector
v	velocity in y direction
V	absolute value of the convection velocity
w	velocity in z direction
W	velocity perturbation in the x direction
\vec{x}	coordinates vector
x,y,z	coordinates
α	coefficient of volumetric thermal expansion
$\alpha_i (i=1,..4)$	coefficients defined in (35)
β	variable defined in (10)
β_1	coefficient defined in (32)
γ	coefficient defined in (4)
Γ	dummy variable

∇	operator defined in (16)
ΔT	difference in temperature between bottom and top of the aquifer
$\underline{\underline{\varepsilon}}$	dispersion tensor resulting from the convection
θ	temperature perturbation
μ	viscosity
ν	kinematic viscosity
ρ	fluid density
ρ_0	density of reference
ρ_s	solid density
σ_1, σ_2	parameter of instability
ϕ	porosity
χ	parameter defined in (25)
ψ	stream function
ω	relaxation coefficient
Ω	function defined in (16)
Ω_1	coefficient defined in (22)

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CHAPTER 7
SUMMARY AND CONCLUSIONS

1. The conceptual design framework of an urban drainage system should be based on adequate attention to legal principles, regulations and management programs.
2. An outlined diagram for urban drainage design is suggested. This diagram consists of three different phases: project feasibility, preliminary design and detailed design. The use of this diagram promotes more effective management through systematic and sequential consideration of design factors.
3. The adequacy of seepage pond to divert storm water to groundwater depends on two, not necessarily related, components: the ability of the pond to seep water and the growth and decay of groundwater mounds. Both of these aspects should be considered by the designers.
4. A simple method based on the Green and Ampt equation is suggested for the analysis of the pond's ability to seep water.
5. The growth and decay of groundwater mounds can be calculated by a method developed by Hantush. However, this method is not always sufficiently accurate. Therefore, numerical solution of the nonlinear differential equation is frequently required. Design charts based on the numerical solution were developed in this study and can be used as an aid for pond designers.
6. Dispersion of pollutants in the Floridan aquifer, as well as possible invalidity of the laminar Darcy law, should be considered.

7. Possible conditions for doubly or singly dispersive convection in the Floridan Aquifer should be considered.
8. Convective motions may significantly affect dispersion of contaminants in the aquifer.
9. A variety of methods may be used for the analysis of dispersive convection in the aquifer. The more efficient methods are those utilizing spectral expansion of flow field perturbations. These methods are, however, usually limited to simple boundary conditions.

ERRATA SHEET

Page 82 Lines 19, 20 should read:

By expanding Ω in the following normal mode,

$$\Omega = \Omega_1 [\sin(\pi z)] \exp[i(a_x x + a_y y) + (\sigma_1 + i\sigma_2)t] \dots \dots \dots (18)$$

Page 84 Line 18 should read:

$$\psi = -i \sum_{\substack{p=-\infty \\ q=1}}^{\infty} \psi_{p,q} [\sin(q\pi z)] \exp(ipay) \dots \dots \dots (36)$$

Page 86 Lines 19, 20 should read:

Equation 44 can be applied for $Re \geq 2.77$. Through the calculation, it was assumed that $Pr = 1$, which seems to be a reasonable value for limestone and

Page 90 Line 16 should read:

$$[w_v \theta_{|p-k-m|}, w_v (v=1,3,5,7; 2,4,6)] \dots \dots \dots (50)$$

Page 92 Line 22 should read:

θ = temperature perturbation;

Page 94 Lines 32, 33 should read:

11. Saffman, P. G., "A Theory of Dispersion in a Porous Medium", Journal of Fluid Mechanics, Vol. 6, Pt. 3, 1959, pp. 321-349.

Page 96 Line 22 should read:

BASIC EQUATIONS

Page 98 Line 18 should read:

Substituting the dimensionless variables of (7) in (1)-(3) and omitting