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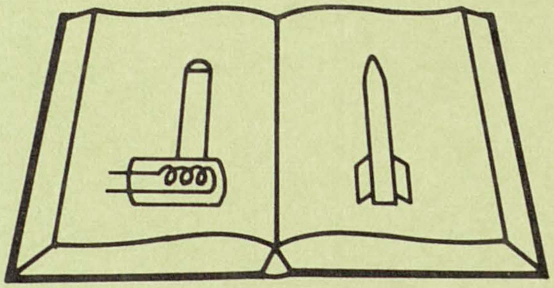
THE JOURNAL OF CHEMICAL ENGINEERING EDUCATION



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PROGRESS

We are now completing the second year of publication. Time does indeed seem to fly.

To date, five issues have appeared containing twenty-two articles plus a number of smaller items. We acknowledge with appreciation the help recieved from many people.

Economy of operation, combined with the indirect support of the University of Cincinnati, has enabled the Journal to give all 1962 subscribers an automatic extension through 1963 at no additional cost. With continued favorable experience, we hope to repeat this two-for-one bonus for 1964 subscribers.

Subscription fees for 1964 are due now. They are still two dollars in the U.S.A. and Canada, and three dollars elsewhere. Remittance should accompany the order since our low cost of operation makes billing awkward.

Checks should be made payable to the University of Cincinnati (which acts as repository for the funds) and mailed with the subscription order directly to the Journal. Prompt remittal will be appreciated and will help greatly in our bookkeeping.

R.L.

INTERPRETATION OF VISCOUS STRESS IN A NEWTONIAN FLUID

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- -

Presented here is an alternative view of some important considerations in fluid mechanics.-- Editor

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This writer has found it possible to picture the viscous stress of an incompressible Newtonian fluid in terms of a rather simple physical model. This is in contrast to a standard approach* which begins in a formal manner assuming eighty one constants to linearly relate the nine components of stress to the nine rate-of-strain components. The eighty one constants are reduced to two which are independent by elaborate arguments entailing invariancy conditions applicable to any isotropic fluid.

As an alternative to this abstract approach the following presents what is thought to be a more physically apparent derivation; the procedure is to make a single, plausible physical statement, then to translate this statement to its mathematical equivalent in a straight forward manner. As a preliminary the kinematics of fluid motion will be reviewed before presenting the rather short derivation.

Review of Kinematics

The vector velocity of fluid at a point P is $\vec{v}_0$ while the vector velocity at a nearby point P' is denoted $\vec{v}$. The vector directed from P to P' is denoted $\vec{\Delta x}$. When the points are close $\vec{v}$ is related to $\vec{v}_0$ by a general expression which results from a Taylor expansion arranged in physically meaningful terms.

$$\vec{v} = \underbrace{\vec{v}_0}_{\text{TRANSLATION}} + \underbrace{\vec{\Omega} \times \vec{\Delta x}}_{\text{ROTATION}} + \underbrace{\boxed{e}\, \vec{\Delta x}}_{\text{STRAIN}} \qquad (1)$$

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*See for example Reference 1. This text is recommended as a fluid-mechanically motivated primer of elementary tensor analysis and contains more details concerning kinematics reviewed herein.

$\vec{\Omega} \times \vec{\Delta x}$ is identical to rigid body rotation about an axis through P with angular velocity specified by $\vec{\Omega}$; i.e. the axis of rotation is along the direction of $\vec{\Omega}$ and the angular velocity is $|\vec{\Omega}|$. All the fluid in the neighborhood of the points share in this rotation motion. $\vec{\Omega}$, the <u>rotation</u>, is equal to half the <u>vorticity</u>.

$$\vec{\Omega} = \frac{1}{2} \vec{\nabla} \times \vec{v} \tag{2}$$

This relationship identifies $\vec{\Omega}$ with spatial derivatives of the velocity components. No relative separation of the points is caused by the rotational motion as may be confirmed by considering the projection of $\vec{\Omega} \times \vec{\Delta x}$ along $\vec{\Delta x}$ which is proportional to $(\vec{\Omega} \times \vec{\Delta x}) \cdot \vec{\Delta x}$ and hence is zero.

The last term in equation 1 being what is left over and hence representing strain motion contains a tensor operator $\underline{|e|}$. $\underline{|e|}$ is a second order tensor and consists of nine elements which may be displayed as its <u>array</u>.

$$\underline{|e|} = \begin{bmatrix} e_{11} & e_{12} & e_{13} \\ e_{21} & e_{22} & e_{23} \\ e_{31} & e_{32} & e_{33} \end{bmatrix}$$

According to the definition of the tensor operator the operational meaning of $\underline{|e|}\, \vec{\Delta x}$ is

$$\begin{aligned} \underline{|e|}\, \vec{\Delta x} = {} & (e_{11} \Delta x_1 + e_{12} \Delta x_2 + e_{13} \Delta x_3)\, \vec{i}_1 \\ & + (e_{21} \Delta x_1 + e_{22} \Delta x_2 + e_{23} \Delta x_3)\, \vec{i}_2 \\ & + (e_{31} \Delta x_1 + e_{32} \Delta x_2 + e_{33} \Delta x_3)\, \vec{i}_3 \end{aligned}$$

Thus the operation of the <u>tensor</u> $\underline{|e|}$ on the <u>vector</u> $\vec{\Delta x}$ yields another <u>vector</u>. The general component of $\underline{|e|}$ is expressed in terms of velocity component derivatives as given by the following.

$$e_{ij} = \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right)$$

In the next section the actual physical modelling is presented.

4

Physical Modelling

It was shown above that the relative motion tending to separate two fluid points whose instantaneous separation is described by the vector $\vec{\Delta x}$ is given by the expression $\boxed{e}\vec{\Delta x}$. It follows that the vector velocity of this strain motion per unit length is $\boxed{e}\vec{\Delta x}/|\vec{\Delta x}|$. Now $\vec{\Delta x}$ may be considered as a normal vector to a fluid plane of small area. Thus $\vec{\Delta x}/|\vec{\Delta x}|=\vec{\eta}$ where $\vec{\eta}$ is the unit normal to the plane and the normalized strain motion may be written simply as $\boxed{e}\vec{\eta}$. Consider that there will exist a vector force $d\vec{F}$ on the fluid plane of area dA whose normal is $\vec{\eta}$, this force being the result of strain motion of the fluid. Accordingly a vector stress is defined as $\vec{\tau}=d\vec{F}/dA$. Up to this point no physical assumptions have been made; now it is desired to relate $\vec{\tau}$ to the motion of the fluid and some fluid property. The crux of this argument is to assume that $\vec{\tau}$ is proportional to $\boxed{e}\vec{\eta}$, i.e. that the vector stress is proportional to the vector rate-of-strain both in magnitude and velocity. Figure 1 illustrates this statement. The constant of proportionality is set equal to 2 μ where μ is the viscosity of the fluid.

$$\vec{\tau} = 2\mu\boxed{e}\vec{\eta} \tag{3}$$

It is to be emphasized that $\boxed{e}\vec{\eta}$ is a relative motion with direction and magnitude and that $\vec{\tau}$ is proportional in the vector sense. Equation 3 contains all the information pertaining to the viscous stresses both tangential stresses and normal stresses. It is rather compact and so, below, it is unfolded in order that its implications may be examined.

Cartesian Form of the Viscous Stress

Equation 3 is the final result of this note. The purpose of the following is simply to demonstrate the correctness of this result.

The viscous stress $\vec{\tau}$ has a component along the direction of the normal $\vec{\eta}$ given by the scalar product $\vec{\tau}\cdot\vec{\eta}$. This represents a stress perpendicular to the small fluid plane.

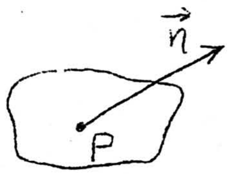
$$\vec{\tau}\cdot\vec{\eta} = (2\mu\boxed{e}\vec{\eta})\cdot\vec{\eta} \tag{4}$$

$$\begin{aligned}\boxed{e}\vec{\eta} &= (e_{11}\eta_1+e_{12}\eta_2+e_{13}\eta_3)\vec{i}_1\\ &+(e_{21}\eta_1+e_{22}\eta_2+e_{23}\eta_3)\vec{i}_2\\ &+(e_{31}\eta_1+e_{32}\eta_2+e_{33}\eta_3)\vec{i}_3\\ &= a_1\vec{i}_1+a_2\vec{i}_2+a_3\vec{i}_3\end{aligned}$$

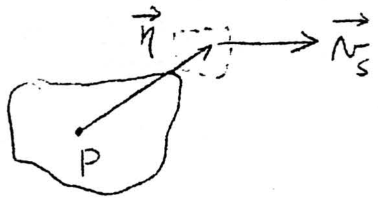
$$\begin{aligned}(\boxed{e}\vec{\eta})\cdot\vec{\eta} &= e_{11}\eta_1\eta_1+e_{12}\eta_1\eta_2+e_{13}\eta_1\eta_3\\ &+e_{21}\eta_1\eta_2+e_{22}\eta_2\eta_2+e_{23}\eta_3\eta_2\\ &+e_{31}\eta_1\eta_3+e_{32}\eta_2\eta_3+e_{33}\eta_3\eta_3\end{aligned}$$

a. Imagine an arbitrary fluid plane through a point P

b. Imagine the unit normal to the plane

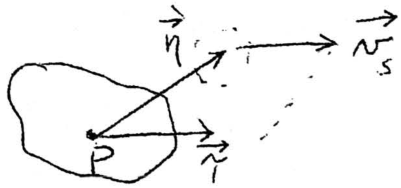


c. Imagine the fluid in the vicinity of the tip of the normal vector.



This fluid is in strain motion $\vec{v}_s$ relative to fluid at point P.
In the text $\vec{v}_s = \boxed{e}\,\vec{\eta}$.

d. The viscous stress vector $\vec{\tau}$ is aligned with $\vec{v}_s$



$\vec{\tau}$ is the vector force per unit area acting on the fluid plane through P.

FIGURE 1. ILLUSTRATION OF THE PHYSICAL ORIGIN OF VISCOUS STRESS.

This latter expression is rather complex due to the generality inherent to it. To obtain a more recognizeable result choose a particular direction. First suppose $\vec{\eta} = \eta_1 \vec{\imath}_1 = \vec{\imath}_1$ or that $\eta_1 = 1 \quad \eta_2 = \eta_3 = 0$. The above expression then reduces to the following.

$$(\boxed{e}\vec{\eta}) \cdot \vec{\eta} = e_{11}\eta_1\eta_1 = e_{11} = \frac{1}{2}\left(\frac{\partial u_1}{\partial x_1} + \frac{\partial u_1}{\partial x_1}\right) = \frac{\partial u_1}{\partial x_1}.$$

Similar expressions follow for the normal stresses in the other two directions. Thus the viscous, normal stress components are

$$\begin{aligned} s_{11} &= 2\mu e_{11} = \mu\left(\frac{\partial u_1}{\partial x_1} + \frac{\partial u_1}{\partial x_1}\right) \\ s_{22} &= 2\mu e_{22} = \mu\left(\frac{\partial u_2}{\partial x_2} + \frac{\partial u_2}{\partial x_2}\right) \\ s_{33} &= 2\mu e_{33} = \mu\left(\frac{\partial u_3}{\partial x_3} + \frac{\partial u_3}{\partial x_3}\right) \end{aligned} \tag{5}$$

These expressions are in agreement with the standard results. Next examine the tangential stresses. The foregoing determined the stress component parallel to the normal $\vec{\eta}$ by use of the scalar product. For finding the perpendicular component to $\vec{\eta}$ and hence the parallel component to the fluid plane we may use the vector cross product which is suitable for the job. The force is $\vec{\Upsilon}$ so that $\vec{\Upsilon} \times \vec{\eta}$ is a vector with the proper magnitude and is in the proper plane though it is ninety degrees rotated from the proper direction in the plane. Another cross product with $-\vec{\eta}$ rotates it to the correct direction. Thus

$$\vec{\Upsilon}_{SHEAR} = -(\vec{\Upsilon} \times \vec{\eta}) \times \vec{\eta} \tag{6}$$

Hence

$$\vec{\Upsilon}_{SHEAR} = -2\mu\left[(\boxed{e}\vec{\eta} \times \vec{\eta}\right] \times \vec{\eta}$$

A cross product may be expanded according to the following vector identity.

$$(\vec{A} \times \vec{B}) \times \vec{C} = (\vec{A} \cdot \vec{C})\vec{B} - (\vec{B} \cdot \vec{C})\vec{A}$$

Applied to the expression for $\vec{\Upsilon}_{shear}$ this gives

$$\begin{aligned} -\frac{1}{2\mu}\vec{\Upsilon}_{SHEAR} &= (\boxed{e}\vec{\eta} \cdot \vec{\eta})\vec{\eta} - (\vec{\eta} \cdot \vec{\eta})\boxed{e}\vec{\eta} \\ &= (\boxed{e}\vec{\eta} \cdot \vec{\eta})\vec{\eta} - \boxed{e}\vec{\eta} \end{aligned}$$

or

$$\vec{\Upsilon}_{SHEAR} = 2\mu\left[\boxed{e}\vec{\eta} - (\boxed{e}\vec{\eta}\cdot\vec{\eta})\vec{\eta}\right]$$

The first term in the brackets gives the original force $\vec{\Upsilon}$. It is easily recognized that the second term is the vector normal force Υ_{normal} found previously. The difference between these must then represent the shear component as found. This result could have been written immediately.

Previously (following equation 4) we made a definition for $\boxed{e}\vec{\eta} = a_1\vec{i}_1 + a_2\vec{i}_2 + a_3\vec{i}_3$

Thus

$$(\boxed{e}\vec{\eta})\times\vec{\eta} = \begin{vmatrix} \vec{i}_1 & \vec{i}_2 & \vec{i}_3 \\ a_1 & a_2 & a_3 \\ \eta_1 & \eta_2 & \eta_3 \end{vmatrix}$$

$$= \vec{i}_1(a_2\eta_3 - a_3\eta_2) + \vec{i}_2(\underline{a_3\eta_1 - a_1\eta_3}) + \vec{i}_3(a_1\eta_2 - a_2\eta_1)$$

And also

$$\begin{aligned}\left[(\boxed{e}\vec{\eta})\times\vec{\eta}\right]\times\vec{\eta} = \vec{i}_1&(a_3\eta_1\eta_3 - a_1\eta_3\eta_3 - a_1\eta_2\eta_2 + a_2\eta_1\eta_2) \\ + \vec{i}_2&(a_1\eta_1\eta_2 - a_2\eta_1\eta_1 - a_2\eta_3\eta_3 + a_3\eta_2\eta_3) \\ + \vec{i}_3&(a_2\eta_2\eta_3 - a_3\eta_2\eta_2 - a_3\eta_1\eta_1 + a_3\eta_1\eta_3)\end{aligned}$$

Consider the component S_{12} representing the shear in the X_2 direction on a plane perpendicular to the X_1 direction. Thus $\vec{\eta}_1 = \eta_1\vec{i}_1 = \vec{i}$ $\rightarrow$ or $\eta_1 = 1$, $\eta_2 = \eta_3 = 0$ and it is the coefficient of i_2 in the foregoing expression which applies. This reduces as follows.

$$(a_1\eta_1\eta_2 - a_2\eta_1\eta_1 - a_2\eta_3\eta_3 + a_3\eta_2\eta_3) = -a_2\eta_1\eta_1 = -a_2$$

$$S_{12} = 2\mu a_2$$

but

$$a_2 = (e_{21}\eta_1 + e_{22}\eta_2 + e_{23}\eta_3) = e_{21} = \frac{1}{2}\left(\frac{\partial u_2}{\partial x_1} + \frac{\partial u_1}{\partial x_2}\right)$$

thus
$$S_{12} = \mu\left(\frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1}\right)$$

This too is the generally accepted result for tangential shear. Detailed verification of S_{13}, S_{23}, S_{31}, and S_{21} is left to the reader. The general result is expressed as follows.
$$S_{ij} = \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$
This applies to both the normal and tangential stresses originating from the mechanism of viscosity.

When Newton's law (f=ma) is applied to a small mass of fluid taking account of pressure forces, the above given viscous forces, and any body forces which act, the result is the Navier-Stokes equations.

Literature Cited;

1. Long, R. R., "Mechanics of Solids and Fluids", Prentice-Hall (1961).

CONVERSION FACTORS

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Abstract: A convenient notation for conversion factors is developed and some properties of conversion factors are discussed. The use of arrays of conversion factors is emphasized.

Introduction

It is often necessary, in scientific or engineering work, to convert data given in one set of units to some other set of units. There is no practical possibility of eliminating this in the immediate future. Even if international agreement could be reached on a standard set of units with a strictly enforced ban on non-conforming publications, it would be a generation before the existing body of literature was replaced or became obsolete. In spite of the simplicity of the underlying principle involved in changing units, almost everyone makes an occasional error in converting. It is hoped that the rather abstract presentation in this paper will appeal to some students and fix the principles in their minds in such a way as to minimize errors.

Conversion Factors

The treatment is based on a notation in which the value of a physical quantity in a certain unit is designated by a lower case letter with the unit (or an abbreviation) written as a superscript. For example, ℓ^{Ft} means a length measured in feet. The numerical value of the measurement is inserted in parentheses, so that ℓ^{Ft} (3) means a length of 3 feet. Conversion factors are written as capital letters with the unit converted from, as a subscript, and the unit converted to, as a superscript. Again, the numerical value is written in parentheses. Thus, L^{Ft}_{yds} (3) means that the conversion factor from yards to feet is 3.

The basic equation for using a conversion factor is

$$\ell^{i} = L^{i}_{j}\ell^{j} \tag{1}$$

A numerical example is

$$\ell^{Ft}(9) = L^{Ft}_{yd}(3)\ell^{yd}(3) \qquad (2a)$$

or

$$9 \text{ ft} = 3 \text{ (3 yards)}. \qquad (2b)$$

There are exceptional cases such as pH, decibel scales, and stellar magnitudes which are defined by a logarithmic relation, i.e.,

$$\text{pH} = \log_{10} \text{ (conc. of } H^{+} \text{ ions in moles/liter)}. \qquad (3)$$

Such quantities do not come within the scope of this treatment.

The numerical value of a conversion factor is found by converting a relation

$$a \text{ (unit i)} = b \text{ (unit j)} \qquad (4)$$

into a ratio

$$L^{i}_{j} = b/a \qquad (5a)$$

or

$$L^{j}_{i} = a/b \qquad (5b)$$

Normally, a relation in which either a or b is unity is used, but this is not essential. The relation

$$1/12 \text{ ft} = 1/36 \text{ yd} \qquad (6a)$$

based on their relation to the inch is obviously equivalent to

$$3 \text{ ft} = 1 \text{ yd}. \qquad (6b)$$

Strictly speaking, some modification of the equality sign to designate "physically equivalent" instead of "numerically equal" should be used in equations 4 and 6, but the usage of the equality sign in both cases is well established.

Equation 1 can be interpreted as a cancellation of the subscript in the conversion factor, and the superscript in the measurement to be converted. Following this approach,

two (or more) conversion factors for the same type of measurement can be multiplied together, provided that the subscripts and superscripts are the same, to define another conversion factor, as

$$L_i^j \quad L_j^k = L_i^k = L_j^k \quad L_i^j \qquad (7a)$$

$$L_i^j \quad L_j^k \quad L_k^\ell = L_i^\ell \qquad (7b)$$

we note that order is unimportant, though in equation 7b if the sequence is changed to $L_k^\ell \; L_i^j \quad L_j^k$, the last two factors must be multipled first as $L_k^\ell \quad L_i^j$ is meaningless.

It can be taken as a basic principle that all expressions for a conversion factor as a product of other conversion factors must give the same value. From this, two important results can be obtained:

I) C_i^i is always unity

II) C_j^i and C_i^j are always reciprocals.

The first result follows from the fact that any number of multiplications of C_i^i gives C_i^i, therefore, it must be 1, the only common root of unity. If only two multiplications were considered, we would have $C_i^i = \sqrt{1}$ which is satisfied by $\pm$ 1, but three multiplications give $C_i^i = \sqrt[3]{1}$ which is satisfied by + 1 and two complex numbers. Similarly, for n multiplications + 1 is always a root, making it the only common root.

The second result follows from

$$C_i^j \quad C_j^i = C_i^i \quad (1) \qquad (8)$$

It would be preferable if the sequence of proof could be reversed and the more complex principle derived from the simple results which are obvious from the definitions of the conversion factor in equation 5.

Arrays

We now transfer our attention from the individual conversion factor to arrays of conversion factors such as are found in handbooks. A simplified example of such an array is

Table I

	cm	in	ft	meter
cm.	L^{cm}_{cm} (1)	L^{in}_{cm} (.394)	L^{ft}_{cm} (.0328	L^{met}_{cm} (.001)
inch	L^{cm}_{in} (2.54)	L^{in}_{in} (1)	L^{ft}_{in} (1/12)	L^{met}_{in} (.0254)
ft	L^{cm}_{ft} (30.48)	L^{in}_{ft} (12)	L^{ft}_{ft} (1)	L^{met}_{ft} (.3048)
meter	L^{cm}_{met} (100)	L^{in}_{met} (39.4)	L^{ft}_{met} (3.28)	L^{met}_{met} (1)

Of course, in a handbook table only the numerical value is given. The table gives the factor for converting from the unit shown in the column in the left to the unit shown in the row on top. It is not necessary to test the units in the same sequence in the row and column, but doing this introduces symmetry into the array. The diagonal elements become unity and elements symmetric with respect to the diagonal are reciprocals. A matrix having these properties is obtained by taking the antilog of each term in an antisymmetric matrix, but the conversion factor array is not a matrix as it does not obey the matrix rules for addition or multiplication.

The array for m units has m^2 terms, of which m are always unity. The remaining m (m-1) entries are determined by (m-1) independent quantities. A set of (m-1) independent conversion factors is a set in which no member can be defined by multiplication of the other members. There are a large number of independent sets. For example, a set consisting of the conversion factors for any one unit to all the other units is obviously an independent set from which the rest of the array can be calculated by equation 7.

Compound Conversion Factors

In a sense, the material presented above is a complete treatment of the problem of changing units. For any physical quantity, a conversion factor array can be set up which enables the necessary conversions to be made. There is, however, the important practical problem of calculating what can be called a compound conversion factor from simple conversion factors. For example, if conversion factors for length and time are considered simple; velocity, area, and volume would be compound. Actually, there is no rigorous rule for distinguishing between simple and compound quantities; to a large extent, the difference is conventional and depends on the current modes of measurement.

The previous notation for measurements and conversion factors is modified by dividing the superscripts and subscripts into parts by commas, as $q^{p',j}$ and $Q^{k,\ell}_{i,j}$. As examples, an area in square feet would be written as $a^{ft,\, ft}$ and $A^{met,\, met}_{ft,\, ft}$ (.0929) would be the conversion factor from square feet to square meters.

When multiplying conversion factors together to form a compound conversion factor, there is no cancellation but rather a merger of subscripts and superscripts. In principle, units can be switched from the top to the bottom and inverted simultaneously.

Thus, we have

$$V^{mile,\ hr^{-1}}_{ft,\ sec^{-1}}\ (.682) = L^{miles}_{ft}\ \left(\frac{1}{5280}\right)\ F^{hr^{-1}}_{sec^{-1}}\ (3600)$$

and this could be rewritten as

$$V^{miles,hr^{-1}}_{ft,sec^{-1}} = \bar{V}^{miles,sec}_{ft,hr} = L^{miles}_{ft}\ \left(\frac{1}{5280}\right)\ T^{sec}_{hr}\ (3600).$$

However, this is not recommended as it is likely to introduce errors.

It would appear that arrays for compound quantities could be obtained by multiplication of corresponding terms in the arrays for simpler quantities, i.e., obtain an array for area conversion by squaring each term in the length conversion array. There are two difficulties. First, many compound quantities have units (such as acre for area), which are not defined directly from the units in the simpler array. Second,

every permutation of products of simple units defines a possible compound unit, and some of these hybrids are encountered in practice. For example, irrigation engineers use acre-ft as a unit of volume.

However, there is a tendency to employ systems of units, such as c. g. s. (centimeter, gram, second,) m. k. s. (meter, kilogram, seconds), or English units (ft, lb, sec) in which compound units are directly related to the simple units. If we limit ourselves to such consistent systems, simple arrays can be multiplied to form arrays for compound quantities by multiplication of corresponding elements. In practice, it is difficult to remain within such consistent systems; for example, in countries employing the metric system, speedometers do not read in cm/sec or meters/sec but in kilometers/hr.

Recommendations

While the use of the notation developed in this paper is suitable for calculating individual conversion factors, the concept of an array greatly simplifies practical work.

It is suggested that students and young engineers prepare a collection of arrays for those conversions which they encounter in their work. Then, when a new unit is encountered for a particular quantity, a row and column can be added to that array. It will be found that such a collection is a valuable time saver, and will more than repay the original effort and the work required to keep it current.

The most complete collection of such arrays that the present writer knows of is Reference 1, which contains 34 arrays, and can, therefore, be a useful starting point. Even this contains some surprising omissions. For example, the Specific Energy array (Table 16) does not contain the units ft^2/sec^2 or $meters^2/sec^2$ which would be essential for gas dynamical calculations. Fortunately,

$$C^{ft^2 sec^{-2}}_{ft\ lb_F Slug^{-1}} \ (1)$$

and

$$C^{meters^2 sec^{-2}}_{joules\ gm^{-1}} \quad (10^{-3})$$

so that it is very easy to add these to the array.

Literature Cited:

1. Kinslow and Majors, "Systems of Units and Conversion Tables", AEDC-TDR 62-6, (Feb. 1962)

RADIANT HEAT EXCHANGE TO TUBES IN ENCLOSED MUFFLE FURNACES

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Abstract:

A method is presented for direct calculation of heat interchange between banks of tubes or rods adjacent to a refractory roof and a radiating planar source or sink at the bottom of a refractory-walled muffle furnace. Use of a derived geometric factor $\overline{F}$ for the configuration eliminates the fictitious plane approach in solving problems of finite geometry furnaces.

In muffle furnaces containing tube banks, radiant heat transfer was calculated by Hottel (5) assuming a fictitious plane just below the tubes. The fictitious emissivity of this grey plane was then computed, taking into account a refractory-backed wall and the area and emissivity of the tube surface. A shape factor $\mathcal{F}_1$ was then calculated which included this grey plane emissivity, the emissivity of the radiating muffle plane, and geometric view factors.

A more realistic approach in terms of avoiding the fictitious plane concept and dealing only with the exchange between the tube bank and muffle plane was presented by Foust et al. (2) but their method, as presented, is limited to infinite geometry.

Both of these methods leave something to be desired in teaching students to visualize real systems. The aim of the approach used in this paper is to base the calculations on interchange between the grey tube surfaces and a grey muffle plane, using only the easily conceived sink-source system.

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* On loan to the Indian Institute of Technology, Kanpur, India, 1963-1965.

Derivation of the Method:

The standard method is used for handling heat exchange in an enclosure in which all of the tube surface can be considered as a single grey-body source. The receiving plane of finite size at a finite distance below and parallel to the tube plane is considered the grey-body sink. The source and sink designations can be reversed depending on the nature of the heat exchange. The well-known formula for the geometric exchange factor is

$$\mathcal{F}_{12} = \frac{1}{\frac{1}{\bar{F}_{12}} + \left(\frac{1}{\epsilon_1} - 1\right) + \frac{A_1}{A_2}\left(\frac{1}{\epsilon_2} - 1\right)} \qquad (1)$$

where: Subscript 1 refers to muffle surface
Subscript 2 refers to tube surface

$\bar{F}_{12}$ = geometric view factor for the sink-source with refractory surfaces

ϵ = emissivity

A = areas involved in the exchange process

The net heat exchange in Btu/hr is then computed as:

$$q_{12} = A_1 \, \mathcal{F}_{12} \, \sigma \, (T_1^4 - T_2^4) \qquad (2)$$

where: $\sigma = 0.173 \times 10^{-8}$
T = Temperature of surface, ^{o}R

The geometric view factor $\bar{F}_{12}$ between the plane and tubes can be obtained by a combination of analytical methods described by Hottel(3,4), but the calculation for tubes plus a plane surface of finite size with multiple emitting refractory surfaces becomes excessively difficult. For the purposes of the present computation of $\bar{F}_{12}$, it is adequate to use a fictitious plane below the tubes coupled with exchange to the real plane, all surfaces being considered black. The values of $\bar{F}_{12}$ are then plotted versus C/d with S/D as a parameter. Here C is the center - center distance between tubes in the row, d is the outside diameter of the tube, S is the side of the square plane geometry, and D is the

distance between the tube plane and the muffle plane. These design curves are given in Figures 1 and 2. The procedure for computing $\bar{F}_{12}$ was based on the formula:

$$\bar{F}_{12} = \frac{1}{\frac{1}{\bar{F}_p} + \left(\frac{1}{\alpha} - 1\right)} \qquad (3)$$

where: $\bar{F}_p$ = geometric view factor between identical parallel black planes.

α = effective emissivity of the plane just below the tube bank.

Equation 3 is derived by application of equation 1 first to the black tubes and a fictitious plane just below the surface. $\bar{F}$ for this case is the geometric view factor derived by Hottel (3) as the effective area for exchange. Reference 2, Fig. 15.33, p. 263 is a source of these data. It is seen that the resultant $\mathcal{F}$ is simply an effective emissivity α for the plane just below the tubes which then exchanges with the black muffle plane. Equation 1 is applied a second time with $\bar{F}_p$, obtained first by Hottel and Keller (4) and plotted as Fig. 15.32, p. 262 of reference 2. $\bar{F}_{12}$ is the net result, rather than $\mathcal{F}_{12}$ since our system is composed of a black sink-source.

A sample calculation follows:

$C/d = 2, \quad \alpha = 0.88$
$S/D = 1, \quad \bar{F}_p = 0.53$

$$\bar{F}_{12} = \frac{1}{\frac{1}{0.53} + \left(\frac{1}{0.88} - 1\right)} = 0.49$$

<u>Use of Method</u>:

The working curves of Figures 1 and 2 yield $\bar{F}_{12}$ for a series of finite geometries with single and double rows of tube banks respectively in square planar array. Other design curves can be computed by use of Equation 3, but in many cases an average $\bar{F}_{12}$ can be obtained by geometric mean of the values of $\bar{F}_{12}$ for square planes of the shorter and the longer size.

Sample Problem (See Reference 1, p.80. Illustration 6):

A muffle type furnace in which the carborundum muffle forms a continuous floor of dimensions 15 by 20 ft. has its ultimate heat-receiving surface in the form of a single row of 4-in. tubes on 9-in. centers above and parallel to the muffle and backed by a well insulated refractory roof; the distance between muffle top and the row of tubes is 10 ft. The tubes fill the furnace top, of area equal to that of the carborundum floor. The average muffle-surface temperature is 2100°F; the tubes are at 600°F. The side walls are assumed to reradiate as much heat as they receive. The tubes of oxidized steel have an emissivity of 0.8, the carborundum has an emissivity of 0.7.

Find the radiant-heat transmission between the carborundum floor and the tubes above, taking into account reradiation from the side walls.

Use Figure 1 with C/d = 2.25.

For 15-ft. squares separated by 10 ft., S/D = 1.5 and $\overline{F}_{12} = 0.550$

For 20-ft. squares separated by 10 ft., S/D = 2.0 and $\overline{F}_{12} = 0.605$

The average $F_{12} = \sqrt{0.550 \times 0.605} = 0.582$

Using Equation 1,

$$\mathfrak{F}_{12} = \frac{1}{\frac{1}{0.582} + \left(\frac{1}{0.7} - 1\right) + \frac{9}{4\pi}\left(\frac{1}{0.8} - 1\right)} = 0.431$$

The fictitious plane method used by Hottel gives $\mathfrak{F}_{12} = 0.433$ which shows excellent agreement between the two procedures. This result is given as Case 1 in Table 1.

Discussion:

Use of Figures 1 and 2 to obtain directly $\overline{F}_{12}$ view factors for finite geometry, tube-muffle combinations enables single step computation of the geometric factor by means of the well-known Equation 1. This procedure was tested for a number of cases, a few of which are reported in Table 1. The agreement between the Hottel fictitious plane method and the direct view factor procedure of this paper is well within the error of graphical read-out and slide rule accuracy. Although the derived curves are useful for nearly all designs of such furnaces, derivation of other $\overline{F}_{12}$ curves may be necessary. For instance, the curves used to obtain α

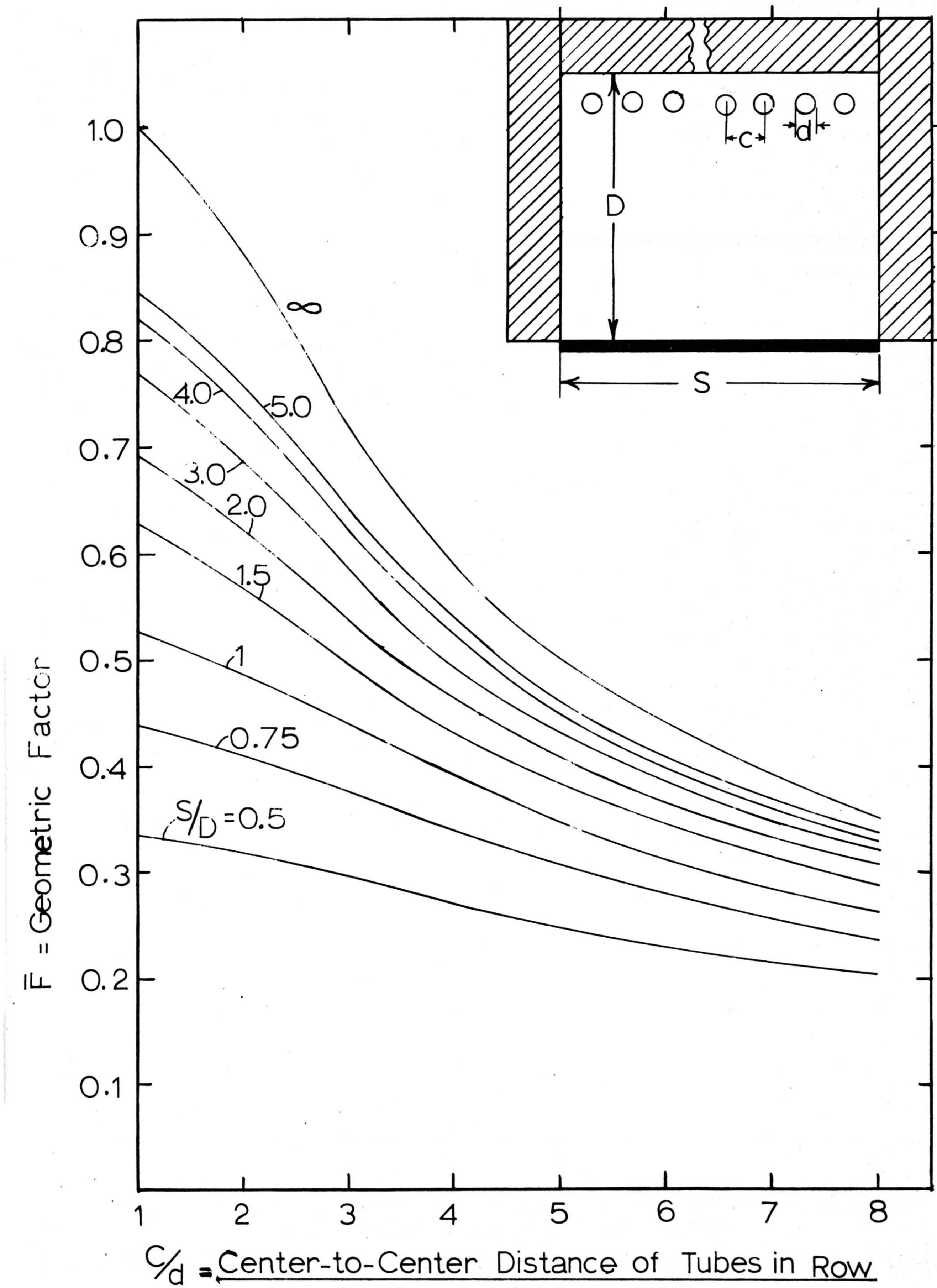

FIGURE-1. View Factors for One Row of Tubes to a Plane Within a Refractory Muffle Furnace

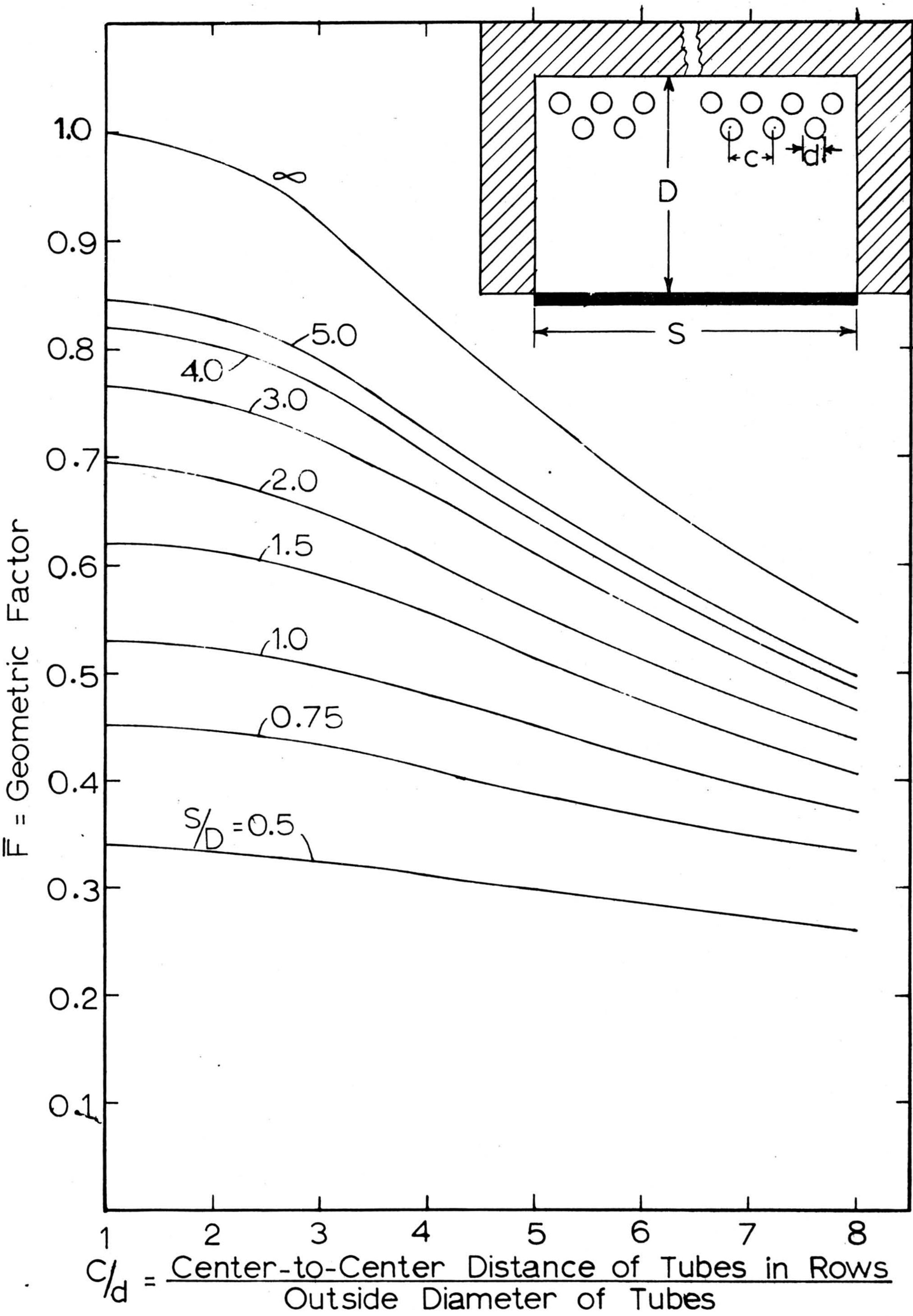

FIGURE. 2. View Factors for Two Rows of Tubes to a Plane Within a Refractory Muffle Furnace

in Equation 3 were based on placing the tubes a distance of greater than d/2 away from the wall. If tubes are placed at a closer distance, then the graph recently reported by Chao (1) can be used to obtain a better value of α. The maximum reduction in α is about 10% when the tubes just touch the refractory wall. Most designers, however, prefer to place the tubes away from the wall at least one diameter to obtain better convection transfer from the tube area (6).

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4. Hottel, H.C., and Keller, J.D., Trans. Amer. Soc. Mech. Engrs., Iron and Steel, 55:39 (1933).

5. Hottel, H.C. in McAdams "Heat Transmission," 3rd edition, pp.80-81, McGraw-Hill (1954).

6. Mathis, H.M., Schweppe, J.L., and Wimpress, R.N., Pet. Ref. 39:No.4,177 (1960).

Table 1. Comparison of the Fictitious Plane Method with the View Factor Method of this Paper

Fixed Conditions:

Muffle size = 15 ft. x 20 ft.
Size of tubes in a single row, d = 4.0 inches diameter
ϵ_1 for muffle = 0.7
ϵ_2 for tubes = 0.8
ϵ_3 = fictitious plane emissivity (calculated)
Temperature of muffle = 2560°R
Temperature of tubes = 1060°R

Variables:	Case No.			
	1	2	3	4
D = Distance between floor and tubes, ft.	10	10	10	20
C = Tube center distance, inches	9	5	27	9
C/d	2.25	1.25	6.75	2.25
Fictitious Plane Method:				
α (ordinate of Figure 1 @ S/D=∞)	0.845	0.975	0.400	0.845
$A_1 / A_2 = A_3 / A_2$	0.714	0.399	2.148	0.714
ϵ_3	0.730	0.890	0.328	0.730
$\bar{F}_{12} = \sqrt{\bar{F}\ (15\ ft) \times \bar{F}\ (20\ ft)}$ (ordinate of Figure 1 @ C/d = 1, S/D)	0.660	0.660	0.660	0.481
$\mathcal{F}_{13}$	0.433	0.483	0.252	0.349
$q_{13} \times 10^{-6}$	9.38	10.57	5.46	7.61
View Factor Method:				
$\bar{F}$ (15') from Fig. 1, this paper	0.550	0.614	0.322	0.478
$\bar{F}$ (20') from Fig. 1, this paper	0.605	0.678	0.340	0.401
$\bar{F}_{12} = \sqrt{\bar{F}\ (15\ ft.) \times \bar{F}\ (20\ ft.)}$ =	0.582	0.645	0.331	0.438
$\mathcal{F}_{12}$	0.431	0.480	0.251	0.347
$q_{12} \times 10^{-6}$	9.37	10.50	5.47	7.57
$\frac{q_{12} - q_{13}}{q_{13}} \times 100$, %	-0.1%	-0.07%	+0.1%	-0.5%

- -

Shorter Communication

THE ROLE OF COMPUTER TRAINING IN UNDERGRADUATE ENGINEERING CURRICULA

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University of Rochester
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In this paper the author's object is to attempt answering three questions for engineering educators, namely:

I. Why should computer training constitute a required discipline of undergraduate engineering curricula?

II. How should computer techniques be taught?

III. When should such training be offered during the undergraduate engineering program?

I. Why?

The widespread use of computers in this country leads educators into considering provision for some sort of computer training at the undergraduate level.

One might argue, however, that many of the graduating engineers do not have to do a great deal of computer programming themselves, but rather they will be directing technicians who will do the actual programming for them. Consequently, those who think in this manner, do not feel that formal computer training is necessary in the undergraduate engineering curriculum.

The author does not believe that this argument is valid, but rather, that a reasonably intimate knowledge of computers is of importance for the graduating engineer. At least two reasons may be given to support the latter point of view:

1. Many times in an engineering office the technicians are too busy to translate the engineer's problem into a computer program. Also, it is not so easy to explain a complex engineering problem to one who really does not know much engineering and, in fact, has no direct interest in the problem himself. This communication problem does exist and although a technician can be helpful at times, there are certain occasions when it would be time saving for the engineer to write his own program or at least to prepare a detailed flow chart for the problem.

2. A fairly intimate knowledge of the computer gives the engineer a better appreciation of its capabilities. In many cases the best approach to the solution of a problem when one has in mind its solution by means of a computer is different from the conventional approach employed by the user of paper-pencil-slide rule. Knowledge of the computer enables the engineer to select this best approach. Usually the technician will not be able to help the engineer in making these preliminary "high level decisions" on the approach to the problem solution.

II. How?

This is a controversial question. Many educators seem to favor the so-called "black box approach," and recommend teaching only compiler language as exemplified by FORTRAN. The author favors teaching first, machine language and symbolic program systems. Three reasons can be presented to justify this opinion:

1. Once machine language is understood the student can pick up relatively easily the use of a compiler, but the reverse is not true.

2. If a compiler source program does not work at first, the knowledge of machine language may become very helpful in "debugging" the program.

3. A third factor which might be labelled as the "psychological factor" may be mentioned. One derives a certain amount of satisfaction in understanding what is going on "inside the black box." Learning only a compiler is a comparable experience to that of one who learns how to use a slide rule without knowing what a logarithm is: true, it can be done, but this approach might not be too appealing to the sophisticated mind.

III. When?

It is not a new idea in engineering curriculum to utilize a few summer weeks for required Engineering courses. Chemical Engineering students at the University of Rochester, for example, take an intensive 3-week summer course in "Chemical Engineering Unit Operations" between the junior and the senior year.

It is believed that the introduction of a 4-week summer course in computer programming between the freshman and sophomore year would be most beneficial. The course could include machine language, symbolic program systems and compilers. The mornings would be devoted to lectures and blackboard exercises. Afternoons could be used to a large extent in actual machine experience.

During the sophomore year in most schools, engineering students take the first professional courses within their chosen fields. In view of freshly acquired acquaintance with computational techniques, these courses could be effectively assisted by computers. Computer-assisted courses should also become a common occurrence during the junior and senior year.

Literature

1. O'Connell, F.P., Chem. Eng. Education 1, 8 (1962).

2. Pehlice, R.D., Sinott, M.J., Journal of Engineering Education 52, 573 (1962).

(continued on next page)

INFORMATION FOR CONTRIBUTORS

Full length articles, shorter communications, and letters to the editor are solicited. Contributions must be original, of course, and must deal with subject matter of interest in chemical engineering education. Naturally, material that has been published elsewhere, or is being considered for publication elsewhere, is not acceptable. (However, a paper that has been merely presented orally at a meeting will be considered provided the author has obtained an appropriate release from the society or other group that sponsored the meeting).

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Appendix

This is an outline for the course "Chemical Engineering Computer Calculations" as taught this year in the Department of Chemical Engineering of the University of Rochester.

Texts used:

1. Germain, C. B. - "Programming the IBM 1620" - Prentice-Hall, Inc. (1962)
2. IBM 1620 FORTRAN Reference Manual - IBM Publication C26-5619-0 (1962)

I. Computing Fundamentals
   a. Evolution of computers
   b. Digital and analog computers
   c. The stored program concept
   d. The fundamental units of a digital computer
   e. Numerical analysis and digital computers
   f. Flow diagrams

II. The IBM 1620 and its Component Parts

III. 1620 Instructions

IV. Operation of the 1620

V. The Symbolic Program System
   a. Pseudo instruction
   b. Declarative operations
   c. Address arithmetic
   d. Unique Mnemonics
   e. Operation of the SPS processor

VI. Program Planning and Debugging

VII. Subroutines and Floating Point Arithmetic

VIII. FORTRAN
   a. Compilers
   b. Writing the 1620 FORTRAN program.
   c. Operating principles
   d. Analysis of the FORTRAN program.
   e. The FORTRAN pre-compiler

IX. Project - Write a computer program for solution of a fairly complex chemical engineering problem. An individual assignment is made to each student.

Shorter Communication

THERE ARE NO "SMALL" MATHEMATICAL ERRORS IN ENGINEERING WORK

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A mathematical error in the solution to a problem is just an error and it will effect only the student's grade for the course. In engineering work, however, the consequences of an error are more serious and may lead to a loss of money and prestige by the design organization and, certainly, to the loss of a job by the unfortunate designer.

Many engineering students are not aware of the seriousness of the problem. When penalized for errors in a calculation problem, they try to involve the instructor in lengthy discussions pointing out the correctness of the procedure used, smallness of the error, etc. To avoid wasting time on this sort of discussion, I present my point of view on this matter to the students in the very first engineering course. In these comments, emphasis is placed on the following points:

1. The procedure, leading to the solution of a given problem, is usually evident from class discussions and text or reference books. It is a result of understanding the statement of the problem but it cannot be substituted for a numerical answer.

2. Using the selected procedure and other tools available (mathematical tables, handbooks, slide rule, computer), the engineer and engineering student must obtain the true numerical answer.

3. Any error in calculations may cause a deviation from the correct answer. Accordingly, there is no room for any mathematical error in engineering calculations.

4. Errors in engineering work must be discouraged by all possible means.

To stress more strongly the undesirability of errors in engineering and to illustrate the serious consequences of error, I usually discuss the following examples.

1. Error in sign.

This is quite a common error but may cause a great deal of confusion. For instance,

- a cooler is installed instead of a reboiler in the distillation column,
- a missile, instead of heading to the moon, goes to the center of the earth,
- a promising chemical reaction does not yield any product (error in sign in free energy calculations),
- multiplication, let's say, by 10^5 instead of by 10^{-5}, as in the rate of chemical reaction, results in a 10^{10} error,
- change of sign in the work term of the mechanical energy balance will result in suction instead of pressure and may reverse the direction of fluid flow in pipes.

2. Division instead of multiplication.

Here the error is of a^2 order where a is the number in question.

3. Error in decimal point.

Let's suppose that a 10-plate column is needed for the requested separation. The error in decimal point makes it either a single-plate or 100-plate column. Or a 10 story house is reduced to a ranch-style house or enlarged to a 100 story sky scraper.

4. Omission of a term in an equation.

This case can readily be illustrated by an omission of reboiler or condenser in the distillation column, the first floor or roof in a house, a span in a bridge, a power house in a plant, etc.

5. Distortion of a term in an equation.

Distortion of a term will be followed by replacement of the required piece of equipment or material by a different item, such as a condenser on the column by a vacuum pump, a jaw crasher by a pulverizer, water by gasolene, etc.

6. Using log instead of ln.

The error is evident from the relationship

$$\ln a = 2.303 \log a$$

7. Use of indefinite integral.

The use of the indefinite integral in calculations, i.e., neglecting the integration constant, introduces the error equivalent to this constant. No error though if $C = 0$.

8. Reversing the limits of integration.

This error results in the change of sign.

9. Dimension checking in the course of calculations is a highly recommendable practice. It may lead to early discovery of errors.

These illustrations, although drastic, are not exaggerated. They may be useful in explaining to the students the significance of the most common mathematical errors.

Try This One

If on Earth an astronaut of the future weighs 200 lb with his space suit and small emergency rocket belt, from approximately how large an isolated asteroid (minor planet) of the same density as Earth could he escape with his belt fueled to provide 100 lb of thrust for 10 seconds?

R.L.

(Solution on following page.)

Solution to Problem on Preceding Page

Applying Newton's Law of Gravity to the center of the masses involved, the gravitational acceleration g_R at the surface of an asteroid of mass M, radius R, and same density as Earth, is

$$g_R = 32.2\left(\frac{M}{M_E}\right)\left(\frac{R_E}{R}\right)^2 = 32.2\left(\frac{R}{R_E}\right)^3\left(\frac{R_E}{R}\right)^2 = 32.2\,\frac{R}{R_E} = \frac{32.2R}{3963} = 0.00812R \quad (1)$$

where M_E is the Earth's mass and R_E is the Earth's radius which is 3963 miles. The gravitational acceleration at a point above the asteroid's surface, x miles from its center, is

$$g_x = g_R\left(\frac{R}{x}\right)^2 = 0.00812\,\frac{R^3}{x^2} \quad (2)$$

Since the rocket belt is small, the change in its mass occasioned by fuel consumption must be even smaller. Therefore, we can take the total mass of 200 lb. as substantially invariant.

The net force in poundals exerted on the 200 lb. of mass during thrust is

$$F = 100 \times 32.2 - 200\,g_x \quad (3)$$

However, the thrust will be completed near the surface so we can substitute g_R for g_x in equation 3, especially since the term will turn out to be only of small influence on the final result. Also, $F = ma = m\,dv/d\gamma$ so that $F\,d\gamma = m\,dv$. Substituting in the latter and integrating,

$$(3220 - 200g_R)\int_0^{10} d\gamma = 200\int_0^{v} dv \quad (4)$$

from which the escape velocity in ft/sec is

$$v = 161 - 10g_R \quad (5)$$

In order to escape, the kinetic energy imparted must equal the work required to move the mass in question from the surface of the asteroid outward against the pull of gravity, theoretically to infinity. Thus,

$$\tfrac{1}{2}\,mv^2 = 5280\int_R^{\infty} g_x\, m\, dx \quad (6)$$

Solution to Problem—con't

Cancelling m, substituting equations 1,2, and 5 into 6, and then integrating, gives

$$1/2\ (161-0.0812R)^2 = 5280 \int_R^{\infty} \frac{0.00812R^3}{X^2}\,dX = 42.9R^2 \qquad (7)$$

Solving equation 7 yields R = 17.2 miles radius. Thus he could escape from an asteroid of up to approximately 34 miles in diameter.

* * * * * *

Try This One Too

A certain neighborhood grocer weighs his pennies 100 at a time, rather than counting them. He claims that because his scale is quite accurate he has "never made an error". If the average deviation in the weight of single pennies in circulation is 1%, would the grocer's claim of near infallibility seem plausible?

R.L.

The solution will appear in the next issue.

INDEX OF TITLES

German Translation by M. Zimmer

Spanish Translation by Saturnino Fanlo

A chronological index for all issues to date is presented below.

As a courtesy to subscribers in other countries, translations of the titles into German and Spanish are included. In the future such translations will be presented at the beginning of each article.

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Volume 2, Number 2, December 1963