

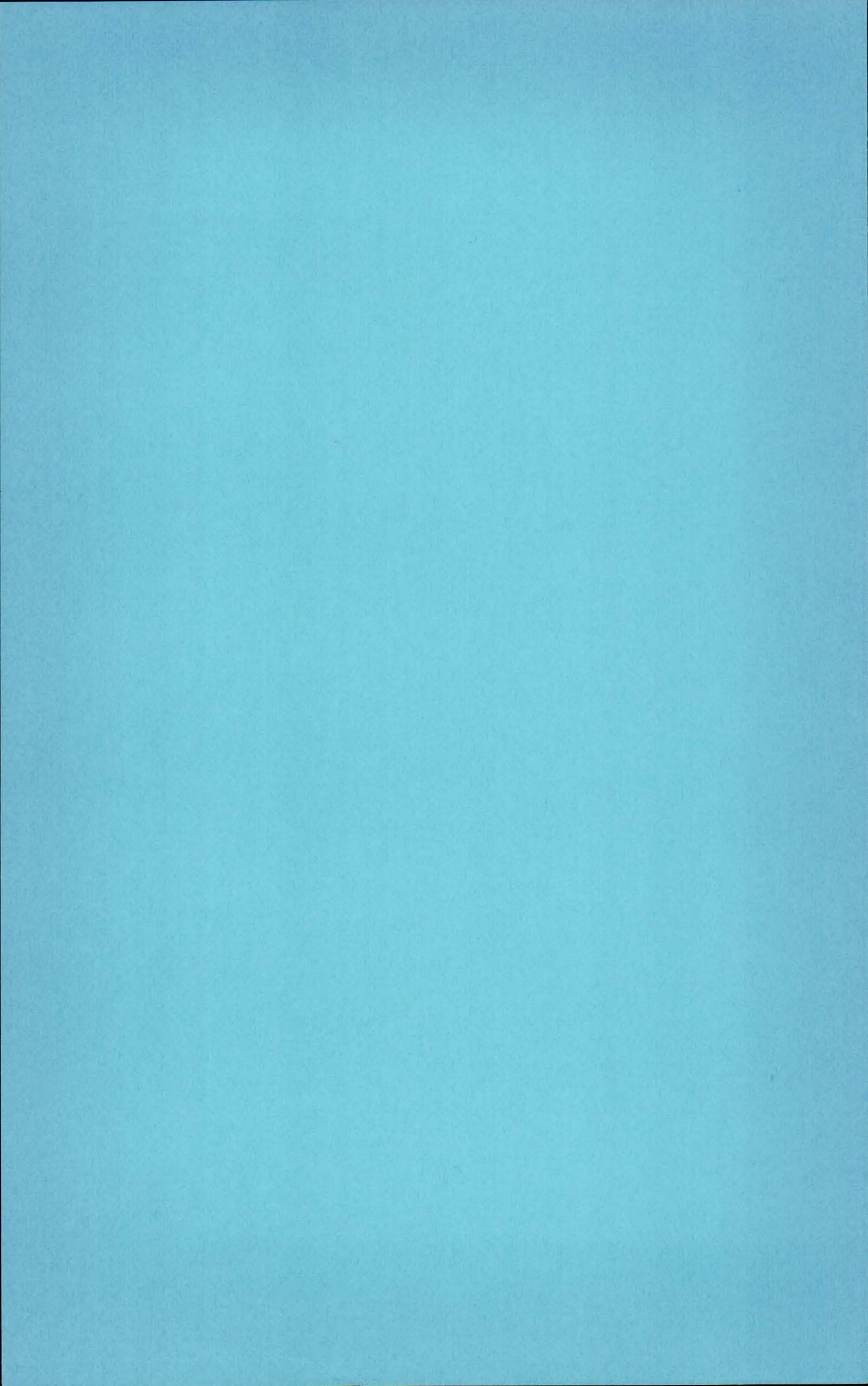
CHEMICAL ENGINEERING EDUCATION



CHEMICAL ENGINEERING DIVISION

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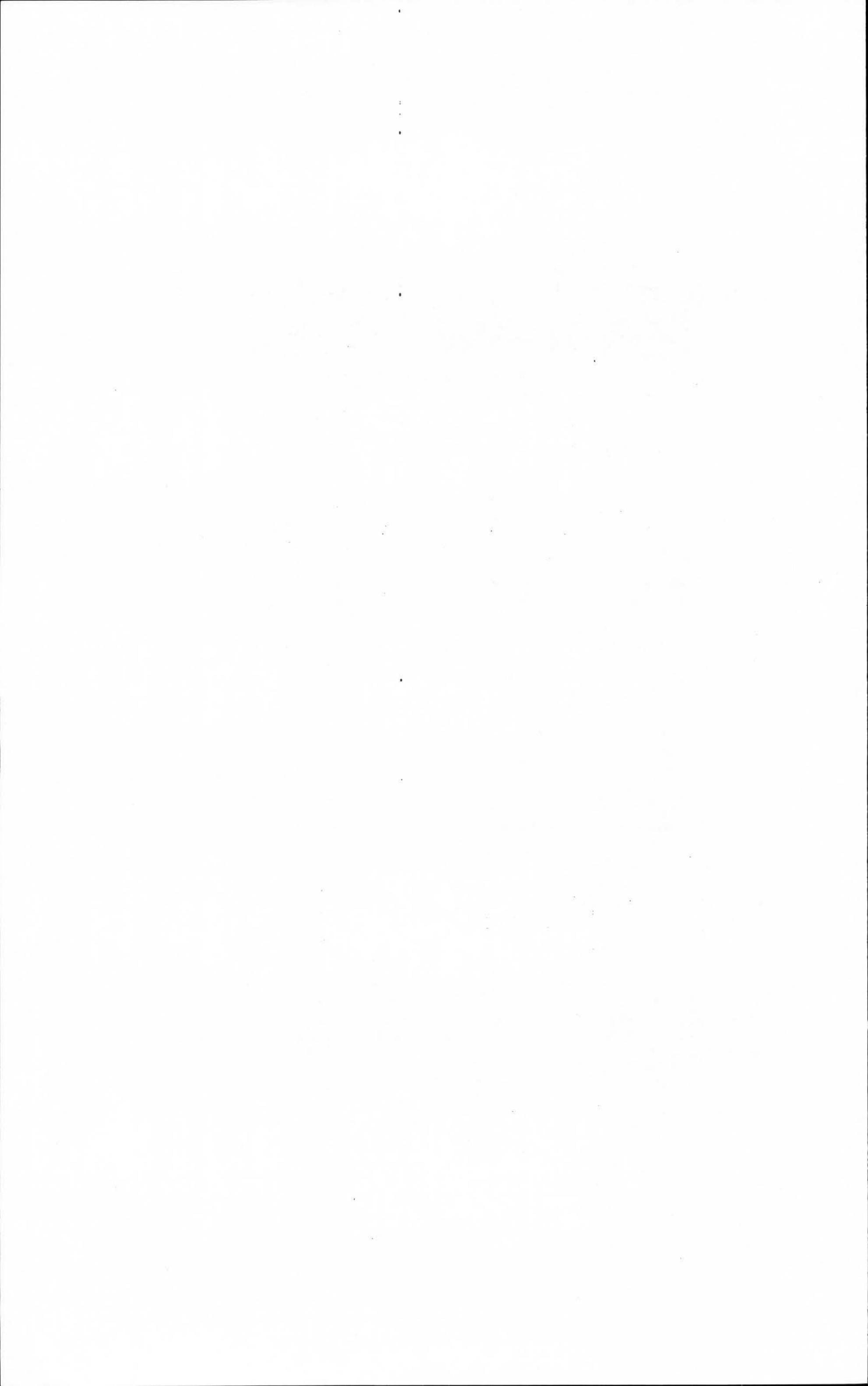
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SOME PHENOMENA FROM FLUID MECHANICS

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Fluid mechanics is a large and rapidly growing field. The first volume of the Journal of Fluid Mechanics covered all of 1956 and contained about 400 pages. Since 1960, in this journal alone, almost 2400 pages have been published each year, about as many as are published in all the American chemical engineering journals. Not only in bulk, but also in diversity has Fluid Mechanics been growing. Imagine a scientific meeting, where the latest advances in fluid mechanics are to be presented, without regard for the professional interest of the author. One would undoubtedly meet there a mathematician working on methods for solving the non-linear flow equations, an aerodynamicist concerned with hypersonic flow, a physicist formulating equations for the flow of plasmas, a meteorologist devising models for the circulation in the atmosphere and another working on the impaction of rain particles, an oceanographer mapping and predicting the ocean currents and the effects of waves, a geologist with improved methods for computing the flow of fluids through oil sands, and many mechanical and chemical engineers expounding on a seemingly endless variety of processing problems. It is, of course, impossible to describe adequately all the significant advances in fluid mechanics in the space available. Instead, I will take a very restricted view and ask what aid the science of fluid mechanics can give to the practicing chemical engineer working mainly in the process industries. One might gain the impression, from skimming recently published texts on fluid dynamics, that the subject has reached maturity, that its laws are understood, and that solutions must follow naturally on applying the deductive method. Let us see how far this is true with regard to the questions in fluid flow that must be answered everyday by some chemical engineer. Typical problems are: After the rupture of the bursting disk on a vessel containing a vaporizing gas-liquid mixture what will be the rate of discharge of liquid through the line? What part of the vessel contents will be carried out through the line? How rapidly will the pressure in the vessel drop to a safe level? If one wishes to protect the particles suspended in an agitated tank from excessive degradation by collision with walls, impellers or other particles, what impeller speeds are tolerable? Will a jet of a gas-liquid mixture entering a distillation column at high velocity splash on the wall opposite and be partially entrained to the tray above? Will the downward deflected portion of that jet stir the liquid in the sump enough to cause troublesome entrainment or vortexing of gas into the pump suction? How thick will be the head of froth on top of a viscous liquid in a reactor and what will happen to its thickness when the vapor velocity in the reactor is raised? How can one calculate the pressure drop in a line carrying two phases and how does one know whether one of the phases may settle out and block the pipe? Will a reactor whose heat evolution is removed by a boiling of a liquid perform smoothly or will it act as a "geyser"? Can one mix non-newtonian fluids in a tank by jetting a stream of liquid into them? How will the dispersion point in a distillation tower using perforated plates be affected by a drastic change in the properties of the fluids, such as a large increase in the liquid viscosity and a large decrease in gas density? Will a conventional pneumatic atomizing nozzle function satisfactorily on a gas of very high density? Ought one to increase or decrease the velocity in a vibrating pipeline if one wishes to reduce the vibration?

In one respect, these problems are simple. For an answer often a simple yes or no will suffice, or if it will not the quantitative answer need not be very accurate. On the other hand, they are very complex in the sense that they have not been formulated neatly like a textbook problem. Nothing warns the engineer to beware of an instability or to beware of a change in regime of atomization. No one gives him the hint that the affects of viscosity are negligible but that capillarity must be considered. In all cases, the first step, and indeed, the greater part of the solution lies in the proper formulation of the problem. To succeed here, with a reasonable score, the engineer needs to know intimately the "things that happen", the "phenomena" of fluid mechanics. Once these are recognized, he can find expert help in the published literature with regard to simplifying concepts, quantitative formulation, methods of solution, or perhaps he may even find the solution to his immediate problem!

With this in mind, we limit ourselves here to describing a few lesser-known phenomena, selected I fear, somewhat capriciously, but with the intent of illustrating the great diversity, fascination and utility of fluid mechanics as viewed by a chemical engineer.

Fluids flowing past solid, or fluid, bodies exert forces on that body - drag and lift. The calculation of these forces, which is probably the most technically important problem in fluid mechanics is now fairly well understood. A few novel studies, throwing light on the sources of the drag, are worth mentioning. Li and Kusukawa (31) have shown that in the absence of viscosity, heat conduction, and diffusion, drag is yet induced by a finite rate of attaining chemical equilibrium.

Other dissipative processes normally increase drag. For example, complete solutions for the very slow motion of a sphere in an electrically conducting and magnetically susceptible fluid are now available.

In calculating the drag on a body one can easily overlook the effects of acceleration or deceleration. If a particle moving through a fluid has no "memory" its kinematic state is determined by its velocity relative to the fluid, or its Reynolds number in dimensionless terms. If it has a "memory" although this memory is short, the drag is, in addition, a function of the derivatives of the velocity with respect to time. One needs to consider, as the next higher approximation, how the drag coefficient depends on the dimensionless group

$$\frac{D^3 p}{u^2} \frac{dv}{dt} .$$

Data on accelerational drag effects are rare. Ingebo's (24) data reproduced here in Figure 1 show that drag coefficients for accelerating drops and solid particles are sometimes vastly different from the drag coefficients for steady flow.

It has been shown by Sproull (43) that the addition of dust to the gas flowing in a pipe can reduce the pressure drop. Two explanations of the effect have been published. That of Sproull attributes the effect to a reduction of the mean free path of the gas molecules because of collisions with the small solid particle. The other, advanced by Saffman (40) is that the particles act to change the stability characteristics of the flow and, by implication, the characteristics of the fully developed turbulence in such a way as to reduce the pressure drop.

Kramer (29) has found that appreciable reductions in the drag that a fluid exerts on a solid body can be achieved by coating the surface with a special type of compliant coating, the artificial "porpoise skin." Benjamin (4) has applied linearized stability theory to the problem to test the assumption that the reduced drag is caused by a stabilization of the boundary layer with respect to the onset of turbulence. He finds that there should be three modes of instability with the flexible surface but only one with the rigid boundary. The possibility of stabilizing the flow depends upon partially stabilizing the mode corresponding to the rigid wall without "letting in" instabilities of the other types.

A surprising feature of the flow of suspensions of solids in liquids is the behavior of turbidity currents in the oceans. It is difficult to reconcile the great range and scouring power attributed by oceanographers to these currents with the rather gentle slope of the terrain over which they are supposed to have occurred. Bagnold (2) has discussed this anomaly and another one related to it associated with the transport of very fine solids by a turbulent stream flowing under gravity. Suspensions containing grains smaller than about 50 microns in diameter are sometimes observed to flow with a concentration gradient which increases upwards, at least in part of the stream. The ordinary model which would attribute the suspending action to eddy diffusion acting in opposition to the gravity settling of the particles is hard pressed to explain this anomalous concentration profile. Bagnold, by a simple semiquantitative argument, shows how gravity can supply the energy to keep the current suspended in this configuration and also keep it flowing over the great distances and small slopes observed in the case of the oceanic turbidity currents. To start the flow, however, it is necessary to postulate a very large scale avalanche.

At times the design engineer has overlooked the electrokinetic effects of flow to his ultimate discomfiture. To quote from the text of Klinkenberg and van der Minne (27): "Time and again there have been mysterious explosions whose cause was subsequently traced to static electricity." The early theories of electrokinetics, often the only ones finding their way into the textbooks, if that, are inadequate to explain the ease with which intense potentials can be produced by flow. Only by introducing modern concepts of turbulent diffusion has it become possible to explain the rapid charging that can occur in the turbulent flow of even very slightly conducting liquids. These interesting effects are due to the fact that the electrical double layer lies partly outside of the laminar sublayer and hence is strongly disrupted by the turbulence. On the theoretical side these effects are interesting because of the dominant role played by entrance effects and because of the extreme sensitivity of the results to conditions very near to the wall. Figure 2 shows how the charging occurs and contrasts the laminar and turbulent flow cases.

As we have seen, a properly directed flow can separate electrical charges. It can also separate energy. The Ranque-Hilsch, or vortex tube, some years ago popularly called a Maxwell's demon, easily separates a gas into hot and cold streams without the use of moving parts. Thermodynamic efficiencies are surprisingly high in a well designed tube. One commercial device can separate 4 cfm of air at 70°F into a cold stream of .8 cfm at -40°F and a hot stream at 90°F. The tube is only 1/4 inch in diameter and 8 inches long. There is, of course, nothing mysterious about such energy separation. Hartnett and Eckert (20) point out that ordinary isothermal Poiseuille flow also causes an energy separation.

In the past sixteen years there have appeared over a hundred publications dealing with the vortex tube effect [Westley (47)]. The recent articles of Reynolds (37) present the most easily understood explanation of the various mechanism leading to the energy separation. Three effects are important. First is the Knoernschild effect, which describes the heat exchange due to a compressible lump of fluid moving rapidly through a pressure gradient and consequently undergoing adiabatic expansion or contraction. Second, there is an Archimedean or buoyancy effect, which makes the lighter clumps of fluid move inwards. Third, the turbulent stresses furnish a means whereby one layer of fluid can do work on the adjacent layers. All of these effects are also important in determining the energy exchange taking place in the atmosphere.

Vibration effects flow in several ways. The streaming currents that flow away from a sound transducer are well known. More spectacular are the interactions of vibrations with a free surface. Figure 3 shows the multinodal columns of fluid that have been observed to form on an ultrasonic transducer (38). A simple laboratory experiment, illustrated in Figure 3, shows the almost explosive nature of ultrasonic atomization. The spray is ejected as discrete bursts of very finely atomized particles.

One of the most intriguing uses of vibrations is in measurement. Eisenmenger (12) has used an ultrasonically excited hydraulic tank to study surface properties of fluids. He measured the surface viscosity of pure water, finding a value of about 10^{-5} dyne sec/cm. Operating on a similar principle is the oscillating jet. In principle one can obtain information about the dynamic surface tension from the spacing of the nodes in a jet issuing from a non-circular orifice. The recent painstaking efforts of van Duyn (46) were not entirely successful in this regard however. He found the spacing between successive nodes on a jet of such a mixture as .01% acetic acid in water to decrease with distance from the orifice rather than to increase as one might at first have expected. Evidently, some factor is still missing from the elaborate theory of the vibrating jet. This effect must be traced down before the vibrating jet can be exploited fully as a measuring device. The attention given to the behavior of a jet of non-newtonian fluids issuing from circular jets may throw some light on this problem [Harris (21)]. Such a jet does not always issue from the orifice with its diameter unchanged but, depending on conditions, may contract shortly after leaving the tube or, more commonly, it may expand. Careful measurements of this effect are potentially capable of giving information about the nature of non-newtonian fluids.

A rather puzzling effect in the realm of vibrations was found by Hughes (23) in measuring uptake of carbon dioxide by water drops falling through a gas. The transfer rates correlate with a postulated "eddy" mixing inside the drops supposedly caused by their vibration as they fall through the gas. For a linearized model of a vibrating fluid sphere one would expect no gross mixing of concentric shells. One wonders whether non-linear effects can account for the degree of mixing observed.

Resonance effects go hand in hand with vibration effects. The pronounced effects of pressure resonance between a chamber and a growing and escaping bubble were pointed out by Hughes et al (22). Perhaps more striking is the observation by Christiansen and Hixson (9) that one can make a dispersion of drops of almost exactly equal sizes the adjusting the velocity with which a liquid jet issues beneath the surface of another liquid. This phenomenon is so precisely controllable as to be useful as a means for measuring dynamic surface tensions.

The mechanics of particles, both fluid and solid, is a subject dear to the heart of the chemical engineer. He wants to know how to make them (usually neither too large nor too small), how to suspend them, transport them, collect and coalesce them. In these areas fluid mechanic is but slowly making the change from art to science.

Dombrowski and Hooper (10) emphasize the rather surprising fact that the average drop size formed in atomization from fan sprays first decreases as the ambient gas density is raised but later increases. This is in accord with predictions made from models of the instability of sheets of liquid moving relative to a gas. Their photographs show clearly changes in the mode of breakup of the sheets as a function of ambient gas density. As shown in Figure 4, the breakup at very low densities, takes place almost exclusively from the edges of the sheet, the drops formed there being relatively large. At moderate gas densities, the leading edge of the sheet becomes unstable, breaks up into drops differing markedly in size from those disrupted from the sides of the sheet.

Let us consider next the single fluid particle moving through another fluid. When the surface tension is sufficiently great as measured by the Laplace number, D^2g/gc_0 , or a related number, the drop is very nearly spherical. When this number is near unity or larger, however, the surface is no longer spherical but is distorted in complex ways depending on flow conditions, drop size and on the

properties of the fluids. Perhaps the least understood of the forms that appear are those with sharp cusps. For drops falling under gravity a "tear drop" with a single cusp at the rear is observed [Fararoui and Kintner (13)]⁷, for a drop in Couette flow a "sigmoid shape" is induced with cusps at either end [Rumscheidt and Mason (39)]⁷.

As a problem in the statics branch of fluid mechanics, the theoretical and experimental work of Allan and Mason (1) on the equilibrium shape and burst of drops subjected to electric fields is interesting. They predict that the drop will be deformed into a prolate spheroid and that it will burst when its eccentricity would otherwise exceed $\sqrt{2}$. By assuming that each half of the burst drop retains its individual charge they are able to estimate the charge separation caused by the splitting. An experimental observation not explained is that some drops are distorted to oblate spheroids rather than to prolate spheroids as required by the theory.

The ways in which atomization can occur are almost bewildering in their complexity. Rumscheidt and Mason (39) have summarized the classes of drop breakup that they have observed in a Couette flow apparatus (concentric counterrotating cylinders). As shown in Figure 5, adapted from their paper, there are three main cases. At very low viscosity ratios, (inside/outside), the drop is progressively stretched out as the shear rate is increased, finally forming a "sigmoidal" drop with two cusplike ends. From these ends continuous streams of very fine particles are ejected. For viscosity ratios near unity, the drop develops a neck, which then rapidly thins down and ultimately ruptures, leaving two almost equal sized drops and several very much smaller satellite drops from the remnants of the neck. A third type of breakup is also observed when the viscosity ratio is near one. Then the drop is drawn out into a long fine filament which breaks up into many drops of almost equal size.

In the same apparatus, Forgacs and Mason (15) have observed the behavior of small flexible fibers (mainly wood fibers). As Figure 6 shows these fibers follow quite complex orbits. The more flexible form single loops or multiple helixes.

The interactions of particles with one another and with the walls of the container holding them are vital to an understanding of the flow of suspensions. Oliver (35) has recently repeated in more detail the experiments of Segre and Silberberg. It is observed that the particles in dilute, initially uniform, suspension of spheres in Poiseuille flow are concentrated eventually into an annular region about halfway between the axis and the wall. Previous theories, which postulate a lag between the particle and the neighboring flow, show that because of induced rotation and the Magnus effect the particle ought to move always to the center of the tube. Oliver's experiments show that particles initially near the wall do indeed move inwards, but that also particles placed near the axis move initially outwards.

The contention of Goldsmith and Mason (17) that solid particles whether spheres or elongated, stick close to the tube wall if initially placed there is explained by Oliver as being due to the very small size of the particles used by them. There is, however, no denying Goldsmith and Mason's observation that fluid particles move away from the wall at velocities which greatly exceed that for a similar solid particle. Related observations bearing on this point are those of Rumscheidt and Mason (39). A fluid particle is placed in a hyperbolic flow apparatus consisting of four cylinders rotating with the proper senses in a liquid substrate. A particle of another fluid suspended in the center of the apparatus exhibits circulation as shown in Figure 8. The circulation is made visible either by fine suspended solids or by similar fine suspended liquid drops. Only in the case where the tracing particles are liquid is the cross-shaped zone indicated on the figure swept free of tracer particles.

Lately, there has been a reawakening to the intriguing role played by surface physics in the flow of fluids. The wide diversity of these phenomena is evident from the review of Scriven et al (42). To cite only two examples, Jepsen-Marwedel (25) has described the erosion of the walls of glass melting furnaces at a series of regularly spaced pits at the glass/gas interface and has attributed this unusual pattern to the flow induced by surface tension variations caused by the solution of the wall material in the melt. Another example is the beautiful experiments of the Langmuirs' (30) on the evaporation of ethyl ether out of water solution. Spontaneous agitation of the surface is very evident and the role of surface active agents in arresting the agitation is demonstrated dramatically by the quenching of the flame formed from burning ether vapors by the addition of a drop of oleic acid. As a result of this reawakened interest there have been produced several very interesting motion pictures [Orell and Westwater (36)]⁷ illustrating "interfacial turbulence". As yet unexplained is the very fast, often almost explosive, growth of nascent convection cells that one often sees in these pictures.

The classical chemical engineering models for mass transfer at a fluid-fluid interface have been somewhat discredited by the recognition of the wide occurrence

of spontaneous agitation of the interface in the case of systems where mass is being transferred. That these models still have their uses can be attributed partly to the equally dramatic effect of surface active materials in arresting the spontaneous agitation. One indeed wonders whether most plant streams may not be sufficiently contaminated to substantially suppress these bizarre effects. In any case, considerable doubt is thrown on the validity of inferences drawn from comparative tests on mass transfer equipment in which care is not taken to reproduce the type and degree of contamination. An urgent requirement for the chemical engineer is one or more reliable "meters" for characterizing the mechanical effects of surface active materials. Any phenomena exhibiting these effects which are sufficiently reproducible and accurately measurable are, therefore, well worth our notice. The work of Haberman (19) on the rise of circulating and non-circulating bubbles through tubes, the measurement of zeta potentials of falling jets (11), and the many phenomena described by Mysels, Shinoda, and Frankel (34) are worth mentioning in this regard.

The latter authors, in their intriguing monograph on soap films, describe many of the phenomena associated with the drainage and rupture of these films. In certain cases, a soap film trapped on a vertical wire frame drains, not by a uniform thinning over the whole film, but rather by the formation of thin, "black" spots at the bottom surface and the upward convection of these spots through the film until they coalesce with a larger region of black film floating atop the ordinary film. The spots, behave as a two dimensional analog of bubbles introduced at the bottom of a vessel filled with liquid. They obey the two dimensional version of Stokes Law!

A determining step in the process of film breakage and of coalescence is the drainage of fluids out of these films formed by the boundaries between two objects. The usual approach taken in attempts at theoretical explanations has been unsatisfying since it is assumed that the shape of the bodies enclosing the thin film is uninfluenced by the force generated by the approach. A quite similar problem arises in the theory of hydrodynamic lubrication where it is found that the deformation of even solid bodies can be important in influencing the rate of drainage of the films. Christensen (8) shows how one may obtain solutions for the flows and pressure induced in the gap between two solid cylinders as they approach one another. Besides considering the case where the solids are perfectly rigid and the viscosity is constant, he considers the effect of pressure on the viscosity of the fluid filling the gap and also the effect of plastic deformation of the solids. The shape of the permanent set given to the solid bodies is very dependent on the "lubrication" conditions. One might have here a model for calculating film drainage in cases where the fluid viscosity depends on layer thickness.

The problem of predicting the conditions of flow on thin films flowing under gravity has long interested chemical engineers. A review of the work on this problem is instructive in showing both the power and the limitations of theoretical fluid mechanics at its present stage of development.

What happens when a liquid film flows down a flat surface under the influence of gravity? At very low flow rates and near the top of the plate it flows with a mirror smooth surface provided the liquid has been well distributed. At flows above a certain critical rate, about 10 pounds per hour per foot of periphery for water, rather regular waves develop on the surface. These waves grow rapidly as the flow rate is raised. At much higher flows, say 1000 pounds per foot of periphery, another change occurs. The appearance of the surface is modified and the trend of average film thickness as a function of flow rate shows a break. The second transition is usually identified with the transition from laminar to turbulent flow since the Reynolds number at which it occurs is near 2000. Between this transition and the lower one the flow is laminar but has pronounced waves on the surface. The waves evidently will affect mass and heat transfer and are therefore of interest to chemical engineers.

It would appear that the determination of the transition from smooth to wavy flow would be amenable to theoretical analysis and indeed this was attempted as early as 1924 by Kapitza (26). His analysis was apparently very successful since it predicted the Reynolds number for transition accurately and showed the effect of surface tension as well (see Figure 8). The paper has not been well received however because of the obscurity of the procedure used. In 1954, Yih (48) made the suggestion that this problem could be solved by the classical method of linearized disturbances and that many of the mathematical difficulties that beset this technique in other problems could be avoided since asymptotic methods appropriate for small Reynolds numbers can be used. Using Yih's suggestion, Benjamin (3) solved the problem and concluded, rather astonishingly, that the flow is unstable at all Reynolds numbers, no matter how small. The theory, in this form, apparently contradicts the experiments showing a fairly well defined transition. Benjamin noted, however, that the rate of growth of disturbances is very small until the Reynolds number attains a value of about 10. He therefore made the suggestion that the flow disturbances, although unstable at low Reynolds

numbers have not had sufficient time to develop to detectable size. Figure 8 shows the results of a quantitative development of this idea. One can define the pseudo-critical Reynolds number as that at which the disturbances grow at a rate which will double their amplitude during the time required for the film to move one wavelength. It is seen that this model agrees with the data and also with the formula of Kapitza obtained by an entirely independent route. One should note, however, that the explanation of the transition given by Benjamin has not been accepted by Tailby and Portalsky (44) and (45) who have made the most recent and extensive series of measurements of the transition.

One is justified, of course, in questioning the assumptions in Benjamin's analysis. One of these, that the ambient phase exerts no drag on the film, has been removed by Graebel (18), who considered the case of countercurrent flow of two fluids of the same viscosity but of different densities. He finds, as did Benjamin, that the flow on a vertical plate is unstable at all Reynolds numbers. However, if the channel is inclined as little as $1/2$ degree off vertical the flow is now stabilized below some definite Reynolds number. This sensitivity of the analyses to small changes points the moral of not generalizing the results of theoretical calculations too fast.

All these analyses assumed the interface to be perfectly compliant. But the experiments by Tailby and Portalsky and others show that the addition of surface active agents suppresses waves formation at low Reynolds numbers. Although this effect is understood qualitatively, a satisfying quantitative formulation has never been made. As Gibbs has pointed out, in any multicomponent system, the phase interface must exhibit an apparent elasticity or resistance to deformation. Such an elasticity undoubtedly suppresses the waves.

As noted above, a falling film shows the classical transition to turbulence at a Reynolds number of about 2000. It is curious that no one has investigated the stability of this type of turbulent flow by the conventional techniques. This configuration might facilitate comparative experiments on turbulent transition.

A flow closely related to that we have just been discussing is that of a thin film driven by friction exerted by another fluid. It occurs in transpiration cooling and in the annular flow regime of gas-liquid pipe flow. Knuth (28) has shown that waves are raised on the film at film Reynolds numbers larger than about 200. The first attempt to explain this observation theoretically was made by Feldman (14). He formulated the stability problem for the case where the driving fluid behaves as if it were laminar near the interface. The results of the analysis do not agree with the data of Knuth, the Reynolds number for transition being about an order of magnitude higher than the experiments would lead one to believe. Since in the experiments, the driving phase was turbulent, it is natural to question and modify the boundary conditions expressing the tractions exerted by the driving phase on the surface. Miles (32) has done this. At first glance, his modification to the shear stress condition at the surface is surprising. He assumes that the stress there has the same value in the disturbed flow as in the undisturbed flow. While cogent arguments can be advanced for making this assumption, they are not entirely convincing. The ultimate test must be comparison of predictions with experiment. In this respect, Miles is moderately successful. He finds that the film is stable provided either the Reynolds number or the Weber number is small enough. As is shown in Figure 10, Knuth's results are substantially confirmed.

One may, I think, draw a moral from the history of these attempts to predict conditions for the raising of waves. Theoretical analyses must be simplified considerably in order to be tractable. It is possible by judicious juggling of the assumptions to force the analyses to agree with data. Such analyses must at present be regarded as secondary, though admittedly they are very powerful tools, to experimental observations.

As a closing comment on this topic it is worth calling attention to the recent paper of Bushmanov (6), who presents two analyses of the problem of the stability of the falling film. By the first approach, substantially the same as Benjamin, he gets the wrong answer! By the second, he improves the analysis of Kapitza, by relaxing some of the assumptions made but in so doing destroys the apparent good agreement of this analysis with the data which was shown in Figure

In view of the comments made by Bushmanov to reconcile his analyses of the stability of thin films with experiment, the article by Caldwell and Donnelly (7) is of great interest. They studied the hysteresis of transition to turbulence in the popular rotating concentric cylinder apparatus. Very precise measurements of torque were made for contrasting series of runs in which the relative speed of rotation was increased or decreased. Within the accuracy of the experiments, which was very good, it appears that there is no hysteresis in this particular transition.

Let us now return to the question posed at the beginning of the paper: To what extent can the chemical engineer use the results of fluid mechanics in the solution of his everyday problems? We must, I think, confess that there are a great many problems where the aid furnished by the conventional literature is slight; many of his problems must be solved by the engineer himself without the aid of the professional fluid mechanic. The approach may be purely empirical or the engineer himself may have to become an expert in fluid flow. Daily, the chemical engineer gives his opinion on problems similar to the ones listed in the introduction. Often the answer is given tacitly, sometimes unfortunately, unknowingly, in the act of making or approving a design. Fortunately, luck is often with him; his design works well without his understanding fully why. Most successful, however, is the engineer who has formulated the relevant problems in his mind, who has considered all the evidence that may bear on the answer, and has designed into his product those features most likely to make it work and designed out the sources of trouble. He has the honesty to recognize what he does not know or cannot economically find out, and has the courage to assess realistically the risks of uncertainty and to take these risks when justified.

In some years, perhaps, the engineer will have more help in solving his flow problems. To advance this day it is important for all of us to look sharply at the "things that happen", examine them critically in the context of related phenomena and of theory and to point them out to one another. As a small contribution I would call your attention to Figure 10, which shows an enlarged view of an air bubble in a sparged vessel containing water. Does not the roiled surface of the bubble strongly resemble the roiled surface of a quiet but turbulent river and ought not the characteristics of the roiling have some bearing on the mass transfer rates calculated on the model of surface renewal?

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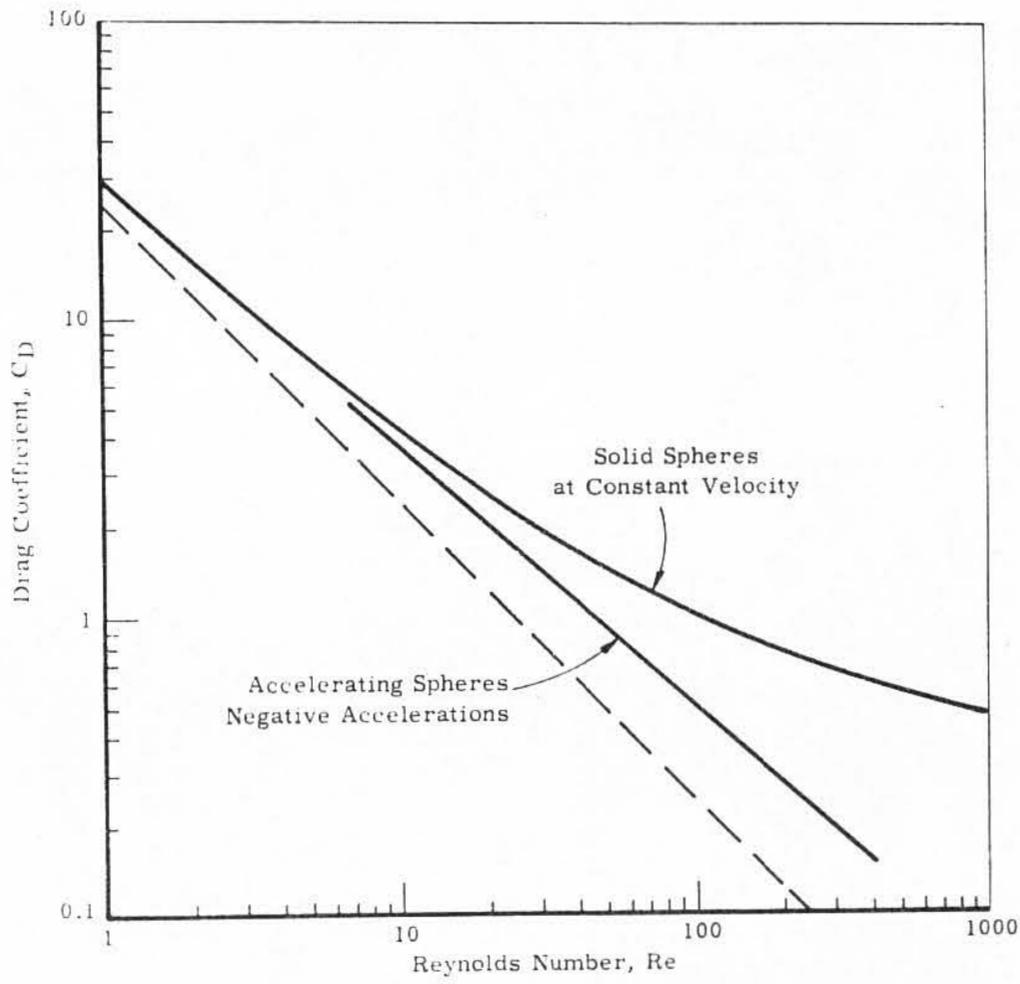


Figure 1. ACCELERATIONAL DRAG DATA OF INGEBO

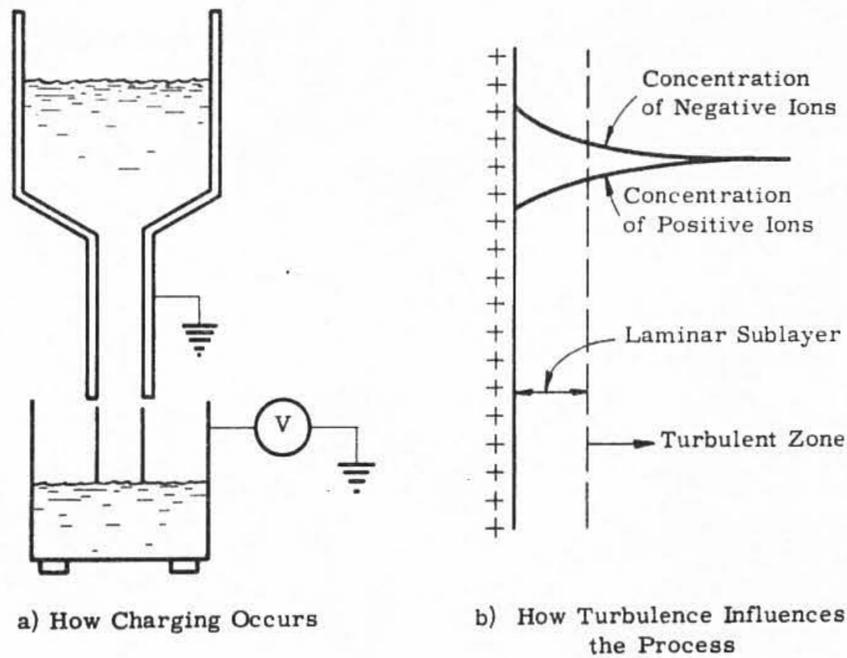


Figure 2. ELECTRIC CHARGING INDUCED BY TURBULENT FLOW
After Klinkenberg and van der Minne

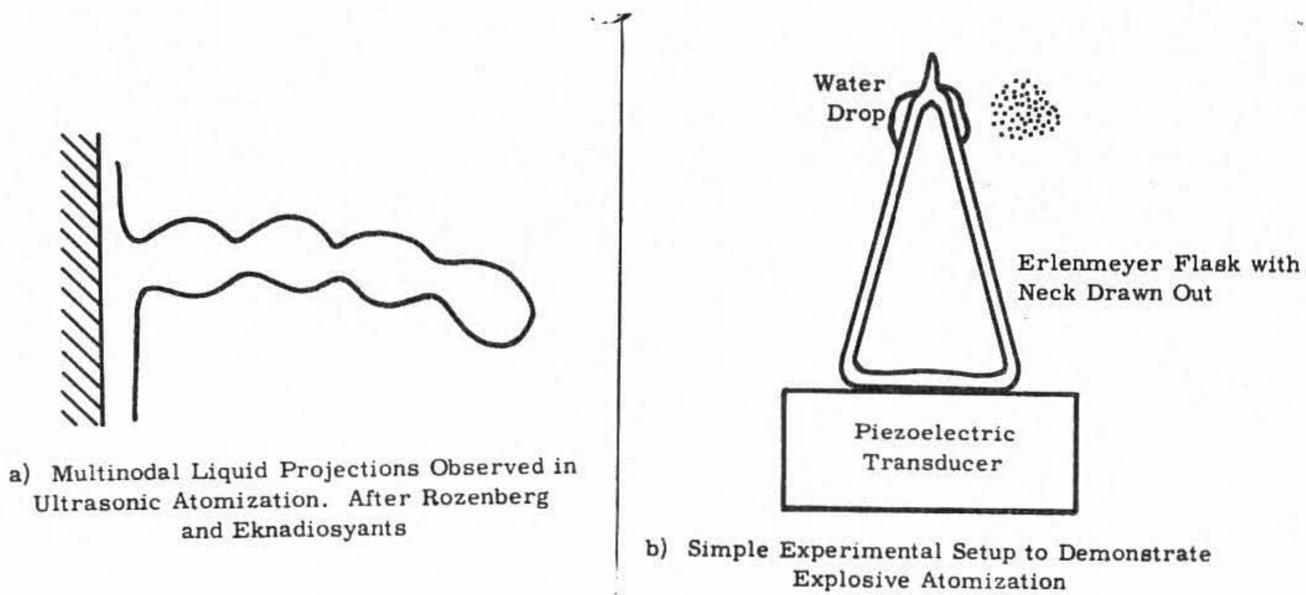


Figure 3. ULTRASONIC ATOMIZATION

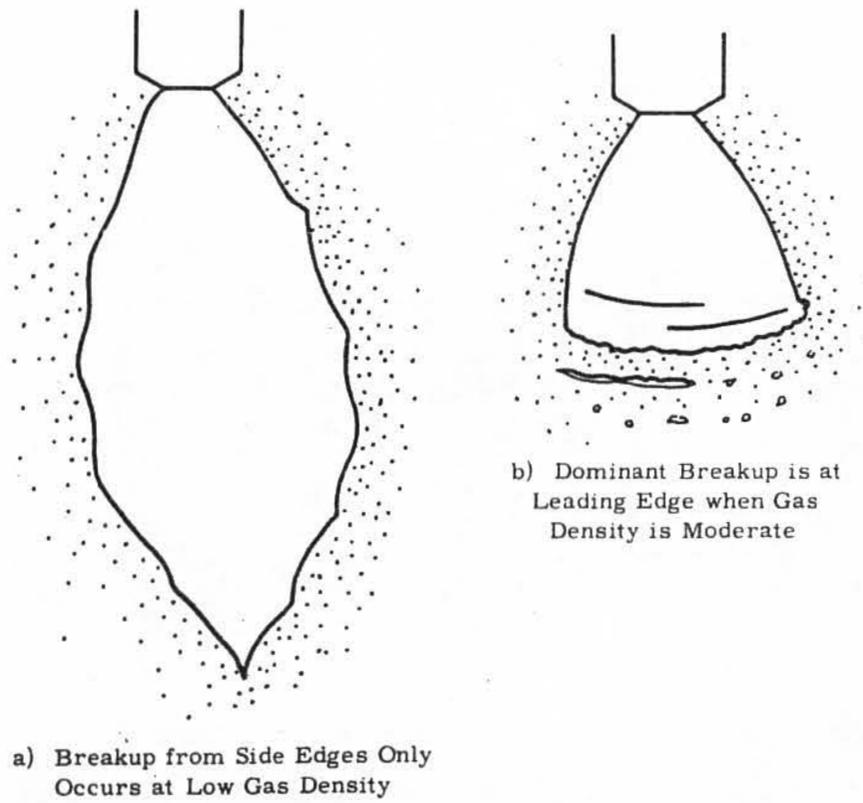


Figure 4. REGIMES OF ATOMIZATION OF FAN SPRAYS
According to Dombrowski and Hooper

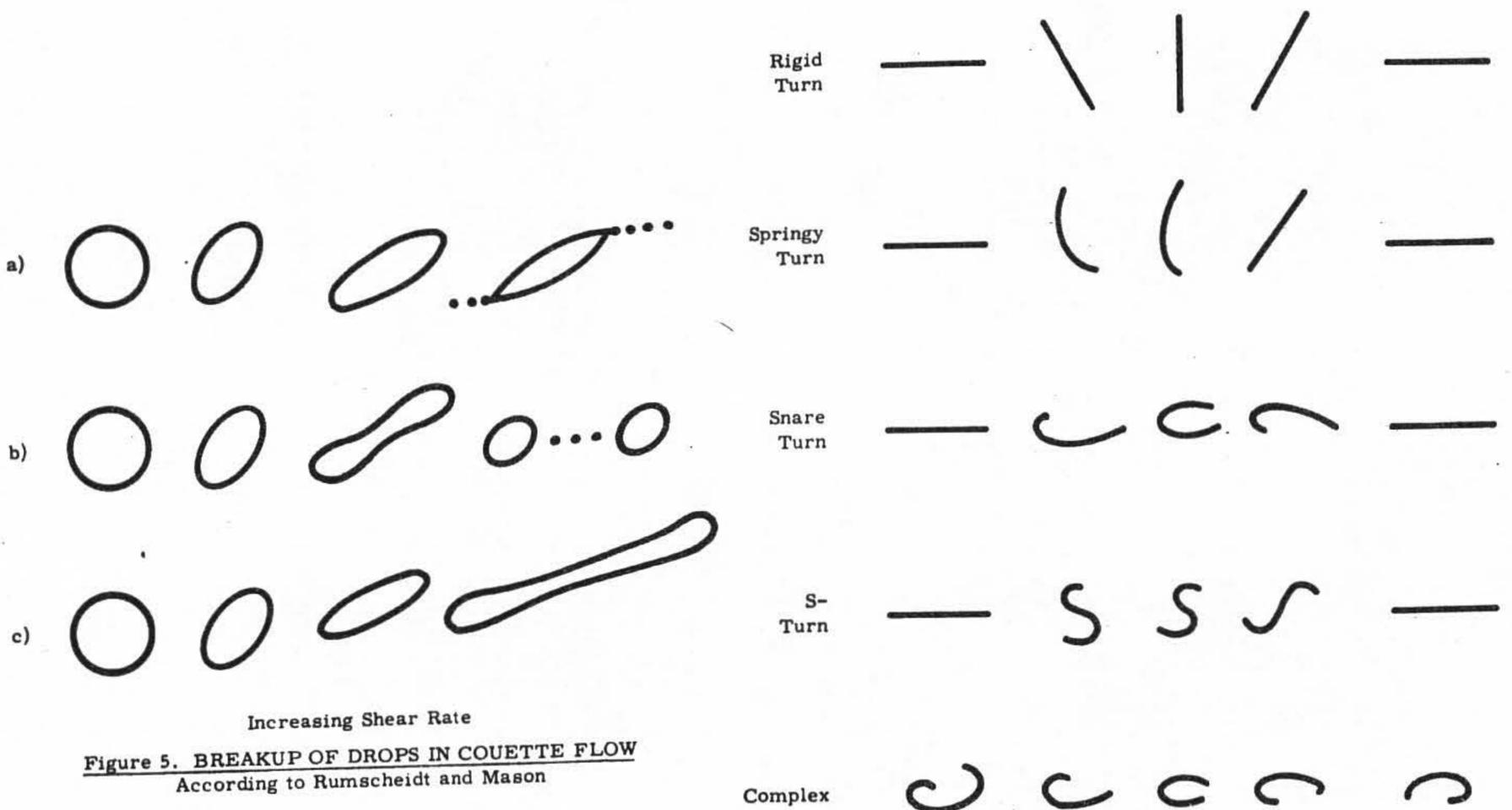


Figure 5. BREAKUP OF DROPS IN COUETTE FLOW
According to Rumscheidt and Mason

Figure 6. ROTATION OF FIBERS OF VARIOUS FLEXIBILITIES
According to Forgacs and Mason

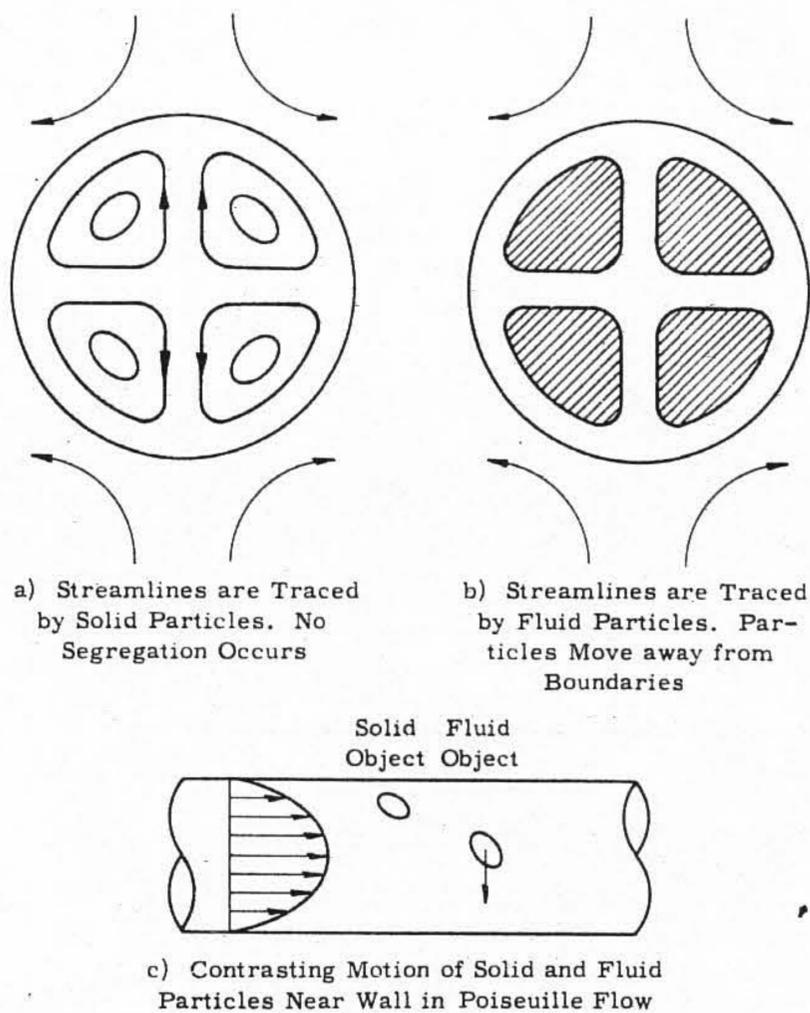


Figure 7. MOTION OF SOLID AND FLUID PARTICLES IN LAMINAR FLOW After Rumscheidt and Mason

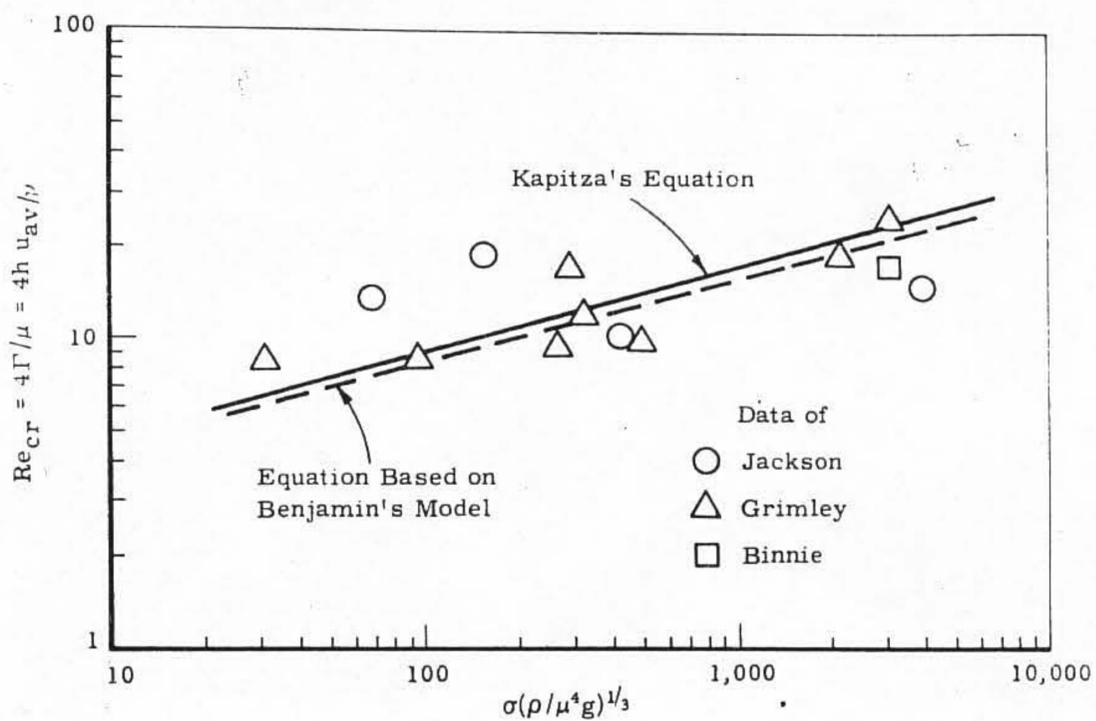


Figure 8. CRITICAL WEBER NUMBER FOR WAVE FORMATION FOR LIQUIDS FLOWING DOWN A VERTICAL WALL

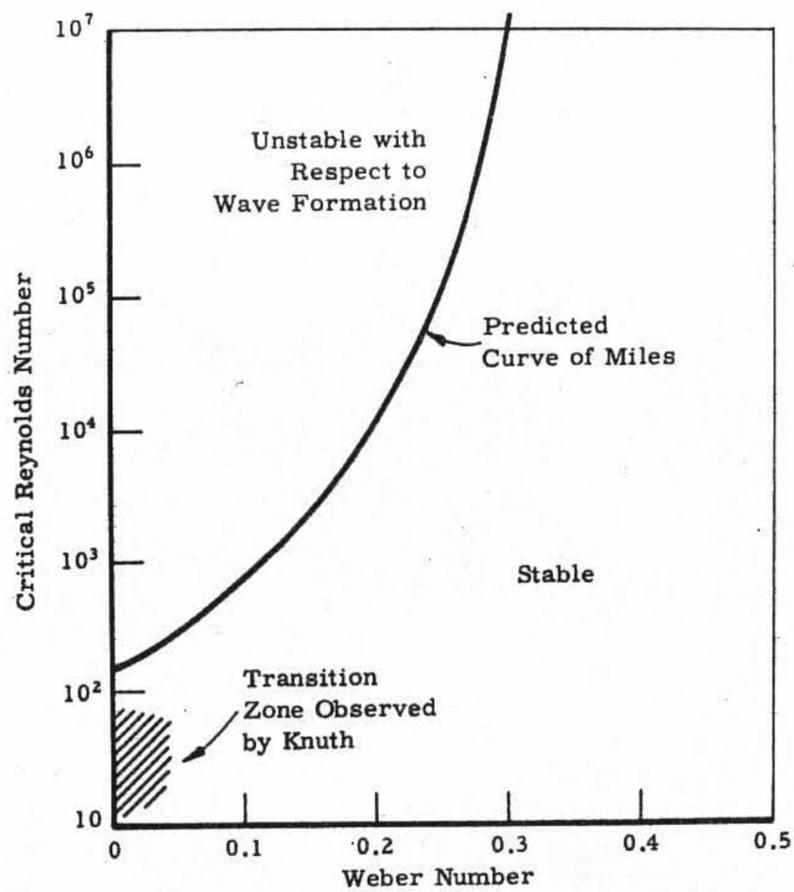


Figure 9. STABILITY OF A THIN DRIVEN FILM

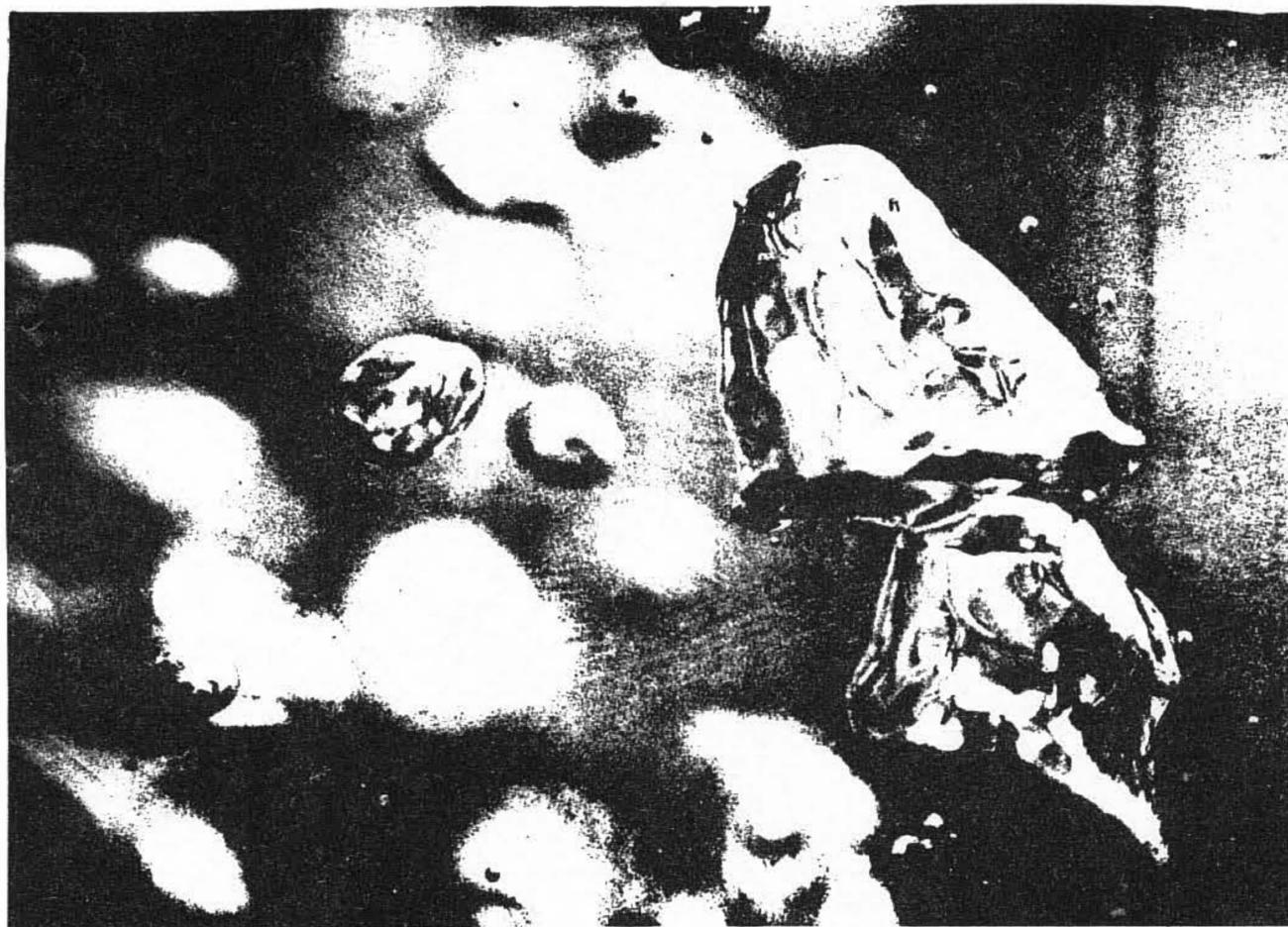


Figure 10. AIR BUBBLES IN WATER SHOWING ROILED SURFACES
 Courtesy of G. D. Towell, Shell Development Co.

ADVANCES IN HEAT AND MASS TRANSFER

by

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Research in heat and mass transfer has received a strong impetus by new engineering developments and has therefore grown considerably in recent years. This is, for instance, evidenced by the fact that the last one of the yearly reviews published in the International Journal of Heat and Mass Transfer contains approximately 500 references selected from more than double the number of published papers (ref. 1). A survey of recent papers and books from the Soviet Union lists 259 references (ref. 2). Consequently, the time available for this lecture permits only to discuss in general terms some highlights in the recent research and in the newer problems presented by engineering developments. The list of references attached to this paper will be helpful for a more detailed information on the subjects discussed. The availability of reference literature (refs. 1 to 3) may also be pointed out.

Heat Conduction

Considerable attention has been directed in recent years to obtain new solutions or to describe new methods of attack on conduction problems. This situation has been created by the fact that new engineering developments required consideration of new and more involved boundary conditions and also that the availability of electronic computers made such solutions possible. The real challenging problems, however, are situations in which heat conduction is interrelated with convection and possibly radiation. An example for such a situation may be discussed with the help of Fig. 1, which is a schematic sketch of an ablation cooling process. In this cooling method, a material which sublimates or decomposes under the influence of heat is used to protect the surface of vehicles re-entering from outer space through the atmosphere to the earth against the heat of friction created in the boundary layer which surrounds the object. Some of the materials used for ablation cooling are composed of a matrix of a temperature-resistant substance like asbestos or ceramics and an ablating substance, for instance, some plastic. Under the influence of a convective heat flux q_c and a radiative heat flux q_r into the surface, the plastic material decomposes and its surface recedes with a velocity v_a , leaving the matrix through which the gas created by the ablation process flows with the velocity v_g . The low heat conductivity of the matrix keeps the heat flow into the interior small. Further cooling of the matrix and a reduction of the convective flow into the surface is provided by the gases created in the ablation process. It is easily seen that in this process conduction, heat convection, and mass transfer processes are interrelated. Additionally, heat sources or sinks are provided by the phase changes and the chemical reactions occurring (ref. 4). In some materials, the radiative heat flux q_r is only gradually absorbed while penetrating into the ablating material. This combination and interrelation of various transfer processes is characteristic for many situations in new applications.

Heat Convection

Channel flows as well as boundary layer flows offer such a variety of boundary conditions that they are far from being completely investigated. An area in channel flow which has recently received attention is connected with ducts of non-circular cross-sections. It was found that the flow and heat transfer characteristics in such a duct are significantly different from those observed in circular pipes, especially when the cross-section contains corners with small opening angle. Fig. 2 shows as an example the results of measurements for fully developed turbulent flow through a duct the cross-section of which has the shape of an isosceles triangle with base to height ratio 1:5 (ref. 5). The measured results indicated by the open circles and triangles are compared with the Nusselt numbers Nu which would be predicted from measurements in a circular pipe of the same hydraulic diameter. It can be recognized that the prediction would overestimate the heat transfer by a factor of two. The actual heat transfer in such a cross-section depends strongly on the boundary condition around the periphery of the duct, for instance, whether a constant wall temperature or a constant heat flux from the duct wall into the fluid is prescribed. The boundary condition for the results in Fig. 2 was between the two extreme cases which have just been measured. Ducts of similar shapes have been used or are considered for coolant passages in nuclear reactors and in gas turbines. The differences between non-circular and circular ducts are less pronounced when the cross-section has only large angle corners (ref. 6).

A situation in boundary layer flow which has found special attention only recently is connected with heat transfer in regions where the flow has separated from the surface. This occurs, for instance, at the downstream part of blunt

objects or behind steps in the surface contour. Fig. 3 sketches such a heat transfer situation. The boundary layer which arrives in its downstream movement at the corner of the step separates from the surface and re-attaches again only further downstream. The region between this boundary layer and the surface is filled with a rotating body of fluid (dead water region). A second boundary layer is created between this dead water and the surface. Heat transfer from the main stream to the body surface behind the step has therefore to overcome two resistances in series: one in the separated boundary layer and one in the attached boundary layer. A large variety of flow situations may exist depending on the Reynolds and Mach number of the flow. The boundary layers may be laminar or turbulent, or transition to turbulence may occur in the boundary layers. The body of separated fluid may also be laminar or turbulent or in fluctuating unsteady motion. This complicates an understanding and analysis of the heat transfer process considerably. Nevertheless, analytical approaches for some of the flow conditions were quite successful (refs. 7, 8).

It is the opinion of this lecturer that the most significant advance in the creation of a science of heat transfer was caused by the concept of a constant property fluid introduced by Wilhelm Nusselt in 1916 and that teaching of heat transfer has to be based extensively on the model of a constant property fluid. New engineering developments, on the other hand, have created many situations where large property variations exist. Such variations cause no principal difficulties in laminar flow but make the equations describing this situation much more complicated. Only the advent of electronic computers made solutions to such problems tractable. For turbulent flow the question arises whether the turbulent transport properties, for instance the diffusivities for momentum and heat, are changed in the presence of local property variations. From the results of analyses it appears that this is not the case even in the presence of strong property variations. Fig. 4 compares the results of experiments with theoretical predictions for heat transfer connected with turbulent flow of carbon dioxide in the critical and super-critical region through a tube (ref. 9). The properties involved in the heat transfer process vary very strongly in the neighborhood of the critical region. The parameter on the curves is a measure of the intensity of the property variations and the good agreement between analysis and measurement supports strongly the assumption that the transfer properties were not influenced by the property variations connected with these temperature differences. This does not hold any more in the immediate neighborhood of the critical state. Investigations on free convection heat transfer with a Zehnder-Mach interferometer, on a vertical plate exposed to carbon dioxide within one degree to its critical state and with temperature differences of order 0.01°C , made it possible to calculate the thermal conductivity (ref. 10). The remarkable conclusion has to be drawn from the results that the thermal conductivity does not only depend on temperature and pressure, but also on the intensity of the heat flux. Similar observations had been made before but had been attributed to convection effects which can be excluded in the present investigation. This is only one indication that an understanding of the critical state is still almost non-existent.

Through many years, convective heat transfer had been studied almost exclusively for steady state situations. This is justified by the fact that very rapid changes of the boundary conditions are required to produce heat transfer coefficients which are significantly different from steady state values. Nevertheless, new applications have raised the question on the limit for the use of steady state values. Fig. 5 shows the results of another interferometric investigation of free convection heat transfer to a vertical plate under the condition that suddenly a locally uniform heat flux from the surface into the surrounding water is started (ref. 11). The measurements essentially indicate that for the initial period, heat is transferred into the fluid by unsteady conduction and that the subsequent transition period to steady state is quite short.

Heat transfer connected with boiling or condensation is an area which is still understood only partially in spite of the intense research effort which has continuously been devoted to this process. The problems and research attempts in this area are actually so large that they cannot even be sketched in this lecture. For a discussion of the physical processes involved and of the analytic attempts which have been published, the reader is referred to the attached references 12 and 13. It is the feeling of this lecturer that the creation of concise and consistent models is still lacking in the analytic investigations. Boiling and condensation of liquid metals, for instance of mercury, and the influence of gravity on free convection boiling are subjects which deserve special attention.

Mass Transfer

A discussion of mass transfer should certainly start with the analogy between heat and mass transfer processes which has originally been pointed out by Wilhelm Nusselt in 1916. This analogy permits to predict mass transfer situations solely from information on an analogous heat transfer process especially when the mass transfer rates and the temperature differences involved are small. Recently this

analogy has been extended by Russian and American scientists to cover situations with large mass transfer rates, large temperature differences, chemical reactions in the fluid, and also especially at high temperature levels (ref. 14). The analogy includes processes in which heat and mass transfer are interrelated. They are especially useful for gases and will be discussed on the example of mass transfer from a surface into a gas flowing over it, thus creating a two-component mixture in the laminar boundary layer. Eq. (1) in Fig. 6 describes a mass flux vector m_1 for the component 1 in the mixture, that is, the mass flux per unit time and unit area of a plane arbitrarily located within the two-component fluid. The first term on the right-hand side of the equation describes the mass flux by diffusion as a consequence of a gradient w_1 of the mass fraction of the component 1. D_{12} is the mass diffusion coefficient and ρ the mixture density. The second term describes the convective transport of the component 1 with a density ρ_1 as a consequence of a movement of the mixture with the velocity V through the plane under consideration. Mass transfer is also created by coupled effects like thermal diffusion, pressure diffusion, or diffusion as a consequence of body forces. These effects are generally small and are not spelled out in Eq. (1). A heat flux vector q (heat flow per unit time through a unit area) will also exist and is described by Eq. (2). The first term on the right-hand side of this equation describes heat transfer by conduction as a consequence of a temperature gradient T . The second term describes the transport of enthalpy h_1 connected with the diffusion mass flux of the components involved. The third term describes transport of the enthalpy h of the mixture as a consequence of the mixture mass flow V through the plane under consideration. Additional terms appear again as a consequence of coupled effects described by irreversible thermodynamics like thermodiffusion. Eq. (2) can be re-written in a different form when the gradient of the temperature T is replaced by the gradient of the mixture enthalpy h (Eq. (3)). This equation is especially useful for a fluid with a Lewis number Le equal to one, because the second term drops out in this case. The Eqs. (1) to (3) have been written in a form which is most useful for mass transfer situations where conduction and convection occur simultaneously.

The boundary layer Eqs. (4) and (5) of Fig. 6 are obtained by a mass balance of the individual components and by an energy balance on a volume element located within the boundary layer. Chemical reactions occurring within the boundary layer destroy or create one or the other component and appear, therefore, in the mass balance of Eq. (4) as a source term K_1 . The energy Eq. (5) has been written in terms of the total or stagnation enthalpy h_s (containing kinetic energy as well as internal energy). The first term on the right-hand side of this equation describes essentially heat transport by conduction; the second term, heat generation by viscous dissipation which becomes important in high velocity flow; and the third term is concerned with enthalpy transport by mass diffusion. From Eqs. (4) and (5) one comes to the similarity considerations by two important steps: The first one entails writing the mass balance for the chemical elements involved instead of the two components; thus w_1 may indicate the mass fraction of the element 1 in the mixture regardless whether the element appears as such or in the chemical compound. No chemical element is created or destroyed in a chemical process, and as a consequence the source term K_1 vanishes in Eq. (4) when it is written for the chemical element. The second step assumes that, for the fluid mixture under consideration, the Prandtl number Pr as well as the Lewis number Le are both equal to one. Eqs. (4) and (5) simplify then to Eqs. (6) and (7) which can be recognized to be completely similar. As a consequence, analysis of mass transfer processes and of combined mass and heat transfer processes becomes much simpler because the most difficult elements in such an analysis can often be taken over from known solutions of an analogous heat transfer situation. Proper boundary conditions have of course to be considered in such an analysis which may include chemical processes occurring at the surface. It should be mentioned that the mass and energy conservation equations alone do not describe the transfer problem completely. A momentum equation and equations for the thermodynamic and transfer properties have to be added. The analogy, however, holds independent of these. It has become especially useful in an analysis of problems like combustion or heat transfer to re-entering vehicles as mentioned in the section on Conduction. Approximate relations have also been developed which extend the analysis to situations with Prandtl and Lewis numbers different from one (ref. 14).

Coupled effects have been neglected in the discussion up to now because they are unimportant in many mass transfer situations. Recent studies (refs. 15, 16), however, have demonstrated that one has to be careful in this respect. This will be discussed with the help of Fig. 7 which presents the results of the following experiments (ref. 16). A cylinder with porous surface was exposed on its outside to a flow of air in axial direction at a Reynolds number which created a turbulent boundary layer. Helium was injected from the inside of the cylinder through the porous surface into this boundary layer. A difference in the temperature with which the helium was fed into the cylinder and the air temperature T_{∞} outside of the boundary layer could be adjusted by pre-heating or pre-cooling of the helium. Fig. 7 presents the heat flux q_w through the porous surface as a function of the difference between the wall surface temperature T_w and the air temperature T_{∞} .

with the specific mass flow \dot{m} of the helium as parameter. One series of measurements made with air instead of helium injection is also entered as dashed line. The heat flux q_w is defined as the sum of the first two terms in Eq. (2). The striking feature in this figure is the observation that the heat flux becomes zero at a finite temperature difference $T_w - T_\infty$ and it is believed that this is a consequence of the fact that concentration differences within the boundary layer appear through thermodiffusion as driving force in addition to conduction. Another consequence of this interplay of driving potentials is the fact that a finite difference between the wall surface temperature T_w and the air temperature T_∞ exists when the helium is admitted into the porous cylinder at a temperature T_∞ equal to the temperature in the outside air flow. This situation is marked in the figure by crosses and it can be recognized that the wall temperature may be up to almost 30 degrees higher than both the helium and the air temperatures, depending on the injection rate of the helium. Similar effects have also been observed in laminar boundary layers for forced and free convection.

An area in which investigations have recently started are transfer processes in a gas plasma. In such a plasma, the temperature is so high that dissociation and ionization occur. Transport processes in such a situation are therefore most involved because mass transfer processes are interrelated with heat transfer, chemical reactions occur, at least three-component mixtures of neutral atoms, ions, and electrons are involved, and electric as well as magnetic body forces influence the flow. An example of a recent experimental investigation in this area is shown in Fig. 8 which presents and analyzes the local heat flux distribution into a water-cooled anode of an electric arc burning in argon (ref. 17). It can be recognized that only a small portion of the specific heat flux q into the anode surface is caused by convection of the atom gas. Convection of the electron gas, which is generated at the cathode and absorbed by the anode, contributes approximately an equal fraction and the rest is due to energy released when the electrons enter the anode material (similar to a heat of condensation). The heat flux q_0 indicates the electrical energy which is converted into heat within the current tube ending at the anode location under consideration. It may be recognized that the majority of the electrically generated heat enters the anode surface. This is the reason for the many burnouts occurring in electrically heated plasma generators at the anode surface. The heat fluxes q occurring at the anode surface on spots at which the arc strikes are among the largest known in any engineering application.

Radiative Heat Transfer

An important tool in all radiative heat transfer calculations is the shape or angle factor. Graphical, mechanical, and optical means have been described, in addition to analytical methods for its calculation. The analysis can, in many cases, be considerably simplified by converting an area integral describing the shape factor into a line integral (ref. 18).

The network method for the solution of radiative heat transfer problems in enclosures, which lumps emitted radiation together with the reflected parts, is of such advantage that it has been introduced into practically all recent books on heat transfer. The analogy to electric circuits, illustrated in Fig. 9, gives in many situations without analysis a feeling for the heat flux distributions occurring in such a transfer process. The network method is applicable to enclosures the surfaces of which emit radiation according to Lambert's cosine law and which reflect diffusely. Many engineering materials, on the other hand, have surfaces, the reflection of which comes closer to a specular than to a diffuse character. Some measured directional distribution curves of reflected radiation are shown in Fig. 10. In enclosures with specularly reflecting surfaces, the analytical approach has to be different and has to consist of summation of the first, second, third, and so on reflections. The analysis is simplified when one introduces the optical images as shown in Fig. 11 in which 1 (3), 2 (3), and 4 (3) denote the optical images of the diffuse surfaces 1, 2, and 3 created by the specularly reflecting surface 3 (ref. 19). This image method can be combined with the network method for enclosures consisting partially of diffusely and partially of specularly reflecting surfaces (ref. 20). The network method is actually an approximation to the integral equations which in principle describe radiative transfer processes. It is important to obtain exact solutions to the integral equations for a few simple situations in order to get a feeling for the errors which may be introduced in the network method. Several recent papers have started to formulate radiative heat processes in this form including the scattering mechanism in a radiating-absorbing medium filling the enclosure (ref. 21). Engineering analyses usually attack problems in which radiative energy occurs simultaneously with other transfer mechanisms like conduction or convection in such a way that the various contributions are calculated separately and that the total energy transfer is obtained by a summation of the individual parts. In reality, situations are encountered in which the various transfer processes interact. Such interactions have been studied for a few cases in the recent past (ref. 22).

At the end of our discussion we will return to the molecular and convective transport processes with a brief review of the similarity between radiative and molecular transport (ref. 23). We can consider radiative transport as caused by the movement of photons in a similar way as energy, mass, or momentum transport is caused by the movement of molecules. Fig. 12 illustrates this similarity by considering Couette flow or heat conduction in a rarefied gas between two parallel walls on one hand, and radiative energy transfer in a radiating and absorbing medium between two parallel walls which are non-transmitting on the other hand. The temperature or velocity variation between the two walls follow a straight line, as indicated in Fig. 12a, as long as the ratio of the mean free path length is very small compared to the distance L of the two walls. With increasing path length, temperature or velocity still exhibits the linear variation within the gas; however, a slip of the velocity or a temperature jump can be observed in the immediate neighborhood of both wall surfaces (fig. 12b). For situations, on the other hand, in which the mean free path length is large compared to the distance L , velocity or temperature in the gas is uniform (Fig. 12d). The terms for the corresponding regimes are indicated above the figures. Completely analogous situations exist for the radiative transfer process. The black-body emissive power e_b has now to be considered instead of the velocity or temperature. This emissive power drops linearly in the absorbing and radiating medium as long as the free path length of the photons is small compared to the distance L . Jumps near the surface of the two walls occur with increasing photon path length. The variation in the absorbing medium itself decreases towards zero when the photon path length gets larger and larger. The terms for these regimes are listed below the figures. This similarity is very helpful in unifying the concepts for transfer processes and such a unification and interrelation of the concepts I would consider as one of the most essential requirements of a good course in transfer processes.

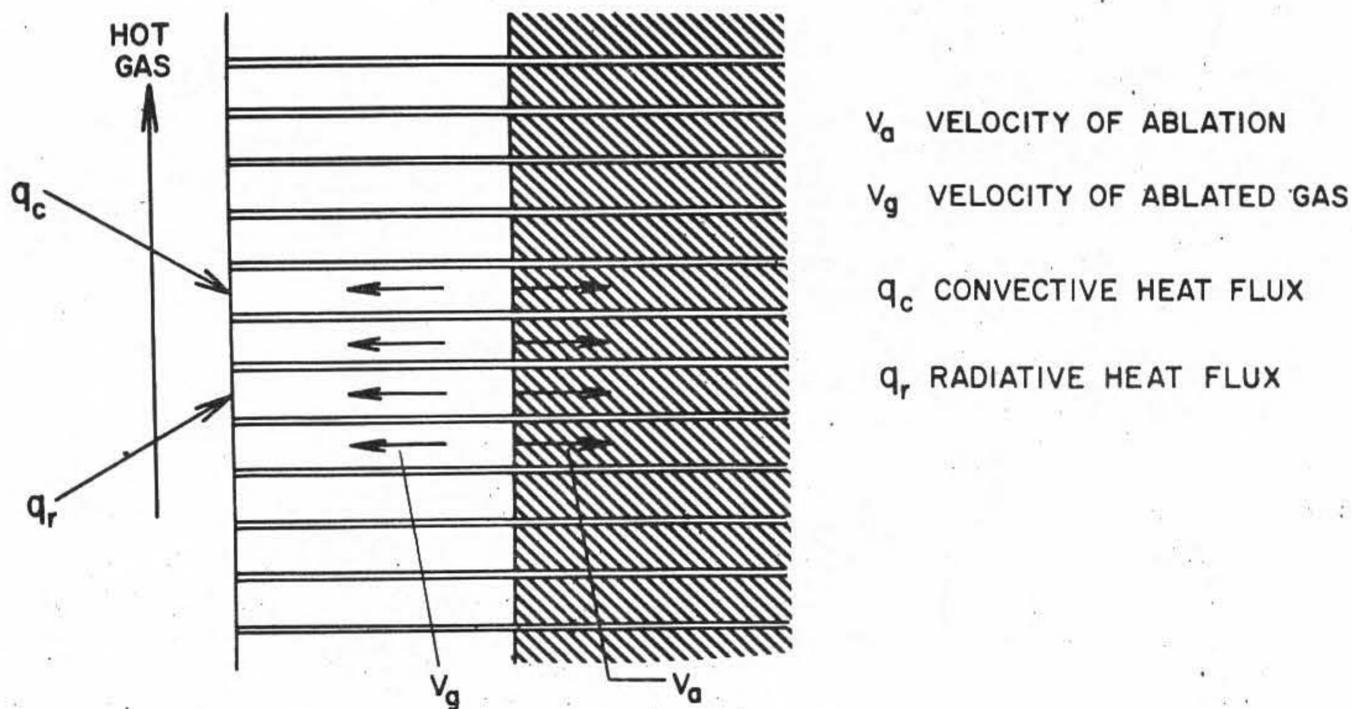


FIG. 1

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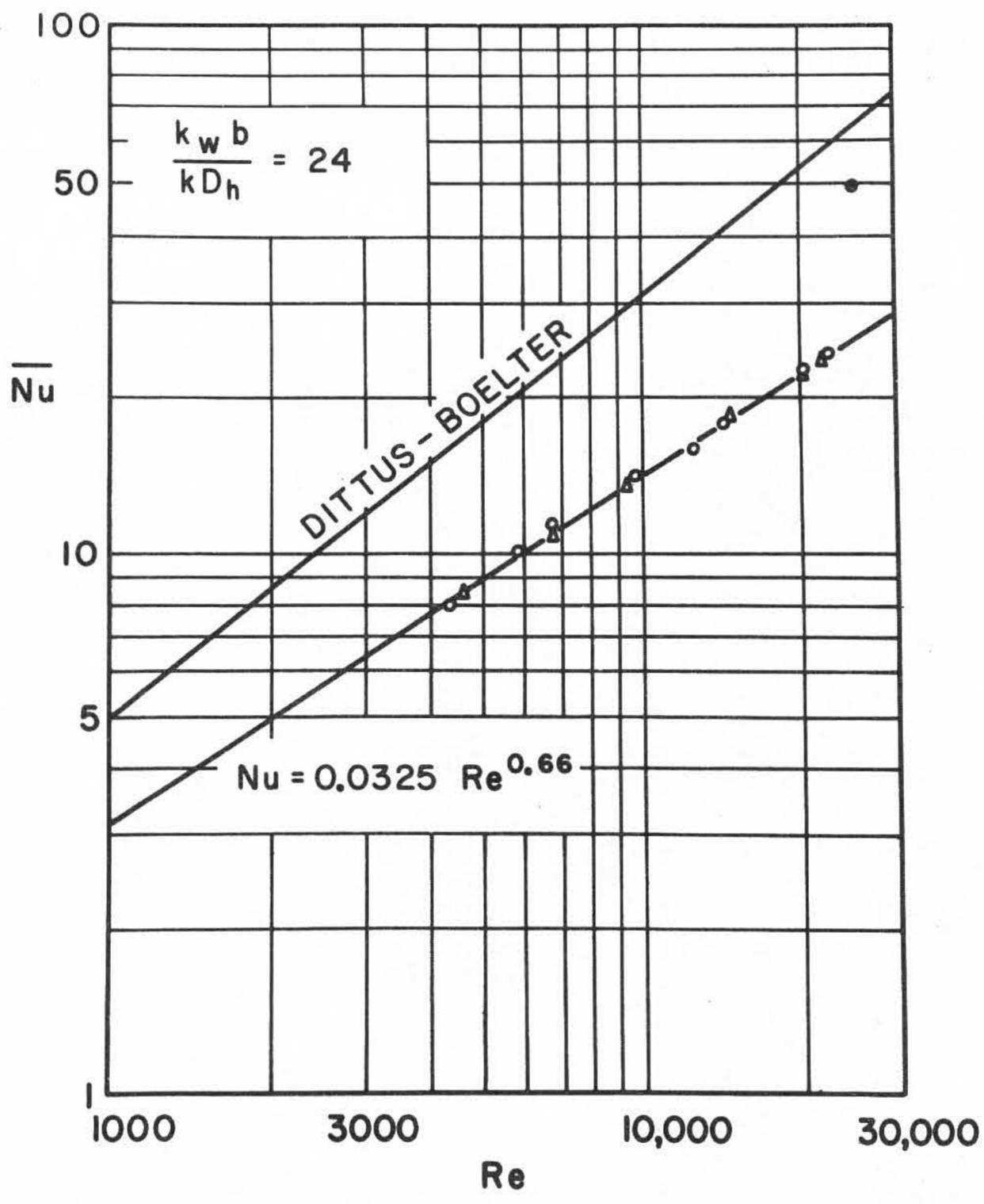


FIG. 2

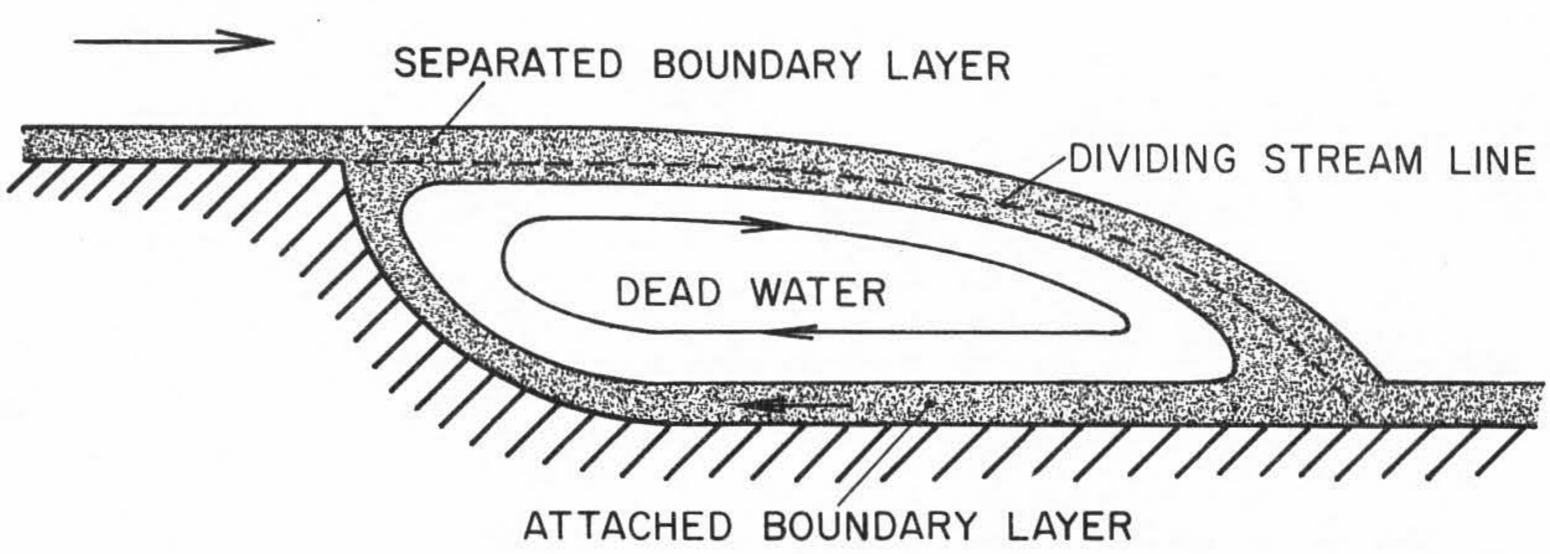


FIG. 3

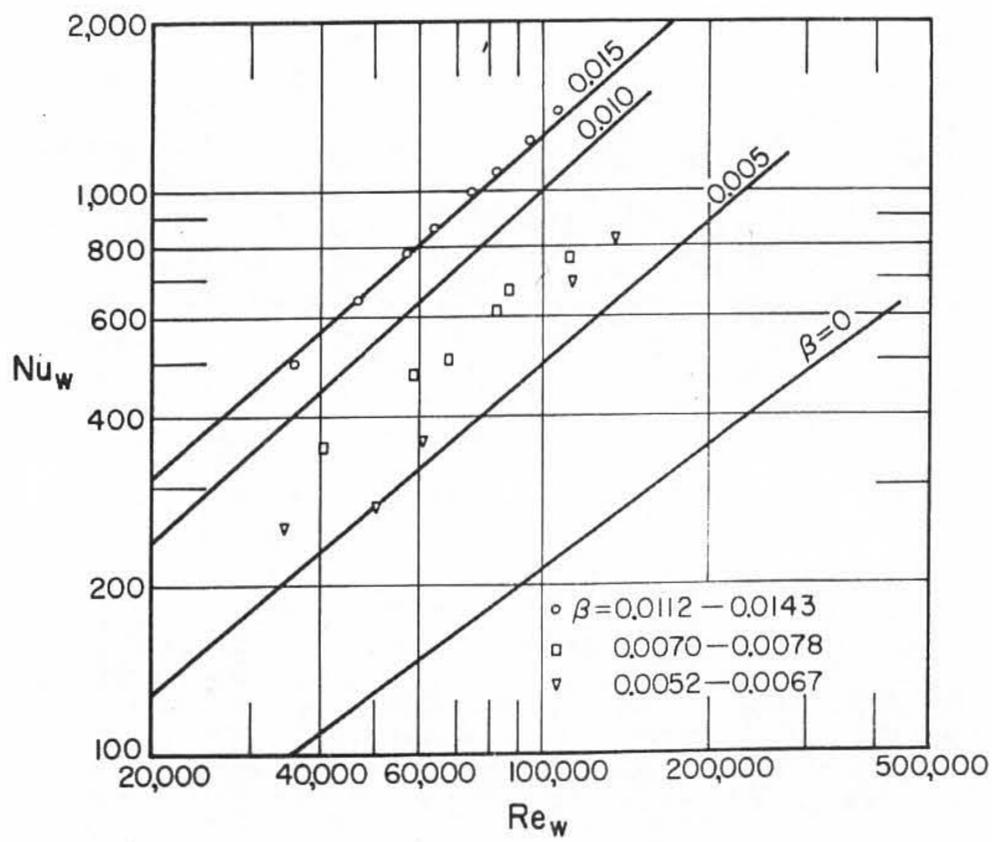


FIG. 4

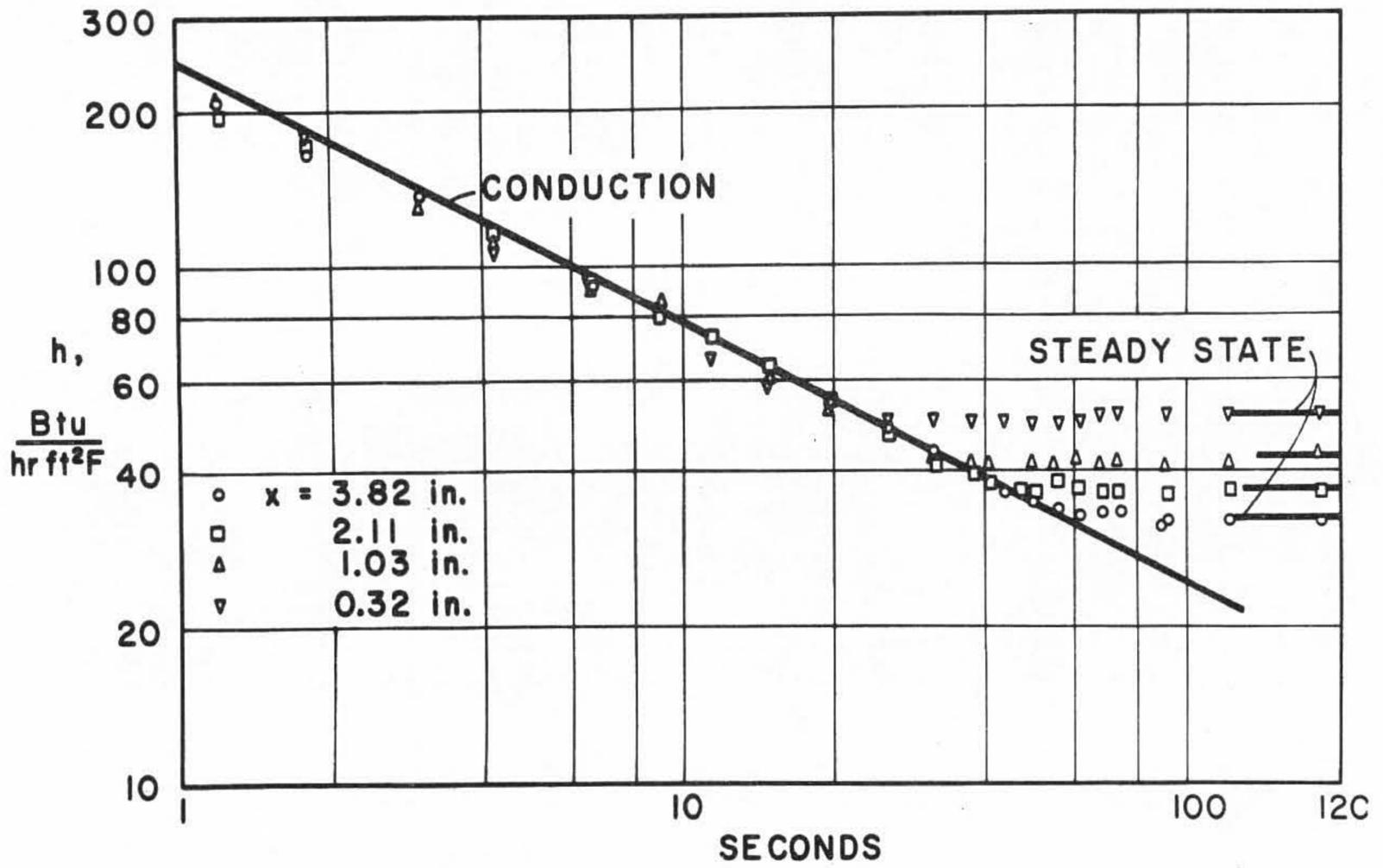


FIG. 5

MASS AND HEAT TRANSFER EQUATIONS :

$$\dot{m}_i = -\rho D_{12} \nabla w_i + \rho_i V + \text{COUPLED EFFECTS} \quad (1)$$

$$\dot{q} = -k \nabla T - \rho D_{12} \sum h_i \nabla w_i - \rho V h + \text{ " " } \quad (2)$$

$$= -\frac{k}{C_p} \nabla h - (Le - 1) \sum h_i \nabla w_i - \rho V h + \text{ " " } \quad (3)$$

BOUNDARY LAYER EQUATIONS :

$$\rho \left(u \frac{\partial w_i}{\partial x} + v \frac{\partial w_i}{\partial y} \right) = \frac{\partial}{\partial y} \left(\rho D_{12} \frac{\partial w_i}{\partial y} \right) + K_i \quad (4)$$

$$\rho \left(u \frac{\partial h_s}{\partial x} + v \frac{\partial h_s}{\partial y} \right) = \frac{\partial}{\partial y} \left(\frac{\mu}{Pr} \frac{\partial h_s}{\partial y} \right) + \frac{\partial}{\partial y} \left[\mu \left(1 - \frac{1}{Pr} \right) \frac{\partial}{\partial y} \left(\frac{u}{2} \right)^2 \right] + \frac{\partial}{\partial y} \left[\rho D_{12} \left(1 - \frac{1}{Le} \right) \sum h_i \frac{\partial w_i}{\partial y} \right] \quad (5)$$

$$\rho \left(u \frac{\partial \tilde{w}_i}{\partial x} + v \frac{\partial \tilde{w}_i}{\partial y} \right) = \frac{\partial}{\partial y} \left(\mu \frac{\partial \tilde{w}_i}{\partial y} \right) \quad (6)$$

$$\rho \left(u \frac{\partial h_s}{\partial x} + v \frac{\partial h_s}{\partial y} \right) = \frac{\partial}{\partial y} \left(\mu \frac{\partial h_s}{\partial y} \right) \quad (7)$$

FIG. 6

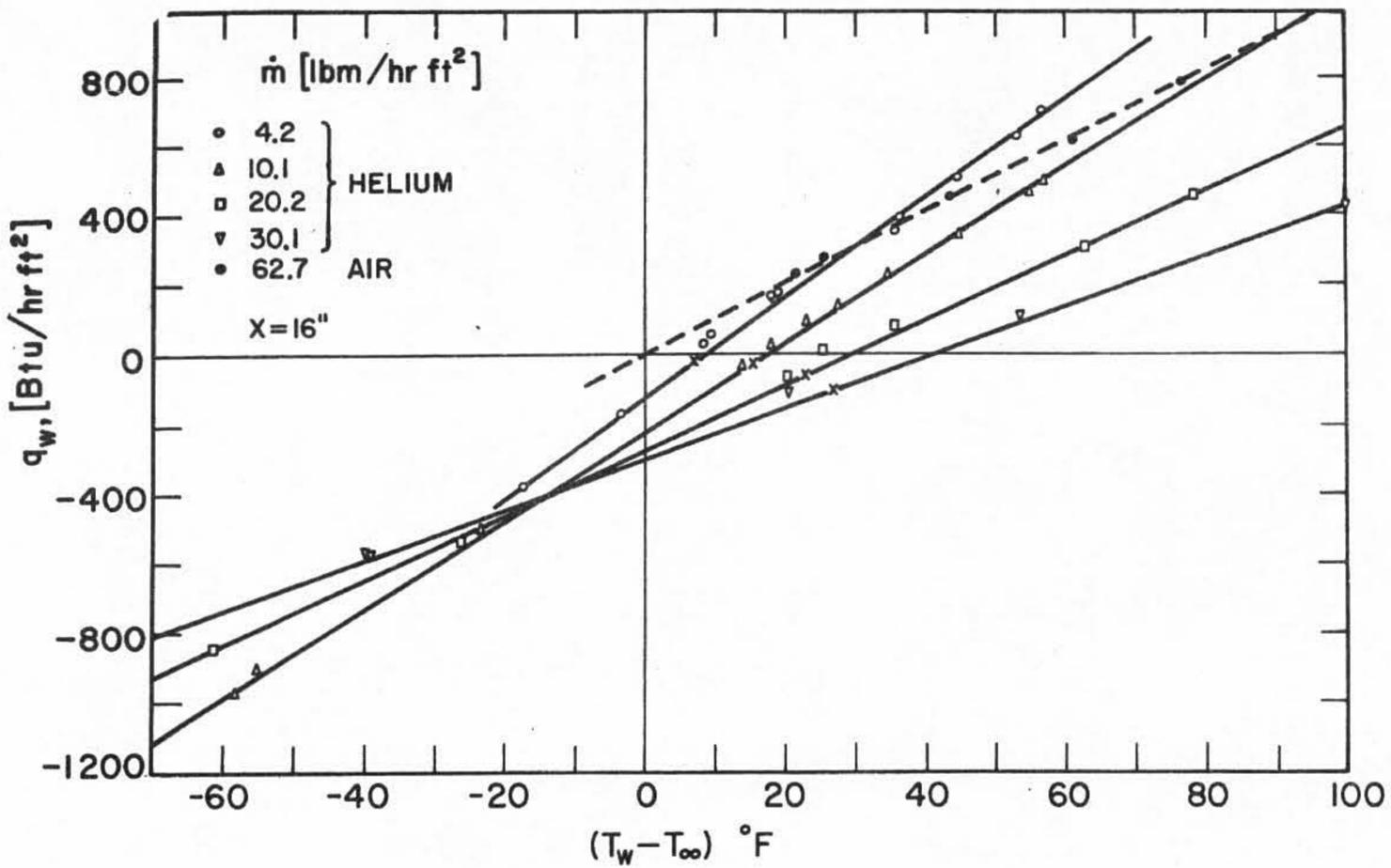


FIG. 7

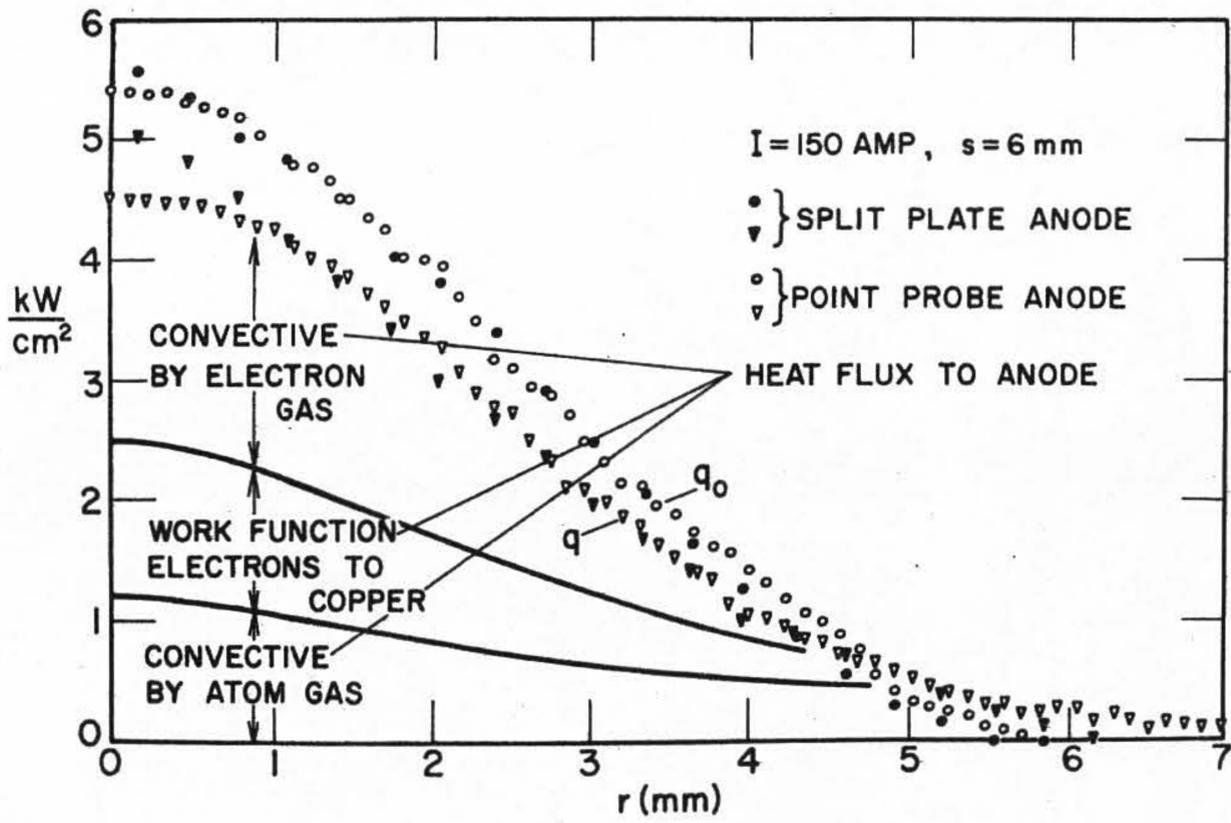


FIG. 8

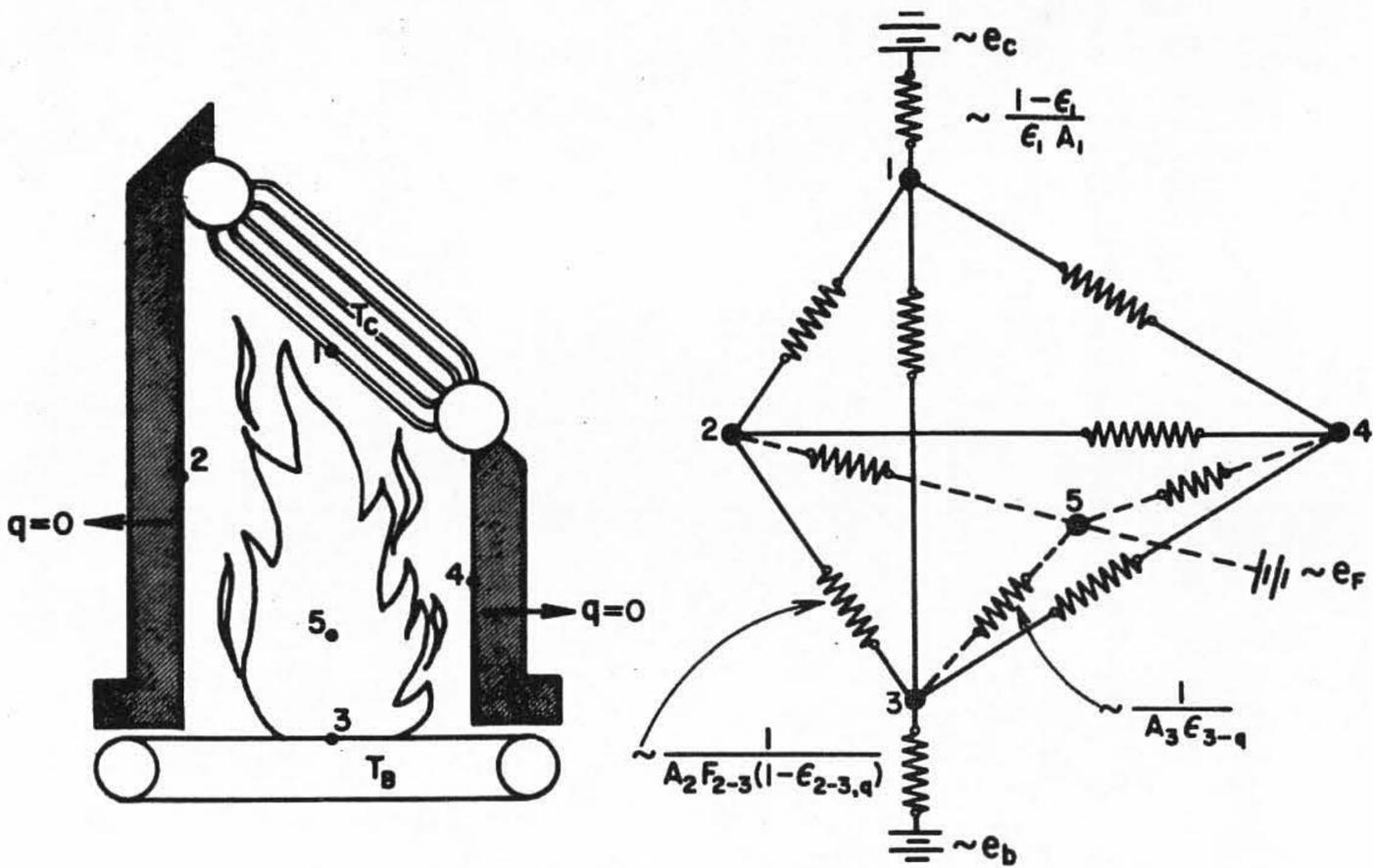
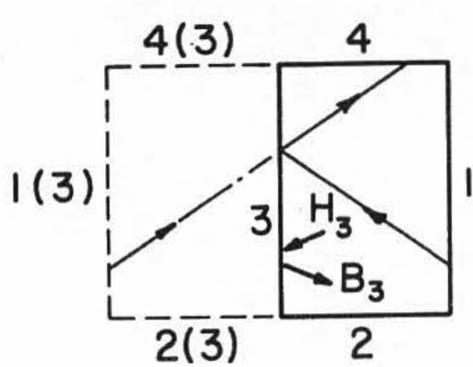


FIG. 9



3: Specular
 1, 2, 4: Diffuse

$$B_3 = \epsilon_3 \sigma T_3^4 + \rho_3 H_3$$

$$H_3 = B_1 F_{3-1} + B_2 F_{3-2} + B_4 F_{3-4}$$

$$B_4 = \epsilon_4 \sigma T_4^4 + \rho_4 H_4$$

$$\begin{aligned}
 H_4 = & B_1 (F_{4-1} + \rho_3 F_{4-1(3)}) \\
 & + B_2 (F_{4-2} + \rho_3 F_{4-2(3)}) \\
 & + \epsilon_3 \sigma T_3^4 F_{4-3}
 \end{aligned}$$

FIG. 11

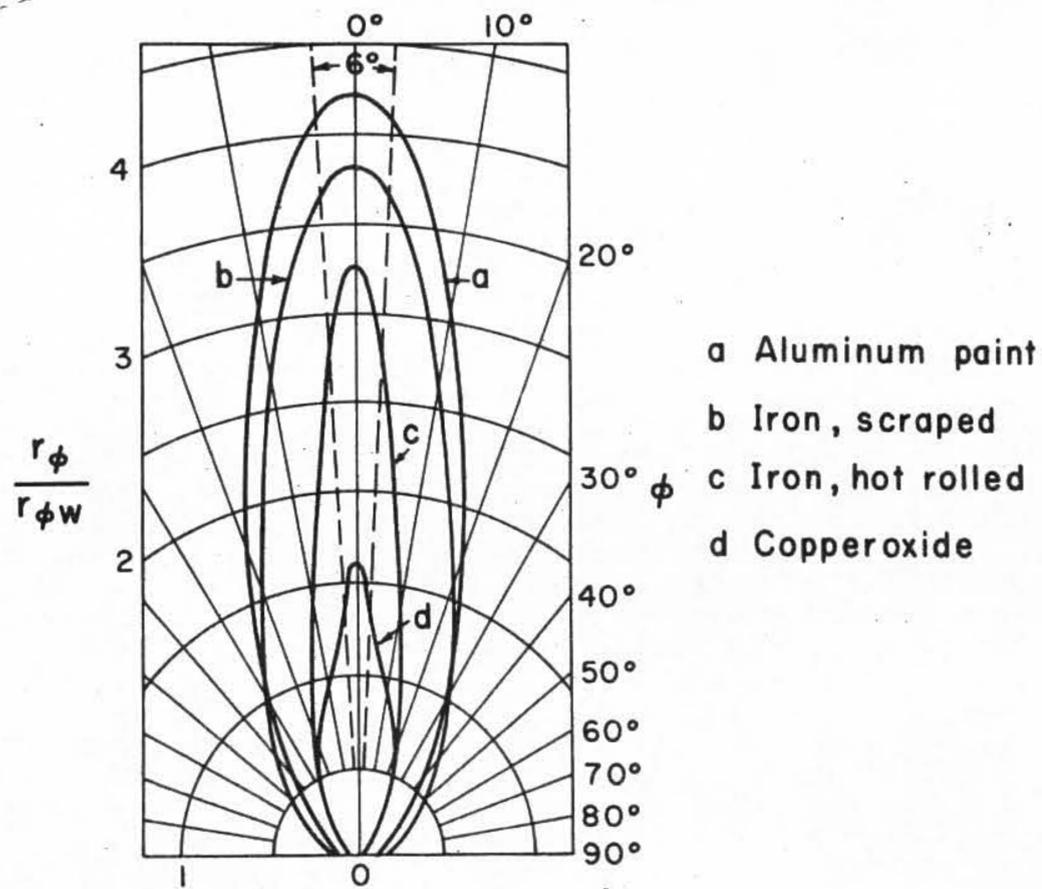


FIG. 10

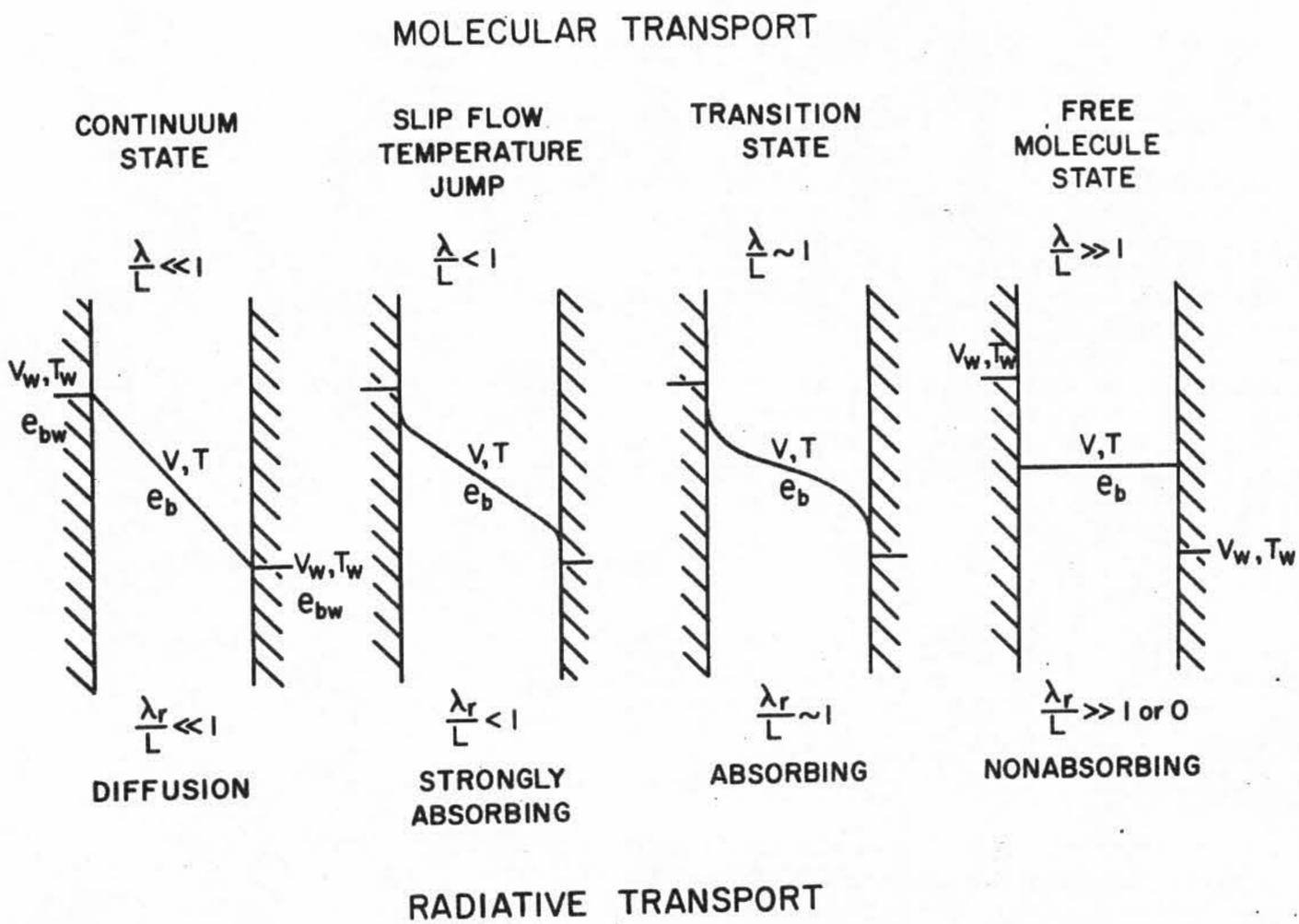


FIG. 12

AN UNDERGRADUATE COURSE IN ANALYSIS
OF MULTISTAGE SEPARATION OPERATIONS

by

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Multistage separation operations have been a part of Chemical Engineering ever since the beginning of Chemical Engineering as a field. The unit operation of distillation appeared in the earliest textbooks on unit operations and has been studied by every undergraduate chemical engineer in the United States. Other separation operations such as absorption and extraction have had less emphasis. Processes such as reboiled absorption and refluxed stripping, as typified by crude oil columns, and processes which exist but are unnamed, have had essentially no attention in undergraduate chemical engineering courses. In addition, the treatment of the whole of these operations has been limited in the usual undergraduate course. If we take distillation as an example, the usual undergraduate course discusses principally the McCabe-Thiele diagram or the Ponchon diagram for the analysis of binary distillation problems. Extraction is limited to problems solved on a triangular diagram or a Jänecke diagram.

The student certainly gains an understanding of separation processes from these problems and he is also well-equipped to solve additional problems of the same type. Unfortunately the chances of his encountering a binary system in later work are very small while his chances of encountering a multicomponent system are very large. Also, many of the processes he meets will not be simple distillation or extraction, and he will need methods of analysis which are general enough to be extended to any problem.

At most schools, this more general material is presently taught in graduate courses. However, there are increasing numbers of B.S. graduates doing the process design work of industry while the M.S. and Ph.D. graduates are increasingly doing less of it, and it is thus the B.S. graduate who needs to know the techniques.

Even without the justification of the need of technique, the undergraduate courses should certainly treat as many areas of the subject matter as possible in as general or all-inclusive a fashion as can be done. All parts of the field are, of course, not susceptible to generalization. Computational procedures or analyses have been worked out which should be taught simply because the single problem for which they are applicable is a highly important problem. However, the fundamental requirements which must be met to create a multistage separation process can be generalized, and the first basic analysis of a given process in terms of what independent or arbitrary numbers can and must be set can be generalized. In ordinary distillation, as soon as the number of components is three or four, many methods of attack on problems are of a general character for any number of components. In addition, methods of calculation which are capable of either a very high degree of generalization or complete generalization have become useable over the past few years because of the routine availability of computers, and these methods should certainly be taught.

I would like to outline here the content of a course we have been offering for the last few years to the first semester seniors in approximately twenty lectures, and which we intend to offer in the future to second semester juniors in approximately thirty lectures. The course assumes that the student already has a knowledge of equilibrium constants and their use in the calculation of simple bubble points, dew points, or flashes. Plate efficiency and the design and capacity of equipment are omitted to be treated in later mass transfer and plant design courses.

The course thus concentrates completely on the calculational analysis of multistage processes, and even within this limitation it is necessarily a collection of a few methods and approaches out of the tremendous bulk of methods which has been published. The choice of the particular methods could easily be different since so much is available. Even the areas of coverage could be different and in the future undoubtedly will be.

If we refer to Figure 1, the course starts out by illustrating a typical multistage separation process in which the separation desired on the mixture of A and B fed to the column of stages is not good enough in a single stage and is in effect, multiplied by a succession of stages to obtain the separation desired. The process shown assumes counter-current flows linking the stages and shows that in the separation between A and B, a section of stages is needed above the feed point in order to produce a purified product of A and a section of stages is needed below the feed point to produce a purified product of B. The individual stage is described as a mixing and separating device in which various streams

may be entered to be mixed, and then separated into new streams which leave the stage. What the stage might be or do is discussed.

The only two requirements which must be met to create a workable multistage separation process are then discussed; namely, 1) that it be possible to separate the mixture of input streams into two phases which can be transported to the adjoining stages, and 2) that the components to be separated must appear in these streams in different ratios of their mole fractions.

There is no requirement as to how the phases are produced as long as the materials to be separated appear in both phases. It is pointed out that the streams may well be vapor and liquid, typical of distillation, absorption, and stripping operations, or they may be liquid-liquid streams typical of liquid-liquid extraction. They may be gas-gas streams in barrier diffusion, or gas-solid streams in adsorption processes. Any mechanism by which two different phases can be produced from the input mixture which can be handled so that they can flow from stage to stage will produce a separation process. As shown in Figure 1, the stages are linked in a counter-current fashion. As far as I know, there is no way to prove that this is the optimum fashion for linkage of stages. But by example, one can easily show that counter-current flow of the phases is better than parallel flow, which is, of course, no good at all, and is better than a linkage scheme as typified by such processes as extraction in which a solvent might be split into parts, each part being introduced in order to further extract the other phase flowing through the stage. An attempt is made to impart an understanding of the purpose of reflux and stripping vapor in a distillation column, but the results are sometimes doubtful.

The second general requirement of the multistage process is that the concentration ratios of the two components to be separated must be different in the two phases produced in the stage. This quantity is usually labeled α_{AB} as shown in Figure 1. It is inevitably called relative volatility in distillation, but is more generally defined as the separation factor. As long as this separation factor remains on the same side of unity throughout the concentration range which is to exist in the separation process, a process is feasible to produce the separation desired.

Figure 1

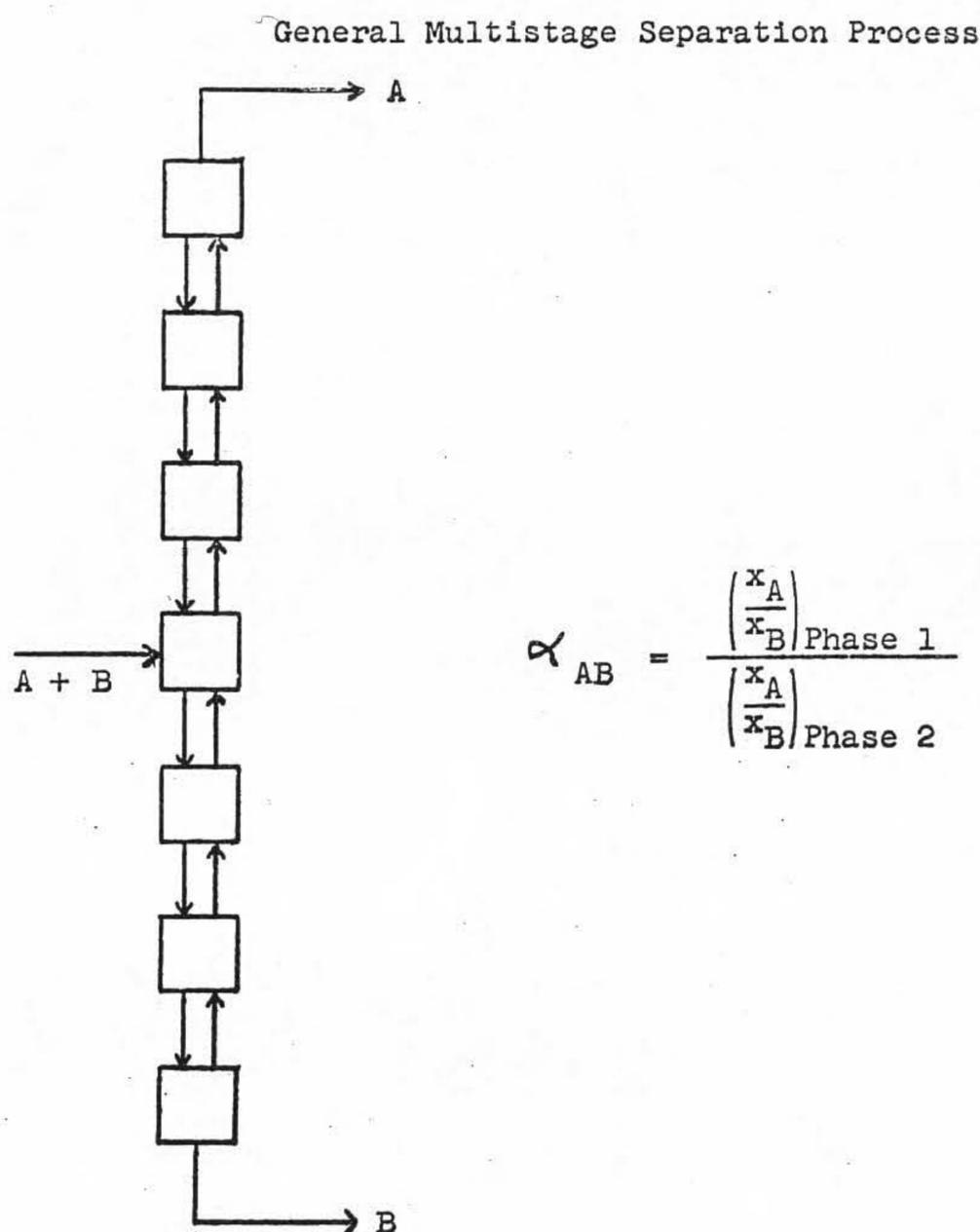
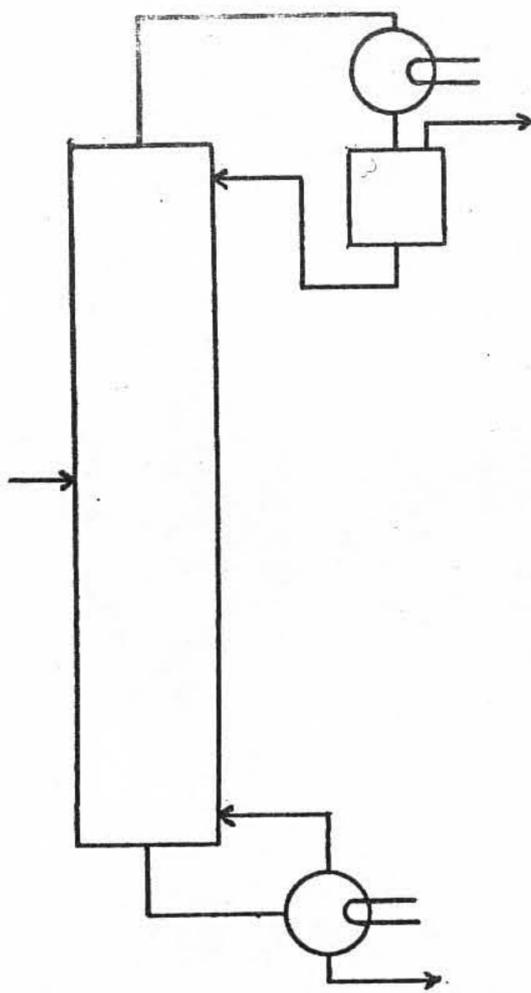


Figure 2.
Equations for Distillation Column
with Partial Condenser



Component Material Balance around Each Stage (xR)

Condenser $V_t y_t - r x_r = D y_D$

$$\begin{cases} V_{t-1} y_{t-1} - L_t x_t = V_t y_t - r x_r \\ V_{t-2} y_{t-2} - L_{t-1} x_{t-1} = V_{t-1} y_{t-1} - L_t x_t \\ \vdots \\ V_f y_f - L_{f+1} x_{f+1} = V_{f+1} y_{f+1} - L_{f+2} x_{f+2} \end{cases}$$

Feed Stage $V_{f-1} y_{f-1} - L_f x_f + F x_F = V_f y_f - L_{f+1} x_{f+1}$

$$\begin{cases} V_{f-2} y_{f-2} - L_{f-1} x_{f-1} = V_{f-1} y_{f-1} - L_f x_f \\ \vdots \\ V_R y_R - L_1 x_1 = V_1 y_1 - L_2 x_2 \end{cases}$$

Reboiler $-b x_b = V_R y_R - L_1 x_1$

Energy Balance around Each Stage

$$\begin{aligned} V_t H_t - r h_r &= D H_D + Q_c \\ V_{t-1} H_{t-1} - L_t h_t &= V_t H_t - r h_r \\ V_{t-2} H_{t-2} - L_{t-1} h_{t-1} &= V_{t-1} H_{t-1} - L_t h_t \\ &\vdots \\ V_f H_f - L_{f+1} h_{f+1} &= V_{f+1} H_{f+1} - L_{f+2} h_{f+2} \\ V_{f-1} H_{f-1} - L_f h_f + F h_F &= V_f H_f - L_{f+1} h_{f+1} \\ V_{f-2} H_{f-2} - L_{f-1} h_{f-1} &= V_{f-1} H_{f-1} - L_f h_f \\ &\vdots \\ V_R H_R - L_1 h_1 &= V_1 H_1 - L_2 h_2 \\ Q_R - b h_b &= V_R H_R - L_1 h_1 \end{aligned}$$

Component Equilibrium Relations (xR)

$$\begin{aligned} y_D &= K_c x_r \\ y_t &= K_t x_t \\ y_{t-1} &= K_{t-1} x_{t-1} \\ &\vdots \\ y_{f+1} &= K_{f+1} x_{f+1} \\ y_f &= K_f x_f \\ y_{f-1} &= K_{f-1} x_{f-1} \\ &\vdots \\ y_1 &= K_1 x_1 \\ y_R &= K_R x_b \end{aligned}$$

Defining Equations for Component Equilibrium Relations (xR)

$$\begin{aligned} K_c &= \phi(T_c, P) \\ K_t &= \phi(T_t, P) \\ K_{t-1} &= \phi(T_{t-1}, P) \\ &\vdots \\ K_{f+1} &= \phi(T_{f+1}, P) \\ K_f &= \phi(T_f, P) \\ K_{f-1} &= \phi(T_{f-1}, P) \\ &\vdots \\ K_1 &= \phi(T_1, P) \\ K_R &= \phi(T_R, P) \end{aligned}$$

Defining Equations for Molal Enthalpy

$$\begin{aligned} H_D &= \phi(T_c, y_D) & h_r &= \phi(T_c, x_r) \\ H_t &= \phi(T_t, y_t) & h_t &= \phi(T_t, x_t) \\ H_{t-1} &= \phi(T_{t-1}, y_{t-1}) & h_{t-1} &= \phi(T_{t-1}, x_{t-1}) \\ &\vdots & & \vdots \\ H_{f+1} &= \phi(T_{f+1}, y_{f+1}) & h_{f+1} &= \phi(T_{f+1}, x_{f+1}) \\ H_f &= \phi(T_f, y_f) & h_f &= \phi(T_f, x_f) \\ H_{f-1} &= \phi(T_{f-1}, y_{f-1}) & h_{f-1} &= \phi(T_{f-1}, x_{f-1}) \\ &\vdots & & \vdots \\ H_1 &= \phi(T_1, y_1) & h_1 &= \phi(T_1, x_1) \\ H_R &= \phi(T_R, y_R) & h_b &= \phi(T_R, x_b) \end{aligned}$$

$$\begin{aligned} \Sigma y_D &= 1 & \Sigma x_r &= 1 \\ \Sigma y_t &= 1 & \Sigma x_t &= 1 \\ \Sigma y_{t-1} &= 1 & \Sigma x_{t-1} &= 1 \\ &\vdots & & \vdots \\ \Sigma y_{f+1} &= 1 & \Sigma x_{f+1} &= 1 \\ \Sigma y_f &= 1 & \Sigma x_f &= 1 \\ \Sigma y_{f-1} &= 1 & \Sigma x_{f-1} &= 1 \\ &\vdots & & \vdots \\ \Sigma y_1 &= 1 & \Sigma x_1 &= 1 \\ \Sigma y_R &= 1 & \Sigma x_b &= 1 \\ & & \Sigma x_F &= 1 \end{aligned}$$

After discussing the general features of multistage separation processes, the course continues with a general discussion of the variables associated with the process. I believe this is one of the most important steps in convincing the student that he can deal with any process. He determines with very little effort what can be done with the process and what possibilities he has to alter or control the separation. In addition, he discovers that the analysis of the process is simply the solution of a rather lengthy and complicated set of equations, for which he must find a method of attack.

For example, in Figure 2, all of the equations which define a distillation column with a partial condenser have been written down. The first set of equations is a set of mass balances for each component around each stage. The second set of equations is a set of energy balances around each stage. The third set of equations defines the relation between mole fractions in the vapor and the liquid for each component in each stage in terms of an equilibrium constant or K-value. The fourth set of equations defines the K-values. The fifth and sixth set define the molal enthalpies, H and h of the vapor and liquid phases leaving each stage. Lastly, the seventh and eighth sets state that the mole fractions in all phases must sum to unity. It might be noted that the pressure has been assumed constant for all stages; it need not be, but would simply require more equations and would leave the basic conclusions unchanged. Also, the equations could have been written differently but with no change in the conclusions.

It is apparent to every student that in order to solve this set of equations one must have as many equations as unknowns. Hence, the number of independent variables which must be assigned values in order to set up a solvable problem on the process can be obtained by counting the number of equations and the number of unknowns. The number of unknowns inevitably exceeds the number of equations and the difference in these numbers is thus the number of independent variables to be assigned values.

If one does this for the process of Figure 2, where R components are fed, n stages exist above the feed stage, and m stages exist below the feed stage, he finds the number of equations is

$$\begin{aligned} & \lfloor R(n+m+3) \rfloor + \lfloor (n+m+3) \rfloor + \lfloor R(n+m+3) \rfloor + \lfloor R(n+m+3) \rfloor + \\ & \lfloor n+m+3 \rfloor - \lfloor n+m+3 \rfloor + \lfloor n+m+3 \rfloor + \lfloor n+m+4 \rfloor \\ & = (3R + 5) (n+m+3) + 1 \end{aligned}$$

Similarly the number of variables is

$$\begin{aligned} & \lfloor R(2) (n+m+3) + R + 2 (n+m+3) + 1 \rfloor + \lfloor 2(n+m+3) + 3 \rfloor + \\ & \lfloor R(n+m+3) \rfloor + \lfloor (n+m+4) \rfloor \\ & = (3R + 5) (n+m+3) + R + 5 \end{aligned}$$

For the column shown in Figure 2, then it appears that $R - 4$ variables must be assigned values in order to describe a meaningful problem. Actually, in writing the set of equations, two other variables, n and m , must also be assigned arbitrary values, so that in defining problems for this column, $R + 6$ variables must be set. One is, of course, free to set any $R + 6$ variables he wishes so long as they are independent, and he is then posed with the problem of solving the set of equations for the remaining dependent variables.

This method of analysis can be extended to any process, but if the process is reasonably complicated, it is quite easy to write the wrong number of equations and hence get a false answer. The student is then shown a far easier way to count the number of independent variables by what we have called the Description Rule. The Description Rule simply says that one must set a number of variables equal to the number of independent choices he can make in construction plus the number of independent choices he can make during operation of the column. It is obvious that during construction one does have independent choices to make which constitute independent variables. After the column is built, he has certain valves and other features on the column which allow him to make arbitrary adjustments. The number of these arbitrary adjustments must also constitute a number of independent variables. Thus, if one simply draws the column of stages and examines it for these independent choices, he can determine in seconds how many independent variables he must set in order to describe a problem on the process. As an example of this, consider in Figure 3 the same distillation column for which the equations were all written down in Figure 2. In drawing the column, one can arbitrarily say there are n stages above the feed stage and m stages below, thus generating two independent variables. It is then apparent that one can feed as much as he wants of any component to the system; so that if he has R components, there are R independent variables in the component feed amounts. He can also arbitrarily set the

enthalpy of the feed, adding one more independent variable, h_f . He can, within limits, operate at any pressure he chooses, generating an independent variable, P . With the column in operation, he can, within limits, change the steam valve and put an arbitrary setting on the reboiler load, generating one more independent variable, Q_R . He can, again within limits, arbitrarily set the cooling water to the condenser and hence the condenser load, creating one last independent variable Q_C . If one sums these variables, as shown in Figure 3, one obtains $R - 6$ variables, which is identical with the result obtained by consideration of the set of equations shown in Figure 2.

As a further illustration, if one has a total condenser on the distillation column instead of a partial condenser, it is apparent that he can put a valve in the reflux line as shown in Figure 3 and split the flow of liquid from the condenser in any proportion he wishes. This constitutes one more independent variable, and if one considers the equations for this process, he will find that there is indeed one more independent variable. I usually tell the students, with an explanation, that this total condenser which contains two independent variables, rather than the one of a partial condenser, is best treated by always setting the condenser temperature. One can arbitrarily say that the reflux coming from the condenser will be saturated reflux, which sets one variable, or alternatively, he can choose an arbitrary temperature.

To illustrate still further what would be a reasonably complicated process, a refluxed stripper with a side stripper which might be considered to be a very basic model of a crude oil column is shown in Figure 3. Again, one can count the variables very simply as the list there shows. No matter how complicated the process, the counting of the variables is quite simple.

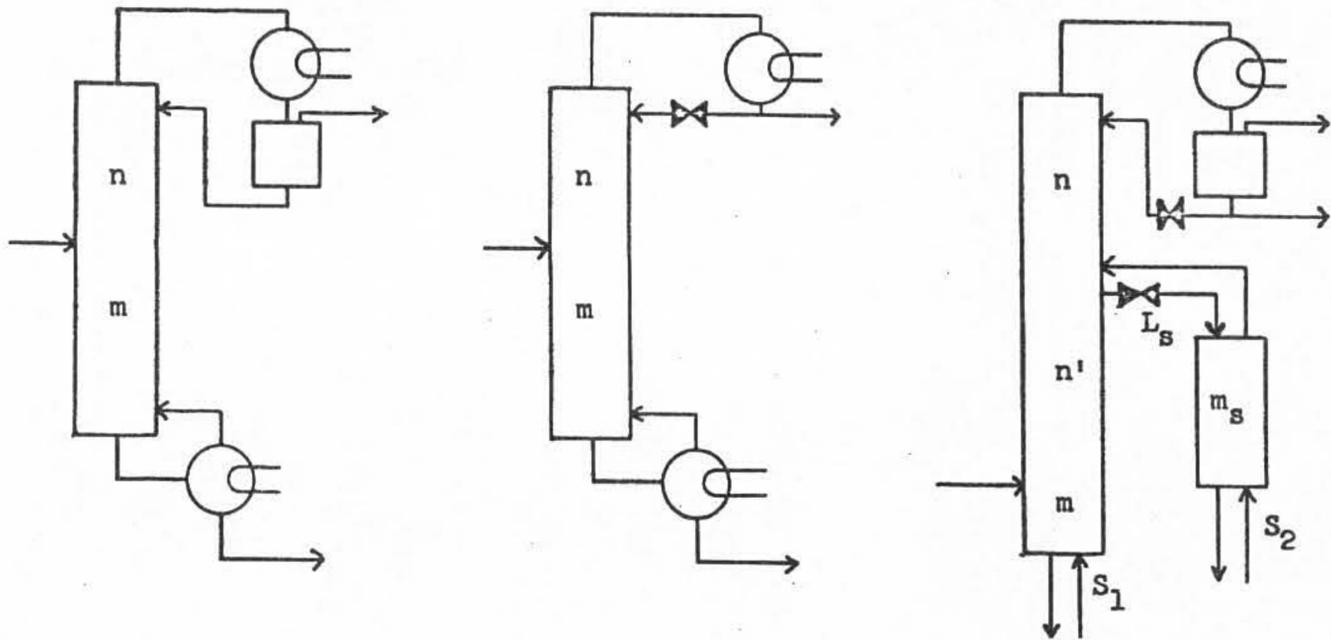
I believe the use of the Description Rule for the counting of variables, because of its simplicity, is of real benefit to the student. He is able to set up a correctly defined problem on any complicated process with real assurance. He has counted the number of independent variables to be set. He replaces as many of those which he counted as he wishes with others for which he wants to specify values in his particular problem. The only requirement left is to find a way to solve the problem for the remaining dependent variables. If he cannot find a way to solve the particular problem he has set up, he can at least set up a series of others which he can solve, and then find the particular one of these which corresponds to the original problem.

Again, I believe the usual undergraduate course dwells too much on one particular type of problem, although there are many problems which can be of interest in any process. If we take distillation as an example, almost inevitably the student is asked to solve a problem in which he sets the separation specifications on two components, sets the reflux and then arbitrarily sets the feed plate location during the course of his calculation. This is the standard design problem, and if the column happens to have a partial condenser, the variables set are shown in the second list of Figure 4. This solution of this problem has a distinct method of attack which is well-understood by students if they read any textbook. However, there can be many other problems of interest on the same distillation column; and in Figure 4, I have added a few which might be of interest and which, incidentally, can also be reasonably easily solved.

Almost every problem encountered sets certain variables. For example, the amount of each component in the feed is always set. The enthalpy of the feed is almost always set. The pressure under which the column is to operate is always set. These variables I have shown in Figure 4 listed above the line drawn in each list, and the remaining variables below. The first list of Figure 4 is the problem described by counting the variables. It would not be easily solved and luckily is not often of interest. In the second list, which is the typical design problem, the recovery fractions in the top and bottom product for two components to be separated have been set, the reflux has been set, and the feed plate location has been set. In the third list, a problem is shown for a column which might already be in existence. The number of stages in the column above the feed point and the number of stages in the column below the feed point are set. As the last two independent variables, one could set the bulk split and the reflux, obtaining then as an answer the separation which will be achieved. This problem, although not in itself often of interest, is the stepping stone to the solution of many problems. Through it one could easily get the solution to the problem of the fourth list in which the last two variables set are the recovery and purity of a component in the top product. Also, through it one could even more easily get the solution to the problem of the fifth list in which the octane number of the bottom product were set and the problem is to find the amount which could be produced under the set reflux. The only requirement would be that it be possible to calculate the octane number from a knowledge of the bottom product composition.

All of these problems and many others could be solved, although the last two would have to be solved by a parametric solution. But it is helpful to point out to the student that many problems exist other than the straightforward design prob-

Figure 3



n
m
R of Fx_F
 h_F
P
 Q_R
 Q_C

R + 6

n
m
R of Fx_F
 h_F
P
 Q_R
 Q_C

r
R + 7

n
n'
m
 m_s
R of Fx_F
 h_F
P
r
 Q_C
Amount of S_1
H of S_1
Amount of S_2
H of S_2
 L_s

R + 13

Figure 4

Set Variables for
Various Problems in Distillation

All Fx_F	All Fx_F	All Fx_F	All Fx_F	All Fx_F
h_F	h_F	h_F	h_F	h_F
P	P	P	P	P
n	(/i) _d	n	n	n
m	(/j) _d	m	m	m
Q_R	r	r	(/i) _d	r
Q_C	Fd Plate Loc.	d	(x_1) _d	Octane no. of b

lem which he normally encounters in the textbooks. A new problem does not present him with a requirement for developing a new understanding. He simply must search for a way in which to solve it. I might say also, that while I have dwelt at some length on the subject of variables here, the discussion in the course occupies only about two lectures, which is easily sufficient for it.

The next two sections of the course consider binary distillation, starting with McCabe-Thiele diagram, which I think is very instructive. The student sees graphically what the column looks like and he obtains an insight into the operation of a typical distillation column by looking at the result of increasing the reflux, or increasing the number of stages, or for example, tightening the separation between the two components. However, I do not intend to discuss the McCabe-Thiele diagram here. Every teacher uses it, I am sure, or uses the Ponchon diagram as a substitute for it. I might point out that the "assumption" of constant molal overflow requires justification. If one has been very careful to ex-

plain to the students that there are only a certain number of variables which can be set and he then finds the instructor setting all liquid flows in the rectifying section at a value equal to the reflux and all liquid flows in the stripping section at a value equal to the reflux plus whatever liquid flow might have been generated due to the introduction of the feed, he is naturally puzzled. The instructor is now setting a new number of independent variables equal to the number of stages in the system. This is easily explained to him by simply postulating that you have introduced a heat exchanger on each stage, and it is possible for you to heat or cool the stage in an arbitrary amount thus creating the necessary new set of independent variables equal to the number of stages. Thus, you can then control the liquid flow off the stage at the quantity you wish. Since the column is never built with this set of heat exchangers on each stage, the answer calculated for the problem cannot correspond to reality. You can then only explain that in many cases the result is close enough to the truth to satisfy you, and if it is not, it is possible to use a heat balance to correct the calculation.

The following section of the course deals with a set of "group" methods, still applied first to binary systems. It is useful to the student to know that there are analytic methods by which he can solve directly for the number of plates required in distillation columns provided he make certain assumptions, namely that the flows through any section of the column are constant and that the relative volatility is constant through the column or at least that an average relative volatility will suffice. He can then avoid the stage-by-stage analysis typified by the McCabe-Thiele diagram and calculate his problem by means of a set of equations such as Underwood's equations (9). I have used Underwood's equations because their derivation is simple and extends to multicomponent systems by simply adding terms, because the equation by which he defines his parameters is easily shown to be simply an infinite section equation, and also, because through them the minimum reflux calculation of Underwood can be proved. The equations are solved for the standard design problem, the second list of Figure 4, and the only problem for which they allow an easy solution.

The concepts of minimum reflux and minimum plates are next developed, but again it is emphasized that they are merely different problems in which the variables set below the line are respectively

$$\begin{array}{ll} \left(\frac{A}{B}\right)_d & \left(\frac{A}{B}\right)_d \\ \left(\frac{A}{B}\right)_b & \left(\frac{A}{B}\right)_d \\ n = \infty & r = \infty \\ m = \infty & \text{Arbitrary feed plate location} \end{array}$$

At this point the course extends naturally into consideration of multicomponent systems. A general equation for minimum reflux in multicomponent systems can be easily developed from infinite section equations under the assumption that the infinite sections meet at the feed plate, a concept the student is already familiar with from the McCabe-Thiele diagram. The variables set are

$$\begin{array}{l} \left(\frac{i}{j}\right)_d \\ \left(\frac{i}{j}\right)_d \\ n = \infty \\ m = \infty \end{array}$$

where i and j are any two components of a multicomponent mixture. If the feed is a saturated liquid feed, the equation is exceedingly simple.

$$L_{\min} = \frac{\left(\frac{i}{j}\right)_d - \frac{\alpha_i}{\alpha_j} \left(\frac{j}{i}\right)_d}{\frac{\alpha_i}{\alpha_j} - 1}$$

The equation applies to any two components and hence, the distribution of all other components can be back-calculated once the value of L_{\min} is known for a particular system and a particular set of $\left(\frac{i}{j}\right)_d$ and $\left(\frac{j}{i}\right)_d$. The students quickly find that by back-calculation of the distribution of the components other than i and j , impossible results are often obtained, e.g. more of a light component will be calculated in the top product than enters in the feed, and it is apparent that the assumption that the infinite sections meet at the feed stage has broken down. Also, with infinite stages, it is possible to reduce the amount of a component in the product to zero.

The next logical step is the derivation of Underwood's equation for minimum reflux for multicomponent systems (10). This equation is derived, along with all other equations in the course so that no method is discussed which is not derived from basic principles. Fenske's equation for minimum stages at total reflux (3) is also derived for multicomponent systems.

The whole problem of stage-by-stage calculations for multicomponent systems is discussed as a logical extension of binary systems. I believe the student is made well aware of the fact that the only new feature is simply that his calculations must now include estimates of the distribution of all the components whose separation he cannot set because of the limited number of independent variables. Also, algebraic calculation replaces graphical methods, and the students calculate design problems for short columns.

The use of Underwood's equations for design problems on multicomponent systems is also discussed although the exact solution is avoided. Instead, time is devoted to showing how the concentrations of components other than the two components whose separation has been set, can be estimated on the feed plate, thus yielding an approximate feed plate composition. In comparisons between solutions of Underwood's equations for multicomponent systems with the approximate feed plate composition and the exact solutions, the errors are of the order of 1%. The student is thus able to use Underwood's equations for multicomponent systems without the extremely tedious solution of the determinants necessary in the exact solution.

The last part of the course is devoted to the iteration and relaxation schemes which have come into use in recent years. Students are asked to solve simple problems, but the time-consuming nature of the methods would allow solution of long problems only if a computer were used and we have not yet used one in the course. However, the very fact that computers are widely available in industry makes the methods important since they are capable of extension and generalization to a degree the other method cannot achieve.

Iteration methods directly solve only problems in which the numbers of stages in all sections of the column are fixed, plus a sufficient number of other independent variables to completely define the problem. The "other" variables are the bulk split and a flow such as the reflux; direct solution for problems described by other variables is at present not worked out, but must be obtained parametrically.

The basic iteration scheme was proposed by Thiele and Geddes in 1932⁽⁸⁾. Many workers have elaborated and changed the scheme since, notably Edmister⁽²⁾, Admundson and Pontinen⁽¹⁾, and Lyster and Holland⁽⁵⁾, and the choice of exact procedure is today quite broad. The particular scheme I teach the students is one I have worked on a considerable amount and I have chosen it to a large degree because of this.

All iteration schemes break the system of equations for the column into groups and solve the groups separately. As an example, consider the simple distillation column of Figure 2. The mass balance equations will be solved first, and the set of equations for each component must be separately solved. If the equations for the first component are written down it is apparent that nothing, or almost nothing, is known about them. However, we could solve the equations for the mole fraction of the component on each stage and in the products if all unknown flows and temperatures were assumed. The set of equations is then a linear set of equations in either x or y , we arbitrarily choose x , and will solve for all values of x .

The technique of solving the set of equations I use was first published by Smith⁽⁷⁾ in essentially the form I use it. Since nothing is known, one assumes a value of bx_b for the component, the amount of the component in the bottom product. The amount is wrong by some unknown error, ϵ_b , where

$$bx_b = (bx_b)_{\text{true}} + \epsilon_b$$

A typical stage by stage calculation of the component can now easily be run up the column. Thus

$$V_R y_R = S_R (bx_b)$$

$$L_1 x_1 = V_R y_R + bx_b$$

$$V_1 y_1 = S_1 (L_1 x_1)$$

etc. to the top of the column where

$$rx_r = V_t y_t + bx_b - Fx_F$$

$$Dy_D = S_c (rx_r)$$

Dy_D is, of course, wrong since bx_b is wrong.

However, we know the error in bx_b is some quantity, ϵ_b . The error in $V_R Y_R$ is then $S_R \epsilon_b$ and the error in $L_1 x_1$ is $\epsilon_{V_R} - \epsilon_b$. None of these errors can be calculated directly but all of their ratios can. Thus

$$\frac{\epsilon_{V_R}}{\epsilon_b} = S_R$$

$$\frac{\epsilon_{L_1}}{\epsilon_b} = \frac{\epsilon_{V_R}}{\epsilon_b} + 1$$

$$\frac{\epsilon_{V_1}}{\epsilon_b} = S_1 \frac{\epsilon_{L_1}}{\epsilon_b}$$

ect., on up the column. The feed has no error associated with it and hence, does not enter the error ratio calculation.

At the top of the column the ratios

$$\frac{\epsilon_r}{\epsilon_b} = \frac{\epsilon_{V_t}}{\epsilon_b} + 1$$

and

$$\frac{\epsilon_d}{\epsilon_b} = S_c \frac{\epsilon_r}{\epsilon_b}$$

are calculated.

Then one can write

$$Dy_D + bx_b = (Dy_D)_{\text{true}} + \epsilon_d + (bx_b)_{\text{true}} + \epsilon_b$$

and, since $(Dy_D)_{\text{true}} + (bx_b)_{\text{true}} = Fx_F$

a slight rearrangement yields

$$\epsilon_b = \frac{Dy_D + bx_b - Fx_F}{\frac{\epsilon_d}{\epsilon_b} + 1}$$

The amount of the component in every flow can then be corrected to the true value through the knowledge of ϵ_b and all of the error ratios.

This procedure is followed for each component (the reverse calculation down the column must be employed for light components) and a complete set of mole fractions is obtained for the column. All of these mole fractions should sum to unity, and if they do not, the assumed temperatures or flows must be corrected to better values. Either could be corrected at this point, but most procedures correct the temperatures. I use a simple bubble point calculation for this purpose, after which the mass balance equations are solved again.

The heat balance equations are used to correct the flows, either at the end of each temperature correction or after several temperature corrections. If the correction methods for the temperatures and flows are effective, the calculations converge rapidly.

The students understandably shy from the tedious work of the method, but they certainly have no trouble understanding it, and it is apparent that with suitable correction methods, the iteration scheme could be applied to essentially any process. Actually, multiple feeds present no problem, side streams only a slight problem, and side strippers a somewhat harder problem. Correction methods are well-developed at the present time for many common processes and the iteration method has been extensively used in industry. The student is impressed when he is told that columns with 40 plates and 20 components can be solved on a computer for \$1.50.

The relaxation method is conceptually the most simple method which one can imagine, and because of this it is also completely general. The basic idea was used first by Rose⁽⁶⁾ for the mass-balance solution to the Steady-state by calculation through the unsteady state period of a column startup. However, if the steady state is the desired answer, no consideration need be given the time behavior and the method can be simplified and at the same time extended⁽⁴⁾. The

relaxation scheme solves the same type problem which the iteration methods solve, more slowly in every case, but with great reliability.

One simply takes each stage in a process and answers the single, simple question: "If I bring in one, two, three, or more streams into the stage of a given amount and composition and with a given energy, what will be composition, amount and energy of the phases leaving the stage?" In most processes which one calculates in the present day, one assumes that the phases leaving the stage are in thermodynamic equilibrium. In a distillation calculation, for example then, an isenthalpic flash will answer the question. If one then loads his column with any material and simply starts to calculate from stage to stage, answering the question in turn for each stage, the calculation will converge asymptotically to the steady state condition of the process. It is not necessary to completely solve the isenthalpic flash on each stage. Commonly two isothermal flashes are used to predict the results of the isenthalpic flash and the calculation moves on to the next stage. There does not appear to be any particular benefit in any order of consideration of the stages, although if one could locate the stage of largest error, it would undoubtedly be beneficial to relax this one and then relax the one which had again the largest error, etc. Such calculations are slow, even on a computer, but they inevitably result in solution. Many fairly complicated processes which have been unsolvable by the present iteration schemes can be very readily solved by the relaxation technique.

This simple concept obviously can be extended to any process. In extraction processes, all one needs to consider is what will the distribution of the components be between the phases leaving the stage. Again, for each relaxation of a stage, it is not necessary to calculate exactly what this distribution will be. Simple prediction of the distribution from the results of one cycle in an iterative solution of the flash equation suffices. There are other and better ways to solve liquid-liquid extraction systems, but the relaxation method works well and shows the complete generality of the method.

As I said initially, the course we are now teaching is a collection of methods which we know particularly well and which cover the general field of multistage separation operations. There is such a tremendous literature in this field that one could easily pick a totally different set of methods. The course will undoubtedly change in the future, hopefully in the direction of increasing generality, and will also perhaps have the benefit of having some computer time available. It is not a simple matter to use a limited amount of computer time effectively, however and the use of the computer will not in itself add to the understanding the student may have gained in the field. The advantage of the computer may well be that it will allow the student to invent a new separation process himself and do a sufficient amount of calculation to test it and thereby gain a little more understanding.

NOMENCLATURE

b	moles of bottom product.
D	moles of vapor top product.
F	moles of feed.
H	molal enthalpy of a vapor leaving a stage.
h	molal enthalpy of a liquid leaving a stage.
K	component equilibrium constant, y/x .
L	moles of liquid flow leaving a stage.
L_{min}	minimum liquid flow in rectifying section for a distillation column, usually taken as minimum external reflux.
m	number of stages in the stripping section of a distillation column.
n	number of stages in the rectifying section of a distillation column.
P	pressure.
Q_C	condenser load.
Q_R	reboiler load.
R	number of components.
r	moles of reflux.
S	stripping factor for a component in a stage, $\frac{VK_i}{L}$.
T	temperature.
\bar{x}	mole fraction of a component in a liquid.
y	mole fraction of a component in a vapor.
$(/i)_d$	fraction of i fed which is removed in the top product.
$(/i)_b$	fraction of i fed which is removed in the bottom product.

SUBSCRIPTS

A	component A
B	component B
b	bottom product
C	condenser
D	vapor top product
d	liquid top product or top product in general
F	feed
f	feed plate
i	component i
j	component j
L ₁	liquid flow leaving plate 1, etc.
R	reboiler
r	reflux
t	top plate
V _R	reboiler vapor, etc.
l	plate 1, numbering upward, etc.

GREEK

α_{AB} separation factor between components A and B in phases leaving a stage.

$$\alpha_{AB} = \frac{\left(\frac{x_A}{x_B}\right)_{\text{Phase 1}}}{\left(\frac{x_A}{x_B}\right)_{\text{Phase 2}}}$$

α_i separation factor or relative volatility of component i based on some unnamed reference component. All α_i are based on same reference component.

ϵ error in the calculated amount of a component in a flow stream leaving a stage.

ϕ function.

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